Inflation Announcements and Social Dynamics*

Kinda Hachem † Jing Cynthia Wu ‡
Chicago Booth Chicago Booth

First Version: August 28, 2012
This Version: November 23, 2012

Abstract

This paper investigates the effectiveness of central bank communication when firms have heterogeneous inflation expectations that are updated through social dynamics. The bank’s credibility evolves with these dynamics and determines how well its announcements anchor expectations. We show that trying to eliminate high inflation by abruptly introducing a low inflation target can lead to short-term overshooting whereas gradually introducing the target directs the economy to the long-term goal more quickly and smoothly. In contrast, avoiding a protracted deflation requires aggressive announcements that could more easily be accommodated under price-level targeting. For an inflation targeting central bank, we find that varying two dimensions of quantitative easing - number of rounds and intensity of announcements - provides an effective way to stem deflationary expectations without altering inflation targets.

Keywords: central bank communication, credibility, expectations heterogeneity, inflation targeting, quantitative easing, social dynamics

---

*We thank Veronica Guerrieri, Anil Kashyap, and Randy Kroszner for helpful suggestions. We have also benefitted from discussions with Jon S. Cohen, Jim Hamilton, Jim Pesando, and seminar participants at the Federal Reserve Board. Both authors gratefully acknowledge financial support from the University of Chicago Booth School of Business. Cynthia Wu also gratefully acknowledges financial support from the IBM Faculty Research Fund at the University of Chicago Booth School of Business.

†The University of Chicago Booth School of Business, 5807 South Woodlawn Avenue, Chicago, IL 60637, USA, Kinda.Hachem@chicagobooth.edu

‡The University of Chicago Booth School of Business, 5807 South Woodlawn Avenue, Chicago, IL 60637, USA, Cynthia.Wu@chicagobooth.edu
1 Introduction

For many central banks, communication is now part of the policy toolkit. This is certainly true among inflation targeters who rely on clear and transparent communication to anchor inflation expectations over various horizons (e.g., Carney (2012)). The rise of communication as a policy tool has also been quite apparent since December 2008 when the Fed Funds Rate hit its zero lower bound and the Federal Reserve began unconventional monetary policy to shore up the financial system and stem deflationary expectations. The problem facing policymakers, however, is that the transmission from communication to expectations and outcomes is still largely a black box (e.g. Kroszner (2012), Boivin (2011), and King (2005)). Our paper investigates this transmission by endogenizing central bank credibility and, therefore, the propensity of agents to center expectations around the communicated goals.

To date, many of the macroeconomic insights regarding central bank communication depend on public uncertainty about current or future central bank actions (e.g. Melosi (2011) and Eggertsson and Pugsley (2006)). A similar dependence also exists in game theoretic work, with both Cukierman and Meltzer (1986) and Stein (1989) using asymmetric information between agents and policymakers to explain the pre-Greenspan Fed’s preference for ambiguity. However, given significant increases in transparency among modern central banks (see Woodford (2005) and Blinder et al. (2008)), public information about central bank goals may now be the relevant baseline. In this environment, the key question is whether people believe the bank’s goals will actually be achieved, not whether they believe these are the goals the bank really wants to achieve. The question of interest thus involves a more fundamental notion of credibility - one related to public confidence in the central bank. To the best of our knowledge, our paper is the first to formalize this notion and apply it to understanding the communication-expectations-inflation nexus.

We start by constructing a simple model of monopolistic competition where firms make decisions before the aggregate price level is realized and thus rely on inflation forecasts. The Philadelphia Fed Survey of Professional Forecasters suggests two natural forecasting
rules: one that centers around the central bank’s known inflation goal and one that does not. We calibrate our model so that each rule is indeed an unbiased forecast of inflation when adopted by all firms. At the end of each period, firms can meet and potentially switch rules based on relative performance. Our measure of credibility is the fraction of firms that center their expectations around the central bank’s inflation goals. This measure evolves through the social dynamics of meetings and switches. A small and/or temporary divergence of realized inflation from the bank’s goals may not have enough momentum to significantly affect credibility. However, prolonged divergence may convince some firms to abandon the central bank’s cues in favor of more successful forecasting rules, thus limiting the extent to which future communications will anchor inflation expectations. We investigate how announcements can be tailored to limit divergence and build credibility with the price-setters who ultimately determine inflation.

As described below, our model yields inflation dynamics which are consistent with empirical evidence yet difficult to generate if expectations are homogeneous and rational as assumed in workhorse models of monetary policy.\(^1\) The use of rule-based agents to bridge the gap between tractability and realism has recently gained attention in economic modeling, with Ellison and Fudenberg (1993) showing that even naive rules-of-thumb can ultimately achieve fairly efficient outcomes. Further work in this area has also established the ability of social interactions between rule-based agents to generate emergent phenomena - that is, aggregate behaviors not displayed at the individual level.\(^2\) In Burnside, Eichenbaum and Rebelo (2011), for example, interactions between agents with heterogeneous beliefs about housing fundamentals are key to understanding why only some housing booms culminate in busts. Moreover, in Arifovic, Bullard and Kostyshyna (2012), the presence of social rather than representative learning generates very different predictions about whether Taylor Rules are necessary for a learnable rational expectations equilibrium. As it pertains to communi-
cation, papers such as Orphanides and Williams (2005), Berardi and Duffy (2007), Eusepi and Preston (2010), and Branch and Evans (2011) have investigated representative learning of central bank goals. However, we are not aware of any papers that have analyzed communication under the more realistic assumption of social learning or that have applied social dynamics to endogenize the credibility of transparent communication.

Our analysis yields three main insights. First, we demonstrate that trying to eliminate high inflation by abruptly introducing a low inflation target can lead to temporary overshooting of the target, even if the central bank is transparent and firms reset prices every period. In contrast, gradually introducing the target (i.e., via interim targets) directs the economy to the long-term goal more quickly and smoothly. Our model thus provides a novel explanation for the different cross-country experiences with inflation targeting described in Mishkin and Schmidt-Hebbel (2007).

Second, we show that gradual strategies are actually less effective in a deflation. Instead, the central bank can eliminate deflation more quickly by communicating an aggressive increase in its short-term inflation goals. While many announcements are aggressive enough to have a prompt impact and ultimately return inflation to the long-run target, we find that virtually none can return it monotonically. Our results thus suggest that price-level targeting may have some communication-based benefits over inflation targeting during a deflation.

Third, we illustrate how two dimensions of quantitative easing - number of rounds and intensity of announcements - can be varied to guide the economy out of deflation without explicitly increasing short-term targets. While many papers have focused on the yield curve response to QE, our model sheds some light on the less pervasive but potentially crucial inflation response defined in Krishnamurthy and Vissing-Jorgensen (2011).  

The paper proceeds as follows. Sections 2 and 3 build our core model of price determination, Section 4 introduces credibility through social dynamics, Sections 5 and 6 present the results, and Section 7 concludes. All proofs are contained in the Appendix.

3 For more on the yield curve effect, see Gagnon et al. (2010), D’Amico and King (2010), Williams (2011), Hamilton and Wu (2012), and the references therein.
2 A Simple Model

2.1 Environment

To fix ideas, we begin with a simple model of price determination when agents have heterogeneous expectations. Consider a continuum of firms, each producing a differentiated perishable good \( i \in [0, 1] \). At date \( t \), firm \( i \) faces marginal cost \( MC_t \) and demand

\[
D (p_t (i), P_t) = \left( \frac{P_t}{p_t (i)} \right)^{\frac{1}{1-\rho}}
\]

where \( P_t \) is the aggregate price level, \( p_t (i) \) is the price charged by firm \( i \), and \( \rho \in (0, 1) \). Essentially, the demand for good \( i \) is decreasing in the relative price \( \frac{p_t (i)}{P_t} \). If goods are highly substitutable (i.e., if \( \rho \) is high), then the demand for good \( i \) decreases very sharply when firm \( i \)'s price increases relative to everyone else’s. Firm \( i \)'s pricing decision thus depends on the aggregate price level which is realized after all such decisions have been made. Since any one firm in the continuum is too small to affect the aggregate, each just conditions on its expectation of \( P_t \). With point expectation \( P^e_t (i) \), firm \( i \) solves:

\[
\max_{p_t (i)} [p_t (i) - MC_t] \times D (p_t (i), P^e_t (i))
\]

The first order condition from this problem implies \( p^*_t (i) = \frac{MC_t}{\rho} \). In other words, regardless of the firm’s price expectation, the optimal price is simply a markup over marginal cost. Closing the model thus requires an expression for \( MC_t \). A natural way to obtain such an expression is through factor market clearing. Normalizing aggregate input supply to 1, the market clearing marginal cost solves

\[
\int_0^1 D \left( \frac{MC^*_t}{\rho}, P^e_t (i) \right) di = 1 \quad \text{or, equivalently:}
\]

\[
MC^*_t = \rho \left[ \int_0^1 P^e_t (i)^{\frac{1}{1-\rho}} di \right]^{1-\rho}
\]

In what follows, we use \( \pi^*_t = \ln \left( \frac{P^*_t}{P^*_t} \right) \) to denote the realized inflation rate. Similarly,
price and inflation expectations are related by \( P_t^e(i) \equiv \exp(\pi_t^e(i)) P_{t-1}^* \). With all firms charging the same price, the aggregate price level is just \( P_t^* = \frac{MC_t}{\rho} \). Combining with equation (3) thus yields the following expression for realized inflation:

\[
\pi_t^* = (1 - \rho) \ln \left( \int_0^1 \exp \left( \frac{\pi_t^e(i)}{1 - \rho} \right) \, di \right)
\]  

(4)

2.2 The Central Bank

As equation (4) shows, our model is one where actual inflation is determined entirely by expected inflation. In other words, a central bank in this environment would only have indirect control over inflation through expectations. While our environment is certainly stark, the basic idea that inflation is not just determined by the monetary authority is realistic: an increase in the money supply will have little effect on prices if agents choose not to spend the extra money (i.e., if velocity slows down sufficiently).

To make the central bank’s role in our model more concrete, suppose it announces the following: based on its knowledge of the economy, inflation will be \( m \). If the bank is highly credible, then the average firm believes \( m \) and we model \( \pi_t^e(i) \sim N(m, \sigma^2) \). Together with equation (4), this distribution implies \( \pi_t^* = m + \frac{\sigma^2}{2(1 - \rho)} \). Notice that \( \pi_t^* = m \) when \( \sigma = 0 \). In other words, if all firms expect an inflation rate of \( m \), then realized inflation will indeed be \( m \). The homogeneous version of our simple model thus exhibits rational expectations.

This example raises two questions. First, what does the heterogeneous version of our model imply? Second, what happens if the central bank is not highly credible? We discuss each in turn and explain how the answers to these two questions take us from the simple model to the full model in Section 3.

2.2.1 Heterogeneity

While the homogeneous version has standard properties, what we are interested in is heterogeneity - that is, \( \sigma > 0 \). In this case, \( \pi_t^* = m + \frac{\sigma^2}{2(1 - \rho)} \) so realized inflation exceeds the
mean expectation and, the more heterogeneity there is, the larger the excess. We refer to this as the substitutability effect and explain it as follows. With high values of $\sigma$, some firms will have very high price expectations and will thus want to produce a lot. Since $\frac{1}{1-\rho} > 1$, the increase in input demand among high expectation firms trumps the decrease among low expectation firms. This puts upward pressure on marginal cost and, therefore, prices. The effect is stronger when goods are more substitutable (i.e., when $\rho$ is closer to 1) because high expectation firms anticipate a huge increase in sales by undercutting the aggregate price level and thus demand a lot of input to supply this increase.

Although we have some intuition for why $\sigma > 0$ drives a wedge between $\pi^*_t$ and $m$, a “mean rational” equilibrium would serve as a useful benchmark. In particular, when the average firm expects $m$, we would like $\pi^*_t = m$ and thus no excess inflation. This will require extending the model to get mean rationality at a positive value of $\sigma$ rather than just $\sigma = 0$.

2.2.2 Credibility

What happens if, unlike our earlier example, the central bank is only partially credible? That is, what happens if only a subset of firms center around $m$? In this case, we can model two subpopulations: the first has $\pi^*_{1,t} (i) \sim N (m, \sigma_1^2)$ and the second has $\pi^*_{2,t} (i) \sim N (x, \sigma_2^2)$, where $x \neq m$. The mean rationality benchmark now requires $\pi^*_t = m$ when all firms are in the first subpopulation and $\pi^*_t = x$ when all firms are in the second. If $\sigma_1 = \sigma_2$, then extending our model to have mean rationality for one $\sigma > 0$ will suffice. However, if $\sigma_1 \neq \sigma_2$, then we will need to extend our model to have mean rationality for two positive values of $\sigma$.

Intuitively, we can accomplish this by having two competing forces: one that puts upward pressure on excess inflation and one that puts downward pressure. A natural candidate for downward pressure is exit. In particular, if some firms choose not to operate, then the demand for inputs will fall, the market clearing marginal cost will also fall, and prices will be lower. Notice that this argument relies on heterogeneity - if all firms were the same, then they would either all operate or all not operate. Therefore, with $\sigma > 0$, exit can dampen the
upward pressure imparted by the substitutability effect and introduce a positive crossing as illustrated by the red line in Figure [I]. However, to get two such crossings out of exit, we would need something that creates excess inflation at $\sigma = 0$. Exit could then push excess inflation to zero for one crossing and the substitutability effect would take over at higher $\sigma$ for a second crossing. Effort - an additional factor of production whose cost is a disutility rather than a market clearing price - would introduce the desired inflation at $\sigma = 0$. In particular, production would be more costly than in the simple model so firms would want to sell less and would thus charge higher prices. The next section formalizes this discussion by incorporating effort and exit into the simple model without changing the basic property that inflation expectations drive actual inflation.

3 Full Model

Suppose good $i$ is produced according to the production function $Y(i) = L(i)^{\alpha} z(i)^{1-\alpha}$, where $L(\cdot)$ denotes labor, $z(\cdot)$ denotes firm effort, and $\alpha \in (0,1)$. Aggregate labor supply is normalized to 1 and the cost per unit of labor is a market clearing wage $w_t$. In contrast, effort is exerted by the firm and imparts (real) disutility $z(i)^{\theta}/\theta$, where $\theta > 1$. All firms have a real outside option $U$, thus allowing for firm exit and entry.

3.1 Price Determination

The demand function is again given by equation (1). If firm $i$ charges $p_t(i)$, it will now need

$$\left( \frac{D(p_t(i), P_t^e(i))}{z_t(i)^{1-\alpha}} \right)^{\frac{1}{\alpha}}$$

units of labor to supply its anticipated demand. So, in real terms, $i$ solves:

$$\max \left\{ \max_{p_t(i), z_t(i)} \left[ \frac{p_t(i)D(p_t(i), P_t^e(i))}{P_t^e(i)} - \frac{w_t}{P_t^e(i)} \left( \frac{D(p_t(i), P_t^e(i))}{z_t(i)^{1-\alpha}} \right)^{\frac{1}{\alpha}} - \frac{z_t(i)^{\theta}}{\theta} \right], U \right\}$$

From the inner maximization problem, the price and effort choices of an operating firm are:
\[ p_t(i) = \left( \frac{w_t}{\alpha \rho} \right)^{\theta \alpha} \left( \frac{P_t^e(i) \theta - \rho}{\rho(1 - \alpha)} \right)^{1 - \alpha} \left( \frac{(1 - \rho)}{\theta(1 - \alpha \rho) - \rho(1 - \alpha)} \right)^{1 - \alpha} \] \tag{6}

\[ z_t(i) = \left[ \rho \left( 1 - \alpha \right) \left( \frac{\alpha P_t^e(i)}{(1 - \alpha) w_t} \right)^{\alpha \rho} \right] \left( \frac{1}{\theta(1 - \alpha \rho) - \rho(1 - \alpha)} \right)^{\theta \alpha (1 - \alpha \rho) - \rho(1 - \alpha)} \] \tag{7}

From the outer maximization problem, the set of operating firms is then:

\[ O_t(w_t) = \left\{ i \mid P_t^e(i) \geq \psi(\alpha, \rho, \theta, U) w_t \right\} \] \tag{8}

where

\[ \psi(\alpha, \rho, \theta, U) \equiv \frac{1}{\alpha \rho} \left[ \frac{1}{\rho(1 - \alpha)} \right]^{1 - \alpha} \left[ \frac{\theta U}{\theta(1 - \alpha \rho) - \rho(1 - \alpha)} \right]^{\theta(1 - \alpha \rho) - \rho(1 - \alpha)} \]

Using the firm’s first order conditions, we can also calculate its labor demand:

\[ L_t(i, w_t) = \left[ \rho^\theta (1 - \alpha)^{\rho(1 - \alpha)} \left( \frac{\alpha P_t^e(i)}{w_t} \right)^{\theta - \rho(1 - \alpha)} \right]^{\theta(1 - \alpha \rho) - \rho(1 - \alpha)} \] \tag{9}

The market clearing wage then solves \( \int_{O_t(w_t^*)} L_t(i, w_t^*) \, di = 1 \), yielding:

\[ w_t^* = \chi(\alpha, \rho, \theta) \left[ \int_{O_t(w_t^*)} P_t^e(i) \left( \frac{\theta - \rho(1 - \alpha)}{\theta(1 - \alpha \rho) - \rho(1 - \alpha)} \right) \, di \right]^{\theta(1 - \alpha \rho) - \rho(1 - \alpha)} \] \tag{10}

where

\[ \chi(\alpha, \rho, \theta) \equiv \alpha \left[ \rho^\theta (1 - \alpha)^{\rho(1 - \alpha)} \right]^{\theta - \rho(1 - \alpha)} \]

As equation (6) reveals, firms no longer charge the same price so we define the realized aggregate price level to be the consumption-weighted average of individual firm prices. In our model, the consumption of each good equals the minimum of its supply and its realized demand. The realized value of \( P_t \) thus solves the fixed point problem below:
\[ P^*_t = \int_0^1 \frac{c_t(i)}{\int_0^1 c_t(j) \, dj} p_t(i) \, di \]  

where

\[ c_t(i) = \begin{cases} \left( \frac{\min(P^*_t, P^e_t(i))}{p_t(i)} \right)^{1/\rho} & \text{if } i \in O_t(w^*_t) \\ 0 & \text{if } i \notin O_t(w^*_t) \end{cases} \]

Note that this aggregator would generate \( P^*_t = \bar{p}_t \) if all firms were to charge the same \( \bar{p}_t \).

### 3.2 Model Properties

The above equations show that operating firms with higher price expectations charge higher prices. They also hire more labor and exert more effort, resulting in more output. Furthermore, for any given wage, firms with higher price expectations are more likely to operate. The higher the wage though, the smaller the set of operating firms, the lower the output of each operating firm, and the higher the prices charged. For operating firms, we also have:

**Proposition 1** If \( U > 1 - \alpha \rho - \frac{\rho(1-\alpha)}{\theta} \), then all operating firms charge \( p_t(\cdot) < P^e_t(\cdot) \). Otherwise, \( \exists a P_t \) such that operating firm \( i \) charges \( p_t(i) < P^e_t(i) \) if and only if \( P^e_t(i) > P_t \).

Proposition 1 says that \( U \leq 1 - \alpha \rho - \frac{\rho(1-\alpha)}{\theta} \) is a necessary condition for the price level to be mean rational: if \( U \) is too high, then \( P^*_t \) will lie below the weighted average of the \( P^e_t(\cdot) \)'s. Moreover, the necessary condition becomes stricter with higher values of \( \rho \). When goods are highly substitutable, a firm anticipates a big increase in its demand if it undercuts the aggregate price level by even a little bit. As a result, even firms expecting a low aggregate price level find it profitable to undercut their expectations. More substitutability between

---

4While our results are robust to alternative price aggregators, using a CES aggregator with our model would yield \( P^*_t \neq \bar{p}_t \) if all firms were to charge \( \bar{p}_t \). Even with homogeneity in firm prices, heterogeneity in expectations leads to heterogeneity in supply and thus heterogeneity in realized consumption. Since the CES consumption weights only sum to 1 when consumption is homogeneous, they would generate \( P^*_t \neq \bar{p}_t \) for our model. We find this property undesirable and thus use the aggregator presented above.
goods thus decreases the marginal $P^e_t(\cdot)$ that justifies undercutting. At the same time, lower $U$ decreases the marginal $P^e_t(\cdot)$ that justifies operating. Therefore, with high $\rho$, even a relatively low value of $U$ is consistent with all operating firms charging $p_t(\cdot) < P^e_t(\cdot)$.

**Proposition 2** If $\pi^*_t(i) \sim N(g, \sigma^2)$ for all $i$, then $\pi^*_t = g + f(\sigma)$.

When all firms center their inflation expectations around the same point $g$, Proposition 2 says that excess inflation depends only on the extent of expectations heterogeneity. The green line in Figure 1 provides some insight into the curvature of $f(\cdot)$ by illustrating how $\pi^*_t - g$ varies with $\sigma$ when $\rho = 0.9$, $\alpha = 0.9$, $\theta = 2$, and $U = 0.141$. Recall that without exit and effort we return to the simple model. In other words, $U = 0$ and $\alpha = 1$ imply $\pi^*_t - g = \frac{\sigma^2}{2(1-\rho)}$ as illustrated by the blue line. Adding in only exit (i.e., $U > 0$ but $\alpha = 1$) yields the red line and the one point of mean rationality described at the end of Section 2. Relative to the simple model, Figure 1 shows that the exit-only model results in lower excess inflation for any $\sigma > 0$. The following proposition helps establish this more formally:

**Proposition 3** With $\pi^*_t(i) \sim N(g, \sigma^2)$ for all $i$, the set of operating firms is shrinking in $\sigma$.

All else constant, less operation decreases aggregate labor demand, pushing down wages and, therefore, prices. Proposition 3 thus tells us that higher values of $\sigma$ have a negative effect on inflation via exit. All that remains now is to unpack the effect of effort. Adding in effort along with exit (i.e., $U > 0$ but $\alpha < 1$) yields the green line and the two points of mean rationality described at the end of Section 2. Notice that $\alpha < 1$ actually does two things: it introduces effort as a costly factor of production and it introduces heterogeneity in individual price setting. The implications are as follows. First, heterogeneity in price setting dilutes the substitutability channel. With $\alpha = 1$, equation (6) says that individual prices are (as in the simple model) proportional to the wage. In contrast, $\alpha < 1$ generates individual prices that are a weighted geometric average of the wage and the firm’s price.

---

5 The parameterization is explained in Subsection 4.1.
expectation. Therefore, wage increases from the substitutability channel do not pass fully into individual price increases, implying that the substitutability channel is weaker when $\alpha$ is small. Turn next to the implications of effort as a costly factor. When production requires both effort and labor and effort involves a disutility, firms want to produce less relative to the case where production requires only labor. This decreases labor demand, resulting in a lower wage. However, the fact that the firm produces less means that it has less to sell and can thus charge a higher individual price.

4 Dynamics

Recall that inflation in our model is ultimately determined by inflation expectations. As a result, any inflation dynamics will be driven by dynamics in expectations. Empirical evidence suggests that inflation expectations are at least partly backward-looking (e.g., King (2005), Rudd and Whelan (2006), and Boivin (2011)). To see what this implies in our model, consider the simplest type of backward-looking expectations, namely $E(\pi^e_t(i)) = \pi^*_t - 1$. Unless our economy is at either point A or point B in Figure 1, inflation will not be stable. In particular, if $\sigma > B$, then realized inflation exceeds the mean expectation so $\pi^*_t > \pi^*_{t-1}$ and inflation is ever-increasing. On the other hand, if $\sigma \in (A, B)$, then realized inflation is lower than the mean expectation so $\pi^*_t < \pi^*_{t-1}$ and inflation is ever-decreasing. In this regard, A and B are our steady states, with A exhibiting more anchored inflation expectations than B.

What we are now interested in is the transition between steady states. One of the primary goals of inflation targeting is creating more anchored inflation expectations so we are particularly interested in how the economy can move from B to A. This movement involves changing people’s expectations and, therefore, requires modeling both the economic and social forces that govern the expectations formation process.

The economic forces are fairly straightforward - optimizing agents whose forecasts are consistently outperformed by their peers will want to change how they forecast. However,
the social forces that bring us to this juncture are less clear. For example, how does an agent discover that he is being outperformed and how much of this outperformance does he attribute to one-time shocks rather than fundamentals? These questions depend on how agents interact and how confident they are in their beliefs. The next subsection parameterizes the full model to pin down A and B. Subsection 4.2 then explains how we incorporate the social forces just described to understand the transition between these two points.

4.1 Parameterization

We set $\rho = 0.9$ to capture an economy where goods are highly but not perfectly substitutable. We also set $\alpha = 0.9$ to introduce effort without eliminating labor (i.e., the more conventional factor of production) as the main input. Following standard assumptions about the disutility of effort, we fix $\theta = 2$. Turning now to inflation expectations, the 2012Q2 Philadelphia Fed Survey of Professional Forecasters (SPF) reveals two groups of inflation forecasts: one that is consistent with the Fed’s long-run goal ($\pi_{LR}$) and one that is consistent with a random walk. Assuming that our firms rely on professional forecasters rather than duplicating forecasting effort, we can map these two groups into the two subpopulations described in Subsection 2.2.2. Letting $\pi_t$ denote the central bank’s date $t$ announcement, we have:

**Definition 1** Fed Followers (FFs): $\pi_t^e (\cdot) \sim N(\pi_t, \sigma_F^2)$.

**Definition 2** Random Walkers (RWs): $\pi_t^c (\cdot) \sim N(\pi_{t-1}^*, \sigma_{R,t}^2)$.

The SPF also reveals that forecasters who self-identify as consistent with the Fed’s goals form a tight distribution around $\pi_{LR} = 2\%$. In contrast, the remaining forecasters form a wider distribution around past inflation. In our primary simulation, we consider a central bank that introduces inflation targets to guide the economy from 20% to 2% inflation. We set $\sigma_F = 0.005$ so that two standard deviations on each side put most FFs between $\pi_t - 1\%$ and $\pi_t + 1\%$. This is consistent with countries such as Canada and New Zealand who experienced
double-digit inflation in the 1980s and subsequently introduced inflation targets with ±1% bands. We then calibrate $U = 0.141$ so that point A in Figure 1 occurs at $\sigma = 0.005$.

The parameter choices so far put point B at $\sigma = 0.0459$. The SPF suggests $\sigma_{R}/\sigma_{F} = 5$ when actual and target inflation are reasonably close so we define $\sigma_{R,t}^2 \equiv \sigma_{R}^2 \max \left\{ 1, \frac{\pi_{t-1}^{*}}{\pi_{LR}} \right\}$ then set $\gamma = 0.264$ to get $\sigma_{R,t} = 0.0459$ when $\pi_{t-1}^{*} = 20\%$ and $\pi_{LR} = 2\%$. Our formulation of $\sigma_{R,t}$ implies higher uncertainty at higher inflation rates, consistent with empirical accounts.

### 4.2 Social Forces

Letting $\xi_t$ denote the fraction of FFs at the beginning of date $t$, we have $\xi_t = 0$ at point B and $\xi_t = 1$ at point A. Our model’s expectations dynamics (and thus its inflation dynamics) are driven by changes in $\xi_t$. In particular, if the central bank wants to move from B to A, it basically needs to increase $\xi_t$ from 0 to 1. Since FFs center their expectations around the central bank’s inflation goals, we interpret $\xi_t$ as the bank’s credibility. We initialize $\xi_1 = \exp \left( - \frac{\left| \pi_{LR} - \pi_0 \right|}{\pi_{LR}} \right)$ to capture the idea that the immediate credibility of an inflation goal is lower when the goal is very different from current inflation. The evolution of $\xi_{t+1}$ is then determined by mutation and tournament selection.

We use tournament selection to simulate the transmission of information in a complex world. As described by [Burnside et al. (2011)](Burnside2011), this transmission parallels the epidemiology literature and provides a useful approach to modeling population dynamics. However, while transmission in [Burnside et al. (2011)](Burnside2011) favors agents that are inherently more confident, we follow [Arifovic et al. (2012)](Arifovic2012) and favor agents that are endogenously more successful. In particular, after the realization of $P_{t}^{*}$ at the end of date $t$, our firms meet in pairs and compare forecast errors. In our baseline specification, these pairs are drawn randomly with replacement so each firm can have zero to many meetings in a given period. Since even exiting firms form price expectations - indeed, it is based on these expectations that they decided to exit - we include them in the meetings. In other words, exiting firms exit the labor market, not the population. Denote firm $i$’s forecasting rule by $Rule_i (i) \in \{ FF, RW \}$ and consider
a meeting between $i$ and $j$. Firm $i$ counts one strike against its rule if $\text{Rule}_t(i) \neq \text{Rule}_t(j)$ and $|P^e_t(i) - P^*_t| > |P^e_t(j) - P^*_t|$. Strikes accumulate across meetings and periods. After $S$ strikes, $i$ switches rules and starts counting strikes against its new rule. We refer to $S$ as stubbornness, with higher values of $S$ amounting to firms that are more stubborn in their beliefs. Experimental evidence suggests that people are very reluctant to contradict their own information, even when Bayesian updating suggests they should (e.g., Weizsacker (2010) and Andreoni and Mylovanov (2012)). We interpret this as evidence of stubbornness and thus begin with a high value of $S$, namely $S = 8$. We then describe how our results change when $S$ is decreased. We also discuss what happens if meetings are not random.

To capture the fact that some changes may not be performance-driven, we incorporate mutations. In particular, at the beginning of date $t+1$, a fraction $\mu \in (0, 1)$ of firms randomly switches rules regardless of strikes. Compared to Arifovic et al. (2012) who use a mutation probability of 0.1, we set $\mu = 0.02$ which is considered very low in the relevant literature.

The timing of our social forces can be summarized as follows: (i) the fraction of FFs at the beginning of date $t$ is $\xi_t$; (ii) mutation transforms this fraction into $\hat{\xi}_t = (1 - \mu) \xi_t + \mu (1 - \xi_t)$ if $t \geq 2$; (iii) firm decisions determine $\pi^*_t$ as described in Subsection 3.1; (iv) tournament selection transforms $\xi_t$ into $\xi_{t+1}$ if $t = 1$ and $\hat{\xi}_t$ into $\xi_{t+1}$ if $t \geq 2$.

## 5 Simulation Results

### 5.1 Introducing Inflation Targets

Using the economic model of Section 3 with social dynamics as per Section 4 we now investigate the effect of introducing inflation targets to combat high inflation. Figure 2 presents the results when the economy starts at 20% inflation. All simulations are based on 1000 firms. Black lines are the average of 100 simulations while the dashed blue lines are 5% and 95% confidence intervals.

We begin with a central bank that introduces its target abruptly, announcing $\pi_t = 2\%$
for all \( t \). Figure 2(a) demonstrates that inflation converges to 2% but is followed by a temporary overshooting of the target. Initially, \( \xi_1 \approx 0 \) and the economy is at point B in Figure 1. The bank’s announcement introduces a new forecasting rule which a small fraction of firms mutate towards. Recall from Section 3 that firms with low expectations (relative to their peers) are less likely to operate. FFs thus do not participate in the labor market early on, putting downward pressure on input prices and lowering inflation.

To see why overshooting emerges, turn to the fraction of FFs just before the economy reaches 2%. With realized inflation near target and \( \sigma_F < \sigma_R \), Fed Following is on average a better forecasting rule than Random Walking. If beliefs were not stubborn, RWs would switch very quickly, \( \xi_t \) would rise sharply, the economy would be very close to point A in Figure 1 and excess inflation would be virtually zero. In contrast, stubbornness implies that the economy reaches 2% with a sizeable fraction of RWs. As several of these RWs draw low expectations from their wider distribution, their exit generates negative excess inflation and pushes realized inflation below 2%. Over time though, RWs accumulate enough strikes to compel them to become FFs, returning inflation to target.

Figure 2(b) shows that overshooting can be avoided by introducing a gradual target - that is, a path which interpolates between initial inflation and the long-run target of 2%. By achieving the interim targets in its gradual path, the central bank is able to convert more firms into FFs on the way down to 2%, putting the economy very close to point A when the long-run target is actually reached. The prediction that gradual targets can avoid the overshooting associated with abrupt targets is consistent with empirical evidence. In particular, data from Mishkin and Schmidt-Hebbel (2007) reveals that countries such as Chile, Mexico, Columbia, and Peru introduced their targets more gradually than countries such as Canada, Sweden, the UK, and the Czech Republic. Incidentally, the first group experienced less overshooting than the second.

Figure 3 reveals broadly similar results when inflation starts at 40% rather than 20%. With \( \gamma \) unchanged, 40% puts \( \sigma_{R,t} \) to the right of point B in Figure 1 so RWs alone would
generate ever-increasing inflation. We interpret this as our model’s version of a hyperinflation. Figure 3(a) shows that the abrupt introduction of targets can still decrease inflation in this environment, albeit much more slowly than before. We also observe some overshooting but, since the extra time it takes to reach target gives FFs more time to accumulate, the overshooting is not as statistically significant as in Figure 2(a). Finally, Figure 3(b) confirms that the use of gradual targets eliminates any overshooting. Indeed, by increasing the fraction of FFs along the way, gradual targets both detract from the upward tendency of RWs in the early stages of the hyperinflation and temper exit once inflation nears 2%. Compared to Figure 3(a) then, the result is a faster achievement of target and no overshooting.

5.1.1 Comparison to Benchmark: Fixed Proportions

The key insight from the above discussion is that the occurrence of overshooting hinges on the fraction of FFs when the economy reaches target. To better appreciate the role of social dynamics in determining this fraction, it will be instructive to compare the results with a benchmark that holds $\xi_t$ fixed for all $t$. Take the first scenario, namely initial inflation of 20% and a target of 2%. If $\xi_t = 0$ for all $t$ (i.e., if the central bank is never credible), then inflation is stable at 20% and the announcement is ineffective as illustrated by the solid grey line in Figure 4. In contrast, if $\xi_t = 1$ for all $t$ (i.e., if the central bank is always fully credible), then inflation drops to 2% immediately and the announcement is very effective as illustrated by the solid blue line. With a constant mix of FFs and RWs, the announcement still succeeds in lowering inflation but achieving 2% may take more than one period. Moreover, if the central bank always has some but not a lot of credibility - say, $\xi_t = 0.25$ for all $t$ - then the economy may actually settle on something noticeably below 2%, even if targets are introduced gradually. Endogenizing credibility thus provides a richer and more plausible set of predictions about the effect of central bank communication on inflation.
5.1.2 Comparison to Benchmark: Mutation Only

Recall that our social dynamics have two elements: mutation and tournament selection. To see the impact of each, Figure 5 compares the full dynamics from Figure 2 against the results that would arise under only mutation. With just mutation, $\xi_{t+1} = (1 - \mu) \xi_t + \mu (1 - \xi_t)$ for all $t \geq 1$ and the fraction of FFs converges to 0.5. Initially, many FF forecasts are outperformed by RWs so tournament selection would slow the accumulation of FFs relative to a model with only mutation. Eventually though, the tables turn and many RW forecasts are outperformed by FFs so tournament selection would accelerate this accumulation. When targets are introduced abruptly, tournaments extend the time to target and amplify overshooting. However, when targets are introduced gradually, tournaments are what allow the central bank to eliminate overshooting. Again then, allowing the bank’s credibility to evolve within the model generates fundamentally different predictions.

5.2 Eliminating Deflation

Having seen how communication can be used to reduce inflation, we now investigate how it can be used to pull the economy out of deflation. Abstracting from policy rates has two virtues here. First (and as before), it allows us to isolate the effect of communication. Second, not being able to use the policy rate to achieve its goals is precisely the situation faced by a central bank at the zero lower bound.

Suppose the economy starts at $-2\%$ inflation and the bank announces that it will keep targeting $2\%$ for all $t$. In what follows, we refer to this strategy as maintaining target. With the parameterization unchanged, initial inflation of $-2\%$ puts $\sigma_{R,t}$ roughly at the trough of the green line in Figure 1 so RWs alone would exacerbate the deflation via exit. We interpret this as our model’s version of a deflationary spiral. The very wide confidence bands in Figure 6(a) suggest that maintaining target returns the economy to 2% with low certainty.

The effect of using gradual interim targets to increase inflation is shown in Figure 6(b). While convergence is much more certain, it is relatively slow and involves additional deflation
in the short-term. The intuition for more short-term deflation is as follows. Recall from Section 3 that firms with higher price expectations set higher prices. The central bank’s gradual path initially implies $\pi_1 = -2\% + \varepsilon$ where $\varepsilon > 0$ is small so the average FF only sets a slightly higher price than the average RW. With a low fraction of FFs, the upward pressure from FF price-setting is thus insufficient to offset the downward pressure from RW exit and curb the deflationary spiral in the first few periods.

Finally, Figure 6(c) shows that aggressive communication may actually be better than gradualism at eliminating deflation. The path we consider is one where the central bank announces short-term targets that are well above the long-run goal of 2%. Aggressive short-term targets induce any FFs to set very high prices, pushing realized inflation upwards. At the same time, however, the big gap between realized and targeted inflation does nothing to help the central bank accumulate more FFs. Therefore, when the target returns to 2% and the economy approaches it from above, we have the same overshooting problem we had in Figure 2(a). In order to eliminate this dip, the central bank would have to implement a gradual path on the way down to 2% and thus keep the economy above 2% for longer. To some extent, these results suggest that price-level targeting - which would indeed require the bank to balance out periods of deflation with periods of high inflation - has some advantages over inflation targeting in dealing with deflations when expectations exhibit some stubbornness.

5.3 Alternative Specifications

5.3.1 Lower Stubbornness

The results so far have considered firms with somewhat stubborn beliefs - that is, firms who do not switch forecasting rules at the first sign of a better rule. We now discuss how things change if firms do indeed switch after just one period of underperformance (i.e., $S = 1$).

Begin with the introduction of inflation targets. If initial inflation is 20%, Figure 7(a) shows that an abrupt introduction does not lead to overshooting when $S = 1$. As explained
in Subsection 5.1, RWs who are not stubborn will switch rules very quickly once inflation approaches 2%, pushing the economy very close to point A in Figure 1 and implying excess inflation of virtually zero. Notice, however, that $S = 1$ converges more slowly than $S = 8$. Faster convergence under higher $S$ stems from more FFs persisting in early tournament selections: even though their forecasts may be outperformed by some RWs, stubbornness means that a few strikes are insufficient to induce a switch so the exit channel plays out more strongly. Stubbornness thus has advantages and disadvantages for the central bank. On one hand, higher stubbornness among FFs yields faster convergence to the bank’s target but, on the other, higher stubbornness among RWs leads to a temporary overshooting of the target if the target is introduced abruptly.

Turn now to the introduction of targets when initial inflation is 40%. Unlike Figure 3(a) which ultimately converged to 2%, Figure 7(b) shows that abruptly introducing inflation targets in a hyperinflation will not work when $S = 1$. Given the huge difference between current inflation and the long-run goal, very few firms actually believe that 2% will be achieved in the near-term. Moreover, with low stubbornness, any mutations in the central bank’s favor will be reversed in tournament selection. The fraction of FFs thus remains too low to offset the upward trend imparted by the RWs. While gradually introducing the target may work better, Figure 7(c) demonstrates the danger of being insufficiently gradual. In particular, if the central bank tries to decrease inflation too quickly, the first few interim targets will be much lower than 40%. Since the bank is not endowed with a large following, there are not enough FFs to help achieve the interim targets so RWs will tend to have lower forecast errors than FFs. Combined with the fact that FFs are not stubborn in their beliefs, this means that they will only persist in early tournament selections if they meet other FFs or if they meet RWs with extreme beliefs. For some simulations, the random interactions are such that this occurs but, for others, they are not. The result is very wide confidence bands as illustrated in the figure.

So what can the central bank announce in a hyperinflation if it believes stubbornness is
low? One option is to pursue even more gradual targets than in Figure 7(c). Another option is illustrated in Figure 8(a): the bank can substantially reduce its required gradualism by making high inflation part of the plan. In the path we consider, the central bank takes ownership of hyperinflation and announces that it wants to maintain 40% for the first few periods. With only RWs, inflation would exceed 40% so maintenance is indeed a contractionary announcement. The advantage of this strategy is that it requires a much smaller fraction of FFs in order to succeed. With a straight path from 40% to 2%, the central bank needs a big enough following to reverse the upward trend of the RWs. With maintenance, however, it only needs enough FFs to flatten this trend. Because our bank is not endowed with a large following, flattening the trend is thus easier to achieve. As before, a central bank that achieves its goals reduces the average forecast error of FFs relative to RWs, allowing it to build a following. By pursuing the strategy in Figure 8(a), the bank is essentially building its credibility at the top. Once it has a large enough following, inflation can be brought down to 2% quite rapidly.

Consider now the elimination of deflation under $S = 1$. Figure 8(b) shows that maintaining target will not work. This failure is similar to the failure of abrupt targets in a hyperinflation when stubbornness is low: very few firms believe that 2% will be achieved in the near-term and tournaments quickly reverse any mutations in the central bank’s favor. The fraction of FFs thus remains too low to exert enough upward pressure on prices and offset the downward trend imparted by the RWs. Once again though, low stubbornness is not all bad for the central bank. In particular, comparing Figures 8(c) and 6(b) reveals that using interim targets to guide the economy out of deflation works better when $S = 1$ than when $S = 8$. This is because RWs with extreme beliefs are more liable to switch under $S = 1$. The somewhat better performance under $S = 1$ also depends crucially on the fact that the early deviations from the targeted path are not very large: as we saw in Figure 7(c), a big deviation when $S = 1$ can wipe out the FFs unless they have a specific pattern of meetings.
5.3.2 Local Interactions

We now explore what happens if tournament selection occurs within neighborhoods rather than randomly throughout the population. Suppose, for example, that firms lie along a circle and that each firm meets its right and left neighbors every period. With interactions set up in this way, firms meet the same people every period. Over time, this creates clusters of FFs and clusters of RWs. Firms at the center of an FF cluster are thus more likely to meet other FFs, increasing their effective stubbornness for any value of $S$. The implication for the central bank is that the same strategies can now achieve a much more certain outcome.

To see this, recall the two scenarios where we previously had very wide confidence bands: eliminating deflation by maintaining target when $S = 8$ (Figure 6(a)) and eliminating hyperinflation by introducing insufficiently gradual targets when $S = 1$ (7(c)). As illustrated in Figure 9, the same announcements generate tight bands when interactions are local rather than random. In other words, maintaining target now has a high probability of eliminating deflation when $S = 8$ while the interim targets in Figure 7(c) are no longer insufficiently gradual. In fact, $S = 1$ with local interactions in Figure 9(a) looks a lot like $S = 8$ with random interactions in Figure 3(b). Moreover, to get the same dynamics as Figure 9(b) under random interactions, we would need $S = 12$. This suggests an alternative interpretation for our $S$: higher values of $S$ are a stand-in for more localized interactions.

6 Quantitative Easing

Our focus thus far has been on how central banks can use precise announcements about their inflation goals and/or forecasts to steer the course of actual inflation. To avoid any premature assumptions about the power of the bank, we restricted the direct effect of its announcements to just the mean of the FF distribution. If the bank has absolutely no followers, then the announcements are irrelevant. If the bank has some followers, then we illustrated how announcements can be tailored to endogenously increase this following by
capitalizing on social dynamics between price-setters.

We now consider what happens if the central bank’s communication tool is more potent than previously assumed. In particular, instead of just changing the mean of the FF distribution, suppose the bank’s announcements can directly affect the skewness of the economy-wide distribution. A practical example is what Krishnamurthy and Vissing-Jorgensen (2011) dub the inflation channel of QE - that is, the curtailment of deflation expectations due to publicity surrounding the Fed’s recent large-scale asset purchases. We introduce this channel into our model via redraws. More precisely, some firms with deflationary expectations redraw their $\pi_t(\cdot)$’s after hearing that the central bank is taking a proactive approach to stimulating the economy. Each redraw comes from the same distribution as the original draw so not all deflationary expectations will be eliminated. However, redraws do have the effect of skewing the RW and FF distributions so that more mass exists to the right of the mean. Since very few FFs actually expect deflation, the skew is stronger for RWs but, either way, the effect of QE communications is to reduce dispersion in expectations, enlarge the set of operating firms (by extension of Proposition 3), and put upward pressure on inflation.

We consider two dimensions of QE: rounds and intensity. In our context, rounds means the number of periods with media coverage about QE and, therefore, the number of periods that have redraws. Intensity means the fraction of deflationary firms that are exposed to this coverage and, therefore, the fraction that redraw in a given period. We start with the baseline of random interactions when $S = 8$. Our central bank again faces $-2\%$ inflation but, as an inflation targeter, would like to return the economy to $2\%$ without changing its short-term targets (i.e., without pursuing the aggressive strategy in Figure 6(c)). Figures 10 to 12 illustrate how this can be achieved by varying rounds and intensity. Figure 13 then shows the implications of lower stubbornness and local interactions for QE communications.
6.1 Rounds

To isolate the effect of rounds, we fix intensity at 1. In other words, QE announcements are so pervasive and authoritative that all firms with deflationary expectations redraw. Figure 10(a) illustrates the dangers of stopping QE right when 2% is reached. Just as redraws skew the distributions and increase operation, the end of redraws unskews the distributions and decreases operation. Therefore, if the fraction of FFs is low when the redraws stop, exit among RWs generates a dip back below target for several periods. While this suggests that we need many rounds to prevent a second dip, Figure 10(b) shows that too many rounds will lead to several periods of above-target inflation. Figure 10(c) then shows the difficulty in eliminating both the increase above 2% and the dip back below by varying only the number of rounds. We can thus conclude the following. First, increasing the number of rounds while holding intensity constant amplifies overshooting on the way up and attenuates it on the way down. Second, full intensity makes it difficult to find a number of rounds that generates a monotonic return to 2% (i.e., that eliminates overshooting in both directions).

6.2 Intensity

Figure 11 shows that monotonicity is easier to achieve when the central bank can also vary the intensity of communication. Figure 11(a) illustrates the outcome of many rounds and moderate intensity. Figure 11(b) illustrates the outcome of many rounds and low intensity. Although too low an intensity delays the return to target, a combination of many rounds and moderate intensity provides a roughly monotonic recovery. The importance of many rounds is confirmed by Figure 12 which shows that decreasing the number of rounds makes it harder to find a monotonicity-inducing intensity. Indeed, decreasing both the number of rounds and the intensity of QE announcements leaves open the possibility that inflation will not return to 2% at all (see Figure 12(b)).

Why do many rounds make it easier to find a monotonicity-inducing intensity when beliefs are somewhat stubborn? To avoid the rise above 2%, the central bank needs to stop
QE right when inflation hits its target. Moreover, to avoid the dip below 2%, the central bank needs to have a large following by the time QE stops. With $T$ rounds of QE, the bank therefore has $T$ periods to accomplish two things: hit 2% and accumulate FFs. As we shorten $T$ to $\tilde{T}$, accumulating a large enough following requires higher intensity. However, higher intensity also hastens the return to 2%. When firms are stubborn in their beliefs, a small increase in intensity will have a weaker effect on the accumulation of FFs than it will on the speed of recovery. The intensity increase needed to accumulate a large enough following in $\tilde{T}$ periods thus exceeds the intensity increase needed to hit 2% in $\tilde{T}$ periods.

6.3 Alternative Specifications

What happens if firm beliefs are not stubborn? Figures 13(a) and 13(b) reveal that the central bank can get away with fewer rounds and higher intensity when $S = 1$. With low $S$, FFs accumulate faster as the economy approaches 2% so the bank does not need as many rounds to build its following. However, in the initial periods, inflation is well below 2% so low $S$ also means that any early mutations towards the central bank are quickly undone by tournament selection. The bank thus cannot rely on FF price-setting to help with the initial push towards 2%. Instead, it has to rely entirely on redraws which means it needs a higher intensity in order to get the economy close enough to 2% before the rounds run out.

Turning next to local rather than random interactions, Figure 13(c) confirms the main conclusion of Subsection 5.3.2: some strategies can achieve much more certain outcomes when firms judge their performance against the same neighbors every period. In particular, Figure 13(c) demonstrates that the one QE scenario which had wide confidence bands under random interactions - namely, Figure 12(b) - has tight bands under local interactions.
7 Conclusion

This paper has investigated the effectiveness of central bank communication when price-setters with heterogeneous inflation expectations are subject to social dynamics. Prolonged periods of divergence between realized inflation and the central bank’s goals can lead to a loss of credibility through these dynamics and make future goals much harder to achieve. In this context, we identified how central bank communications can be tailored to endogenously build credibility. We demonstrated that the abrupt introduction of a low inflation target can lead to a temporary overshooting of that target. In contrast, gradually introducing the target (i.e., via interim targets) directs the economy to the long-term goal more quickly and smoothly. We also found that gradualism is crucial for guiding the economy out of a hyperinflation. Unless the central bank is willing to make high inflation part of its initial plan, the degree of gradualism must be very high when addressing a hyperinflation, especially if agent interactions are not confined to neighborhoods. Our next set of results concerned communications to guide the economy away from deflation. We found that avoiding a protracted deflation requires an aggressive rather than gradual approach, with price-level targeting conferring some communication-based benefits over inflation targeting. We then studied two dimensions of quantitative easing (QE): number of rounds and intensity of announcements. Our results indicated that the inflation channel of QE is an effective way for an inflation targeting central bank to guide the economy out of deflation without announcing higher short-term targets. However, the mix of rounds and intensity needed to guide the economy out monotonically depends on the stubbornness of agents’ beliefs.
References


Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack, “Large-Scale Asset Purchases by the Federal Reserve: Did They Work?,” 2010. Federal Reserve Bank of New York Staff Reports.


Figure 1: Full Model vs Simple Model

Excess Inflation ($\pi^* - g$)

- **Simple Model (U=0 and $\alpha=1$)**
- **Simple Model + Exit (U>0 and $\alpha=1$)**
- **Simple Model + Exit + Effort (U>0 and $\alpha<1$)**

Points A and B indicate specific values of the excess inflation at different levels of $\sigma$.
Figure 2: Introduction of IT (High Inflation)

(a) Inflation Rate, Abrupt Strategy

(b) Inflation Rate, Gradual Strategy
Figure 3: Introduction of IT (Hyperinflation)
Figure 4: Comparison to Fixed Proportions

(a) Abrupt Strategy

(b) Gradual Strategy
Figure 5: Comparison to Mutation-Only

(a) Abrupt Strategy

(b) Gradual Strategy
Figure 6: Eliminating Deflation

(a) Inflation Rate, Maintaining Target

(b) Inflation Rate, Gradual Strategy

(c) Inflation Rate, Aggressive Strategy

36
Figure 7: Effect of Lower Stubbornness

(a) Inflation Rate, Introduction of IT (High Inflation), Abrupt Strategy

(b) Inflation Rate, Introduction of IT (Hyperinflation), Abrupt Strategy

(c) Inflation Rate, Introduction of IT (Hyperinflation), Gradual Strategy
Figure 8: Effect of Lower Stubbornness, Continued

(a) Inflation Rate, Introduction of IT (Hyperinflation), Alternative Strategy

(b) Inflation Rate, Eliminating Deflation, Maintaining Target

(c) Inflation Rate, Eliminating Deflation, Gradual Strategy
Figure 9: Effect of Local Interactions

(a) Inflation Rate, S=1, Introduction of IT (Hyperinflation), Gradual Strategy

(b) Inflation Rate, S=8, Eliminating Deflation, Maintaining Target
Figure 10: Effect of QE Rounds (Intensity = 1)
Figure 11: Effect of QE Intensity (Rounds = 20)

(a) Inflation Rate, Intensity = 0.4

(b) Inflation Rate, Intensity = 0.2
Figure 12: Effect of QE Intensity (Rounds = 12)

(a) Inflation Rate, Intensity = 0.4

(b) Inflation Rate, Intensity = 0.2
Figure 13: Effect of Alternative Specifications Under QE

(a) Inflation Rate, S=1, Random, Rounds=12, Intensity=1

(b) Inflation Rate, S=1, Random, Rounds=12, Intensity=0.6

(c) Inflation Rate, S=8, Local, Rounds=12, Intensity=0.2
Appendix - Proofs

Proof of Proposition 1

Notice that $p_t(i) < P_t^e(i)$ is equivalent to $P_t^e(i) > \frac{1}{\alpha \rho} \left[ \frac{1}{\rho(1-\alpha)} \right]^{1-\alpha} w_t$. The latter inequality is guaranteed by the operation constraint $P_t^e(i) \geq \bar{\psi}(\alpha, \rho, \theta, U) w_t$ if $U > 1 - \alpha \rho - \frac{\rho(1-\alpha)}{\theta}$. If instead $U < 1 - \alpha \rho - \frac{\rho(1-\alpha)}{\theta}$, then there will be low values of $P_t^e(i)$ that satisfy the operation constraint but not the condition for $p_t(i) < P_t^e(i)$. ■

Proof of Proposition 2

Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal PDF and CDF respectively. From Pezzey and Sharples (2007), the moment generating function of a truncated normal random variable with mean 0 and variance $\sigma^2$ is:

$$\int_{x \geq c} \exp(rx) \phi(x, \sigma^2) dx = \exp\left(\frac{r^2\sigma^2}{2}\right) \Phi\left(r\sigma - \frac{c}{\sigma}\right)$$  \hspace{1cm} (A.1)

Using $P_t^e(i) \equiv \exp(\pi_t^e(i)) P_{t-1}$ and $\pi_t^e(i) = g + \varepsilon_t(i)$ with $\varepsilon_t(i) \sim N(0, \sigma^2)$ in equation (8), we can rewrite the operation constraint as:

$$\varepsilon_t(i) \geq \ln\left(\frac{\bar{\psi} w_t}{P_{t-1}}\right) - g \equiv X$$  \hspace{1cm} (A.2)

To ease notation, define the following constants:

$$\kappa_1 \equiv \frac{\theta - \rho(1-\alpha)}{\theta(1-\alpha)(1-\rho)}, \quad \kappa_2 \equiv \frac{\theta \alpha}{\theta(1-\alpha)(1-\rho)}, \quad \text{and} \quad \kappa_3 \equiv \frac{\theta - \rho(1-\alpha)}{\theta(1-\alpha)(1-\rho)}$$

Combining equations (A.1) and (A.2) with the wage equation in (10) then yields an implicit definition of $X$ which is independent of $g$:

$$X = \frac{1}{\rho \kappa_2} \ln\left(\frac{\kappa_2 U}{\alpha}\right) + \frac{1-\alpha}{\theta - \rho(1-\alpha) \kappa_2} \ln\left(\frac{1}{\rho(1-\alpha)}\right) + \kappa_1 \sigma^2 \kappa_2 + \frac{1}{\kappa_1} \ln\Phi\left(\frac{\kappa_1 \sigma - X}{\sigma}\right)$$  \hspace{1cm} (A.3)
Turn now to inflation. Substitute the firm pricing equation (6) into the price aggregator (11) and simplify to get:

\[
\int \frac{\exp\left(\min\left\{ \frac{\pi_t}{1-\rho} - \frac{\pi_t(1)}{1-\rho} \right\}\right)}{\exp\left(\frac{\rho(\theta-\rho)(1-\alpha)}{\theta(1-\alpha)-\rho(1-\alpha)} \pi_t(i)\right)} \, di
\]

Combining equations (A.3) and (A.4) then taking logs yields:

\[
\pi_t = \frac{(1-\alpha)(1-\rho)}{\theta-\rho(1-\alpha)} \ln \left( \frac{1}{\rho(1-\alpha)} \right) + \frac{\theta(1-\rho)}{\theta(1-\alpha)-\rho(1-\alpha)} g + \frac{\theta(1-\rho)[\theta(1-\alpha)]^2}{[\theta(1-\alpha)-\rho(1-\alpha)]^2}\frac{\alpha^2}{2} \]

Now use \( \pi_t(i) = g + \varepsilon_t(i) \) and \( \varepsilon_t(i) \sim N(0, \sigma^2) \) with equation (A.4) to simplify (A.5). There are two cases. If \( X \) is greater than or equal to the right hand side of (A.6) below, then:

\[
\pi_t - g = \left( \kappa_1 \kappa_2 \right) (1-\rho)^2 - \frac{(1+\rho)(\theta-\rho)^2(1-\alpha)^2}{\theta(1-\alpha)-\rho(1-\alpha)} \frac{\alpha^2}{2(1-\rho)} \]

Otherwise, \( \pi_t - g \) is the solution to the following fixed point problem:

\[
\pi_t - g = \left( \frac{\theta(1-\alpha)-\rho(1-\alpha)}{\theta(1-\alpha)-\rho(1-\alpha)} - \kappa_2^2 \right) (1-\rho) \frac{\sigma^2}{2} \]

\[
+ \frac{(1-\alpha)(1-\rho)}{\theta-\rho(1-\alpha)} \ln \left( \frac{1}{\rho(1-\alpha)} \right) + \frac{\theta(1-\rho)}{\theta(1-\alpha)-\rho(1-\alpha)} \ln \Phi \left( \kappa_1 \sigma - \frac{X}{\sigma} \right) + \ln \left( \frac{\Phi\left( -\rho \kappa_2 \sigma - \frac{X}{\sigma} \right) - \Phi\left( \kappa_1 \sigma - \frac{X}{\sigma} \right)}{\Phi\left( -\rho \kappa_2 \sigma - \frac{X}{\sigma} \right) - \Phi\left( -\kappa_3 \sigma - \frac{X}{\sigma} \right)} \right) \]
Either way, we have a definition of $\pi_t - g$ which is independent of $g$. ■

Proof of Proposition 3

The fraction of firms not operating is $\Delta \equiv \Phi \left( \frac{X}{\sigma} \right)$. Taking derivatives yields $\frac{d\Delta}{d\sigma} \propto \frac{dX}{d\sigma} - \frac{X}{\sigma}$ so what we want to show is $\frac{dX}{d\sigma} > \frac{X}{\sigma}$. Using equation (A.3) from the proof of Proposition 2:

$$\frac{dX}{d\sigma} = \kappa_1 \sigma + \frac{\frac{X}{\sigma}}{1 + \frac{\Phi(\kappa_1 \sigma - \frac{X}{\sigma})}{\phi(\kappa_1 \sigma - \frac{X}{\sigma}) \kappa_1 \sigma}}$$

The desired inequality is thus $\left( \kappa_1 \sigma - \frac{X}{\sigma} \right) \frac{\Phi(\kappa_1 \sigma - \frac{X}{\sigma})}{\phi(\kappa_1 \sigma - \frac{X}{\sigma}) \kappa_1 \sigma} > -1$. Using $x \Phi (x) > -\phi (x)$ as shown next completes the proof: $x \Phi (x) = x \int_{-\infty}^{x} \phi (t) dt > \int_{-\infty}^{x} t \phi (t) dt = - \int_{-\infty}^{x} \phi' (t) dt = -\phi (x)$. ■