Dispersed Information and CEO Incentives∗

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Abstract
This paper shows how the stock market’s capacity to aggregate dispersed information is connected to CEO incentives. In particular, I highlight several informational inefficiencies associated with giving the CEO stock-based compensation and thus discretion when choosing corporate investment. While this scheme leads to high effort provision in equilibrium, it suffers from amplified noise in the stock price and an excessive use of price information (relative to the constrained efficient benchmark). As a result, stock prices under this compensation mechanism are excessively volatile and exposed to non-fundamental noise. I then compare this incentive structure to a flat wage & fixed investment rule environment in which the firm owners specify an ex ante optimal rule for the CEO. It follows, that the equilibrium outcome demonstrates undistorted price efficiency, but no incentive for the CEO to invest in a profitable growth opportunity. I characterize conditions for the optimal compensation structure and show how a log-linear tax on the firm’s dividend can improve upon the equilibrium outcome.

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1 Introduction

In this paper, I study optimal corporate investment decisions in the presence of noisy, but informative stock prices. I show that a performance-based contract (stock-based compensation) incentivizes the CEO to exert high effort to open up a growth opportunity. However, it also leads to informational inefficiencies as traders transmit noise associated with the growth opportunity and the CEO overuses price information. Consequently, the stock markets’ capacity to aggregate dispersed information about the firm’s “assets in place” is weakened which lowers the CEO’s ability to extract information from the stock price. As a result, the firm’s initial shareholders can be better off with a contract that pays a flat wage for the CEO, if it also specifies a fixed investment rule. While this fixed rule prevents the new growth opportunity from being started, it also maximizes price efficiency such that the CEO can optimally extract information from the financial market.

The idea that efficient stock markets aggregate private and public information into stock prices is a central topic in financial economics. The extent to which prevailing prices are informative about the future value of a firm is important for both, traders and real decision makers such as firm managers, central bankers, or politicians. It determines the amount of information these agents can extract from stock prices to improve on their economic decisions such as trading, corporate investment, and policy interventions. This paper focuses on the case where a firm manager (the CEO) can learn from the financial market to improve corporate investment decisions. This in turn leads to a “feedback effect” from stock prices to the firm value and therefore the terminal payoff. I explore the interplay of CEO incentives and stock market efficiency in this framework. In particular, I highlight the importance to consider both aspects together as certain compensation schemes lead to inefficiencies in the use of information and thus economic decisions.

In this paper, I consider a static model where a firm hires a CEO to make a capital investment decision that determines (together with two productivity shocks) the firm’s terminal value. As the CEO maximizes his own utility, the owners of the firm have to provide suitable incentives to ensure a diligent investment decision. The firm’s ability to transfer capital to output (i.e. terminal firm value) depends on two independent productivity shocks: factors $\tilde{a}$ and $\tilde{g}$. The first factor reflects uncertainty associated with the firm’s existing business or “assets in place”. Households trading in the financial market
possess private information about $\tilde{a}$. The second factor, $\tilde{g}$ represents a “growth opportunity” that is only “activated” if the CEO chooses to exert effort. If the CEO provides effort, he observes the realization of $\tilde{g}$ such that he has an informational advantage vis-a-vis individual households regarding the growth opportunity. However, all agents in the economy (the CEO and households) make a small, correlated error when forming their expectation about $\tilde{g}$. Therefore, in some states of the world the profitability of $\tilde{g}$ is believed to be too high or too low.

At $t = 0$ the firm is owned by a continuum of risk-neutral households trading claims to the firm’s terminal cash-flow in a financial market at $t = 1$. In equilibrium, the firm’s stock price reflects households’ aggregated dispersed information about $\tilde{a}$ such that all agents can extract information from it.

I first consider the case of stock-based compensation. Thus, the CEO gets endowed with a certain number of shares at $t = 0$ and receives a wage proportional to the firm’s liquidating dividend. This cash flow in turn contains both the terminal firm value as well as the costs associated with the corporate investment decision. Under this compensation scheme the CEO enjoys discretion: when choosing effort and capital investment the CEO takes into account the implications on his expected future wage, i.e. the firm value and investment costs. I show that in this case, the CEO always chooses to exert high effort such that the new growth opportunity is started. Then, households’ expected dividend depends on both productivity shocks ($\tilde{a}$ and $\tilde{g}$) and the CEO’s investment decision. While, trading on information about the first productivity shock renders prices more informative, households’ correlated errors about $\tilde{g}$ lead to aggregate noise in the stock price. What is more, this noise gets doubled as households predict the firm’s capital decision which depends on $\tilde{g}$ itself. Interestingly, this direct source of noise gets amplified in two ways: first, by trading on price information, households transmit these small correlated errors. Secondly, households also try to predict noise in the CEO’s investment decision. As this error is correlated with the stock price, they have an incentive to increase their weight on price information, which crowds out private information in equilibrium. Thus, although stock-based compensation incentivizes the CEO to exert high effort, it diminishes the financial market ability to transmit information.

I then consider the welfare consequences for the firm’s initial owners (i.e. households). First, I highlight inefficiencies associated with households’ trading behavior. As price efficiency depends on the weight, $\alpha_1$, traders attach to their private signal about $\tilde{a}$, I
show that investor welfare monotonically increases in $\alpha_1$. Intuitively, a higher degree of price efficiency allows the CEO to extract more information about $\tilde{a}$ from the financial market which improves investment efficiency. It then follows that households are better off ignoring their information about the CEO’s capital investment decision as trading on it reduces price informativeness.

I then turn to the firm side and analyze inefficiencies associated with the CEO’s decisions. While in the first-best without information asymmetries stock-based compensation (and therefore high effort) is efficient, I highlight inefficiencies relative to the constrained first-best. In this benchmark, private information remains decentralized, households follow their equilibrium strategies, and all agents commit a small error when forming their expectation about $\tilde{g}$. Relative to this benchmark, I first show that stock-based compensation leads to an overuse of price information for the CEO. This in turn renders stock prices excessively volatile and exposed to non-fundamental noise.

Next, I let the initial shareholders choose how to compensate the CEO. In particular, I allow them to use either stock-based compensation as analyzed above or a fixed wage with a pre-determined investment rule (depending on public information). While the fixed rule does not incentivize the CEO to start the growth opportunity, it maximizes price efficiency as households solely act on their private information about the assets in place. Consequently, the CEO can make a precise investment decision about $\tilde{a}$. I construct the optimal fixed investment rule and show that it is linear in the log stock price. Then, I characterize conditions when shareholders are better off with having a flat wage and fixed rule as opposed to giving the CEO discretion about investment.

Then, I turn to possible policy instruments that can improve upon the equilibrium outcome. More specifically, I show that a positive log-linear tax on the firm’s dividend improves upon the equilibrium under stock-based compensation. Intuitively, a positive tax reduces the response of the asset price to households’ expectations about the terminal payoff. Consequently, households and the CEO have lower incentives to rely on the stock price when forming their expectations about the productivity shocks. It follows that stock prices become less volatile and the constrained first best is attained. Interestingly however, there are circumstances where shareholders are still better off with a fixed investment rule. In that case, the government cannot improve investor welfare with a the type of tax considered.
1.1 Related Literature

This paper relates to two large literatures in finance and economics: that on optimal managerial incentives and the one on dispersed information in financial markets. A number of papers has studied the relationship between stock-based managerial contracts and the efficiency of corporate investment decisions, including Stein (1989), Diamond and Verrecchia (1982), Benmelech, Kandel, and Veronesi (2010), Strobl (2014), and Peng and Roell (2014). Relative to this literature my contribution is to highlight the dependence of efficient managerial contracts on the financial markets capacity to aggregate information.

Furthermore, this paper contributes to the large literature on noisy rational expectations equilibria following Hellwig (1980) and Diamond and Verrecchia (1981) in which the informational content of stock prices is clouded by exogenous noise trading. As in Hassan and Mertens (2011a) and Hassan and Mertens (2011b), the introduction of near-rational errors instead of noise traders allows me to make inferences on investor welfare. The notion of near-rationality puts discipline on the amount of noise in equilibrium asset prices which is consistent with the idea that losses to individual traders causing this noise must be economically small. Some recent papers in the literature model a feedback effect from the financial market to firm decisions as e.g. Subrahmanyam and Titman (2001), Subrahmanyam and Titman (2013), Goldstein, Ozdenoren, and Yuan (2013), and Goldstein and Yang (2014). Relative to this literature my contribution lies in the analysis of the optimal managerial incentive contract. Lastly, the notion that decision makers rely on stock market information has been supported empirical in Chen, Goldstein, and Jiang (2007), Foellalt and Fresard (2013), and Bakke and Whited (2010).

The remainder of the paper is structured as follows. Section 2 sets up the model. Section 3 contains the model solution in the case of stock-based compensation. I analyze the amplification of noise and welfare consequences in Section 4. Section 5 shows corporate governance schemes and policy tools that can improve upon the equilibrium outcome.

2 Model

There is a continuum of risk-neutral agents indexed by $i \in [0,1]$ and one firm issuing claims to its terminal earnings. At $t = 0$ one of these agents (the “CEO”) is hired to decide on the firm’s invested capital. The terminal payoff per unit of capital depends on invested
capital and realized productivity. Time is discrete and there are three time periods: in 
$t = 0$ the fundamentals (productivity shocks) are drawn, but remain unobserved until 
$t = 2$. In $t = 1$ households trade shares of the firm in a financial market and the CEO 
makes an investment decision. Both, households and the CEO base their conditional 
espectation of the fundamentals on the stock price and private signals. In the last period, 
productivity shocks are realized, the firm value is determined, and all agents get paid.

2.1 Firm Decision

The representative firm has access to the following production technology:

$$V = e^{\tilde{\alpha} + \tilde{g}(e)} K$$

where $K$ denotes capital investment, $\tilde{\alpha}$ represents the firm’s profitability associated with 
it’s “assets in place”, and $\tilde{g}(e)$ should be interpreted as a growth opportunity (independent 
of $\tilde{\alpha}$). The profitability of the growth opportunity depends on the CEO’s effort choice 
e $\in \{0, 1\}$. If he chooses to exert low effort, $e = 0$, the growth opportunity vanishes, i.e. 
$\tilde{g}(0) = 0$ with certainty. Otherwise, if $e = 1$, $\tilde{g}(1)$ is Normally distributed with mean 
$\mu_g \geq 0$ and variance $\pi_g^{-1}$:

$$\tilde{\alpha} \sim N(\mu_{\alpha}, \pi_{\alpha}^{-1}) \text{ and } \tilde{g}(1) \equiv \tilde{g} \sim N(\mu_g, \pi_g^{-1})$$

where $\pi_x$ denotes the precision for the generic random variable $\tilde{x}$.

Therefore, $\tilde{\alpha}$ and $\tilde{g}$ represent two independent sources of uncertainty that affect the firm’s 
cash flow. The first factor, $\tilde{\alpha}$, should be interpreted as the productivity shock associated 
with the firm’s current projects and the overall market conditions. Information about 
$\tilde{\alpha}$ is dispersed among households and is partially reflected in the equilibrium stock price 
through their trading behavior. As a result, the CEO is able to update his beliefs about 
$\tilde{\alpha}$ from the stock price. Households do not have any private information about $\tilde{g}$, while 
the CEO is precisely informed about it.

This two-factor structure allows me to highlight the importance of higher-order uncer-
tainty. The key idea is that one group of agents (households) has more information about 
one factor, while the CEO is more precisely (here perfectly) informed about the other 
one.

\footnote{I also solved a version of the model where the CEO receives private information about $\tilde{\alpha}$. As long as, 
there remains some informational asymmetry between both types of agents all results remain valid.}
In \( t = 1 \), the CEO chooses capital investment \( K \) and effort \( e \) to maximize his expected terminal wealth \( W_{CEO} \) net of effort costs:

\[
\max_{e \in \{0,1\}, K} E_{CEO} [W_{CEO}] - c_0 e
\]

(3)

where \( c_0 \geq 0 \) governs the CEO’s disutility from exerting high effort.

Without loss of generality, I assume that the manager starts with zero initial wealth, such that his only source of income is his wage. For now, this wage is stock-based such that the CEO owns \( \omega_{CEO} \) shares of the firm initially. As a result, the final wage is proportional to the firm’s terminal profits, i.e. the firm value \( V \) net of capital investment costs \( C \). The latter are increasing in the CEO’s capital investment decision:

\[
C = C(K) = \frac{1}{2} K^2
\]

(4)

Then, the CEO’s wage is given by:

\[
W_{CEO} = \omega_{CEO} (V - C)
\]

(5)

As a result, the CEO chooses \( \{e, K\} \) to maximize:

\[
\max_{e \in \{0,1\}, K} E_{CEO} \left[ \omega_{CEO} \left( e^{\tilde{a} + \tilde{g}(e)} K - \frac{1}{2} K^2 \right) \right] - c_0 e
\]

(6)

taking as given the stock price \( P \) as well as any private signals about the productivity shocks. For simplicity, I assume that the CEO’s outside option is equal to zero, i.e. the expected wage (net of effort costs) needs to be weakly positive to satisfy his participation constraint.

2.2 Information Structure

The CEO’s information set includes the asset price \( P \), as well as the prior distribution of the first productivity shock \( \tilde{a} \). Moreover, he perfectly observes the realization of the growth opportunity \( \tilde{g}(e) \) once the effort decision has been made.\(^2\) Household \( i \in [0, 1] \) only receives a private signal about the productivity shock \( \tilde{a} \):

\[
x_i = \tilde{a} + \tilde{\nu}_i
\]

(7)

\(^2\)This implies that households can perfectly infer the CEO’s effort decision from the ex ante known parameters. Shutting down this dimension of uncertainty allows me to highlight the role of informational inefficiencies given the CEO’s effort choice.
where $\tilde{\nu}_i$ is i.i.d. across households and normally distributed with mean zero and precision $\pi_x$. In addition to $x_i$ households also observe the stock price and know the prior distributions of $\tilde{a}$ and $\tilde{g}$.

### 2.3 Households

At $t = 1$ households submit price-dependent orders to trade claims to the terminal firm value $V$. They can buy or sell shares inside the limits $\omega_i \in [\omega, \bar{\omega}]$ with $\omega < 1 < \bar{\omega}$. These position limits are necessary to keep optimal portfolio shares finite and can be interpreted as borrowing or short-selling constraints. In general, the specific values for these limits do not matter and it is sufficient to rule out unlimited positions. As households are risk-neutral, they choose the portfolio share $\omega_i$ that maximizes their expected terminal wealth conditional on information set $\mathcal{I}_i$.

I assume that every household is endowed with one share of the firm in the beginning, such that $W_0 = P$. Then, households choose their portfolio share in the risky asset to maximize expected wealth:

$$\max_{\omega_i} \mathbb{E}_i[W_i]$$

with $W_i = W_0 \left(\omega_i \frac{V}{P} + (1 - \omega_i)\right) - C - W_{CEO}$. Therefore, households receive the portfolio return weighted by their initial wealth less capital investment costs and the CEO's wage. Due to linear preferences and zero discounting, the risk-free rate is equal to zero and each agent either purchases up the upper limit $\bar{\omega}$ or goes to the lower bound $\omega$.

I assume that all agents (households and the CEO) commit a “near-rational” error as in Hassan and Mertens (2011b). When forming their expectation about the growth opportunity $\tilde{g}$, all agents make a small correlated error. As a result, their posterior probability density function is shifted by $\tilde{\varepsilon} + \tilde{\varepsilon}_i$. I assume a positively correlated error, so that $\tilde{\varepsilon} \sim N(0, \pi_{\varepsilon}^{-1})$ is the common component. The idiosyncratic part is distributed as $\tilde{\varepsilon}_i \sim N(0, \hat{\mu} \pi_{\varepsilon}^{-1})$ where $\hat{\mu}$ calibrates the size of the correlation. Thus the near-rational expectation is given as:

$$\mathcal{E}_i[g] = E_i[g] + \tilde{\varepsilon} + \tilde{\varepsilon}_i$$

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3In Section 6 I analyze a setting with risk-averse households as a robustness check. The assumption of risk-neutrality allows me solve the model in closed form.

4Note that both, investment cost and the CEO's wage are paid as lump-sum transfers. This assumption allows me to obtain closed-form solutions due to the log-normal structure.

5Without loss of generality, I set $\hat{\mu} = 0$ such that the common error is perfectly correlated.
where $E_i [\tilde{g}]$ denotes the rational, Bayesian expectation. Importantly, even though near-rational households commit a mistake in their expectation, they fully understand the structure of the economy, the equilibrium mapping of information into the stock price, and all higher moments of $\tilde{g}$. Moreover, every agent is aware of the fact that everybody loads on the common error $\tilde{\varepsilon}$.\textsuperscript{6}

Here, $\pi_\varepsilon$ governs the magnitude of the near-rational error, i.e. the distance between the optimal and near-rational expectation. As $\pi_\varepsilon \to \infty$ all agents become fully rational.

Market clearing requires that aggregate demand equals the fixed supply of the asset. I normalize the supply of stocks to 1, thus:

$$\omega_{CEO} + \int_0^1 \omega_i di = 1 \quad (10)$$

The market clearing condition then determines the equilibrium stock price, $P$.

### 2.4 Equilibrium Definition

**Definition 1** An equilibrium consists of a price function, $P (\tilde{a}, \tilde{\varepsilon}) : \mathbb{R}^2 \to \mathbb{R}$, an investment policy for the CEO, $K (\tilde{g}(e), P, \tilde{\varepsilon}) : \mathbb{R}^3 \to \mathbb{R}$, an effort decision, $e \in \{0, 1\}$, and a trading strategy for households, $\omega (x_i, P, \tilde{\varepsilon}) : \mathbb{R}^3 \to [\omega_-, \omega_+]$ such that:

(a) The CEO maximizes his expected utility;
(b) Households maximize expected wealth; and
(c) The stock market clears.

### 3 Model Solution

In this section, I first solve for the CEO’s optimal effort provision, I then characterize households’ and the CEO’s equilibrium expectations. Subsequently, I describe how these expectations map into optimal trading behavior and the capital investment decision, respectively.

#### 3.1 Effort Provision

After the CEO has been hired and endowed with $\omega_{CEO}$ shares, he has to decide whether to choose low or high effort and how much capital to invest.\textsuperscript{7}

\textsuperscript{6}When I quantitatively analyze the model in Section 6, I calibrate $\pi_\varepsilon$ such that the individual utility loss from near-rational behavior is below a given threshold.

\textsuperscript{7}All proofs can be found in the Appendix.
Lemma 1 The CEO’s expected wage is equal to:

\[ E_{\text{CEO}}[W_{\text{CEO}}] = \frac{1}{2} \omega_{\text{CEO}} \exp \left( E_{\text{CEO}}[\bar{a} + \bar{g}(e)] + \frac{1}{2} V_{\text{CEO}}[\bar{a} + \bar{g}(e)] \right) \]  

(11)

First, note that the manager’s participation constraint is satisfied for any non-negative asset share and any effort decision. Moreover, remember that the CEO only observes \( \bar{g} \) if he chooses to exercise effort. Hence, the CEO needs to compare his expected wage under low effort to that under high effort and choose \( e \) accordingly.

Lemma 2 Under stock-based compensation, the CEO chooses to exert high effort, \( e = 1 \), iff \( \mu_g \geq \pi \) where the constant \( \pi \) is defined in the Appendix.

Therefore, quite intuitively, the CEO’s expected wage is increasing in the effort choice \( e \). As a result, the CEO chooses high effort if the project is sufficiently profitable to compensate for his effort costs. In the following, I assume that this assumption is met such that giving the CEO a stake in the firm’s terminal cash-flow always leads to high effort choice such that the growth opportunity \( \bar{g} \) is started.

3.2 Equilibrium Expectations

As argued above, households’ optimal trading behavior amounts to demanding \( \omega \) if their conditional expectation of the payoff, \( V \), exceeds the price, and to go down to \( \omega \) otherwise. Hence, the market clearing condition implies that portfolio share \( \omega \) demanded by the “pessimists” plus the quantity demanded by the “optimists” \( \overline{\omega} \) equals the unit supply:

\[ Pr(\mathcal{E}_i[V] \leq P) \omega + Pr(\mathcal{E}_i[V] > P) \overline{\omega} = 1 - \omega_{\text{CEO}} \]  

(12)

Here the probability is taken conditional on aggregate shocks. The implied equilibrium stock price is characterized in the following proposition.

Proposition 1 The equilibrium stock price is solely a function of households’ average expectation of the log payoff, \( \bar{q} \equiv \int_0^1 \mathcal{E}_i[\log V] \, di \) and fixed parameters:

\[ P = \exp(\kappa_0 + \bar{q}) \]  

(13)

where \( \kappa_0 \geq 0 \) is a constant defined in the Appendix.

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\(^8\)Here I rule out “insider trading” by the CEO, i.e. he is not allowed to implicitly alter his amount of shares in the firm through trading in the stock market.

\(^9\)I.e. the two probabilities are CDFs over the distribution of \( \bar{\nu} \sim N \left( 0, \pi^{-1}_\nu \right) \).
The fact that households’ demand for the firm’s shares is solely based on the expected payoff is reflected in this proposition. As the asset supply is fixed by assumption, the only random variable that affects the asset price is the average conditional expectation of the (log) payoff, \( \hat{q} \). This average expectation in turn is driven by two unknown, aggregate variables: the aggregate productivity shock, \( \tilde{a} \), and the near-rational error, \( \tilde{\varepsilon} \). Note that the equilibrium stock price does not reveal any information about \( \tilde{g} \) as households do not receive informative signals about the growth opportunity by assumption.

**Corollary 1** Learning from the stock price \( P \) is equivalent to learning from the average expectation \( \hat{q} \), i.e. \( P \) and \( \hat{q} \) span the same \( \sigma \)-algebra.

As the stock price, \( P \), is a monotonically increasing function of \( \hat{q} \), both variables contain the same amount of information about the productivity shock, \( \tilde{a} \). Mathematically, using \( \hat{q} \) instead of \( P \) simplifies the analysis significantly as it allows me to use the standard projection theorem for normally distributed variables.\(^{10}\) In the following, I refer to learning from \( \hat{q} \) as “learning from the stock price”.

Consequently, the conditional expectation for household \( i \) is a linear function of prior information about \( \tilde{a} \) and \( \tilde{g} \), the private signal \( x_i \), and price information captured through \( \hat{q} \):

\[
E_i [\log V] = \alpha_0 + \alpha_1 x_i + \alpha_2 \hat{q} + \alpha_3 \tilde{\varepsilon} \tag{14}
\]

where \( \{\alpha_0, \alpha_1, \alpha_2, \alpha_3\} \) are constants determined in equilibrium. Plugging in the definition of \( \hat{q} \) and integrating over all households implies:

\[
\hat{q} = \frac{\alpha_0}{1 - \alpha_2} + \frac{\alpha_1}{1 - \alpha_2} \tilde{a} + \frac{\alpha_3}{1 - \alpha_2} \tilde{\varepsilon} \tag{15}
\]

Therefore the firm’s log stock price is Normally distributed with mean \( \kappa_0 + E_0 [\hat{q}] \) and variance \( V_0 [\hat{q}] \).

Thus the transformed stock price (or equivalently households’ average expectation), \( \hat{q} = \log P - \kappa_0 \), is informative about the productivity shock, \( \tilde{a} \). The aggregation is however not perfect as it is clouded by households’ near-rational errors \( \tilde{\varepsilon} \). As a result, households

\(^{10}\)Note that the fundamentals \( \{\tilde{a}, \tilde{g}\} \), private signals \( \{x_i\} \), near-rational errors \( \tilde{\varepsilon} \), and the average expectation \( \hat{q} \) are all Normally distributed.
and the CEO can extract information about $\bar{a}$ from the unbiased signal $s_p$:

$$s_p = \frac{1 - \frac{\alpha_2}{\alpha_1} \hat{q} - \frac{\alpha_0}{\alpha_1}}{\frac{\alpha_3}{\alpha_1}} = \bar{a} + \frac{\alpha_3}{\alpha_1} \bar{\varepsilon} \quad (16)$$

This signal’s precision, $\pi_p \equiv \frac{\alpha_2^2}{\alpha_3^2} \pi_\varepsilon$, depends positively on two factors: the degree of rationality ($\pi_\varepsilon$) and the weight households attach to their private signal ($\alpha_1$). On the other hand, price efficiency decreases if households’ equilibrium expectations are more heavily influenced by the near-rational error, i.e. if $\alpha_3$ is large.

As a result, all agents in the economy can extract more precise information from the stock price if each individual trader allocates relatively more weight towards his private signal. Below I show that in equilibrium traders optimally put too little weight on $x_i$ and too much weight on the price signal $\hat{q}$. This is due to the fact that households try to forecast the CEO’s near-rational error by the equilibrium stock price.

### 3.3 Optimal Firm Investment

The solution to the CEO’s optimization problem is given by:

$$K = \mathcal{E}_{\text{CEO}} [\exp(\bar{a} + \bar{g}(e))] \quad (17)$$

As shown above, it is optimal for the CEO to provide high effort, $e = 1$. Due to the log normal structure of the problem, log investment is a linear function of $\{\bar{g}, \hat{q}, \bar{\varepsilon}\}$, i.e. the CEO’s informative signals and the common near-rational error:

$$k = \log K = \beta_0 + \beta_1 \hat{q} + \bar{g} + \bar{\varepsilon} \quad (18)$$

where $\{\beta_0, \beta_1\}$ follow from the standard projection theorem:

$$\beta_0 = \frac{\pi_a \mu_a + \frac{1}{2} - \frac{\alpha_0}{\alpha_1} \pi_p}{\pi_a + \pi_p} \quad (19)$$

$$\beta_1 = \frac{1 - \frac{\alpha_2}{\alpha_1} \pi_p}{\frac{\alpha_1}{\alpha_1} \pi_a + \pi_p} \quad (20)$$

Intuitively, the CEO relies a lot on price information, if the price signal is highly informative about the aggregate shock, i.e. if $\pi_p$ is high.

### 3.4 Optimal Trading Behavior

In this setting, households face higher order uncertainty; the firm’s terminal cash flow depends on both, fundamentals and capital investment. The latter in turn depends on
the CEO’s expectation of both fundamentals. Hence, the conditional expectation of the log payoff is given by:

$$E_i[v] = E_i[\tilde{a} + \tilde{g} + k] = \alpha_0 + \alpha_1 x_i + \alpha_2 \tilde{g} + \alpha_3 \tilde{\varepsilon}$$  \hspace{1cm} (21)

Using the projection theorem for normally distributed random variables together with the expression for optimal capital investment leads to:

$$\alpha_0 = \frac{\alpha_1 \pi_a \mu_a + (\beta_0 + 2 \mu_g) \pi_x}{\frac{1}{2} \pi_x + \pi_p}$$ \hspace{1cm} (22)

$$\alpha_1 = \frac{\pi_x}{\pi_a + \frac{3}{2} \pi_x + \pi_p}$$ \hspace{1cm} (23)

$$\alpha_2 = \frac{\pi_p + \pi_x (\beta_1 + \frac{1}{2})}{\frac{3}{2} \pi_x + \pi_p}$$ \hspace{1cm} (24)

$$\alpha_3 = 2$$ \hspace{1cm} (25)

where $\pi_p = \frac{\alpha_2^2 \pi_x}{\pi_p}$ denotes the precision of the endogenous price signal.

As a result, I get five equations for the weights determining expected payoffs and capital investment, $\{\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1\}$. Solving these equations leads to a complete and unique characterization of the economy and the corresponding equilibrium expressions are given in the Appendix.

From the expression for households’ average expectation of the log payoff in (15) it is clear that the equilibrium stock price is driven by the fundamental, $\tilde{a}$, and the common near-rational error, $\tilde{\varepsilon}$. The impact of the latter depends on the weight households attach to the stock price, $\alpha_2$. As this weight is a number between zero and one, this multiplier is always larger than one such that the stock price transmits the common error together with information about $\tilde{a}$.

### 4 Shareholder Welfare

#### 4.1 Amplification of Noise

The discussion above shows that tying the CEO’s compensation to firm performance, incentivizes the CEO to provide effort and to invest in the growth opportunity $\tilde{g}$. Next, I show that at the same time, it also leads to an amplification of noise in the stock price so that movements in $P$ are more highly exposed to changes in the correlated error $\tilde{\varepsilon}$ and less to the fundamental $\tilde{a}$.

First note that if the CEO does not choose to exert high effort, the growth opportunity
vanishes and $\tilde{g} = 0$ with certainty. As a result, traders only need to forecast $\tilde{a}$ and the CEO’s capital investment decision which in turn only depends on prior information and the publicly observable stock price. It follows that in this setup, the stock price perfectly reveals the fundamental $\tilde{a}$ to all households and the firm CEO.\textsuperscript{11}

**Lemma 3** If the CEO chooses $e = 0$, the stock price perfectly reveals $\tilde{a}$.

If, however, the CEO invests in the growth opportunity, there are three different channels that introduce aggregate noise into the stock price. As pointed out above, when traders forecast the log payoff, they need to forecast the fundamentals as well as the firm’s investment decision:

$$E_i[\log V] = E_i[\tilde{a} + \tilde{g} + k]$$

While forecasting $\tilde{a}$ does not lead to any direct source of noise in the stock price, traders make a correlated error when forecasting $\tilde{g}$. As, the CEO’s capital investment decision is equal to $E_{\text{CEO}}[\tilde{a}] + \frac{1}{2} V_{\text{CEO}}[\tilde{a}] + \tilde{g}$, traders make the same correlated error again when forecasting $k$. Moreover, this direct effect of $2\tilde{e}$ gets further amplified in two ways: on the one hand, as traders use the stock price when predicting $\tilde{a}$ they also proliferate the common error $\tilde{e}$. This is reflected in the factor of $\frac{1}{1-\alpha_2}$ in equation (15) such that the contribution of noise to the stock price is particularly strong if traders rely a lot on the $\hat{q}$ when predicting the fundamental, i.e. if $\alpha_2$ is large. On the other hand, traders do not only forecast productivity growth, but they also try to predict the CEO’s error associated with choosing capital investment, $\tilde{e}$. This common near-rational error can be partially inferred from the asset price and the private signal about $\tilde{a}$:

$$E_i[\tilde{e}] = \frac{1}{2} (-\alpha_0 - \alpha_1 E_i[\tilde{a}] + (1 - \alpha_2) \hat{q})$$

Therefore, to forecast the aggregate near-rational error, $\tilde{e}$, traders optimally put a positive weight on the price signal and a negative weight on the conditional expectation of $\tilde{a}$. This in turn implies that traders’ conditional expectation of the asset payoff becomes more exposed to the price signal $\hat{q}$ and less exposed to the private signal $x_i$. Consequently, the equilibrium asset price becomes noisier and less informative about $\tilde{a}$.

\textsuperscript{11}The assumption that the stock price becomes perfectly revealing in the absence of $\tilde{g}$ renders the exposition simple. As the focus lies on the welfare differential between low and high effort, any constant noise in the stock price that is present in both scenarios does not alter the qualitative results.
**Proposition 2** *Forecasting capital investment renders prices more exposed to noise and less informative about \( \tilde{a} \).*

This proposition highlights a key difference between traders’ forecasting of fundamentals and capital investment. While the first renders prices more informative as traders rely on their private signals, forecasting the latter crowds out information and leads to an amplification of noise in the asset price. Consequently, traders could increase the informational content of the asset price by ignoring the part of the cash flow related to capital investment when forming their expectation.

### 4.2 Investor Welfare

For the normative analysis, I first define the social welfare function as the unconditional expected utility of an arbitrary agent at \( t = 0 \):\(^\text{12}\)

\[
SWF = E_0[W_i] \propto E_0[\mathcal{V} - C]
\] (28)

Plugging in the expressions for the payoff \( \mathcal{V} \) and the investment costs \( C \) gives social welfare solely as a function of households’ weight attached to their private signal, \( \alpha_1 \).

**Proposition 3** *Social welfare can be written as:*

\[
SWF = e^{2(\mu_a + \mu_f + \pi_a^{-1} + \pi_f^{-1})} \left[ e^{\frac{1}{2} \pi_x^{-1} - \frac{1}{2} \frac{\pi_a^{-1} + \pi_p^{-1}}{\pi_a + \pi_p}} - \frac{1}{2} e^{2 \pi_x^{-1} - \frac{1}{2} \frac{1 - 2 \alpha_1}{\pi_a + \pi_p}} \right]
\] (29)

Therefore, investor’s ex ante expected wealth depends on the fixed model parameters as well as the weight they attach to their private signal, \( \alpha_1 \) (which itself depends on the parameters \( \pi_a, \pi_x, \pi_e \)). Intuitively, welfare increases in the means of the two fundamentals, \( \mu_a \) and \( \mu_f \) as well as in their variances.\(^\text{13}\) The following proposition shows that investor welfare also increases in the informational content of stock prices, i.e. in \( \alpha_1 \):

**Proposition 4** *Social welfare increases in the weight traders attach to their private signal:*

\[
\frac{\partial SWF}{\partial \alpha_1} > 0
\] (30)

if \( \pi_p > \pi_a \).

---

\(^{12}\)This utilitarian social welfare function corresponds to the conventional definition of “real efficiency” as e.g. in Goldstein, Ozdenoren, and Yuan (2013) and Goldstein and Yang (2014).

\(^{13}\)This holds if and only if \( \frac{1}{2} \log 2 > \pi_e^{-1} + \frac{\alpha_1}{\pi_a + \pi_p} \) which I assume throughout.
This result is intuitive, as households rely more on their private information the stock price transmits more dispersed information about $\tilde{a}$ to the CEO. As a result, capital is more precisely allocated and investor welfare rises. This immediately leads to the following implication:

**Corollary 2** Households’ forecasting of capital investment is inefficient if $\pi_p > \pi_a$.

Consequently, households would be ex ante better off if they did not try to forecast the CEO’s capital investment decision, but instead solely focused on predicting the fundamental about which they have private information.

In this environment, a social planner could clearly achieve a Pareto superior solution by collecting all dispersed information about $\tilde{a}$ and announcing it to all agents. As a result, the stock price perfectly reveals both $\tilde{a}$ and $\tilde{\varepsilon}$ so that all asymmetric information vanishes.

**Proposition 5** In the first best, all agents perfectly observe $\{\tilde{a}, \tilde{g}, \tilde{\varepsilon}\}$. Optimal capital investment is given by $K_{fb} = \exp (\tilde{a} + \tilde{g})$ and investor welfare by:

$$SWF_{fb} = \frac{1}{2} e^{2(\mu_a + \mu_g + \pi_a^{-1} + \pi_g^{-1})}.$$  

Therefore, with perfect information investors’ welfare increases in the two projects’ fundamentals’ mean and variance. As a result, motivating the firm CEO to exert high effort is always desirable without information asymmetry.

**Corollary 3** In the first best, stock-based compensation is efficient.

The previous corollary confirms the standard notion of stock-based compensation. Giving the CEO an incentive to invest in growth opportunities is welfare improving. Next, I show that this is not necessarily true in the constrained efficient solution that respects households’ private information. Therefore, I highlight the importance of informational inefficiencies as a result of managerial discretion about capital investment.

### 4.3 Constrained First-Best Investment Policy

In practice, it seems implausible that the government (acting as a social planner) is able to collect households’ private information. Therefore, I consider a more restrictive efficiency benchmark that addresses the question: what is the optimal corporate structure that maximizes shareholders’ ex ante expected utility? More specifically, private information
remains decentralized and all agents still commit a correlated error when forecasting $\tilde{g}$.
Moreover, households still follow their equilibrium policies, such that the social planner only controls the CEO’s investment behavior.

Next, I characterize the constraint efficient benchmark in more detail:

**Definition 2** A feasible allocation is a collection of portfolio choices $\omega_i$, one for each household, and an investment decision $K$ for the CEO that jointly satisfy the following constraints:

(a) resource feasibility:

$$\int_0^1 C_i \, di = V - \frac{1}{2} K^2 - W_{CEO}$$  (32)

where $C_i = W_i$ denotes terminal consumption of agent $i \in [0, 1]$.

(b) informational feasibility: for each household $i$, $\omega_i$ is contingent on the private signal $x_i$ and the stock price $P$, for the CEO $K$ is contingent on $\tilde{g}$ and $P$.

(c) near-rational feasibility: each household $i$ and the CEO commit a near-rational error when forming their expectation about $\tilde{g}$.

**Definition 3** A constrained efficient investment allocation is a feasible allocation that is not Pareto dominated by any other feasible allocation taking as given the utility maximizing portfolio choices for each trader.

In particular, I show that in this economy, both the CEO and households optimally extract less information from the stock price compared to the competitive equilibrium. Specifically, I keep the information structure as before and allow the CEO to extract information from the stock price to predict the productivity shock $\tilde{a}$. I restrict the discussion to log-linear investment rules such that $k = \beta_0 + \beta_1 \tilde{q} + \beta_2 \tilde{g} + \tilde{\varepsilon}$. Then the social planner chooses the $\beta$ coefficients to maximize investor welfare. The solution is described in the following proposition.

**Proposition 6** The investment policy that maximizes social welfare is given by $k = \ldots$
\[ \beta_0 + \beta_1 \hat{q} + \beta_2 \bar{g} + \varepsilon \] where

\[
\begin{align*}
\beta_0 &= \mu_a + \frac{1}{2} \pi_a^{-1} + \frac{\beta_1}{1 - \alpha_2} \left( \alpha_1 \pi_a^{-1} - \alpha_0 - \alpha_1 \mu_a - \frac{3}{2} \frac{\beta_1}{1 - \alpha_2} \alpha_1 \pi_a^{-1} \right) \\
&\quad - \frac{3}{2} \left( 1 + 2 \frac{\beta_1}{1 - \alpha_2} \right)^2 \pi_e^{-1} \\
\beta_1 &= \frac{1 - \alpha_2}{\alpha_1} \frac{\pi_p - 2 \pi_a}{4 (\pi_a + \pi_p)} \\
\beta_2 &= 1
\end{align*}
\] (33) (34) (35)

First, note that here the social planner takes trading behavior as given. As a result, the \( \alpha \) coefficients from the competitive equilibrium in (22)-(25) still apply. Then, it follows that the weight traders attach to their private information, \( \alpha_1 \) determined in (23), remains unchanged such that price informativeness, \( \pi_p \), in the constraint efficient allocation is unchanged.

**Lemma 4** Price informativeness in the constrained efficient allocation is equal to that in the competitive equilibrium.

At the same time, it follows that the firm CEO efficiently uses his knowledge about the growth option in equilibrium as \( \beta_2 \) is still equal to 1. However, it also follows that the CEO relies too much on price information in the competitive equilibrium, i.e. \( \beta_2 \) is inefficiently high.

**Proposition 7** Relative to the constrained efficient investment rule, the CEO efficiently uses knowledge about \( \bar{g} \), but relies too much on price information.

The fact that the firm CEO bases the capital investment decision too heavily on stock prices has a direct spill-over effect to households. From the expression for \( \alpha_2 \) in (24) it follows that all households also place a lower weight on price informativeness in the constrained efficient allocation which leads to the following corollary.

**Corollary 4** Relative to the constrained efficient investment rule, households efficiently use private information, but rely too much on price information.

Therefore, the firm CEO as well as each household should optimally react less to changes in the stock price and focus relatively more on prior and private information.

The discussion above shows that agents overuse information transmitted through the
stock price. This in turn implies that households’ portfolio decisions are more exposed to correlated noise, \( \bar{\varepsilon} \) which then gets passed through into the stock price, \( P \).

**Corollary 5** In the constrained efficient allocation, the (log) stock price:

a) is less volatile and

b) displays less non-fundamental variance.

Note that the log stock price is given by 

\[
p = \kappa_0 + \hat{q}
\]

where \( \kappa_0 \) is a positive constant and \( \hat{q} \) is equal to the average expectation of the log payoff. Then, using the definition of \( \hat{q} \), the log price variance is given by:

\[
V_0[p] = \left( \frac{\alpha_1}{\alpha_1 - \alpha_2} \right)^2 \pi_a^{-1} + \left( \frac{1}{\alpha_1 - \alpha_2} \right)^2 \pi_{\bar{\varepsilon}}^{-1}
\]

As \( \alpha_2 \) is inefficiently high in the competitive equilibrium, it immediately follows that the asset price is too volatile in general and in particular too much exposed to non-fundamental noise, \( \left( \frac{1}{1 - \alpha_2} \right)^2 \pi_{\bar{\varepsilon}}^{-1} \).

### 5 Model Implications

The discussion above shows that stock-based compensation leads to inefficiencies due to informational feedback effects. Next, I analyze mechanisms that aim at improving shareholder welfare. First, I allow the shareholders to decide on the optimal corporate governance at \( t = 0 \). More specifically, they can decide on the compensation package of the CEO which in turn determines his capital investment decision. Afterwards, I focus on possible policy tools that can improve upon the equilibrium allocation.

#### 5.1 Optimal Corporate Governance

In this section, I allow the shareholders to decide on corporate governance at \( t = 0 \). More specifically, they can choose between stock-based compensation or a flat wage. As shown above under the first scheme, the CEO is incentivized to exert high effort and to use discretion when making the capital investment decision. As the CEO does not have an incentive to exert high effort, he optimally chooses \( e = 0 \) under a flat wage. As the CEO does not have an incentive to invest optimally under a flat wage, investors need to design an optimal fixed rule (depending on publicly available quantities). The following proposition characterizes the optimal contract in this scenario.
Proposition 8 Under a flat wage:

- The CEO chooses $e = 0$
- The stock price is perfectly revealing about $\tilde{a}$
- The optimal capital investment rule is given by:

$$k = \beta_0 + \beta_1 \hat{q}$$  (37)

As the CEO has no incentive to start the growth opportunity, agents have no reason to use their expectations about $\tilde{g}$. Consequently, the stock price does not contain any correlated noise which implies that it perfectly reveals $\tilde{a}$ to everybody. This in turn gets rid of any informational asymmetry in the economy such that there are no informational inefficiencies and investor welfare simply depends on the fundamentals of $\tilde{a}$:

Corollary 6 With a flat wage and fixed investment rule, investor welfare is given by:

$$SWF = \frac{1}{2} e^{2(\mu_g + \pi g^{-1})}$$  (38)

While there are no informational inefficiencies in this setup, the (profitable) growth opportunity is not started. As a result, whether this scenario dominates the previous setup with stock-based compensation and discretion depends on the parameters of the model.

Proposition 9 Shareholders prefer a fixed investment rule (with a flat wage) iff:

$$2e^{2(\mu_g + \pi g^{-1})} \left[ e^{\pi g^{-1} \frac{1-2\alpha_1}{\alpha_1 + \gamma p}} - \frac{1}{2} e^{2\pi g^{-1} \frac{1-2\alpha_1}{\alpha_1 + \gamma p}} \right] < 1$$  (39)

Intuitively, the fixed rule is particularly attractive if the growth opportunity has a low expected return such that the opportunity costs of not exercising high effort are low. Similarly, the larger the error variance $\pi g^{-1}$, the more costly it is to give the CEO discretion about investment by endowing him with stock-based compensation.

5.2 Tax on Dividend

The preceding analysis suggests that optimally structured policies can improve upon the equilibrium outcome even though the “government” does not enjoy any informational advantage. I now show how a log-linear tax on the firm’s dividend reduces price volatility and establishes the constrained-first best outcome under discretionary capital investment.
Afterwards, I highlight the limited scope of this instrument in the presence of a fixed investment rule.

Consider a log-linear tax on the dividend at \( t = 1 \). This tax is meant to capture more generally the idea that a given policy introduces a “wedge” between the asset price. As in Angeletos, Lorenzoni, and Pavan (2010) I allow the tax to depend on \( P \) to capture the idea that policy interventions may depend on the level of stock prices:

\[
\tau = \tau(p) = \tau_0 + \tau_1 p
\]  

(40)

where \( \tau_0, \tau_1 \) are scalars chosen by the “government” and \( p = \log P \) denotes the log stock price.

As a result, the expected log cash flow for trader \( i \) is given by:

\[
E_i [\log V] = \alpha_0 + \alpha_1 x_i + \alpha_2 \hat{q} - (\tau_0 + \tau_1 p)
\]  

(41)

Then it follows, that a positive \( \tau_1 \) effectively reduces the weight agents attach to their price signal. As the CEO’s optimal weight \( \beta_1 \) is decreasing in \( \alpha_2 \) it follows that there always exists a combination of \( \tau_0, \tau_1 \) that implements the constrained efficient allocation.

**Proposition 10** The optimal \( \alpha \) coefficients with a log-linear transaction tax are given by:

\[
\alpha_0 = \frac{\alpha_1 \pi a \mu a + (\beta_0 + 2 \mu_f - \tau_0 - \kappa_0 \tau_1) \pi x}{\frac{3}{2} \pi x + \pi p}
\]  

(42)

\[
\alpha_1 = \frac{\pi x}{\pi a + \frac{3}{2} \pi x + \pi p}
\]  

(43)

\[
\alpha_2 = \frac{\pi p + \pi x (\beta_1 - \tau_1 + \frac{1}{2})}{\frac{3}{2} \pi x + \pi p}
\]  

(44)

The CEO’s optimal \( \beta \) coefficients are unchanged, but depend on \( \{\alpha_0, \alpha_1, \alpha_2\} \) of course.

As a result, the government should optimally choose a pro-cyclical transaction tax, \( \tau_1 > 0 \), such that \( \alpha_2 \) is reduced to the constrained efficient value.

**Corollary 7** There always exists a log-linear tax with \( \tau_1 > 0 \) that implements the constrained efficient allocation under stock based compensation.

Even though a positive transaction improves upon the equilibrium allocation, it might still be welfare improving to introduce a fixed investment rule for the CEO. As shown
above, this depends on both the attractiveness of the growth opportunity ($\mu_g$) and the importance of the informational inefficiencies ($\pi_x$ and $\pi_x$).

6 Quantitative Model

Still in progress.

7 Conclusion

This paper highlights the importance to consider the effects of a managerial incentive contract together with its implications on price efficiency. By decreasing the financial market’s capacity to aggregate dispersed information a contract that leads to high effort provision can actually be inefficient ex ante. While the main scope of the paper is to qualitatively show the benefits and drawbacks of certain compensation packages a quantitative extension of the model would be useful to get a better feeling for the magnitudes of the effects described.
References


A Technical Appendix

A.1 Proofs

Proof of Lemma 1

Note that the expected wage is given by:

\[ \omega_{CEO} \left[ e^{\mathcal{E}_{CEO}[\tilde{a} + \tilde{g}(e) + k]} + \frac{1}{2} V_{CEO}[\tilde{a} + \tilde{g}(e) + k] \right] - \frac{1}{2} e^{2\mathcal{E}_{CEO}[k] + 2V_{CEO}[k]} \]  

(A.1)

Then, using \( k = \mathcal{E}_{CEO}[\tilde{a} + \tilde{g}(e)] \) gives the result.

Proof of Lemma 2

The CEO chooses high effort if his utility from setting \( e = 1 \) is higher than that under \( e = 0 \). Then,

\[ \frac{1}{2} \omega_{CEO} \exp \left( \mathcal{E}_{CEO}[\tilde{a} + \tilde{g}] + \frac{1}{2} V_{CEO}[\tilde{a} + \tilde{g}] \right) - c_0 \geq \frac{1}{2} \omega_{CEO} \exp \left( \mathcal{E}_{CEO}[\tilde{a}] + \frac{1}{2} V_{CEO}[\tilde{a}] \right) \]  

(A.2)

Then the expected wage under \( e = 1 \) can be written as \( \frac{1}{2} \omega_{CEO} \exp \left( \mathcal{E}_{CEO}[\tilde{a}] + \frac{1}{2} V_{CEO}[\tilde{a}] \right) \) + \( \hat{q} \) by the definition of the average expectation \( \hat{q} \) and therefore:

\[ \Pr \left( \hat{q} + \alpha_1 \pi_i \leq \frac{1}{2} V_{i}[v] \right) \omega + \Pr \left( \hat{q} + \alpha_1 \pi_i + \frac{1}{2} V_{i}[v] > p \right) \omega = 1 - \omega_{CEO} \]  

(A.3)

Define the standard normal random variable \( u_i \equiv \pi_{x}^{-1/2} \nu_i \):

\[ \Pr \left( \hat{q} + \alpha_1 \pi_x^{1/2} u_i + \frac{1}{2} V_{i}[v] \leq p \right) \omega + \Pr \left( \hat{q} + \alpha_1 \pi_x^{1/2} u_i + \frac{1}{2} V_{i}[v] > p \right) \omega = 1 - \omega_{CEO} \]

\[ \Phi \left( \frac{p - \hat{q} - \frac{1}{2} V_{i}[v]}{\alpha_1 \pi_x^{1/2}} \right) \left( \omega - \omega \right) + \omega = 1 - \omega_{CEO} \]

where \( \Phi(\cdot) \) denotes the standard normal CDF. Then, it follows that the log stock price is given by:

\[ p = \alpha_1 \pi_x^{1/2} \Phi^{-1} \left( \frac{1 - \omega_{CEO} - \omega}{\omega - \omega} \right) + \frac{1}{2} V_{i}[v] + \hat{q} \]  

(A.4)
where \( V_i[v] = V_i[a] = (\pi_a + \pi_x + \pi_p)^{-1} \).

**Proof of Corollary 1**

From the expression for the stock price \( P = \exp(\kappa_0 + \tilde{q}) \) it follows that \( P \) is monotonically increasing in the average expectation \( \tilde{q} \). As a result, agents can equivalently learn from the transformation \( \tilde{q} = \log P - \kappa_0 \).

**Proof of Lemma 3**

This follows directly from the assumption that traders do not make a correlated error when forming their expectation about \( \tilde{a} \). As a result, the average expectation of the payoff only depends on the average expectation of the productivity shock which in turn reveals the true value as \( \int_0^1 x_i \, di = \tilde{a} \).

**Proof of Proposition 2**

If households did not forecast capital investment, their weight on the private signal is given by:

\[
\alpha_1 = \frac{\pi_x}{\pi_a + \pi_x + \pi_p} \quad \text{(A.5)}
\]

This is larger than the corresponding \( \alpha_1 \) in the benchmark model. As price informativeness solely depends on this weight, the result immediately follows.

**Proof of Proposition 3**

This follows from using the expressions for \( V = e^{\tilde{a} + \tilde{g} + k} \) and \( C = e^{2k} \) together with the expressions for \( k \). I then plug in the equations for all endogenous variables excluding \( \alpha_1 \) and simplify.

**Proof of Proposition 4**

Taking the partial derivative of the social welfare function in Proposition 3 gives the result.

**Proof of Corollary 2**

This is a direct consequence of Proposition 2 together with Proposition 4.

**Proof of Proposition 5**

Here, the social planner collects all private signals about the productivity shock \( \tilde{a} \) and the growth opportunity \( \tilde{g} \) and announces these to all agents. As a result, the near-rational error can be backed out from the equilibrium stock price such that all agents can adjust their near-rational expectations accordingly. As a result, the CEO’s investment decision follows from the FOC without uncertainty. Plugging in the value for \( k \) yields the expres-
Proof of Corollary 3
In the first-best welfare increases in $\mu_g$ and $\pi_g^{-1}$. As both variables are positive by assumption, adding the growth opportunity increases households’ ex ante welfare.

Proof of Proposition 6
The expressions follow from maximizing social welfare by choosing $\{\beta_0, \beta_1, \beta_2\}$ and taking households’ trading behavior as given.

Proof of Lemma 4
As price informativeness only depends on $\{\alpha_1, \alpha_3\}$ and both of these variables are unaffected in the constrained first-best, the result follows.

Proof of Proposition 7
This immediately follows from comparing the expressions for $\beta_1$ and $\beta_2$ in the competitive equilibrium to those in the constrained efficient allocation.

Proof of Corollary 4
This follows from the expressions for $\alpha_1$ and $\alpha_2$ together with the results in 7. In particular, $\alpha_1$ is independent of the CEO’s choice for $\beta$ and $\alpha_2$ increases in $\beta_1$.

Proof of Corollary 5
This follows from the expression for $V_0[p]$ given in the text together with the result in 4 that $\alpha_2$ in the competitive equilibrium is higher than in the constrained efficient allocation.

Proof of Proposition 8
The first claim, $e = 0$, follows from the fact that here the CEO’s expected utility (weakly) declines in his effort choice. As a direct consequence, the growth opportunity is not started such that no aggregate noise ($\bar{\varepsilon}$) is reflected in the stock price which then perfectly reveals $\bar{a}$. The manager can then infer the productivity shock from the stock price by choosing $\beta_0$ and $\beta_1$ accordingly.

Proof of Corollary 6
This directly follows from the definition of investor welfare together with the expression for capital investment in 8.

Proof of Proposition 9
This expressions follows from using the results for investor welfare in both scenarios and simplifying the expressions.

Proof of Proposition 10
This follows from the expression for household i’s expectation of the log payoff after subtracting the tax from $v$ and matching coefficients to $E_i[v] = \alpha_0 + \alpha_1 x_i + \alpha_2 \hat{q}$.

**Proof of Corollary 7** As $\alpha_2$ is strictly decreasing in $\tau_1$ and $\beta_1$ in turn is decreasing in $\alpha_2$ it follows that a rise in $\tau_1$ decreases $\beta_1$. Together with the result that $\beta_1$ in the competitive equilibrium is inefficiently high, this implies that there always exists a positive value for $\tau_1$ that implements the constrained first best under stock-based compensation.