Do Financial Factors Drive Aggregate Productivity? Evidence from Indian Manufacturing Establishments*

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Abstract

Numerous countries have implemented financial reforms in the past three decades, but how these reforms affect economic growth has not been established. I develop a novel dynamic equilibrium model with heterogeneous firms and endogenous leverage to isolate the effects of financial liberalization on aggregate productivity. Changes in financial frictions affect aggregate productivity by shifting the allocation of resources across firms, while leaving productivity within firms constant. However, common shocks to productivity also change the allocation of resources across firms, because more-productive firms respond to shocks by changing leverage. I use India, which underwent large-scale financial reforms beginning in 1991, as a laboratory to test my model. Using establishment-level micro-data, I find that financial factors explain 71% of Indian labor productivity growth from 1990–1995, and between 2–8% from 1995–2011. My work suggests that other factors besides financial frictions might explain why developing economies lag behind the U.S. in growth and productivity.

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1 Introduction

Although dozens of countries have instituted financial reforms in the past three decades, economists still debate whether and how these reforms affect economic growth. One channel through which better-functioning financial markets might increase economic growth is through a superior allocation of resources, which raises aggregate productivity. While plausible, research focusing on this channel has not tied improved financial markets directly to aggregate productivity.

In this paper, I propose a methodology to gauge the quantitative importance of financial shocks in increasing aggregate productivity. I derive a dynamic equilibrium model in which the allocation of resources across firms is endogenous, depends in part on the degree of financial frictions, and affects aggregate productivity. Tracking the endogenous size-productivity distribution over time allows the model to distinguish financial shocks from other shocks that affect aggregate productivity. In so doing, the model links financial improvements directly to aggregate economic performance.

To illustrate the quantitative applicability of the model, I calibrate it to establishment-level microdata from Indian manufacturing plants. I use my model to identify financial vs. aggregate-productivity shocks in India over the 1990–2011 period. India is an ideal laboratory for testing the effects of financial shocks because it began a major series of financial reforms in 1991, after the start of my sample period. Although the biggest changes occurred near the beginning of my sample, reforms continued over the entire sample period. Importantly, the labor productivity of the Indian manufacturing sector has grown tremendously over my sample period (Figure 1). My methodology allows me to quantify how much of this improved performance is attributable to the increased efficiency of the financial sector.

The basic workhorse behind my quantification exercise is a model that identifies shocks to financial frictions from common productivity shocks by exploiting their distinct effects on the joint distribution of size and productivity. I use an identity that decomposes aggregate productivity into a within-firm component and an across-firm component representing the cross-sectional covariance between size and productivity. The second term of the decomposition is commonly interpreted as a measure of allocative efficiency. I derive a structural model that formalizes this intuition and use it to understand what underlying economic forces affect aggregate productivity.
I find that financial shocks explain a substantial part of productivity growth only in the first few years after the reforms, from 1990 to 1995. From 1995 to 2011, common shocks to labor productivity can explain both the within-firm and the across-firm components of aggregate productivity growth. Changes in financial frictions after 1995 are not as important for understanding productivity growth. Thus my work suggests that factors other than financial frictions may be important for understanding why India and other developing countries continue to lag behind the United States in labor productivity.

Reductions in financial frictions increase aggregate productivity by reducing the cost of capital for borrowing firms, causing these firms to grow faster than other firms. Because in equilibrium borrowing firms are also more productive, reductions in financial frictions increase the covariance between size and productivity without affecting productivity within firms directly. However, common shocks to productivity also affect the covariance between size and productivity, because larger and more-productive firms increase their leverage in response to productivity shocks, while smaller and less-productive firms will not.

Two important assumptions lead to more-productive firms choosing higher leverage ratios: (1) borrowers can default on their debt, and (2) borrowers want to smooth consumption. Borrowers pay a credit spread for any leverage beyond the collateral rate sufficient to cover the lender’s losses should they default. For any level of leverage, more-productive firms represent a better credit risk, and thus enjoy a lower interest rate. In addition, because borrowers have a finite elasticity of intertemporal substitution and productivity is persistent, at any interest rate more-productive firms want to borrow more. Taken together, in equilibrium, more-productive firms will choose higher leverage ratios, and grow faster, than less-productive ones. This mechanism allows me to isolate financial shocks from common productivity shocks by analyzing changes in the joint distribution of size and productivity.

My paper furthers our understanding of how financial markets affect economic growth. Since the pioneering work of King and Levine (1993), a vast literature has shown that economic growth is robustly correlated with larger and more-efficient financial sectors; however, this literature struggles to impute causality. Rajan and Zingales (1998) note that “One way to make progress on causality is to focus on the details of theoretical mechanisms through which financial development affects economic growth, and document their working.” Levine (2005) goes further: “If finance is to
explain economic growth, we need theories that describe how financial development influences resource allocation decisions in ways that foster productivity growth.” In this paper I follow that approach: I propose a model in which a simple, one-parameter representation of financial factors vies with other factors—represented by a common shock to productivity—to determine aggregate productivity. I then apply the model to establishment-level microdata to infer which shocks were most important, and when, in a large developing economy.

2 Literature Review

I contribute to a literature that uses cross-country regression evidence to show that economic growth is highly correlated with the size of the financial sector. Since its beginnings, this literature has struggled to show that causality runs from finance to growth. King and Levine (1993) argued that because financial development predicts subsequent economic growth over the next thirty years, causality does not run from growth to finance. Subsequent work has deepened the basic finance-growth correlation using sophisticated panel-IV methods (Levine, Loayza and Beck 2000) and by showing that more financial intermediation is primarily associated with higher productivity, and not more savings or capital accumulation (Beck, Levine and Loayza 2000). La Porta, Lopez-De-Silanes and Shleifer (2002) show that in addition to the indicators used in Beck, Levine and Loayza (2000), higher government ownership of the banking sector is strongly associated across countries with lower aggregate productivity. This is important because India had a very high percentage of government ownership and control of the banking sector before reforms began, and has worked continually over the sample period to reduce this percentage.

My paper also contributes to a related literature that seeks to draw out the mechanisms whereby financial innovation affects economic growth using industry- and firm-level data, rather than country-level aggregate variables. In a seminal paper Rajan and Zingales (1998) argue that better-functioning financial markets should disproportionately affect industries which are more dependent on external finance, by lowering their cost of capital; they extract a measure of this external dependence from U.S. firm-level data, and show that measures of financial development do exert a greater influence on the growth rates of externally-dependent industries across countries. Wurgler (2000) also uses industry-level data to show that the elasticity of investment growth to value-added
growth is higher in more financially-developed countries, suggesting that better-functioning financial markets improve the allocation of resources. Beck, Demirgüç-Kunt and Maksimovic (2005) use cross-country firm-level survey data to show that financial factors are an important obstacle to firm growth, especially for small firms. The results of this literature suggest that improvements in financial markets remove constraints on the growth of some industries and firms, enhancing allocative efficiency and through it, aggregate productivity and economic growth. I contribute to this literature by linking financial frictions at the firm level directly to aggregate productivity, using both theory and empirical evidence.

This paper speaks to a growing literature that measures changes in allocative efficiency over time. In an influential paper Olley and Pakes (1996) decompose aggregate productivity into a within-plant term and an across-plant term representing the extent to which larger plants are more productive. They then relate changes in the second term to regulatory changes that affected the telecommunications industry in the United States. Bartelsman, Haltiwanger and Scarpetta (2013) perform the same decomposition for all manufacturing industries in a number of European countries and interpret the across-plant term as an overall measure of allocative efficiency, showing that it is high for more-developed economies like the United States and Germany, but lower—though growing over time—for some formerly-Communist countries in Eastern Europe. I derive an equilibrium model that relates the across-plant term directly to economic shocks—in particular changes in financial frictions. I then apply the model to another country in which the across-plant term has grown over time, and use it to infer the sources of that growth.

My paper also contributes to a related literature that ties productivity differences across countries to the misallocation of resources. In a seminal paper, Hsieh and Klenow (2009) use a parsimonious model to measure how much of the productivity difference between the U.S. and two large developing economies is due to a poor allocation of resources. My paper is also related to recent work by Banerjee and Moll (2010) and Midrigan and Xu (2014), who propose models with financial frictions to explain this poor allocation. They find that any misallocation due to financial frictions disappears quickly over time because firms can grow their way out of binding financial constraints. Their result depends on highly persistent shocks and rapidly decreasing returns to scale at the firm level. I also find that financial frictions had little impact, at least in India from 1995 to 2011, but through a different channel: the shocks in my model are persistent, too, but because returns to
scale are constant, firms never grow out of financial constraints. Thus, financial shocks have the potential in my model to exert a strong influence on aggregate productivity. Nevertheless, I find that changes in the size-productivity distribution in Indian manufacturing plants since 1995 are consistent with shocks other than financial shocks.

My paper relates to a new wave of macroeconomic research that adds financial factors to macroeconomic models. Much of this research, including Gertler and Kiyotaki (2010), Gertler, Kiyotaki and Queralto (2012), Buera and Moll (2012), and Jermann and Quadrini (2012), assumes a maximum leverage constraint as a parsimonious way to model financial frictions. In these models debt is assumed riskless by requiring that borrowing be fully collateralized; often this is justified as satisfying an incentive-compatibility constraint. My paper enriches the financial side of these models by noting that lenders are not interested in whether individual loans are incentive-compatible; in equilibrium they only care about the return on their entire portfolio. Lenders can earn the same expected return by charging borrowers a higher interest rate to compensate them for default risk. Borrowers, rather than facing a “hard” borrowing constraint, instead face an interest rate schedule that depends on their productivity and their choice of leverage. In equilibrium, this makes the size-productivity distribution across firms highly sensitive to financial shocks.

Finally, I contribute to the corporate finance literature on capital structure by analyzing the leverage decisions of small, privately-owned firms. Most empirical work on capital structure focuses on publicly-traded firms, for example firms in the Compustat database. This may be why the canonical theories of optimal capital structure in a dynamic setting, such as Leland (1994, 1998) and Hennessy and Whited (2005, 2007), focus on the trade-off between debt and external equity, usually from the standpoint of a borrower balancing the tax advantages of debt against the costs of bankruptcy and (frequently) some equity-issuance costs. This trade-off is most appropriate for large firms with access to public equity markets. Other quantitative and empirical studies of capital structure, such as Strebulaev (2007), Bhamra, Kuehn and Strebulaev (2010), Chen (2010), Korteweg (2010), and Glover (2014) also focus on the tax treatment of debt vs. equity and on firms large enough to be able to exploit it. In contrast, I analyze data on the financial decisions of firms from a representative cross-section of manufacturing firms, most of which are small and unlikely to have access to public equity markets, and how they affect aggregate outcomes.
3 Empirical Evidence

In this section I describe the Indian economic reforms mentioned in Section 1, and use microdata from Indian manufacturing plants to decompose aggregate productivity growth into a common factor within plants and a factor representing the allocation of resources across plants. In Section 3.1 I describe the Indian macroeconomic and financial reforms in more detail. I describe the data in Section 3.2, and perform the decomposition of aggregate productivity in Section 3.3. For more details on the data, see Appendix A.

3.1 Background

India instituted a number of economic reforms in 1991 after a severe balance of payments crisis. Many of the reforms affected the manufacturing sector directly. For instance, the government essentially abolished the system of industrial licensing, a major constraint on investment and output for registered manufacturing firms (Aghion et al. 2008). In addition, a large number of industries that had been reserved solely for the government were opened to up to private entry (Ahluwalia 2002). Reforms in 1991 that took immediate effect also include a massive drop in tariffs, especially for capital goods, and freeing up of foreign direct investment restrictions (Joshi and Little 1996). Both of these reforms are likely to have made it easier for productive firms to expand production.

In addition to reforms targeted directly at the manufacturing sector, the government also instituted various financial reforms in 1991. Chief among these were changes to the Indian banking system, which was dominated by poorly-performing government-owned banks. Accounting rules that allowed banks to hide non-performing assets were changed, and public banks with negative net worth were recapitalized by the government. High reserve ratios, whose main purpose was to pre-empt banking resources to finance the government deficit, fell dramatically in 1991. The government also allowed for more private entry into the banking sector, including allowing the public-sector banks to issue raise capital in public equity markets, diluting the amount of government ownership (Joshi and Little 1996).

Not all Indian financial reforms occurred immediately after the 1991 crisis. The Recovery of Debts Act, which established tribunals in several major cities to aid in the process of recovering bad debts, was passed in 1993 but did not become effective immediately due to challenges in the court
system (Joshi and Little 1996). The Securitisation and Reconstruction of Financial Assets and Enforcement of Security Interest Act of 2002 (known as the Sarfaesi Act) allowed for the creation of asset reconstruction companies to which banks could auction non-performing loans (Rajan et al. 2009). Finally, the process of removing the government from control of the banking sector has also proceeded slowly since 1991 (Rajan et al. 2009).

3.2 Data

I combine data from two sources, the Annual Survey of Industries (ASI) and the National Sample Survey (NSS). The ASI is a survey of manufacturing establishments in India that are registered under the 1948 Factories Act. These establishments generally have 10 or more employees (20 or more if they don’t use power) and include the largest factories in India. By contrast, the NSS is a survey of unregistered plants, which are typically much smaller than those in the ASI.¹ Because the NSS is only administered to manufacturing plants roughly every five years, I only have five years of combined ASI-NSS data: for 1989–1990, 1994–1995, 1999–2000, 2005–2006, and 2010–2011 (data years refer to the Indian fiscal year, which runs from April to March).

[Table 1 about here.]

Table 1 reports various summary statistics for the NSS and ASI datasets separately. Panel A reports the total number of manufacturing plants and employees, as well as the NSS shares of firms, employment, and output (real value-added in 1993–1994 rupees) over time. The Indian manufacturing sector employs between 27 and 46 million people, and this number has been rising over time. Over 99% of the 12 to 17 million manufacturing plants in India are in the unregistered, informal sector covered by the NSS. These plants account for about fourth-fifths of manufacturing employment in India, but only between a sixth and a quarter of manufacturing output. Thus, NSS plants must be much smaller, and substantially less productive, than their ASI counterparts.

Panel B of Table 1 reports the employment distribution of plants in the NSS and ASI. NSS plants employ on average only two employees, compared to 67-135 employees on average in the ASI. Median employment in the ASI is much lower at about twenty employees, reflecting the substantial

¹Although I will refer to plants in the NSS as “informal” plants or representing the “informal” sector, NSS respondents are not informal in the sense of operating illegally or evading taxes. They are merely manufacturers that are not covered by the 1948 Factories Act, and as a result are not part of the sampling frame of the ASI.
positive skewness in the plant-size distribution in the ASI. The employment measure used in this paper is average number of employees over the course of the year, including administrative and part-time employees.

Plants in the ASI are not only larger, but much more productive than plants in the NSS. Panel C of Table 1 reports the distribution of (log) labor productivity in the two datasets. Plants in the ASI are roughly 200 log-points more productive than plants in the NSS, though productivity is rising over time for both types of plants. I define productivity as the natural logarithm of real value-added, in 1993-1994 rupees, per employee. Value-added is total revenue, including revenue from non-production activities, less total costs, including materials inputs and other expenses but excluding the cost of labor. I deflate nominal value-added using the industry-level Wholesale Price Indices; see Appendix A for details.

I focus on revenue, rather than physical, productivity in this paper in order to include the NSS plants in my analysis. Revenue productivity can differ from physical productivity when plants in the same industry charge different prices; see Foster, Haltiwanger and Syverson (2008) for an analysis of this phenomenon in the US. Physical productivity would be a preferable measure because the model in section 4 is based on physical productivity. Although I do observe plant-level prices for a subset of plants in the ASI, I do not observe plant-level prices in the NSS.\footnote{Not all ASI respondents report the units of the output good to which the price refers, making the reported price almost worthless.} Because the NSS plants are such a substantial fraction of employment and output in India, I opt to include them and focus on revenue productivity for the empirical analysis of this paper.

Although plants in the NSS are much smaller than those in the ASI, they do have access to external sources of credit that were affected by the Indian financial reforms described in section 3.1. Table 2 reports statistics on borrowing across the two datasets. Between six and ten percent of NSS plants have loans outstanding, though this number is decreasing over time. ASI plants are much more likely to borrow; between two-thirds and three-quarters of ASI plants report having loans. Because ASI plants are so much bigger than NSS plants, and are more likely to have a loan at all, this means that plants in NSS account for between 3% and 7% of total debt outstanding in
the combined ASI-NSS data. The tiny fraction of total borrowing accounted for by NSS plants is consistent with the model developed in section 4, because NSS plants are so much less productive than their ASI counterparts.

[Figure 2 about here.]

Figure 2 plots the sources of credit for NSS plants over time. About 60% of NSS loans come from commercial banks, including state-owned banks and other government sources (such as the Khadi & Village Industries Commission). This is significant because many of the financial reforms covered in section 3.1 were specifically targeted at making the banking sector more efficient. The remaining 40% of NSS credit is about evenly split between money-lenders, loans from friends, family, and business partners, and other sources.

3.3 Aggregate Productivity

In this section I decompose aggregate manufacturing productivity according to an identity derived by Olley and Pakes (1996):

\[
\text{Aggregate Productivity} \equiv \sum_i w_i z_i = \frac{1}{N} \sum_{i=1}^{N} z_i + \sum_i (z_i - Z)(w_i - \overline{w})
\]

(1)

where \( z_i \) is log labor productivity at plant \( i \), \( w_i \) is plant \( i \)'s share of total industry employment, and \( \overline{w} \) is the unweighted average of \( w_i \), \( 1/N \). I define aggregate productivity in this identity as a weighted average of plant-level productivities, which because the weights \( \{w_i\} \) sum to one can be decomposed into an unweighted average \( Z \) across plants plus a term \( C \) representing the covariance between the weights (plant size) and productivity. The level of \( C \) indicates how much aggregate productivity would drop if, holding all plant productivities fixed, employment were re-allocated uniformly across plants. In this sense \( C \) is a measure of allocative efficiency.

[Table 3 about here.]

\(^3\)ASI plants do not break down their sources of credit.
Table 3 reports values of the three terms in equation (1) over time in India according to the combined ASI-NSS data. Over the twenty-one years in my sample, aggregate labor productivity in India has increased by over 150 log-points. A substantial portion of this increase is due to changes in the covariance term $C$, which has increased by 33 log-points. In section 4 I derive a model to interpret the changes over time in these aggregates in terms of structural aggregate shocks.

[Figure 3 about here.]

The rise in $C$ over time is mainly due to changes within industries, and not to changes in the industry composition. Figures 3 and 4 plot the values of $Z$ and $C$, respectively, for different sets of aggregation weights across industries. The top and bottom panels of Figure 3 plot the values from Table 3 as solid black lines. These values are computed at each date by averaging across industries using each industry’s share of aggregate employment at that date. The dotted lines in Figure 3 repeat the calculation but using constant industry weights, using either the 2010–2011 weights (top dotted line) or the 1989–1990 weights (bottom dotted line). Some of the change in average $C$ is due to a shift over time towards industries with a higher value of $C$, since the 2010–2011-weights line is higher than the other two, but all three lines feature a dramatic increase in $C$ over the sample period. In addition, $C$ rises very quickly in the first five years of the sample, while $Z$ is essentially flat, regardless of which weights are used; this is the primary feature which leads the model to infer large changes in financial frictions early in the sample.

[Figure 4 about here.]

The main pattern of evolution over time for $Z$ and $C$ in Figure 3 also holds for subsets of industries. Figure 4 uses dynamic weights but splits industries into three groups, based on their share of NSS employment. Splitting industries according to their share of NSS output yields similar results. I chose the breakpoints used in Figure 4 so that each group of industries represents roughly the same fraction of aggregate employment. Industries with a smaller share of NSS employment have higher levels of $C$, though in all three groups $C$ rises most quickly over the first five years of the sample, just as in Figure 3. Average productivity $Z$ is falling during the first five years for

\footnote{Tables 5, 6, and 7 in Appendix A report individual-industry values of these measures.
industries with high NSS shares, and it rises no more quickly for other industries during those years than in other years.

The level of the covariance term $C$ in India (averaged across all industries) is quite high when compared to the values reported in Table 1 of Bartelsman, Haltiwanger and Scarpetta (2013). Bartelsman, Haltiwanger and Scarpetta (2013) compare the levels of $C$ across developed and developing countries. They find that while in the U.S., often considered the peak of allocative efficiency (see for example Hsieh and Klenow 2009, 2014), $C$ is 0.51, in the “transition” economies of Eastern Europe, such as Romania and Slovenia, $C$ is closer to 0 and rising over time.

However, the levels of $C$ reported in Table 3 are not directly comparable to those reported in Table 1 of Bartelsman, Haltiwanger and Scarpetta (2013), for two reasons. First, Bartelsman, Haltiwanger and Scarpetta (2013) construct a harmonized dataset that ensures consistency across countries in definitions and key concepts. Differences in the level of $C$ could be due in part to definitional changes across the data, or even the inclusion of informal establishments, which may be a less important feature of the economies they study. Second, Bartelsman, Haltiwanger and Scarpetta (2013) use the U.S. industry weights when they average the $C$ term across industries in all countries, so that any difference in $C$ must be due to within-industry differences and not the industry composition in a particular country. As I show in Table 5 in Appendix A, there is substantial dispersion across industries in the level of $C$, and the industries of primary importance in India (Weaving, Tobacco, Apparel, and Grain Milling) are quite different from the high-employment industries in the U.S. (Plastic Products, Motor Vehicles, Meat Processing, and Printing, according to the 2013 Annual Survey of Manufactures).

Although comparing the level of $C$ across countries is not appropriate, it could still be the case that changes in $C$ within a country over time indicate changes in allocative efficiency. In the next section I derive a structural model of firm investment and growth in which financial frictions have a direct impact on allocative efficiency. I calibrate the model to match moments from the Indian data and to infer interpretable fundamental shocks from the observed values of $Z$ and $C$.

A key moment for the model to match is the covariance between size and productivity. In India this covariance is extreme: the top panel Figure 5 plots the distribution of real log labor productivity across plants in six employment categories. Although there is substantial dispersion in productivity across plants, the entire distribution of productivity is shifted upwards for plants
with more than five employees. The bottom panel of Figure 5 reports the shares of aggregate employment accounted for by each size bin, for reference. Each bin accounts for a large share of aggregate employment, although over 60% of Indian manufacturing employees (leftmost three categories) work in low-productivity establishments with less than five employees.

[Figure 5 about here.]

In order for my model to match the strong correlation between size and productivity in the Indian data, it departs from standard heterogeneous-agent models—such as Banerjee and Moll (2010), Hsieh and Klenow (2009, 2014), and Midrigan and Xu (2014)—that rely on decreasing returns to scale, either in the the physical production function or through monopolistic competition. In such models the covariance between size and productivity should be zero, because in an optimal allocation all firms equalize their productivity to a common cost of capital. In fact, this modeling choice leads Hsieh and Klenow (2009) to infer that the allocation of resources in India actually got worse from 1987 to 1994, despite using similar data to that in this analysis (the ASI). They note that this is somewhat puzzling given the economic reforms that took place.

In contrast, the model derived in section 4 will naturally generate a positive covariance between size and productivity, because the marginal productivity of firms does not diminish as they grow, and equilibrium occurs by varying the cost of capital across firms according to their leverage. Furthermore, this covariance will depend strongly on the level of financial frictions; an increasing covariance, such as that seen in India over my sample period or in the developing economies studied by Bartelsman, Haltiwanger and Scarpetta (2013), reflects a reduction in financial frictions.

4 Model

In this section I derive a model in which entrepreneurs borrow to smooth consumption in the face of persistent idiosyncratic productivity shocks, but cannot commit to repaying their debt. I then put many such entrepreneurs together in industry equilibrium and compute the endogenous size-productivity distribution, as well as model-implied values of the terms from equation (1). In the model, reductions in financial frictions increase the covariance term $C$ because they primarily affect more-productive firms, who borrow more and grow faster on average than less-productive firms.
Sections 4.1, 4.2, and 4.3 derive the model, solve it, and characterize the equilibrium. In section 4.3 I assume, for tractability, that agents have log utility; this assumption along with a constant returns to scale production technology ensures that the endogenous joint distribution of wealth and productivity does not enter as a state variable in individual’s agents’ maximization problems, as tends to happen in models with heterogeneous agents and aggregate shocks, such as Krusell and Smith (1998). Section 4.4 derives the law of motion of the joint distribution of size and productivity, and describes the calculation of aggregate capital and the model-implied values of $Z$ and $C$. For details on the formulas, see Appendix C.

4.1 Environment

Time is countable and there is a consumption good (the numeraire) and an output good with price $P$. There is a continuum of agents, each of whom is risk-averse and discounts the future at a rate $\beta \leq 1$. Each agent $i$ has idiosyncratic productivity $z_{i,t}$ which evolves according to the AR(1) process

$$z_{i,t+1} = \rho z_{i,t} + \sigma \varepsilon_{i,t+1},$$

(2)

where $\varepsilon_{i,t}$ is a standard normal random variable that is independent across agents and time. In addition to their mean-zero idiosyncratic productivity, agents have a time-constant average productivity $\bar{z}_i$ that may differ across agents and is common knowledge. Specifically there are $N$ types of agents, each a fraction $\mu_i$ of the unit measure of agents. Agents have their $\bar{z}_i$ fixed for life and can never change it.\footnote{From this point on I will omit the $\bar{z}_i$ term in expressions where it is not necessary to understand the derivation, and where it is especially cumbersome.} This will allow the model to capture the disparity between the formal and informal sectors discussed in Section 3.

There are two aggregate exogenous variables: the first, $Z_t$, represents a common component in productivity shared by all agents. In addition, all agents share a common recovery rate $\zeta_t < 1$, which represents the fraction of their wealth that the agent can credibly promise to creditors.

Agent $i$ owns a stock of productive assets $k_{i,t}$ at time $t$, and can operate a production technology...
that produces the output good according to

$$y_{i,t} = A \epsilon^{z_{i,t} + \tau_i + Z_i k_{i,t}}.$$  

(3)

Agents can freely convert the consumption good into units of capital, and vice versa, at a relative price of 1; there are no capital adjustment costs. Capital does not depreciate, and must be non-negative.

Agents face an exogenous demand curve for their output good, so that the output price $P$ satisfies

$$\log P = -\eta \log Y + D$$  

(4)

where $\eta$ and $D$ are parameters and $Y$ in equation (4) is aggregate output from all agents. Agents take the equilibrium output price $P$ as given when making their investment and financing decisions.

Financial markets are incomplete and the only asset that agents may trade are one-period zero-coupon bonds. In order to match the dispersion in productivity in the combined ASI-NSS data—see section 5.1—I assume that agents cannot lend to each other; lenders in this economy are agents outside this production sector. Let $q$ denote the price of one unit of face value of the agent’s debt, and the agent’s chosen face value $b$. $b < 0$ denotes that the agent is borrowing funds.

Agents are free to borrow as much as they wish, but they cannot commit to repaying their debt in the future. The timing of the default decision is as follows: at the beginning of each period, the agent learns her own productivity value $z_{i,t}$. At that moment she decides whether she will pay back her borrowing (if $b < 0$) or default. If she defaults, she retains a fraction $1 - \zeta$ of her own capital stock $k$, may not produce using the production function (3), and sets $b = 0$. The lender recovers a total amount $\zeta k$, that must be distributed among a total face value $b$, so that per bond the lender recovers

$$\chi \equiv \min \left\{ 1, \frac{\zeta k}{-b} \right\}$$

$$= \min \left\{ 1, \frac{\zeta}{b} \right\}.$$  

(5)
where the last line defines the agent’s leverage as \( \ell \equiv \frac{-b}{k} \). The min ensures that lenders do not recover more than they were owed; if agents default with enough assets such that lenders would recover more than they were owed, the extra resources are lost. This will not occur in equilibrium.

Lenders are perfectly competitive and require an exogenous expected return of \( r \) on their portfolios. Lenders understand that borrowers may default and that if they do, the lenders only recover \( \chi \leq 1 \) per bond, and they incorporate this default risk into the zero-coupon bond price \( q \). For their part borrowers understand that the bond price they pay per unit of face value depends on the current state and their choices for investment and borrowing.

In equilibrium the realized return to each lender’s portfolio of bonds is riskless at an exogenous level \( r \). This is because lenders can lend to a sufficiently diverse cross-section of borrowers such that a law of large numbers applies and their return is riskless. A key tractability assumption behind this result is that the shocks to \( Z \) and \( \zeta \) occur after default decisions are made and production has occurred, but before new borrowing, investment, and consumption decisions are made. This means that when agents borrow, the only unknowns are the realizations of the idiosyncratic shocks \( \varepsilon_{i,t+1} \), which are diversifiable by lenders. This is possible because the bond price \( q \) charged to any individual borrower can perfectly offset that borrower’s default risk, so that the lender’s expected return is constant across loans and that is the return they realize on their entire portfolio.

Each period agents choose consumption \( c \), capital next period \( k' \), and financial assets \( b' \). The budget constraint for an individual agent is then

\[
c + q(k', b', z; Z, \zeta, P) b' + k' = \begin{cases} 
PY + k + b & \text{if debt is repaid} \\
(1 - \zeta) k & \text{if defaulted on debt}
\end{cases}
\]

where the bond price \( q \) depends on the agent’s choices for capital and savings, their productivity \( z \), aggregate productivity \( Z \), the aggregate recovery rate \( \zeta \), and the equilibrium output price \( P \).

In order to ensure a stationary distribution across agents I assume that they exit the economy exogenously with probability \( \pi \) each period.\(^6\) This assumption is necessary for modeling an equilib-

\(^6\)For technical reasons I assume that agents learn whether they will exit early in the period, immediately after their productivity \( z \) is realized. This timing assumption, which is also used by Gilchrist, Sim and Zakrajšek
rium with many agents because each agent’s log wealth will be a random walk with drift, so that if
agents live forever the log difference in wealth between any two agents will also be a random walk
and one agent will quickly make up 100% of the economy.

All of the timing assumptions of the model are illustrated in Figure 6.

4.2 Individual Agent’s Problem

Agents choose consumption, investment, and borrowing to maximize the expected discounted flow
of utility. The description above leads to the following recursive representation of a single agent’s
problem:

\[
V\left(k, b, z; Z, \zeta, P, \tilde{F}\right) = \max_{k' \geq 0, b' \leq 0} \left[ u(c) + \beta \left(1 - \pi_d\right) E\left\{V\left(k', b', z'; Z', \zeta', P', \tilde{F}'\right)\right\} + \pi_d E\left\{V^d\left(k', b', z'; Z', \zeta', P', \tilde{F}'\right)\right\}\right]
\]

s.t.

\[
c \equiv (PAe^z + 1) k + b - q\left(k', b', z; Z, \zeta, P\right) b' - k'
\]

where \(u\) is the agent’s utility function, \(Z\) is the current level of aggregate productivity, \(\tilde{F}\) is the
joint distribution function of \((k, b, z)\) across all agents, \(V\) is the agent’s value function conditional
on surviving to the next period, \(V^d\) is their value when they receive the exogenous exit shock at
the beginning of the period, and the laws of motion for the agent’s indiosyncratic productivity \(z\)
is given in equation (2). \(P\) and the wealth distribution function \(\tilde{F}\) are endogenous and potentially
important state variables to individuals; I describe their calculation and law of motion below. The
value function in equation (6) is the agents’ value after the default decision and the aggregate shock
have been realized, right before deciding how much to invest, borrow, and consume (just to the
right of the large red line in Figure 6). Thus the expectation in equation (6) is taken with respect
to the idiosyncratic shock \(\varepsilon_i\) alone, and \((Z, P)\) refers to the aggregate productivity and output price
that will apply to production next period.

There are several simplifications to the basic recursive relation (6) that can be performed at
\(2014\) and Khan, Senga and Thomas (2014), ensures that agents cannot exit the model exogenously with out-
standing debts. Otherwise, the exogenous death risk would enter the bond-price equation, where it would create
non-differentiabilities that complicate the model solution without adding anything substantive.
this point. The value function $V^d$ is straightforward: because she will not continue to the next period, the agent has no desire to save any resources and lenders know that she cannot pay back any debt were she to borrow. Therefore she optimally consumes all her resources. The dependence of $V$ on $k$ and $b$ separately in equation (6) is not necessary; agents need only know their net wealth $x \equiv [PAe^z + 1]k + b$ (or $(1 - \zeta)k$ if they had chosen to default), not how it separates into real and financial assets. This means that the endogenous distribution of interest is no longer $F(k, b, z)$, but $F(x, z)$. At the same time the bond price does not depend on $k'$ and $b'$ separately, but only on their ratio $\ell = \frac{b}{k}$.

Finally, the bond price can be derived in closed-form as follows: suppose the agent’s productivity in the next period (inclusive of aggregate productivity) is $z'$. If she does not default, her net wealth will be $x' = (PAe^{z'} + 1)k + b$, whereas if she defaults her net wealth will be $x' = (1 - \zeta)k$. Equating these two values and plugging in the law of motion (2) leads to a cutoff rule for default: the agent will default on her debt if her productivity shock is less than or equal to $\xi$ that satisfies

\[
\xi = \begin{cases} 
\frac{1}{\sigma} \left[ \log (\ell - \zeta) - \log (A) - \bar{z} \right] & \ell > \zeta \\
-\infty & \text{otherwise}
\end{cases}
\] (7)

where $\bar{z} \equiv \rho z + Z + \log P$ is the agent’s forward-looking productivity.

From equation (7) it is straightforward to show that lenders earn an expected return of $1 + r$ if the bond price satisfies

\[
q(k', b', \bar{z}; \zeta) = q(\ell, \bar{z}; \zeta) = \frac{1}{1 + r} \left[ 1 - (1 - \chi(\ell, \zeta)) \Phi \{\xi\} \right],
\] (8)

where $\chi(\ell, \zeta)$ is given by equation (5) and $\Phi \{\cdot\}$ is the CDF of the standard normal distribution.
These changes lead to a simplified recursive representation:

\[
V (x, z; Z, \zeta, P, F) = \max_{k' \geq 0, b' \leq 0} u(c) + \beta \left[ (1 - \pi) E \left\{ V (x', z'; Z', \zeta', P', F') \right\} + \pi E \left\{ u (x') \right\} \right]
\]

s.t.

\[
\begin{align*}
\epsilon & \equiv x - qb' - k' \\
q & \equiv q (\ell, \tilde{z}; \zeta) \\
\tilde{z} & \equiv \rho z + Z + \log P \\
x' & = \max \left\{ [A e^{\tilde{z} + \sigma \epsilon} + 1] k' + b', (1 - \zeta) k' \right\},
\end{align*}
\]

where \( F \) now denotes the joint distribution across all agents of \((x, z)\).

### 4.3 Equilibrium

In order to ensure a stationary equilibrium, agents will have to enter the model exogenously to "replace" those that exit. Assume a continuum of agents and normalize its measure to 1. Because each period a measure \( \pi \) of agents will exit the model, a measure \( \pi \) of agents must enter the model each period to maintain stationarity. Let these new agents have net wealth \( x \) drawn from the lognormal distribution \( \log x \sim \mathcal{N} \left( 0, \sigma^2 x_0 \right) \) and their independent idiosyncratic productivity \( z \) is drawn as \( z \sim \mathcal{N} \left( 0, \sigma^2 z_0 \right) \). Fixing the cross-sectional entry distribution for \( x \) and assuming \( \pi > 0 \) ensures a stationary distribution for \( x \) across agents even though each agent’s \( \log x \) will be a random walk with drift in equilibrium.

Agents take the output price \( P \) as given when making their borrowing and investment decisions, but in equilibrium \( P \) will be determined according to the demand curve \( (4) \). Define equilibrium as

1. Each agent of type \( i \) chooses \( k' = k_i (x, z; Z, \zeta, P, F) \) and \( b' = b_i (x, z; Z, \zeta, P, F) \) to solve their individual problem \( (9) \), appropriately extended to include the fixed productivities \( \tilde{z}_i \). All agents take \( P \) and the function \( q (\cdot, \tilde{z}; \zeta) \) as given (the latter depends on \( \tilde{z}_i \) as well).

2. Aggregate output (next period) is

\[
Y = \sum_i \mu_i \int A \exp \left\{ \rho z + \tilde{z}_i + Z + \sigma \epsilon \right\} k_i (x, z; Z, \zeta, P, F) \phi (\epsilon) d\epsilon dF_i (x, z)
\]
where \( F_i(X, z) \) is the joint distribution function of wealth and productivity for agents with average idiosyncratic productivity \( \overline{z}_i \).

3. The output price \( P \) satisfies

\[
\log P = -\eta \log Y + D.
\]

Note that aggregate output next period \( Y \) (and the output price \( P \)) are predetermined because the only uncertainty is the idiosyncratic shock \( \varepsilon \).

To further simplify the solution of the model, assume that the agent’s utility function is \( u(c) = \log(c) \). This leads to the following proposition:

**Proposition 1.** Suppose \( u(c) = \log(c) \). Then \( V(x, z; Z, \zeta, P, F) = a_0 + a_1 \log x + f(z; Z, \zeta, P, F) \) solves equation (9) for constants \( a_0 \) and \( a_1 \) and some function \( f(\cdot) \), and the optimal policies \( (c, k', b') \) satisfy

\[
k' + q(\ell, \tilde{z}; \zeta) + b' = \beta^* x
\]

\[
c = (1 - \beta^*) x
\]

for some \( \beta^* \leq \beta \).

**Proof.** See Appendix B.

Proposition 1 greatly simplifies the analysis of the effects of aggregate shocks, because it means that optimal policies can be solved “state by state” rather than all at once, and removes any dependence of policies on the complicated state variable \( F \), which is a distribution function. This is because optimal consumption under log utility is independent of everything except net wealth \( x \). This ensures that agents are “optimally myopic” and have no hedging motives; that is, their decisions are affected by the current state only through its effect on current variables, not through what information (if any) the current state implies about future variables. In other words, the law of motion of the aggregate state (including the joint distribution of size and productivity) do not enter agents’ decision rules. Without log utility and Proposition 1, I would have to solve the
individual’s agent’s problem given an assumed law of motion for the endogenous aggregate state, and then use the implied policy functions to check whether the assumed laws of motion are correct.

The next proposition characterizes the investment and financing decision of agents in more detail.

**Proposition 2.** The agent’s optimal policy satisfies a threshold rule in \( \tilde{z} \equiv \rho z + \tilde{z}_i + Z + \log P \):

- If \( \tilde{z} \leq \bar{z}(r) \), then \( b' = 0 \) and \( k' = \beta^* \).
- If \( \tilde{z} > \bar{z}(r) \), then \( k' = \frac{\beta^*}{1 - q}\ell \), \( b' = -k'\ell \), and \( \ell > 0 \) solves

\[
q + \frac{\partial q}{\partial \ell} \frac{\ell}{1 - q\ell} = \int_{\tilde{z}(\ell)}^{\infty} \frac{\phi(\varepsilon) \, d\varepsilon}{A e^{\tilde{z} + \sigma\varepsilon} + 1 - \ell},
\]

(12)

- \( \bar{z}(r) \) is the value of \( \tilde{z} \) that solves

\[
1 = (1 + r) \int_{-\infty}^{\infty} \frac{\phi(\varepsilon) \, d\varepsilon}{A e^{\tilde{z} + \sigma\varepsilon} + 1},
\]

(13)

**Proof.** See Appendix B. \( \square \)

Proposition 2 characterizes the investment and borrowing decisions of all agents in the economy as a function of their forward-looking productivity \( \tilde{z} \). The left-hand side of equation (12) represents the marginal benefit of an additional unit of leverage, which depends not only on the bond price \( q \) but also on how quickly that price is changing as the agent increases leverage. Agents optimally equate this marginal benefit with the marginal cost of an additional unit of leverage, on the right-hand side of equation (12). The marginal cost integrates over the idiosyncratic shock \( \varepsilon \), but is only paid if agents do not default on their debt, so the lower limit of integration is \( \tilde{z} \).

Equation (13) is equation (12) with \( \ell = 0 \); the level of \( \tilde{z} \) for which this equation is satisfied is the cutoff value for forward productivity at which agents are just indifferent between borrowing and not. For productivity just below this value, they do not borrow and set \( k' = \beta^* \).

### 4.4 Aggregation

In this section I close the model by describing the time-series evolution of the endogenous wealth-productivity distribution and the calculation of the various aggregates used in Section 5. The latter
include quantities used in the calibration, such as aggregate output and capital for agents of each type \( i \), as well as the cross-sectional standard deviation of productivity. I also compute the OP decomposition terms \( Z \) and \( C \) in this section. For detailed formulas, see Appendix C.

I characterize the cross-sectional joint distribution of size and productivity for agents of type \( i \) at time \( t \) as a cumulative distribution function \( F_{i,t}(\log x^*, z^*) \), where \( i \) indexes the fixed-effect in idiosyncratic productivity \( \tau_i \). This function is the probability that a randomly-drawn firm at time \( t \) with fixed effect \( \tau_i \) has net wealth \( x < x^* \) and idiosyncratic productivity \( z < z^* \). Because agents never change their value of \( \tau_i \), I can compute the CDFs for each type of agent separately and aggregate across them according to the measure \( \mu \) later.

I parameterize \( F \) in terms of \( \log x \) instead of \( x \) because in equilibrium this will behave better computationally. For fixed values of \( P_{t+1}, \zeta_t, Z_{t+1} \), and the function \( F_{i,t} \), the function \( F_{i,t+1} \) is given by

\[
F_{i,t+1}(\log x^*, z^*) = (1 - \pi) \int \Phi \left\{ E_i \left( \frac{x^*}{x}, z^*, z; Z_{t+1}, \zeta_t, P_{t+1} \right) \right\} dF_{i,t}(\log x, z) + \pi F^e(\log x^*, z^*) ,
\]

where \( F^e(\cdot, \cdot) \) is the CDF of \( (\log x, z) \) for the agents that enter exogenously each period, and the function \( E_i \) is the conditional transition CDF for agents of type \( i \), which depends on the decision rules of individual agents and the law of motion of the idiosyncratic productivity \( z \) (see Appendix C for details). I use equation (14) to compute the evolution over time of the size-productivity distribution, and also to compute the steady-state distribution where \( F_{i,t+1} = F_{i,t} \).

Given a distribution \( F_t \), I compute aggregate output \( Y \) and capital \( K \) as integrals over \( dF_{i,t} \), the idiosyncratic productivity shocks \( \varepsilon \), and the agent-type distribution. The aggregate capital stock and output at \( t + 1 \), \( K_{t+1} \) and \( Y_{t+1} \), are given by

\[
K_{t+1} = \sum_i \mu_i \int \left[ k(\tilde{z}; \zeta_t) x \left[ 1 - \Phi \{ \xi(\tilde{z}, \zeta_t) \} \right] \right] dF_{i,t}(\log x, z)
\]

\[
Y_{t+1} = \sum_i \mu_i \int \left[ A \exp \left\{ \tilde{z} + \frac{1}{2} \sigma^2 + \log x \right\} k(\tilde{z}; \zeta_t) \left[ 1 - \Phi \{ \xi(\tilde{z}, \zeta_t) - \sigma \} \right] \right] dF_{i,t}(\log x, z)
\]

\[
\tilde{z} \equiv \rho z + \tau_i + Z_{t+1} + \log P_{t+1}
\]

(15)
The capital that will be used in production at $t + 1$ is chosen by all firms at time $t$, so that $K_{t+1}$ is time-$t$ measureable. This is true even though I don’t know at $t$ which firms will default; by the law of larges numbers I know the measure of firms which will default. This is the role of the $1 - \Phi\{\cdot\}$ term in equation (15). Total output $Y_{t+1}$, and thus the output price $P_{t+1}$, are also be time-$t$ measureable, because the only unknown at time $t$ after investment occurs is the idiosyncratic shock $\varepsilon$ (the next aggregate productivity shock occurs after production takes place; see Figure 6).

From equation (15) it is straightforward to calculate the shares of agents of type $i$ in aggregate output and capital.

To compute the cross-sectional disperion in productivity and the OP decomposition terms $Z$ and $C$ in equation (1), I define the log-productivity of an agent of type $i$ with idiosyncratic productivity $z$ as

$$\log \frac{y}{k} = \log A + Z + z_i + \rho z + \sigma \varepsilon.$$  \hspace{1cm} (16)

I compute the unweighted-average productivity $Z$ by integrating equation (16) over the densities $F_{i,t}$, the distribution of $\varepsilon$, and the distribution of agent types; aggregate productivity is similar except that I weight each agent by their share of aggregate capital, $k (\zeta_i, \zeta_t) x/K_{t+1}$. Then $C$ is the difference between aggregate productivity and $Z$. To compute the standard deviation of log productivity across agents I square equation (16) and integrate to get the uncentered second moment of log productivity, from which the standard deviation is straightforward. For details see Appendix C.

5 Results

In this section I solve the model derived in section 4, calibrate it, and use it to back out the fundamental shocks that drove Indian manufacturing productivity growth from 1989–1990 to 2010–2011. I find that improving financial conditions explain a large fraction of productivity growth in the period after India’s major economic reforms in the early 1990s, but that I can explain the behavior of both of the Olley and Pakes (1996) terms in equation (1) in India with common shocks to labor productivity alone after 1994–1995. The reason is that common productivity shocks affect
more-productive agents more than less-productive ones because the former adjust their leverage to reflect their higher productivity. In equilibrium this makes the larger, more-productive firms grow faster on average than other firms, even though they receive the same productivity shock.

The results in this section are based on a numerical approximation to the model’s solution conditional on a set of parameters. In section 5.1 I calibrate the model, and in section 5.2 I describe the leverage and investment decisions of individual agents in the model given the calibrated parameters. In section 5.3 I use the calibrated model to back out a time-series of \((Z, \zeta)\) to match the observed values of \((Z, C)\).

5.1 Calibration

Table 4 reports the parameters for the quantitative application of the model. Some parameters can be set to match empirical targets without solving the model. I set \(N = 2\) as the number of agent-types in the model; \(i = 1\) corresponds to “informal” agents with \(\bar{z}_i = -2\). A measure 0.99 of agents in the economy are of this type, while the remaining 1% of agents have \(\bar{z}_i = 0\). This not only matches these shares of informal and formal firms in the NSS and ASI data, but allows the model to match the unconditional standard deviation of productivity in the data without setting \(\rho\) or \(\sigma\) too high.

I set the exogenous exit probability \(\pi\) to 0.1, which means that the average plant age in the model is 10 years. This is a bit lower than the average age of 12 years in the data, but lowering \(\pi\) to match average age in the data leads to instability. In particular, if \(\pi\) is too low then it is possible for plants to grow on average faster than the exit rate, leading to a stationary firm size distribution with infinite output. In the context of Gabaix (2011) these densities would resemble a power law with tail parameter less than 1. I avoid this issue by setting the exit rate such that the average age of plants in the model is somewhat lower than what is observed in the data.

I set the persistence of productivity shocks to be \(\rho = 0.95\) following Midrigan and Xu (2014). Given this value and the calibrated values for the share and productivity differences between the formal and informal sectors, I set the idiosyncratic productivity volatility \(\sigma\) to match the cross-sectional standard deviation of log productivity. A key assumption to match this moment is that
agents are restricted from lending, so that in particular the informal agents with $z_i = -2$ must produce with their low-productivity technology. Absent the lending restriction, most of these agents would choose to lend to more-productive agents, lowering the standard deviation of productivity (and raising the interest rate, which would have to be endogenous). Raising $\sigma$ to combat this selection effect mainly serves to increase the average growth rate of producing firms, which leads to explosive behavior as described in the previous paragraph. Such a model also seems at odds with the data, which features a massive number of very low-productivity manufacturers. Surely such agents would be better off investing (or working!) in higher-productivity plants, but understanding why they do not do so is beyond the scope of the present paper.

I choose the riskless interest rate $r$, subjective discount rate $\beta$, and the (gross) level of productivity $A$ to match the share of plants reporting zero leverage, the share of informal firms in total output, and the median leverage ratio of plants that do borrow, respectively. Of course each of these parameters affects all three moments, but I have chosen the mapping between parameters and moments in Table 4 to loosely convey which moment is most affected by which parameter. The fit is not perfect, particularly the median leverage ratio, but I have verified that the qualitative results of the paper are not sensitive to this parameter.

Finally, I set the parameters of the exogenous demand curve to normalize the steady-state price to 1 and to match the evolution of the informal share of inputs. The demand-curve intercept $D$ is not separately identified without price information, so I set it to ensure that the steady-state price of output is 1. I then set the price elasticity of demand $\eta$ to match the change in the NSS share of employment, which drops by roughly eleven percentage points from 1989–1990 to 2010–2011.

5.2 Policy Rules

To solve the model I solve equation (13) on a univariate grid of values for $\{r_k\}$ and equation (12) on a 3-dimensional grid of values for $\{z_i, \zeta_j, r_k\}$. I interpolate the policy functions between gridpoints as described in Appendix D.1. Figure 7 plots the agent’s decision rules as a function of future productivity $Ae^{\tilde{z}}$ for various values of $\zeta$.

[Figure 7 about here.]
Optimal leverage is increasing in productivity, as can be seen in the top right panel of Figure 7. The graph plots optimal leverage $\ell = \frac{-b'}{k'}$ as a function of $Ae^{\bar{z}}$ for $r = 1\%$ and $\zeta$ ranging from 0 to 0.35, whenever $\ell > 0$. Higher leverage goes hand in hand with higher investment, as can be seen in the bottom-left panel of Figure 7. Thus more-productive firms will grow faster than less-productive ones, generating a positive correlation between size and productivity in equilibrium that depends strongly on the value of $\zeta$.

5.3 Steady-State and Aggregate Productivity Growth

In this section I use the calibrated model to determine the aggregate shocks that drive the observed aggregate productivity decomposition in Table 3. First, I compute the steady-state size-productivity distribution implied by the parameters in Table 4, setting $\zeta$ to yield an OP covariance term $C = 0.16$, the value in India in 1989–1990. For this date I normalize $Z = 0$. Then for each subsequent year to 2010–2011 I compute new values of $(Z, \zeta)$ so that $C$ in the model economy matches the value in Table 3, and the change in $Z$ matches the change in the value of $Z$ in Table 3. I linearly interpolate the target values of $(Z, C)$ between the 5-year observation years.

Given the policy functions computed in Section 5.2 and illustrated in Figure 7, I compute the model’s implied steady-state distributions of size and productivity $F_i (\log X, z)$ according to equation (14) for a given value of $\zeta$, assuming that $F_{i,t+1} = F_{i,t}$ and $Z_{t+1} = 0$ for all $i, t$. I then search over values of $\zeta$ to match the value of $C = 0.16$ using equation (26). I choose the intercept in the demand curve $c$ so that the market-clearing price in this year is $P = 1$, according to equation (4). See Appendix D.2 for details on how I represent and solve for the endogenous density, and how I approximate integrals numerically.

For each subsequent year after 1989–1990 I solve for the implied values of $(P, Z, \zeta)$ that match the observed values of $(Z, C)$ and that clear the goods market according to equation (4). The values of $(P, Z, \zeta)$ determine the investment and borrowing decisions of agents in the current period, which affect the future joint distribution of size and productivity according to equation (14). This distribution, along with the values of $(Z, \zeta)$, determine aggregate output $Y$ and the OP decomposition terms $(Z, C)$. I match the levels of $C$ to those in Table 3 but I normalize $Z$ to zero in 1989–1990, so that the model values equal the data values in log differences. Finally, the price $P$ must be consistent with aggregate output $Y$ and the aggregate demand curve.
Figures 8 and 9 plot the results of this exercise. Figure 8 plots the observed OP decomposition terms as well as the implied values of the fundamental shocks over this time period. The top two panels of Figure 8 plot the observed values of $Z$ and $C$ from Table 3, linearly interpolating at the annual frequency and normalizing the initial value of $Z$ to 0. The bottom two panels plot the values of $(Z, \zeta)$ that generate the observed OP decomposition according to the calibrated model.

The bottom two panels of Figure 8 show that the history of manufacturing productivity growth in India can be divided into two periods. In the first period, from 1989–1990 to 1994–1995, labor productivity growth was driven by changes in financial frictions $\zeta$, and common productivity shocks were small or even negative. The model identifies this period by the very high change in the OP covariance term, relative to almost no change in unweighted average productivity; the model matches this by drastically increasing $\zeta$.

Figure 9 converts the values plotted in the bottom panels of Figure 8 into contributions to aggregate productivity growth. Each bar represents the contribution to aggregate productivity growth from each shock over the indicated time span. I compute these contributions as counterfactuals: in each time span, I compute average productivity growth holding one shock constant at its beginning-of-period value, and allowing the other shock to change as in the bottom panels of Figure 8. Because the effect of shocks to $\zeta$ depend on the level of $Z$, and vice versa, the sum of the two contributions may not equal realized productivity growth. I plot the difference in Figure 9 as the “joint” contribution.

For the first five years of the sample, improvements in financial conditions (the black bars in Figure 9) contributed almost four percentage points per year to labor productivity growth; this is 71% of total productivity growth in that time frame. In other years common shocks to productivity explain nearly all the change in aggregate productivity; financial shocks were even a slight drag on productivity growth in 2006–2011. The average contribution of financial shocks to total productivity growth from 1995 to 2011 is between 2.4% and 7.5%, depending on whether one includes the joint term or not.
Although I find that changes in financial frictions are not important drivers of aggregate productivity since 1994–1995, the level of financial frictions from 1994–1995 onwards is important for allowing common productivity shocks $Z$ to affect the covariance term $C$. In a simplified version of the model without borrowing, common shocks to productivity lead to uniform changes in growth rates across the productivity distribution: all firms grow faster, by the same amount. This leaves the covariance term unchanged. But when firms can borrow, their leverage depends on the level of productivity. Thus larger, more-productive firms respond to the common shock to productivity by choosing higher leverage ratios, and grow faster than smaller, less-productive firms. In equilibrium, this mechanism increases the covariance between size and productivity without any changes in financial frictions.

A crucial parameter driving the constancy of $\zeta$ after 1994–1995 is the price elasticity of demand, $\eta$. This parameter controls the degree to which shocks to physical productivity affect the decisions of firms. In the corner case where $\eta = 1$, the price of output adjusts to automatically cancel any changes in physical productivity, all else equal, so agents do not respond to common shocks to productivity (although aggregate output does). For $\eta < 1$ the price of output does not adjust fully to shocks to $Z$, so agents respond to positive aggregate productivity shocks by increasing investment. If $\eta$ were lower than the value in Table 4, $\zeta$ would decrease over time after 1994–1995 because agents would respond so much to the implied changes in $Z$ that the OP covariance term predicted by the model for constant $\zeta$ would be higher than what is observed. However, for such values of $\eta$ the employment share of informal firms would drop faster than the values given in Panel A of Table 1.

6 Conclusion

In this paper I derive a novel dynamic equilibrium model that identifies aggregate financial shocks from common shocks to productivity by comparing their distinct effects on the joint distribution of firm size and productivity. Financial shocks affect the allocation of resources across firms, but

\footnote{This is only approximately true, because the amount of undepreciated capital that firms carry over into the next period is unaffected by their productivity. For firms with very low levels of productivity, their future wealth comes entirely from undepreciated capital and their growth is unaffected by a productivity shock. The reverse is true for extremely productive firms, for whom left-over capital is a small share of their future wealth. Thus productivity shocks will have a bigger effect on the growth rates of more-productive firms, and the covariance term $C$, even in a model without financial frictions. However, this effect is quantitatively negligible.}
hold productivity within firms constant; common shocks to productivity affect both the allocation of resources and within-firm productivity. I use the calibrated model and establishment-level microdata from India to show that despite numerous financial reforms since 1991, financial factors only affected aggregate Indian manufacturing labor productivity until 1994–1995.

Because Indian labor productivity has increased dramatically since 1994–1995, my model suggests that factors other than financial frictions are important in explaining India productivity growth. One such factor might be frictions in adopting modern management practices. Bloom et al. (2013) show that management practices are important factors driving productivity in large Indian textile firms. They argue that informational barriers, rather than financial frictions, are what prevent firms from adopting efficiency-enhancing management practices. Another factor that may have increased Indian manufacturing productivity is the gradual dismantling of the small-scale reservation policy (see Mohan 2002 for an overview), which began in 2003 and has only recently finished (The Gazette of India 1991–2015). Alternatively, financial frictions that primarily affect within-firm productivity could also be driving my results. For example, it may be that entry into a higher-productivity sector requires large sunk costs that are difficult to finance, as in the model of Buera, Kaboski and Shin (2011). Uncovering the exact underlying factors behind the common productivity growth identified by my model is an avenue that I hope to explore in future research.
A Data

Tables 5, 6, and 7 list the 100 four-digit NIC 2008 industries used in the paper, in decreasing order of average (over time) employment share. I compute all aggregates in the paper within each of these industries in each year, and then report the overall aggregate as a weighted average of the industry values, using each industry’s employment share at time $t$ as the weights.

[Table 5 about here.]

All aggregate statistics use the reported sampling weights in both the ASI and NSS; I verify that these aggregates match the publicly-available aggregates for each dataset when possible (for example, for employment, gross value added, number of firms, etc).

[Table 6 about here.]

I define productivity as annual real gross value added per employee, where value added is gross of depreciation and defined as the ex-factory value of goods produced less cost of material inputs, plus other income (from services, goods sold in same condition as purchased, value of own construction, and rent received for fixed assets) less other expenses (work done by others, repair and maintenance of machinery and equipment, operating expenses, and insurance, interest, and rent expenses). In the 2005–2006 and 2010–2011 NSS many gross value added figures are reported at monthly (rather than annual) rates. The 2005–2006 NSS reports individual-observation reference periods; I infer reference period in the 2010–2011 NSS by whether the respondent used a reliable book of accounts to answer the survey, as described in the document “Key Results of the Survey on Unincorporated Non-Agricultural Enterprises (Excluding Construction) in India” for the NSS 67th round. The other NSS years report annualized figures for all observations. I verify these data manipulations by comparing aggregate values with the publicly-available aggregates for the NSS.

[Table 7 about here.]

I deflate value-added using the Indian Whole Price Index (WPI) for each industry. To do so, I manually link the NIC-2008 industries (for the 2010–2011 data) with the NIC-2004 industries (for the 2005–2006 data), the NIC-1998 industries (for the 1999–2000 data), and the NIC-1987
industries (for the 1989–1990 and 1994–1995 data). To make these links I have to merge some NIC-2008 industries together, for example Fertilizers with Agrochemical Products, because they are not separated in some years. I then link these merged industry codes with the manufacturing commodities in the 1982, 1993-1994, and 2004-2005 WPI series. I construct a single price index for each industry by extending the 1993-1994 WPI series before its initial year (1993-1994) and beyond its final year (2009-2010) using the appropriate growth rates from the 1982 and 2004-2005 WPI, respectively, so that real value added in the paper is at constant 1993-1994 prices. The 1982 WPI is monthly, which I convert to the annual frequency by taking the average value of the index over each year (April–March).

I drop from the sample any observations from non-manufacturing industries, including recycling plants and repair shops, and any observation missing industry information. I also drop any observation missing employment information or all components of gross value-added, because I cannot infer productivity for these observations and including them would invalidate the identity in equation (1). I drop one firm in the 2005–2006 NSS that reports 8,000 employees; this is almost twenty times as large as the next-largest plant, though his inclusion chiefly affects the value of the standard deviation of log employment in that year (raising it to 6 from 3.0). My final sample consists of 612,907 (unweighted) plant-year observations over the five years 1989–1990, 1994–1995, 1999–2000, 2005–2006, and 2010–2011.

I do not include the 2000-2001 NSS and ASI because the former appears to be based on a different Indian Economic Census, as described on page 18–19 of NSS Report 477 “NSS 56th Round Key Results.” The problem with including this year of data is that the 2000–2001 NSS reports 17 million plants and over 37 million employees (compared to 14 million plants the year before and five years later), which if included would make for a sharp drop in aggregate productivity and the covariance term in that year, only one year away from the 1999–2000 NSS and ASI. I believe these values reflect differences in the Economic Census sampling frames, as suggested in Report 477, and not drastic changes in real Indian manufacturing plants or employment.
B Proofs

B.1 Proof of Proposition 1

Proof. For ease of exposition, suppose that \( \pi_i = 0 \).

The proof consists of guessing that \( V(x, z; Z, \zeta, P, F) \) of the form \( a_0 + a_1 \log(X) + f(z, Z; \zeta, P, F) \) solves equation (9), and then verifying that \( a_0, a_1, \) and \( f(\cdot) \) exist. The value of \( a_1 \) will also be important for characterizing the decision rules.

Thus, suppose \( V(x', z'; Z', \zeta', P', F') = a_0 + a_1 \log(X') + f(z'; Z', \zeta', P', F') \). Plugging into equation (9) and rearranging yields

\[
V(x, z; Z, \zeta, P, F) = \max_{k' \geq 0, b' \leq 0} \log(x - qb' - k')
\]

\[
+ \beta \left[ (1 - \pi) \left( a_0 + E \left\{ f(z'; Z', \zeta', P', F') \right\} \right) + ((1 - \pi)a_1 + \pi) E \left\{ \log(x') \right\} \right].
\]

Because agents are restricted from lending, it must be the case that \( k' > 0 \) strictly, otherwise net wealth next period would be negative. Thus I can rewrite the original problem in terms of choosing \( k' > 0 \) and \( \ell = \frac{-b'}{k'} \geq 0 \). Net wealth becomes \( x' = k' \max\left\{ PAe^{z'} + 1 - \ell, 1 - \zeta \right\} \), and equation (17) becomes

\[
V(x, z; Z, \zeta, P, F) = \max_{k' \geq 0, \ell \geq 0} \log \left[ x - (1 - q\ell) k' \right] + \beta \left[ (1 - \pi) \left( a_0 + E \left\{ f(z'; Z', \zeta', P', F') \right\} \right) \right]
\]

\[
+ ((1 - \pi)a_1 + \pi) \left( \log(k') + E \left\{ \log \max\left\{ PAe^{z'} + 1 - \ell, 1 - \zeta \right\} \right\} \right).
\]

The first-order condition for \( k' \) is

\[
\frac{1 - q\ell}{x - (1 - q\ell) k'} = \beta \frac{(1 - \pi)a_1 + \pi}{k'},
\]
which I rewrite as

\[
(1 - q\ell) k' = \frac{\beta}{1 + \beta} x \\
\equiv \beta^* x,
\]  

(19)

where \( \beta \equiv \beta \left( 1 - \pi \right) a_1 + \pi \) and the second line defines \( \beta^* \). Plugging equation (19) back into the definitions for consumption and \( \ell \) then yields that \( c = (1 - \beta^*) x \) and \( k' + qb' = \beta^* x \).

It remains to verify the guess. The first-order condition for leverage \( \ell \) is given by

\[
\frac{\left( q + \frac{\partial q}{\partial \ell} \right) k'}{x - (1 - q\ell) k'} = \beta \int_{\xi}^{\infty} \frac{\phi(\epsilon) d\epsilon}{A e^{\tilde{z} + \sigma \epsilon + 1 - \ell}},
\]

which can be written using equation (19) as

\[
\frac{q + \frac{\partial q}{\partial \ell}}{1 - q \ell} = \int_{\xi}^{\infty} \frac{\phi(\epsilon) d\epsilon}{A e^{\tilde{z} + \sigma \epsilon + 1 - \ell}},
\]

(20)

Equations (19) and (20) imply that optimal capital \( k' \) and consumption \( c \) are proportional to wealth \( x \), while optimal leverage \( \ell \) is independent of \( x \). Plugging into equation (18) yields

\[
V (x, z; Z, \zeta, P, F) = \log \left[ (1 - \beta^*) x \right] + \beta (1 - \pi) \left( a_0 + E \left\{ f (z'; Z', \zeta', P', F') \right\} \right)
\]

\[
+ \beta \log (\beta^* x) - \beta \log (1 - q\ell) + \beta E \left\{ \log \max \left\{ A e^{\tilde{z} + \sigma \epsilon + 1 - \ell, 1 - \zeta} \right\} \right\}
\]

\[
= \log (1 - \beta^*) + \beta (1 - \pi) a_0 + \beta \log (\beta^*) + \left( 1 + \beta \right) \log (x) \]

\[
- \beta \log (1 - q\ell) + \beta E \left\{ f (z'; Z', \zeta', P', F') \right\} + \beta E \left\{ \log \max \left\{ A e^{\tilde{z} + \sigma \epsilon + 1 - \ell, 1 - \zeta} \right\} \right\}
\]

(21)

Equating terms when the left-hand-side of equation (21) equals \( a_0 + a_1 \log (X) + f (z; Z, \zeta, P, F) \) yields

\[
a_1 = 1 + \beta \left[ (1 - \pi) a_1 + \pi \right] \]

\[
= \frac{1 + \beta \pi}{1 - \beta (1 - \pi)},
\]
from which $\tilde{\beta}$ and $\beta^*$ can be calculated.

B.2 Proof of Proposition 2

Proof. Suppose that $z$ is large enough that $\ell > 0$. Then by Proposition 1, $k'$ and $\ell$ satisfy

$$k' = \frac{\beta^*}{1 - q\ell}$$

and

$$q + \frac{\partial q}{\partial \ell} \ell = \frac{1}{1 - q\ell} \int_{\tilde{z}}^{\infty} \frac{\phi(\varepsilon)}{Ae^{\tilde{z} + \sigma\varepsilon} + 1 - \ell'}$$

where $\tilde{z} \equiv \rho z + z_i + Z + \log P$. It is straightforward to show that the optimal policy for $\ell$ is continuous in $z$. Now suppose I lower $z$ until $\ell = 0$ is optimal. At this point $\ell < \zeta$ so $q = \frac{1}{1+r}$ and $\frac{\partial q}{\partial \ell} = 0$, so equation (20) becomes

$$\frac{1}{1+r} = \int_{-\infty}^{\infty} \frac{\phi(\varepsilon)}{Ae^{\tilde{z} + \sigma\varepsilon} + 1},$$

as was to be shown.

C Formulas

In this section I derive the formulas for the aggregate variables and the endogenous size-productivity transition density described in Section 4.4.

The conditional transition CDF for agents of type $i$ in equation (14) is given by

$$\mathcal{E}_i \left( \frac{x^*}{x}, z^*, z, Z, \zeta_t, P_{t+1} \right) \equiv \min \left\{ \mathcal{E}_1 \left( \frac{x^*}{x}, \rho z + z_i + Z_{t+1} + \log P_{t+1}; \zeta_t \right), \mathcal{E}_2 \left( z^*, z \right) \right\}$$

$$\mathcal{E}_1 \left( \frac{x^*}{x}, \zeta_t \right) \equiv \begin{cases} \frac{1}{\sigma} \left( \log \left[ \frac{z^* - b(\tilde{z}, \zeta_t)}{k(\tilde{z}, \zeta_t)} \right] - 1 \right) - \log A - \tilde{z} & \text{if } \frac{x^*}{x} > (1 - \zeta) k(\tilde{z}, \zeta_t) \\ -\infty & \text{otherwise} \end{cases}$$

$$\mathcal{E}_2 \left( z^*, z \right) \equiv \frac{z^* - \rho z}{\sigma},$$

(22)
where $\tilde{z}$ is defined in equation (15) and depends in part on the productivity fixed-effect $\bar{z}_{i}$. The functions $k(\cdot)$ and $b(\cdot)$ in equation (22) are the investment and borrowing decisions from Proposition 2, equation (12), expressed as shares of net wealth $x$. Heuristically, $E_{1}$ inverts the policy function for net wealth $x$ and $E_{2}$ inverts the law of motion for $z$, so that $\Phi\{E\}$ returns the probability of receiving a shock $\varepsilon$ sufficiently large to move from $(\log x, z)$ to a value lower than $(\log x^{*}, z^{*})$. Such an $\varepsilon$ may not exist, as the default option puts a lower bound on the value of $x$ in the next period. Thus the policy functions are not strictly invertible, though the transition distribution function is still well defined.

Aggregate capital $K_{t+1}$ and output $Y_{t+1}$ are integrals over the densities $dF_{i,t}$ and the distribution of $\varepsilon$, omitting the agents who default by receiving a low value of $\varepsilon$. The formula for $K_{t+1}$ in equation (15) follows immediately, but total output $Y_{t+1}$ is given by

$$Y_{t+1} = \sum_{i} \mu_{i} \int_{(\log x, z)}^{\infty} A e^{\tilde{z} + \sigma \varepsilon} k(\tilde{z}; \zeta_{t}) x \phi(\varepsilon) d\varepsilon dF_{i,t}(\log x, z)$$

$$= \sum_{i} \mu_{i} \int_{(\log x, z)}^{\infty} A e^{\tilde{z} k(\tilde{z}; \zeta_{t})} x \int_{\log z}^{\infty} e^{\sigma \varepsilon} \phi(\varepsilon) d\varepsilon dF_{i,t}(\log x, z)$$

$$= \sum_{i} \mu_{i} \int_{(\log X, z)} A \exp \left\{ \tilde{z} + \frac{1}{2} \sigma^{2} + \log x \right\} k(\tilde{z}; \zeta_{t}) \left[ 1 - \Phi \{ \varepsilon(\tilde{z}; \zeta_{t}) - \sigma \} \right] dF_{i,t}(\log x, z)$$

where $\tilde{z}$ is defined in equation (15), the limit of integration over $\varepsilon$ in the first line reflects the assumption that defaulting firms do not produce, the function $\varepsilon(\tilde{z}, \zeta)$ is given by equation (7) after plugging in optimal leverage $\ell(\tilde{z}, \zeta)$, and the third line applies the identity $\int_{a}^{b} e^{\sigma \varepsilon} \phi(\varepsilon) d\varepsilon = \exp \left\{ \frac{1}{2} \sigma^{2} \right\} \left[ \Phi \{ b - \sigma \} - \Phi \{ a - \sigma \} \right]$.

To compute the distribution function of leverage conditional on borrowing, I change variables from $\log x$ to $y \equiv \log k = \log k(\tilde{z}; \zeta_{t}) + \log x$. With this change of variables, the distribution function for leverage conditional on borrowing is

$$F_{t+1}^{\ell}(\ell) \propto \int_{-\infty}^{\infty} \int_{\pi(r)}^{\ell^{-1}(\ell; \zeta_{t})} \left[ 1 - \Phi \{ \varepsilon(\tilde{z}; \zeta_{t}) \} \right] dF_{t}(y - \log k(\tilde{z}; \zeta_{t}), z),$$

where $\pi(r)$ given in equation (13) is the cutoff in forward-looking productivity where agents begin to borrow, and where $\ell^{-1}(\ell; \zeta_{t})$ returns the level of productivity $\tilde{z}$ such that leverage is $\ell$; this function exists because the leverage function is strictly increasing in $\tilde{z}$ for $\tilde{z} > \pi(r)$. The “$\propto$”
symbol indicates that I divide the function in equation (23) by its value at \( \infty \) so that it ranges from 0 to 1, as a distribution function should. Inverting \( F^\ell \) yields the quantile function for leverage, from which I compute its median.

In the rest of this section I derive the formulas for the first and second weighted and unweighted moments of log-productivity, which I need for the two terms in equation (1) and the cross-sectional dispersion of log productivity. Averaging the log productivity of an agent of type \( i \) with idiosyncratic productivity \( z \) in equation (16) over log \( x \), \( z \), and \( \varepsilon \) yields

\[
Z_i = \log A + Z + \bar{z}_i + \rho \int_{(\log x, z)} zdF_i(\log x, z) + \sigma \int_{(\log x, z)} \lambda(\varepsilon) dF_i(\log x, z) \tag{24}
\]

where \( \varepsilon \) is defined in equation (7), and \( \lambda(\cdot) \) is defined as

\[
\lambda(\varepsilon) \equiv \begin{cases} 
\phi(\varepsilon) / \left(1 - \Phi(\varepsilon)\right) & \varepsilon > -\infty \\
0 & \text{otherwise}
\end{cases}
\]

The penultimate term in equation (24) is zero for all types, because the marginal distribution of idiosyncratic productivity is mean-zero. The last term in equation (24) corrects for the bias coming from agents with low draws for \( \varepsilon \) who default and thus do not produce; it depends on the agent’s type \( i \) as well as the current value of \( \zeta \). This means that changes in \( \zeta \) will affect unweighted-average productivity, though this effect is quantitatively small relative to the effect of changes in \( Z \) for the parameters given in Table 4. Aggregate unweighted log productivity then just averages over the different agent types:

\[
Z \equiv \sum_i \mu_i Z_i. \tag{25}
\]

Equation (25) gives the first moment of aggregate log productivity: to compute its standard
deviation, I need its second moment. Squared log productivity is given by
\[
\left[ \log \frac{y}{k} \right]^2 = \left[ \log A + Z + \rho z + \bar{z}_i + \sigma \varepsilon \right]^2
\]
\[
= \left[ \log A + Z \right]^2 + \rho^2 z^2 + \bar{z}_i^2 + \sigma^2 \varepsilon^2
\]
\[
+ 2 \left( \log A + Z \right) \rho z + 2 \left( \log A + Z \right) \bar{z}_i + 2 \left( \log A + Z \right) \sigma \varepsilon + 2 \rho z \bar{z}_i + 2 \rho \sigma \varepsilon + 2 \bar{z}_i \sigma \varepsilon.
\]

Taking the integral over the distributions of \( \varepsilon, \bar{z}_i, \) and \( (\log x, z) \) and simplifying gives
\[
E \left\{ \left[ \log \frac{y}{k} \right]^2 \right\} = \left( \log A + Z \right)^2 + \sigma^2 + \sum_i \mu_i \left[ \bar{z}_i^2 + 2 \left( \log A + Z \right) \bar{z}_i \right]
\]
\[
+ \sum_i \mu_i \int_{(\log x, z)} \left[ \rho^2 z^2 + 2 \left( \log A + Z \right) \rho z + 2 \rho \sigma \varepsilon \right]
\]
\[
\lambda \left( \varepsilon \right) \left( 2 \sigma \left( \log A + Z + \rho z + \bar{z}_i \right) + \sigma^2 \varepsilon \right) dF \left( \log x, z \right).
\]

Then the standard deviation of log productivity is given by
\[
\sigma \{ Z \} = \sqrt{E \left\{ \left[ \log \frac{y}{k} \right]^2 \right\} - Z^2}.
\]

Finally, to compute the model-implied value of the OP covariance term \( C \), I compute weighted-average productivity as
\[
Z_{i, wt} = \log A + Z + \bar{z}_i + \rho \int_{(\log x, z)} k \frac{\bar{z}_i \zeta}{K} dF_i \left( \log x, z \right) + \sigma \int_{(\log x, z)} \lambda \left( \varepsilon \right) \frac{k \left( \bar{z}_i \zeta \right)}{K} dF_i \left( \log x, z \right)
\]
where \( K \) is aggregate capital across all agent types from equation (15). Aggregate productivity is then given by
\[
Z_{wt} = \sum_i \mu_i Z_{i, wt}
\]
so that the OP covariance \( C \) is
\[
C = Z_{wt} - Z.
\]
D Computation

D.1 Policy Interpolation

For this section \(k\) and \(b\) refer to capital and borrowing as proportions of net wealth \(X\), e.g. \(k/X\) and \(b/X\). Also for ease of exposition assume that \(z_i = Z = \log P = 0\); relaxing this assumption is straightforward. Rather than interpolate equation (12) in \(\tilde{z}\), I use \(\tilde{A} \equiv Ae^{\tilde{z}}\) which is more computationally convenient. Thus \(A(r) \equiv Ae^{\tilde{z}(r)}\).

Let \(\{r_k\}\) be the 15 Chebyshev zeroes stretched to fit the interval \([1, 1.25]\); this is the \(r\)-grid. Let \(\{\zeta_j\}\) be 15 evenly-spaced points between 0 and \(\tilde{\zeta}\); this is the \(\zeta\)-grid. For each value of \(r_k\), I define the \(A\)-grid as the fifteen Chebyshev zeroes stretched to fit the interval \([A e^{\rho \tilde{z}(r_k)}, A_{\text{max}}]\); this is the \(A_k\) grid. I compute \(A_{\text{max}}\) as the highest value of \(\tilde{A}\) for which \(1 - q(\ell) \ell \geq 0\) when \(r = 0\) and \(\zeta = 0.45\), e.g. when financial conditions are most conducive to high leverage. Values of \(\tilde{A}\) larger than \(A_{\text{max}}\) present a problem for the model, because under these conditions the marginal cost of leverage is negative, so that an agent can borrow more, consume more, and invest more without any negative consequence. Fortunately, for the parameters in Table 4 the probability that \(\tilde{A} > A_{\text{max}}\) in steady-state is 0.00021, so that not much mass is lost in the truncation.

For each value of \(r_k\) I then solve equation (12) at each point \((A_k, \zeta_j)\) \(i, j = 1, 2, \ldots 15\). I linearly interpolate between these values as a set of 15 functions \(\ell(z, \zeta, r)\) for \(k = 1, 2, \ldots 15\). Interpolation between values of \(r\) requires some care, because the policy rule must respect the bounds set by \(\tilde{z}(r)\) as described in Proposition 2. First, I linearly interpolate \(\tilde{z}(r)\) in \(r\), for the fifteen values \(\{r_k\}\). I then compute \(B(\tilde{A}, \zeta, r)\) and \(k(\tilde{A}, \zeta, r)\) for values of \(r\) between \(r\)-points as described in Figure 10 and below, where \(\tilde{z} \equiv \frac{1}{\rho} \log \left( \frac{\tilde{A}}{A} \right)\).

[Figure 10 about here.]

First fix a value for \(\zeta\), and suppose that \(r \in [r_1, r_2]\). Define \(t_r = \frac{r - r_1}{r_2 - r_1}\); this value interpolates between values indexed by \(r\). If the interpolation point corresponds to a point like \(x_1\), then \(k' = \beta^*\) and \(B = 0\) as in Proposition 2. This is the same value I would get if I took \(k = t_r k(r_1) + (1 - t_r) k(r_2)\). The same is not true of point \(x_2\): here, \(A < A(r)\), so \(k = \beta^*\). But a straightforward linear interpolation between \(k(r_1)\) and \(k(r_2)\) would yield an incorrect \(k > \beta^*\), since \(k(r_1) > \beta^*\).

I solve this problem as follows: first, compute \(A(r)\). If \(\tilde{A} \leq A(r)\) then the values of \(k\) and \(b\)
follow immediately from Proposition 2. The more-difficult case is when \( \widetilde{A} > A(r) \), for example at point \( x_3 \). To interpolate these points, I find the points \( A_1 \) and \( A_2 \) such that the line between them passes through \( x_3 \) and has the same slope as the boundary line between \( A(r_1) \) and \( A(r_2) \). I then calculate implied leverage at this point as 

\[
\ell (\widetilde{A}, \zeta, r) = t_r \ell (A_1, \zeta; r_1) + (1 - t_r) \ell (A_2, \zeta; r_2).
\]

To compute \( k \) and \( b \) at this point, I first compute 

\[
q = q\left( \ell (\widetilde{A}, \zeta, r), \widetilde{A}, \zeta, r \right)
\]

and then infer \( k \) and \( b \) using 

\[
k = \frac{\beta^*}{1 - \pi} \quad \text{and} \quad b = -\ell k.
\]

This method ensures that \( k + qb = \beta^* \) at all points and that the interpolated policies satisfy the hurdle rate defined by \( \overline{A}(r) \).

### D.2 Endogenous Distribution Approximation

The model implies an equilibrium distribution across log net wealth log \( x \) and productivity \( z \). To compute aggregate quantities such as total output and capital, or the excess demand for savings, I need to integrate over this distribution. In this section I describe how I represent the endogenous distribution of \((x, z)\) and solve for it in steady-state equilibrium, as well as how it varies over time according to the aggregate shocks to \( Z \) and \( \zeta \).

To solve for the distribution \( F \) and its density \( dF = f \) in equation (14), I define grids for log \( x \) and \( z \) and represent \( F \) and \( f \) as vectors with values at a discrete number of points. This reduces equation (14) to a set of linear equations, as follows: let \( \vec{F}, \vec{f}, \) and \( \vec{F}^e \) be vectors containing the values of the CDF, pdf, and exogenous-entry CDF at the \( N^2 \) gridpoints (the tensor product of \( N \) points for log \( x \) and \( N \) points for \( z \)), respectively. Then equation (14) becomes, for a single grid value \( \vec{F}_i \),

\[
\vec{F}_i = \Pr \{ \log x < \log x_i, z < z_i \}
= (1 - \pi) \sum_{j=1}^{N^2} g_{ij} (Z, \zeta, P) \vec{f}_j + \pi \vec{F}^e_i,
\]

where

\[
g_{ij} (Z, \zeta, P) \equiv \Phi \left\{ \mathcal{E} \left( \frac{x_i}{x_j}, z_i, z_j, Z, \zeta, P \right) \right\}
\]
is the discretized version of the transition CDF $\Phi \{ \mathcal{E} \}$ from equation (22). Stacking all $N^2$ equations (27) yields the system of linear equations

$$\vec{F}_{t+1} = (1 - \pi) G (Z_{t+1}, \zeta_t, P_{t+1}) \vec{f}_t + \pi \vec{F}^e.$$  

(28)

There are two complications that arise when solving equation (28). First, the left-hand side is the CDF, while the right-hand side contains the pdf. To correct for this, define

$$T = t \otimes t,$$

where $t$ is a lower-triangular $N \times N$ matrix of 1’s, as the matrix that transforms a vector pdf into a vector CDF. The matrix $t$ transforms a univariate pdf into a CDF by summing across values; $T$ extends this matrix to the two-dimensional case. Then $T^{-1}$ is the matrix that converts a CDF into a pdf.

The second complication when using equation (28) is that care must be taken to choose at what points to evaluate functions. When defining the gridpoints, the CDF returns the probability of being less than a given gridpoint; it is evaluated at the right boundary of each interval. However, evaluating the pdf at the right endpoints leads to bias that significantly distorts the solution to equation (14), as can be verified using simple examples with known solutions, such as a normal AR(1). To get around this problem, I evaluate all pdfs at the midpoint of each interval; this means losing one point in each grid, so that $N^2$ CDF points become $(N - 1)^2$ pdf points. If the grid is wide enough so that the CDF is effectively zero at all the dropped points, this truncation is computationally negligible. Define

$$S \equiv s \otimes s$$

$$s \equiv \begin{bmatrix} \vec{0}_{1,N} \\ I_{N-1} \end{bmatrix},$$

so that $S \vec{f}$ effectively chops off the bottom-right series of values of the 2-dimensional grid. Although $S$ is not invertible (it is not square), notice that $SS' = I$. The matrix $S$ chops off the bottom values of the grid, and the matrix $S'$ adds back a row and column of zeroes to make things conformable.
again. With the two matrices $T$ and $S$ and their inverses in hand, equation (28) becomes
\[
\tilde{f}_{t+1} = ST^{-1}\tilde{F} = (1 - \pi) ST^{-1} G (Z_{t+1}, \zeta_t, P_{t+1}) S' \tilde{f}_t + \pi ST^{-1} F^e
\] (29)
whose solution for the steady-state density $\tilde{f}$ is (recall that the parameter $D$ in equation (4) is chosen so that $P = 1$ in steady-state)
\[
\tilde{f} = \pi \left[ I_{N-1} - (1 - \pi) ST^{-1} G (0, \zeta, 1) S' \right]^{-1} ST^{-1} F^e.
\] (30)

The $(N - 1)^2$ values in $\tilde{f}$ are density values at the midpoints of each square in the grid, not the upper-right corners at which the CDFs $G$ and $\tilde{F}$ are evaluated.

I apply equation (30) on grids of size $N = 64$ for log $X$ and $z$. The marginal distribution of $z$ for incumbent agents is normal with mean 0 and variance $\sigma_z^2 = \frac{\sigma^2}{1 - \rho^2}$; I define the $z$-grid to be the 63 evenly-spaced points on $[-7\sigma_z, 7\sigma_z]$, plus the hurdle point $\overline{z}(r)$, where I convert from $\tilde{A}$ to $z$ according to $\tilde{A} \equiv A e^{\rho z}$. For the log $X$-grid I choose the 64 evenly-spaced points on $[-12, 12]$, and verify ex-post that this maximum value for log $X$ is not binding.

To solve for the steady-state $\tilde{f}$, I solve equation (30) for $\zeta = \overline{\zeta}$ and an arbitrary value of $P$, and compute the implied aggregate output $Y$. I then search over $P$ until I find a value such that $\log P = -\eta \log Y + D$.

I then compute all integrals in this paper, apart from equation (12), by summing across the density grid-points described above.
References


Khan, Aubhik, Tatsuro Senga, and Julia K. Thomas. 2014. “Credit Shocks in an Economy with Heterogeneous Firms and Default.” Working paper. 17


Figure 1. Aggregate Labor Productivity
The figure aggregate log labor productivity in India from 1989–1990 to 2010–2011. I define aggregate productivity as an employment-weighted average of industry-level aggregate productivity, where the weights are each industry’s share of total employment at each date. Industry-level aggregate productivity is a employment-weighted average of plant-level log labor productivity, as in equation (1). The data values plotted in this figure are reported in the second column of Table 3.
Figure 2. Sources of Credit
The figure plots the sources of credit (loans) reported by NSS plants over time. Plants in the ASI do not report their sources of credit. Each line plots the percentage of total credit reported from each source: banks and state financial corporations (including loans from government-owned commercial banks and Khadi & Village Industries Commission), money-lenders, friends, family, & business partners, and other.
Figure 3. OP Decomposition over Time

The top panel plots the average productivity $Z$ term from equation (1) over the five years in the combined ASI-NSS data. The solid black line plots the values reported in Table 3, which average the within-industry $Z$ terms using each industry’s share of total employment at each date. The upper and lower dotted lines represent averages using constant industry employment shares in 2010–2011 and 1989–1990, respectively. The bottom panel is identical except that it plots the OP covariance term $C$ for the three sets of weights.
Figure 4. OP Decomposition for Industry Groups
The top panel plots the average productivity $Z$ term from equation (1), and the bottom panel plots the OP covariance $C$ term, for three industry groups. I split the 100 industries into three groups based on their average share of NSS employment; the three groups are industries with less than 78%, between 78% and 90%, and greater than 90% of industry employment accounted for by NSS plants. These breakpoints ensure that each group is responsible for roughly the same share of total employment on average.
Figure 5. Productivity and Employment Shares by Size
The top panel plots the distribution of log labor productivity, in 1993-1994 rupees of value-added per employee, across six size categories: plants with 1 employee, 2 employees, 3–4 employees, 5–20 employees, 21–250 employees, and more than 250 employees. The edges of each box represent the 25th and 75th percentiles, while the middle line inside each box is the median. The bars represent the 1st and 99th percentiles. The data are pooled across industries and dates, and I use the sample weights. The bottom panel plots the percentages of total employment accounted for by each size category.
Figure 6. Timing
The figure plots the timing of shocks and decisions within the period. Agents realize their idiosyncratic productivity and exit shocks at the start of the period, and must immediately decide whether to default on their outstanding debt. Firms that default keep a fraction $1 - \zeta$ of their capital stock and cannot produce, but set $B = 0$. Firms that repay their debt may produce but must pay back their debt. After production occurs, but before investment and borrowing decisions are made, agents learn the values of $\zeta$ that will obtain for future loans (the aggregate financial shock, if there is one) as well as the expected value of their productivity next period (the aggregate productivity shock, if there is one) and the equilibrium price of output $P$. After choosing consumption, borrowing, and investment, agents consume and continue to the next period if they did not receive the exit shock.
Figure 7. Policy Functions
The top panel plots the optimal leverage policy $\ell \equiv \frac{\nu}{\kappa'}$, and the bottom panel plots the optimal investment policy $k'$, both as a function of productivity $Ae^{\tilde{z}}$, for $r = 1\%$ and values of $\zeta$ ranging from 0 to 0.35. Each line represents a different value of $\zeta$. The bottom panel plots the optimal investment policy $k'$. 
Figure 8. Aggregate Productivity Decomposition
The top two panels plot the values of $Z$ and $C$ from Table 3. The bottom two panels plot the implied fundamental aggregate shocks $Z$ and $\zeta$ that, when combined with the parameters in Table 4 and assuming the economy is in steady-state in 1989–1990, lead to the observed values of $(Z, C)$. 
Figure 9. Aggregate Productivity Decomposition
The figure plots the average contributions to aggregate productivity growth, in annualized percentage points, coming from changes in the two fundamental shocks $Z$ and $\zeta$ for the four time intervals in the sample period. The black bars represent the contributions to productivity growth coming from shocks to financial frictions $\zeta$, and the dark brown bars the contribution from common shocks to productivity $Z$. Each bar represents an average value over the indicated years. I compute contributions from each source by solving the model over each time frame, evolving the indicated shock as in the bottom panels of Figure 8, and holding the other shock fixed at its beginning of period value. Then each bar is the average growth in aggregate productivity from the appropriate counterfactual. The light brown bars labeled “joint” are the difference between actual aggregate productivity growth, and the sum of the two counterfactual values.
Figure 10. Interpolation Scheme

The figure illustrates an interpolation method between $r$-points that ensures that the interpolated policies respect the hurdle rates for investment and borrowing given by $\zeta(r)$ defined in Proposition 2.
### Table 1. NSS and ASI Summary Statistics.

The first two columns of Panel A report the total estimated number of firms and employees (in millions) in the combined ASI-NSS dataset over time. Columns 3 through 5 of report the aggregate shares of plants, employment, and output (real value added in 1993-1994 rupees) accounted for by respondents in the NSS data. Panel B reports the distribution of employment in the NSS and ASI, while Panel C reports the distribution of log labor productivity (real value-added per employee) in the two datasets. The first two columns in Panels B and C report the mean and standard deviation, respectively, while the last three columns report the 1st, 50th, and 99th percentiles.
<table>
<thead>
<tr>
<th>Year</th>
<th>% that Borrow</th>
<th>NSS</th>
<th>ASI</th>
<th>NSS % of Total Borrowing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>NSS</td>
<td>ASI</td>
<td></td>
</tr>
<tr>
<td>1989–1990</td>
<td>10.7</td>
<td>10.6</td>
<td>66.2</td>
<td>2.8</td>
</tr>
<tr>
<td>1994–1995</td>
<td>9.2</td>
<td>8.9</td>
<td>77.8</td>
<td>5.0</td>
</tr>
<tr>
<td>1999–2000</td>
<td>8.1</td>
<td>7.6</td>
<td>73.0</td>
<td>3.2</td>
</tr>
<tr>
<td>2005–2006</td>
<td>9.0</td>
<td>8.4</td>
<td>74.2</td>
<td>6.9</td>
</tr>
<tr>
<td>2010–2011</td>
<td>6.6</td>
<td>6.0</td>
<td>73.9</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Table 2. Borrowing
The first three columns report percentages of plants that borrow in the combined ASI–NSS data. The first column reports the percentage across all plants, the second column reports the percentage of NSS plants that borrow, and the third column reports the percentage of ASI plants that borrow. The last column reports the percentage of all borrowing that is accounted for by NSS plants.
<table>
<thead>
<tr>
<th>Year</th>
<th>Log Labor Productivity</th>
<th>Average Productivity</th>
<th>Covariance</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1989–1990</td>
<td>8.46</td>
<td>8.30</td>
<td>0.16</td>
<td>1.30</td>
</tr>
<tr>
<td>1994–1995</td>
<td>8.71</td>
<td>8.37</td>
<td>0.34</td>
<td>1.14</td>
</tr>
<tr>
<td>1999–2000</td>
<td>9.30</td>
<td>8.87</td>
<td>0.43</td>
<td>1.15</td>
</tr>
<tr>
<td>2005–2006</td>
<td>9.54</td>
<td>9.05</td>
<td>0.49</td>
<td>1.16</td>
</tr>
<tr>
<td>2010–2011</td>
<td>10.05</td>
<td>9.56</td>
<td>0.49</td>
<td>1.08</td>
</tr>
</tbody>
</table>

**Table 3.** Olley and Pakes (1996) Decomposition Over Time.  
The table reports aggregate (employment-weighted average) log labor productivity in the combined ASI-NSS dataset over time in column 2, the average productivity term $Z$ from equation (1) in column 3, the OP covariance term $C$ from the same equation in column 4, and the standard deviation of log labor productivity in the last column. Each value is computed within 100 different NIC industries and then aggregated using each industry’s share of aggregate manufacturing employment in the indicated year.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.1</td>
<td>Average Age</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>$r$</td>
<td>1%</td>
<td>Percentage with Leverage = 0</td>
<td>91%</td>
<td>84%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.97</td>
<td>NSS Share of Output</td>
<td>31%</td>
<td>25%</td>
</tr>
<tr>
<td>$A$</td>
<td>0.024</td>
<td>Median Leverage</td>
<td>Borrow</td>
<td>24%</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.95</td>
<td>Midrigan and Xu (2014) Value</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.34</td>
<td>Std Dev of Log Productivity</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.99</td>
<td>NSS Share of Plants</td>
<td>99%</td>
<td>99%</td>
</tr>
<tr>
<td>$z_{1}$</td>
<td>-2</td>
<td>ASI-NSS Log Productivity Difference</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.47</td>
<td>Change in NSS Share of Inputs</td>
<td>-11.2%</td>
<td>-11.1%</td>
</tr>
<tr>
<td>$D$</td>
<td>-1.52</td>
<td>$P = 1$ in Steady-State</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

**Table 4. Calibration**

The table reports the parameter values of the model solved in the text. The first two columns report the parameter and its value. The third column reports the target moment used to set the given parameter, and the last two columns report the value of the target moment in the data and in the model, respectively.
<table>
<thead>
<tr>
<th>Industry</th>
<th>% of Total Employment</th>
<th>Log Labor Productivity</th>
<th>Covariance Term C</th>
<th>NSS % of Employment</th>
<th>NSS % of Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1311 Weaving &amp; Fiber</td>
<td>10.30</td>
<td>9.62</td>
<td>0.71</td>
<td>77.08</td>
<td>30.07</td>
</tr>
<tr>
<td>1200 Tobacco</td>
<td>9.37</td>
<td>8.13</td>
<td>0.11</td>
<td>88.00</td>
<td>39.38</td>
</tr>
<tr>
<td>1410 Apparel</td>
<td>8.59</td>
<td>9.46</td>
<td>0.28</td>
<td>85.76</td>
<td>57.59</td>
</tr>
<tr>
<td>1061 Grain Milling</td>
<td>6.65</td>
<td>8.83</td>
<td>0.02</td>
<td>91.13</td>
<td>59.25</td>
</tr>
<tr>
<td>1623 Wooden Container</td>
<td>5.15</td>
<td>7.96</td>
<td>-0.03</td>
<td>99.65</td>
<td>96.04</td>
</tr>
<tr>
<td>1629 Other Wood Products</td>
<td>3.50</td>
<td>8.03</td>
<td>-0.10</td>
<td>99.56</td>
<td>93.28</td>
</tr>
<tr>
<td>3211 Jewelry</td>
<td>3.20</td>
<td>9.61</td>
<td>0.16</td>
<td>95.78</td>
<td>77.65</td>
</tr>
<tr>
<td>1399 Other Textiles</td>
<td>3.14</td>
<td>8.51</td>
<td>0.26</td>
<td>98.58</td>
<td>83.31</td>
</tr>
<tr>
<td>1075 Prepared Meals</td>
<td>3.07</td>
<td>8.89</td>
<td>0.50</td>
<td>69.69</td>
<td>25.43</td>
</tr>
<tr>
<td>2393 Other Ceramic Products</td>
<td>3.03</td>
<td>8.42</td>
<td>-0.01</td>
<td>96.47</td>
<td>69.98</td>
</tr>
<tr>
<td>3100 Furniture</td>
<td>2.67</td>
<td>9.37</td>
<td>0.01</td>
<td>97.24</td>
<td>84.58</td>
</tr>
<tr>
<td>3240 Toys &amp; Other Articles</td>
<td>2.52</td>
<td>8.12</td>
<td>0.20</td>
<td>94.60</td>
<td>58.69</td>
</tr>
<tr>
<td>2391 Brick &amp; Ceramic</td>
<td>2.42</td>
<td>9.18</td>
<td>0.33</td>
<td>77.10</td>
<td>47.96</td>
</tr>
<tr>
<td>1622 Carpentry &amp; Joinery</td>
<td>2.04</td>
<td>9.36</td>
<td>-0.01</td>
<td>99.47</td>
<td>97.70</td>
</tr>
<tr>
<td>1392 Non-Apparel Textile</td>
<td>1.85</td>
<td>9.08</td>
<td>0.18</td>
<td>93.67</td>
<td>68.09</td>
</tr>
<tr>
<td>1072 Sugar</td>
<td>1.66</td>
<td>9.43</td>
<td>1.02</td>
<td>56.80</td>
<td>10.33</td>
</tr>
<tr>
<td>1394 Thread and Rope</td>
<td>1.54</td>
<td>8.22</td>
<td>0.36</td>
<td>91.66</td>
<td>58.33</td>
</tr>
<tr>
<td>2593 Tools and Hswres</td>
<td>1.45</td>
<td>9.21</td>
<td>0.25</td>
<td>89.48</td>
<td>49.29</td>
</tr>
<tr>
<td>2410 Basic Iron &amp; Steel</td>
<td>1.25</td>
<td>11.64</td>
<td>1.83</td>
<td>7.59</td>
<td>1.24</td>
</tr>
<tr>
<td>1073 Candy</td>
<td>1.11</td>
<td>9.50</td>
<td>0.09</td>
<td>96.37</td>
<td>73.38</td>
</tr>
<tr>
<td>1520 Footwear</td>
<td>1.08</td>
<td>9.58</td>
<td>0.36</td>
<td>74.82</td>
<td>43.23</td>
</tr>
<tr>
<td>2511 Str Metal Prod</td>
<td>0.97</td>
<td>10.25</td>
<td>0.26</td>
<td>79.35</td>
<td>50.77</td>
</tr>
<tr>
<td>2220 Other Plastic Products</td>
<td>0.95</td>
<td>10.18</td>
<td>0.77</td>
<td>64.95</td>
<td>25.95</td>
</tr>
<tr>
<td>1313 Textile Finishing</td>
<td>0.95</td>
<td>9.98</td>
<td>0.99</td>
<td>62.97</td>
<td>31.07</td>
</tr>
<tr>
<td>2599 Fabr Metal Prod nec</td>
<td>0.92</td>
<td>10.16</td>
<td>0.36</td>
<td>79.82</td>
<td>45.73</td>
</tr>
<tr>
<td>2910 Motor Vehicles</td>
<td>0.90</td>
<td>11.45</td>
<td>1.20</td>
<td>14.42</td>
<td>2.41</td>
</tr>
<tr>
<td>1820 Recorded Media</td>
<td>0.87</td>
<td>9.70</td>
<td>0.29</td>
<td>85.92</td>
<td>55.03</td>
</tr>
<tr>
<td>2023 Soap</td>
<td>0.83</td>
<td>8.71</td>
<td>0.84</td>
<td>79.59</td>
<td>12.47</td>
</tr>
<tr>
<td>1050 Dairy Products</td>
<td>0.73</td>
<td>9.48</td>
<td>0.54</td>
<td>75.77</td>
<td>22.30</td>
</tr>
<tr>
<td>2029 Other Chemical Products</td>
<td>0.73</td>
<td>9.25</td>
<td>1.15</td>
<td>50.81</td>
<td>7.92</td>
</tr>
<tr>
<td>1071 Bakery Products</td>
<td>0.69</td>
<td>9.66</td>
<td>0.19</td>
<td>86.28</td>
<td>48.53</td>
</tr>
<tr>
<td>2100 Pharmaceuticals</td>
<td>0.65</td>
<td>11.16</td>
<td>1.59</td>
<td>10.31</td>
<td>1.51</td>
</tr>
<tr>
<td>2396 Stone Finishing</td>
<td>0.59</td>
<td>9.23</td>
<td>0.33</td>
<td>74.18</td>
<td>43.41</td>
</tr>
<tr>
<td>1010 Meat Processing</td>
<td>0.59</td>
<td>9.72</td>
<td>-0.03</td>
<td>97.54</td>
<td>87.25</td>
</tr>
<tr>
<td>1610 Wood Milling</td>
<td>0.54</td>
<td>9.86</td>
<td>0.02</td>
<td>94.40</td>
<td>94.35</td>
</tr>
<tr>
<td>2710 Ele Mtrs &amp; Distrib</td>
<td>0.51</td>
<td>11.22</td>
<td>0.88</td>
<td>30.25</td>
<td>9.23</td>
</tr>
<tr>
<td>2592 Metal Trtmt &amp; Coating</td>
<td>0.50</td>
<td>9.96</td>
<td>0.09</td>
<td>90.84</td>
<td>76.55</td>
</tr>
</tbody>
</table>

**Table 5. Industry Detail**

The table lists the NIC-2008 4-digit manufacturing industries used in the paper in decreasing order of average employment share. The first column reports the average percentage of total employment, across both the NSS and ASI, accounted for by the indicated industry. The second column reports weighted-average log labor productivity. The third column reports the average OP covariance term C. The fourth and fifth columns report the average percentage of employment and output accounted for by NSS firms, respectively. Averages are equal-weighted across the five data years from 1989–1990 to 2010–2011.
Table 6. Industry Detail (continued 2)
The table lists the NIC-2008 4-digit manufacturing industries used in the paper in decreasing order of average employment share. The first column reports the average percentage of total employment, across both the NSS and ASI, accounted for by the indicated industry. The second column reports weighted-average log labor productivity. The third column reports the average OP covariance term $C$. The fourth and fifth columns report the average percentage of employment and output accounted for by NSS firms, respectively. Averages are equal-weighted across the five data years from 1989–1990 to 2010–2011.

<table>
<thead>
<tr>
<th>Industry</th>
<th>% of Total Employment</th>
<th>Log Labor Productivity</th>
<th>Covariance Term $C$</th>
<th>NSS % of Employment</th>
<th>NSS % of Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2819 Machinery nec</td>
<td>0.48</td>
<td>11.06</td>
<td>0.63</td>
<td>42.06</td>
<td>18.87</td>
</tr>
<tr>
<td>1702 Cardboard Products</td>
<td>0.47</td>
<td>9.26</td>
<td>0.68</td>
<td>77.01</td>
<td>42.92</td>
</tr>
<tr>
<td>1049 Food Oil Mfg</td>
<td>0.47</td>
<td>10.05</td>
<td>0.74</td>
<td>56.26</td>
<td>12.47</td>
</tr>
<tr>
<td>1101 Liquor</td>
<td>0.46</td>
<td>9.04</td>
<td>0.48</td>
<td>83.06</td>
<td>14.48</td>
</tr>
<tr>
<td>1104 Non-Alcoholic Beverages</td>
<td>0.40</td>
<td>9.28</td>
<td>0.44</td>
<td>83.01</td>
<td>34.86</td>
</tr>
<tr>
<td>1391 Knitted Goods</td>
<td>0.39</td>
<td>9.64</td>
<td>0.89</td>
<td>62.64</td>
<td>34.86</td>
</tr>
<tr>
<td>2821 Ag &amp; Forest Machinery</td>
<td>0.39</td>
<td>9.72</td>
<td>0.82</td>
<td>74.87</td>
<td>21.95</td>
</tr>
<tr>
<td>2813 Mechanical Equipment</td>
<td>0.39</td>
<td>11.18</td>
<td>0.83</td>
<td>27.34</td>
<td>8.64</td>
</tr>
<tr>
<td>1811 Printing</td>
<td>0.37</td>
<td>10.38</td>
<td>0.40</td>
<td>60.19</td>
<td>37.77</td>
</tr>
<tr>
<td>2420 Non-Ferrous Metals</td>
<td>0.35</td>
<td>11.08</td>
<td>1.57</td>
<td>44.35</td>
<td>4.65</td>
</tr>
<tr>
<td>2310 Glass Products</td>
<td>0.34</td>
<td>9.78</td>
<td>0.68</td>
<td>62.59</td>
<td>16.91</td>
</tr>
<tr>
<td>1512 Luggage</td>
<td>0.33</td>
<td>9.46</td>
<td>0.25</td>
<td>86.27</td>
<td>61.08</td>
</tr>
<tr>
<td>2395 Cement &amp; Plaster Products</td>
<td>0.32</td>
<td>9.99</td>
<td>0.56</td>
<td>68.61</td>
<td>25.01</td>
</tr>
<tr>
<td>2394 Cement &amp; Plaster</td>
<td>0.31</td>
<td>11.27</td>
<td>2.32</td>
<td>26.43</td>
<td>0.97</td>
</tr>
<tr>
<td>1701 Primary Paper Materials</td>
<td>0.28</td>
<td>11.46</td>
<td>1.93</td>
<td>13.16</td>
<td>2.86</td>
</tr>
<tr>
<td>1030 Fruit Processing</td>
<td>0.28</td>
<td>9.17</td>
<td>0.47</td>
<td>75.58</td>
<td>37.84</td>
</tr>
<tr>
<td>2011 Basic Chemical Compounds</td>
<td>0.27</td>
<td>11.77</td>
<td>1.60</td>
<td>13.37</td>
<td>1.13</td>
</tr>
<tr>
<td>1709 Other Paper Products</td>
<td>0.26</td>
<td>9.27</td>
<td>0.49</td>
<td>81.15</td>
<td>35.00</td>
</tr>
<tr>
<td>2219 Other Rubber Products</td>
<td>0.25</td>
<td>10.37</td>
<td>1.13</td>
<td>48.30</td>
<td>22.16</td>
</tr>
<tr>
<td>2432 Non-Fe Casting</td>
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<td>Industry</td>
<td>% of Total Employment</td>
<td>Log Labor Productivity</td>
<td>Covariance Term C</td>
<td>NSS % of Employment</td>
<td>NSS % of Output</td>
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</table>

Table 7. Industry Detail (continued 3)
The table lists the NIC-2008 4-digit manufacturing industries used in the paper in decreasing order of average employment share. The first column reports the average percentage of total employment, across both the NSS and ASI, accounted for by the indicated industry. The second column reports weighted-average log labor productivity. The third column reports the average OP covariance term $C$. The fourth and fifth columns report the average percentage of employment and output accounted for by NSS firms, respectively. Averages are equal-weighted across the five data years from 1989–1990 to 2010–2011.