STICKY PRICES AND COSTLY CREDIT

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Abstract

We construct a framework where money and credit are alternative payment instruments, use it to endogenize nominal price stickiness, and confront the data. Frictions generate equilibria with price dispersion, where sellers set nominal terms that some keep fixed when aggregate conditions change. Buyers use cash and credit, with the former (latter) subject to inflation (transaction) costs. We provide strong analytic results and closed-form solutions for money demand. The calibrated model matches price-change data well, generating realistic durations, large average price changes, plus many small and many negative changes, while staying consistent with macro and micro data on money and credit. Policy implications are discussed.

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1 Introduction

This paper has two related goals: (i) construct a framework where money and credit serve as alternative payment instruments; (ii) pursue within this setting a theory of endogenously sticky prices that we can take to the data. It builds on the analysis of price dispersion in frictional goods markets by Burdett and Judd (1983), integrated into the general equilibrium monetary model of Lagos and Wright (2005). This environment generates monetary equilibria with a distribution of prices, and that means sellers can set prices in nominal terms that they may keep fixed when aggregate conditions change. Buyers may use both cash and credit because the former is subject to the inflation tax while the latter involves a fixed or variable transaction cost, making the choice of payment method nontrivial and, we think, realistic. Also, introducing costly credit allows us to avoid an indeterminacy of equilibrium than plagues some similar models, as we now explain.

To begin, consider Diamond (1971), where each seller (firm) of an indivisible good posts a price $p$, then buyers (households) sample sellers one at a time until finding one below their reservation price $p^*$. Clearly, for any seller, the profit maximizing strategy is $p = p^*$. Hence, there is a single price in the market. The Burdett-Judd model makes one change in Diamond’s specification: buyers sometimes sample multiple prices simultaneously, and when they do they obviously choose the lowest. This implies there cannot be a single $p$, nor even a set of sellers with positive measure charging the same $p$, since that would leave open a profitable deviation to $p - \varepsilon$ (see fn. 10). In fact, one can compute explicitly the Burdett-Judd distribution $F(p)$, where any $p$ in the support $\mathcal{F}$ yields the same profit: lower-price sellers earn less per unit, but make it up on the volume, by selling with higher probability.

If one embeds Burdett-Judd in a monetary economy, it makes sense for sellers to post prices in dollars. Then, if the money supply $M$ increases, $F(p)$ shifts so that the real distribution stays the same. Some firms can keep $p$ fixed after $M$ and $\mathbb{E}p$ increase, however, and hence we get sticky prices, according to the usage adopted
here. This is so even though sellers are allowed to adjust whenever they like at no cost. For sellers that stick to their nominal prices, real prices fall, but profits do not, since sales increase. This is similar to Head et al. (2012), but that paper has a technical problem: as discussed in Section 2, the combination of indivisible goods and posting in monetary economies entails an indeterminacy (i.e., a continuum) of stationary equilibria. Hence, that model uses divisible goods, but then another problem pops up: it is not obvious what firms should post. That paper simply assumes linear menus – a seller sets \( p \) and lets buyers choose any \( q \) as long as they pay \( pq \) – but there is no reason to think that this maximizes profit.

Incorporating costly credit eliminates the indeterminacy that obtains with indivisible goods and posting, as discussed in Section 2, so we can avoid ad hoc restrictions like linear pricing. Intuitively, holding more cash reduces the probability of using credit. This delivers a well-behaved money demand function, and hence a unique stationary monetary equilibrium. Thus we can revert to the original Diamond-Burdett-Judd specification, with indivisible goods, and get by with fewer “delicate” assumptions.\(^1\) Moreover, in contrast to theories where stickiness is imposed exogenously, changing \( M \) in our model is neutral – i.e., it has no affect real output. To be clear, the point is not that neutrality holds in reality; it is that nominal stickiness does not logically imply nonneutrality. While we like this point, there are reasons other than making it to integrate money and credit: (i) there is a long tradition of trying to build models along these lines (see fn. 8); (ii) it is relevant for monetary policy analysis because it allows substitution between cash and credit as this policy changes (Section 8); and (iii) it allows us to confront the data in novel ways (Section 7).

In terms of confronting data, Head et al. (2012) calibrate the model to match some features of price-change behavior, then show it accounts for other features, too. We perform a similar, but more disciplined, exercise, calibrating to some features of

\(^1\)We do not take a stand on whether divisible or indivisible goods are more “realistic,” as that depends on the context, but we would certainly argue that indivisibility is an assumption on the physical environment and hence less “delicate” than a restriction on pricing strategies.
pricing behavior, plus the fractions of cash and credit in the micro data and macro observations on money demand (the relation between real balances and interest rates), then check how the model matches other observations. One specification involves a fixed-cost of credit. It performs well in terms of money demand and can match key facts in the pricing data, including long price durations, large average price changes, many small or negative changes, and a decreasing hazard – but it cannot match these plus the shares of cash and credit in micro data. Then we consider a variable-cost model, which is in some respects more flexible. It can simultaneously match the pricing, money demand and payment data.

Section 2 reviews the literature. Section 3 describes our environment. Sections 4 and 5 characterize equilibrium with a fixed and a variable cost of credit, respectively. Section 6 discusses stickiness. Section 7 presents the quantitative analysis. Section 8 discusses an extension and policy implications. Section 9 concludes.

2 Literature

Many sticky-price papers follow Taylor (1980) or Calvo (1983) by letting sellers adjust \( p \) only at certain points in time. Others follow Rotemberg (1982) or Mankiw (1985) by letting them change only at a cost. We allow sellers to change any time for free, but sometimes they choose not to. A few papers push imperfect-information or rational-inattention theories, including Mackowiak and Wiederhold (2009), who provide references to other work. While we are not against these devices, the concentration here is on search, mainly because when Burdett and Menzio (2014) combine this with menu costs, the analysis is more complicated and delivers similar results, and because they find the majority of price dispersion in the data (about 70\%) is due to search. Hence we abstract from menu costs and related devices.\(^3\)

\(^2\)While the model in Head et al. (2012) can match the price-change distribution and average duration, it does not match macro money demand data at all well, and cannot match the shares of money and credit in the micro payment data at all.

\(^3\)Nonmonetary search models with menu costs, where prices are sticky in unit of account, as in Benabou (1988,1992) or Diamond (1993), are special cases of Burdett and Menzio (2014).
The literature on Burdett-Judd pricing is large, including many labor-market applications following Burdett and Mortensen (1998). In monetary economics, prior to Liu (2010), Wang (2011) and Head et al. (2012) putting Burdett-Judd in Lagos and Wright (2005), Head and Kumar (2005) and Head et al. (2010) put it in the related model of Shi (1997). Alternatives models of price dispersion include Albrecht and Axel (1984) and Diamond (1987), where buyers differ not in terms of what they observe, but in their intrinsic (e.g., preference) type. A monetary version in Curtis and Wright (2004) generically delivers a two-point \( p \) distribution, for any number of types, which is less useful for our purposes. Also, as in Shi (1995) or Trejos and Wright (1995), and diametrically from us, that paper assumes goods are divisible while money is not, which limits applicability for policy and empirical issues.

Caplin and Spulber (1987) and Eden (1994) analyze models that are similar in spirit yet also very different. First, they do not try to match the data as we do. Second, we use Burdett-Judd to get endogenous price dispersion. Third, we build on the foundation for monetary economics in Lagos-Wright, which itself builds on Kiyotaki and Wright (1989,1993), Kochelakota (1998), Wallace (2001) etc. See Wallace (2010), Williamson and Wright (2010), Nosal and Rocheteau (2011) or Lagos et al. (2014) for surveys of the approach, sometimes called New Monetarist Economics, that strives to provide relatively clean descriptions of how agents trade and specifications for specialization, commitment and information that give money and related institutions essential roles. We adopt this approach because we believe it is best to analyze monetary phenomena in environments that are explicit about the frictions that money-like institutions are meant to ameliorate.\(^4\)

There is much empirical work on price adjustment. Campbell and Eden (2014) report that in grocery-store data the average duration between price changes is 10.3 weeks, but we do not want to focus exclusively on groceries. In BLS data, Bils and Klenow (2005) find at least half of prices last less than 4.3 months, or 5.5 months

\(^4\)Moreover, while many of our points can also be made in OLG, CIA or MUF models, we find it natural to use monetary theory based on search, as search is also the friction that drives price dispersion and stickiness in our environment.
excluding sales. Klenow and Kryvtsov (2008) report durations from 6.8 to 10.4 months. while Nakamura and Steinsson (2008) report 8 to 11 months excluding substitutions and sales. These papers also find large fractions of small and negative price changes, plus some evidence of a decreasing hazard. Eichenbaum et al. (2011) report a duration of 6 months for “reference” prices, those most often quoted in a quarter (presumably to avoid, e.g., recording Saturday Night Specials as two changes in a week). Cecchetti (1986) finds durations for magazine prices from 1.8 months to 14 years, while Carlton (1986) finds durations for wholesale prices ranging from 5.9 months for household appliances to 19.2 for chemicals, and also finds many small changes, with 2/3 below 2%.

An issue for simple Mankiw-style models is that average price changes are fairly big, suggesting high menu costs, but there are also many small changes, suggesting low menu costs. Midrigan (2011) accounts for this by having firms sell multiple products, and paying a cost to change one price lets them change the rest for free. This is reasonable, but it is still worth considering alternatives. Our model accounts for realistic durations, large average changes, many small changes and many negative changes. It also implies pricing behavior depends on inflation, which is an obvious problem for standard Calvo models. It also generates price dispersion at low or zero inflation, consistent with evidence (e.g., Campbell and Eden 2014), while dispersion in many models requires inflation. Based on this, we think our framework should be part of the conversation on sticky prices and their implications.

In terms of these implications, consider Ball and Mankiw (1994): “We believe that sticky prices provide the most natural explanation of monetary nonneutrality since so many prices are, in fact, sticky.” Also, “As a matter of logic, nominal

5 More empirical work is surveyed by Klenow and Malin (2010). In conversations with people in the area, they agree on the following list of facts: (1) Prices change slowly, although durations vary across studies. (2) The frequency and size of price changes vary across goods. (3) Two sellers changing at the same time do not typically change to the same price. (4) About 1/3 of changes are negative. (5) Hazard rates are slightly declining with duration. (6) There are many small (below 5%), and many big (above 20%) changes. (7) The frequency, size and fraction of negative changes vary with inflation. (8) There is price dispersion even at low inflation. The models presented below are consistent with all of these stylized facts.
stickiness requires a cost of nominal adjustment.” Golosov and Lucas (2003) similarly say that “menu costs are really there: The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing.” We agree with, and indeed embrace, the notion that prices are in fact sticky, but respectfully disagree with the other statements: in reality, menu costs may or may not be important and money may or may not be neutral, but our analysis proves that sticky prices logically imply neither menu costs nor nonneutrality.

The paper may also be considered a contribution to pure monetary economics. Our environment delivers nice money demand functions expressing real balances in terms of the nominal interest rate. While the details are different, the economics is closely related to classic results in Baumol (1952), Tobin (1956), Miller and Orr (1966) and Whalen (1966), although those are partial-equilibrium models, or, more accurately, decision-theoretic analyses of how to manage one’s money assuming that it (as opposed to credit, barter or something else) must be used for transactions. While such models are still being applied to good effect (e.g., Alvarez and Lippi 2009), we admit to a preference for theories where the roles of money and credit are not taken for granted, and, as argued by Wallace (2013), we think this is relevant for understanding important aspects monetary policy.\footnote{Also, incorporating Burdett-Judd pricing is a step further in understanding this particular monetary-theoretic framework, which has become a workhorse in the literature. As discussed in the surveys cited above, previous analyses use Nash, Kalai or strategic bargaining, posting with random or directed search, competitive price taking, auctions and pure mechanism design.}

The above-mentioned multiplicity of monetary equilibria in models with indivisible goods and posting occurs in a series of papers spawned by Green and Zhou (1998). Jean et al. (2010) provide citations and further discussion, but here is the intuition: If all sellers post $p$ then buyers’ best response is to bring $m = p$ dollars to the market as long as $p$ is not too big. If all buyers bring $m$ then sellers’ best response is post $p = m$ as long as $m$ is not too small. Hence, any $p = m$ in some range is an equilibrium, and the selection matters for payoffs. Note that this in-
volves a continuum of stationary equilibria, very different from the multiplicity of dynamic equilibria in other monetary models. Head et al. (2012) avoid this because, even if all sellers post \( p \), buyers need not bring \( m = p \) when \( q \) is divisible, but as we said, that leads to other problems. In our alternative approach, intuitively, even if all sellers post \( p \), buyers need not bring \( m = p \) when they have access to credit.\(^7\)

Before proceeding, we mention heterogeneity. As is well understood in other applications (e.g., Mortensen and Pissarides 1999), if firms are homogeneous as they are here, theory does not pin down which one charges which \( p \), but only the distribution \( F(p) \). With heterogeneity, however, low cost firms prefer low \( p \) since they are more interested in high volume. Still, for any subset of firms with the same marginal cost, it does not matter which one posts which \( p \). Hence, heterogeneity eliminates neither dispersion nor stickiness within sets of similar sellers. This is especially relevant for retail, where the marginal cost is the wholesale price, because even if a few retailers get better deals – e.g., Kmart has a quantity discount – many others face the same wholesale terms. Irrespective of fixed costs, wages etc., these sellers are homogeneous for our purposes, and our theory of price dispersion and price stickiness applies to them without qualification.\(^8\)

3 Environment

Each period in discrete time has two subperiods: first there convenes a decentralized market, called BJ for Burdett-Judd; then there convenes a frictionless centralized market, called AD for Arrow-Debreu. There is a set of firms interpreted as retailers with measure 1, and a set of households with measure \( \bar{b} \). Households consume a

\(^7\)One can imagine other solutions, as in some models the multiplicity can be refined away, but the outcome is sensitive to details like who moves first. In any case, as we say, there are many reasons to study models with money and credit in addition to solving this technical problem.

\(^8\)As a final note on literature, we alluded to the venerability of work on money and costly credit. Papers include Prescott (1987), Freeman and Huffman (1991), Chatterjee and Corbae (1992), Lacker and Schreft (1996), Freeman and Kydland (2000), Dong (2011) and Lucas and Nicolini (2013). Nosal and Rocheteau (2011) provide more discussion and references. See also Gomis-Porqueras and Sanches (2013), Li and Li (2013), Lotz and Zhang (2013), Araujo and Hu (2014), Gu et al. (2014) and Bethune et al. (2015). None of these consider our application: a theoretical and empirical study of sticky prices.
divisible good \( x_t \) and supply labor \( \ell_t \) in AD, while in BJ they consume an indivisible good \( y_t \) produced by firms at unit cost \( \gamma \geq 0 \). As agents are anonymous in the BJ market, they cannot use credit, unless they access a technology to authenticate identity and record transactions at a cost. By incurring the cost, households can get BJ goods in exchange for commitments to deliver \( d_t \) dollars in the next AD; otherwise cash is required at point of sale.\(^9\) We consider both a fixed cost of credit \( \delta \) and a proportional cost \( \tau \). To nest these, let the transaction cost in general be 
\[
\chi(d_t) = \delta \mathbf{1}(d_t) + \tau d_t,
\]
where \( \mathbf{1}(d_t) \) is an indicator function giving 1 iff \( d_t > 0 \).

Household utility within a period is 
\[
U(x_t) + \mu \mathbf{1}(y_t) - \ell_t,
\]
where \( U'(x_t) > 0 > U''(x_t) \), \( \mu > \gamma \) is a parameter and \( \mathbf{1}(y_t) \) is again an indicator function. Let \( \beta = 1/(1 + r) \), with \( r > 0 \), be a discount factor between AD today and BJ tomorrow (any discounting between BJ and AD can be subsumed in the notation). We impose \( \pi > \beta - 1 \), where in stationary equilibrium \( \pi \) is the inflation rate, and the nominal interest rate is given by the Fisher equation 
\[
1 + i = (1 + \pi)(1 + r).
\]
Notice \( i > 0 \), and the Friedman rule is the limiting case \( i \to 0 \). As usual, \( 1 + i \) is the amount of money agents require in the next AD market to give up a dollar in the current AD market, whether or not such trades occur in equilibrium. Let \( x_t \) be AD numeraire, an assume it is produced one-for-one with \( \ell_t \), so the wage is 1 (purely for simplicity, as the results do not depend on this). The AD price of money in numeraire is \( \phi_t \).

Firms enter the BJ market for free, but households must pay cost \( k \), which in general determines participation \( b_t \leq \bar{b} \). However, in the baseline model \( k = 0 \), so \( b_t = \bar{b} \). Firms use BJ profits to buy AD goods, over which they have linear utility, but noting of interest changes if instead they disburse profits to households as dividends. Each firm posts a price taking as given household behavior and the CDF of other firms’ prices, \( F_t(p) \), with support \( \mathcal{F}_t \). Every household in BJ randomly samples \( n \) firms – i.e., sees \( n \) independent draws from \( F_t(p) \) – with probability \( \alpha_n = \alpha_n(b_t) \), generally depending on the buyer-seller ratio, or market tightness, \( b_t \).

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\(^9\) While the cost is paid by households, it is not hard to show the allocation is identical if it is instead paid by firms, as in elementary tax-incidence theory. Also, it does not matter if debt due in the frictionless AD market is denominated in dollars or numeraire.
For our purposes it suffices to have $\alpha_1, \alpha_2 > 0$ and $\alpha_n = 0 \ \forall n \geq 3$, but this can be generalized easily enough (see, e.g., Burdett et al. 2014). Finally, in terms of policy, let the money supply per capita evolve according to $M_{t+1} = (1 + \pi) M_t$, with changes occurring in AD via lump-sum transfers, although the main results are the same if instead the new money is used to buy AD goods.

### 3.1 The Firm Problem

Expected real profit for a firm posting $p$ at date $t$ is

$$\Pi_t(p) = b_t \{\alpha_1 (b_t) + 2\alpha_2 (b_t) [1 - F_t (p)]\} (p\phi_t - \gamma).$$

(1)

Thus, net revenue per unit is $p\phi_t - \gamma$, and the number of units is determined as follows: The probability a household contacts this firm and no other is $\alpha_1 (b_t)$. Then the firm makes a sale. The probability a household contacts this firm plus another is $2\alpha_2 (b_t)$, as this can happen in two ways, this one first and the other second or vice versa. Then the firm makes a sale iff it beats the other’s price, which occurs with probability $1 - F_t (p)$. This is all multiplied by tightness $b_t$ to convert buyer probabilities into seller probabilities.

Profit maximization means every $p \in F_t$ yields the same profit and no $p \notin F_t$ yields higher profit. As is standard in this kind of model, $F_t (p)$ is continuous and $F_t = [p_t, \bar{p}_t]$ is an interval.\(^{10}\) Taking as given for now the upper limit $\bar{p}_t$, profit from any $p \in F_t$ must equal the profit from $\bar{p}_t$, which is

$$\Pi_t (\bar{p}_t) = b_t\alpha_1 (b_t) (\bar{p}_t\phi_t - \gamma).$$

(2)

Equating (1) and (2) $\forall p \in F_t$, we can solve for

$$F_t (p) = 1 - \frac{\alpha_1 (b_t) \phi_t \bar{p}_t - \phi_t p}{2\alpha_2 (b_t) \phi_t p - \gamma}.$$  

(3)

\(^{10}\)There cannot be a mass of firms posting the same $p$ in equilibrium, because any one of them would have a profitable deviation to $p - \varepsilon$, since he would lose only $\varepsilon$ per unit and make discretely more sales by undercutting others at the mass point. Similarly, if there were a gap between say $p_1$ and $p_2 > p_1$, a firm posting $p_1$ can deviate to $p_1 + \varepsilon$ and earn more per unit without losing sales.
This is the BJ distribution. For \( p_l < p < \bar{p}_t \) it is easy to see \( F'_t(p) > 0 \) and \( F''_t(p) < 0 \).

Since the lower limit of \( F_t \) satisfies \( F(p_l) = 0 \), we have

\[
p_l = \frac{\alpha_1(b_t) \phi_t \bar{p}_t + 2\alpha_2(b_t) \gamma}{\phi_t [\alpha_1(b_t) + 2\alpha_2(b_t)]}. \tag{4}
\]

There are different possibilities for \( \bar{p}_t \), as detailed below. To translate from dollars to numeraire, let \( q_t = \phi_t p_t \) and write the distribution of real BJ prices as

\[
G_t(q) = F_t(\phi_t p_t) = 1 - \frac{\alpha_1(b_t)}{2\alpha_2(b_t)} \frac{\bar{q}_t - q}{q - \gamma}. \tag{5}
\]

### 3.2 The Household Problem

We focus on stationary equilibrium, where real variables are constant and nominal variables grow at rate \( \pi \). This makes it convenient to frame the household problem in real terms. The state variable in AD is net worth, \( A = \phi m - \phi d - \chi(d) + \bar{A} \), where \( \phi m \) and \( \phi d \) are real money balances and debt carried over from the previous round of BJ trade, \( \chi(d) \) is the cost of having used credit, and \( \bar{A} \) is any other source of purchasing power, including transfers. Since preferences are linear in \( \ell \), we can assume with no loss in generality that all obligations are settled in AD, so households start BJ debt free. The state variable in BJ is real balances carried into that market, \( z \). The AD and BJ value functions are \( W(A) \) and \( V(z) \).

For a household that decides to enter the next BJ market,

\[
W^1(A) = \max_{x,\ell,z} \{ \mu(x) - \ell + \beta V(z) \} \quad \text{st} \quad x = A - k + \ell - (1 + \pi) z,
\]

where \( k \) is the entry cost, and due to inflation the cost of having \( z \) next period in terms of current numeraire is \( (1 + \pi) z \). Eliminating \( \ell \) and letting \( x^* \) solve \( U'(x^*) = 1 \), we get

\[
W^1(A) = A + U(x^*) - x^* - k + \beta \max_z O_i(z), \tag{6}
\]

where the objective function for the choice of \( z \) is \( O_i(z) = V(z) - (1 + i) z \), with \( i \) again given by the Fisher equation. For a household that decides to skip the next BJ
market, \( W^0 (A) = A + U (x^*) - x^* + \beta W (\bar{A}) \), and \( W (A) \equiv \max \{ W^1 (A), W^0 (A) \} = W^1 (A) \) as long as BJ is open. As in Lagos and Wright (2005), \( W' (A) = 1 \) and the choice of \( z \) does not depend on \( A \), which greatly simplifies the analysis.

The BJ value function satisfies\(^{11}\)

\[
V (z) = W (z + \bar{A}) + [\alpha_1 (b) + \alpha_2 (b)] \{ \mu - \mathbb{E}_{j} q - [1 - J (z)] \delta - \tau (\mathbb{E}_{j} q - z) \} \tag{7}
\]

where \( \mathbb{E}_{j} q = \int_{\frac{q}{2}}^{q} q d J (q) \) and \( J (q) \) is the CDF of transaction prices,

\[
J (q) = \frac{\alpha_1 (b) G (q) + \alpha_2 (b) \{ 1 - [1 - G (q)]^2 \}}{\alpha_1 (b) + \alpha_2 (b)}. \tag{8}
\]

which differs from the CDF of posted prices \( G (q) \), since buyers seeing multiple \( q \) pick the lowest. The first term in (7) is the payoff from going on to AD without trading, while the second is \( \alpha_1 + \alpha_2 \) times the expected surplus, given by BJ utility minus the payment and (fixed plus proportional) cost of credit. Clearly, from (7), the benefit of higher \( z \) is that it reduces the expected cost of using credit.

For the BJ entry decision, it is easy to check \( \Phi \equiv (1 + r) [W^1 (A) - W^0 (A)] \) is independent of \( A \) and satisfies

\[
\Phi = [\alpha_1 (b) + \alpha_2 (b)] \{ \mu - \mathbb{E}_{j} q - [1 - J (z)] \delta - \tau (\mathbb{E}_{j} q - z) \} - \kappa - iz, \tag{9}
\]

where \( \kappa = k / \beta \). The first term is the expected benefit of participating in BJ, as in (7), while \( \kappa + iz \) is the cost. Then

\[
b = \bar{b} \Rightarrow \Phi \geq 0; \ b = 0 \Rightarrow \Phi \leq 0; \ b \in (0, \bar{b}) \Rightarrow \Phi = 0. \tag{10}
\]

\(^{11}\)To derive (7), consider \( \delta > 0 = \tau \). First write

\[
V (z) = W (z) + \alpha_1 (b) \int_{\frac{q}{2}}^{q} (\mu - q) d G_1 (q) + \alpha_1 (b) \int_{z}^{q} (\mu - q - \delta) d G_1 (q)
+ \alpha_2 (b) \int_{\frac{q}{2}}^{q} (\mu - q) d G_2 (q) + \alpha_2 (b) \int_{z}^{q} (\mu - q - \delta) d G_2 (q),
\]

where \( G_n (q) = 1 - [1 - G (q)]^n \) is the CDF of the lowest of \( n \) independent draws from \( G (q) \). The first term is the continuation value if a buyer does not trade. The second is the probability of meeting a seller with \( q \leq z \), so cash is used, times the expected surplus. The third is the probability of meeting a seller with \( q > z \), so credit is used, which has an additional cost \( \delta \). The last two terms are similar except the buyer meets two sellers. The rest is algebra.
3.3 Equilibrium

The above discussion characterizes the pricing behavior of sellers, given an upper bound $\bar{q}$, as well as the participation and money demand decisions of buyers.

**Definition 1** A stationary equilibrium is a list $\langle G(q), b, z \rangle$ solving (5), (10) and the maximization problem in (6), with $\bar{q}$ determined as in Lemma 1 below.

**Definition 2** (i) A nonmonetary equilibrium, or NME, has $z = 0$, so all BJ trades use credit. (ii) A mixed monetary equilibrium, or MME, has $0 < z < \bar{q}$, so BJ trades use cash if $q \leq z$ and credit if $q > z$. (iii) A pure monetary equilibrium, or PME, has $z \geq \bar{q}$, so all BJ trades use cash.

Implicit in these definitions is that the outcome is non-degenerate – i.e., payoffs are positive, so the BJ market does not shut down – as we check below.

**Lemma 1** (i) NME implies $\bar{q} = (\mu - \delta) / (1 + \tau)$. (ii) MME implies $z < \mu - \delta$ and $\bar{q} = (\mu - \delta + \tau z) / (1 + \tau)$. (iii) PME implies $\bar{q} = z \geq \mu - \delta$.

**Proof:** (i) In NME, buyers’ BJ surplus is $\Sigma = \mu - q - \delta - \tau q$. Note $\Sigma = 0$ at $q = (\mu - \delta) / (1 + \tau)$, so no buyer pays more than this. If $\bar{q} < (\mu - \delta) / (1 + \tau)$ then the highest price seller has profitable deviation toward $(\mu - \delta) / (1 + \tau)$, which increases profit per unit without affecting sales. Hence $\bar{q} = (\mu - \delta) / (1 + \tau)$. (ii) In MME, for $q > z$, as it is near $\bar{q}$, $\Sigma = \mu - q - \delta - \tau(q - z)$. Note $\Sigma = 0$ at $q = (\mu - \delta + \tau z) / (1 + \tau)$, and repeat the argument for NME to show $\bar{q} = (\mu - \delta + \tau z) / (1 + \tau)$. The definition of MME has $z < \bar{q} = (\mu - \delta + \tau z) / (1 + \tau)$, which reduces to $z < \mu - \delta$. (iii) In PME, given buyers bring $z$ to BJ, they would pay $z$. Hence $\bar{q} \geq z$, as $\bar{q} < z$ implies the highest price seller has profitable deviation. We also have to be sure there is no profitable deviation to $q > z$, which requires buyers using some credit. The highest such $q$ a buyer would pay solves $\Sigma = \mu - q - \delta - \tau (q - z) = 0$, or $q = (\mu - \delta + \tau z) / (1 + \tau)$. There is no profitable deviation iff $(\mu - \delta + \tau z) / (1 + \tau) \leq z$, which reduces to $z \geq \mu - \delta$. ■
The above style of argument is standard in BJ models, although it is complicated here by the possibility of paying with money and the transaction costs of paying without money. Also, notice PME entails $z = \bar{q}$, related to the above-mentioned indeterminacy of equilibrium without credit – basically, nothing pins down $z$ and $\bar{q}$ as long as they are equal, and this is why we focus on MME below. In any case, in NME, prices must be described in numeraire $q$, while in MME or PME they can equivalently be described in numeraire or dollars, with $F_t(p) = G(\phi_t p)$ where $\phi_t = z_t/M_t$. Other variables that can be computed include AD consumption $x$ and labor supply $\ell$, but we do not need these for what follows. Also, for now $k = 0$, so $b = \bar{b}$ is fixed and we omit the argument from the arrival rates $\alpha_1$ and $\alpha_2$.

4 Fixed-Cost Model

Consider first $\tau = 0$ and $\delta > 0$. Assuming $\delta < \mu - \gamma$ guarantees that when $k = 0$ there is a nonmonetary equilibrium with $b = \bar{b}$, where all households participate and all transactions use credit – basically, the original BJ equilibrium.

**Lemma 2** Consider the fixed-cost model. For $\underline{q} < z < \bar{q}$, $V(z)$ is smooth, with $V'(z) > 0$ and $V''(z) < 0$. For $z < \underline{q}$ or $z > \bar{q}$, $V'(z) = 1$.

**Proof:** For $z \in (\underline{q}, \bar{q})$, $V'(z) = 1 + (\alpha_1 + \alpha_2) \delta J'(z)$ and $V''(z) = (\alpha_1 + \alpha_2) \delta J''(z)$, by virtue of (7). Using (8), we have

$$J'(q) = \frac{\alpha_1 G'(q) + 2\alpha_2 [1 - G(q)] G'(q)}{\alpha_1 + \alpha_2} > 0$$

$$J''(q) = \frac{\alpha_1 G''(q) + 2\alpha_2 [1 - G(q)] G''(q) - 2\alpha_2 G'(q)^2}{\alpha_1 + \alpha_2} < 0$$

where $G$ is smooth by (5). The rest is obvious. ■

These results deliver a unique $z_i = \operatorname{arg\ max}_{z \in [\underline{q}, \bar{q}]} O_i(z)$. If $z_i \in (\underline{q}, \bar{q})$, as required for MME, it satisfies the FOC

$$(\alpha_1 + \alpha_2) \delta J'(z_i) = i. \quad (11)$$
To check $z_i \in (q, \bar{q})$, first notice $O_i'(z) = -\gamma \forall z > \bar{q}$ and $\forall z < q$, as shown in Figure 1. Let $\hat{z}_i$ be the global maximizer of $O_i(z)$, and let $O_i^-(z)$ and $O_i^+(z)$ be the left and right derivatives. If $O_i^+(\bar{q}) \leq 0$ then $\hat{z}_i = 0$, as in the left panel of Figure 1. If $O_i^+(\bar{q}) > 0$ we check $O_i^-(\bar{q})$. If $O_i^-(\bar{q}) \geq 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = \bar{q}$, as in the center panel. If $O_i^-(\bar{q}) < 0$ then either $\hat{z}_i = 0$ or $\hat{z}_i = z_i$, as in the right panel.

Figure 1: Possible Types of Equilibria with a Fixed Cost

**Proposition 1** In the fixed-cost model there exists a unique MME if $\gamma < \tilde{\gamma}$ and $i \in (\bar{i}, \bar{\iota})$, where $\tilde{\gamma} = \mu - (2\alpha_2^2 + 2\alpha_1\alpha_2) / (2\alpha_2^2 + 2\alpha_1\alpha_2 - \alpha_1^2)$, $\bar{i} = \delta \alpha_1^2 / 2\alpha_2 (\mu - \delta - \gamma)$ and $\bar{\iota} \in (\bar{i}, \infty)$; otherwise there is no MME.

**Proof:** MME exists iff three conditions hold: (i) $O_i^-(\bar{q}) < 0$; (ii) $O_i^+(\bar{q}) > 0$; and (iii) $O_i(z_i) > O_i(0)$. Now (i) is equivalent to $(\alpha_1 + \alpha_2) \delta J^-(\bar{q}) < i$, which holds iff $i > \bar{i}$. And (ii) is equivalent to $(\alpha_1 + \alpha_2) \delta J^+(\bar{q}) > i$, which holds iff $i < \bar{\iota}$ where

$$\bar{\iota} = \frac{\delta (\alpha_1 + 2\alpha_2)^3}{2\alpha_1\alpha_2 (\mu - \delta - \gamma)} > \bar{i}.$$

And (iii) is equivalent to $(\alpha_1 + \alpha_2) \delta J(z_i) - iz_i > (\alpha_1 + \alpha_2) \delta J(0)$, which holds iff $\Delta(i) > 0$ where

$$\Delta(i) = -i\gamma + \frac{\delta (\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i^2\delta \frac{\alpha_1^2}{\alpha_2} \alpha_2^{-\frac{\gamma}{2}} (\mu - \delta - \gamma)^{\frac{\gamma}{2}} (2 - \frac{\gamma}{2} + 2 - \frac{\gamma}{2}).$$

Notice $\Delta(0) > 0 > \Delta(\bar{i})$ and $\Delta'(i) < 0$. Hence $\exists! \bar{i}$ such that $\Delta(\bar{i}) = 0$, and $\Delta(i) > 0$ iff $i < \bar{i}$. It remains to verify that $\bar{i} > \bar{i}$, so that (i) and (iii) are not
mutually exclusive. It can be checked that $\bar{i} > \hat{i}$ iff $\delta < \tilde{\delta}$. Hence a MME exist under the stated conditions. It is unique because $\bar{q} = \mu - \delta$, which pins down $G(q)$, and then $\hat{z}_i = \arg\max_{z \in (q, \bar{q})} O_i(z)$. ■

The simplicity of MME arises because buyers have a unique best response $\hat{z}_i$ to $G(q)$, and sellers’ best response conditions hold by construction with $G(q)$ and $\bar{q} = \mu - \delta$ independent of $\hat{z}_i$. Although our interest is mainly in MME, for completeness, notice NME is basically the original Burdett-Judd outcome, with positive payoffs given our maintained assumption $\delta < \mu - \gamma$.

**Proposition 2** In the fixed-cost model there exists a unique NME.

**Proof:** With fiat currency $\phi = 0$ is always self-fulfilling, so we simply set $G(q)$ according to (5). ■

Also for completeness note that while there is a continuum of PME we can still provide sharp conditions for existence.

**Proposition 3** In the fixed-cost model PME exists iff either: $\bar{\delta} < \delta < \mu - \gamma$ and $i < \hat{i} = \delta (\alpha_1 + \alpha_2) / (\mu - \delta)$; or $\delta < \tilde{\delta}$ and $i < \bar{i}$.

**Proof:** From Figure 1, necessary and sufficient conditions for NME are: (i) $O_i^-(q) > 0$; (ii) $O_i^+(q) > 0$; and (iii) $O_i(q) > O_i(0)$. Now (i) holds iff $i < \hat{i}$ and (ii) holds iff $i < \bar{i}$. Condition (iii) holds iff $i < \hat{i}$. For $\delta > \tilde{\delta}$, it is easily checked that $\hat{i} < \bar{i}$ and $i < \hat{i}$ by (??), so the binding condition is $i < \hat{i}$. For $\delta < \tilde{\delta}$, it is easily checked that $\hat{i} > \bar{i}$, and $\bar{i} < \hat{i}$, so the binding condition is $i < \bar{i}$. ■

Propositions 1-3 are illustrated in Figure 2. Again, MME is the case of interest, and it exists for intermediate $(\delta, i)$. In this case, we can insert $G(q)$ into the FOC for $\hat{z}_i$ and rearrange to get the explicit solution for money demand:

$$\hat{z}_i = \gamma + \left[\frac{\alpha_2^2 \delta (\mu - \delta - \gamma)^2}{2 \alpha_2}\right]^{1/3} i^{-1/3} \tag{12}$$

This expresses real balances in terms of the cube-root of $1/i$, reminiscent of Baumol (1952), Tobin (1956), Whalen (1966) or Miller and Orr (1966). The results derive from the first two get a square- rather than cube-root; we get something more like a square-root in Section 5.
Figure 2: Equilibria in Parameter Space with Fixed Cost

from similar economic forces. The usual story behind Baumol-Tobin has an agent sequentially incurring expenses assumed to need currency, with a fixed cost of rebalancing \( z \). The decision rule compares \( i \), the opportunity cost of cash, with the benefit of reducing the number of financial transactions interpretable as trips to the bank. Our buyers make at most one transaction in BJ before rebalancing \( z \) in AD, but the size is random, and credit (spending beyond \( z \)) is costly. Still, they compare \( i \), again the cost of cash, with the benefit of reducing the likelihood of using costly financial services, again interpretable as trips to the bank, although one might say in this environment that they go there for a loan and not a withdrawal.

5 Variable Cost

Now consider \( \tau > 0 \) and \( \delta = 0 \). One but not the only interpretation of \( \tau \) is a proportional tax that can be avoided using cash. Regardless of interpretation, a variable cost is in some respects easier, and it avoids a technical issue that we waited until now to raise. In economies with nonconvexities, as we have with fixed cost \( \delta \), it can be desirable to use a lottery (Berentsen et al. 2002). Thus, a seller can post: “you get my good for sure if you pay \( p \); if you pay \( \tilde{p} < p \) then you get my good with probability \( P = P(\tilde{p}) \).” In Section 8, when a buyer with \( m = p - \varepsilon \).
meets a seller posting \( p \), he pays \( p - \varepsilon \) in cash, \( \varepsilon \) in credit and \( \delta \) in fixed cost; if \( \varepsilon \) is small, however, both parties prefer to trade using cash only, to avoid \( \delta \), and have the good delivered with probability \( P < 1 \). Now, one could try to argue that lotteries are infeasible, unrealistic, or otherwise undesirable, but that would be awkward, because ruling out randomized exchange may be construed as uncomfortably close to imposing linear menus, something we criticized above.

We cover fixed cost models since they have a history in the literature (see fn. 8), but do not want to tackle lotteries here. Lest one considers this problematic, we also consider a variable cost, which has no such technical problem. The price distribution emerging from the firms’ problem is the same as above. For households, as a special case of Lemma 1 and the formula for the lower bound,

\[
\bar{q} = \frac{\mu + z\tau}{1 + \tau} \quad \text{and} \quad q = \frac{\alpha_1 (\mu + z\tau) + 2\alpha_2 \gamma (1 + \tau)}{(\alpha_1 + 2\alpha_2) (1 + \tau)}.
\]

Conveniently, \( O_i(z) \) is now differentiable at \( \bar{q} \) and \( q \), as described in the following analog to Lemma 2, stated without proof as it involves a simple calculation.

**Lemma 3** In the variable-cost model, \( O_i(z) \) is smooth \( \forall z > 0 \), with \( O_i''(z) < 0 \) \( \forall z \in (q, \bar{q}) \), \( O_i'(z) = (\alpha_1 + \alpha_2)\tau - i \ \forall z < q \) and \( O_i'(z) = -i \ \forall z > \bar{q} \).

As Figure 3 shows, there are now only two types of equilibria, NME and MME. If \( i > (\alpha_1 + \alpha_2)\tau \) there is a unique candidate NME, which is a NME as long as payoffs are not negative, which holds if \( \tau \leq \mu/\gamma - 1 \); otherwise, without money the BJ market shuts down. If \( i < (\alpha_1 + \alpha_2)\tau \) there is a unique candidate MME, which satisfies the FOC \( i = \tau(\alpha_1 + \alpha_2)[1 - J(\hat{z}_i)] \). Inserting \( J(z) \), after some algebra, one can again solve for the explicit money demand function\(^{13}\)

\[
\hat{z}_i = \gamma + \frac{(\mu - \gamma) \left[ \tau + (1 + \tau) \sqrt{1 + 4\alpha_2 i / \alpha_1^2 \tau} \right]}{1 + 2\tau + 4\alpha_2 (1 + \tau)^2 i / \alpha_1^2 \tau}.
\]

\(^{13}\)Compared to (12) from the fixed-cost model, here money demand is closer to a classic square-root rule, except \( i \) also appears in the denominator.
It is clear that $\hat{z}_i < \bar{q}$, and $\hat{z}_i > q$ iff $i < \tau(\alpha_1 + \alpha_2)$. This candidate is a MME as long as payoffs are not negative, which holds iff

$$\Phi = (\alpha_1 + \alpha_2) [\mu + \tau \hat{z}_i - (1 + \tau) \mathbb{E} q] - i \hat{z}_i$$

$$= \alpha_2 [\mu + \tau \hat{z}_i - \gamma (1 + \tau)] - i \hat{z}_i \geq 0.$$ 

It can be shown that $i \hat{z}_i$ increases and $\Phi$ decreases with $i$. Letting $i^* = i^* (\tau)$ be the $i$ that solves $\Phi = 0$, we have proved:

**Proposition 4** In the variable-cost model there exists a unique NME iff $\tau \leq \mu / \gamma - 1$, there exists a unique MME iff $i < \min \{\tau(\alpha_1 + \alpha_2), i^*\}$, and there is no PME.

Figure 4 illustrates the results. For any $\tau$ there is a unique MME as long as $i$ is not too big. Notice that credit is used at high prices, since $\bar{q} > \hat{z}_i$, and that the maximum debt $\bar{q} - \hat{z}_i$ increases with $i$. One can also show $\partial \hat{z}_i / \partial i < 0$, $\partial \hat{z}_i / \partial \tau > 0$ and $\partial (\bar{q} - \hat{z}_i) / \partial \tau < 0$. As $\tau$ gets bigger, $\bar{q} \to \hat{z}_i$, so buyers eventually stop using credit. Based on these results, we conclude that the variable-cost model is also tractable, delivers a different but still nice money demand function, has natural properties and avoids the issue of lotteries. Moreover, as suggested by comparing Figures 2 and 4, MME tend to exist for a larger set of parameters in the variable-cost model, which as we show below affects their quantitative performance.
6 Sticky Prices

With either a fixed or variable cost, the nominal price distribution $F_t(p)$ is uniquely determined $\forall t$, but individual-firm price dynamics are not. Consider Figure 5, drawn for the calibrated parameters in Section 7. With $\pi > 0$, the density $F'_{t+1}$ is a right shift of $F'_t$. Firms with $p < p_{t+1}$ at $t$ (Region A) must reprice at $t+1$, because, while $p$ maximized profit at $t$, it no longer does so at $t+1$. But as long as the supports $F_t$ and at $F_{t+1}$ overlap, there are firms with $p > p_{t+1}$ at $t$ (Region B) that can keep the same $p$ at $t + 1$ without reducing profit, since they earn less per unit in real terms, but make it up on the volume.

Since equilibrium puts weak restrictions on individual price behavior, we consider two additional conditions: (i) we add a payoff-irrelevant tie-breaking rule; and (ii) we focus on symmetric behavior. According to (i), firms use strategies of the following class: if $p_t \notin F_{t+1}$ then $p_{t+1}(p_t) = \hat{p}$ where $\hat{p}$ is a new price; and if $p_t \in F_{t+1}$ then

$$p_{t+1}(p_t) = \begin{cases} p_t & \text{with prob } \sigma \\ \hat{p} & \text{with prob } 1 - \sigma \end{cases}$$

so those that are indifferent stick with probability $\sigma$ to their current $p$. In case it is not obvious, we emphasize that this is very different from Calvo pricing, where firms can be desperate to change $p$ but are simply not allowed. In our environment,
any firm that wants to can and does change its price, but there are some that are indifferent, and they are happy to randomize.\textsuperscript{14}

Then (ii) says all changers pick \( \hat{p} \) from the same repricing distribution \( H_{t+1}(\hat{p}) \).

Given \( \sigma \) and \( F_t(p) \), the unique \( H_{t+1}(\hat{p}) \) generating \( F_{t+1}(p) \) is:

\[
H_{t+1}(p) = \begin{cases} 
\frac{F_t\left(\frac{p}{\bar{p}_t}\right) - \sigma[F_t(p) - F_t(\bar{p}_t+1)]}{1 - \sigma + \sigma F_t(\bar{p}_t+1)} & \text{if } p \in [\bar{p}_t, \bar{p}_t+1) \\
\frac{F_t\left(\frac{\bar{p}_t+1}{p}\right) - \sigma[1 - F_t(p)]}{1 - \sigma + \sigma F_t\left(\frac{\bar{p}_t+1}{p}\right)} & \text{if } p \in [\bar{p}_t, \bar{p}_t+1] 
\end{cases}
\]  

(15)

Different \( \sigma \) capture a range of price dynamics, and it is routine to compute statistics comparable to those studied in the empirical literature, including duration, the distribution of changes and the hazard – e.g., see Head et al. (2012) for derivations of these formulae as well as (15).

7 Quantitative Results

While one can in principle examine \( F_t(p) \) directly, and this may be worth future exploration, here we examine the distribution of price changes, \((p_{t+1} - p_t)/p_t\), because that is what is emphasized in the literature. We focus on MME, which is the \textsuperscript{14}In the calibrations below, only about 10% of indifferent sellers change \( p \).
realistic scenario. Given this, an important statistic for our purposes is the fraction of monetary transactions, \( J(\hat{z}_i) \). Also important are statistics derived from an empirical notion of money demand, which (following Lucas 2000 and many others) we take to be \( L_i = \hat{z}_i / Y \), where \( Y = x + (\alpha_1 + \alpha_2) E_t q \) is real output aggregated across AD and BJ markets. These objects are important because the observations of interest concern monetary phenomena, so it is natural to impose discipline using micro and macro monetary data.

The only exogenous functional form needed is \( U(x) = \log(x) \), which normalizes AD output to \( x^* = 1 \).\(^{15}\) To derive the endogenous functional forms, consider the fixed-cost model (the other case is similar but more complicated). In MME, after inserting \( \bar{q} \) and \( q \), the real posted-price and transaction-price CDF’s reduce to

\[
G(q) = 1 - \frac{\alpha_1 \mu - \delta - q}{2\alpha_2} \frac{q - \gamma}{},
\]

\[
J(q) = 1 - \frac{\alpha_1^2 (\mu - \delta - q)(\mu - \delta + q - 2\gamma)}{4\alpha_2 (\alpha_1 + \alpha_2) (q - \gamma)^2}.
\]

From (12), the fraction of monetary transactions is

\[
J(\hat{z}_i) = \frac{(\alpha_1/2 + \alpha_2)^2 - [\alpha_1\alpha_2(\mu - \delta - \gamma)]^{2/3} i^{2/3}}{\alpha_2 (\alpha_1 + \alpha_2)(4\delta)^{2/3}}.
\]

The money demand function is

\[
L_i = \gamma + \frac{[\alpha_1^2 \delta (\mu - \delta - \gamma)^2 / 2\alpha_2]^{1/3} i^{-1/3}}{1 + \alpha_1 (\mu - \delta) + \alpha_2 \gamma},
\]

which implies an elasticity

\[
\eta_i = \frac{1}{3 + 3\gamma [\alpha_1^2 \delta (\mu - \delta - \gamma)^2 / 2\alpha_2]^{-1/3} i^{1/3}}.
\]

Below we try to match \( J(\hat{z}_i) \), \( L_i \) and \( \eta_i \) evaluated at the sample mean in the data, \( E \). Another key statistic for this exercise is the average BJ markup, price over marginal cost, defined by \( E_{q}/\gamma \). This is especially relevant in BJ models,

\(^{15}\)In general, period utility is \( \log(x) + \mu 1(y) - \psi \ell \), with \( \mu \) capturing the relative importance of BJ and AD goods and \( \psi \) the importance of leisure. We can set \( \psi \) to match average hours \( \ell \), as is standard (e.g., Cooley 1995), but as the results below do not dependent on this, we set \( \psi = 1 \).
where as the arrival rates vary equilibrium can deliver anything being monopoly
and marginal-cost pricing. Given $G(q)$, the average markup is
\[
\frac{\mathbb{E}_G q}{\gamma} = 1 + \frac{\alpha_1 (\mu - \delta - \gamma) \log (1 + 2\alpha_2 / \alpha_1)}{2\alpha_2 \gamma}.
\] (19)

These are the main statistics targeted; others are discussed below.

7.1 Data

We focus on 1988-2004, as we are going to evaluate the theory by its ability to
account for the empirical price-change distribution from that period, but other pe-
riods can be used to calibrate some parameters. For money we use the M1J series
in Lucas and Nicolini (2012) that adjusts M1 for money-market deposit accounts,
similar to the way M1S adjusts for sweeps in Cynamon et al. (2006). Lucas-Nicolini
provide an annual series from 1915 to 2008, and a quarterly series from 1984 to
2013, and make the case that there is a stable relationship between these series and
(3-month T-Bill) nominal interest rates. We use the quarterly series, which better
 corresponds to the price-change sample, and has an average annualized nominal rate
of $\mathbb{E}_i = 4.80\%$, implying $L_{\mathbb{E}_i} = 0.279$ and $\eta_{\mathbb{E}_i} = -0.149$.\textsuperscript{16}

Following several related studies, markup data comes from the U.S. Census Bu-
In these data, the low end, in Warehouse Clubs, Superstores, Automotive Dealers
and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high
end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42
and 1.44. Our target is for the gross margin is 1.3, the middle of these data. A gross
margin of 1.3 implies a markup 1.39, as in Bethune et al. (2014). However, the exact
value for this target does not matter very much over a reasonable range (similar to
the findings in Aruoba et al. 2011, although they use bargaining, not posting).

\textsuperscript{16}In the longer annual sample, $\mathbb{E}_i = 3.83\%$, $L_{\mathbb{E}_i} = 0.257$ and $\eta_{\mathbb{E}_i} = -0.105$. Using this instead
does not affect the results much. We also tried truncating the sample in 2004, to better match the
pricing sample, and to eliminate the financial crisis; that did not affect the results much either.
On the fraction of transactions using money and credit, there are various micro data sources. First, in terms of concept, we follow much of the literature by interpreting monetary transactions broadly to include cash, check and debit card purchases. As a rationale, first, checks and debit cards use demand deposits, which very much like currency are quite liquid and pay basically no interest, and it does not matter for the theory whether the money is in your pocket or bank account. Second, for us, a key feature of credit is that it allows buyers to pay for BJ goods by working in the next AD market, while cash, check and debit purchases require working in the previous AD market, and this can matter a lot when transaction are random. Third, this notion of money in the micro data is consistent with our use of M1J in the macro data.

Older calibrations of monetary models (see Cooley 1995, chapter 7) use a target of 16% for credit purchases, but these days there is much more information. In highly detailed grocery-store data from 2001, Klee (2008) finds credit cards account for about 12% of purchases, but that is only for groceries. In 2012 Boston Fed data discussed by Bennett et al. (2014) and Schuh and Stavins (2014), credit cards account for 22% of purchases in the survey and 17% in the diary sample. In Bank of Canada data discussed by Arango and Welte (2012), the fraction of credit card purchases is 19%. While there are some differences across these studies, the numbers are not especially far apart, and we target 20%. Also, this number does not vary too much over time, where the bigger evolution has been out of checks and into debit cards, not between money and credit as defined here.17

The calibration also uses two moments from the pricing data: the average duration and the average absolute change. Heuristically, these observations pin down the

17To be clear, the numbers discussed above are for the share of credit transactions by volume, not by value. In Bank of Canada data, the fraction of credit card purchases by value is 40%, double the fraction by volume, since cash tends to be used for smaller transactions as predicted by theory: “Cash [accounts] for 76 per cent of all transactions below $15, and for 49 per cent of those in the $15 to $25 dollar range ... whereas credit cards clearly dominate payments above $50” (Arango and Welte 2012). But in Boston Fed data the fraction by value is 16%, actually less than by volume. There is no consensus as to why American and Canadian data differ on volume, but in any case, we use volume, where they agree.
arrival rates $\alpha_1$ and $\alpha_2$, although there is not a simple mapping from each parameter to a moment, and moreover we cannot hit all the targets exactly, but minimize the distance between model and data. For the price-change data, we focus mostly on Klenow and Kryvtsov (2008), where average duration ranges between 6.8 and 10.4 months, with an average of 8.6. We benchmark 8.6, but also consider alternatives, since there is variability across studies (e.g., Nakamura and Steinsson 2008 report 8 to 11 months). The average absolute price change is 11%, above average inflation because there are many negative changes. Also, since the price data are monthly, we let the period in the model be a month, and aggregate model-generated money demand to a quarterly frequency for comparison with the Lucas-Nicolini data.\footnote{We mention that our method is different from Head et al. (2012), where they calibrate the analog to $\sigma$ jointly with the curvature of $u(y)$ to match the $p$-change distribution, then see what the model predicts for duration. In principle, we thought there was more discipline in evaluating model performance by its ability to match the price-change distribution after calibrating $\sigma$ to duration, although in practice it does not matter much.}

### 7.2 Findings

Calibration results for the fixed- and variable-cost models are in Table 1. In choosing parameters we are constrained to stay in the region where MME exists. Consider first the fixed-cost model, this allows us to hit most of the targets, but not the fraction of credit trade. In particular, trying to set $\delta$ small enough to get 20% credit transactions in BJ implies MME does not exist at $E\hat{\delta} = 4.80\%$. Hence, we use the smallest $\delta$ that admits MME, which only gives 6.5% credit transactions. In any case, the utility parameter $\mu$ is much larger than $\delta$, which intuitively comes from matching average real balances. The value of $\gamma$, about half of $\mu$, comes primarily from the markup. The probability of sampling one price is $\alpha_1 = 0.18$, and the probability of sampling two is $\alpha_2 = 0.20$, which are low, due to the monthly calibration.\footnote{As usual, a convenient feature of search models is they can be fit to different frequencies simply by scaling parameters like arrival and discount rates.}

Consider next the model with a variable cost of credit. The parameters are calibrated similarly, but this model can simultaneously match all of the targets, including the 20% share of credit transactions, without violating conditions for the
existence of MME. The values of utility $\mu$ and cost $\gamma$ are smaller than in the fixed-cost model, while the probabilities of sampling one and two prices are similar to their fixed-cost counterparts. Both models generate almost the same tie-breaking rule, which implies that sellers who are indifferent to changing prices stick to their incumbent price 90% of the time.

<table>
<thead>
<tr>
<th></th>
<th>BJ utility $\mu$</th>
<th>BJ cost $\gamma$</th>
<th>credit cost $\delta/\tau$</th>
<th>$pr(n = 1)$ $\alpha_1$</th>
<th>$pr(n = 2)$ $\alpha_2$</th>
<th>tie-break $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fix</td>
<td>97.39</td>
<td>55.45</td>
<td>0.912</td>
<td>0.18</td>
<td>0.20</td>
<td>0.90</td>
</tr>
<tr>
<td>Var</td>
<td>21.47</td>
<td>11.53</td>
<td>0.063</td>
<td>0.12</td>
<td>0.17</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table 1: Baseline Calibrations

Figure 6 shows predicted money demand, with the solid curve coming from the fixed-cost model and the dashed curve coming from the variable-cost model. The fit is good in both cases, as is key for evaluating the model from a macro perspective, although note that the curves are somewhat different. In particular, the variable cost model yields lower $z$ at small $i$, which could be important for some issues (e.g., the welfare cost of inflation) but seems less important for our purposes. In terms of micro evidence, Figure 7 shows the predicted $p$ change distribution along with the Klenow-Kryvtsov data. Both models more or less capture the overall shape,
although the fit is obviously not perfect. Importantly, however, the models are broadly consistent with several observations deemed important in the literature.

Figure 7: Distribution of Price Changes

In particular, the average absolute change is 11.1% in the fixed-cost and 12.3% in the variable-cost model vs 11% in the data (again, we cannot hit the calibration targets exactly, but get close). The fraction of small changes (below 5%) is 44% in the data, 32% in the fixed- and 30% in the variable-cost model, not dramatically different.\textsuperscript{20} The fraction of negative changes is 37% in the data, 42.9% in the fixed cost model, and 43.4% with variable cost, again not too far off. As mentioned above these observations are hard to match with other models. The search model does fairly well without complications, like heterogeneity, idiosyncratic shocks, etc. Basically, a shifting $F_t$ combined with our tie-breaking rule calibrated to average duration naturally generate large average, many small and many negative adjustments.

Figure 8 shows the hazard (the probability of changing $p$ as a function of duration since the last change) from Nakamura and Steinsson (2008) and from the model,\textsuperscript{26}Eichenbaum et al. (2015) find a fraction of small (below 5%) prices changes much lower than other studies, and suggest that this is because others did not correct for measurement error. For this exercise we take the Klenow-Kryvtsov numbers at face value, but it is worth mentioning that their fraction of small changes may be too high for this reason.
Figure 8: Price Change Hazard over 18 Months

with either fixed or variable cost, as they are basically identical on this dimension. Theory does not generate enough action at low duration, clearly, but at least the model hazard slopes downward, something that Nakamura and Steinsson say is hard to get in other models. One should not expect to explain every detail, of course, as there is presumably a lot going on in the market that is not in the model – e.g., experimentation by sellers trying to learn market conditions – that can yield more changes at low durations. Our hazard decreases for a while, without such complications, although eventually it turns up since with positive inflation any $p$ falls off the support $\mathcal{F}_t$ in the long run. Yet even at 10 years, the model hazard is only up to 14%, so some firms can stick to prices for a very long time, easily as long Cecchetti’s (1986) mucilaginous magazines mentioned in Section 2.

Because there is a range of findings in the empirical literature, Figure 9 shows results of matching average durations of 1, 4, and 24 months, as well as choosing $\sigma$ to minimize the sum of squared errors between the model-predicted and empirical distributions, which implies a duration of 15.9 months. The overall shape of the distribution does not change a lot, but the fraction of small changes goes up and the fraction of negative changes goes down at higher durations, contributing to a better overall fit. Figure 10 shows results of different inflation rates. The overall shape of the distribution does not change a lot, but the fraction of small changes
goes up with inflation while the fraction of negative changes goes down. The bigger point is that in this model pricing behavior is not invariant to inflation.

Figure 9: Distribution of Price Changes for Different $\sigma$

Figure 10: Distribution of Price Changes for Different $\pi$
We conclude from this that, while the models miss some details, they perform fairly well in terms of the evidence. It would be hard to say there is anything especially puzzling about the behavior of \( \pi \) changes in the data – it is pretty much what to expect from basic search theory. It would be even harder to say there is anything informative about Mankiw costs or Calvo arrivals in the data, given the outcomes generated by models with no such ingredients. We also emphasize the quantitative discipline imposed in this exercise by the macro and micro facts on money and credit. At least the variable-cost model matches these facts well.

8 Participation and Policy

In the model with \( k = 0 \), all buyers enter the BJ market, and so the arrival rates \( \alpha_n(b) \) are fixed exogenously. Since \( y \) is indivisible BJ output in each transaction is fixed, too, and AD output is fixed by \( U'(x) = 1 \). Hence changes in the level of \( M \), as well as changes in \( \pi \) and \( i \), have no impact on output: monetary policy is neutral, as well as superneutral. However, the specification can be amended in various ways to make output endogenous and show how the source of price stickiness might matter. A simple approach, following Liu et al. (2011), is to consider \( k > 0 \), so that the measure of BJ buyers \( b \) is endogenous, with arrival rates adjusting until the marginal entrant is indifferent.

First note that if prices were sticky for Calvo or Mankiw reasons, a one-time unanticipated jump in \( M \) can have real effects. This is because at least some firms could not (with Calvo) or would not (with Mankiw) adjust \( p \), and so the nominal distribution \( F(p) \) may not change enough to keep the same real distribution \( G(q) \). If \( M \) increases, one should expect the real distribution to turn in favor of buyers. This increases \( b \), as households embark on a shopping spree, and this stimulates output. By contrast, in our model, jumps in \( M \) shift \( F(p) \) but not \( G(q) \), and hence they affect neither participation nor output. A policy pundit in our economy, seeing only a fraction of sellers adjusting \( p \) each period while \( E\pi \) rises, may well conclude
that changes in $M$ will have real effects. That would be a mistake. And recognizing this as just another example of the Lucas critique makes it no less relevant.

![Figure 11: Real Balances and Free Entry](image)

While $M$ is irrelevant, with endogenous entry, $\pi$ and $i$ are not. To see this, consider parameterizing the arrival rates in a simple way by having BJ buyers attempt to solicit two price quotes, and succeed in each try with probability $\lambda(b)$, where $\lambda(0) = 1$, $\lambda(\bar{b}) = 0$, $\lambda'(b) < 0$ and $\lambda''(b) > 0$. This captures the standard congestion effects in search models, and implies $\alpha_1(b) = 2\lambda(b)[1 - \lambda(b)]$ and $\alpha_2(b) = \lambda(b)^2$. Inserting these into (12), we get

$$\hat{z}_i = \gamma + (2\delta)^{1/3}[1 - \lambda(b)]^{2/3}(\mu - \delta - \gamma)^{2/3}i^{-1/3},$$

(20)

defining a relation between $\hat{z}_i$ and $b$ called the RB (real balance) curve. Similarly, (10) defines a relation called the FE (free entry) curve,

$$\kappa = \lambda(b)^2(\mu - \delta - \gamma) - \frac{\delta[1 - \lambda(b)]^2(\mu - \delta - \gamma)^2}{(\hat{z}_i - \gamma)^2} + \delta - i\hat{z}_i.$$

(21)

As Figure 11 shows, RB is increasing and convex, with $\hat{z}_i = \gamma$ at $b = 0$, while FE is upward sloping to the left and downward sloping to the right of RB, with $b \in (0, \bar{b})$ at $\hat{z}_i = 0$. Hence there is a unique intersection of FE and RB. Now consider an increase in $i$, which ultimately comes from higher inflation and hence higher monetary expansion. This shifts both curves toward the origin, reducing $b$
and output. Monetary policy matters, although this has nothing to do with nominal rigidities – it is in fact due to higher $\pi$ and $i$ taxing households’ participation in decentralized exchange. Indeed, we think this is a good environment to study such effects, in general, since consumers in this model can substitute out of cash and into credit as $\pi$ and $i$ increase. However, at the risk of repetition, the bigger points are these: (i) whether money is neutral or superneutral in reality, observations of sticky nominal prices do not constitute definitive evidence on the matter; and (ii) the underlying reason for sticky prices can make a huge difference.\textsuperscript{21}

9 Conclusion

One contribution of search theorists generally is to show how some observations that appear anomalous from the perspective of “standard” theory can be understood once frictions are modeled explicitly. A leading example is price dispersion, with deviations from the law of one price emerging naturally in some versions of search equilibrium. Something similar is true for price rigidity. It is a serious puzzle for “standard” theory when sellers let their real prices vary in arbitrary ways by not responding to changes in monetary or other conditions. Yet once one understands how frictions lead to price dispersion, price stickiness is not a big leap. Theory predicts that letting real prices vary in arbitrary ways over some range does not affect profits. This is not because of a knife-edge specification where the profit function just happens to be flat; it is because equilibrium forces shape the price distribution so that profit is endogenously the same for any $p$ over an interval. This paper exploits that insight to build general equilibrium monetary models where sticky nominal prices emerge endogenously.

Constructing rigorous monetary models is nontrivial – indeed, money is another anomaly for “standard” theory that can be better understood once frictions are

\textsuperscript{21}We can also calibrate the model with endogenous participation if we specify a matching technology $\lambda(b)$. This can introduce a new parameter, plus there is the entry cost $k$, but we also lose two parameters, since $\alpha_1$ and $\alpha_2$ are now endogenous. Of course, given same $\alpha_1$ and $\alpha_2$ as in the case where we calibrated these directly, the results will be identical.
modeled carefully. The approach here is based on lessons from the literature. We build on Lagos-Wright because it is easy and flexible, and it has proved useful in other applications. Into this setting we inserted a Burdett-Judd goods market. Then we added costly credit, because it naturally avoids the indeterminacy of monetary equilibrium, and of course because it is interesting for its own sake. With either a fixed or variable cost of credit, there emerge nice, novel money demand functions that fit macro data well. We can also do a reasonable job of emulating the empirical pricing behavior, with long durations, large average absolute changes, many small and negative changes, a decreasing hazard and, importantly, repricing decisions that change with inflation.

One finding that we did not anticipate is that the fixed-cost model performs less well quantitatively, mainly because it is harder to satisfy the parameter conditions for MME. Consequently, in calibrating that version, we gave up on matching the share of credit in the payments data. To say it differently, given the other parameters, if we increase inflation very much in the fixed-cost model, consumers stop using money and switch entirely to credit. While some people see this as the wave of the future, at least for now, money is still very much in circulation.\textsuperscript{22} We do not think it is good that the fixed-cost model it predicts a flight from money at moderate inflation, even if this can happen at high inflation, as in dollarization episodes where the payment instrument switches from local to foreign currency. However, we do not take that prediction literally. In reality there are some buyers that always use cash and sellers that only take cash, for a variety of reasons not included in the model, and they will continue to do so at reasonably high inflation. This suggests that it would be useful to incorporate heterogeneity into the framework. While we emphasize that the variable-cost model does not have this problem, as it allows MME at relatively high inflation, but even in that version it may be interesting to add heterogeneity in future research.

\textsuperscript{22}This is especially the case when one interprets monetary exchange to include check and debit card usage, as discussed above, but it is also true for money more narrowly defined as coins and currency.
References


