Arrested Development: Theory and Evidence of Supply-Side Speculation in the Housing Market*

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Abstract

This paper incorporates speculation into the standard supply-and-demand framework used to analyze housing booms and busts. Speculation reverses the common intuition that elastic housing supply attenuates housing booms. Housing market frictions make land a more attractive speculative investment than housing. As a result, undeveloped land both facilitates construction and intensifies the speculation that causes booms and busts in house prices. This insight explains the frequent housing booms and busts that coincide with high construction activity (e.g. Las Vegas, 2000-2010). These episodes are most likely to occur when a housing market nears but has not yet reached a long-run development constraint. Consistent with the recent U.S. housing experience, the model predicts higher price volatility in neighborhoods where housing is more easily rented. Land is an asset whose price volatility can increase with its float.

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1 Introduction

A major puzzle of the 2000-2006 U.S. housing boom is that several cities that experienced the largest house price increases were also cities in which housing supply was the most flexible. Figure 1 plots house price increases against construction activity for all major cities in the United States. Several cities that led the nation in construction also experienced the largest booms in house prices. Why did rapid construction fail to hold down house prices?

In this paper, we explain these puzzling episodes using models of disagreement and speculation developed in the finance literature. Real estate speculation has always been an integral part of booms and busts in house prices. Turbulent house prices usually coincide with uncertainty about future growth. Investors who disagree about this future growth trade with each other in the land and housing markets. It is exactly this type of trading behavior that leads to booms and busts in asset prices. Yet speculation is absent from the current models of housing supply and housing booms.\footnote{Current models of housing supply and housing booms include Saks (2008), Glaeser et al. (2008), Mian and Sufi (2009), and Charles et al. (2013). Kindleberger and Aliber (2005), Shiller (2005), and Glaeser (2013) provide historical discussions of the role of uncertainty in housing booms and busts. Examples of models of disagreement in the finance literature are Miller (1977), Harrison and Kreps (1978), Chen et al. (2002), and Scheinkman and Xiong (2003).}

The main idea in this paper is to show the strong link between speculation in housing and speculation in land, and what such speculation implies for house prices. Speculating in the housing market is difficult because investors must rent out the housing if they aim to turn a profit. Renting is fraught with costs, such as effort spent monitoring tenants to prevent property damage, tax losses, and difficulty renting properties like single-family homes that are designed for owners.\footnote{See Henderson and Ioannides (1983) on monitoring costs, Poterba (1984) on tax losses, and Sinai and Souleles (2005) and Glaeser and Gyourko (2007) on residents who prefer owning to renting.} In contrast, investors who buy land can simply hold it and hope for its price to rise. Land is a pure, frictionless bet on real estate.

This insight can reverse the conventional intuition about housing supply and house price booms. Land affects house price booms in two opposing ways: the classical channel and the speculative channel. Through the classical channel, undeveloped land facilitates new house construction; this construction response attenuates house price increases. Through the speculative channel, undeveloped land provides a market for speculation, which leads to a boom in land prices. Because land is a necessary input into house construction, the land price boom causes a boom in house prices.

We present a model of a housing boom in which both the speculative and the classical channels are present. We consider a city that faces a long-run growth barrier. Construction occurs as more and more people move to the city. Because of this growth, the city will
eventually run out of space for development. The price of undeveloped land reflects investors’ beliefs about the future flows of people to the city.

The long-run growth barrier comprises many factors—for example, geography, regulation, and transportation costs—that arrest development. For instance, Las Vegas faces a development boundary put in place by Congress in 1998. The Southern Nevada Public Land Management Act forbids the federal government from selling land to developers outside the boundary shown in Figure 2. During the 2000-2006 housing boom, many investors believed the city would soon run out of land. Land prices within this boundary rose from $150,000 per acre to $650,000 per acre from 2001 to 2006.

Consider a stylized depiction of a housing boom in a city with arrested development. A shock increases the current inflow of people to the city. This shock also creates uncertainty

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Notes: The figure includes all metro areas with populations over 500,000 in 2000 for which we have data. We measure each metro area’s housing stock from 2000-2006 using the Census 2000 housing stock estimate and the 2000-2005 Census housing permit data. We measure each metro area’s 2000 and 2006 house price using the FHFA housing price index deflated by the CPI-U. Both housing stock growth and house price return are reported in log points (1% = 1 log point). The highlighted metro areas are those in Arizona, California, Florida, and Nevada.

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3A New York Times article published in 2007 cited investors who believed the remaining land would be fully developed by 2017 (McKinley and Palmer, 2007).
about future inflows of people, and investors disagree about this future growth. As in Miller (1977), Chen et al. (2002), and Hong and Sraer (2012), short-sales constraints lead only the most optimistic investors to speculate in the land and housing markets. Because of rental frictions, investors suffer a loss when speculating directly in the housing market.

Our first result is that the classical channel dominates in cities that are either far from the constraint or already at it. When a city is far from the constraint, undeveloped land allows the city to accommodate the current influx of people with new construction. House prices remain flat because reaching the constraint is so far in the future. In cities at the constraint, new housing cannot be built, and therefore the shock raises house prices. However, the absence of undeveloped land also makes speculation difficult. Because of rental frictions, city residents own and occupy housing and speculative investors do not. As a result, ownership is dispersed and speculation does not raise house prices very much. In the universe of these two types of cities, the house price boom is small when housing supply is flexible and large when supply cannot adjust.

Our second and key result is that speculation leads to a high construction house price boom when the city nears but has not yet reached the long-run development barrier. When a city nears this constraint, undeveloped land allows the city to accommodate the current shock through new construction. At the same time, investors take to the land market to speculate about the price of housing in the near future when the city will have run out of land. Because the price of housing reflects the current cost of land, speculation about future house prices drives up prices *today*.

The speculative channel dominates in cities nearing the constraint. Speculation in land markets amplifies the boom in house prices relative to what the classical channel would predict. When disagreement is strong enough, the house price boom in these areas *exceeds* that in fully developed cities. This surprising result demonstrates the power of speculation to completely reverse the classical intuition about housing supply and housing booms.

Las Vegas during the 2000-2010 boom and bust provides a striking case of a high construction house price boom in an area approaching a development constraint. The ample raw land available in the short-run allowed Las Vegas to build more houses per capita than any other large city in the United States. At the same time, rampant speculation in the land markets caused land prices to quadruple between 2000 and 2006 and then lose those gains, leading to a boom and bust in the house prices. A single land development fund, Focus Property Group, outbid all other firms in every large parcel land auction between 2001 and 2005 conducted by the federal government in Las Vegas, obtaining a 5% stake in the undeveloped land within the barrier. Focus Property Group declared bankruptcy in 2009. As in our model, an optimistic investor crowded out pessimistic investors, thereby pushing
An alternative explanation for this and other high construction house price booms is that housing supply faces binding short-run constraints during these episodes. According to this hypothesis, homebuilders face shortages of inputs, such as drywall and labor, as they attempt to rapidly scale up housing production. We address this hypothesis by measuring both construction cost changes and land price changes at the city level during the 2000-2006 boom. Our theory predicts that land prices should account for the house price boom, whereas the short-run constraint hypothesis holds that house price increases can be traced to changes in construction costs. Construction costs simply did not rise very much during the boom. Furthermore, cities where house prices boomed the most saw construction cost increases on par with the rest of the United States. In contrast, land price changes were much larger, even larger than house prices changes. The cities with the largest house price increases experienced the largest increases in land prices.
We also find empirical support for our model of land speculation from the balance sheets of large public homebuilders. Our model predicts that optimists increase their land holdings during the boom, and then suffer capital losses on those holdings during the bust. Consistent with these predictions, large homebuilders tripled their land holdings between 2000 and 2006, an increase far in excess of their additional construction needs. Their market equity then fell 74%, with most of the losses accounted for by write-downs to their land portfolios. These firms were land investment funds with a side business of construction.

A new prediction of our model is that variation in rental frictions will predict house price booms within a city. For instance, speculators invest in condo units that can be rented out easily instead of in single-family housing that cannot. Similarly, speculators prefer neighborhoods that attract renters to neighborhoods that attract owner-occupants. These results are consistent with the stylized fact that house prices increased more from 2000-2006 in neighborhoods with a higher share of rental housing.

Our paper’s results depend on short-sales constraints in the land and housing markets. Short-sales constraints are especially relevant in this setting. Asset interchangeability in the stock market (e.g. all shares of IBM common stock are equivalent) fails to hold in the real estate market, where all land parcels and housing units are unique. Without asset interchangeability, it is essentially impossible to cover a short.

We contribute to the literature on speculation by providing an example where the price volatility of an asset increases with its float. In our setting, the asset is land, and additional land facilitates speculation which can amplify the boom and bust in land and house prices. The usual asset pricing logic, e.g. Hong et al. (2006), holds that greater asset float should lower price volatility by allowing pessimists to enter the market. The housing market is special because extra land creates a new market. When land is scarce, all land is used for housing, and the housing market resembles a standard goods market where prices are determined by supply and demand. However, when land is less scarce, undeveloped land remains, creating a new market in which investors can speculate on real estate. In this sense, the real estate market is an example of the phenomenon described by Simsek (2013), in which new markets increase the price volatility in existing markets by giving investors a new avenue to speculate.

The paper proceeds as follows. We document the importance of land speculation in the 2000-2010 American housing boom and bust in Section 2. In Section 3, we model the housing market environment. Section 4 contains our analysis of the housing boom and bust in this environment. We calibrate the model to the 2000-2006 American boom in Section 5. We discuss the implications of our framework for the rental market in Section 6 and various ways to extend the model in Section 7. Section 8 concludes.
2 Stylized Facts of the 2000-2010 Boom and Bust

In this section, we document stylized facts of the 2000-2010 boom and bust that motivate the theory we present in this paper. These facts fall into three categories. First, we show that there was wide variation across cities in the size of the housing boom, and that many of the high boom areas also experienced high construction activity. House prices dropped more during the bust in booming cities with high construction that in booming cities with low construction. Second, we show that land prices and not construction cost increases account for the house price booms during this period. This evidence supports our theory of land market speculation, and undermines theories in which shortages of non-land inputs caused house prices to rise. Finally, we document speculative behavior in the land markets among one class of investors for which we have data: large public homebuilders.

2.1 House Price Booms, Busts, and Construction Across Cities

We observe house prices and quantities at the metropolitan area level. Throughout this section, we focus on the 115 metropolitan areas for which the 2000 population exceeds 500,000. Following Mian et al. (2013), we use 2006 as the break point between the boom and the bust.

The first fact we draw attention to is the large variation across metropolitan areas in the size of the boom and bust. Figure 3 simply plots the bust against the boom. As is apparent from this figure, both the boom and the bust varied from about 0% to 80% (log points) across cities.

This large variation provides a natural setting to test various theories of housing booms against each other. Differences in the boom reflect differences in housing supply and housing demand across cities. As we explained in the Introduction, the dominant theme in the literature is that the cities with larger booms had less elastic housing supply.\(^4\)

The problem with this idea is that it predicts that cities with larger booms should have less construction activity. Figure 1 in the Introduction documents the failure of this prediction in the data. In that figure, we plot the same cities as in Figure 3, but we now plot the boom against construction levels. The correlation between these two series is actually positive at 0.07, and a regression of the boom on construction yields a positive and significant coefficient (0.35, standard error 0.07). Of course, this positive relationship may just reflect that variation in housing demand shocks across cities is more important than variation in

\(^4\)Following this argument, several papers have used Saiz (2010)’s geographic measure of housing supply elasticity to instrument for the size of the housing boom. Examples are Mian and Sufi (2009), Mian and Sufi (2011), Mian and Sufi (2012), Mian et al. (2013), Ganong and Shoag (2013), Charles et al. (2013), Chetty and Szeidl (2012), and Diamond (2012).
Notes: House price changes are measured in logs. The figure includes all metro areas with populations over 500,000 in 2000 for which we have data. We measure each MSA’s annual house prices using the second quarter FHFA housing price index deflated by the CPI-U.

housing supply. We are skeptical that differential demand shocks are more important than housing supply variation during the 2000-2006 period. The demand shock of subprime credit expansion was essentially national in nature (although it surely varied to some degree across metro areas), whereas there is significant variation in local housing supply conditions that has been extensively documented by the urban economics literature.5

The puzzle, then, consists of the high construction housing booms in the top-right of Figure 1. Explaining these types of booms is particularly important because the resulting busts are unusually severe. As shown in Figure 3, the size of the bust conditional on the boom varies widely. Consider Las Vegas and Honolulu, which are both visible in Figure 3. They both had the same boom at 60%, but prices then fell 80% in Las Vegas but only 15% in Honolulu. Las Vegas built more houses than any other city during the boom (5.0% annual growth), whereas Honolulu was one of the least active construction markets (0.9% annual

5See Gyourko (2009) for an overview.
Table 1: Construction Predicts the Bust Conditional on the Boom

<table>
<thead>
<tr>
<th></th>
<th>Bust (1)</th>
<th>Bust (2)</th>
<th>Bust (3)</th>
<th>Bust (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>0.67</td>
<td>0.66</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.19)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>Construction</td>
<td>3.9</td>
<td></td>
<td>9.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td></td>
<td>(2.0)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.04</td>
<td>-0.02</td>
<td>-0.29</td>
<td>-0.36</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.12)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td></td>
<td>60%</td>
<td>62%</td>
</tr>
<tr>
<td>Observations</td>
<td>114</td>
<td>112</td>
<td>45</td>
<td>44</td>
</tr>
<tr>
<td>Restrict to Boom &gt; 0.4</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The bust is the total log decrease in real house prices from 2006-2010. The boom us the total log increase in real house prices from 2000 to 2006. Construction equals the log annual growth in the housing stock between 2000 and 2006. We compute the housing stock using the Census 2000 figures and the 2000-2005 housing permits from the Census. Annual house prices are taken from the second quarter FHFA housing price index deflated by the CPI-U.

growth). More generally, construction predicts the bust conditional on the boom, as shown in Table 1. The predictive value of construction for the bust is highest among cities with large booms.

2.2 The Central Importance of Land Prices

In this paper, we argue that speculation in land markets is the missing link in explaining the full variation of booms and construction across cities. In this section, we demonstrate that land price movements, as opposed to construction cost changes, explain nearly all of the house price boom in an accounting sense.

The relative importance of land price changes and construction cost changes distinguishes between our theory and alternate theory of housing supply, which we call “time-to-build.” In this alternate formulation, homebuilders face shortages of inputs, such as drywall and labor, as they attempt to rapidly scale up housing production. This theory predicts that areas with the largest housing booms should be the areas where the local prices of these inputs rise the most.

Our land-based explanation, in contrast, is agnostic on such input prices. We predict instead that a significant portion of house price booms come through land price booms that are capitalizing the beliefs of optimistic investors.
To assess these competing hypotheses, we gather separate data on construction cost and land price increases at the metropolitan area level. Following the urban economics literature,\textsuperscript{6} we measure construction costs using the R.S. Means construction cost survey.\textsuperscript{7} This survey asks homebuilders in each metropolitan area to report the marginal cost of building a square foot of housing, inclusive of all labor and materials costs. The responses to this survey reflect real differences in construction costs across cities. In 2000, the lowest cost is $54 per square foot and the highest is $95; the mean is $67 per square foot and the standard deviation is $9.

The data we use on land prices are the land price indices developed by Nichols et al. (2010). These authors assemble land parcel transaction data and construct metropolitan-level indices as the coefficients on time-dummies in hedonic regressions that control for parcel characteristics.

To get at the relative importance of construction cost and land price increases, we calculate the real increase in each series between 2000 and 2006 and plot that against the corresponding increase in house prices for each city in Figure 4. This exercise is motivated by the following calculations. Competition among homebuilders means that house prices must equal land prices plus marginal construction costs: $p^h_t = p^l_t + C_t$. Log-differencing this equation between 2000 and 2006 yields

$$\Delta \log p^h = \alpha \Delta \log p^l + (1 - \alpha) \Delta \log C,$$

where $\Delta$ is the difference between 2000 and 2006, and $\alpha$ is land’s share of house prices in 2000. Whichever factor increase is more important should vary more closely with house prices increases across cities, and should rise more than 1-for-1 with respect to house price increases (because $\alpha$ and $1 - \alpha$ are less than 1).

Figure 4 provides clear evidence that land prices, not construction costs, are the important factor for understanding the rise in house prices from 2000 to 2006. Construction costs simply did not go up that much during this period, and construction cost increases display very little variation across cities relative to the variation in house price increases. Land price increases display the opposite pattern. We have purposefully drawn the $x$- and $y$-axes to be proportional, so that is is clear that land prices increase more than house prices. Furthermore, land price increases are highly correlated with house price increases. The land market, not short-run limitations on construction, is central for understanding the recent housing boom.

\textsuperscript{6}See Gyourko and Saiz (2006) for an early example.

\textsuperscript{7}We use the reported figures on the marginal cost of an “average quality” home.
Figure 4: Land Prices, not Construction Costs, Explain House Price Boom Variation

Notes: We measure construction costs for each city using the R.S. Means survey figures for the marginal cost of a square foot of an average quality home, deflated by the CPI-U. Land prices changes come from the hedonic indices calculated in Nichols et al. (2010) using land parcel transactions, and house prices come from the second quarter FHFA housing price index deflated by the CPI-U. All changes are in logs.

2.3 Land Market Speculation by Homebuilders

We now argue that the land price increases just documented were driven by speculation in the land markets. By “speculation,” we mean the process described by Miller (1977) and Chen et al. (2002) in which optimists buy up an asset that cannot be shorted. Chen et al. (2002) describe two aspects of this behavior: (1) the optimists who hold the asset increase their positions, as they crowd out the pessimists, and (2) the optimists then suffer capital
losses when their beliefs are revealed to be more optimistic than reality.

We document both of these features among a class of landholders for which we have rich data: public homebuilders. These firms report their land holdings and losses on these land holdings on their annual financial statements (10-Ks). We focus on the eight largest firms over this period.8

Figure 5: Homebuilder Land Holdings and Home Sales, 2001-2010

![Graph showing homebuilder land holdings and home sales from 2001 to 2010.]

Notes: “Lots Held” equals the sum of lots directly owned and those controlled by option contracts. Data come from the 10-K filings of Centex, Pulte, Lennar, D.R. Horton, K.B. Homes, Toll Brothers, Hovnanian, and Southern Pacific.

Consistent with speculative behavior, these firms nearly tripled their land holdings between 2001 and 2005, as shown in Figure 5. These additional land holdings do not appear to reflect increasing construction needs. As shown in Figure 5, annual home sales increased by 120,000 between 2001 and 2005, while land holdings increased by 1,100,000 lots. One lot can produce one house, so the increase in land holdings is nine times larger than the increase in annual home sales. In 2005, Pulte changed the description of its business in its 10-K to include the statement “We consider land acquisition one of our core competencies.” This language appeared until 2008, when it was replaced by “Homebuilding operations represent

8The firms we examine are Centex, Pulte, Lennar, D.R. Horton, K.B. Homes, Toll Brothers, Hovnanian, and Southern Pacific.
our core business.”

Having amassed these large land portfolios, the firms we study then proceeded to suffer large capital losses. Figure 6 documents the quite dramatic rise and fall in the total market equity of these homebuilders between 2001 and 2010. The homebuilder stocks rose 430% and then fell 74% over this period. These large changes are specific to these firms and not the stock market generally. The S&P 500 index fell -2.5% from 2001-2005 and then fell -9.6% from 2005 to 2010.

The majority of the losses from 2005 to 2010 borne by these homebuilders arise from losses on the land portfolios they built up from 2001 to 2005. Starting in 2006, these firms report write-downs to their land portfolios in their annual financial reports. We aggregate these losses across firms, and report the total annual write-downs and the total annual market equity losses in Table 2. The value of the land losses between 2006 and 2010, $29 billion, is 73% of the size of the market equity losses over this time period.

Figure 6: Homebuilder Market Equity, 2001-2010

Notes: Data come from the 10-K filings of Centex, Pulte, Lenmar, D.R. Horton, K.B. Homes, Toll Brothers, Hovnanian, and Southern Pacific.
Table 2: Homebuilder Equity Losses and Land Impairments, 2006-2010

<table>
<thead>
<tr>
<th></th>
<th>Market Equity Loss ($Billions)</th>
<th>Land Impairments ($Billions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>2007</td>
<td>23</td>
<td>13</td>
</tr>
<tr>
<td>2008</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>2009</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2010</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>29</td>
</tr>
</tbody>
</table>

Notes: Data come from the 10-K filings of Centex, Pulte, Lennar, D.R. Horton, K.B. Homes, Toll Brothers, Hovnanian, and Southern Pacific. Market equity loss is defined as the dollar change in market equity from the previous year.

3 Housing Market Environment

We now write down a general model of the housing market in which house price booms are characterized by the stylized facts of Section 2. This section shows that the availability of undeveloped land in a city makes housing supply elastic, but also facilitates real estate speculation. We demonstrate these results in a general model in which beliefs about prices are exogenous. In Section 4, we use this general framework to analyze a demand shock that causes uncertainty about future growth.

3.1 Housing Demand

There is a population of consumers that includes both city residents and investors in the city’s real estate. Each consumer $i$ maximizes her expected present value of utility, given by

$$E_i \sum_{t=0}^{\infty} \beta^t u_{i,t}(h_t, c_t),$$

(2)

where $\beta$ is the common discount factor, $h$ is housing consumed in the city, and $c$ is non-housing consumption.

Flow utility $u_{i,t}$ takes one of two forms. In one case, the consumer $i$ receives a “residency
shock" that compels her to live in the city at time $t$. In this case her utility is

$$u_{i,t}(h, c) = \begin{cases} v(h) + a_i h + c & \text{if owning } h \\ v(h) + c & \text{if renting } h, \end{cases}$$

(3)

where $v$ is an increasing, concave function for which $v'(0) = \infty$, and $a_i$ is drawn from some atomless distribution $F_a$ whose support includes 0. If consumer $i$ does not receive this shock, then she is an “outsider” who derives no benefit from living in the city, and her utility is given by

$$u_{i,t} = c.$$  

(4)

The function $v(\cdot)$ represents intensive housing demand. In this context, owner-occupied and rental housing are perfect substitutes. The parameter $a_i$ is a reduced form that captures reasons some residents prefer owning to renting, or vice-versa. As we discussed in the Introduction, explanations for this preference offered by the literature have relied on maintenance costs, tax advantages, risk management, moving costs, and liquidity. We show in Section 7 that a moral hazard formulation, similar to Henderson and Ioannides (1983), in which maintenance is not fully contractible can exactly generate the linear functional form used here. Residents derive more flow utility from owner-occupancy when $a_i > 0$, and more flow utility from renting when $a_i < 0$.

There are three assets consumers may hold: housing $H$, land $L$, and bonds $b$. Housing and land cannot be shorted, and can only be held in non-negative quantities. A consumer’s housing asset holding $H$ need not equal her housing consumption level $h$. If $H > h$, then the consumer is a landlord: she owns more housing than she consumes. If $H < h$, then the consumer is a renter because she owns less housing than she consumes. If $H = h$, then the consumer is an owner-occupant. In all three cases, the net rental income is $r_t(H - h)$, where $r_t$ is the market rent for housing at time $t$.

Bonds are traded on a global market, and the gross interest rate $R_t$ on bonds equals $1/\beta$.

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9The extensive margin of living in the city is exogenous. This extensive margin can be added in and the results stay the same. The difference is that the demand curve for housing that comes out of the model has a broader interpretation that includes both the intensive and extensive margin. What matters for our results is that this demand curve is smooth. The smoothness comes here from the continuity of housing, but it could also come from an extensive margin in which city amenities for each resident are drawn from a continuous distribution.

10Shorting is impossible because of a lack of asset interchangeability in the real estate market that we do not model explicitly here. As explained in the Introduction, short sales are possible only when there exist comparable assets with which to cover one’s short. Land parcels and houses are all unique. In Section 7, we discuss the robustness of our results to the introduction of additional real estate securities, such as the housing futures proposed by Shiller (1998), that allow investors to short the housing market indirectly.

11The resident may be an owner-occupant in addition to a landlord in the case $H > h$. The resident rents out $H - h$ units of housing, and owner-occupied the remaining $h$ units.
for all $t$, where $\beta$ is the common discount factor. Residents and outsiders can borrow or lend in unlimited quantities (in the background is some income stream that guarantees that these loans will always be repaid).\footnote{The lack of credit constraints will mean that in markets where investors participate, the most optimistic investor will always hold all of the asset. In Section 7, we introduce a borrowing constraint to derive additional results about the effect of credit on the housing boom and bust.}

Thus, consumers have five choices to make at time $t$. They choose their asset holdings $H_t$ and $L_t$ of housing and land. They also choose their consumption levels of housing $h_t$, as well as non-housing consumption $c_t$. Finally, they choose bond holdings $b_t$. The resulting Bellman equation corresponding to equation (2) is

\[
V(H_{t-1}, L_{t-1}, b_{t-1}) = \max_{h_t, c_t, H_t, L_t, b_t} u_{t,t}(h_t, c_t) + \beta \mathbb{E}_t V(H_t, L_t, b_t),
\]

where the maximization is subject to the short-sale constraint

\[
0 \leq H_t, L_t
\]

and the budget constraint

\[
p^h_t(H_t - H_{t-1}) + p^l_t(L_t - L_{t-1}) + c_t \leq r_t(H_t - h_t) + b_t - R_t b_{t-1},
\]

where $p^h_t$ and $p^l_t$ denote the prices of housing and land, respectively.

One feature of this set-up is that both residents and outsiders can invest in the land and housing markets. This feature captures the behavior of many homeowners who purchased second homes for investment purposes during the boom. Haughwout et al. (2011) examine detailed credit data and conclude that 50% of the mortgages originated between 2004-2006 in Arizona, California, Florida, and Nevada were to investors purchasing second homes; Bayer et al. (2013) construct a similar measure and find that investment purchases comprised 30% of home buying in Los Angeles. Of course, many of the investors in land markets were not individual homeowners but large firms, as we documented in Section 2. Shareholders of such firms are the outsiders in our model.

The number of consumers who receive the residency shock at time $t$ is given by $N_t$.\footnote{We only allow an intensive margin of housing choice. As house prices rise, residents buy smaller houses. In principle we could model an extensive margin where higher prices induce residents to move out of the city. In each case we would get an aggregate demand curve that would be isomorphic and would not change our results.}
This aggregate housing demand evolves according to a stochastic process

\[
\log N_t = \log N_{t-1} + \zeta_t.
\]  

(8)

Residents (may) hold heterogeneous beliefs about future demand innovations \(\zeta_t\). Specifically, the expectation \(E_i(\zeta'_{t'} | \zeta_t, \zeta_{t-1}, ... )\) may vary across residents \(i\) for \(t' > t\). We will assume a specific functional form for these innovations when we introduce the demand shock in Section 4, and will explain why heterogeneous beliefs are reasonable in the setting of that shock.

It is not necessary to specify the process through which individual consumers receive the residency shocks. For example, it is not necessary to model any serial correlation in being a resident. Consumers have quasi-linear utility in all states of the world, so their marginal utility is always the same. Therefore, a consumer solving equation (5) does not care about whether she is a resident or an outsider in future periods. The future prices of the assets she holds are sufficient statistics for her welfare.\(^{14}\)

### 3.2 Housing Supply

A unit of housing can be produced with one unit of land, plus a construction cost of \(C\) (measured in terms of a composite commodity \(c\) with a price normalized to 1). A Leontief production function captures this cost structure and is given by

\[
H = \min(L, c/C).
\]  

(9)

Housing construction is irreversible, and the housing stock does not depreciate. Construction is performed by a perfectly competitive industry of homebuilders.\(^{15}\)

The only constraint on housing supply is a long-run constraint on the total amount of land in the city, which we denote \(\bar{L}\). This development constraint reflects a variety of barriers to unlimited growth that cities or neighborhoods face. Saiz (2010) documents how geography (e.g. mountains and water) limits the growth of metropolitan areas. Often development restrictions are man-made, taking the form of zoning and “growth management” policies that manage the long-term size of a city or neighborhood.\(^{16}\) A final source of this constraint,

\(^{14}\)Quasi-linear utility eliminates risk aversion and any hedging motives for owning versus renting housing. Household risk management is clearly important. Piazzesi et al. (2007) provide an asset pricing model in which housing and other consumption are non-separable and individuals are risk-averse, and Sinai and Souleles (2005) show that owning housing acts as a hedge against future rent risk. Abstracting from these considerations allows us to focus on the precise role that flow utility plays in aggregating resident and investor beliefs.

\(^{15}\)The largest 100 builders had less than 45% market share in each year from 2001-2006, according to Martin and Whitlow (2012), which draws data from Builder Magazine.

\(^{16}\)These regulations also restrict the amount of housing that can fit on a parcel of land. Therefore, it most
which we do not explicitly model, is transportation costs that establish a commuting radius around a city center. Even with no geographic or regulatory constraints, the scope of a city will be limited if all residents have to commute to the same central location each day, and commuting takes time that scales linearly with distance.

One feature of this approach is that the short-run supply elasticity will be endogenous to the model, depending on the city’s level of development relative to the long-run constraint. The literature on housing supply (Topel and Rosen, 1988; Saiz, 2010; Glaeser et al., 2013) largely assumes an exogenous static supply elasticity, which may vary across cities. Our set-up generalizes this formulation by jointly modeling the short-run supply elasticity and land prices.

Another feature of our production function is constant construction cost $C$. The constant construction cost setting allows us to clearly explain how a housing boom can occur in an area with no short-term barriers to construction. Furthermore, as discussed in Section 2 and shown in Figure 4, constant construction costs largely corresponds to the cross-city housing boom experience in the latest U.S. housing episode.

As we explain below, short-run housing supply will be perfectly elastic when land remains for construction, and perfectly inelastic when all land has already been used for housing. In reality, even very dense areas experience some housing construction, and cities do not face such a discontinuous transition from having perfectly elastic to perfectly inelastic housing supply. A simple extension to our model, which we discuss in Section 7, addresses both points. We allow land to have a productive use other than house construction which enters the resident utility function. As long as an Inada condition is satisfied, then these unrealistic discontinuities disappear, but our main results are preserved.

3.3 Equilibrium

Equilibrium holds when consumers are optimizing, the construction industry is operating at cost, and housing and land markets clear:\footnote{The equilibrium condition we provide for builder optimization is that house prices equal land prices plus construction costs. This condition holds as long as housing demand does not fall too much relative to the existing housing stock, creating an “over-building” scenario. As our attention is on the boom and not the bust, we choose not to focus on this case. Our results hold as long as optimistic investors who price the assets believe that the city will not be in this over-building scenario in the subsequent period.}

Definition 1. Let $H^*_i(p^h_t, p^l_t, r_t)$, $L^*_i(p^h_t, p^l_t, r_t)$, and $h^*_i(p^h_t, p^l_t, r_t)$ denote the solutions to the Bellman equation (5). Then prices $p^h_t$, $p^l_t$, and $r_t$ constitute an equilibrium if the following...
market-clearing conditions hold:

housing market: \[ \sum_i H^* = \sum_i h^*_i; \quad (10) \]

land market: \[ L = \sum_i H^* + \sum_i L^*; \quad (11) \]

and if the competitive construction industry makes zero profits:

\[ p^h_t = p^l_t + C. \quad (12) \]

We denote the housing stock at time \( t \) by \( H_t \):

\[ H_t \equiv \sum_i H^*_i. \quad (13) \]

There are three broad points about the equilibrium we would like to make in this general setting. The first is that house prices can be written as the sum of current and future rents. We can specifically write

\[ p^h_t = r_t + \beta \tilde{E} p^h_{t+1}, \quad (14) \]

where \( \tilde{E} \) represents the beliefs of residents and outsiders who are landlords. This equation results from landlord arbitrage: they must be indifferent between selling their houses and renting them out.

The second point is that the equilibrium falls into two cases: when demand is high, all land is used for housing, and when demand is low, some land remains undeveloped. This dichotomy will be quite important for analyzing the model, so it is worth spending some space building up the intuition with equations. The basic idea is that when housing demand is high, it is inefficient to let any land sit idle, so investors build houses on the land and sell them to residents. To see this, consider the first-order condition for resident housing consumption:

\[ 0 = \max(\nu'(h^* - r_t, \nu'(h^* - (p^h_t - \beta \tilde{E} p^h_{t+1} - a_t)) \quad (15) \]

If the first argument is larger, the resident rents, and if the second argument is larger, the residents owns. We let \( D \equiv (\nu')^{-1} \) denote each resident’s demand curve for housing. We can
substitute (14) into (15) to derive the market-clearing condition

\[
H_t = \sum_{a_i < \beta(\tilde{E}p_{t+1}^h - E_i p_t^h)} N_t D(r_t) + \sum_{a_i > \beta(\tilde{E}p_{t+1}^h - E_i p_t^h)} N_t D \left( r_t - a_i + \beta(\tilde{E}p_{t+1}^h - E_i p_{t+1}^h) \right),
\]

(16)

where \( H_t \) is the stock of housing. When this total demanded housing stock \( H_t \) is less than the total land supply \( L \), some raw land remains. The total stock demanded depends on aggregate demand \( N_t \) and market rents \( r_t \). Market rents, however, are pinned down by construction costs when land remains. If land remains, then a landowner must be indifferent between building a house and selling today or doing the same tomorrow. Therefore, \( p_t^h - C = \beta(\tilde{E}p_{t+1}^h - C) \); together with equation (14), this arbitrage condition means that

\[
r_t = (1 - \beta)C
\]

(17)

when raw land remains. The quantity \((1 - \beta)C\) equals the “flow costs” of construction, i.e. the costs of building today as opposed to next period.

The level of demand \( N_t^* \) at which all land is used for housing is the level at which housing demand given by (16) equals the total land \( L \) when rents equal \( r_t = (1 - \beta)C \). When \( N_t \) is below this threshold, the amount of housing demanded when it is supplied at cost is not enough to fill up the city. When \( N_t \) is above this threshold, the amount of housing demanded when it is supplied at cost exceeds the space available in the city. Lemma 1 summarizes this result.

**Lemma 1** (Land Exhaustion). There exists a level of aggregate demand \( N_t^* \) such that when demand \( N_t \) exceeds this threshold, all land is used for housing: \( H_t = L \). When demand \( N_t \) falls short of this threshold, some land remains undeveloped: \( H_t < L \).

The third point and final point is that pass-through of demand shocks to prices is higher when demand is higher, but the pass-through of investor beliefs to prices is higher when demand is lower. Speculation has the largest impact on prices precisely when the elasticity of housing supply is high.

Let us turn first to the direct effect of a demand shock on prices and housing quantities. Recall that an innovation to log housing demand \( \log N_t \) is given by \( \zeta_t \), as specified by equation (8). In theory this innovation can affect future price expectations because it may convey information about future shocks. Here we will consider just the direct effects of the shock \( \zeta_t \) holding future price expectations constant.

The effect of the shock \( \zeta_t \) on house prices and quantities is actually quite simple in our
model. We can most easily describe the effect of the shock by examining the city in the two separate cases of full development \((N_t > N_t^*)\) and continuing development \((N_t < N_t^*)\).

By assumption, the shock we are considering does not affect future price expectations, and therefore affects prices only through current rents \(r_t\) (see equation (14)). When \(N_t < N_t^*\), these rents are fixed at the flow construction cost \(r_t = (1 - \beta)C\) (equation (17)). Therefore, the shock \(\zeta_t\) has no affect on rents when the city is undeveloped. Furthermore, the shock passes through to quantities 1-for-1 in this undeveloped case. The housing demand equation (16) may be written \(\mathcal{H}_t = N_t D_t(r_t)\), where \(D_t\) aggregates individual housing demands \(D_t\).

Because \(r_t\) is fixed, innovations in \(N_t\) pass through perfectly to the housing stock \(\mathcal{H}_t\).

The effect of the shock is completely the opposite when the city is fully developed \((N_t > N_t^*)\). Now, the housing stock is fixed at \(\mathcal{H}_t = \mathcal{L}\), the amount of space available. The shock therefore has no effect on housing supply. In contrast, rents are no longer fixed, and are determined by the market-clearing equation (16) with the housing stock fixed at \(\mathcal{L}\). We may write this condition as \(\mathcal{L} = N_t D_t(r_t)\). Differentiating this equation with respect to a log change in \(N_t\) yields \(\partial \log r_t / \partial \zeta_t = 1 / \epsilon_t\), where \(\epsilon_t\) is the elasticity of demand \(D_t\) given by \(\epsilon_t = -r_t D'_t(r_t) / D(r_t)\).

Proposition 1 summarizes these results.

**Proposition 1 (Supply-and-Demand Effect of Land).** Consider a shock \(\zeta_t\) to demand log \(N_t\) that does not affect beliefs about future shocks. The shock raises house prices if and only if all land is being used for housing. The shock leads to new construction if and only if some of the land is still undeveloped. The shock affects house prices only through rents, and its affects on house quantity \(\mathcal{H}_t\) and rents \(r_t\) are given by

\[
\frac{\partial \log \mathcal{H}_t}{\partial \zeta_t} = \begin{cases} 
1 & \text{if } N_t < N_t^* \\
0 & \text{if } N_t \geq N_t^* 
\end{cases}
\tag{18}
\]

\[
\frac{\partial \log r_t}{\partial \zeta_t} = \begin{cases} 
0 & \text{if } N_t < N_t^* \\
1/\epsilon_t & \text{if } N_t \geq N_t^* 
\end{cases}
\tag{19}
\]

where \(N_t\) is the level of housing demand, and \(N_t^*\) is the threshold at which the city fully develops given by Lemma 1.

Proposition 1 states the classic supply-and-demand theory of a housing boom. When land is still available, housing supply is completely elastic (pass-through is unity), and the shock has no affect on house prices. When land is no longer available, housing supply is completely inelastic (pass-through is 0), and the shock raises rents according to a demand elasticity. Land availability means that housing supply is more elastic and demand shocks have smaller
effects on prices.

Land availability has exactly the opposite effect on the pass-through of investor beliefs to house prices. As we now show, land availability facilitates speculation, allowing speculators to raise prices more. Again, the analysis proceeds most logically if we separately consider the case where land is available and the case where all land is used for housing.

When all land is used for housing, landlords face frictions from trying to rent out “too much” of the housing stock. There is a natural market for renting, namely the residents for whom ownership utility \( a_i \) is negative. If landlords buy up more housing than they can rent to these residents, they exert downward pressure on rents, cutting into their returns.

To show how this friction emerges from the model, we differentiate (16) with respect to the discounted investor belief \( \beta \tilde{E}_{p_{t+1}} \), keeping in mind that \( \bar{H}_t \) is fixed. What we obtain is

\[
\frac{\partial r_t}{\partial \beta \tilde{E}_{p_{t+1}}} = -\frac{\sum_{a_i < \beta(\tilde{E}_{p_{t+1}} - E_{p_{t+1}})} D_i'}{\sum a_i D_i'},
\]

where \( D_i'(r_t) \equiv D(r_t) \) if \( a_i < \beta(\tilde{E}_{p_{t+1}} - E_{p_{t+1}}) \) and \( D_i'(r_t) \equiv D(r_t - a_i + \beta(\tilde{E}_{p_{t+1}} - E_{p_{t+1}})) \) otherwise. Equation (20) can be given an intuitive interpretation. It is, on the margin, the fraction of the housing market that is owner-occupants. When speculative investors buy up housing, they push up prices, leading to expected capital losses for owner-occupants. On the margin, the rental and owner-occupied populations are distinct (substitution effects between groups are second-order). Therefore, for the housing market to clear, rents must fall to offset the lower demand from owner-occupants coming from expected capital losses.

We may substitute (20) into (14) to derive the total effect of speculators on house prices when the housing stock is fixed:

\[
\frac{\partial p^h_t}{\partial \beta \tilde{E}_{p_{t+1}}^h} = \frac{\sum_{a_i < \beta(\tilde{E}_{p_{t+1}} - E_{p_{t+1}})} D_i'}{\sum a_i D_i'},
\]

\[
\equiv \chi.
\]

The price effect of speculators is between 0 and 1. If all of the market is rental housing, then their beliefs pass through 1-for-1 to prices. If all of the housing is owner-occupied, they have no effect on house prices.

In contrast, investor beliefs pass through 1-for-1 to house prices when land is still available. To see this, we need only combine equations (14) and (17). Rents are fixed when land
is still available. Therefore, investor beliefs pass-through completely to house prices.

Intuitively, investor beliefs determine land prices without any frictions, because they do not need to rent out land to capture its value. But house prices are determined by land prices: to build a house, a homebuilder must buy land from one of these optimistic investors. This argument only holds when investors are holding land in equilibrium. If demand is high \((N_t > N^*_t)\), it is always more efficient for investors to build on their land than to hold it, and investor no longer price the land as they do when demand is low.

Proposition 2 summarizes these results.

**Proposition 2** (Speculation Effect of Land). When rental frictions are present \((\chi < 1)\), speculative investors affect house prices more when undeveloped land remains than when the city is fully developed. The pass-through of investor beliefs to house prices is given by

\[
\frac{\partial p^h_t}{\partial \beta \tilde{E}p^h_{t+1}} = \begin{cases} 
1 & \text{if } N_t < N^*_t \\
\chi & \text{if } N_t \geq N^*_t
\end{cases}
\]  

(22)

where \(\chi \in (0, 1)\) is the ratio of rental market depth to total market depth given by equation (21), \(N_t\) is the level of housing demand, and \(N^*_t\) is the threshold at which the city fully develops given by Lemma 1.

Proposition 2 shows that the effect of speculation on prices is perfectly negatively correlated with the direct effect of a demand shock on prices given by Proposition 1. Speculation has the largest effect on prices when the city is undeveloped. The direct effect of a demand shock on prices is smallest when the city is undeveloped, and in that case the supply response is the largest.

One shortcoming of our approach so far is that we have taken investor beliefs \(\tilde{E}p^h_{t+1}\) to be exogenous, in particular not to depend on the level of development \(N_t\). This level of generality has allowed us to derive broad results about the relationship between housing supply and speculation in real estate markets. In order to close the system, we turn now to a model of a housing boom and bust in which specify beliefs more carefully.

## 4 Housing Boom

### 4.1 Demand Shock and Beliefs

Our goal is to consider the types of housing booms described by Glaeser (2013). In his account, the hallmarks of a housing boom and bust are an initial period of surging housing demand that coincides with uncertainty about future housing demand, followed by a
subsequent period in which that uncertainty is resolved. To match this framework, we embed a simple two-period model of uncertainty the dynamic model of the housing market we developed in Section 3.

The period of the initial shock is $t = 0$. Before this period, all residents and outsiders agree about the future housing market. At time 0, log housing demand rises unexpected by $x$:

$$\log N_0 = x + E\log N_0,$$

where $E$ denotes the common, time $t = -1$ expectation of residents. The arrival of this shock creates new uncertainty about future housing demand. In particular, consumers disagree about the shock’s persistence, that is, whether the current shock $x$ indicates that future housing demand will also rise. Resident $i$’s perceived persistence is $\mu_i$, so that

$$E_i \log N_t = \mu_i x + E \log N_t,$$

where $E$ again denotes the common, time $t = -1$ expectation of residents. At time $t = 1$, residents learn the true value of this persistence $\mu$. The shock $x$ and the resultant uncertainty are unanticipated by residents at time $t = -1$.

As in several recent finance papers,\textsuperscript{18} the actors in our model “agree to disagree,” in this case about $\mu$. They form their beliefs solely on the basis of subjective priors, and draw no inferences from the actions of other residents. As argued by Morris (1996), this heterogeneous prior assumption is most appropriate when investors face an unprecedented situation in which they have not yet had a chance to collect information and engage in rational updating. The events surrounding housing booms are precisely these types of situations. Glaeser (2013) takes great pains to show that in each of the historical boom episodes he analyzes, reasonable investors could agree to disagree about future real estate prices. In the case of the 2000-2010 housing boom and bust, we follow Mian and Sufi (2009) in thinking of the shock $x$ as the arrival of new securitization technologies that expanded credit to low-income borrowers. The $\mu$ in our model reflects the degree to which this expansion of credit in 2000-2006 persisted after 2006.

Because disagreement about $\mu$ is resolved after a single period of uncertainty, residents have no incentive to forecast the beliefs of other residents. The “resale option” effect of Harrison and Kreps (1978) and Scheinkman and Xiong (2003) in which residents buy an asset to sell it to a future optimist is absent here.\textsuperscript{19}

\textsuperscript{18}See Geanakoplos (2009), Simsek (2013), and Brunnermeier et al. (2013).

\textsuperscript{19}Our set-up also differs from Scheinkman and Xiong (2003) in the sense that residents do not infer any information from other residents, whereas investors in Scheinkman and Xiong (2003) only update partially from the actions of others. Our residents are like the investors in that paper if they had infinite overconfidence.
4.2 Future Price Expectations

The shock parametrization we have just written down lets us solve for the future price expectations $E_i p^h_t$ that we treated as exogenous in Section 3. Because we are interested in the effect of the shock $x$ on house prices, it is sufficient to calculate the derivative $\partial E_i p^h_t / \partial x$, as opposed to computing the level of this expectation.

A helpful decomposition is to write $p^h_t$ as the sum of rents:

$$p^h_t = \sum_{j=0}^{\infty} \beta^j r_{j+1}. \tag{25}$$

Consumer $i$ believes that the shock will raise log housing demand $\log N_t$ by $\mu_i x$ in all future periods. By Proposition 1, this future shock will have no effect on rents in the future periods in which the city is undeveloped, and will raise log rents by $1/\epsilon_t$ in future periods where the city is developed. The development cutoff in all future periods $N_t^*$ is the same because all residents have the same beliefs in those periods; we denote it $N^*$. Therefore, the future shock raises rents only in periods in which $N_t > N^*$. The total effect of the shock on future price expectations is the fraction of the city’s future that is constrained times the average elasticity in those periods. Lemma 2 states this result precisely.

Lemma 2 (Long-Run Constraint Measure). The effect of the shock $x$ on resident $i$’s future price expectation is

$$\frac{\partial \log E_i p^h_t}{\partial x} = \frac{\rho \mu_i}{\epsilon^c_0}. \tag{26}$$

Here $\rho \in [0, 1]$ is the fraction of future rents that accrue when the city is constrained:

$$\rho \equiv \frac{\sum_{j \geq J} \beta^j r_j}{\sum_{j \geq 1} \beta^j r_j}, \tag{27}$$

where $J = \min(j \mid N_j > N^*)$ is the number of periods until the city runs out of land. The term $\epsilon^c_0$ is the average elasticity of housing demand in the city’s constrained future:

$$\epsilon^c_0 \equiv \left( \frac{\sum_{j \geq J} \beta^j r_j / \epsilon_j}{\sum_{j \geq J} \beta^j r_j} \right)^{-1}. \tag{28}$$

In equation (16), the term $\tilde{E}_i p^h_{t+1} - \tilde{E}_i p^h_t = 0$ for all $i$ when residents hold the same beliefs. Therefore, the solution $N^*$ to the equation $L = N_t D((1 - \beta)C)$ does not depend on $t$.

This formulation assumes that once the city runs out of land, it never can tear down houses and re-enter the state in which land remains. This result holds if house construction is irreversible or if demand $N_t$ is always increasing for $t \geq 1$. 

20

21
Lemma 2 shows how the long-run constraints play a role in future price expectations. The parameter $\rho$ describes how supply-restricted the city is in the long-run, on a scale from 0 to 1. At $\rho = 0$, the city has enough land to develop for decades (in fact for eternity), and so whether the shock $x$ persists is irrelevant for future prices. In this case, the speculation goes away, even though there is ample land which investors can use for trading. The reason is that future prices are completely certain when the city will never run out of land; house prices equal construction costs $C$ for the foreseeable future.

Larger values of $\rho$ that are less than 1 correspond to cases in which the city is approaching its long-run development constraint. An example would be Las Vegas as depicted in Figure 2 in the Introduction. Half of the available land had been developed in 2008, and at its recent growth rate, Las Vegas would use up the remaining land in the next 10-20 years. A back-of-the-envelope calculation suggests that a reasonable value for $\rho$ in this setting is 0.5. The larger $\rho$, the greater the scope for disagreement about future house prices.

Finally, $\rho = 1$ when a city is already fully developed.

4.3 Impact of Shock on Prices

We now put together the general results of Section 3 with the characterization of future price expectations of Lemma 2 to derive the effect of the shock on house prices at time 0.

The shock has two effects on prices. First, it affects current rents through the supply- and-demand channel explained in Proposition 1. The shock raises log demand by $x$ at time 0; this demand shock raises rents today if the city is fully developed and does not raise rents if the city still has raw land.

The shock also affects prices by changing expectations $\mathbb{E}_t p_{t+1}^h$ of future prices, which are embedded in today’s price $p_0^h$. Proposition 2 shows that when the city is undeveloped, investor beliefs solely determine the future price expectations capitalized into prices at time 0. Following the notation used in Section 3, we denote the investor belief about $\mu$ by $\tilde{\mu}$.

When the city is fully developed, investors face frictions in the housing market, and their beliefs no longer fully determine prices. Proposition 2 shows that their beliefs account only for a fraction $\chi$ of the beliefs capitalized into prices at time 0. The remaining $1 - \chi$ comprise the beliefs of the owner-occupants who are also holding housing. We have not formally proved this result yet, but it follows easily from the general market-clearing condition for housing given in equation 16 in Section 3. We take the derivative of this equation with respect to each owner-occupant’s belief $\mathbb{E}_t p_{t+1}^h$ and then aggregate these effects appropriately over all
owner-occupants. What we find is that
\[
\frac{\partial r_t}{\partial \beta \mathbb{E}_{P_{t+1}^h}} = 1 - \chi,
\] (29)
where \(\chi\) is given by equation (21) and \(\mathbb{E}_{P_{t+1}^h}\) is the average owner-occupant belief given by
\[
\mathbb{E}_{P_{t+1}^h} \equiv \sum_{a_i > \beta} \frac{\rho_{\tilde{\mu} \epsilon_0}}{\epsilon_0} \left\{ \sum_{a_i > \beta} \frac{\rho_{\tilde{\mu} \epsilon_0}}{\epsilon_0} \left( \frac{\mathbb{E}_{P_{t+1}^h} - E_{i_{t+1}}}{D_i'} \right) \right\}.
\] (30)
We let \(\pi\) denote the average owner-occupant belief about \(\mu\) that comes from substituting the expression for \(E_{i_{t+1}}\) from Lemma 2 into equation (30).

Putting together the speculation effect and the supply-and-demand effect we have just discussed yields the total effect of the shock \(x\) on house prices \(p_0^h\). Following the Lemmas and Propositions up to this point, we express this effect as a change in log prices \(\log p_0^h\). Doing so creates one new log-linearization parameter, \(\lambda\), which expresses the share of house prices attributable to current rents instead of to future price expectations.\(^{22}\)

**Proposition 3** (Price Impact of Shock). Consider a shock that raises current housing demand \(\log N_0\) by \(x\) and future demand by \(\mu x\), where \(\mu\) has an uncertain value about which consumers disagree. The total effect of this shock on house prices \(p_0^h\) at time 0 is given by
\[
\frac{\partial \log p_0^h}{\partial x} = \begin{cases} 
\text{future price expectations} & \text{if } N_t < N_t^* \\
\frac{\rho_{\tilde{\mu} \epsilon_0}}{\epsilon_0} (1 - \lambda) & \\
\lambda \frac{1}{\epsilon_0} + (1 - \lambda) \frac{\chi \tilde{\mu} + (1 - \chi) \pi}{\epsilon_0} & \text{if } N_t \geq N_t^*,
\end{cases}
\] (31)
where \(N_t\) is the level of housing demand, and \(N_t^*\) is the threshold at which the city fully develops given by Lemma 1. Only the optimistic investor belief \(\tilde{\mu}\) matters when undeveloped land remains in the city. When the city is fully developed, the average belief of owner-occupants \(\pi\) matters in addition to the investor beliefs, as long as rental frictions are present (\(\chi < 1\)). The shock raises current rents only when the city is fully developed.

We put off analyzing this result until we have derived a similar expression for construction. This result will allow us to compare the size of the housing boom in different types of cities.

\(^{22}\)More precisely, \(\lambda = r_0/p_0^h\) conditional on the shock \(x = 0\), and satisfies \(\frac{\partial \log p_0^h}{\partial x} = \lambda \frac{\log r_0}{\partial x} + (1 - \lambda) \frac{\partial \log \mathbb{E}_{P_{t+1}^h}}{\partial x}\).
4.4 Impact of Shock on Quantities

We began this paper by describing why high construction housing booms are a puzzle. To explain this puzzle, we analyze the effect of the shock $x$ on construction, supplementing the formula we just gave for the shock’s effect on prices. The construction caused by the shock is given by the shock’s effect on the level of the housing stock at time 0, $H_0$.

As in our analysis of prices, the shock $x$ has two effects on house quantities, corresponding to the current effect of the shock $x$ at time 0 and the changes in beliefs about future demand. The direct effect at time 0 is like the demand shock $\zeta_t$ considered in Proposition 1. It passes through 1-for-1 to quantities when land remains, but has no effect on construction when the city is fully developed.

Speculation in land markets also has an effect on construction. We have not considered this effect up until now, but it comes out of the equations we have written down. More intense speculation in land markets (i.e. higher values of $\tilde{\mu}$, or $\tilde{E}_{ph1}$ in the general model) curtails construction. The reason is that this speculation raises house prices, making housing more expensive for owner-occupants and decreasing the amount of housing they demand. To derive this effect, we differentiate the market-clearing equation (16) with respect to investor and owner-occupant beliefs, and then substitute equations (26), (30), and the definitions of the demand elasticity $\epsilon_0$ and the log-linearization parameter $\lambda$. The result is Proposition 4.

Proposition 4 (Quantity Impact of Shock). Consider a shock that raises current housing demand $\log N_0$ by $x$ and future demand by $\mu x$, where $\mu$ has an uncertain value about which consumers disagree. The total effect of the shock $x$ on the housing stock $H_0$ at time 0 is given by

$$
\frac{\partial \log H_0}{\partial x} = \begin{cases} 
\frac{1}{\epsilon_0} & \text{construction attenuation} \\
1 - \frac{\epsilon_0}{\epsilon_0} \frac{1 - \lambda}{\lambda} (\bar{\mu} - \mu) \rho (1 - \chi) & \text{if } N_t < N_t^* \\
0 & \text{if } N_t \geq N_t^*,
\end{cases}
$$

(32)

where $N_t$ is the level of housing demand, and $N_t^*$ is the threshold at which the city fully develops given by Lemma 1. The shock increases construction when undeveloped land remains.

---

23 Differentiating the market-clearing equation (16), noting that $r_0$ stays constant at $(1 - \beta)C$, yields $\partial H_0/\partial x = \sum N_0 D_t(\partial \beta \tilde{E}_{ph1}/\partial x - \partial \beta \tilde{E}_{ph1}/\partial x)$, where the sum is taken just over owner-occupants. Factoring out $\sum N_0 D_t$, diving both sides of the equation by $H_0$, and using the definition of $\chi$ given by equation (22) transforms this equation to $\partial \log H_0/\partial x = (1 - \chi)(D_0/D_0)(\partial \beta \tilde{E}_{ph1}/\partial x - \partial \beta \tilde{E}_{ph1}/\partial x)$, where $D_0$ is the sum of all resident demands $D_i$ and $\tilde{E}$ is the average owner-occupant belief given by equation (30). Finally, we multiply by $1 = r_0(\beta \tilde{E}_{ph1}/r_0)/(\beta \tilde{E}_{ph1})$, where $\tilde{E}$ denotes the common expectation before the shock that holds when $x = 0$, to reduce the last equation to $\partial \log H_0/\partial x = (1 - \chi)(\epsilon_0/\epsilon_0)(\bar{\mu} - \mu) \rho (1 - \lambda) / \lambda$, in which we use the definition $\epsilon_0 \equiv r D_0/D_0$, the formula (26) for the effect of the shock on belief, and the definition $\lambda = r_0/(r_0 + \beta \tilde{E}_{ph1})$. 

28
as long as the current housing demand elasticity \( \epsilon_0 \) is small relative to the demand elasticity \( \epsilon_0^c \) in the future when the city is developed. The shock has no effect on construction in fully developed cities.

We call the negative term “construction attenuation” because it shows that the construction response to the shock \( x \) is curtailed. In principle, this term could lead a city with undeveloped land not to meet the shock with very much construction. However, we are skeptical that this term is of much empirical relevance. The reason is that this term is small when \( \epsilon_0 / \epsilon_0^c \) is small, that is, when housing demand is more inelastic when house prices are cheap. The term \( \epsilon_0 \) represents housing demand elasticity when housing is cheap (rents equal construction costs \( (1 - \beta)C \)), and \( \epsilon_0^c \) represents housing demand elasticity in the future when the city is constrained and rents become expensive. We imagine a world in which most residents receive consumer surplus from cheap housing. In this case, very few residents are on the margin when housing is cheap, but residents start getting rationed out of an area when rents begin to rise. In this case, \( \epsilon_0 / \epsilon_0^c \) will be small.

We now use Propositions 3 and 4 jointly to analyze housing booms in different types of cities.

## 4.5 A Tale of Three Cities

In this section, we show that the joint behavior of prices and quantities can be usefully described as falling into one of three possibilities, which depend on the level of development of the city. The first city is one that has an unlimited amount of land for future development. The second city is one which is still developing, but will run out of available space soon. The final city is one that has already fully developed. As we will show, high construction housing booms occur precisely in the intermediate case.

### 4.5.1 City 1: Unlimited Land

In the first type of city, the available space \( \mathcal{L} \) is very large relative to \( N_t \). Land remains for development, and will continue to remain for the foreseeable future. The parameter \( \rho \), which measures how supply-restricted the city is in the long-run, equals 0.

Propositions 3 and 4 show that the effect of the shock on prices is 0, while the shock passes through perfectly to quantities: \( \partial \log p_0^h / \partial x = 0 \) and \( \partial \log H_0 / \partial x = 1 \). The unlimited amount of land means that house prices just equal construction costs \( C \) at all times. They don’t depend on current or future housing demand. By the same token, the city can always accommodate additional housing demand by building new houses.
These results correspond perfectly to the classic supply-and-demand intuition. In a location with no current or future restrictions on housing supply, house prices stay constant and construction can absorb all shocks.

In practice, a city or neighborhood that can be characterized as having unlimited land must have some of the following features. First is a perceived lack of future regulation. Investors must believe that future regulators will not prohibit housing supply. The second feature is flat geography. If the city can expand forever, then there cannot be natural barriers such as water or mountains on the horizon. Finally, a city with unlimited land in this framework likely involves homogeneous sprawl. Sprawl means that commuting time does not limit the growth of the metropolitan area, because workers are able to work near their homes instead of in a central business district. Furthermore, within sprawl, neighborhoods are good substitutes for each other, meaning that individual neighborhoods are not land-constrained.

In the recent national housing boom and bust, we think of the sprawling metropolitan areas of Texas as prime examples of locations with unlimited land. These areas are characterized by flat geography, a history of low levels of regulation, and sprawl. They experienced very small booms and busts and high levels of construction.

4.5.2 City 2: Approaching the Constraint

The second type of city is one in which undeveloped land is still present, but will be exhausted in the future. The level of available space $L$ relative to demand $N_t$ is smaller than in the previous type of city, but it is still large enough that raw land remains. The value of the long-run development measure $\rho$ in the city we are now considering is positive.

Propositions 3 and 4 show that a high construction housing boom occurs in this sort of city. The price boom equals $\partial \log p_h^b/\partial x = (1 - \lambda)\rho\tilde{\mu}/\epsilon c_0$. This boom reflects the beliefs of optimistic investors about future housing demand. Construction also booms. Proposition 4 shows that when demand is relatively inelastic, the present shock passes through completely to construction: $\partial \log H_0/\partial x = 1$. When demand is more elastic, some of this construction response is attenuated.

The boom in this area occurs purely through the speculation channel. Optimistic investors speculate in the land markets about future housing demand, pushing up house prices today. Note that only the beliefs of the most optimistic investors matter for house prices, because these are the investors who are holding the land (and for whom $\mu_i = \tilde{\mu}$). Speculation means that house prices in this city are governed by the beliefs of a few fringe optimists who can take large positions in the land markets.

One implication of this result in particular is that homeowner beliefs are irrelevant for
prices. Only the beliefs of real estate investors matter, and only the most optimistic beliefs at that. Surveys of homeowner beliefs, such as Case et al. (2012), are actually uninformative about house prices in this type of city. More useful would be surveys of the investors who hold the land. A promising area of future research is to measure these beliefs using the National Association of Homebuilder surveys of homebuilder sentiment, as these homebuilding firms were prominent land investors during the recent boom.

The available land allows the city to accommodate the current demand shock \( x \) in new construction. As long as homeowner demand is inelastic at the low initial price levels in the city, homeowners do not balk at the rising prices wrought by the optimist investors, and construction is high.

It is useful to walk through the anatomy of the boom in a specific example, given that describing high construction housing booms is the motivation of the paper. We focus on Las Vegas during the recent episode.

Around 2000, new technologies are developed that expand credit to low-income borrowers, raising housing demand throughout the country and in Las Vegas. Due to the regulations described in the Introduction and shown graphically in Figure 2, Las Vegas will run out of land in the next 20-30 years. Investors, anticipating this future development, scramble to get a hold of this remaining land, knowing that it will appreciate as the city continues to develop. These land investments are akin to buying property in an up-and-coming neighborhood before it gets popular. In Las Vegas, the belief was that once it ran out of land, it would transition to being a glamorous, expensive city akin to those in coastal California from which many of its migrants came. The mayor of North Las Vegas summed up this perception in 2007, when he said

\[ \text{I believe in 100 years they’ll be studying this [boundary] as a great piece of public policy. I think it will go right up beside Robert Moses building New York.}^{24} \]

Investors disagree about Las Vegas’s future transition to an expensive city. They speculate in the land market, where speculation occurs quite easily because there are no frictions. The most optimistic investors buy up all of the land, driving up land prices. House prices rise because they are tied to land prices. Construction is high because the current subprime shock, which got everyone speculating in the first place, has increased housing demand, which builders can meet by building on the available land. Homeowners were getting a lot of consumer surplus from the cheap housing, so they continue to buy as prices rise. People were moving from California to Las Vegas because Las Vegas was much cheaper than California. The higher prices just made Las Vegas housing seem like a good deal, instead of an excellent

\[^{24}\text{Quotation comes from McKinley and Palmer (2007).} \]
Consistent with this intuition, Las Vegas led all metropolitan areas in construction rates between 2000 and 2006.

Eventually, the surging housing demand starts to revert. Mian et al. (2013) provide evidence that various measures of consumer credit health fell between 2006 and 2009. We think of this period as the time in which the true value of the persistence of the initial shock is revealed. Optimistic investors learn that the truth is less rosy than the one behind their valuations of land, and land and house prices fall, causing the bust.

The general take-away from this specific example is that approaching a development constraint is the necessary setting for a high construction housing boom. The upcoming constraint creates uncertainty, leading to speculation in land markets. These same land markets accommodate new construction. As we now discuss, both speculation and new construction go away in the final city we consider, the fully developed one.

### 4.5.3 City 3: Fully Developed

The final category of city is the fully developed one in which no raw land remains. The level of available space $\mathcal{L}$ is low relative to demand $N_t$.

Proposition 4 shows that there is no construction in this city. All available space already has houses on it, so no additional construction is possible. Proposition 3 shows that there is a house price boom, consisting of two effects: the direct effect on rents $\lambda/\epsilon_0$, and the effect on future price expectations $(1 - \lambda)(\chi\tilde{\mu} + (1 - \chi)\tilde{p})/\epsilon_0$. As in the city with unlimited land, the housing boom here corresponds well to the predictions of the classic supply-and-demand story.

What is notable about the boom in this city is that the beliefs that are capitalized into prices are less optimistic than the beliefs in the developing city we just considered. In this fully developed area, investors have only the housing markets; there are no land markets for speculation. Optimistic investors can only partially effect prices. This partial effect is represented by $\chi$, the depth of the rental market relative to the entire housing market. The larger the market for rental housing, the smaller the frictions and the more direct the effect of speculators on prices.

The total market belief is $\chi\tilde{\mu} + (1 - \chi)\tilde{p}$. It combines the average owner-occupant belief and the optimistic investor belief. This combination is less affected by the beliefs of an optimistic fringe. The non-transferable ownership utility means that home ownership is dispersed in equilibrium among many individual residents. Their aggregate belief is given by $\bar{\mu}$, and it guarantees that the market belief cannot get too optimistic.

The boom in this area is more about fundamental supply-and-demand than about speculation by optimistic investors. When rental frictions are complete ($\chi = 0$), no speculation
occurs, and only the supply-and-demand effects remain.

We think of a tony, exclusive neighborhood that forbids new construction as a pure example of such a city. At the level of a larger metropolitan area, a region where local governments all forbid new construction would fit this pattern. Coastal cities like Boston, New York, and San Francisco, where construction has been nearly regulated out of existence for the past 30 years, are well-described by this model.

### 4.5.4 Summary

We may summarize the key differences between these three cities with a table in which we turn off many of the model’s parameters. In this section, all residents prefer owner-occupancy, so that $\chi = 1$. Housing demand is completely inelastic at low price levels (everyone receives consumer surplus), so that $\epsilon_0/\epsilon_c = 0$. Finally, all residents believe that the shock will fully revert except for a few optimistic investors. In terms of parameters, this belief means that the average owner-occupant belief about $\mu$ is $\overline{\mu} = 0$.

<table>
<thead>
<tr>
<th></th>
<th>Boom $\Delta \log p_h^b$</th>
<th>Construction $\Delta \log H_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unlimited Space</td>
<td>$0$</td>
<td>$x$</td>
</tr>
<tr>
<td>Approaching Constraint</td>
<td>$(1 - \lambda) \rho \tilde{\mu} x_0/\epsilon_0$</td>
<td>$x$</td>
</tr>
<tr>
<td>Fully Developed</td>
<td>$\lambda x_0/\epsilon_0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

**Notes:** “Unlimited Space” refers to a city where $N_t/L = 0$, “Approaching Constraint” is a city where $N_t < N_t^*$, and “Fully Developed” is a city where $N_t \geq N_t^*$. The expressions in this table are given by the formulas in Propositions 3 and 4 with the following parameter values: $\chi = 1$, $\overline{\mu} = 0$, and $\epsilon_0/\epsilon_0' = 0$.

Table 3 lists the boom and construction under these parameter restrictions. The high construction housing boom occurs in the intermediate case. The boom there comes entirely from the beliefs of optimistic investors in the land markets. In contrast, the boom in the fully developed location is entirely a direct effect of the shock on rents. There is no speculation. The supply-and-demand story perfectly explains the first and third cities, but the middle city is anomalous. Speculation is needed to explain a high construction housing boom.
4.6 Where is the Boom the Largest?

Because of frictions in the rental market encapsulated by $\chi$, speculation can affect prices more in undeveloped areas than in fully developed ones. If investors are sufficiently optimistic, this speculative effect can overwhelm the direct effect of the shock on rents in the fully developed areas. The price boom can actually be larger in undeveloped areas that have relatively more land. To see this result, we need only turn to Proposition 3. The value of $\rho$ can get as high as 1 in unconstrained areas as they approach the development barrier. Therefore, there are undeveloped areas with a boom exceeding the one in the fully developed cities if the top term in equation (31) exceeds the bottom two terms when $\rho = 1$. Proposition 5 states the condition under which this result holds.\(^{25}\)

**Proposition 5 (Boom Increases with Land Float).** Consider a shock that raises current housing demand $\log N_0$ by $x$ and future demand by $\mu x$, where $\mu$ has an uncertain value about which consumers disagree. If there are frictions in the rental market ($\chi < 1$), then the housing boom is largest in undeveloped areas when there is sufficient dispersion of beliefs about $\mu$. If

$$(1 - \chi)(\bar{\mu} - \mu) > \lambda/(1 - \lambda),$$

then there exists $N_t^{**} < N_t^*$ such that the boom is larger for $N_t^{**} < N_t < N_t^*$ than for $N_t \geq N_t^*$. Here $N_t$ is the level of housing demand, and $N_t^*$ is the threshold at which the city fully develops given by Lemma 1.

The intuition of Proposition 5 can be gleaned most readily from Table 3. In the special case shown there, the boom in the intermediate city is larger than in the fully developed city as long as the investors are optimistic enough. These investors are crowded out of the fully developed city because of rental frictions.

Usually, we think that asset float decreases volatility from speculation. The usual logic, offered for instance by Hong et al. (2006), is that larger float allows pessimists to participate, blunting the effect of optimists and attenuating price volatility. Proposition 5 shows that this intuition can be reversed in the real estate market. Larger land float brings speculators into the real estate market. At low levels of float, they are crowded out of the market by residents who receive ownership utility from housing. They face a tax of $1 - \chi$ from buying housing. At high levels of float, there are not enough homeowners to hold all the land, so a land market emerges in equilibrium, allowing speculators to have a greater impact on house prices.

\(^{25}\)In this proposition, we assume that the current demand elasticity $\epsilon_0$ in the fully developed area equals the average future constrained elasticity $\epsilon_c^0$ in all cities.
This story is similar to the model offered by Simsek (2013), in which new markets increase the price volatility of existing securities by giving investors a new means to speculate. Larger land float leads to a new market, raw land, in equilibrium. Speculation is easier in this market.

5 Calibration

In this section, we make clear the mechanics of our model by working through a specific numeric example. We will draw many of the parameter values from outside sources and calculate the two sufficient statistics characterizing beliefs, $\tilde{\mu}$ and $\bar{\mu}$, from match moments of the data. Our resulting estimates should not be taken to represent deep structural parameters. Rather, they provide some intuition about the quantitative magnitudes the model can explain.

We calibrate our model against the cross-section of house price booms and construction from 2000 to 2006, which is shown in Figure 1 in the Introduction. Propositions 3 and 4 provide formulas for the boom and construction in a city as a function of parameter values and belief statistics. As we explain in Section 4, cities can be usefully described in three mutually exclusive categories based on those formulas. The first is a developing city ($N_t < N_t^*$) in which the long-run constraint measure $\rho = 0$, the second is a developing city in which $\rho$ is positive, and the third is a fully developed city ($N_t \geq N_t^*$).

To categorize the actual cities in the data in this way, we divide them into size-weighted terciles according to the supply elasticity calculated by Saiz (2010). This elasticity combines both geographic constraints and local regulation, both of which are relevant to the constraint measure in our model. Table 4 shows that, as is predicted by our model, high price booms occur in low and middle elasticity areas, while high construction activity occurs in the middle and high elasticity areas.

We normalize the data provided in the first two rows of Table 4 before using it to solve for the belief statistics. First, we normalize house price growth by subtracting away construction cost increases, which we have abstracted from in our model. Rather than use the construction costs increases we have calculated for each city in Figure 4, we just subtract off the 14% price growth in the high elasticity areas, which is similar to the construction costs increases in Figure 4. That figure shows that these construction cost increases are relatively uniform across cities, so this normalization is fairly accurate and has the advantage of bringing the model closer to the data, as the model predicts zero price growth in the high elasticity areas.

The second normalization we make is by subtracting off the construction in low elasticity areas from reported construction rates. Our model predicts zero construction in the fully
Table 4: Inputs into Calibration: 
Housing Statistics by Saiz Supply Elasticity Tercile

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Middle</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Price Growth 2000-2006</td>
<td>52%</td>
<td>43%</td>
<td>14%</td>
</tr>
<tr>
<td>Total Construction 2000-2006</td>
<td>7%</td>
<td>11%</td>
<td>11%</td>
</tr>
<tr>
<td>Supply Elasticity</td>
<td>0.8</td>
<td>1.3</td>
<td>2.7</td>
</tr>
<tr>
<td>Income 2000</td>
<td>$37,000</td>
<td>$32,000</td>
<td>$29,000</td>
</tr>
<tr>
<td>Observations</td>
<td>21</td>
<td>40</td>
<td>49</td>
</tr>
</tbody>
</table>

Notes: Supply elasticity comes from Saiz (2010). Terciles are weighted by each metropolitan area’s housing stock in 2000, which is taken from the Census. Price growth is the log change in the FHFA house price index deflated by the CPI-U. Total construction is the log change in the imputed housing stock, formed using Census 2000 housing stock figures and annual permit counts. Income comes from the BEA.

developed area. We readily admit that this result is counterfactual, and as we noted in Section 3, it resulted from simplifying assumptions that we relax in Section 7. Rather than use a more complicated model to capture this construction, we assume that the 7% construction represents a baseline amount that occurs in all cities (such as from replacing depreciating housing).

With these normalizations, the four equations the model matches are

\begin{align}
0.04 &= x \\
0.04 &= \left(1 - \frac{\epsilon_0}{\epsilon_0^c} \frac{1 - \lambda}{\lambda} \left(\bar{\mu} - \overline{\mu}\right) \rho (1 - \chi)\right) x \\
0.28 &= \left(\frac{1 - \lambda}{\lambda}\right) \rho \bar{\mu} x \\
0.38 &= \lambda \left(\frac{x}{\epsilon_0} + (1 - \lambda) \frac{\chi \bar{\mu} + (1 - \chi) \overline{\mu}}{\epsilon_0^c} \right) x.
\end{align}

In this exercise, we are assuming that the shock $x$ is the same across the low, middle, and high supply elasticity cities on average. There may be empirical reasons why this assumption is false. What are model offers is an ability to explain the variation of price
booms and construction levels as just a function of supply conditions. Therefore, to get a sense of the quantitative nature of our model, it makes sense to assume that the shocks are the same across areas. Furthermore, Table 4 provides some bare bones evidence that variation in the shocks across areas is not too bad of an empirical concern. As shown by Mian and Sufi (2009), the housing demand shock during this period likely came from an expansion of credit to subprime borrowers. The fraction of subprime borrowers in an area is correlated with income. Table 4 shows that the middle elasticity areas are no poorer elasticity areas, undermining the claim that the middle elasticity areas simply were hit by a bigger shock than the high elasticity areas.

We work with the following parameter values. We use $\beta = 0.65$, which is taken from the annual homeowner discount factor of 0.93 estimated in Carrillo (2012) with a time scale of 6 years. This value of $\beta$ yields $\lambda = 0.35$, which means that 35% of house prices come from rents over the next 6 years, and the remainder comes from future rents. We set the elasticities in fully developed areas equal to 0.1, which means that a permanent demand shock that leads an developing city to build 1% more housing would raise prices in a constrained place permanently by 10%. We let $\chi = 0.35$, which corresponds to 1 minus the national homeownership rate, which we show in Section 6 can proxy for $\chi$ when all housing is identical and each resident has the same elasticity demand. We also let $\rho = 0.5$ in the middle elasticity categories. Finally, we assume that $\epsilon_0/\epsilon_c = 0$ in the middle elasticity category. We could actually derive this equation as a result by solving the equations, because the construction levels in the middle and high elasticity areas are the same.

We turn now to calculating $x, \tilde{\mu}, \bar{\mu}$ from the data using these parameter values. Using the above equations, we find that $\tilde{\mu} = 2.2$ and $\bar{\mu} = 0.2$. These numbers imply that the most optimistic investors thought not just that the shock would be persistent, but they extrapolated to believe that the future shock would be greater than the present shock. Average homeowners were much less optimistic and thought that most of the shock would go away, but 20% would persist in the long-run.

These estimates allow us to run a counterfactual where we investigate the importance of speculation in each area. We consider a counterfactual in which all residents and investors hold the same belief. The common belief we use for this exercise is the average owner-occupant belief $\overline{\mu} = 0.2$. Because beliefs only enter through this statistic and the optimistic investor belief $\tilde{\mu}$, we can produce the counterfactual just by replacing the estimated value of $\tilde{\mu} = 2.2$ with $\mu = 0.2$.

Table 5 displays the results of this counterfactual. Net of construction costs, nearly all

\footnote{Our calculate of $\lambda = 1 - \beta$ presumes no long-run growth. Adding in growth will decrease $\lambda$ but also cause it to vary across areas depending on their level of development.}
Table 5: Implied Importance of Speculation Across Areas

<table>
<thead>
<tr>
<th></th>
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<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Price Growth 2000-2006</td>
<td>52%</td>
<td>43%</td>
<td>14%</td>
</tr>
<tr>
<td>Homogeneous Beliefs</td>
<td>33%</td>
<td>17%</td>
<td>14%</td>
</tr>
<tr>
<td>Counterfactual</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of Boom Attributed to Speculation</td>
<td>0.37</td>
<td>0.60</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: We calculate the counterfactual by replacing the estimated investor belief with the estimated average owner-occupant belief. Supply elasticity comes from Saiz (2010). Terciles are weighted by each metropolitan area’s housing stock in 2000, which is taken from the Census. Price growth is the log change in the FHFA house price index deflated by the CPI-U.

of the boom in the middle category comes from speculation. This result is clear because the boom drops nearly to the boom in the high elasticity area without speculation. Thus, according to our model, the middle and high elasticity areas would be approximately observationally equivalent in terms of prices and construction when investors have homogeneous beliefs.

In contrast, much of the boom remains in the low elasticity areas when we remove speculation. The remaining boom reflects the direct effect of rents of the shock $x$, as opposed to optimistic beliefs about future demand. Furthermore, speculators had only a blunt effect on the empirical boom of 52% due to the rental frictions that are the focus of our model. Only 37% of the boom in these areas came from speculation, while the majority of the boom in the middle elasticity areas did.

6 Variable Rental Frictions

One result from our model is that frictions in the rental market impede speculation in areas without land markets. Places where these rental frictions are higher should experience less speculation and smaller booms and busts. A natural empirical question to ask is whether booms and busts do indeed vary with some measure of these rental frictions.

To be clear on the precise mechanism in our model, we draw attention to Proposition 3.
According to this result, the house price boom in a fully developed area is given by

\[
\lambda \frac{1}{c_0} + (1 - \lambda) \frac{c_{\hat{\mu}}}{c_{\mu}} + (1 - \chi) \frac{\mu}{c_0},
\]

(38)

where \(\chi\) is the measure of rental market frictions given by equation (22). The term \(\hat{\mu}\) denotes the beliefs of the landlords, who are the most optimistic investors because of short-sale constraints in the housing market. In particular, \(\hat{\mu}\) is higher than \(\bar{\mu}\), the average belief of owner-occupants in the area. Therefore, higher values of \(\chi\) lead to a larger boom, all else equal.

The goal in any empirical work should therefore be isolate exogenous variation in \(\chi\) and examine whether this variation negatively predicts the size of a housing boom and the extent of participation by speculators. The precise definition of \(\chi\) is given in equation (22). It is the ratio of rental market depth to the total depth of the housing market, where “depth” is the derivative of the aggregate demand curve. When the resident housing demands \(D_{i}'\) all have the same elasticity with respect to rents, then \(\chi\) may be written as

\[
\chi = \frac{\sum\limits_{a_i < \beta (E_{p_{t+1}} - E_{p_{t+1}})}^{\text{rental housing}} D_i}{\sum\limits_{a_i}^{\text{all housing}} D_i},
\]

(39)

that is, as the share of housing that is rental as opposed to owner-occupied. This expression has the advantage that it is expressed in terms of measurable quantities.

There are two ways to approach measuring this statistic \(\chi\): one involves variation across neighborhoods, and the other involves variation across types of housing.

First we consider variation across neighborhoods. Suppose that all housing units are identical. Neighborhoods differ in terms of the local amenities they offer, and they therefore attract different types of residents. For instance, a neighborhood near a university will attract a mix of students and faculty, whereas a neighborhood with high property taxes funding public schools will attract families with children. The relative depth of the rental market (\(\chi\)) in each area is then just the share of housing that is rented as opposed to owner-occupied.

The advantage of this measure of \(\chi\) is that is easily computed from available housing occupancy data. The downside is that the local share of rental housing is correlated with a host of other variables which are undoubtedly important in explaining housing dynamics.
Notes: “Low owner-occupancy” is the smallest decile of ZIP-level owner-occupancy, and “high owner-occupancy” is the top decile. We average annual log returns for each group and report the cumulative sums. Owner-occupancy data comes from the 2000 Census, and house price indices come from Zillow.com. We deflate these indices with the CPI-U.

Figure 7 plots the average cumulative housing returns in areas where the owner-occupancy share of housing is low and in areas where it is high. We select the lowest and highest deciles of ZIP-code level owner-occupancy in the 2000 Census, and use Zillow.com’s publicly available ZIP-code level house price indices. The figure shows that house prices did indeed boom more in the areas where a greater share of housing was rented. We want to stress that Figure 7 is simply the raw data, and that several other papers suggest mechanisms that could likely explain this difference. Mian and Sufi (2009) show that house prices rose more in areas where borrowers had worse credit ratings; Guerrieri et al. (2013) and Landvoigt et al. (2013) document larger house price booms in areas with a lower price level in 2000. Both the initial price level and the share of subprime borrowers are likely correlated with the rental share of housing. The data are also consistent with our explanation.

The second approach to measuring $\chi$ is to exploit variation across different types of housing structures. For example, Glaeser and Gyourko (2007) show that single-family detached
housing is much more likely to be owner-occupied than rented. Multifamily housing, such as units in apartment buildings, are more likely to be rented. Our theory predicts that structure types that are rented more frequently would attract more speculators and experience larger booms and busts in prices. This structure-based test is promising, because there is likely variation in $\chi$ among structures even holding initial price and subprime share constant. There are both condos and single-family housing for credit-worthy residents buying expensive housing.

7 Robustness

7.1 Additional Securities

In this section, we allow residents to trade housing futures. These assets are traded at time 0, the period in which the demand shock occurs. The seller of the asset agrees to pay the buyer of the asset the market value of a house at time 1, the period in which uncertainty is resolved. We study these housing futures for two reasons. First, they allow investors to short the real estate market. Specifically, an investor can short the value of a house at time 1 by selling a housing future at time 0. Our model’s main results depend on the absence of short-selling in the real estate market. By studying housing futures, we can learn which of the model’s results are robust to this investment behavior. The second reason we study housing futures is because such assets were actually introduced during the housing boom. As described in Shiller (2008), housing futures for ten different cities began to be traded on the Chicago Mercantile Exchange in 2006.

We denote the price of the future at time 0 to be $p_{f0}$. The liquidation value of this future at time 1 is the price of housing at that time. Therefore, $p_{f1} = p_{h1}$. Consumer $i$’s holding of the future at time 0 is denoted $F_{i,0}$. Unlike housing and land, the future can be held in negative quantities: $F_{i,0}$ can be positive or negative. The net supply of futures is 0, because each futures contract is originated with one resident as a buyer and one as a seller. Therefore, in equilibrium

$$\sum_i F_{i,0} = 0. \quad (40)$$

Residents must post collateral to buy or sell a future. Instead of paying the seller the price $p_{f0}$, a futures buyer posts collateral with a centralized futures market. The seller of the future also posts collateral. These collateral constraints will determine the price of the future because each resident has a fixed amount of credit. Without any collateral constraints, residents could buy and sell an unlimited amount of futures. In practice, such collateral
constraints take the form of margins that specify a fixed fraction of the notional futures that must be deposited. We need not specify the exact nature of these collateral requirements in this section. All that we need is that the collateral constraint is symmetric across buyers and sellers. As a result, the collateral constraint will not favor either side of the market.

The introduction of housing futures has two effects. The first effect is that homeowners and landowners can now perfectly hedge their real estate risk. Suppose a homeowner owns \( H \) units of housing. If he sells \( H \) units of the future, then the value of his portfolio at 1 is \( p^h_1 - p^f_1 = 0 \). The price of this portfolio at 0 must equal the market rent \( r_0 \) by the law of one price. Indeed, a landlord who purchases this portfolio receives the rent \( r_0 \) and a certain payoff of 0 at time 1. This portfolio allows owner-occupants to purchase housing just at time 0 for \( r_0 \), while still receiving ownership utility. The owner-occupant still owns the house at time \( t^* \), but he is no longer exposed to the house price risk. His financial position is thus equivalent to that of a renter. Landowners can similarly hedge risk by selling \( L \) units of the future when holding \( L \) units of land. In this case, the landowner faces a certain payout at 1 of \( p^l_1 - p^f_1 = p^h_1 - C - p^h_1 = -C \).

The second effect of the introduction of housing futures is to change the price of housing and land. By allowing residents and investors to hedge, the futures markets move all speculation from the housing and land markets into the futures markets. House prices are

\[
p^h_0 = r_0 + \beta p^f_0.
\]

(41)

The market rent \( r_0 \) is determined by housing supply and demand, and is independent of future beliefs. The reason is that futures allow residents to consume housing purely as a flow good. Because the future can be shorted, its price reflects the beliefs of all consumers, not just the most optimistic ones. The next proposition makes this point clear.

**Proposition 6.** Let \( \mu_{med} \) equal the median belief of \( \mu \) among all consumers. Then the futures price reflects this median belief:

\[
p^f_0 = E(p^h_1 \mid \mu = \mu_{med}).
\]

(42)

It follows that the house price \( p^h_0 \) reflects this median belief as well.

Proposition 6 shows that house prices reflect the median belief of all residents when futures are available. Without futures, house prices reflected the beliefs of the most optimistic investors. Futures should thus reduced the boom and bust by allowing house prices to include the beliefs of pessimists as well as optimists. Furthermore, the futures market eliminates the variation across cities in the aggregation of beliefs. Without futures, optimists influence
prices more in unconstrained cities because land markets there facilitate their speculation. With futures, speculation moves from the land and housing markets into the futures market. Futures markets should eliminate the amplification effect of the house price boom in unconstrained cities which we have documented and explained in this paper.

The analysis in this section adds to the puzzle of why participation in the housing futures markets introduced in the Chicago Mercantile Exchange in 2006 has been extremely low (Shiller, 2008). These markets should allow homeowners to eliminate house price risk. Furthermore, to the extent that housing market booms and busts have negative spillovers, policymakers might favor these futures markets because they would lead house prices to reflect pessimist beliefs in addition to optimist beliefs by relaxing short-sale constraints. Ideally, the Chicago Mercantile Exchange markets would have existed earlier, say in 2000, because most of the land market speculation had already occurred by 2006.

7.2 Microfounding Ownership Utility

In this section, we microfounded the ownership utility from Section 3. The key properties used there were that the ownership utility varied across residents, and was positive for some residents and negative for others. Our particular functional form was linear. Here, we present a moral hazard framework in which ownership utility will match these characteristics. Our framework follows the spirit of Henderson and Ioannides (1983)'s treatment of tenure choice, in which maintenance frictions lead some residents to own instead of rent.

Residents derive utility from the particular way their house is “customized”: e.g. the color of the walls, the way the lawn is maintained, et cetera. The customization choices are often vague in nature, so we think of precise customization as being non-contractible. However, the right to customize one’s house is contractible. A landlord can sell the customization rights to a tenant, but she cannot contract with the tenant on the particular customization. If the landlord retains the customization rights, then the house is not customized at all. Customization only occurs when it is performed by the resident. This case corresponds to a landlord preforming just routine maintenance on the property, which is enjoyed equally by all residents.

Each customization incurs a cost, which comes about from depreciation to the property. For example, planting an ugly tree in the backyard may depreciate the value of the house. This cost is thus borne by the owner of the property. Because each particular customization is not contractible, the resultant costs are also not contractible. A landlord can prohibit a tenant from making any changes to the property (customization rights are contractible), but the landlord cannot recoup the losses to the property from the ugly tree (customization...
costs are not contractible) and the landlord cannot sell the specific right to put up that ugly tree (specific customizations are not contractible).

We model this set-up by with a set $K$ of possible customizations. A resident who holds customization rights may choose a customization $k \in K$. This customization delivers resident-specific utility $\theta_i(k)h$, where $h$ is the quantity of housing consumed. The cost of this customization is $\eta(k)h$. Consider the case of a landlord renting to a tenant. Suppose the landlord sells the customization rights to the tenant. Then the tenant will choose the customization $k_{rent}^*$ to maximize $\theta_i(k)$: she chooses her preferred customization, ignoring any resultant costs. The landlord bears these costs, specifically $\eta(k_{rent}^*)$. We assume that the resultant costs always exceed the utility benefits:

**Assumption 1 (Customization Moral Hazard).** For all residents $i$, if $k_{rent}^* = \arg \max_k \theta_i(k)$, then $\theta_i(k_{rent}^*) < \eta(k_{rent}^*)$.

Assumption 1 states that customization involves a moral hazard problem. When residents do not internalize the costs of the customization, they always choose an inefficient customization, i.e. one whose costs exceed its benefits. Due to this moral hazard problem, a landlord never sells the customization rights to a tenant. The price of these rights would have to exceed the cost to the landlord $\eta(k_{rent}^*)h$ but not exceed the cost to the tenant $\theta_i(k_{rent}^*)h$, and there is no such price. Hence, in equilibrium, landlords always retain customization rights, and leave their units un-customized. This result accords with the evidence presented in Glaeser and Gyourko (2007) that rental units have different characteristics than owner-occupied units.

Now consider a resident who both owns and occupies her housing. Unlike a renting tenant, this resident bears the costs of any customization she chooses. She therefore chooses a customization to maximize $\theta_i(k) - \eta(k)$. The additional utility she receives from the customization equals $a_i h$, where

$$a_i \equiv \max_k \theta_i(k) - \eta(k). \quad (43)$$

If $a_i > 0$, then resident $i$ derives a net benefit from customization. She can choose a customization whose utility benefits exceed the costs. If $a_i < 0$, then resident $i$ suffers from the burden of customization. The cost of each customization exceeds the benefits. This resident prefers to leave the customization to the landlord. This case corresponds to a resident who wants the landlord to maintain the house, because she receives no utility from being creative with maintenance. The landlord maintains the house in a standard way.
7.3 Smoothing out the Transition to Being Constrained

In Section 3, the city discontinuously transitions from being unconstrained (having some land available) to being constrained (having houses on all the land). In this section, we show that this discontinuity is not critical to the model’s main result: that the market’s aggregation of beliefs is more optimistic when the city is less developed. In this continuous case, the intuition is the same: the less developed the city, the more land markets facilitate speculation, increasing the market aggregate belief.

We smooth out the transition by allowing land to have a productive use other than residential housing. We have in mind commercial real estate, parks, and farming. We denote consumption of this composite alternate good by \( l \). Resident flow utility from real estate is now \( v(h, l) \) instead of \( v(h) \). The utility function \( v \) has the standard properties that deliver the existence of demand curves: \( v_1, v_2 > 0 \), \( v_1(0) = v_2(0) = \infty \), and \( v \) is strictly quasi-concave.\(^{27}\) Each unit of land can produce either a unit of housing \( h \) at a cost of \( C \) or a unit of this composite alternate good \( l \) at a cost of 0. The smaller cost of producing \( l \) reflects the utility that open space, such as parks, can provide with minimal development costs.

We quickly describe how to solve this generalized model, and then reproduce the paper’s main results. The first-order conditions that emerge from the Bellman equation (5) are

\[
\begin{align*}
v_1(h, l) &= \min(r^h_t, p^h_t - \beta \tilde{E} p^h_{t+1} - a_i); \quad (44) \\
v_2(h, l) &= r^l_t, \quad (45)
\end{align*}
\]

where \( r^h_t \) is the rental price of housing and \( r^l_t \) is the spot price for the alternate good \( l \). Because \( v \) is strictly quasi-concave, unique solutions \( h^*_i(t, \min(r^h_t, p^h_t - \beta \tilde{E} p^h_{t+1} - a_i)) \) and \( l^*_i(r^l_t) \) exist that give housing and alternate good consumption as functions of prices.

Two supply-side arbitrage conditions reduce the prices in these demand equations to the single price \( r^h_t \), the rental price of housing. First, as in the model of Section 3, landlord arbitrage results in \( p^h_t = r^h_t + \beta \tilde{E} p^h_{t+1} \), where \( \tilde{E} \) denotes the belief of the most optimistic investor. A second arbitrage equation ties the spot price of the alternate good to the rental price of housing. A landowner must be indifferent between building a house and selling today or doing the same tomorrow. Therefore, \( p^l_t - C = r^l_t + \beta (\tilde{E} p^h_{t+1} - C) \); together with the landlord arbitrage equation, this intertemporal indifference condition means that

\[
r^h_t = r^l_t + (1 - \beta)C. \quad (46)
\]

\(^{27}\)Strict quasi-concavity holds if \( v \) is concave in each of its arguments and if \( h \) and \( l \) are complements.
Equation (46) generalizes equation (17) from Section 3 to the case in which the spot price \( r_l \) for the alternate good can be positive. In Section 3, this spot price was implicitly assumed to be 0.

Using these arbitrage equations, we denote the aggregate demand for the alternate good by \( D_l(r_h) \equiv \sum_i l_i^*(r_h - (1 - \beta)C) \), the aggregate demand for rental housing by \( D_{h,r}(r_h) \equiv \sum_{a_i < \beta(\tilde{E}_{h,t+1} - E_{h,t+1})} h_i^*(r_h) \), and the aggregate demand for owner-occupied housing by \( D_{h,o}(r_h) \equiv \sum_{a_i > \beta(\tilde{E}_{h,t+1} - E_{h,t+1})} h_i^*(r_h - \beta(\tilde{E}_{h,t+1} - E_{h,t+1}) - a_i) \). The equilibrium rental price \( r_h \) is pinned down by market-clearing. The total space in the city must all go to production of the alternate good, rental housing, or owner-occupied housing:

\[
\bar{L} = N_t \left( D_l(r_h) + D_{h,r}(r_h) + D_{h,o}(r_h) \right),
\]

(47)

where \( N_t \) denotes the number of consumers who want to live in the city at \( t \). Equation (47) closes the model.

We impose two regularity conditions on these demand functions. First, as the city develops, more land is used for housing and less is used for the alternate good.

**Assumption 2** (Housing Stock Growth). As the demand for the city increases, the share of land used for housing increases:

\[
\frac{D_h(r_h)}{D_l(r_h) + D_{h,r}(r_h)}
\]

(48)

is an increasing function of \( r_h \).

Second, as the city develops, the price elasticity of alternate good demand is strictly decreasing in magnitude, while the price elasticity of housing demand is weakly increasing.

**Assumption 3** (Monotone Elasticities). The alternate good demand elasticity

\[
\epsilon_i^l = -\frac{(D_l')^*(r_h)}{r_h D_l'(r_h)}
\]

(49)

strictly decreases in magnitude with \( r_h \). The housing demand elasticity

\[
\epsilon_i^h = -\frac{(D_h')^*(r_h)}{r_h D_h'(r_h)}
\]

(50)

weakly increases in magnitude with \( r_h \).

Assumption 2 simply states that the city builds more housing in response to demand growth as opposed to knocking them down. This assumption is mathematically equivalent to the
statement that housing demand is less elastic than alternate good demand ($\epsilon_h < \epsilon_l$). Assumption 3 states that the difference between these elasticities monotonically decreases as the city grows. If some alternate good remains as the city demand gets large (e.g. even New York has Central Park), then these elasticities converge to each other as demand goes to infinity. Assumption 3 just states that this convergence is monotonic.

The following proposition replicates the results of the paper in this general framework.

**Proposition 7.** Suppose Assumptions 2 and 3 hold, and that the rental market depth $\chi = (D^h_{t,r})'/(D^h_t)'$ does not vary with the size of the city. The more developed the city is, the smaller the increase in house prices resulting from a shock to the beliefs of optimists:

$$\frac{\partial^2 p^h_t}{\partial N^i_t \partial \tilde{E}^p_{h,t+1}} < 0,$$

and the larger the increase in house prices resulting from a shock to the average beliefs of homeowners:

$$\frac{\partial^2 p^h_t}{\partial N^i_t \partial E^p_{h,t+1}} > 0,$$

where the average homeowner belief is defined as

$$\overline{E}^p_{h,t+1} = \frac{\sum_{a_i > \beta} (\tilde{E}^p_{h,t+1} - E^p_{h,t+1}) (h^*_{i,t})'}{\sum_{a_i > \beta} (\tilde{E}^p_{h,t+1} - E^p_{h,t+1}) (h^*_{i,t})'}.$$

Furthermore, the higher the rental market depth $\chi$, the greater the price sensitivity to optimist beliefs:

$$\frac{\partial^2 p^h_t}{\partial \chi \partial \tilde{E}^p_{h,t+1}} > 0$$

and the smaller the price sensitivity to the average homeowner belief:

$$\frac{\partial^2 p^h_t}{\partial \chi \partial \overline{E}^p_{h,t+1}} > 0.$$

**Proof.** Totally differentiating the market-clearing equation (47) with respect to the optimist belief yields $\partial r^h_t / \partial \tilde{E}^p_{h,t+1}$, the effect of these beliefs on rents. Using this derivative, we find that the total effect of optimist beliefs on prices $p^h_t = r^h_t + \beta \tilde{E}^p_{h,t+1}$ is

$$\frac{\partial p^h_t}{\partial \beta \tilde{E}^p_{h,t+1}} = \frac{(D^h_{t,r})' + (D^h_{t})'}{(D^h_{t,r})' + (D^h_{t,o})'} = \chi + (1 - \chi) \frac{(D^h_{t})'}{(D^h_{t,r})' + (D^h_{t,o})'}.$$

Similarly, by Taylor-expanding the owner-occupancy demands $h^*_{i,t}$ with respect to changes
in individual beliefs and then aggregating the result, we find that the effect of the average homeowner belief on prices is

\[
\frac{\partial p_t^h}{\partial \beta E_{t+1}^h} = \frac{(D_t^{h,o})'}{(D_t')'} + \frac{(D_t^{h,r})'}{(D_t')'} = (1 - \chi) \frac{(D_t^h)'}{(D_t')'}. \tag{57}
\]

These last two equations have intuitive interpretations. The total effect of optimistic investor and average homeowner beliefs on prices is 1, as they are the only individuals holding real estate. The relative effect of optimistic investor beliefs is the share of market depth coming from the alternate good and from rental housing, which they supply. The relative effect of average homeowner beliefs comes from the share of market depth coming from owner-occupied housing.

Equations (54) and (55) from the proposition follow immediately. The higher is \(\chi\), the greater the influence of the optimistic investors in prices and the smaller the influence of the average homeowner. The other two equations of the proposition state that the relative influence of the optimistic investor decreases with city development. Given (54) and (55), we must show that \((D_t)^'/((D_t')' + (D_t^h)'\), the share of market depth going to the alternate good, decreases with city development. Intuitively, this result holds because at low levels of development, all of market depth is the alternate good, given that all of the city is land and none is housing. The challenge is showing that share of the market depth going to the alternate good declines monotonically.

To show this monotonic decline, we rely on Assumptions 2 and 3. Differentiating the first Assumption with respect to \(r_t^h\) yields

\[
\frac{(D_t')'}{D_t} < \frac{(D_t^h)'}{D_t^h}. \tag{58}
\]

Differentiating the second Assumption with respect to \(r_t^h\) yields

\[
\frac{r_t^h(D_t')'}{D_t^h} + 1 > \frac{r_t^h(D_t)^''}{(D_t')'} \tag{59}
\]

and

\[
\frac{r_t^h(D_t^h')'}{D_t^h} + 1 \leq \frac{r_t^h(D_t^h)^''}{(D_t^h')'}. \tag{60}
\]
Therefore,

\[
\frac{(D_l')''}{(D_l')'} = \frac{1}{r_t^h} \frac{r_t^h(D_l')''}{(D_l')'} \leq \frac{1}{r_t^h} \left( \frac{r_t^h(D_l')'}{D_t^l} + 1 \right) < \frac{1}{r_t^h} \left( \frac{r_t^h(D_h')'}{D_t^h} + 1 \right) \leq \frac{1}{r_t^h} \frac{(D_h')''}{(D_h')'} = \frac{(D_h')''}{(D_h')'} \cdot
\]

This inequality is precisely equivalent to the statement that \((D_l')'/((D_l')' + (D_h')')\) is decreasing.

\[\square\]

### 7.4 Credit Constraints

In Section 3, residents could borrow an unlimited amount. Here we investigate changes the model when residents face a credit constraint \(k\). Due to this constraint, the bond holdings of each resident are limited to \(b_t \geq -k\) each period.

Without credit constraints, the most optimistic residents became the investors and comprised the entire population of landlords and landowners. Credit constraints stop the most optimistic investors from buying this entire quantity of assets if they do not have enough capital. The investor who prices the assets is now the marginal investor, who is the least optimistic investor holding these assets. More optimistic investors also hold the asset, but they are credit constrained and therefore cannot hold the quantities that they wish.

It follows that the formulas in Sections 3 and 4 continue to hold, if we re-interpret \(\tilde{E}\) as the belief of the marginal investor as opposed to the belief of the most optimistic investor. This re-interpretation calls into question the relationship that delivered all of the results about speculation in those sections: \(\tilde{\mu} > \bar{\mu}\). This condition states that the belief of the marginal investor is more optimistic than that of the average homeowner. The inequality holds trivially when \(\tilde{\mu}\) is the most optimistic resident belief. We show in this section that the inequality holds in general as long as the credit constraint is sufficiently slack (if \(k\) is sufficiently large).

As available credit increases, the marginal investor belief \(\tilde{\mu}\) increases. Credit allows optimists to take larger investment positions in land and housing markets. With no credit
constraints, the marginal investor belief becomes the most optimistic belief in the population:

\[
\lim_{k \to \infty} \tilde{\mu} = \max_i \mu_i. \tag{66}
\]

Equation (66) is the classic result from Miller (1977). When short-sales are not possible, credit increases asset prices by increasing the belief of the marginal investor.

In contrast, credit does not raise the average homeowner belief \( \bar{\mu} \), past a certain point. The reason is that homeowner demand is an interior solution to the first-order condition given by (15). With enough credit, all homeowners can reach this interior solution. Therefore, additional credit has no impact on this average belief of homeowners. This result may seem counterintuitive: shouldn’t credit encourage optimistic homeowners to buy larger homes, thus increasing the belief of the average homeowner? This effect is second order; the first-order effect is that optimists use additional credit as investors. Optimists start flipping condos instead of buying a larger house, due to diminishing marginal utility. As mentioned earlier, Haughwout et al. (2011) and Bayer et al. (2013) calculate that this investment behavior constituted a large fraction (30%-50%) of new mortgages in cities with large booms and busts. We summarize these results in the following lemma:

**Proposition 8** (Credit and Beliefs). If credit availability \( k \) is larger than some \( k^* \), then the marginal investor belief is higher than the average homeowner belief: \( \tilde{\mu} > \bar{\mu} \). As credit increases, the marginal investor belief increases but the average homeowner belief stays constant: \( \partial \tilde{\mu} / \partial k > 0 \) and \( \partial \bar{\mu} / \partial k = 0 \).

Proposition 8 holds because it is efficient for the ownership of some of the housing stock to be dispersed. Ownership utility, together with diminishing marginal utility of housing, lead the housing stock to be efficiently dispersed among individual homeowners. In contrast, no dispersion effect holds for the ownership of land and rental housing. Therefore, when credit is large, a small number of optimists can hold much of the outstanding land and rental housing, whereas owner-occupied housing remains dispersed among many homeowners. Credit availability increases the optimism of the marginal investor but leaves the optimism of the average homeowner unchanged.

Whether \( \tilde{\mu} \) or \( \bar{\mu} \) was larger during the 2000-2006 boom is ultimately an empirical question. In Section 5, our calibration suggested that \( \tilde{\mu} = 2.2 \) and \( \bar{\mu} = 0.2 \). The static supply elasticity story mentioned in the Introduction assumes the opposite: that \( \tilde{\mu} < \bar{\mu} \). For instance, Glaeser et al. (2008) write a model in which homeowners are optimistic about housing prices; landholders are less optimistic, and build as much housing as the static supply elasticity allows.
Proposition 8 highlights a channel by which credit amplifies a housing boom that the literature has neglected. In our model, as long as \( k \) is large enough, credit has no effect on housing demand. With enough credit, residents are at an interior optimum for their housing consumption choices. Credit’s only effect is to provide leverage to real estate investors.

8 Conclusion

In this paper, we have argued that real estate speculation facilitated by land markets is an important part of housing booms and busts. This speculation channel reverses the intuition linking housing supply and housing booms. A less developed area has vacant land that permits new construction, but this land also creates a market for speculation that can amplify a house price boom.

Our model succeeds in explaining why cities during the 2000-2010 housing boom and bust experienced high construction levels and high price increases and decreases. We provide direct evidence that optimistic investors took large stakes in land markets in these cities, leading to a boom and bust in the land market that was ever larger than the simultaneous boom and bust in the housing market. Our calibrated model suggests that 60% of the booms in such areas resulted from optimistic investors in the land markets. In contrast, the boom and bust in more supply constrained areas resulted primarily from the fact that those cities could not build additional housing.

From a policy angle, our paper shows that expectations of future constraints matter in a housing boom even more than short-run constraints. An interesting extension to our model is to allow investors to have uncertainty about the future constraint \( L \). We suspect that even if there are no short-run constraints, and even if most investors believe there will never be long-run constraints, a boom and bust can occur if some investors believe that future regulatory policy will limit city growth. City managers who wish to promote house price stability may want to be especially clear about future regulatory policy for this reason.

References


