Intangibles, Inequality and Stagnation

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Abstract

We examine how aggregate output and income distribution interact with accumulation of intangible capital over time and across generations. We consider an overlapping generations economy in which skill of managers (intangible capital) is essential for production along with labor, and managerial skill is acquired by young workers when they are trained by old managers on the job. Because training is costly, it becomes investment in intangible capital. We show that, when young trainees face financing constraint, a small difference in initial endowment of young workers leads to a large inequality in the assignment and accumulation of intangibles. A negative shock to endowment can generate a persistent stagnation and a rise in inequality.

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1 Introduction

In the last few decades, especially after the global financial crisis of 2007-9, we observe two major concerns: secular stagnation of many countries and rising inequality across households within a country. In Japan, there are heated debates on why Japan stopped growing and what causes the rising inequality after it entered into a prolonged financial crisis in 1992 with collapse of asset prices. Although specific mechanism differs across authors, the key phenomena to explain appear to be worsening labor market condition for young workers and slowing down of the growth rate of total factor productivity.

In this paper, we explore a hypothesis that the worsening youth labor market and slower productivity growth are entwined with accumulation of intangible capital. For this purpose, we consider an overlapping generations economy in which skill of managers (intangible capital) is essential for production along with labor, and managerial skill is acquired by young workers when they are trained by old managers on the job. We consider relatively general technology of accumulating intangible capital: The outcome is the managerial skill acquired by young trainees, and inputs include goods (or resources), the skill of the old manager, the initial skill of the young trainee (innate ability or ability acquired by earlier education) and the skill level of the society. The skill level of the society is included to represent externality in accumulation of intangible capital. Because training is costly, it becomes investment in intangible capital. Young workers are heterogeneous in initial endowment and skill.

In a competitive economy, old managers offer workers two options: Simple labor contract, which pays competitive wage without training; the other is a career path offering apprentice wage and training to be future managers.[1] Without financing constraint, the present value of lifetime income would be the same across the two options for young workers with the same initial skill. So, there is no inequality in permanent income (aside from the difference in initial endowment). When young workers face financing constraint, we show the aggregate intangible investment is lower than the unconstrained economy for any given interest rate. Moreover, inequality in initial endowment of young leads to diverse career path and unequal distribution of income. At the extensive margin, rich young workers with large initial endowment accept the lower apprentice wage and opt for the career path to become future

[1]Consistent with the specification, Heckman, Lochner and Taber (1998)[16] show that to account for skill premium, it is important to differentiate the potential income and the actual income during on-the-job training.

managers, while poor young workers receive no training and work as simple workers for life. At the intensive margin, more skilled managers train a smaller number of richer young workers more intensively, which leads to large inequality even among workers who receive training. Over time, a temporary decrease in initial endowment is followed by persistent fall in intangible capital investment, aggregate production and rise in inequality. We further endogenize the endowment distribution through bequest and show that the results our robust. In this extension, bequest is persistent over time, implying reduced social mobility because of wealth inequality.

Our theory provides some guidance for public policy. With financing constraint, the competitive economy exhibits a misallocation in matching between old managers and young workers with heterogeneous initial endowment and skill. Rich young workers receive more training regardless of their talent while poor but talented young workers receive less training under financing constraint. If the government is better than private lenders in enforcing debt repayment so that it can relax the financing constraint, then the government can provide loan for workers to receive training, which improves the resource allocation. If government is no better than private lenders in enforcing debtors (old managers) to pay, the policy becomes more delicate. Government can provide subsidy for training poor young. But because government has difficulty in enforcing old managers to pay their debt and tax above a certain limit, the subsidy must be financed by taxing workers (like payroll tax). Then the training subsidy may lead to too much training compared to the efficient allocation, which must be offset by the rationing of training based on the initial skill of young workers.[2]

Our paper is related to a few lines of literature. First is the literature on intangible capital accumulation of firms including Boyd and Prescott (1987)[26], Chari and Hopenhayn (1991)[10] and Kim (2006)[18]. Our model is based on Boyd and Prescott (1987)[26], about firms as dynamic coalitions for intangible capital accumulation. Chari and Hopenhayn (1991)[10] apply Boyd and Prescott (1987) for endogenous technology adoption, while Kim (2006) introduces financing constraint to Chari and Hopenhyn (1991) to show how differences in financing constraint lead to a large gap in TFP across countries. We introduce

[2]If people can change the initial skill level at the start of working life through education, then people would start investing earlier to acquire better initial skill. Young people with larger initial endowment would have an advantage of acquiring initial skill through better education.

Government can improve basic education to enhance the initial skill, to create equal opportunity instead of equal outcome across all workers. This is related to Benabou(2002)[3] for an example.

occupational choice, financing constraint and heterogeneous initial endowment and skill of young workers to Boyd and Prescott (1987). With these additional ingredients, we can study how small differences in initial endowment lead to large inequality across workers through assortative matching between old managers and young workers and how a small shock to endowment leads to a persistent decrease in intangible capital accumulation and aggregate production.

Secondly related is a vast literature on wealth distribution, human capital accumulation and occupational choices in the presence of financial frictions. Restricting the attention to a most closely related literature, Galor and Zeira (1993)[13] examine how indivisible human capital accumulation and financial friction lead to endogenous wealth distribution when parents care about their children and leave bequest. Banerjee and Newman (1993)[2] show rich dynamics of wealth distribution and growth as a result of occupational choices. Although we have similar extensive margin of human capital accumulation through occupational choices, we introduce a richer technology for accumulating intangible capital which uses resources as well as skills of managers, trainees and society as input for accumulating intangible capital.[3] This leads to a richer distribution dynamics through the assortative matching between skilled managers and heterogeneous young workers. On the other hand, we abstract from the endogenous bequest until the last section. The mechanism in our paper is related to those in Lochner and Monge-Naranjo (2011)[21]. They study the effect of financing constraint of households on their education decisions. They find empirical evidence that college attendance is an increasing function of family income conditional on ability and develop a partial equilibrium model in which relaxing financing constraint reduces misalignment between human capital accumulation and talent. More recent developments in this line of research include Córdoba and Ripoll (2013)[11] and Castro and Ševčík(2016)[9]. We use a similar mechanism to further study the general equilibrium implications of the on-the-job training.[4][5]

[3] We owe Rothschild and White (1995)[27] for the idea that input for human capital accumulation includes initial skill of trainee themselves.

[4] Michelacci and Quadrini (2009)[24] present theory and evidence that financially constrained firms grow by offering a steep wage profile to employees. While Michelacci and Quadrini (2009) emphasize the effect of firm's financing constraint, assuming workers cannot borrow, we stress the effect of workers' financing constraint on their occupational choice and income profile. In our model, investment in intangible capital can actually increase the profit of firms, so their financing constraint is not as relevant.

[5] See Banerjee and Duflo (2005)[1] and Matsuyama (2007)[23] for survey of more literature. There is an

The third related literature is the macro literature on financial friction and capital misallocation. Kiyotaki (1998)[19], Buera (2009)[4], Buera, Kaboski and Shin (2011)[5] and Buera and Shin (2013)[6] and Moll (2014)[25] for example study how financial frictions affect misallocation of capital and economic growth. Our research is complementary to theirs because they focus on the allocation and accumulation of tangible capital and we focus on intangible capital. This addition is relevant because financial frictions may be more severe for intangible capital and a large component of skilled workers' asset is the intangible capital. Caggese and Perez-Orive (2017)[7] study the implication of intangible capital as less collateralizable assets on capital misallocation across firms. Caselli and Gennaioli (2013)[8] feature a similar mechanism but focus on the allocation of the control right of dynastic firms. Complementary to their research, we study implications of employees' financial constraint on both the accumulation and allocation of intangibles both cross-sectionally and over time.[6]

Our theory is consistent with empirical findings on the level and the slope of workers' income profile in recent papers. Kambourov and Manovski (2009)[17] find that an increase in occupational mobility explains substantially why life-cycle earning profile becomes flatter, the experience premium becomes smaller and the inequality rises within group for more recent cohort. While they emphasize the role of increasing occupation specific risks, we attribute the flattening life-cycle earning profile to the slowdown in investment in intangibles. Guvenen, Karahan, Ozkan and Song (2016)[15] find that there is a strong positive association between the level of lifetime earning and how much earning grow over the life cycle.[7]

extensive literature of endogenous financing constraints due to hidden information and hidden action. See Lucas (1992)[22] and Ljungqvist and Sargent (2012)[20] for example for the literature.

[6]As a specific form of intangible capital, organization capital has also been shown to have important implications on asset pricing. See, for example, Eisfeldt and Papanikolaou (2013)[12].

[7]Guiso, Pistaferri and Schivardi (2013)[14] find that firms operating in less financially developed markets offer lower entry wages but faster wage growth than firms in more financially developed markets, which is consistent with Michelacci and Quadrini (2009) in the earlier footnote. Guiso et. al. (20013) also find managers' income profile is steeper in financially underdeveloped market, which is consistent with our theory.

2 Basic Model

2.1 Set-up

Consider an overlapping generations model in which a unit measure of agents are born every period and lives for two periods.

When young, each agent is endowed with consumption goods e, initial skill κ and one unit of labor. The initial endowment of goods is heterogeneous and distributed according to

$$e \sim G(e) \text{ on } [0, \overline{e}].$$

A particular example we use is

$$G(e) = 1 - \omega + \omega \frac{e}{\overline{e}}, \text{ for } [0, \overline{e}],$$

$G(e) = 0$ for $e < 0$, and $G(e) = 1$ for $e \geq \overline{e}$. Thus a fraction $1 - \omega$ of young agents has no endowment, while the endowment of a fraction ω of young agents is distributed uniformly on $[0, \overline{\omega}]$. In the Basic Model, the initial skill is the same across workers. We will consider the heterogeneous initial skill in the Full Model.

When a manager with skill k hires n measure of workers, he or she can produce output according production function,

$$\begin{aligned} y &= Ak^{\alpha} f(n) \\ &= Ak^{\alpha} n^{1-\alpha}, \end{aligned}$$

where $A > 0$ and $\alpha \in (0, 1)$ are technological parameters. We can think of α as share of managerial skill, or intangible capital, in production.

Each manager chooses the number of young workers to train n_t^m to acquire skill to become future managers. Inputs for training are current manager's skill k, the skill input of society $\overline{K} = \bar{k} n^m$ (where $\bar{k}$ is the average skill of current managers), the initial total skill of future managers $K_0 = \kappa n^m$ and goods input i for training (possibly output loss due to training). The outcome is number times skill - total skill level - of the future managers as

$$n^m k' = \frac{1}{b} \left[i^{\gamma} + \psi \left(\hat{k} \right)^{\gamma} \right]^{1/\gamma}.$$

$$\hat{k} = \left[\eta k^{\rho} + \overline{\eta} \overline{K}^{\rho} + (1 - \eta - \overline{\eta}) K_0^{\rho} \right]^{\frac{1}{\rho}},$$

where $b, \psi > 0,\ \rho, \gamma < 1$, and $\eta, \overline{\eta}, 1 - \eta - \overline{\eta} \in (0, 1)$. $\hat{k}$ is skill composite - CES aggregate of skills of current manager and society and aggregate initial skill of future managers. Training investment cost i and $\hat{k}$ are complement input if $\gamma < 0$. If $\gamma = 0$, we get the same Cobb-Douglas skill production function. Here we assume that the skill k' of future managers is the same as long as they are trained by the same current manager. This is for tractability and will discuss the limitation later. If not trained, young workers loose the initial skill and become simple workers when old.

With this production function of training, the training cost function takes the following form

$$\begin{aligned} i &= \left[(bn^m k')^\gamma - \psi \left(\hat{k} \right)^\gamma \right]^{\frac{1}{\gamma}} \\ &= \Phi \left(n^m, k', k, \overline{K}, K_0 \right), \qquad (1) \end{aligned}$$

where $\Phi_{n^m}, \Phi_{k'} > 0$ and $\Phi_k, \Phi_{\overline{K}}, \Phi_{K_0} < 0$.

The utility function of agent born at date t is given by

$$U_t = U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o,$$

where c_t^y and c_{t+1}^o are consumption when young at date t and when old at date t+1, and $\beta \in (0, 1)$ is a utility discount factor.

Presently, each of m measure of old agents are managers with some skill, while $1 - m$ measure of old agents are simple workers who is endowed with one unit of labor. In the aggregate, total employment equals population who are not managers, $2 - m$. Aggregate output plus endowment equals aggregate consumption of young and old agents and investment for training.

2.2 Social Optimum

Because all young workers have the same level of initial skill and the production and training technology are convex, we restrict our attention to the case in which the skill of managers are the same within the generation. Let (m, k) be the number and skill of current managers and (m', k') be the number and skill of the future managers. Aggregate output under symmetry equals

$$Y = A(mk)^\alpha (2 - m)^{1-\alpha}. \qquad (2)$$

We consider the social welfare function as discounted sum of utility of present and future as

$$V_t = \delta_t^{-1}\beta \ln c_t^o + (\ln c_t^y + \beta \ln c_{t+1}^o) + \delta_{t+1}\left(\ln c_{t+1}^y + \beta \ln c_{t+2}^o\right) + \delta_{t+1}\delta_{t+2}\left(\ln c_{t+2}^y + \beta \ln c_{t+3}^o\right) + ...,$$

where $\delta_t \in (0,1)$ is relative utility weight of generation born in t and t-1. Let $V^s(m,k;\underline{\delta})$ be the social welfare when the number and skill of current managers are (m,k) and $\underline{\delta} = (\delta_t, \delta_{t+1}, ...)$. The symmetric social optimal allocation should maximize the social welfare as

$$V^s(m,k;\underline{\delta}) = \underset{n^m,k',m',c^o,c^y}{Max} \left\{\delta^{-1}\beta \ln c^o + \ln c^y + \delta' V^s(m',k';\underline{\delta}')\right\},$$

subject to the resource constraint

$$\begin{aligned} c^o + c^y + m\Phi\left(n^m, k', k, \bar{k}n^m, \kappa n^m\right) &= A(mk)^\alpha (2-m)^{1-\alpha} + e^a, \\ m' &= n^m m, \end{aligned}$$

where $e^a = \int_0^{\overline{e}} edG(e) = \omega\frac{\overline{e}}{2}$ is the aggregate endowment. We refer the detail of the condition for the social optimal in section A.1 of the Appendix.

2.3 Overlapping Generations without Financial Friction

Without financial friction, we can achieve a similar allocation with the social optimum (aside from the effect of externality) by using dynamic coalition, following Boyd and Prescott (1987)[26]. We continue restrict our attention that the skill of managers are the same within the generation. Each manager with skill k takes the wage rate of simple workers w, the real gross interest rate R and the average skill of the other managers $\overline{k}$ as given. We denote the aggregate state of the economy as z. He or she chooses the number of simple workers n^n, and the number, wage and skill level of future managers with endowment e, $(n^m(e), w^m(e), k'(e))$ to maximize the profit

$$\pi(k,z) = \max_{n^n, n^m(e), w^m(e), k'(e)} [Ak^\alpha f(n^n + \int n^m(e)de) - wn^n - \int w^m(e)n^m(e)de - \Phi],$$

subject to the constraint that each future manager enjoys at least the discounted utility from working as a simple worker $V(w,w';e)$

$$V(w^m(e), \pi(k'(e), z'); e) \geq V(w, w'; e). \tag{3}$$

Without financial friction, (3) becomes

$$w^m(e) + e + \frac{\pi(k'(e), z')}{R} \geq w + e + \frac{w'}{R}, \text{ or}$$

$$\frac{\pi(k'(e), z') - w'}{R} \geq w - w^m(e). \tag{4}$$

Because this participation constraint and the training cost are the same across young workers, we get $w^m(e) = w^m$ and $k'(e) = k'$. Denoting $\int n^m(e)de = n^m$, the Lagrangian of the problem of manager becomes

$$\begin{aligned}\mathcal{L} &= Ak^\alpha(n^n + n^m)^{1-\alpha} - wn^n - w^m n^m + \Phi(n^m, k', k, \overline{k}n^m, \kappa n^m) \\ &\quad + \lambda\left[\frac{\pi(k', z') - w'}{R} - (w - w^m)\right].\end{aligned}$$

We guess the participation constraint is binding and that (4) holds with equality, and we verify it later for a set of parameters.

The first order conditions for n^n and n^m are

$$(1-\alpha)A\left(\frac{k}{n}\right)^\alpha = w, \tag{5}$$

$$\begin{aligned}(1-\alpha)A\left(\frac{k}{n}\right)^\alpha &= w^m + \left(\frac{\partial \Phi}{\partial n^m} + \frac{\partial \Phi}{\partial \overline{K}}\overline{k} + \frac{\partial \Phi}{\partial K_0}\kappa\right) \\ &= w^m + \frac{i}{n^m}\left(1 + q - q\overline{s} - q(1 - s - \overline{s})\right) \\ &= w^m + \frac{i}{n^m}\left(1 + qs\right), \end{aligned} \tag{6}$$

where

$$q = \frac{\psi\left(\widehat{k}\right)^\gamma}{(bn^m k')^\gamma - \psi\left(\widehat{k}\right)^\gamma} = \frac{\psi\left(\widehat{k}\right)^\gamma}{i^\gamma} \tag{7}$$

$$s \equiv \frac{\eta k^\rho}{\eta k^\rho + \overline{\eta}\overline{K}^\rho + (1 - \eta - \overline{\eta})K_0^\rho}, \tag{8}$$

$$\overline{s} \equiv \frac{\overline{\eta}\overline{K}^\rho}{\eta k^\rho + \overline{\eta}\overline{K}^\rho + (1 - \eta - \overline{\eta})K_0^\rho}. \tag{9}$$

We cam think of q as a ratio of the importance of skill composite $(\widehat{k})$ to goods input (i) in training, and s and $\overline{s}$ as the share of manager's skill and sociality's skill in skill composite. From $(5, 6)$, we learn

$$w^m = w - \frac{i}{n^m}\left(1 + qs\right).$$

The wage rate of a trainees is lower than that of simple worker by the marginal cost of training an additional worker in extensive margin - which equals the total training cost plus the contribution of the current manager's skill per trainee.

The first order condition for career path package w^m and k' are

$$\begin{aligned} n^m &= \lambda, \\ \frac{\partial \Phi}{\partial k'} &= \lambda \frac{1}{R} \frac{\partial \pi(k', z')}{\partial k'}. \end{aligned}$$

Putting together, we get

$$\frac{1}{n^m} \frac{\partial \Phi}{\partial k'} = \frac{i}{n^m k'}(1+q) = \frac{1}{R} \frac{\partial \pi(k', z')}{\partial k'} = \frac{1}{R}\left(\alpha \frac{y'}{k'} + \frac{i'}{k'} q' s'\right), \text{ or}$$

$$\frac{i}{n^m}(1+q) = \frac{1}{R}\left(\alpha \frac{Y'}{m'} + i' q' s'\right). \tag{10}$$

The left hand side (LHS) and the right hand side (RHS) are proportional to the marginal cost and marginal benefit of increasing skill of future managers at the intensive margin.

From (5), the marginal product of labor is equalized across producers, and we get

$$\begin{aligned} n &= \frac{k}{K} N \\ Y &= A K^\alpha N^{1-\alpha} = A\left(\bar{k} m\right)^\alpha (2-m)^{1-\alpha} \\ w &= (1-\alpha) A K^\alpha N^{1-\alpha} = (1-\alpha) \frac{Y}{2-m}. \end{aligned}$$

From (6), we have

$$\begin{aligned} \pi(k, z) &= y - wn + (w - w^m) n^m - i \\ &= \alpha y + iqs. \end{aligned}$$

The second term in the RHS is the cost-saving by offering training over the training cost, which equals to the share of contribution of current manager's skill in training future managers. Then from (4) with equality, we get

$$w - w^m = \frac{\pi' - w'}{R} \tag{11}$$

or

$$\frac{i}{n^m}(1+qs) = \frac{1}{R}\left[\left(\frac{\alpha}{m'} - \frac{1-\alpha}{2-m'}\right) Y' + i' q' s'\right]. \tag{12}$$

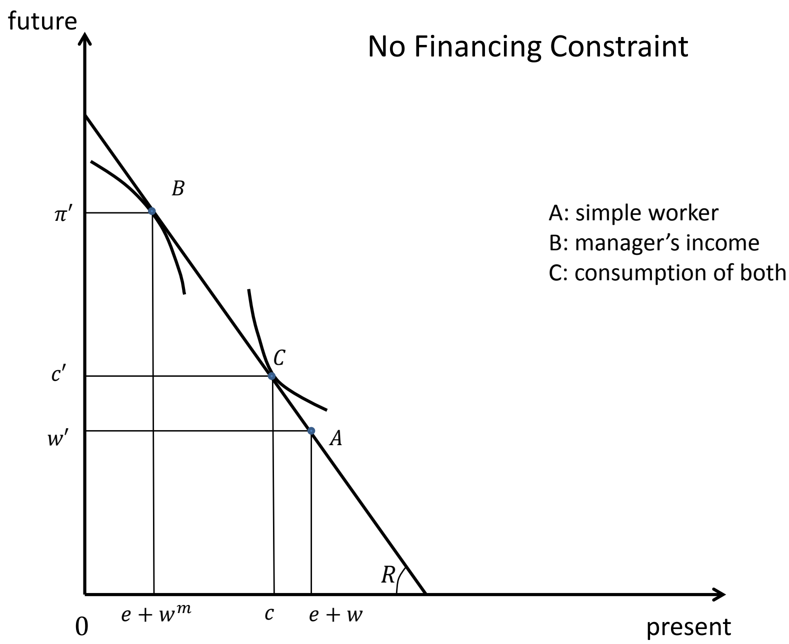

Figure 1: Income/consumption profile in the competitive equilibrium without financing constraint of the basic model.

Figure 1 illustrates income and consumption of young workers with initial endowment e over their lifetime. The horizontal axis is present income and consumption, while the vertical axis is future income and consumption. The point A, $(w+e, w')$ is the income of the simple labor contract. The line ACB is the lifetime budget constraint of the simple worker whose slope is equal to $-R$, the gross real interest rate. This line is also the participation constraint for the career path package because the package has to provide the same present value of income as the simple worker as in (11). The income of the future manager in equilibrium is given by point B, $(w^m + e, \pi')$. Both simple worker and future manager enjoys the same consumption at Point C, (c, c'), as they have the same permanent income.

From the utility maximization condition of household and (11), we get

$$\begin{aligned} \frac{\beta c^y(e)}{c^{o\prime}(e)} &= \frac{1}{R} = \beta \frac{c^y}{c^{o\prime}}, \\ c^y(e) + \frac{c^{o\prime}(e)}{R} &= e + w + \frac{w'}{R} = e + w^m + \frac{\pi'}{R}. \end{aligned} \tag{13}$$

(13) implies the ratio of consumption between when young and when old is the same across

agents of the same generation, and thus the ratio of individual agent equals that of aggregate consumption $(c^y, c^{o\prime})$. Noting that all the borrowing and lending are among agents of the same generation, we have from the budget constraint of each generation as

$$\begin{aligned} c^y &= w(1-m') + w^m m' + e^a \\ &= w - mi\,(1+qs) + e^a, \end{aligned}$$

$$\begin{aligned} c^o &= w(1-m) + m\pi \\ &= \left[(1-\alpha)\frac{1-m}{2-m} + \alpha\right] Y + miqs. \end{aligned}$$

In section A.1 of the Appendix, we show that, aside from the effect of externality through $\overline{k}'$, a competitive equilibrium achieves a very similar allocation with a particular social optimal allocation through the dynamic coalition when there is no financial friction. Such close relationship between the social optimum and the competitive equilibrium will no longer hold once there is a financing constraint as below.

2.4 Competitive Equilibrium with Financing Constraint

In this section we consider each future manager faces a financing constraint, so that he or she can borrow against only up to $\theta \in (0,1)$ fraction of future profit. So, the consumption and saving choice of the future manager with initial endowment e is

$$\begin{aligned} V(w^m, \pi'; e) &= \underset{c^m, c^{m\prime}}{Max} \left(\ln c^m + \beta \ln c^{m\prime}\right), \text{ subject to} \\ c^m + \frac{c^{m\prime}}{R} &= e + w^m + \frac{\pi'}{R}, \\ c^m - w^m - e &\leq \frac{\theta}{R}\pi'. \end{aligned}$$

Restricting the attention to the case of small θ and e, the borrowing constraint is binding and we have

$$\begin{aligned} c^m &= w^m + e + \frac{\theta}{R}\pi', \ c^{m\prime} = (1-\theta)\,\pi' \text{ and} \\ V(w^m, \pi'; e) &= \ln\left(w^m + e + \frac{\theta}{R}\pi'\right) + \beta \ln\left[(1-\theta)\,\pi'\right]. \end{aligned}$$

Once there is a financing constraint, we expect the distribution of skill is no longer degenerate. Denote the distribution of skill of current managers in equilibrium as

$$k \sim F(k), \text{ on } [k_{\min}, k_{\max}].$$

Manager of skill k hires $n^m(k)$ number of workers with endowment $e(k)$. Conjecture that there is an assortative matching between the skill of current managers and endowment of the young trainees, so that more skilled managers hire richer young trainees. We later verify this conjecture. Then the matching function is

$$1 - G(e(k)) = \int_k^{k_{\max}} n^m(\tilde{k}) dF(\tilde{k}).$$

The individual manager of skill k chooses the number of simple workers, the number, wage and training level of future managers $(n(k), n^m(k), w^m(k), k' = k^*(k))$ to maximize the profit subject to the participation constrain. Since young agents can choose the best contract offered by many managers with different skills including a simple labor contract, the participation constraint for a career package $(w^m(k), k' = k^*(k))$ becomes

$$\begin{aligned} V(w^m(k), \pi(k'); e(k)) &\geq V(e(k)) \\ &= Max \left\{ \max_{\widetilde{k}} V\left(w_t^m\left(\widetilde{k}\right), \pi_{t+1}\left(k^*\left(\widetilde{k}\right)\right); e\right), V(w, w'; e) \right\}, \end{aligned}$$

where $V(e(k))$ is the competitive level of discounted utility of young worker with endowment e.

The Lagrangian for the choice of the individual manager of skill k is given by

$$\begin{aligned} \mathcal{L}(k) &= Ak^\alpha f(n^n + n^m(k)) - wn^n - w^m(k)n^m(k) - \Phi(n^m(k), k', k, \overline{k}n^m(k), \kappa n^m(k)) \\ &\quad + \lambda(k) \left[V(w^m(k), \pi(k'); e(k)) - V(e(k))\right] + \nu\left(e(k)\right) n^m(k). \end{aligned}$$

The variable $\nu(e(k)) \geq 0$ is Lagrangian multiplier for the non-negativity constraint for $n^m(k)$. So, the first order conditions for the choice of the manager $(n(k), n^m(k), w^m(k), k' = k^*(k))$ are

$$Ak^\alpha f'(n) = (1-\alpha)A\left(\frac{k}{n(k)}\right)^\alpha = w, \text{ or} \tag{14}$$

$$n(k) = \left[\frac{(1-\alpha)A}{w}\right]^{\frac{1}{\alpha}} k,$$

$$Ak^{\alpha}f'(n) = w^m(k) + \frac{\partial\Phi(k)}{\partial n^m} + \bar{k}\frac{\partial\Phi(k)}{\partial \overline{K}} + k_0\frac{\partial\Phi(k)}{\partial K_0} - \nu\left(e(k)\right), \text{ and} \tag{15}$$

$$0 = \nu\left(e(k)\right)\cdot n^m(k) \tag{16}$$

$$n^m(k) = \lambda(k)\frac{\partial V(w^m(k), \pi(k', z'); e(k))}{\partial w^m(k)}, \tag{17}$$

$$\frac{\partial\Phi}{\partial k'} = \lambda(k)\frac{\partial V(w^m(k), \pi(k', z'); e(k))}{\partial \pi'}\frac{\partial\pi(k', z')}{\partial k'}. \tag{18}$$

Denoting $X(k) = i(k)/w = \Phi(n^m(k), k', k, \overline{k}, \kappa n^m(k))/w$, we get

$$w_m(k) = w\left\{1 - [1 + q(k)s(k)]\frac{X(k)}{n^m(k)}\right\}$$

$$\begin{aligned}\pi(k) &= Ak^{\alpha}n(k)^{1-\alpha} - wn(k) + (w - w^m(k))n^m(k) - \Phi(k)\\ &= w\frac{\alpha}{1-\alpha}n(k) + q(k)s(k)i(k)\\ &= w\left[\frac{\alpha}{1-\alpha}\frac{2-m}{m}\frac{k}{\overline{k}} + q(k)s(k)X(k)\right]\end{aligned} \tag{19}$$

Denote e^* as the lowest endowment of young workers who receive the training. In the next period, the number of managers, aggregate skill and distribution of skill becomes

$$\begin{aligned}m' &= \omega\left(1 - \frac{e^*}{\overline{e}}\right),\\ K' &= m'\overline{k}' = \int_{k_{\min}}^{k_{\max}} n^m(k)k^*(k)dF(k)\\ F_{t+1}(k') &= \int_{k_{\min}}^{k^{*-1}(k')} n^m(k)dF(k),\end{aligned}$$

where we denote $k^{*-1}(k') = k_{\min}$ if $k' \leq k^*(k_{\min})$ and $k^{*-1}(k') = k_{\max}$ if $k' \geq k^*(k_{\max})$.

From the market clearing condition for funds, the aggregate saving of simple workers equals the aggregate borrowing of future managers as

$$\int_0^{e^*}\left(e + w - \frac{e + w + \frac{w'}{R}}{1+\beta}\right)dG(e) = \frac{\theta}{R}\int_{k_{\min}}^{k_{\max}} n^m(k)\pi(k^*(k), z')dF(k), \text{ or}$$

$$\beta\left[(1-m')w + (\omega - m')\frac{e^*}{2}\right] = (1-m')\frac{w'}{R} + (1+\beta)\frac{\theta}{R}\int_{k_{\min}}^{k_{\max}} n^m(k)\pi(k^*(k), z')dF(k), \text{ or}$$

$$\beta = \frac{w'}{wR}\Omega, \text{ where}$$

$$\left[1 - m' + (\omega - m')\frac{e^*}{2w}\right]\Omega \equiv 1 - m' + (1+\beta)\theta \int_{k_{\min}}^{k_{\max}} n^m(k) \frac{\pi(k^*(k), z')}{w'} dF(k) \tag{20}$$

$$= 1 - m' + (1+\beta)\theta \int_{k_{\min}}^{k_{\max}} n^m(k) \left\{ \frac{\alpha}{1-\alpha} \frac{2-m'}{m'} \frac{k^*(k)}{\bar{k}'} + q(k^*(k))s(k^*(k))X(k^*(k)) \right\} dF(k),$$

using (19).

We know

$$V(w, w'; e) = (1+\beta)\ln\left[\frac{w}{1+\beta}\left(1 + \frac{e}{w} + \frac{w'}{wR}\right)\right] + \beta\ln(\beta R).$$

$$V(w^m(k), \pi(k^*(k), z'); e(k)) = \ln\left(e(k) + w^m(k) + \frac{\theta}{R}\pi(k^*(k), z')\right) + \beta\ln\left[(1-\theta)\pi(k^*(k), z')\right]$$

$$= (1+\beta)\ln w + \ln\left\{1 + \frac{e(k)}{w} - (1+qs)\frac{X}{n^m} + \frac{\theta}{R}\frac{w'}{w}\frac{\pi'}{w'}\right\} + \beta\ln\left\{(1-\theta)\frac{w'}{w}\frac{\pi'}{w'}\right\}.$$

Thus the indifference condition for workers with endowment e^* is that

$$\begin{aligned} &(1+\beta)\ln\left[\frac{1}{1+\beta}\left(1 + \frac{e^*}{w} + \frac{w'}{wR}\right)\right] + \beta\ln(\beta\frac{Rw}{w'}) \\ &= \ln\left\{1 + \frac{e^*}{w} - [1 + q(k_{\min})s(k_{\min})]\frac{X(k_{\min})}{n^m(k_{\min})} + \frac{\theta}{R}\frac{w'}{w}\frac{\pi(k^*(k_{\min}), z')}{w'}\right\} \\ &+ \beta\ln\left\{(1-\theta)\frac{\pi(k^*(k_{\min}), z')}{w'}\right\}. \end{aligned} \tag{21}$$

For workers with endowment $e > e^*$, we consider the occupational choice as,

$$V(e) = \max_k \ln\left(e + w^m(k) + \frac{\theta}{R}\pi(k^*(k), z')\right) + \beta\ln\left[(1-\theta)\pi(k^*(k), z')\right]$$

The first order condition becomes

$$\frac{1}{e + w^m(k) + \frac{\theta}{R}\pi(k^*(k), z')}\frac{d}{dk}\left[w^m(k) + \frac{\theta}{R}\pi(k^*(k), z')\right] + \frac{\beta}{\pi(k^*(k), z')}\frac{d}{dk}\pi(k^*(k), z') = 0$$

Because $\frac{\partial\pi(k', z')}{\partial k'} > 0$, we learn sorting occurs, i.e., $\frac{dk^*(k)}{dk} > 0$, if

$$\frac{d}{dk}\left[w^m(k) + \frac{\theta}{R}\pi(k^*(k), z')\right] < 0.$$

From the envelope theorem, we have

$$V'(e) = \frac{1}{e + w^m(k) + \frac{\theta}{R}\pi(k^*(k), z')}$$

and for $e > e^*$, we get

$$V(e) = V(e^*) + \int_{e^*}^{e} \frac{1}{\tilde{e} + w^m(k(\tilde{e})) + w\frac{\theta w'}{Rw}\frac{\pi(k^*(k(\tilde{e})),z')}{w'}} d\tilde{e}.$$

where $k(e) = e^{-1}(k)$. Thus for all types $k \geq k_{min}$,

$$\begin{aligned}
V(e) &= (1+\beta)\ln\left[\frac{w}{1+\beta}\left(1+\frac{e^*}{w}+\frac{w'}{wR}\right)\right] + \beta\ln(\beta R) + \int_{e^*}^{e}\frac{1}{\tilde{e}+w^m(k(\tilde{e}))+\frac{\theta}{R}\pi(k^*(k(\tilde{e})),z')}d\tilde{e} \\
&= (1+\beta)\ln w + \beta\ln\left\{(1-\theta)\frac{w'}{w}\frac{\pi(k^*(k(e)),z')}{w'}\right\} \\
&+ \ln\left\{1+\frac{e}{w} - [1+q(k(e))s(k(e))]\frac{X(k(e))}{n^m(k(e))} + \frac{\theta}{R}\frac{w'}{w}\frac{\pi(k^*(k(e)),z')}{w'}\right\}.
\end{aligned}$$

From (17) and (18), we have

$$MRS(k) = \frac{\frac{\partial V(w^m(k),\pi(k^*(k),z');e(k))}{\partial \pi'}}{\frac{\partial V(w^m(k),\pi(k^*(k),z');e(k))}{\partial w^m(k)}} = \frac{\frac{\partial \Phi(n^m,k^*(k),k,\bar{k},K_0)}{\partial k'}}{n^m(k)\frac{\partial \pi(k^*(k),z')}{\partial k'}} = MRT(k). \tag{22}$$

The LHS is the marginal rate of substitution between future profit and apprentice wage to achieve the same utility for the trainee and given by

$$\begin{aligned}
MRS(k) &= (1+\beta)\frac{\theta}{R} + \beta\frac{e(k)+w^m(k)}{\pi(k^*(k),z')} \\
&= (1+\beta)\frac{\theta}{R} + \beta\frac{w}{w'}\frac{w'}{\pi(k^*(k),z')}\left(\frac{e(k)}{w} + 1 - [1+q(k)s(k)]\frac{X(k)}{n^m(k)}\right)
\end{aligned}$$

The RHS is the ratio of marginal cost to marginal benefit of increasing training, and given by

$$MRT(k) = \frac{w}{w'}\frac{\frac{1}{n^m(k)}(1+q(k))X(k)}{\frac{\alpha}{1-\alpha}\frac{2-m'}{m'}\frac{k^*(k)}{k'} + q'(k^*(k))s'(k^*(k))X'(k^*(k))}.$$

Thus we have the marginal condition for future skill at intensive margin as

$$\begin{aligned}
&\frac{\frac{1}{n^m(k)}(1+q(k))X(k)}{\frac{\alpha}{1-\alpha}\frac{2-m'}{m'}\frac{k^*(k)}{k'} + q'(k^*(k))s'(k^*(k))X'(k^*(k))} \\
=&(1+\beta)\frac{\beta}{\Omega}\theta + \beta\frac{\frac{e(k)}{w} + 1 - [1+q(k)s(k)]\frac{X(k)}{n^m(k)}}{\pi(k^*(k),z')/w'}
\end{aligned} \tag{23}$$

The equilibrium can be solved by equations (20), (21) and (23).

Figure 2 illustrates income and consumption of young workers who are have a larger initial endowment than the minimum endowment to receive training, $e > e^*$. When they choose

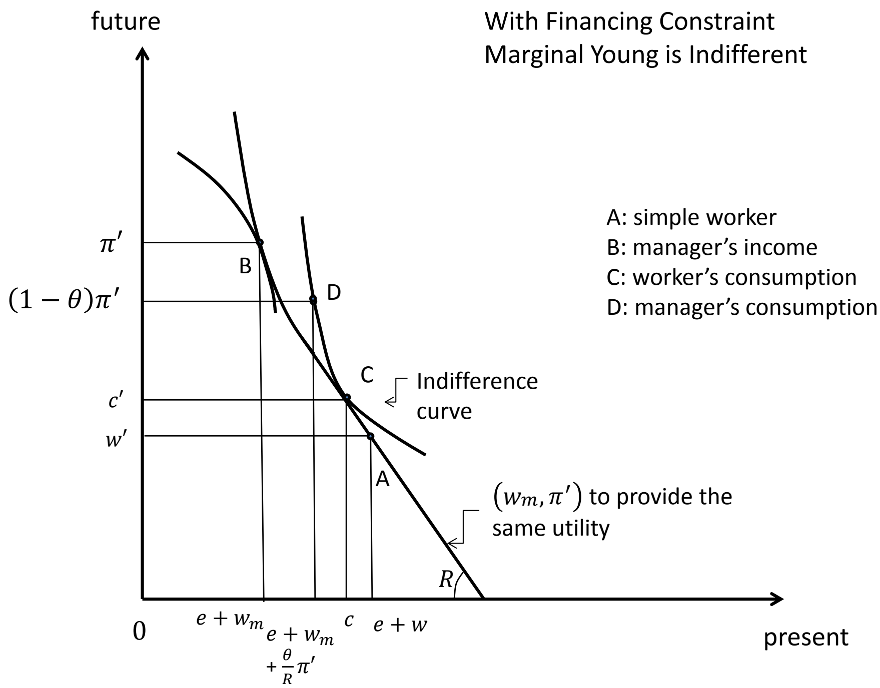

Figure 2: Income/consumption profile in the competitive equilibrium with financing constraint of the basic model.

to become simple workers, the income would be $(w + e, w')$ at point A, and consumption would be at point C. If they choose to become future managers, their income is $(w^m + e, \pi')$ at point B and their consumption is $\left(w^m + e + \frac{\theta}{R}\pi', (1-\theta)\pi'\right)$ at point D because of the binding financing constraint. Because rich young agents can use a large endowment to smooth consumption better than poor young agents, they enjoy a higher utility at point D than becoming a simple worker at point C.

2.5 Results

2.5.1 Parametrization

The parameter values we use in the numerical exercise is reported in Table 1. Most parameters are standard. We think of a period as 20 years. So the annualized discount factor is 0.986. The income share of the intangible capital is set to $\alpha = 0.3$. We choose $\gamma = 0.1 < \rho = 0.2$, so that the substitutability between k, $\overline{k}$ and K_0 in skill composite $\widehat{k}$ is higher than the

fraction of positive endowment ω	0.7
upper bound of endowment $\overline{e}$	1
initial skill κ	1
share of intangibles α	0.3
elasticity parameter in skill production γ	0.1
elasticity parameter in skill composite ρ	0.2
share parameter of skill composite ψ	2
share parameter of manager's skill η	0.4
share parameter of society's skill $\overline{\eta}$	0.2
utility discount β	0.75
borrowing constraint θ	0.1

Table 1: Parameter values used in the basic model simulation.

substitutability between goods input i and skill composite $\widehat{k}$ for training. We assume that a household can only pledge 10% of their future income. ($\theta = 0.1$) The limited borrowing capacity reflects that the intangible capital is less collateralizable.

2.5.2 Inequality in the Steady State

Without the financing constraint, we can show there is a competitive equilibrium in which all managers receive the same training. The allocation is efficient if there is no externality, $\bar{\eta} = \bar{s} = 0$. When there is no initial skill heterogeneity, the present value of labor income is equal across agents when there is no skill heterogeneity.

Figure 3 illustrate the present value of income and consumption, the income and consumption of agents when young and when old for the economy without financing constraint. Without binding financing constraint, the inequality of permanent income and consumption is entirely due to the inequality of the initial endowment. The income of managers is higher than income of simple workers when old, because managers had lower apprentice wage than simple workers when young. In contrast 4 illustrates the present value of income and consumption, the income and consumption of agents when young and when old for the economy with financing constraint. If young agents' endowment is lower than $e^* = .43$, they will not receive training and work as simple workers for life. Income and consumption are smooth and the inequality is due to the difference in initial endowment. If young agents' initial endow-

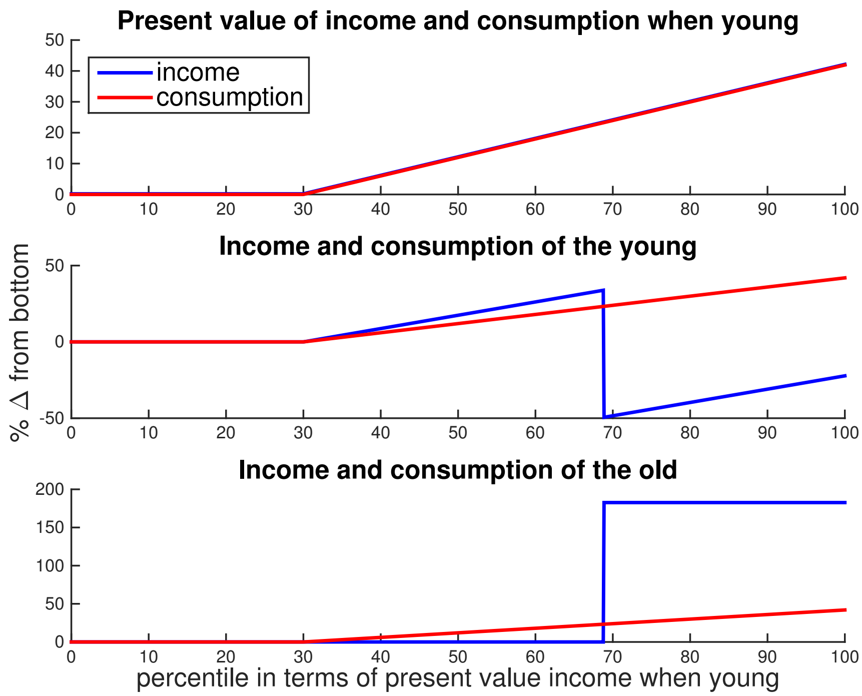

Figure 3: Income distribution, basic model, without financing constraint.

ment is lager than e^*, they receive the training and the present value of income is distinctively higher than the simple workers in order to compensate the non-smooth consumption due to the borrowing constraint. Moreover, the larger is the initial endowment, better training a young agent receives. which leads to a larger permanent income and consumption. On the other hand, the trainees have lower income and consumption when young due to financing constraint while enjoy very large income and consumption when they become managers in old age.

The income inequality across trainees arises because of the sorting between the initial endowment of trainees and the level of intangible capital of managers. Figure 5 illustrates the sorting. Rich young agents are trained by productive managers. This is because the most productive managers train more intensively, which corresponds to the steepest income profile, as is illustrated in Figure 4. Rich young agents, who can obtain a less steep consumption profile by consuming his endowment, therefore has a comparative advantage to be trainees of productive managers. Young agents with intermediate initial endowment remains in the middle class in terms of the present value of income because they are trained by managers

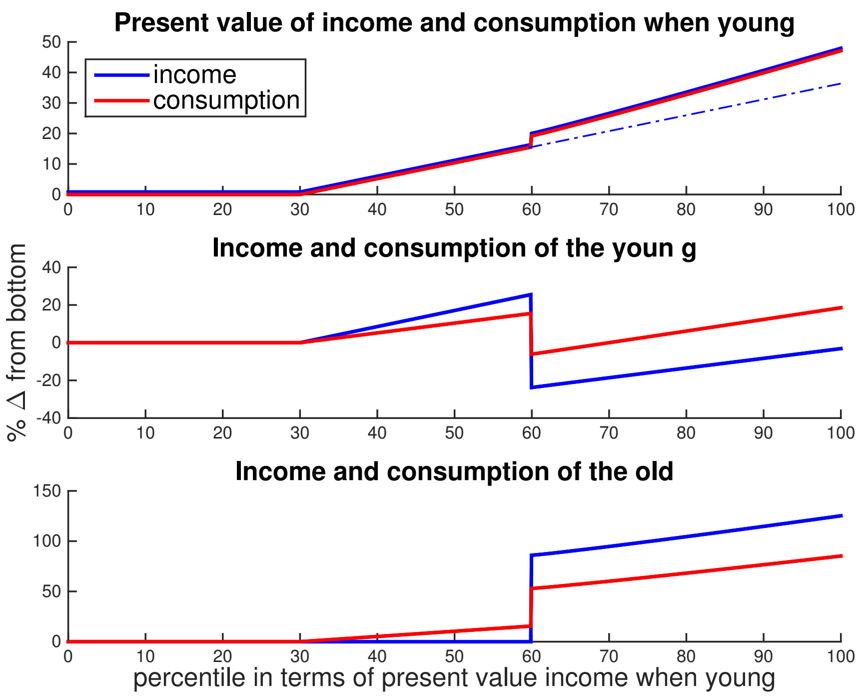

Figure 4: Income distribution, basic model, with financing constraint.

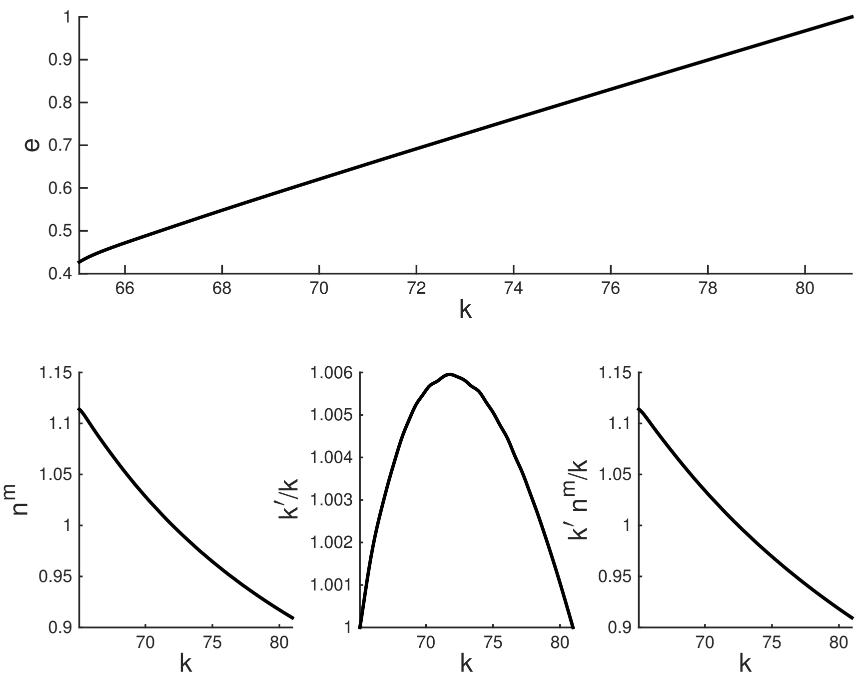

Figure 5: Sorting between managers and trainees; training on the extensive and intensive margin.

with less intangible capital. The poorest young agents, although equally talented as the rich, become workers.

When young trainees with heterogeneous initial endowment are matched with managers with heterogeneous intangible capital, the heterogeneity is passed on and amplified through training and leads to a dispersed distribution of intangible capital across managers. In the current numerical example, the endowment of the richest trainee is 0.57 unit higher than the endowment of the poorest trainee, while the most productive manager has about 15 units more of intangible capital. The distribution of intangible is a sign of inefficiency because of financing constraint. The aggregate stock of intangible capital with financing constraint is lower.

2.5.3 Dynamic Response to Negative Shock to Endowment

In this section, we study the effect of a negative shock to the endowment distribution: the total measure of young agents with positive endowment, denoted ω, drops unexpectedly, while

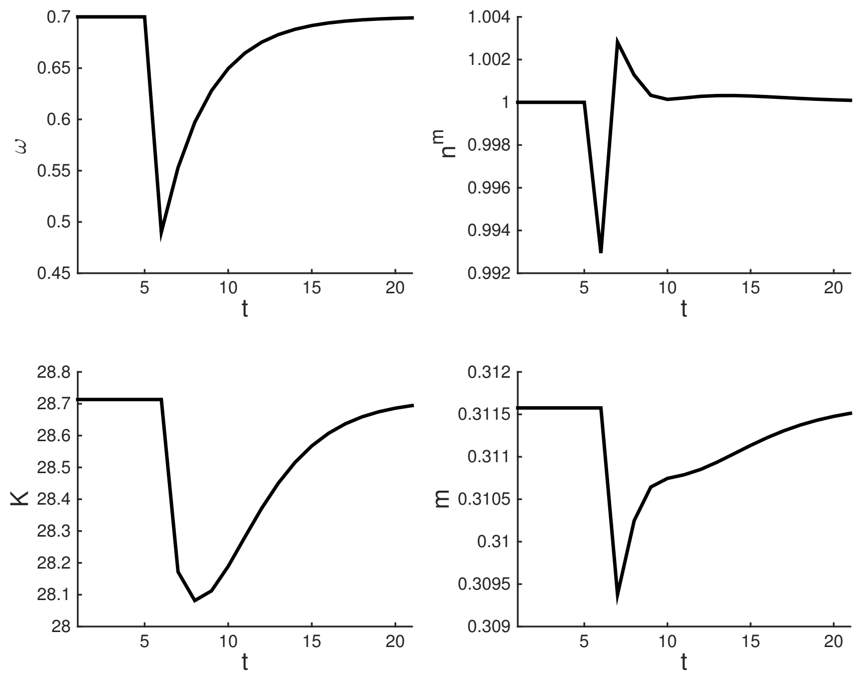

Figure 6: Dynamics in the basic model when financing constraint is not binding.

the conditional distribution of young agents with positive endowment remain unchanged. After the initial shock, the measure of young agents with positive endowment converges gradually to the original level. The shock process is illustrated in the sub-figure on the northwest corner of Figure 6, 7 and 8. It is meant to capture the effect of a negative financial shock. With the shock, all agents receive less endowment on the transition path. Then, if young agents were to receive the same amount of training, they may be more financially constrained.

Figure 6 shows the corresponding transition dynamics of the basic model when financing constraint is not binding. Without financing constraint, the shock affects little mangers' training decision on both margins. The dynamic response of the full model is also small without financing constraint.

In contrast, the impulse response is much more significant when the financing constraint binds. Figure 7, 8 and 9 illustrate the transition dynamics in the basic model when the financing constraint is binding. Figure 7 illustrates the training decisions of managers. Because agents are more financially constrained on the transition path, most productive managers

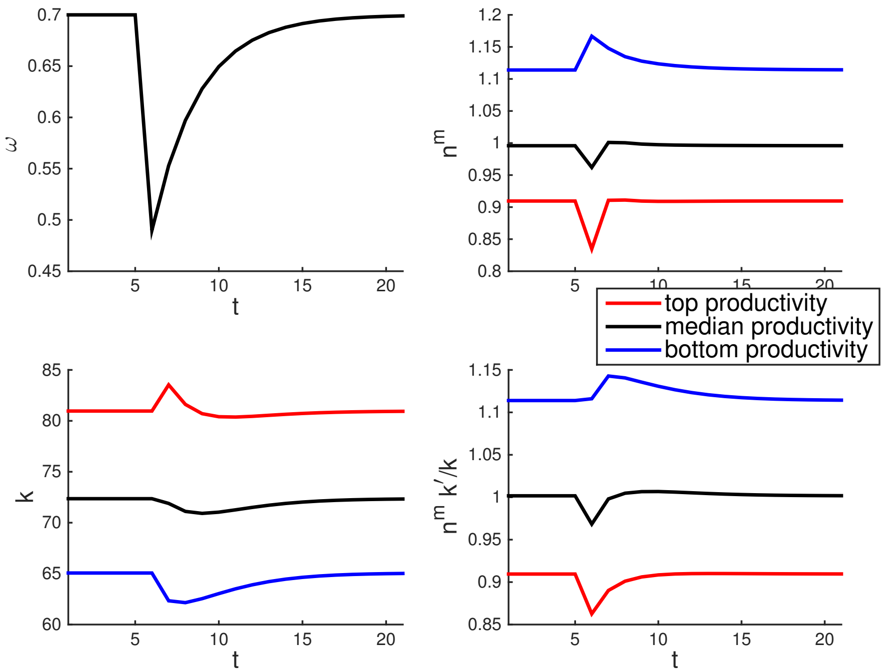

Figure 7: Training decisions on the transition path of the basic model when financing constraint is binding.

train less rich young agents but more intensively. Meanwhile, least productive managers train more poorer young agents but less intensively. As a result, the distribution of income and intangible capital is more dispersed on the transition path.

Figure 8 illustrates the transition dynamics of aggregate variables. Because of tighter financial constraints on the transition path, training on both margins fall. As a result, in the following periods, the wage rate falls, the aggregate stock of intangible capital decreases, and the total measure of managers (who is capable training future generations) falls. The fall in training on both margins implies that it takes a long time for the intangible capital stock to recover from the recession. The persistent slowdown, we believe, contributes to secular stagnation we observe in the past decades.

Figure 9 illustrates the dynamics of earnings inequality. The earnings inequality within the young generation is reduced along the transition path, because less young agents are trained and trainees are on average trained less intensively. The earning inequality within the old generation increases on the transition path, because to attract trainees managers

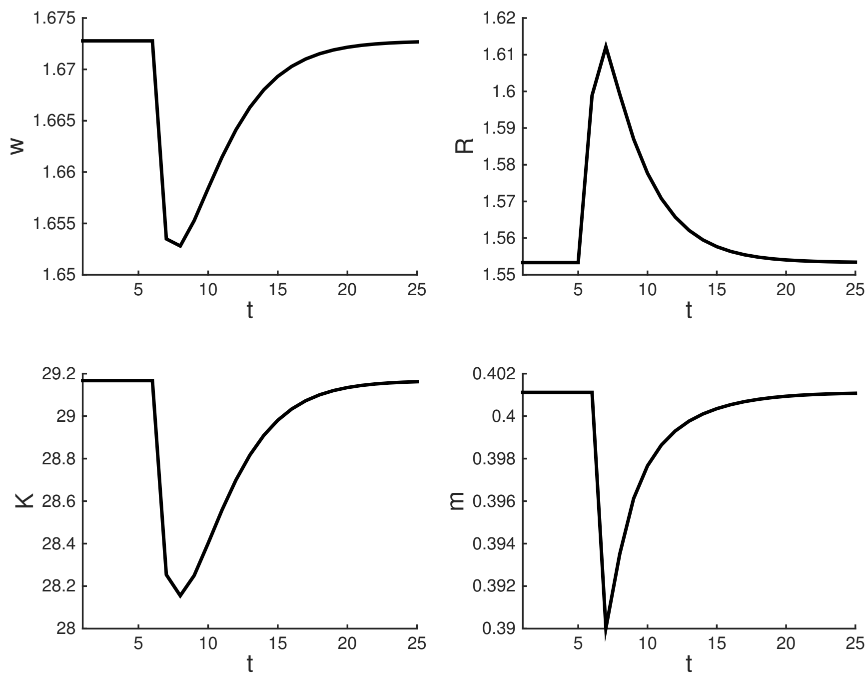

Figure 8: Aggregate variables on the transition path of the basic model when financing constraint is binding.

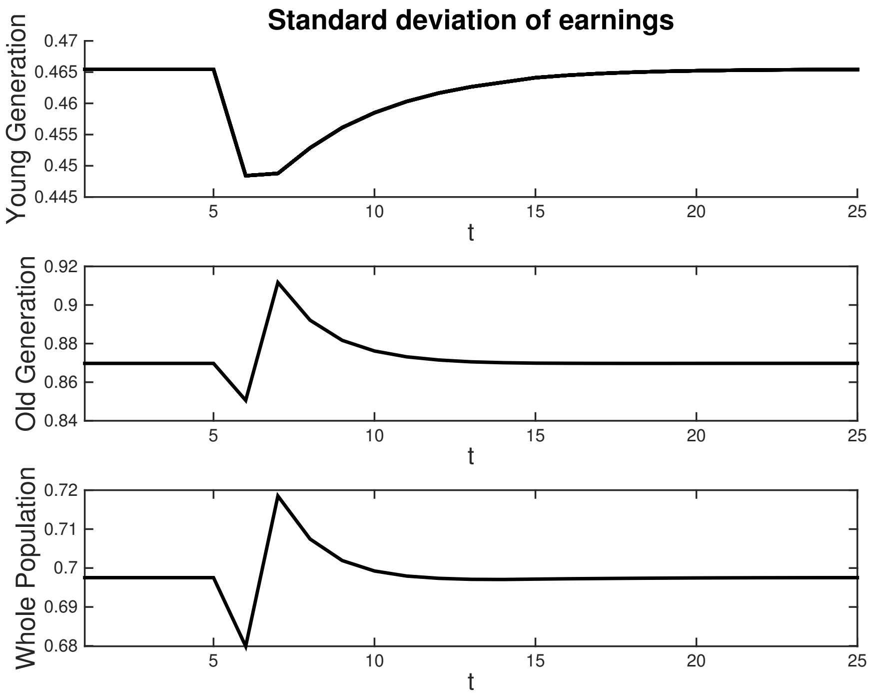

Figure 9: Standard deviation of earnings distribution on the transition path.

have to offer career paths with higher present value of income, which implies even steeper income profile. Overall, the income inequality across the whole population may increase on the transition path.

3 Model with Only Skill Heterogeneity

$$
\begin{aligned}
n^m k' &= \left(\frac{1}{b} i\right)^{\frac{1}{1+\psi}} \left(k^\eta (n^m \bar{k})^{\bar{\eta}} (n^m \kappa)^\nu\right)^{\frac{\psi}{1+\psi}} \\
i &= b \frac{(n^m)^{1+\eta\psi} (k')^{1+\psi}}{\left(k^\eta \bar{k}^{\bar{\eta}} \kappa^\nu\right)^\psi} \\
&= \frac{b (n^m)^{1+\eta\psi} (k')^{1+\psi}}{\hat{k}^\psi} \\
&= b n^m k' (n^m)^{\eta\psi} \left(k'/\hat{k}\right)^\psi
\end{aligned}
$$

where $\hat{k}$ is a new notation,

$$\hat{k} = k^{\eta}\bar{k}^{\bar{\eta}}\kappa^{\nu},$$

where $\eta + \bar{\eta} + \nu = 1$.

$$\frac{\partial \Phi}{\partial \kappa} = -\psi\nu\frac{i}{\kappa},$$
$$\frac{\partial \Phi}{\partial k} = -\psi\eta\frac{i}{k},$$
$$\frac{\partial \Phi}{\partial k'} = (1+\psi)\frac{i}{k'},$$
$$\frac{\partial \Phi}{\partial n^m} = (1+\eta\psi)\frac{i}{n^m}.$$

$$\pi(k) = \max_{k',n^n,n^m,w^m,\kappa} Ak^{\alpha}n^{1-\alpha} - wn + (w - w^m(k))n^m(k) - \Phi\left(n^m, k', k, n^m\bar{k}, n^m\kappa\right)$$
$$+ \lambda\left[V(w^m(k), \pi(k', z'); e) - V^*(\kappa, e)\right]$$

FOCs:

$$n :(1-\alpha)Ak^{\alpha}n^{-\alpha} = w$$
$$n^m :w - w^m = \frac{\partial \Phi}{\partial n^m} = (1+\eta\psi)\frac{i}{n^m}$$
$$w^m :n^m(k) = \lambda\frac{\partial V}{\partial w^m}$$
$$k' :\frac{\partial \Phi}{\partial k'} = \lambda\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k', z')}{\partial k'}$$
$$\kappa : -\frac{\partial \Phi}{\partial \kappa} = \lambda\frac{\partial V^*(\kappa, e)}{\partial \kappa}$$
$$V(w^m(k), \pi(k', z') = V^*(\kappa, e)$$

Because $n = \left(\frac{(1-\alpha)A}{w}\right)^{1/\alpha} k$, $2 - m = \left(\frac{(1-\alpha)A}{w}\right)^{1/\alpha} K$,

$$\pi(k, z) = \alpha Ak^{\alpha}n^{1-\alpha} + (1+\eta\psi)i - i$$
$$= \alpha A\left(\frac{2-m}{K}\right)^{1-\alpha} k + \eta\psi i$$

$$\frac{\partial \pi(k, z)}{\partial k} = \frac{\alpha Ak^{\alpha}n^{1-\alpha}}{k} - \frac{\partial \Phi}{\partial k}$$
$$= \frac{\alpha Ak^{\alpha}n^{1-\alpha}}{k} + \eta\psi\frac{\Phi}{k}$$
$$= \frac{\pi(k, z)}{k}$$

$$
\begin{aligned}
\frac{1}{n^m}\frac{\partial \Phi}{\partial k'} &= \frac{\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k',z')}{\partial k'}}{\frac{\partial V}{\partial w^m}} \\
\frac{\frac{\partial \Phi}{\partial k'}}{-\frac{\partial \Phi}{\partial \kappa}} &= \frac{\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k',z')}{\partial k'}}{\frac{\partial V^*(\kappa,e)}{\partial \kappa}} \\
V(w^m(k),\pi(k',z')) &= V^*(\kappa,e) \\
w &= w^m + \frac{\partial \Phi}{\partial n^m}
\end{aligned}
$$

Simplifying the conditions, we have

$$
\begin{aligned}
\frac{1}{n^m}(1+\psi)\frac{i}{k'} &= \frac{\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k',z')}{\partial k'}}{\frac{\partial V}{\partial w^m}}, \\
\frac{(1+\psi)\frac{i}{k'}}{\psi\nu\frac{1}{\kappa}} &= \frac{\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k',z')}{\partial k'}}{\frac{\partial V^*(\kappa,e)}{\partial \kappa}}, \\
V(w^m(k),\pi(k',z')) &= V^*(\kappa,e), \\
w &= w^m + (1+\eta\psi)\frac{i}{n^m},
\end{aligned}
$$

which will help us pin down $n^m(k)$, $k^+(k)$, w^m, κ (or $\frac{\partial V^*(\kappa,e)}{\partial \kappa}$).

$$
\begin{aligned}
V &= \ln(w^m(k) + \frac{\theta}{R}\pi(k^+(k),z') + e) + \beta\ln((1-\theta)\pi(k^+(k),z')) \\
\frac{\partial V}{\partial \pi'} &= \frac{\theta}{R}\frac{1}{w^m(k) + \frac{\theta}{R}\pi(k^+(k),z') + e} + \frac{\beta}{\pi(k^+(k),z')} \\
\frac{\partial V}{\partial w^m} &= \frac{1}{w^m(k) + \frac{\theta}{R}\pi(k^+(k),z') + e}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{n^m}(1+\psi)\frac{i}{k'} &= \frac{\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k',z')}{\partial k'}}{\frac{\partial V}{\partial w^m}}, \\
&= \left[\frac{\theta}{R} + \beta\frac{w^m(k) + \frac{\theta}{R}\pi(k',z') + e}{\pi(k',z')}\right]\frac{\pi(k',z')}{k'} \\
&= \frac{\theta}{R}\frac{\pi(k',z')}{k'} + \beta\frac{w^m(k) + \frac{\theta}{R}\pi(k',z') + e}{k'} \\
\frac{1+\psi}{n^m}i &= \beta\left[w^m(k) + e\right] + (1+\beta)\frac{\theta}{R}\pi(k',z').
\end{aligned}
$$

$$
\begin{aligned}
i &= \frac{n^m}{1+\psi}\left\{\beta\left[w^m(k)+e\right]+(1+\beta)\frac{\theta}{R}\pi(k',z')\right\}.\\
&= \frac{n^m}{1+\psi}\left\{\beta\left[w-(1+\eta\psi)\frac{i}{n^m}+e\right]+(1+\beta)\frac{\theta}{R}\pi(k',z')\right\}.
\end{aligned}
$$

$$
\frac{i}{n^m}\left[1+\beta+\psi(1+\beta\eta)\right]=\beta\left(w+e\right)+(1+\beta)\frac{\theta}{R}\pi(k',z')
$$

Market clearing conditions.

Since $(1-\alpha)Ak^\alpha(n^n+n^m(k))^{-\alpha}=w$

$$
\frac{n^n+n^m(k)}{k}=\left(\frac{(1-\alpha)A}{w}\right)^{\frac{1}{\alpha}}
$$

The market clearing condition of the labor market implies that

$$
\begin{aligned}
2-m &= \int(n^n+n^m(k))dF(k)\\
&= \left(\frac{(1-\alpha)A}{w}\right)^{1/\alpha}\int kdF(k)\\
&= \left(\frac{(1-\alpha)A}{w}\right)^{1/\alpha}\bar{k}m\\
w &= (1-\alpha)A\left(\frac{m}{2-m}\bar{k}\right)^{\alpha}
\end{aligned}
$$

Therefore,

$$
\frac{n}{k}=\frac{2-m}{m\bar{k}}.
$$

$$
\frac{\partial\pi(k,z)}{\partial k}=\alpha A\left(\frac{2-m}{m\bar{k}}\right)^{1-\alpha}+\eta\psi\frac{i}{k}.
$$

$$
w^m(k)=(1-\alpha)A\left(\frac{m}{2-m}\bar{k}\right)^{\alpha}-(1+\eta\psi)\frac{i}{n^m}.
$$

The equations can be simplified as

$$\frac{1}{n^m}(1+\psi)\frac{i}{k^+(k)} = \frac{\frac{\partial V}{\partial \pi'}}{\frac{\partial V}{\partial w^m}}\left[\alpha A'\left(\frac{2-m'}{m'\bar{k}'}\right)^{1-\alpha} + \psi\eta\frac{1}{k^+(k)}\right]$$

$$\frac{\partial V^*(\kappa,e)}{\partial \kappa} = \frac{\psi\nu}{\kappa}\frac{\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k',z')}{\partial k'}}{(1+\psi)\frac{i}{k^+(k)}} = \frac{\psi\nu}{\kappa}\frac{\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k',z')}{\partial k'}}{n^m\frac{\frac{\partial V}{\partial \pi'}\frac{\partial \pi(k',z')}{\partial k'}}{\frac{\partial V}{\partial w^m}}},$$

$$= \frac{\psi\nu}{\kappa}\frac{1}{n^m}\frac{\partial V}{\partial w^m},$$

$$V(w^m(k), \pi(k', z')) = V^*(\kappa, e),$$

$$w = w^m + (1+\eta\psi)\frac{i}{n^m},$$

Because $i = bn^m k' \left(n^m\right)^{\eta\psi}\left(k'/\hat{k}\right)^{\psi}$

$$b(1+\psi)\left(n^m\right)^{\eta\psi}\left(k^+(k)/\hat{k}(k)\right)^{\psi} = \frac{\frac{\partial V}{\partial \pi'}}{\frac{\partial V}{\partial w^m}}\left[\alpha A'\left(\frac{2-m'}{m'\bar{k}'}\right)^{1-\alpha} + \psi\eta\frac{1}{k^+(k)}\right]$$

$$\ln(w^m(k) + \frac{\theta}{R}\pi(k^+(k), z') + e) + \beta\ln((1-\theta)\pi(k^+(k), z')) = V^*(\kappa, e)$$

$$w^m(k) = (1-\alpha)A\left(\frac{m}{2-m}\bar{k}\right)^{\alpha} - (1+\eta\psi)bn^m k'\left(n^m\right)^{\eta\psi}$$

At $\hat{\kappa}(k)$,

$$1 - G(\hat{\kappa}(k_{\min})) = \int n^m(\tilde{k})dF(\tilde{k})$$

$$G(\hat{\kappa}(k)) - G(\hat{\kappa}(k_{\min})) = \int_{k_{\min}}^{k} n^m(\tilde{k})dF(\tilde{k})$$

The market clearing condition for saving is

$$(1-m')(w + e - \frac{1}{1+\beta}(w + \frac{w'}{R} + e)) = \frac{1-m'}{1+\beta}(\beta(w+e) - \frac{w'}{R})$$

$$= \theta\int_{k_{\min}}^{k_{\max}} n^m(\tilde{k})\frac{\pi(k^+(\tilde{k}))}{R}dF(\tilde{k})$$

$$\frac{\pi(k)}{w} = \frac{\alpha}{1-\alpha}\frac{2-m}{m}\frac{k}{\bar{k}} + \psi\eta\frac{i}{w}$$

$$(1-m')\left(\beta(1+\frac{e}{w}) - \frac{w'}{wR}\right) = (1+\beta)\frac{w'}{wR}\theta\int_{k_{\min}}^{k_{\max}} n^m(\tilde{k})\frac{\pi(k^+(\tilde{k}))}{w'}dF(\tilde{k})$$

$$\frac{w'}{wR} = \frac{(1-m')\,\beta(1+\frac{e}{w})}{1-m'+(1+\beta)\theta\int_{k_{\min}}^{k_{\max}} n^m(\tilde{k})\frac{\pi(k^+(\tilde{k}))}{w'}dF(\tilde{k})}$$

4 Full Model: Heterogeneous Initial Skill and Endowment

Suppose the initial skill of workers is heterogeneous, and that

$$\begin{aligned} k_0 &= \kappa_h, \text{ for a fraction } \varepsilon \text{ of workers,} \\ k_0 &= \kappa_l < \kappa_h, \text{ for a fraction } 1-\varepsilon \text{ of workers.} \end{aligned}$$

The distribution of initial endowment is independent of initial skill and is distributed according to $G(e)$ as before. Let n_h^m and n_l^m be the numbers of workers of high and low initial skills. Then

$$\begin{aligned} n^m &= n_h^m + n_l^m, \\ K_0 &= \kappa_h n_h^m + \kappa_l n_l^m. \end{aligned}$$

and

$$\begin{aligned} i &= \left[(bn^m k')^\gamma - \psi\left(\widehat{k}\right)^\gamma\right]^{\frac{1}{\gamma}} = \Phi(n^m, k', k, \overline{K}, K_0) \\ \widehat{k} &= \left[\eta k^\rho + \overline{\eta}\overline{K}^\rho + (1-\eta-\overline{\eta})K_0^\rho\right]^{\frac{1}{\rho}}. \end{aligned}$$

Here we assume again the skill of workers (future managers) will be the same when they are trained by the same manager.

4.1 Social Optimum Allocation

We assume a fraction of workers with high initial skill ε is small. We have the following proposition. (Details are in the Appendix):

(a) There are only two levels of skill levels: m_h numbers of current managers have skill k_h and m_l number of current mangers have skill k_l;

(b) $m_h = \varepsilon$ and all the workers with high initial skill are trained by managers with high skill level one-to-one to acquire skill k'_h;

(c) Each managers of skill k_l trains n_l^m numbers of workers of low initial skills to acquire skill k'_l.

Thus aggregate number and skill of current managers are

$$\begin{aligned} m &= \varepsilon + n_l \\ K &= \varepsilon k_h + n_l k_l \end{aligned}$$

and $\overline{k} = K/m$. The social planner's problem is

$$V(m_l, k_h, k_l; \underline{\delta}) = Max \left\{ \begin{array}{c} \frac{\beta}{\delta} \ln c^o + \ln c^y + \delta' V(m_l', k_h', k_l'; \underline{\delta}') \\ +\lambda \left[\begin{array}{c} AK^\alpha (2-m)^{1-\alpha} - c^o - c^y \\ -\varepsilon \Phi(1, k_h', k_h, \overline{k}, \kappa_h) - m_l \Phi(n_l^m, k_l', k_l, \overline{k} n_l^m, \kappa_l n_l^m) \end{array} \right] \end{array} \right\}.$$

4.2 Competitive Equilibrium without Financing Constraint

Without financing constraint, we have a similar equilibrium as the social optimum as:

(a) There are only two levels of skill levels: m_h numbers of current managers have skill k_h and m_l number of current mangers have skill k_l;

(b) $m_h = \varepsilon$ and all the workers with high initial skill are trained by managers with high skill level one-to-one to acquire skill k_h';

(c) Each managers of skill k_l trains n_l^m numbers of workers of low initial skills to acquire skill k_l'.

Each manager with high skill k_h chooses $(n_h^n, n_h^m, w_h^m, k_h')$ to maximize the profit

$$\pi(k_h, z) = Ak^\alpha (n^n + n_h^m)^{1-\alpha} - wn^n - w_h^m n_h^m - \Phi(n_h^m, k_h', k_h, \overline{k} n_h^m, \kappa_h n_h^m),$$

subject to the constraints

$$V(w_h^m, \pi(k_h', z'); e) \geq V_h(e).$$

Without the financing constraint, this constraint becomes

$$\frac{1}{R} \left[\pi(k_h', z') - w' \right] \geq w - w_h^m. \tag{24}$$

Using Lagrangian

$$\begin{aligned} \mathcal{L}(k_h) &= Ak_h^\alpha (n^n + n_h^m)^{1-\alpha} - wn^n - w_h^m n_h^m - \Phi(n_h^m, k_h', k_h, \overline{k} n_h^m, \kappa_h n_h^m) \\ &\quad + \lambda_h \left\{ \frac{1}{R} \left[\pi(k_h', z') - w' \right] - (w - w_h^m) \right\}, \end{aligned}$$

we get first order conditions for $(n_h^n, n_h^m, w_h^m, k_h')$ as

$$(1-\alpha) A \left(\frac{k_h}{n_h} \right)^\alpha = w = w_h^m + \frac{i_h}{n_h^m} (1 + q_h s_h)$$

$$\begin{aligned} n_h^m &= \lambda_h \\ \frac{i_h}{k_h'} (1 + q_h) &= \lambda_h \frac{1}{R} \frac{\partial \pi(k_h', z')}{\partial k_h'}, \end{aligned}$$

where $q_j = \psi \left(\widehat{k}_j\right)^\gamma / (i_j)^\gamma$, $s_j = \eta \left(k_j\right)^\rho / \left(\widehat{k}_j\right)^\rho$ for $j = h, l$. Noting $n_h^m = 1$, the last two equations imply

$$\begin{aligned} \frac{i_h}{k_h'}(1+q_h) &= \frac{1}{R}\frac{\partial \pi(k_h', z')}{\partial k_h'} = \frac{1}{R}\left(\alpha \frac{Y'}{K'} + \frac{i_h'}{k_h'} q_h' s_h'\right), \text{ or} \\ i_h(1+q_h) &= \frac{1}{R}\left(\alpha \frac{Y'}{K'} k_h' + i_h' q_h' s_h'\right). \end{aligned} \tag{25}$$

Each manager with low skill k_l chooses $(n_l^n, n_l^m, w_l^m, k_l')$ to maximize the profit

$$\pi(k_l, z) = Ak^\alpha (n_l^n + n_l^m)^{1-\alpha} - wn^n - w_l^m n_l^m - \Phi(n_l^m, k_l', k_l, \overline{k} n_l^m, \kappa_l n_l^m),$$

subject to the constraints

$$V(w_l^m, \pi(k_l', z'); e) \geq V_l(e).$$

We conjecture that this constraint holds with equality and thus

$$\frac{1}{R}\left[\pi(k_l', z') - w'\right] = w - w_l^m. \tag{26}$$

Using Lagrangian

$$\begin{aligned} \mathcal{L}(k_l) &= Ak^\alpha (n_l^n + n_l^m)^{1-\alpha} - wn_l^n - w_l^m n_l^m - \Phi(n_l^m, k_l', k_l, \overline{k} n_l^m, \kappa_l n_l^m) \\ &\quad + \lambda_l \left[\frac{1}{R}\left[\pi(k_l', z') - w'\right] - (w - w_l^m)\right], \end{aligned}$$

we get first order conditions for $(n_l^n, n_l^m, w_l^m, k_l')$ as

$$(1-\alpha) A \left(\frac{k_l}{n_l}\right)^\alpha = w = w_l^m + \frac{i_l}{n_l^m}(1 + q_l s_l)$$

$$\begin{aligned} n_l^m &= \lambda_l \\ \frac{i_l}{k_l'}(1+q_l) &= \lambda_l \frac{1}{R}\frac{\partial \pi(k_l', z')}{\partial k_l'}. \end{aligned}$$

The last two equations imply

$$\frac{i_l}{n_l^m}(1+q_l) = \frac{1}{R}\left(\alpha \frac{Y'}{K'} k_l' + i_l' q_l' s_l'\right). \tag{27}$$

Also from (26), we get

$$\frac{i_l}{n_l^m}(1 + q_l s_l) = \frac{1}{R}\left(\alpha \frac{Y'}{K'} k_l' + i_l' q_l' s_l' - \frac{1-\alpha}{2-m'} Y'\right). \tag{28}$$

Denote $(c_h^y(e), c_h^{o\prime}(e))$ as the consumption of an agent with high initial skill and endowment e when young and old, and denote $(c_l^y(e), c_l^{o\prime}(e))$ as that of low initial skill. From the utility maximization condition of household, we get

$$\begin{aligned}
\frac{\beta c_h^y(e)}{c_h^{o\prime}(e)} &= \frac{\beta c_l^y(e)}{c_l^{o\prime}(e)} = \frac{1}{R} = \beta \frac{c^y}{c^{o\prime}}, \\
c_h^y(e) + \frac{c_h^{o\prime}(e)}{R} &= e + w_h^m + \frac{\pi(k_h', z')}{R} \\
c_l^y(e) + \frac{c_l^{o\prime}(e)}{R} &= e + w + \frac{w'}{R} = e + w_l^m + \frac{\pi(k_l', z')}{R}.
\end{aligned} \tag{29}$$

In the Appendix, we show that the competitive equilibrium without financing constraint achieves a very similar allocation with the social optimum aside from the effect of the externality through the skill level of the society.

4.3 Competitive Equilibrium with Financing Constraint

Once there is a financing constraint, we expect the distribution of skill is no longer degenerate and we denote the distribution of skill of current managers in equilibrium as

$$k \sim F(k), \text{ on } [k_{\min}, k_{\max}].$$

Each manager with skill k chooses $(n^n(k), n_h^m(k), n_l^m(k), w_h^m(k), w_l^m(k), k' = k^*(k))$ to maximize the profit

$$\pi(k, z) = Ak^\alpha (n^n + n_h^m + n_l^m)^{1-\alpha} - wn^n - w_h^m n_h^m - w_l^m n_l^m - \Phi(n^m, k', k, \overline{K}, K_0),$$

subject to the constraints

$$\begin{aligned}
V(w_h^m, \pi(k', z'); e) &\geq V_h(e), \\
V(w_l^m, \pi(k', z'); e) &\geq V_l(e),
\end{aligned}$$

where $V_h(e)$ and $V_l(e)$ are the competitive level of discounted utility of high and low type workers with initial endowment of e.

Consider that present manager of skill of k hires future manager with initial skill of κ_j and endowment of $e = e_j(k)$. We show the following propositions in Appendix:

(a) There is assortative matching between productivity of current managers and initial endowment of future managers of the same initial skill;

(b) Because workers with lower initial skill are more costly to acquire skill, they need to have higher initial endowment to be trained.

Thus we have the property of equilibrium such that

$$\begin{aligned} e_j'(k) &> 0, \text{ for } j = h, l \\ e_h(k_{\max}) &= \bar{e}, \\ e_h(k_{\min}) &= e_h^* < \bar{e}, \\ e_l(k_l^*) &= \bar{e}, \text{ where } k_l^* \in (k_{\min}, k_{\max}), \\ e_l(k_{\min}) &= e_l^* > e_h^*. \end{aligned}$$

Thus we obtain the property:

(a) current manager $k \in (k_l^*, k_{\max}]$ trains $n_h^m(k)$ number of workers with initial endowment of $e_h(k)$ and high initial skill, offering package of $(w_h^m, k') = (w_h^m(k), k_h^*(k))$.

(b) current manager $k \in [k_{\min}, k_l^*]$ either trains $n_h^m(k)$ number of high-type workers with initial endowment of $e_h(k)$, offering package of $(w_h^m, k') = (w_h^m(k), k_h^*(k))$, or,

trains $n_l^m(k)$ number of low-type workers with initial endowment of $e_l(k)$, offering package of $(w_l^m, k') = (w_l^m(k), k_l^*(k))$.

In the next period, the number of managers, aggregate skill and distribution of skill becomes

$$\begin{aligned} m' &= \omega \left[1 - \varepsilon \frac{e_h^*}{\bar{e}} - (1-\varepsilon) \frac{e_l^*}{\bar{e}} \right], \\ K' &= m'\overline{k}' = \int_{e_h^*}^{\bar{e}} Pr_h(k) n_h^m(k^h(e)) k_h(k^h(e)) dG(e) + \int_{e_l^*}^{\bar{e}} Pr_l(k) n_l^m(k^l(e)) k_l(k^l(e)) dG(e) \\ F_{t+1}(k') &= \int_{k_{\min}}^{k_h^{-1}(k')} Pr_h(k) n_h^m(k) dF(k) + \int_{k_{\min}}^{k_l^{-1}(k')} Pr_l(k) n_l^m(k) dF(k), \end{aligned}$$

where $\Pr_j(k)$ denotes the probability that the manager with skill k hires workers of initial skill j, and $n_j^m(k)$ is the number of j-type workers manager k hires conditional on hiring such workers. Also $k_h^{-1}(k') = k_{\min}$ if $k' \leq k_h(k_{\min})$, and $k_h^{-1}(k') = k_{\max}$ if $k' \geq k_h(k_{\max})$ and similar for $k_l^{-1}(k')$.

initial skill of talented κ_h	1.1
initial skill of less talented κ_l	1
fraction of talented ε	0.36

Table 2: Additional parameter values used in the full model simulation.

4.4 Results

4.4.1 Parametrization

We use the same parametrization as in the basic model in the numerical simulation. Values for additional parameters of the full model are listed in Table 2. The heterogeneity across agents is assumed to be small. The talented are endowed with 10% more skill than the less talented. 36% of the population is talented.

4.4.2 Inequality and Misallocation in the Steady State

In the full model with skill heterogeneity, when the financing constraint is not binding, there are two income levels in terms of the present value, a lower level for the workers not receiving training and less talented workers receiving training, and a higher level for talented workers receiving training.

With financing constraint, sorting between rich young trainees and productive managers shows up as in the Basic Model. In addition, allocation between the initial skill and training is distorted relative the social optimal. In Figure 10, the most productive group of managers ($k \geq k_l^* = 79.5$) only train rich and talented young agents. The group of less productive managers ($k \leq k_l^* = 79.5$) train either rich and less talented young or poorer but talented young workers. Although talented workers are more likely to be trained, the poorest talented workers are not trained.

Figure 11 further shows distortion at the extensive and intensive margins. When the group of less productive managers ($k \leq k_l^* = 79.5$) train either poorer but talented young or rich and less talented young workers, they train more extensively talented workers ($n_h^m(k) > n_l^m(k)$) less intensively ($k' = k_h^*(k) < k_l^*(k)$) than less talented workers. This misallocation arises because the group of less productive managers trains talented but poor young workers, who prefer to get paid earlier in life-cycle instead of receiving better training due to the financing constraint.

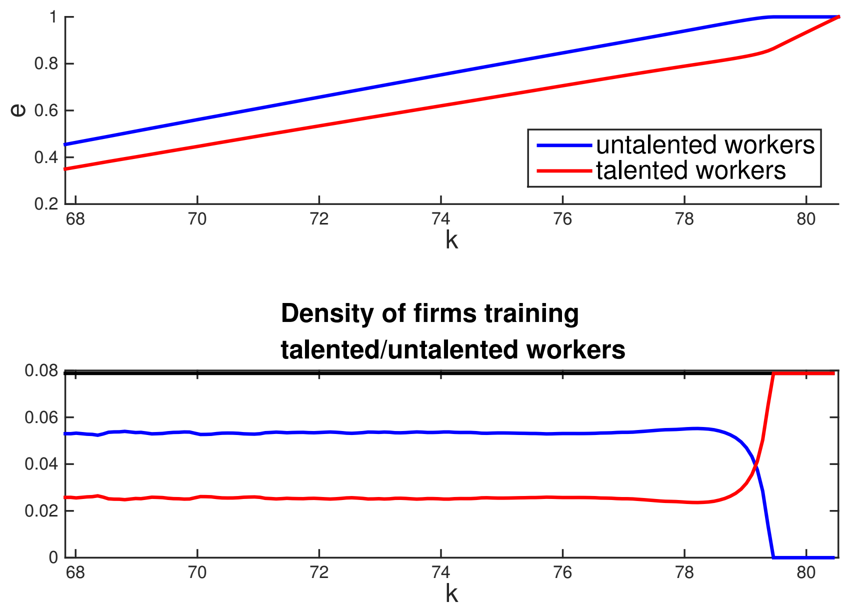

Figure 10: Sorting between trainees and managers.

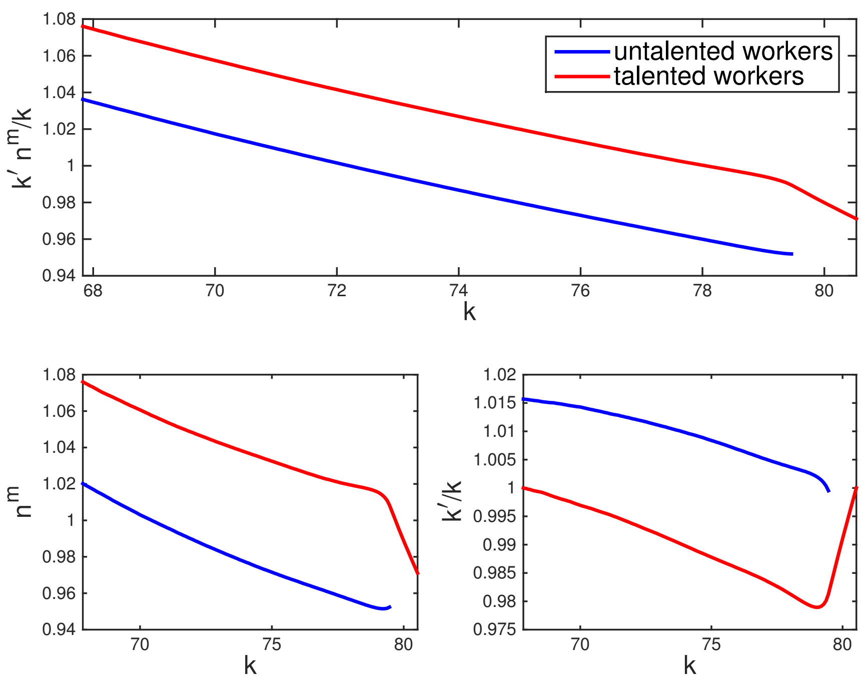

Figure 11: Training in the full model.

5 Endogenous Wealth Distribution through Bequest

5.1 Set-up

So far, our analysis is based on exogenous endowment distribution. In this section, we endogenize the endowment distribution by allowing bequest. The utility function of agent born at date t is given by

$$U_t = U(c_t^y, c_{t+1}^o, b_{t+1}) = \ln c_t^y + \beta \left[(1-\delta) \ln c_{t+1}^o + \delta \ln b_{t+1} \right],$$

where c_t^y and c_{t+1}^o are consumption when young at date t and when old at date t+1, b_{t+1} is the bequest to their children, $\beta \in (0,1)$ is a utility discount factor and δ captures the preference for bequest.

Denote the indirect utility function of a trader to be $V_t(y_t, y_{t+1}, b_t; \delta)$. Regardless of whether the agent is financially constrained or not, we can show that

$$V_t(y_t, y_{t+1}, b_t; \delta) = V_t(y_t, y_{t+1}, b_t; 0) + \beta \left[\delta \ln \delta + (1-\delta) \ln(1-\delta) \right], \tag{30}$$

$$c_{t+1}^o / b_{t+1} = (1-\delta)/\delta, \tag{31}$$

$$c_t^y / (c_{t+1}^o + b_{t+1}) \text{ is independent of } \delta. \tag{32}$$

The proof is in section A.3 of the Appendix. Because of (30) and (32), the equilibrium characterization on the consumption profile, $(c_t^y, c_{t+1}^o + b_{t+1})$, does not depend on δ. Because of (31), the law of motion of bequest is $b_{t+1} = \delta(c_{t+1}^o + b_{t+1})$. Further, we assume that δ is an i.i.d. draw from a distribution.

With this setup, we can show that bequest does not affect the first order conditions of the manager's problem and workers' occupational choice. Then, it is easy to extend the basic and full model to allow bequest. The results are robust and qualitatively similar. The additional result from this extension is that bequest is persistent over time, which implies reduced social mobility because of wealth inequality and financial constraint.

A Appendix

A.1 Social Optimum in Basic Model

Defining λ as the Lagrangian multiplier, we get the Lagrangian as

$$V^s(m,k;\underline{\delta}) = \underset{n^m,k',c^o,c^y}{Max} \left\{ \begin{array}{c} \frac{\beta}{\delta}\ln c^o + \ln c^y + \delta' V^s(n^m m, k'; \underline{\delta}') \\ +\lambda\left[A(mk)^\alpha(2-m)^{1-\alpha} + e^a - c^o - c^y - m\Phi\left(n^m, k', k, \overline{k}, \kappa n^m\right)\right] \end{array} \right\}.$$

The first order condition for consumption is

$$\lambda = \frac{1}{c^y} = \frac{\beta}{\delta}\frac{1}{c^o}. \tag{33}$$

The first order conditions for (n^m, k') are

$$\begin{aligned} \delta' \frac{\partial V^s(m',k';\underline{\delta})}{\partial m'} &= \lambda\left[\frac{\partial \Phi}{\partial n^m} + \frac{\partial \Phi}{\partial \overline{K}}\bar{k} + \frac{\partial \Phi}{\partial K_0}\kappa\right] \\ &= \lambda \frac{i}{n^m}\left[1 + q - q\bar{s} - q(1-s-\bar{s})\right] = \lambda\frac{i}{n^m}(1+qs), \end{aligned} \tag{34}$$

$$\delta' \frac{\partial V^s(m',k';\underline{\delta})}{\partial k'} = \lambda m \frac{\partial \Phi}{\partial k'} = \lambda m \frac{i}{k'}(1+q), \tag{35}$$

From the envelope theorem, we get

$$\begin{aligned} \frac{\partial V^s(m,k;\underline{\delta})}{\partial m} &= \lambda\left[\alpha\frac{Y}{m} - (1-\alpha)\frac{Y}{2-m} - i\right] + n^m\delta'\frac{\partial V^s(m',k')}{\partial m'} \\ &= \lambda\left[\left(\frac{\alpha}{m} - \frac{1-\alpha}{2-m}\right)Y + iqs\right],\ (\because (34)), \end{aligned}$$

$$\begin{aligned} \frac{\partial V^s(m,k;\underline{\delta})}{\partial k} &= \lambda\left[\alpha\frac{Y}{k} - m\left(\frac{\partial\Phi}{\partial k} + n^m\frac{\partial \Phi}{\partial \overline{K}}\right)\right] \\ &= \lambda\left[\alpha\frac{Y}{k} + mi\left(\frac{qs}{k} + n^m\frac{q\bar{s}}{\overline{K}}\right)\right] \\ &= \lambda\left[\alpha\frac{Y}{k} + m\frac{i}{k}q(s+\bar{s})\right]. \end{aligned}$$

Substituting these into (34), we get

$$\begin{aligned} \frac{i}{n^m}(1+qs) &= \frac{\delta'\lambda'}{\lambda}\left[\left(\frac{\alpha}{m'} - \frac{1-\alpha}{2-m'}\right)Y' + i'q's'\right] \\ &= \frac{\beta c^y}{c^{o\prime}}\left[\left(\frac{\alpha}{m'} - \frac{1-\alpha}{2-m'}\right)Y' + i'q's'\right]. \end{aligned} \tag{36}$$

using $\delta'\lambda'/\lambda = (\beta/c^{o\prime})/(1/c^y) = \beta c^y/c^{o\prime}$ where $c^{o\prime}$ is consumption of present young when he or she gets old in the next period. For the first order condition of future skill (35), we get

$$\lambda m \frac{i}{k'}(1+q) = \delta'\lambda'\left[\alpha\frac{Y'}{k'} + m'\frac{i'}{k'}q'(s'+\overline{s}')\right], \text{ or}$$
$$\frac{i}{n^m}(1+q) = \frac{\beta c^y}{c^{o\prime}}\left[\alpha\frac{Y'}{m'} + i'q'(s'+\overline{s}')\right]. \tag{37}$$

The relationship between m and m' is given by

$$m' = n^m m \tag{38}$$

The resource constraint implies

$$Y + e^a = c^o + c^y + mi. \tag{39}$$

The allocation of unconstrained social optimum is given by ten variables $(i, Y, c^o, c^y, q, s, \overline{s}, n^m, k', m')$ as a function of state variable (m, k) that satisfy ten equations $(1, 2, 33, 7, 8, 9, 36, 37, 38, 39)$ together with $\overline{k} = k$.

(12) together with (13) in the text is equivalent to the social optimal condition (36). Through the choice of whether to train workers or not, the competitive equilibrium through the dynamic coalition achieve the social optimal condition for the choice of number of future managers.

By comparing (10) and (13) in the text with (37), we learn the competitive equilibrium without financing constraint is similar to the social optimum, except that it fails to take into account the marginal benefit of future skill through the externality $\overline{k}'$. The ratio of consumption between young and old is equal between the competitive equilibrium and social planner's economy if we choose δ to satisfy

$$\frac{c^y}{c^o} = \frac{\delta}{\beta} = \frac{\frac{1-\alpha}{2-m}Y - mi\,(1+qs) + e^a}{\left[(1-\alpha)\frac{1-m}{2-m} + \alpha\right]Y + miqs}.$$

Thus aside from the effect of externality through $\overline{k}'$, a competitive equilibrium achieves a very similar allocation with a particular social optimal allocation through the dynamic coalition when there is no financial friction.

A.2 Full Model

A.2.1 Social Optimum of Full Model

The first order conditions for consumption is

$$\lambda = \frac{\beta}{\delta}\frac{1}{c^o} = \frac{1}{c^y}. \tag{40}$$

The first order condition for n_l^m is

$$\delta' \frac{\partial V(m_l', k_h', k_l'; \underline{\delta}')}{\partial m_l'} = \lambda \frac{i_l}{n_l^m}\left(1 + q_l s_l\right), \tag{41}$$

where $q_l = \psi\left(\widehat{k}_l\right)^{\gamma}/(i_l)^{\gamma}$, $s_l = \eta(k_l)^{\rho}/\left(\widehat{k}_l\right)^{\rho}$ and $\overline{s}_l = \overline{\eta}(\overline{k})^{\rho}/\left(\widehat{k}_l\right)^{\rho}$. The envelope theorem implies

$$\begin{aligned}\frac{\partial V(m_l, k_h, k_l; \underline{\delta})}{\partial m_l} &= \lambda\left[\left(\frac{\alpha k_l}{K} - \frac{1-\alpha}{2-m}\right)Y - i_l\right] + n_l^m \delta' \frac{\partial V(m_l', k_h', k_l'; \underline{\delta}')}{\partial m_l'} \\ &= \lambda\left[\left(\frac{\alpha k_l}{K} - \frac{1-\alpha}{2-m}\right)Y + i_l q_l s_l\right].\end{aligned}$$

Together with the first order condition for n_l^m, we get

$$\frac{i_l}{n_l^m}\left(1 + q_l s_l\right) = \frac{\beta c^y}{c^{o\prime}}\left[\left(\frac{\alpha k_l'}{K'} - \frac{1-\alpha}{2-m'}\right)Y' + i_l' q_l' s_l'\right],$$

using $\frac{\delta'\lambda'}{\lambda} = \frac{\beta c^y}{c^{o\prime}}$ by (40). The first order condition for k_h' is

$$\delta' \frac{\partial V(m_l', k_h', k_l'; \underline{\delta}')}{\partial k_h'} = \lambda \varepsilon \frac{i_h}{k_h'}(1 + q_h).$$

The envelope theorem implies

$$\begin{aligned}\frac{\partial V(m_l, k_h, k_l; \underline{\delta})}{\partial k_h} &= \lambda\left[\frac{\alpha\varepsilon}{K}Y + \varepsilon\frac{i_h}{k_h}q_h(s_h + \overline{s}_h\chi_h)\right], \text{ where} \\ \chi_h &= \frac{\varepsilon k_h}{\varepsilon k_h + m_l k_l}.\end{aligned}$$

Together with the first order condition for k_h', we have

$$i_h(1 + q_h) = \frac{\beta c^y}{c^{o\prime}}\left[\frac{\alpha Y'}{K'}k_h' + i_h' q_h'(s_h' + \overline{s}_h'\chi_h')\right]. \tag{42}$$

Similarly the first order condition for k_l' becomes

$$\frac{i_l}{n_l^m}(1 + q_l) = \frac{\beta c^y}{c^{o\prime}}\left[\frac{\alpha Y'}{K'}k_l' + i_l' q_l'(s_l' + \overline{s}_l'\chi_l')\right], \tag{43}$$

where $\chi_l = \frac{m_l k_l}{\varepsilon k_h + m_l k_l}$. The resource constraint is

$$Y + e^a = AK^\alpha(2-m)^{1-\alpha} + e^a = c^o + c^y + \varepsilon i_h + m_l i_l.$$

The allocation of unconstrained social optimum is given by $(i_h, i_l, Y, c^o, c^y, q_h, q_l, s_h, \bar{s}_h, s_l, \bar{s}_l, n_l^m, k_h', k_l', m_l')$ as a function of state variable (m_l, k_h, k_l) that satisfies the equilibrium conditions.

A.2.2 Competitive Equilibrium without Financing Constraint

(28) is equivalent with (41) in the social planner's condition. (25, 27) are comparable with (42, 42) of the social planner's problem, except that they fail to take into account the external effect through $\overline{k}$. Therefore, the competitive equilibrium without the financing constraints achieve a very similar allocation with a particular social planner's economy.

A.2.3 Competitive Equilibrium with Financing Constraint

Using the Lagrangian

$$\begin{aligned}\mathcal{L}(k) &= Ak^\alpha(n^n + n_h^m + n_l^m)^{1-\alpha} - wn^n - w_h^m n_h^m - w_l^m n_l^m - \Phi(n^m, k', k, \overline{K}, K_0) \\ &\quad + \lambda_h(k)\left[V(w_h^m, \pi(k', z'); e) - V_h(e)\right] \\ &\quad + \lambda_l(k)\left[V(w_l^m, \pi(k', z'); e) - V_l(e)\right],\end{aligned}$$

the first order condition (FOC) for n^n is

$$\begin{aligned}(1-\alpha)Ak^\alpha n^{-\alpha} &= w, \text{ or} \\ n &= \left[(1-\alpha)\frac{A}{w}\right]^{\frac{1}{\alpha}} k. \quad (44)\end{aligned}$$

The FOC for n_j^m for $j = h, l$ is

$$\begin{aligned} w &\leq w_j^m + \frac{\partial \Phi}{\partial n^m} + \bar{k}\frac{\partial \Phi}{\partial \overline{K}} + \kappa_j \frac{\partial \Phi}{\partial K_0}, \text{ and} \quad (45) \\ 0 &= n_j^m \left[w_j^m + \frac{\partial \Phi}{\partial n^m} + \bar{k}\frac{\partial \Phi}{\partial \overline{K}} + \kappa_j \frac{\partial \Phi}{\partial K_0} - w\right].\end{aligned}$$

The FOC for w_j^m is

$$n_j^m = \lambda_j(k)\frac{\partial V(w_j^m, \pi(k', z'); e)}{\partial w_j^m}. \quad (46)$$

and the FOC for k' is

$$\frac{\partial \Phi}{\partial k'} = \left[\lambda_h(k)\frac{\partial V(w_h^m, \pi(k',z');e)}{\partial \pi(k',z')} + \lambda_l(k)\frac{\partial V(w_l^m, \pi(k',z');e)}{\partial \pi(k',z')}\right]\frac{\partial \pi(k',z')}{\partial k'}.$$

Putting the last two equations together, we get

$$\frac{\partial \Phi}{\partial k'} = \left[n_h^m \frac{\frac{\partial V(w_h^m,\pi(k',z');e)}{\partial \pi(k',z')}}{\frac{\partial V(w_h^m,\pi(k',z');e)}{\partial w_h^m}} + n_l^m \frac{\frac{\partial V(w_l^m,\pi(k',z');e)}{\partial \pi(k',z')}}{\frac{\partial V(w_l^m,\pi(k',z');e)}{\partial w_l^m}}\right]\frac{\partial \pi(k',z')}{\partial k'}. \quad (47)$$

We know as before

$$\begin{aligned}
\frac{\partial \Phi}{\partial n^m} &= \frac{w}{n^m(k)}(1+q(k))X(k), \\
\frac{\partial \Phi}{\partial \overline{K}} &= -\frac{w}{\overline{K}(k)}q(k)\overline{s}(k)X(k), \\
\frac{\partial \Phi}{\partial K_0} &= -\frac{w}{K_0(k)}q(k)(1-s(k)-\overline{s}(k))X(k), \\
\frac{\partial \Phi}{\partial k'} &= \frac{w}{k'}(1+q(k))X(k),
\end{aligned}$$

where

$$\begin{aligned}
X(k) &= \frac{\Phi(k)}{w}, \\
q(k) &= \frac{\psi \widehat{k}(k)^\gamma}{i(k)^\gamma}, \\
s(k) &= \frac{\eta k^\rho}{\eta k^\rho + \overline{\eta}\overline{K}^\rho + (1-\eta-\overline{\eta})K_0(k)^\rho}, \\
\overline{s}(k) &= \frac{\overline{\eta}\overline{k}^\rho}{\eta k^\rho + \overline{\eta}\overline{K}^\rho + (1-\eta-\overline{\eta})K_0(k)^\rho}.
\end{aligned}$$

In equilibrium, we know as before

$$\begin{aligned}
w &= (1-\alpha)AK^\alpha N^{-\alpha} \\
&= (1-\alpha)A\left(\frac{m\overline{k}}{2-m}\right)^\alpha
\end{aligned}$$

Thus from (44), we have

$$\begin{aligned}
y(k) &= A\left(\frac{2-m}{m\overline{k}}\right)^{1-\alpha} k \\
&= w\frac{1}{1-\alpha}\frac{2-m}{m}\frac{k}{\overline{k}}
\end{aligned}$$

Thus from (46) , we have

$$w_j^m(k) \geq w\left\{1 - [1+q(k)]\frac{X(k)}{n^m(k)} + q(k)[1-s(k)-\overline{s}(k)]\frac{\kappa_j}{K_0(k)}X(k) + q\overline{s}(k)\frac{\bar{k}}{\bar{K}}X(k)\right\},$$

and

$$\begin{aligned}\pi(k,z) &= y(k) - wn(k) + [w - w_h^m(k)]n_h^m(k) + [w - w_l^m(k)]n_l^m(k) - \Phi \\ &= w\left\{\frac{\alpha}{1-\alpha}\frac{2-m}{m}\frac{k}{\bar{\bar{k}}} + q(k)s(k)X(k)\right\},\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi(k,z)}{\partial k} &= \alpha\frac{y(k)}{k} - \frac{\partial \Phi}{\partial k} \\ &= \frac{w}{k}\left[\frac{\alpha}{1-\alpha}\frac{2-m}{m}\frac{k}{\bar{\bar{k}}} + q(k)s(k)X(k)\right].\end{aligned}$$

Define $k^j(e) = e_j^{-1}(k)$. (NOTE $k = k^j(e)$ is different from $k' = k_j(k)$.) The discounted utility of future manager with initial skill of κ_j and endowment of e is given by

$$V_j(e) = \ln\left[e + w_j^m(k^j(e)) + \frac{\theta}{R}\pi\left(k_j\left(k^j(e)\right), z'\right)\right] + \beta\ln[(1-\theta)\pi\left(k_j\left(k^j(e)\right), z'\right)].$$

Thus we have

$$\frac{\frac{\partial V_j(e)}{\partial \pi(k_j(k^j(e)),z')}}{\frac{\partial V_j(e)}{\partial w_j^m(k^j(e))}} = (1+\beta)\frac{\theta}{R} + \beta\frac{e + w_j^m(k^j(e))}{\pi\left(k_j\left(k^j(e)\right), z'\right)}$$

Thus for the manager who hires only j type of workers ($n_j^m(k) > 0$, and $n_{j'}^m(k) = 0$ for $j\prime \neq j$), the condition (47) becomes

$$\frac{w'}{w}\frac{\frac{\partial V_j(e)}{\partial \pi(k_j(k^j(e)),z')}}{\frac{\partial V_j(e)}{\partial w_j^m(k_j(e))}} = \frac{\frac{1}{n_j^m(k)}\frac{\partial \Phi}{\partial k'}/w}{\frac{\partial \pi(k_j(k^j(e)),z')}{\partial k'}/w'},$$

or

$$\begin{aligned}&(1+\beta)\theta\frac{w'}{Rw} + \beta\frac{\frac{e_j(k)}{w} + 1 - [1+q(k)(s(k)]\frac{X(k)}{n_j^m(k)}}{\frac{\alpha}{1-\alpha}\frac{2-m'}{m'}\frac{k_j(k)}{\bar{\bar{k}}'} + q'(k_j(k)s'(k_j(k))X'(k_j(k))} \\ = \;& \frac{(1+q(k))\frac{X(k)}{n_j^m(k)}}{\frac{\alpha}{1-\alpha}\frac{2-m'}{m'}\frac{k_j(k)}{\bar{\bar{k}}'} + q'(k_j(k))s'(k_j(k))X'(k_j(k))}. \qquad (48)\end{aligned}$$

e_h^* and e_l^* satisfy the indifference conditions as

$$\begin{aligned} V_h(e_h^*) &= (1+\beta)\ln\left[\frac{1}{1+\beta}\left(\frac{e_h^*}{w}+1+\frac{w'}{Rw}\right)\right]+\beta\ln(\beta R) \\ &= \beta\ln\left[(1-\theta)\frac{w'}{w}\frac{\pi\left(k_h(k_{\min}),z'\right)}{w'}\right] \\ &\quad +\ln\left\{\frac{e_h^*}{w}+1-\left[1+q(k_{\min})s(k_{\min})\right]\frac{X(k_{\min})}{n_h^m(k_{\min})}+\frac{\theta}{R}\frac{w'}{w}\frac{\pi\left(k_h(k_{\min}),z'\right)}{w'}\right\} \end{aligned} \tag{49}$$

and

$$\begin{aligned} V_l(e_l^*) &= (1+\beta)\ln\left[\frac{1}{1+\beta}\left(\frac{e_l^*}{w}+1+\frac{w'}{Rw}\right)\right]+\beta\ln(\beta R) \\ &= \beta\ln\left[(1-\theta)\frac{w'}{w}\frac{\pi\left(k_l(k_{\min}),z'\right)}{w'}\right] \\ &\quad +\ln\left\{\frac{e_l^*}{w}+1-\left[1+q(k_{\min})s(k_{\min})\right]\frac{X(k_{\min})}{n_l^m(k_{\min})}+\frac{\theta}{R}\frac{w'}{w}\frac{\pi\left(k_l(k_{\min}),z'\right)}{w'}\right\}. \end{aligned} \tag{50}$$

For the current manager $k\in[k_{\min},k_l^*]$, the profit from training $n_h^m(k)$ number of high-type workers by offering $(w_h^m,k')=(w_h^m(k),k_h(k))$ is the same as the profit from training $n_l^m(k)$ number of low-type workers by offering $(w_l^m,k')=(w_l^m(k),k_l(k))$, iff

$$\begin{aligned} q(k)s(k)X(k) &= \frac{\psi\widehat{k}^{\gamma}}{i^{\gamma}}\frac{\eta k^{\rho}}{\widehat{k}^{\rho}}\frac{i}{w} \\ &= \frac{\psi}{w}\eta k^{\rho}\left[(bn^m k')^{\gamma}-\psi\widehat{k}^{\gamma}\right]^{\frac{1-\gamma}{\gamma}}\widehat{k}^{\gamma-\rho} \end{aligned}$$

is the same between the two training strategies. This is equivalent to

$$\begin{aligned} &\left\{\frac{\left[bn_h^m(k)k_h(k)\right]^{\gamma}-\psi\left[\eta k^{\rho}+\overline{\eta}\left(\overline{k}n_h^m(k)\right)^{\rho}+(1-\eta-\overline{\eta})\left(\kappa_h n_h^m(k)\right)^{\rho}\right]^{\frac{\gamma}{\rho}}}{\left[bn_l^m(k)k_l(k)\right]^{\gamma}-\psi\left[\eta k^{\rho}+\overline{\eta}\left(\overline{k}n_l^m(k)\right)^{\rho}+(1-\eta-\overline{\eta})\left(\kappa_l n_l^m(k)\right)^{\rho}\right]^{\frac{\gamma}{\rho}}}\right\}^{\frac{1-\gamma}{\gamma}} \\ = &\left\{\frac{\eta k^{\rho}+\overline{\eta}\left(\overline{k}n_l^m(k)\right)^{\rho}+(1-\eta-\overline{\eta})\left(\kappa_l n_l^m(k)\right)^{\rho}}{\eta k^{\rho}+\overline{\eta}\left(\overline{k}n_h^m(k)\right)^{\rho}+(1-\eta-\overline{\eta})\left(\kappa_h n_h^m(k)\right)^{\rho}}\right\}^{\frac{\gamma-\rho}{\rho}}. \end{aligned}$$

We assume that $\gamma>\rho$, i.e., substitutability between k, $\overline{k}$ and K_0 in skill composite $\widehat{k}$ is higher than the substitutability between goods input i and skill composite $\widehat{k}$ for training. Because $\kappa_l<\kappa_h$, we guess

$$\kappa_l n_l^m(k)<\kappa_h n_h^m(k).$$

Then from the above equality, we learn that the total investment cost is lower when the manager $k \in [k_{\min}, k_l^*]$ trains the high-type workers:

$$i_h < i_l.$$

We also conjecture

$$k_h(k) > k_l(k).$$

From the market clearing condition for funds, we have

$$\beta = \frac{w'}{wR}\Omega, \text{ where}$$

$$\Omega \equiv \frac{(1-m') + (1+\beta)\theta \int_{k_{\min}}^{k_{\max}} [Pr_h(k) n_h^m(k) \pi(k_h(k), z') + Pr_l(k) n_l^m(k) \pi(k_l(k), z')] dF(k)}{(1-m') + \omega\varepsilon \frac{e_h^*}{\bar{e}} \frac{e_h^*}{2w} + \omega(1-\varepsilon) \frac{e_l^*}{\bar{e}} \frac{e_l^*}{2w}}. \tag{51}$$

A.3 Bequest and Endogenous Wealth Distribution

When the financing constraint is not binding,

$$V_t(y_t, y_{t+1}, b_t) = \max_{c_t^y, c_{t+1}^o, b_{t+1}} \ln c_t^y + \beta \left((1-\delta) \ln c_{t+1}^o + \delta \ln b_{t+1} \right),$$
$$\text{s.t.} c_t^y + \frac{1}{R_t} \left(c_{t+1}^o + b_{t+1} \right) = b_t + y_t + \frac{y_{t+1}}{R_t}.$$

The first oder conditions for the problem include

$$\frac{1}{c_t^y} = \lambda$$
$$\frac{(1-\delta)\beta}{c_{t+1}^o} = \frac{1}{R_t}\lambda$$
$$\frac{\delta\beta}{b_{t+1}} = \frac{1}{R_t}\lambda$$

where λ refers to the Lagrangian multiplier for the budget constraint. Then

$$\frac{b_{t+1}}{c_{t+1}^o} = \frac{\delta}{1-\delta},$$
$$\frac{b_{t+1} + c_{t+1}^o}{c_t^y} = \beta R_t,$$

$$V_t(y_t, y_{t+1}, b_t) = \ln\left(\frac{b_t + y_t + \frac{y_{t+1}}{R_t}}{1+\beta R_t}\right) + \beta \ln\left(\frac{R_t\left(b_t + y_t + \frac{y_{t+1}}{R_t}\right)}{1+\beta R_t}\right) + \beta\left[\delta \ln \delta + (1-\delta)\ln(1-\delta)\right]$$

When the financing constraint is binding,

$$V_t(y_t, y_{t+1}, b_t) = \max_{c^o_{t+1}, b_{t+1}} \ln\left(y_t + b_t + \frac{\theta y_{t+1}}{R_t}\right) + \beta\left[(1-\delta)\ln c^o_{t+1} + \delta \ln b_{t+1}\right],$$
$$\text{s.t.} c^o_{t+1} + b_{t+1} = (1-\theta) y_{t+1}.$$

The first oder conditions for the problem include

$$\lambda = \frac{\beta(1-\delta)}{c^o_{t+1}} = \frac{\beta\delta}{b_{t+1}},$$

where λ refers to the Lagrangian multiplier for the budget constraint. Then

$$V_t(y_t, y_{t+1}, b_t) = \ln\left(y_t + b_t + \frac{\theta y_{t+1}}{R_t}\right) + \beta \ln\left((1-\theta) y_{t+1}\right) + \beta\left[\delta \ln \delta + (1-\delta)\ln(1-\delta)\right]$$

Compared with the model without the preference for bequest, the indirect utility function is only different by a constant. From the first order conditions, it is easy to confirm other statements in Section 5.1.

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