

Wealth distribution and social mobility in the US: A quantitative approach

Jess Benhabib Alberto Bisin Mi Luo
NYU and NBER NYU and NBER NYU

First draft: July 2015; this draft: February 2017

INCOMPLETE DRAFT PRESENTLY UNDER REVISION*

Abstract

This paper attempts to quantitatively identify the factors that drive wealth dynamics in the U.S. and are consistent with its observed skewed cross-sectional distribution and with observed social mobility. We concentrate on three critical factors: i) skewed and persistent distribution of earnings, ii) differential saving and bequest rates across wealth levels, and iii) stochastic returns on wealth (possibly also differential across wealth levels). All of these factors are fundamental for matching both distribution and mobility, with a distinct role in inducing wealth accumulation near the borrowing constraints, contributing to the thick top tail of wealth, and affecting upward and/or downward social mobility. The stochastic process for capital income risk which best fits the cross-sectional distribution of wealth and social mobility in the U.S. shares several statistical properties with those of the returns to wealth uncovered by Fagerang et al. (2016) from tax records in Norway.

Key Words: wealth distribution; thick tails; inequality; social mobility

JEL Numbers: E13, E21, E24

*Thanks to seminar audiences at Duke, NYU, Minneapolis Fed, SED-Warsaw, Lake Baikal Summer School, SAET-Cambridge, University College of London, Wharton School, NBER Summer Institute. Special thanks to Alberto Alesina, Orazio Attanasio, Laurent Calvet, Mariacristina De Nardi, Pat Kehoe, Dirk Krueger, Per Krusell, Ben Moll, Andrew Newman, Tom Sargent, Ananth Shesadri, Gianluca Violante, Daniel Xu, Fabrizio Zilibotti. Special thanks to Luigi Guiso, for many illuminating discussions and for spotting a mistake in a previous version, to the Editor and the referees for their exceptional work on the paper. Corresponding author's email: alberto.bisin@nyu.edu.

1 Introduction

Wealth in the U.S. is unequally distributed, with a Gini coefficient of 0.82. It is skewed to the right, and displays a thick, right tail: the top 1% of the richest households in the United States hold over 33.6% of wealth.¹ At the same time, the U.S. is characterized by a relatively fast social mobility, with an inter-generational Shorrocks mobility index in the range of .88 – .98.²

This paper attempts to quantitatively identify the factors that drive wealth dynamics in the U.S. and are consistent with the observed cross-sectional distribution of wealth and with the observed social mobility. We first develop a macroeconomic model displaying various distinct wealth accumulation factors. Once we allow for an explicit demographic structure, the model delivers implications for social mobility as well as the cross-sectional distribution. We then match the moments generated by the model to several empirical moments of the observed distribution of wealth as well as of the social mobility matrix. Indeed various distinct wealth accumulation factors have qualitatively different effects on social mobility which then help identify the relative importance of the factors in driving wealth dynamics: savings rates which increase steeply with wealth, e.g., deliver the thick tails of the wealth distribution but also imply too little intergenerational mobility relative to the data.

Many recent studies of wealth distribution and inequality focus on the relatively difficult task of explaining the thickness of the upper tail. We shall concentrate mainly on three critical factors previously shown, typically in isolation from each other, to affect the tail of the distribution, empirically and theoretically. First, a skewed and persistent distribution of *stochastic earnings* translates, in principle, into a wealth distribution with similar properties. A large literature in the context of Aiyagari-Bewley economies has taken this route, notably

¹See Diaz-Gimenez et al. (2011), Table 6, elaborating data from the Survey of Consumer Finances (SCF) 2007.

²The range is computed, respectively, from the social mobility matrix in Charles and Hurst (2003), Table 2, from PSID data, and the matrix we construct from SCF 2007-2009 data, in Section 3.2. See below Section 3.2 for a discussion of the different methodologies adopted).

Castañeda et al. (2003) and Kindermann and Krueger (2015).³ Another factor which could contribute to generating a skewed distribution of wealth is *differential saving rates* across wealth levels, with higher saving and accumulation rates for the rich. In the literature this factor takes the form of non-homogeneous bequests, bequests as a fraction of wealth that are increasing in wealth; see for example Cagetti and De Nardi (2006).⁴ Finally, stochastic idiosyncratic returns on wealth, or *capital income risk*, has been shown to induce a skewed distribution of wealth, in Benhabib et al. (2011); see also Quadrini (2000), which focuses on entrepreneurial risk.⁵

Finally allowing rates of return on wealth to be increasing in wealth might also add to skewedness of the distribution. This could be due e.g., to the existence of economies of scale in wealth management, as in Kacperczyk et al. (2014); see Saez and Zucman (2016), Fagereng et al. (2016) and Piketty (2014, p. 447) for evidence.

While all these factors possibly contribute to produce skewed wealth distributions, their relative importance remains to be ascertained.⁶

[...]

Saving rate differentials (as well as returns on wealth increasing in wealth) help match the top tail but they reduce social mobility as the rich get richer accumulating at higher rates (as well as facing higher returns). Stochastic returns on wealth, or capital income risk, also contributes to the thick top tail while allowing for social mobility, especially in terms of speeding up downward mobility. Finally, we extend our analysis to allow for returns to

³Several papers in the literature include a stochastic length of life (typically, “perpetual youth”) to complement the effect of skewed earnings on wealth. We do not include this in our model as it has manifestly counterfactual demographic consequences.

⁴See also Piketty (2014) which directly discusses the saving rates of the rich.

⁵Krusell and Smith (1998) instead introduce heterogeneous discount factors. However, such discount factors are non-measurable. Micro data allowing estimates of capital income risk are instead rapidly becoming more available; see e.g., the tax records for Norway studied by Fagereng et al. (2016) and the Swedish data studied by Bach et al (2016).

⁶Other possible factors which qualitatively would induce skewed wealth distributions include a precautionary savings motive for wealth accumulation. In fact, the precautionary motive, by increasing the savings rate at low wealth levels under borrowing constraints and random earnings, works in the opposite direction of savings rates increasing in wealth. We do not exploit this channel for simplicity, assuming that life-cycle earnings profiles are random across generations but deterministic within lifetimes.

depend on wealth and show that this indeed helps in matching wealth and social mobility, though without directly observing return data their dependence on wealth is somewhat poorly identified.

The rest of the paper is structured as follows. Section 2 lays out the theoretical framework. Section 3 explains our quantitative approach and data sources we use. Section 4 shows the baseline results with the model fit for both targeted and un-targeted moments. Section 5 presents several counterfactual exercises, where we re-estimate the model shutting down one factor at a time. Section 6 reports on a robustness exercise, allowing for non-stationarity of the wealth distribution and measuring the transition speed our model delivers. Section 7 concludes.

2 Wealth dynamics and stationary distribution

Most models of the wealth dynamics in the literature focus on deriving skewed distributions with thick tails, e.g., Pareto distributions (power laws).⁷ While this is also our aim, we more generally target the whole wealth distribution and its intergenerational mobility properties. To this end we study a simple micro-founded model - a standard macroeconomic model in fact - of life-cycle consumption and savings. The model exploits the interaction of the factors identified in the Introduction that tend to induce skewed wealth distributions: stochastic earnings, differential saving and bequest rates across wealth levels, and stochastic returns on wealth. We then extend the model to allow for differential rates of return across wealth levels.

Each agent's life span is finite and deterministic, T years. Every period t , consumers choose consumption c_t and accumulate wealth a_t , subject to a no-borrowing constraint. Consumers leave wealth a_T as a bequest at the end of life T . Each agent's preferences are

⁷We only discuss here those models which are directly relevant to our present analysis, referring to Benhabib and Bisin (2015) for an extensive survey of the theoretical and empirical literature on the wealth distribution.

composed of a per-period utility from consumption, $u(c_t)$, at any period $t = 1, \dots, T$, and a warm-glow utility from bequests at T , $e(a_T)$. Their functional forms display Constant Relative Risk Aversion:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad e(a_T) = A \frac{a_T^{1-\mu}}{1-\mu}.$$

Wealth accumulates from savings and bequests. Idiosyncratic rates of returns r and lifetime labor earnings profiles $w = \{w_t\}_{t=1}^T$ are drawn from a distribution at birth, possibly correlated with those of the parent, deterministic within each generation life.⁸ We emphasize that r and $\{w_t\}$ are stochastic over generations only: agents face no uncertainty within their life span. Lifetime earnings profiles are hump-shaped, with low earnings early in life. As a consequence, borrowing constraints limit how much agents can smooth lifetime earnings.

Let $\beta < 1$ denote the discount rate. Let $V_t(a_t)$ denote the present discounted utility of an agent with wealth a_t at the beginning of period t . Given initial wealth a_0 , earnings profile w , and rate of return r , each agent's maximization problem, written recursively, then is:

$$\begin{aligned} V_t(a_{t-1}) &= \max_{c_t, a_{t+1}} u(c_t) + \beta V_{t+1}(a_{t+1}) \\ \text{s.t.} \quad & a_t = (1+r)a_{t-1} - c_t + w_t \\ & 0 \leq c_t \leq a_t, \quad t = 1, \dots, T-1 \\ V_T(a_T) &= \frac{1}{\beta} e(a_T) \end{aligned}$$

The solution of the recursive problem can be represented by a map

$$a_T = g(a_0; r, w).$$

It can be shown that under our assumptions, the map $g(\cdot)$ can be characterized as follows:

if $\mu = \sigma$, then $g(a_0; r, w) = \alpha(r, w)a_0 + \beta(r, w)$;

⁸As we noted, assuming deterministic earning profiles amounts to disregarding the role of intra-generational life-cycle uncertainty and hence of precautionary savings. While the assumption is motivated by simplicity, see Keane and Wolpin (1997), Huggett et al. (2011), and Cunha et al. (2010) for evidence that the life-cycle income positions tend to be determined early in life.

if $\mu < \sigma$, then $\frac{\partial^2 g}{\partial a_0^2}(a_0; r, w) > 0$.

In the first case, $\mu = \sigma$, the savings rate is $\alpha(r, w)$ and it is independent of wealth. If instead $\mu < \sigma$ differential savings rate emerge, increasing with wealth.

The map $g(\cdot)$ serves as the building block to construct the stochastic wealth process across generations. Adding an apex n to indicate the generation and slightly abusing notation, we denote with $\{r^n, w^n\}_n$ the stochastic process over generations for the rate of return on wealth r and earnings w . We assume it is a finite irreducible Markov Chain. We assume also that r^n and w^n are independent, though each is allowed to be serially correlated, with transition $P(r^n \mid r^{n-1})$ and $P(w^n \mid w^{n-1})$. The life-cycle structure of the model implies that the initial wealth of the n 'th generation coincides with the final wealth of the $n - 1$ 'th generation: $a^n = a_0^n = a_T^{n-1}$. We can then construct then a stochastic difference equation for the initial wealth of dynasties, induced by $\{r^n, w^n\}_n$, and mapping a^{n-1} into a^n :

$$a^n = g(a^{n-1}; r^n, w^n),$$

The map $g(\cdot)$ represents the solution of the life-cycle consumption-saving problem, as characterized above, and it induces a stochastic process $\{a^n\}_n$ for initial wealth a . When $\mu = \sigma$, the wealth process across generations is represented then by a linear stochastic difference equation in wealth, which has been closely studied in the math literature; see de Saporta (2005). Indeed, if $\mu = \sigma$, under general conditions,⁹ the stochastic process $\{a^n\}_n$ has a stationary distribution whose tail is independent of the distribution of earnings and asymptotic to a Pareto law:

$$Pr(a > \underline{a}) \sim Q\underline{a}^{-\gamma},$$

where $Q \geq 1$ is a constant and $\lim_{N \rightarrow \infty} E \left(\prod_{n=0}^{N-1} (\alpha(r^{-n}, w^{-n}))^\gamma \right)^{\frac{1}{N}} = 1$.¹⁰ If instead,

⁹More precisely, the tail of earnings must be not too thick and furthermore $\alpha(r^n, w^n)$ and $\beta(r^n, w^n)$ must satisfy the restrictions of a *reflective process*; see Grey (1994), Hay et al. (2011), and Benhabib et al., 2011, for a related application.

¹⁰While a denotes initial wealth, it can be shown that when the distribution of initial wealth has a thick

keeping σ constant, $\mu < \sigma$, a stationary distribution might not exist; but if it does,

$$Pr(a > \underline{a}) \geq Q(\underline{a})^{-\gamma},$$

and hence it displays a thick tail.

Finally, the model is straightforwardly extended to allow for the Markov states of the stochastic process for r to depend on the initial wealth of the agent a . In this case, the intergenerational wealth dynamics have properties similar to the $\mu < \sigma$ case: a stationary distribution might not exist; but if it does, it displays a thick tail.

3 Quantitative analysis

The objective of this paper, as we discussed in the Introduction, consists in measuring the relative importance of various factors which determine the wealth distribution and the social mobility matrix in the U.S. The three factors are stochastic earnings, differential saving and bequest rates across wealth levels, and stochastic returns on wealth. These are represented in the model by the properties of the dynamic process and the distribution of (r^n, w^n) and by the parameters μ and σ , which imply differential savings (the rich saving more) when $\mu < \sigma$.

3.1 Methodology

We estimate the parameters of the model described in the previous section using a Method of Simulated Moments (MSM) estimator: i) we fix (or externally calibrate) several parameters of the model; ii) we select some relevant moments of the wealth process as target in the estimation; and iii) we estimate the remaining parameters by matching the targeted moments generated by the stationary distribution induced by the model and those in the data. The

tail, the distribution of wealth also does; see Benhabib et al., 2011 for the formal result.

quantitative exercise is predicated then on the assumption that the wealth and social mobility observed in the data are generated by a stationary distribution.¹¹

More formally, let θ denote the vector of the parameters to be estimated. Let m_h , for $h = 1, \dots, H$, denote a generic empirical moment; and let $d_h(\theta)$ the corresponding moment generated by the model for given parameter vector θ . We minimize the deviation between each targeted moment and the corresponding simulated moment. For each moment h , define $F_h(\theta) = d_h(\theta) - m_h$. The MSM estimator is

$$\hat{\theta} = \arg \min_{\theta} \mathbf{F}(\theta)' W \mathbf{F}(\theta).$$

where $\mathbf{F}(\theta)$ is a column vector in which all moment conditions are stacked, i.e. $\mathbf{F}(\theta) = [F_1(\theta), \dots, F_H(\theta)]^T$. We choose an identity matrix for the weighting matrix in the baseline, $W = \mathbf{I}_N$.¹²

In our quantitative exercise we proceed as follows.

- i) We fix $\sigma = 2$, $T = 36$, $\beta = 0.97$ per annum. We feed the model with a stochastic process for individual earnings profiles, w^n and its transition across generations, $P(w^n | w^{n-1})$. Both the earning process and its transition are taken from data; respectively from the PSID and ????
- ii) We target as moments:

the bottom 20%, 20 – 39%, 40 – 59%, 60 – 79%, 80 – 89%, 90 – 94%, 95 – 99%, and the top 1% wealth percentiles; and

the diagonal of the social mobility Markov chain transition matrix defined over the same

¹¹Very few studies in the literature deal with the transitional dynamics of wealth and its speed of transition along the path, though this issue has been put at the forefront of the debate by Piketty (2014). Notable and very interesting exceptions are Gabaix et al. (2015), ?, and Hubmer et al. (2015). We extend the analysis to possibly non-stationary distributions in Section 6 as a robustness check. Our preliminary results are encouraging, in the sense that the model seems to be able to capture the transitional dynamics with parameters estimates not too far from those obtained under stationarity.

¹²The objective function is highly nonlinear in general and therefore, following Guvenen (2016), we employ a global optimization routine for the MSM estimation. Further details on the estimation can be found in Appendix A.

percentile ranges as states.

iii) We estimate:

the preference parameters μ, A ; and

a parametrization of the stochastic process for r defined by 5 states r_i and 5 diagonal transition probabilities, $P(r^n = r_i | r^{n-1} = r_i)$, $i = 1, \dots, 5$ (restricting the 5×5 transition matrix to display constantly decaying off-diagonal probabilities except for the last row for which we assume constant off-diagonal probabilities).¹³

In total, therefore, we target 15 moments and we estimate 12 parameters.

Finally, in Section 4.4 we study the case in which the Markov states of the stochastic process for r depend on the initial wealth a of the agent.

3.2 Data

Our quantitative exercise requires data for labor earnings, wealth distribution, and social mobility.

Labor earnings. We use 10 deterministic life-cycle household-level earnings profiles at different deciles, as estimated by Heathcote et al. (2010) from the Panel Study of Income Dynamics (PSID). These profiles are drawn in Figure 1.¹⁴ In our quantitative exercise we collapse earnings levels into six-year averages, as in Table 1.¹⁵

¹³Formally, $P(r^n = r_i | r^{n-1} = r_j) = P(r^n = r_i | r^{n-1} = r_i)e^{-\lambda j}$, $i = 1, 2, 3, 4$, $j \neq i$, λ such that $\sum_{j=1}^5 P(r^n = r_i | r^{n-1} = r_j) = 1$; and $P(r^n = r_5 | r^{n-1} = r_j) = \frac{1}{4}(1 - P(r^n = r_5 | r^{n-1} = r_5))$. We adopt a restricted specification in order to reduce the number of parameters we need to estimate. This particular specification performs better than one with constant off-diagonal probabilities as well as one with decaying off-diagonal probabilities in all rows.

¹⁴The panel data on earnings from the U.S. Social Security Administration (SSA) are not yet generally available. However, the crucial aspect of earnings data, for our purposes, is that they are far from skewed enough to account by themselves for the skewedness of the wealth distribution. This is in fact confirmed on SSA data directly by Guvenen et al. (2016), Section 7.2.II, and by De Nardi, Fella, Pardo (2016).

¹⁵In the actual quantitative exercise we also replace negative earnings levels for the first decile (first column) with a small value (0.001), as our theoretical model does not allow for negative wealth positions (borrowing).

Table 1: Life-cycle earnings profiles

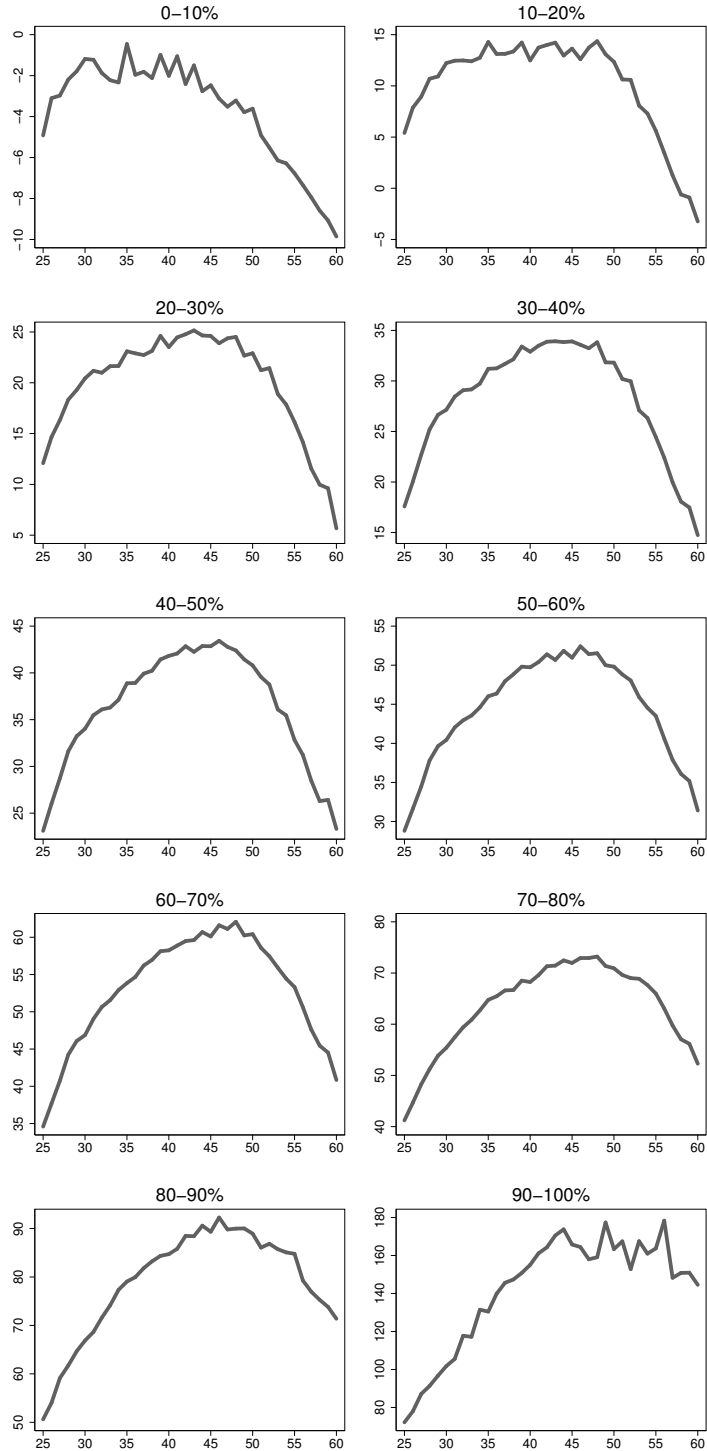
Age range / %	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
1 [25-30]	-2.689	9.356	16.87	23.23	29.47	35.48	41.71	49.12	59.52	87.90
2 [31-36]	-1.683	12.90	21.88	29.78	37.10	44.21	52.06	61.69	75.01	123.5
3 [37-42]	-1.733	13.48	23.84	32.88	41.35	49.64	57.95	68.42	84.67	153.8
4 [43-48]	-2.734	13.59	24.54	33.73	42.76	51.46	60.73	72.46	90.04	165.5
5 [49-54]	-4.973	10.47	20.95	29.68	38.81	47.98	57.98	69.65	87.23	165.2
6 [55-60]	-8.226	1.047	11.31	19.63	28.21	37.60	47.20	59.23	77.07	156.5

Notes: Earnings are in thousand dollars.

The intergenerational transition matrix for earnings we use is from Chetty et al. (2014). The data in Chetty et al. (2014) refers to the 1980-82 U.S. birth cohort and their parental income. We reduce it to a ten-state Markov chain.¹⁶

¹⁶We relegate to Appendix B a detailed discussion of several issues with our measure of labor earnings.

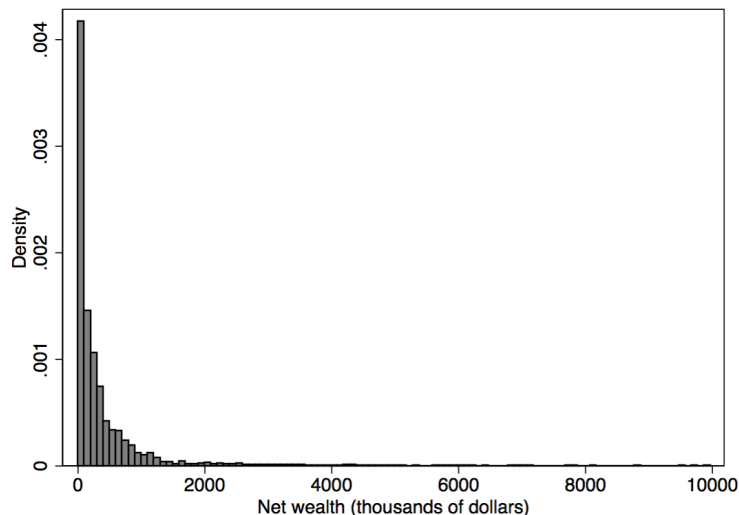
Figure 1: Life-cycle earnings profiles by deciles



Wealth distribution. We use wealth distribution data from the Survey of Consumer

Finances (SCF) 2007.¹⁷ The wealth variable we use is *net wealth*, the sum of net financial wealth and housing minus any debts. The distribution is very skewed to the right. We take the fractile shares from the cleaned version in Díaz-Giménez et al. (2011). Figure 2 displays the histogram for the wealth distribution.

Figure 2: Wealth distribution in the SCF 2007 (weighted)



Notes: Net wealth, from 2007 SCF, truncated at \$0 on the left and at \$10 million on the right.

Table 2 displays the wealth share moments we use.

Table 2: Wealth distribution moments

Share of wealth	0-20%	20-40%	40-60%	60-80%	80-90%	90-95%	95-99%	99-100%
	-0.002	0.001	0.045	0.112	0.120	0.111	0.267	0.336

Social mobility. As for wealth transition across generations, we estimate an inter-generational mobility matrix from the 2007-2009 SCF 2-year panel as follows. We first construct age-dependent 2-year transition matrices for age groups running from 30 – 31 to 66 – 67.¹⁸ We

¹⁷As noted, the wealth distribution in our methodology is to be interpreted as stationary. Choosing 2007 avoids the non-stationary changes due to the Great Recession.

¹⁸Because of limited sample dimension, we average the left and right matrices obtained using, respectively,

then multiply these age-dependent 2-year transition matrices for all age groups, to construct the intergenerational social mobility matrix, which we report in Table 3.¹⁹

Table 3: Intergenerational social mobility transition matrix

Percentile	0-20%	20-40%	40-60%	60-80%	80-90%	90-95%	95-99%	99-100%
0-20%	0.223	0.222	0.215	0.187	0.081	0.038	0.029	0.006
20-40%	0.221	0.220	0.215	0.188	0.082	0.039	0.029	0.006
40-60%	0.208	0.209	0.210	0.194	0.090	0.046	0.036	0.008
60-80%	0.199	0.201	0.207	0.198	0.095	0.052	0.040	0.009
80-90%	0.175	0.178	0.197	0.207	0.110	0.067	0.054	0.012
90-95%	0.182	0.184	0.200	0.205	0.106	0.062	0.050	0.011
95-99%	0.125	0.125	0.166	0.216	0.141	0.114	0.094	0.021
99-100%	0.086	0.084	0.142	0.228	0.170	0.143	0.121	0.028

It displays substantial social mobility: the Shorrocks mobility index²⁰ is .98.²¹

the left-middle ages and the middle-right ages to define the age group in the 2-year panel; for instance, the 30 – 31 age-group is constructed using the average of the transitions of the 29 – 30 and the 31 – 32 groups in the data.

¹⁹Effectively, this construction computes the transition matrix for a synthetic agent over his/her age profile. It accounts for the wealth transitions along the whole working life of agents. As a consequence, it accounts for any transition induced by bequests (as well as in-vivos transfers) the agents receive in this period. This is a defining element of our quantitative strategy, since the model relies importantly on the bequest motive. Indeed this is the reason why we preferred not to adopt the inter-generational social mobility matrix estimated by Charles and Hurst (2003) on PSID data. This matrix is in fact constructed by means of pairs of simultaneously alive parent and child and, as a consequence, cannot account for bequests.

²⁰Formally, for a square mobility transition matrix A of dimension m , the Shorrocks index given by $s(A) = \frac{m - \sum_j a_{jj}}{m-1} \in (0, 1)$, with 0 indicating complete immobility.

²¹Measurement error in wealth can by itself induce spurious mobility in the transition matrix; Jappelli and Pistaferri (2006) discuss this issue with regards to consumption mobility and account explicitly for measurement error in the analysis. We leave this for a future analysis of social mobility in itself. The qualitative implications regarding social mobility we obtain are similar to those we obtain from Kennickell and Starr-McCluer (1997)'s 6-years transition matrix from SCF (1983-89). In this case, the inter-generational (36-years) matrix is constructed by raising the 6-years matrix to the 6 – th power; see Appendix B for details. Our method, besides using more recent data, exploits the more precise information contained in age-dependent transitions. Similar implications also hold when adopting Klevmarken et al. (2003) and Charles and Hurst (2003) estimates with the PSID data (though Charles and Hurst (2003) mobility matrix displays moderately less mobility, with a Shorrocks index of .88); see Appendix B where these matrices are reported

4 Estimation results

The baseline estimation results are reported in Section 4.1, Table 4. The targeted simulated moments of the estimated model are reported and compared to their counterpart in the data in Section 4.2, Table 5. Some independent evidence which bears on the fit of the model is discussed in Section 4.3.

4.1 Parameter estimates

The upper part of Table 4 reports the estimates of the preference parameters. The lower part of Table 4 reports the estimated state space and diagonal of the transition matrix of the 5-state Markov process for r we postulate. It also reports, to ease the interpretation of the estimates, the implied mean and standard deviation of the process, $E(r)$, $\sigma(r)$; as well as its auto-correlation, $\rho(r)$, computed fitting an $AR(1)$ on simulated data from the estimated process.²²

Table 4: Parameter estimates: Baseline

	Preferences				
	σ	μ	A	β	T
	[2]	1.0108 (0.0010)	0.0084 (0.0011)	[0.97]	[36]
	Rate of return process				
state space	0.0060 (0.0019)	0.0125 (0.0022)	0.0217 (0.0027)	0.0401 (0.0077)	0.0962 (0.0091)
transition diagonal	0.0121 (0.1015)	0.4229 (0.1270)	0.3467 (0.1112)	0.2132 (0.1775)	0.0226 (0.1781)
statistics	$\mathbb{E}(r)$ 2.76% (0.80%)	$\sigma(r)$ 2.54% (0.87%)	$\rho(r)$ 0.078 (0.063)		

Notes: Standard errors in (); fixed parameters in [].

Most of the parameters are estimated quite precisely with a t statistic greater than 2. To

²²The standard errors, also reported in the Table, are obtained by bootstrapping, details are in Appendix A.

elaborate, the curvature parameter μ is statistically very significant and so is the bequest intensity parameter A . Furthermore, while a subset of the parameters of the transition diagonal are statistically insignificant, they are jointly significant (F-test value is ?????) and so are the mean $\mathbb{E}(r)$ and the variance $\sigma(r)$ of the rate of return process. The correlation $\rho(r)$ is not surprisingly imprecisely estimated since it is not a statistics pertaining directly to the r process we estimate.

4.2 Model fit

The simulations of our estimated model seem to capture the targeted moments reasonably well. Table 5 compares the moments in the data with those obtained simulating the model.

Table 5: Model fit: Baseline

	0-20%	20-40%	40-60%	60-80%	80-90%	90-95%	95-99%	99-100%
	Wealth distribution moments							
Data	-0.002	0.001	0.045	0.112	0.120	0.111	0.267	0.336
Model	0.005	0.065	0.106	0.163	0.121	0.071	0.149	0.321
	Social mobility moments							
Data	0.223	0.220	0.210	0.198	0.110	0.062	0.094	0.028
Model	0.202	0.215	0.209	0.205	0.100	0.114	0.043	0.060

Due to the borrowing constraint, in our simulation agents necessarily display non-negative wealth holdings throughout their lifetime, and thus the simulated wealth distribution is slightly less skewed than the data's: too much wealth is concentrated in the bottom 40% of the population. We match rather precisely the top 1% share, yet we underestimate the 90 – 99%. We will see in Section 4.4 that allowing the return process r to depend on wealth substantially improves our fit on this margin.

We reasonably match the diagonal of the social mobility as well, though we over-estimate the probability of staying in the top 1% of the distribution and under-estimate the probability

of staying in the 95 – 99% percentile.

4.3 Discussion and interpretation

We discuss and interpret here the estimates we obtain. We also put them in the context of independent evidence which bears on non-targeted moments regarding savings, bequests, rates of return, and wealth mobility.

Differential savings and bequests. Our estimates point to the existence of the differential saving factor as a component of the observed wealth dynamics in the U.S. Indeed, Our estimate of μ is 1.0108, which is significantly lower than 2, the value of σ we fixed; therefore $\mu < \sigma$ and, as we noted, savings out of wealth increase with wealth itself: the rich save proportionally more than the poor.

Of course, the strength of this factor depends on the intensity parameter A as well.²³ To better evaluate the quantitative role of differential savings and bequests in our estimation, we calculate the average savings rates implied by our model at the estimated parameters for different wealth percentiles. It turns out the savings rates schedule is quite steep: rates range from the –5.65% of the bottom 90% to the 42.2% of the top 1% of the population. These values are roughly consistent with the empirical values calculated by Saez and Zucman (2016) using 2000-2009 data on wealth accumulation with the capitalized income tax method, though our model clearly over-estimates the savings rate in the 90 – 99% range; see Table 6.

Table 6: Savings rates

	0 – 90%	90 – 99%	Top 1%
Our estimates	-5.65%	29.3%	42.2%
Saez and Zucman (2016)	-4%	9%	35%

²³The existence of a stationary distribution of wealth imposed by our methodology limits the possible strength of this factor. We will gauge at this issue better when we discuss counterfactuals in the next section.

To gain a more precise sense of the mechanism driving differential savings, we also look at bequest, since in our model differential savings are mostly motivated by a bequest motive.²⁴ The distribution of bequests implied by our model at the estimated parameters is very skewed, mapping closely the stationary wealth distribution. This is consistent with Health Retirement Survey (HRS) data studied by Hurd and Smith (2003). In particular, retirement savings in the data do not decline along the age path and, furthermore, this pattern is more accentuated for the 75% percentile, as our estimates also imply.²⁵

Returns on wealth. The rate of return process we estimate displays a standard deviation which is significantly different than 0. This indicates a role for capital income risk as a factor driving wealth and mobility. The standard deviation $\sigma(r) = 2.54\%$ is however smaller than previous direct estimates. This is the case, e.g., for the return estimates by Case and Shiller (1989) and Flavin and Yamashita (2002) on the housing market, by Campbell and Lettau (1999), Campbell et al. (2001) on individual stocks of publicly traded firms, and by Moskowitz and Vissing-Jørgensen (2002) on private equity and entrepreneurship. A wide dispersion in returns to wealth is also documented by Fagerang et al. (2016) and Bach et al. (2015) using, respectively Norwegian and Swedish data. Such comparisons require however great caution. First of all, in our model, r is assumed constant throughout each agent's lifetime, disregarding the whole variation across the life-cycle. The rate of return we estimate should ideally be then compared with the permanent components of individual returns across generations, which are hardly available. Furthermore, rate of returns heterogeneity is in part a consequence of differences in the risk composition of investment portfolio; see Calvet and Sodini (2014) and Bach et al. (2015) for evidence in Swedish data. For our purposes, the most appropriate outside validation perspective is then provided by Fagerang et al.

²⁴The bequest motive stands on relative solid grounds: it is well documented that retirees do not run down their wealth as predicted by the classical life-cycle consumption-savings model (Poterba et al., 2011).

²⁵Our model does not have a role for accidental bequests. Therefore, while the literature on retirement savings distinguishes between precautionary saving motives for uncertain medical expenses (De Nardi et al., 2010), uncertain and potentially large long-term care expenses (Ameriks et al., 2015a), family needs (Ameriks et al., 2015b) and the genuine bequest motive, we necessarily lump all these into aggregate bequests.

Table 7: Rate of return process

Statistics	$E(r)$	$\sigma(r)$	$\rho(r)$
Our estimates	2.76%	2.54%	.078
Fagerang et al. (2016)	2.98%	2.82%	.1

Notes: Fagerang et al. (2016)'s permanent component has zero-mean by construction: we report their mean of returns. ???their $\rho(r)$???

(2016), in that their Norwegian administrative data allows them to estimate the permanent components of individual returns across generations and to control for portfolio composition. The consistency of our estimates with the Fagerang et al. (2016)'s data is impressive; see Table 7.²⁶

Social mobility. The implied non-targeted moments (the off-diagonal cells) of the social mobility matrix we obtain align quite well with the mobility we constructed from the SCF data, with the important exception, however, of the last row, which captures the inter-generational flows from the top 1% of the distribution of wealth. We have already noted that our model over-estimates the probability of staying in the top 1%. Indeed the model overestimates the probability that children of parents in the top 1% stay in the top 10% percentile (41.5% against 29.2% in the data) as well as the probability that they move to the bottom 40% (31.5% against 17%).

4.4 Rate of return dependent on wealth

A positive correlation between the rate of return on wealth and wealth has been documented by Piketty (2014)'s analysis of university endowments, see especially p. 447, and by Fagerang et al. (2016)'s careful study of Norwegian administrative data.²⁷ Such a correlation of course does not imply that the rate of return increases with wealth. Even in the context of our

²⁶Fagerang et al. (2016) also find rate of returns increasing in wealth. We shall discuss this in the next section.

²⁷See also Kacperczyk, Nosal and Stevens (2015). On the other hand no correlation is apparent in Saez and Zucman (2016). Also, Bach et al. (2015) find that the correlation is largely due, in the Swedish administrative data they observe, to the portfolio composition by risk class changing with wealth.

model, agents with relatively high wealth would have experienced on average high realizations of the rate of return r . Indeed, the simulated model at the parameters estimates in the previous section, produces a strong positive correlation of r and a : a regression between r and wealth a in percentiles produces a small but strongly significant coefficient of .01 (standard error .0004).

Allowing rates of return on wealth to be increasing in wealth might however add to the skewedness of the distribution. In this section we therefore extend our analysis to allow for the rate of return process r to depend on wealth, explicitly introducing a dependence of the stochastic rate of return r on wealth percentiles. The functional form we introduce allows for r to depend on wealth a as follows:

$$r = r_0 + b \times p(a) \tag{1}$$

where $p(a) = 1, 2, \dots, 8$ numbers the wealth percentiles we identify as moments and r_0 is a 5-state Markov process along the lines what we assumed in the baseline model for r . This formulation maps a positive slope b into a convex relationship between r and a . We then estimate the parameters of our model as well as the wealth dependence parameter b that enters the stochastic rate of return process. The results of our estimation are reported in Tables 8 and 9.

Table 8: Parameter estimates: r dependent on wealth

	Preferences				
	σ	μ	A	β	T
	[2]	1.0123 (0.0010)	0.0183 (0.0011)	[0.97]	[36]
	Rate of return process				
state space	0.0020 (0.0004)	0.0087 (0.0018)	0.0152 (0.0148)	0.0439 (0.0369)	0.0652 (0.0062)
transition diagonal	0.2681 (0.2169)	0.0464 (0.1342)	0.0405 (0.2859)	0.1585 (0.1312)	0.0464 (0.2401)
wealth dependence, b	0.000252 (0.0105)				
statistics	$\mathbb{E}(r_0)$ 2.28% 1.53%	$\sigma(r_0)$ 2.16% 0.87%	$\rho(r_0)$ 0.169 0.007	$\mathbb{E}(r)$ 2.67%	$\sigma(r)$ 2.37%

Notes: Standard errors in (); fixed parameters in [].

Table 9: Model fit; r dependent of wealth

	0-20%	20-40%	40-60%	60-80%	80-90%	90-95%	95-99%	99-100%
	Wealth distribution							
Data	-0.002	0.001	0.045	0.112	0.120	0.111	0.267	0.336
Model	0.003	0.031	0.085	0.123	0.127	0.101	0.203	0.327
	Social mobility							
Data	0.223	0.220	0.210	0.198	0.110	0.062	0.094	0.028
Model	0.239	0.201	0.178	0.195	0.103	0.044	0.053	0.125

The estimate of the parameter b , which captures the dependence of the rate of return on wealth is positive but not significantly different than 0. The point estimate implies that going from the bottom 20% to the top 1% in the wealth distribution would increase the annual return by 0.2 percentage points. Reassuringly, however, the estimates of the preference parameters are not much changed with respect to the baseline. On the other hand, the fit of the wealth distribution is improved: while the distribution of wealth implied by the model

is still less skewed than the data's, we continue to match precisely the top 1% share and, most importantly, we do substantially better in matching the 90-99%. With regards to social mobility, this specification loses fit on the top 1% but gains in on the 90-99%.

Fagerang et al. (2016) also estimate the dependence of the rate of return r on wealth. However they work with a very rich and detailed Norwegian data set, and can directly control for the effects of a variety of factors like age, education and portfolio composition. Their findings provide stronger evidence of dependence than ours. Indeed, their data display not only evidence for significant heterogeneity in returns, but also average returns within generations which are increasing in wealth with a significant coefficient equal to 0.028, and at an increasing speed at the top of the distribution.

5 Counterfactual estimates

In this section we perform a set of counterfactual estimations of the model, under restricted conditions. More in detail, we perform three sets of counterfactuals, corresponding to shutting down each of the three main factors which can drive the distribution of wealth: (1) capital income risk, (2) differential savings rates, and (3) stochastic earnings.

The objective of this counterfactual analysis is twofold. First of all we aim at gauging at the relative importance of the various mechanisms we identified as possibly driving the distribution of wealth. In particular, we aim at a better understanding of which mechanism mostly affects which dimension of the wealth distribution and mobility. Second, we interpret the counterfactuals as informal tests of identification of these mechanisms, lack of identification implying that shutting down one or more of the mechanism has limited effects on the fit for the targeted moments.

5.1 Re-estimation results

We examine the counterfactual estimates in detail in the following. The estimated parameters are in Table 10.²⁸ Table 11 reports the fit of the estimates.

In the counterfactual with no capital income risk, we re-estimate the model under the constraint that the rate of return is constant, which is estimated at 2.99%, just above its mean in the baseline. The differential savings factor substitutes for capital income risk, as the lower estimate of μ implies. Furthermore, the bequest motive is higher due to a much higher relative preference for bequests reflected in the estimate for A . This uniformly increases the savings rate. Nonetheless, the model with r constant misses in matching the top 1% of the wealth distribution, which is reduced to more than 1/2 of the baseline (and the data). More generally, the wealth distribution implied by the model is much less skewed: it has less mass on the top 20% and more of the bottom 80%, especially so in the bottom 40%. This is an indication that capital income risk is not only a factor in spreading the tail, but also, importantly, in facilitating the escape from low levels of wealth close to the borrowing constraint. This is conformed in the match of social mobility: the top 1% has an even higher probability of staying than the baseline and so does the bottom 20%, implying that capital income risk increases churning both at the bottom as well as at the top of the distribution.

In the counterfactual with no stochastic earnings we feed the model an average earnings path. The resulting estimates of the preference parameters and of the rate of return process r reveal a strengthening of both the savings factor (μ is reduced, and A increased) and of capital income risk (both the mean and the auto-correlation of r are increased, while the standard deviation is slightly smaller). Interestingly, in this case as in the counterfactual with constant r , the model misses in matching the top 1% of the wealth distribution and produces distribution which is much less skewed than the baseline's (and the data's). We interpret

²⁸We report only the mean, standard deviation and auto-correlation statistics for r , to save space. The estimates for the state space and the diagonal of the transition matrix are in Appendix C.

this as suggesting that stochastic earnings and capital income risk are complementary in facilitating the escape from low levels of wealth. The very high probabilities of staying in the bottom 20% and in the 20 – 40% (respectively 57.3% and 34.5%) implied by the model with constant w clearly reinforce this point.

Table 10: Parameter estimates: Counterfactuals

	Preferences				
	σ	μ	A	β	T
baseline	[2]	1.0108 (0.0010)	0.0084 (0.0011)	[0.97]	[36]
constant r	[2]	0.7669 (0.0145)	0.5094 (0.0498)	[0.97]	[36]
constant w	[2]	0.6414 (0.0042)	0.0710 (0.0014)	[0.97]	[36]
$\mu = 2$	[2]	2 -	0.2119 (0.0147)	[0.97]	[36]
	Rate of return process				
	$\mathbb{E}(r)$	$\sigma(r)$	$\rho(r)$		
baseline	2.76% (0.80%)	2.54% (0.87%)	0.078 (0.063)		
constant r	2.99% (1.70%)				
constant w	3.13% (1.65%)	2.34% (1.48%)	0.160 (0.008)		
$\mu = 2$	2.99% (2.10%)	2.97% (1.76%)	0.112 (0.006)		

In the counterfactual with homogeneous saving rates, we set $\mu = 2$, that is, we set the curvature parameter of the bequest utility equal to the curvature of consumption utility, so that agents with different wealth save at the same rate. In terms of the estimates, preferences for bequests are greatly increased (though not as much as in the counterfactual with constant r), so as to increase the (now constant) saving rate in the economy. Once again, the model misses to match the top 1% of the wealth fractiles, which is greatly reduced to about 1/5th of the baseline (and the data). Apart from the last percentile, the simulated wealth distribution is not really less skewed, displaying even excessive mass on the top 60%. This is evidence,

in our interpretation, that the heterogeneous savings factor’s role is mainly that of fattening the tail, with limited effects on the rest of the distribution. The reasonably good match in mobility is further evidence for this point.

Table 11: Model fit: Counterfactuals

	0-20%	20-40%	40-60%	60-80%	80-90%	90-95%	95-99%	99-100%
	Wealth distribution							
<i>Data</i>	-0.002	0.001	0.045	0.112	0.120	0.111	0.267	0.336
<i>Model</i>								
(1) Baseline	0.006	0.065	0.107	0.132	0.122	0.092	0.147	0.328
(2) Constant r	0.070	0.138	0.156	0.180	0.115	0.074	0.107	0.160
(3) Constant w	0.053	0.116	0.128	0.142	0.112	0.147	0.204	0.099
(4) $\mu = 2$	0.012	0.098	0.156	0.241	0.174	0.120	0.147	0.051
	Social mobility							
<i>Data</i>	0.223	0.220	0.210	0.198	0.110	0.062	0.094	0.028
<i>Model</i>								
(1) Baseline	0.202	0.215	0.209	0.205	0.100	0.114	0.043	0.060
(2) Constant r	0.452	0.263	0.187	0.267	0.123	0.052	0.028	0.090
(3) Constant w	0.573	0.345	0.251	0.198	0.065	0.010	0	0
(4) $\mu = 2$	0.230	0.211	0.211	0.209	0.096	0.064	0.033	0.028

In summary, all the factors we study in our quantitative analysis, stochastic earnings, differential savings, and capital income risk, are crucial for generating the thick right tail of the wealth distribution and sufficient mobility. Furthermore, the factors seems to have a distinct role. Stochastic earnings avoid poverty traps and allow for upward mobility near the borrowing constraints. Random returns on capital (capital income risk) induces churning all over the distribution of wealth, complement stochastic earnings at low levels of wealth and differential savings at high level of wealth to produce a thick tail. Saving rate differentials indeed mainly help match the top tail but they reduce social mobility as the rich get richer accumulating at higher rates. Stochastic returns on wealth, or capital income risk, contributes to the thick top tail while allowing for social mobility, especially in terms of speeding up downward mobility.

5.2 More on the earnings mechanism

In apparent contrast with our results, several previous papers in the literature have obtained considerable success in matching the wealth distribution in the data with simulated models fundamentally driven by the stochastic earnings mechanism. These simulated models however appear driven by extreme assumptions either about the skewness of earnings (adding an *awesome* state) or about the working life of agents.

A successful simulation exercise driven by skewed earnings is Díaz et al. (2003).²⁹ They also use an excessively skewed earning process where roughly 6% of the top earners have 46 times the labor endowment of the median. In the World Top Income Database 2013-14 the average income of the top 5% is \$367,000. Even if it were all earnings, it would correspond to no more than 7.5 times the median income. Similarly, in Castañeda et al. (2003), at the stationary distribution for labor earnings in the simulation, the top 0.039% earners have 1000 times the average labor endowment of the bottom 61%. Even if the bottom 61% earns a low \$25,000 on average, the top 0.039% would have to earn \$25,000,000. This also appears implausibly high according to the World Top Income Database 2013-14.

In Kindermann and Krueger (2015), instead, earnings are endogenously driven by a seven state Markov chain for labor productivity. In their stationary distribution, 0.036% of the population is in the *awesome* productivity state with average earnings of about 20 million dollars when calibrated to median earnings, about 3 times the earnings reported in the WWID for the same share of the population.³⁰

A feature complementing skewed earnings in producing heterogeneity in wealth accumulation rates across agents is variable and random working life-span. Variable working life-spans produce a fraction of agents with high sojourn times in high earnings states, which implies in turn a better fit to the wealth distribution with less extreme skewness. For example even though their awesome state is less extreme than in the above cited literature,

²⁹Dávila et al. (2012), Section 5.2, adopt this same calibration.

³⁰We thank the authors for a personal communication which clarified some issues in these calculations.

Kaymak and Poschke (2015) calibrate expected working lives to 45 years, with a constant exponential decay rate into retirement of $\mu = 1/45$. This implies a substantial fraction of agents with an excessive working life-span at the stationary distribution: over 100 years for 11% of the working population (of these 11%, those at the high earnings states populate the tail of the wealth distribution).³¹

We conclude that without extremely skewed labor earnings or extreme working life-span variability, it may not be possible for enough agents to accumulate enough wealth to populate the tail of wealth distribution via saved earnings alone. More generally, barring other mechanisms contributing to thick tails in wealth, the skewness of the earnings distributions tends to translate one-for-one to the distribution of wealth; see Benhabib and Bisin (2015). But in the data wealth is substantially more skewed than earnings,³² making it difficult to match the data with stochastic earnings only.

6 Transitional dynamics of the wealth distribution

Our quantitative analysis so far is predicated on the assumption that the observed distribution of wealth is a stationary distribution, in the sense that our estimates are obtained by matching the data with the moments of the stationary distribution generated by the model. In this section we instead begin studying the implications of our model for the transitional dynamics of the distribution of wealth.

The exercise we perform is as follows: using the observed SCF 1962-1963 distribution of wealth as initial condition, we estimate the parameters of the model by matching the implied distribution after 72 years (two iterations of the model) with the observed SCF 2007 distribution and the transition matrix adopted in the previous quantitative analysis.³³

³¹Kaplan et al. (2015) also use the same perpetual youth calibration as in Kaymak and Poschke (2015).

³²In the SCF 2007, the Pareto tail, an inverse measure of skewness of the distribution, is estimated to be 1.09 for wealth (net worth), 1.71 for total income, 2.13 for earnings.

³³While the analysis does not require nor imposes any stationarity of the distribution of wealth over time, it does postulate that the model structure and parameter values stay constant after 1962. Importantly, we

The new parameter estimates that we obtain, μ and A are not very different than their counterparts in the benchmark model. The rate of return process has higher mean and is slightly more volatile. The growth of inequality over 1962-2007 however generates wealth fractiles more skewed towards the rich: the 90%-99% and especially the top 1% hold higher proportions of the total wealth than in data.

Table 12: Parameter estimates: Transitional dynamics

	Preferences				
	σ	μ	A	β	T
	[2]	1.1965 (0.0208)	0.0016 (0.0003)	[0.97]	[36]
	Rate of return process				
state space	0.0048 (0.0011)	0.0085 (0.0011)	0.0244 (0.0032)	0.0710 (0.0021)	0.0844 (0.0098)
transitional diagonal	0.2972 (0.3672)	0.0139 (0.0539)	0.1233 (0.3805)	0.1167 (0.4357)	0.0452 (0.1892)
statistics	$\mathbb{E}(r)$ 3.27% (0.96%)	$\sigma(r)$ 3.03% (0.42%)	$\rho(r)$ 0.192 (0.010)		

Notes: [] indicates fixed parameters. σ is the CRRA elasticity of consumption, μ is the CRRA elasticity of bequest, and A is the intensity of bequest. β is the annual discount factor, and T is the number of working periods.

Table 13: Model fit: Transitional dynamics

	0-20%	20-40%	40-60%	60-80%	80-90%	90-95%	95-99%	99-100%
<i>Data</i> : SCF 1962-63	0.009	0.043	0.094	0.173	0.142	0.115	0.190	0.242
<i>Data</i> : SCF 2007	-0.002	0.001	0.045	0.112	0.120	0.111	0.267	0.336
<i>Model</i>	0.000	0.000	0.001	0.022	0.075	0.143	0.341	0.418
	Social mobility							
<i>Data</i>	0.223	0.220	0.210	0.198	0.110	0.062	0.094	0.028
<i>Model</i>	0.347	0.162	0.261	0.443	0.376	0.218	0.320	0.200

In fact our parameter estimates overshoot the actual increase in the top shares of wealth

do not feed in the analysis the observed fiscal policy reforms since the '60's. Doing so should improve the fit.

relative to the data from 1962 to 2007 (two periods or generations in our model), in large part due to the volatility of the estimated rate of return that fattens and fills the tail. A small increase in the volatility of the rate of return, therefore not only helps generate a fat tail, but also speeds up the transition to the tail, as in Gabaix et al (2016). To the extent that we abstract from exogenous factors like tax changes that can account for the fatter tail of wealth,³⁴ the estimated rate of return volatility may in fact be a bit upward biased, and explain the overshoot in the thickness of the tail relative to data. We note that along with increased estimated inequality, mobility also seems reduced relative to the data. The diagonal elements of all fractiles of the transition matrix are higher except for the 20%-40% fractile which is slightly lower, indicating that agents stay within their fractiles and are less mobile relative to the data.

7 Conclusions

We estimate a macroeconomic model of the distribution of wealth in the U.S. While we assign special emphasis on the tail of the distribution, the model performs well in fitting the whole distribution of wealth in the data. Importantly, the model is also successful in fitting the social mobility of wealth in the data.

Our analysis allows us to distinguish the contributions of three critical factors driving wealth accumulation: a skewed and persistent distribution of earnings, differential saving and bequest rates across wealth levels, and capital income risk in entrepreneurial activities. All of these three factors are necessary and empirically relevant in matching both distribution and mobility, with a distinct role for each, which we identify.

Finally, we begin studying the implications of the model for the transitional dynamics of the distribution of wealth. The estimates are qualitatively similar to those in the baseline, and our model delivers fast transitional dynamics. While more work is obviously necessary

³⁴See Aoki and Nirei (forthcoming).

in this respect, our results are quite encouraging.

References

- Ameriks, John, Joseph Briggs, Andrew Caplin, Matthew D. Shapiro, and Christopher Tonetti**, “Long-Term Care Utility and Late in Life Saving,” 2015.
- , – , – , **Mi Luo, Matthew D. Shapiro, and Christopher Tonetti**, “Measuring and Modeling Inter-generational Wealth Transfers: the Precautionary Transfer Motives,” 2015.
- Benhabib, Jess, Alberto Bisin, and Shenghao Zhu**, “The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents,” *Econometrica*, 01 2011, *79* (1), 123–157.
- and – , “Skewed Wealth Distributions: Theory and Empirics,” 2015.
- Cagetti, Marco and Mariacristina De Nardi**, “Entrepreneurship, Frictions, and Wealth,” *Journal of Political Economy*, October 2006, *114* (5), 835–870.
- Campbell, John Y. and Martin Lettau**, “Dispersion and Volatility in Stock Returns: An Empirical Investigation,” NBER Working Papers 7144, National Bureau of Economic Research, Inc May 1999.
- , – , **Burton G. Malkiel, and Yexiao Xu**, “Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk,” *The Journal of Finance*, 2001, *56* (1), 1–43.
- Case, Karl E. and Robert J. Shiller**, “The Efficiency of the Market for Single-Family Homes,” *American Economic Review*, March 1989, *79* (1), 125–37.
- Castañeda, Ana, Javier Díaz-Giménez, and José-Víctor Ríos-Rull**, “Accounting for the U.S. Earnings and Wealth Inequality,” *Journal of Political Economy*, August 2003, *111* (4), 818–857.
- Charles, Kerwin Kofi and Erik Hurst**, “The Correlation of Wealth across Generations,” *Journal of Political Economy*, December 2003, *111* (6), 1155–1182.
- Chetty, Raj, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez**, “Where is the land of Opportunity? The Geography of Intergenerational Mobility in the United States,” *The Quarterly Journal of Economics*, 2014, *129* (4), 1553–1623.

- Cunha, Flavio, James J. Heckman, and Susanne M. Schennach**, “Estimating the Technology of Cognitive and Noncognitive Skill Formation,” *Econometrica*, 05 2010, *78* (3), 883–931.
- Dávila, Julio, Jay H. Hong, Per Krusell, and José-Víctor Ríos-Rull**, “Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks,” *Econometrica*, November 2012, *80* (6), 2431–2467.
- De Nardi, Mariacristina, Eric French and John B. Jones**, “Why Do the Elderly Save? The Role of Medical Expenses,” *Journal of Political Economy*, 02 2010, *118* (1), 39–75.
- de Saporta, Benoîte**, “Tail of the stationary solution of the stochastic equation with Markovian coefficients,” *Stochastic Processes and their Applications*, 2005, *115* (12), 1954 – 1978.
- Díaz, Antonia, Josep Pijoan-Mas, and José-Víctor Ríos-Rull**, “Precautionary savings and wealth distribution under habit formation preferences,” *Journal of Monetary Economics*, September 2003, *50* (6), 1257–1291.
- Díaz-Giménez, Javier, Andrew Glover, and José-Víctor Ríos-Rull**, “Facts on the distributions of earnings, income, and wealth in the United States: 2007 update,” *Quarterly Review*, 2011.
- Fagerang, Andreas, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri**, “Heterogeneity and Persistence in Returns to Wealth,” 2016.
- Flavin, Marjorie and Takashi Yamashita**, “Owner-Occupied Housing and the Composition of the Household Portfolio,” *American Economics Review*, 2002, pp. 345–362.
- Gabaix, Xavier, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll**, “The Dynamics of Inequality,” 2015.
- Grey, D. R.**, “Regular Variation in the Tail Behaviour of Solutions of Random Difference Equations,” *Ann. Appl. Probab.*, 02 1994, *4* (1), 169–183.
- Güvenen, Fatih**, *Quantitative Economics with Heterogeneity: An A-to-Z Guidebook*, Princeton University Press, 2016.

- Hay, Diana, Reza Rastegar, and Alexander Roitershtein**, “Multivariate linear recursions with Markov-dependent coefficients,” *Journal of Multivariate Analysis*, 2011, *102* (3), 521 – 527.
- Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante**, “Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006,” *Review of Economic Dynamics*, January 2010, *13* (1), 15–51.
- Hubmer, Joachim, Per Krusell, and Anthony A. Smith**, “The historical evolution of the wealth distribution: A quantitative-theoretic investigation,” 2015.
- Huggett, Mark, Gustavo Ventura, and Amir Yaron**, “Sources of Lifetime Inequality,” *American Economic Review*, December 2011, *101* (7), 2923–54.
- Hurd, Michael and James P. Smith**, “Expected Bequests and Their Distribution,” Working Papers 03-10, RAND Corporation Publications Department April 2003.
- Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante**, “Monetary Policy According to HANK,” 2015.
- Kaymak, Barış and Markus Poschke**, “The evolution of wealth inequality over half a century: The role of taxes, transfers and technology,” *Journal of Monetary Economics*, 2016, *77*, 1 – 25. Inequality, Institutions, and Redistribution held at the Stern School of Business, New York University, April 24-25, 2015.
- Keane, Michael P. and Kenneth I. Wolpin**, “The Career Decisions of Young Men,” *Journal of Political Economy*, June 1997, *105* (3), 473–522.
- Kennickell, Arthur B. and Martha Starr-McCluer**, “Household Saving and Portfolio Change: Evidence from the 1983-89 SCF Panel,” *Review of Income and Wealth*, December 1997, *43* (4), 381–99.
- Kindermann, Fabian and Dirk Krueger**, “High Marginal Tax Rates on the Top 1%? Lessons from a Life Cycle Model with Idiosyncratic Income Risk,” 2015.
- Klevmarken, Anders, Joseph P. Lupton, and Frank P. Stafford**, “Wealth Dynamics in the 1980s and 1990s: Sweden and the United States,” *Journal of Human Resources*, 2003, *38* (2).
- Krusell, Per and Anthony A. Smith**, “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy*, October 1998, *106* (5), 867–896.

- Moskowitz, Tobias J. and Annette Vissing-Jørgensen**, “The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?” *American Economic Review*, September 2002, *92* (4), 745–778.
- Piketty, Thomas**, *Capital in the 21st Century*, Cambridge, MA: Harvard University Press, 2014.
- Poterba, James, Steven Venti, and David Wise**, “The Composition and Drawdown of Wealth in Retirement,” *Journal of Economic Perspectives*, Fall 2011, *25* (4), 95–118.
- Quadrini, Vincenzo**, “Entrepreneurship, Saving and Social Mobility,” *Review of Economic Dynamics*, January 2000, *3* (1), 1–40.
- Saez, Emmanuel and Gabriel Zucman**, “Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data,” *The Quarterly Journal of Economics*, 2016.