# Estimating Airlines' Dynamic Price Competition

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**Abstract**

The paper studies implications of dynamic price competition in differentiated products markets in which firms are capacity constrained. The analysis is performed in the context of the airline industry. The paper develops a structural dynamic oligopoly model in which firms perform dynamic pricing to: (i) price discriminate across heterogeneous buyers arriving at different points in time, (ii) smooth the impact of stochastic demand fluctuations on capacity utilization. The supply and demand are jointly estimated using a unique daily-level data on flight prices and capacity utilization. The identification leverages a natural experiment, which involves carrier exit and monopolization in several focal airline markets. The estimates show that demand for airline seats exhibits large degree of temporal heterogeneity and stochastic variability. The counterfactual experiments show that, in competitive airline markets, the ability to perform dynamic pricing increases total welfare. In particular, (i) the ability to price discriminate significantly increases profits. This increase highlights that the price discrimination in the airline industry is predominantly driven by the collective incentives to "expand the market", and less so by the private incentives to "steal each others business". (ii) The ability to smooth demand fluctuations has small and mixed effects on profits, but significantly benefits both early- and late-arriving consumers. This is because selling the capacity "too soon" softens competition in the early market and decreases supply in the late market. I find that extra profits resulting from the ability to price discriminate are significantly larger than the extra profits resulting from throttling capacity. This signifies the importance of exploring temporal consumer heterogeneity, in addition to temporal stochastic demand fluctuations, when designing revenue management systems.

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# 1. Introduction

Many industries, such as airlines, car rental, hospitality, and event tickets, are characterized by the following features: (i) firms are capacity constrained, (ii) products are perishable and (iii) demand uncertainty is significant compared to the scale of production. In these industries, capacities are frequently predetermined and marginal cost is low relative to consumer's valuation, thus unsold products are wasted. To improve profitability, firms have developed sophisticated dynamic pricing strategies. As the leading example, airline industry pioneered a well-known dynamic pricing practice called *revenue management*. In early 1980s, American Airlines developed a dynamic pricing system called Dynamic Inventory Optimization and Maintenance Optimizer, which maximizes revenue through real-time capacity control. Over the next year, American's revenue increased by 14.5%. Soon after American's success, all other major airlines and even firms in other industries implemented similar systems. Recently, the revenue management industry is estimated to grow from $9.27 billion in 2015 to $21.92 billion by 2020, at a compound annual growth rate of 18.8%.

In addition to price patterns designed to smooth capacity utilization, the prevalent fact about airlines price paths is that airfares increase as the departure date approaches. In particular, the same seat is priced more than two times higher one week before departure than five weeks before departure suggesting that the value of the seat increases over time. This is in contrast with pure capacity-based pricing, in which the value of a seat should decrease as the departure date approaches, because of the diminishing option value to sell it in the future. This discrepancy suggests that early-arriving customers may be different from those arriving late, and the increasing price pattern is a result of price discrimination.

This paper attempts to provide a better understanding on profit and welfare implications of airlines' dynamic price competition. Both revenue management and price discrimination clearly benefit monopoly firms. However, in a competitive environment, the consequences of both practices for firms' profits and consumers' surplus are theoretically ambiguous or unknown.[1] Since the impact of dynamic pricing on profits and consumer surplus have clear managerial and regulatory implications, I approach this question empirically. In particular, I estimate a rich demand and supply model that encompasses both capacity-management and price discrimination in the competitive setting. Subsequently, I use the estimated model to perform counterfactual experiments quantifying the impact of both capacity-management and price discrimination on profits and consumer welfare.

For the purpose of answering my research question, I manually collected real-time airfares

[1] Sophistication is typically good in a system of only one strategic player, but may lead to undesired outcomes when multiple sophisticated players compete against each other. See Cabral and Villas-Boas (2005) on Bertrand Supertrap, and Holmes (1989) on price discrimination.

and capacity utilization from the official websites of several major airlines. My data set covers 225,704 observations of daily prices and inventories from 4,550 flights in 10 directional routes for a period of 6 months. The time period for the data collection was chosen to contain a natural experiment of carrier exit, which resulted in route monopolization. This variation enables me to obtain identification by implementing a structural version of difference-in-differences research design. Exogenous price variation in a small time window before and after carrier exit identifies the demand elasticity in a similar way as in the regression discontinuity design.

Conveniently, the exit was scheduled one year before it actually took place. Thus, unless the carrier was able to predict aggregate market shocks one year beforehand, the exact timing of the exit should be uncorrelated with such shocks. The exit may, however, be correlated with aggregate demand trend. To control for the demand trend before and after the exit, I supply the market with exit (a treated market) with a control market. The control market is a duopoly that had the closet average price path to the treated market before the exit event. Consequently, I employ a two-by-two design with four conditions (control/treatment$\times$before/after the exit). The real-time data on pricing and capacity utilization enables me to estimate rich heterogeneity in consumer demand, which is necessary to separate the price variation driven by revenue management from that driven by price discrimination.

I start by examining the raw data to show that the prices before the exit event follow similar paths in both control and treated market before the exit. Next, I demonstrate that after the exit, the price increases significantly in the treated market, but not in the control market. The reduced form analysis also demonstrates rich intertemporal variation in prices, quantities and capacity utilization.

The estimation results show that the demand elasticity varies significantly across time as the departure date approaches. For a typical flight, own demand elasticity is equal to 1.7, measured 7 weeks before departure. The elasticity decreases to approximately 1.0, measured 1 week before departure. Industry demand elasticity, and cross elasticities also decrease over time, ranging from 0.6 to 1.3, and from 0.12 to 0.75, respectively. These results suggest that flights by different carriers departing on the same day are far from perfect substitutes. Moreover, the numbers imply that the market power increases over time, with decreased incentives for business stealing and less substitution to the outside option. I also find that the demand elasticities vary across firms, for instance, Alaska tends to have bigger market power in late markets than JetBlue. Lastly, on average, high willingness-to-pay (high type) consumers also have stronger brand preference.

I use these estimates to perform a series of counterfactual experiments to study how airlines' dynamic price competition affect consumers' surplus and firms' profits. In the first counterfactual, I shut down the dynamic pricing by assuming that the airline must charge a constant price as departure date approaches. I find that constant price restriction would increase consumers' surplus by \$419.32 (or 1.27%) per flight. This is because high type consumers gain from a uniform price more than low type consumers lose from such policy. Charging a constant price would, however, decrease the airlines profits by \$2126 (or 14.62%) per flight and \$1607 (or 8.32%) per flight. Overall, dynamic pricing increases total welfare by 4.96%.

The above counterfactual estimates joint impact of capacity-smoothing and price discrimination. In a second counterfactual, I isolate the effects of airlines' ability to smooth demand fluctuations. I find that when airlines are unable to adjust prices based on stochastic demand, one airline's profit decrease by \$297.79 (or 2.05%) per flight and the others increases by \$81.05 (0.42%) per flight. Total consumer surplus decreases by as much as \$3567.45 per flight (10.80%). These findings on firms' profits and consumers' surplus are driven by two factors: (i) inefficient allocation of seats to consumers and (ii) reduced competition. The variance of demand is large, thus, without capacity throttling, the probability of selling out "too soon" is high. This possibility lowers firms' incentives to increase sales by competing in early market prices. As a result, early market prices are higher, and early arriving consumers are worse off. Moreover, because firms cannot throttle capacity, some flights do sell out too soon, thus, the supply to high type consumers is under-allocated. Consequently, late-arriving customers (high types) are worse off as well.

In a third counterfactual, I single out the effects of airlines' competitive price discrimination by removing airlines' capacity constraints. In a world without capacity constraints, the only motivation for dynamic pricing is price discrimination. I find that in this world, price discrimination decreases consumers surplus by \$1460 (or 3.3%) per flight. However, it increases firms' profits by \$2422 (20.0%) per flight and \$1021.92 (5.7%) per flight. All in all, the ability to price discriminate has much larger impact on profits than the ability to throttle capacity. This has significant managerial implications, since both scientific and popular literature on dynamic pricing tend to emphasize capacity throttling over price discrimination. My results suggest that increasing emphasis on price discrimination may be beneficial.

The literature on capacity-constrained dynamic price competition is scant. "*This is due, in part, to the challenges imposed by the complex game of capacitated intertemporal price competition, and even more thorny problems of time-varying demands*" (Gallego and Hu (2014)). To the best of my knowledge, the most related structural model is Sweeting (2015) who investigates dynamic price competition with demand uncertainty in event ticket resale market from a platform design perspective. Sweeting (2015)'s model is not immediately applicable to the airline context. First, the event ticket resale markets typically have large numbers of sellers with little market power, whereas the airline markets typically have a small number of players with considerable market power.[2] Second, Sweeting (2015) abstracts from consumer heterogeneity and price discrimination. This choice reflects the reality of the resale market for event tickets, but is likely to be inadequate for modeling the airline industry.

To encompass the competition amongst a relatively small number of strategic firms, this paper develops an equilibrium model of dynamic oligopoly following Maskin and Tirole (1988) and Ericson and Pakes (1995). The model can be used to investigate the strategic interactions of oligopoly firms when selling perishable goods to heterogeneous consumers under capacity constraint and demand uncertainty. In the model, multiple differentiated and perishable asset providers start from initial capacities and compete simultaneously in prices at each period until a common deadline. The firms play Markov strategies that are contingent on time and capacities. The model's demand system is built on two classic literature: (i) Berry et al. (1995) (BLP hereafter) for modeling consumer heterogeneity through latent taste shocks and price endogeneity through unobserved product-specific demand shocks; (ii) Gallego and Van Ryzin (1994) for modeling demand uncertainty through stochastic Poisson arrival process. The model maintains simplicity but still captures the first order effects in airlines dynamic price competition including product differentiation, discriminatory pricing, and scarcity pricing. In each period, consumers arrive as an exogenous Poisson process whose distribution is known to firms. Importantly, firms do not observe the number of consumers. Conditional on arrival, each consumer's preference is a random draw from an exogenous distribution. Consumers can differ in both their price sensitivities and their valuations towards each firm. Firms do not observe consumers' types. However, since the distribution of consumers' types is a function of time known to firms, firms can still price discriminate basing on consumers' arrival times. Random coefficient preference under stochastic arrival can easily be untrackable both conceptually and computationally. In solving this, the paper recognizes that a multinomial choice process (such as BLP) conditional on a Poisson arrival process is equivalent to mutually independent Poisson processes of sales (Proposition 1). This says that one can analytically integrate the arrival process and the choice process into closed-form sales processes. Perhaps counterintuitively, these sales processes are mutually independent. It reduces a problem of order $(N^K)^J$ to $N^J$, where $N$ is the number of arriving consumers, $K$ is the number of consumer types, and $J$ is the number of alternatives. This observation makes it computationally feasible to incorporate rich heterogeneity in consumer types.

To account for price endogeneity, the demand model allows for product specific demand

[2] Sweeting (2015) uses Non-stationary Oblivious Equilibrium (Weintraub et al. (2008)).

shocks observed by firms but not by researchers. When doing so, the paper cannot apply standard tool for demand estimation, that is, BLP market share inversion. The market share inversion works well in cases, where the sales data is aggregated, usually over time, so that firms markets shares contain little measurement error. However, since the airline demand and pricing changes daily, and one of the goals of this paper is to capture these changes, normal levels of aggregation may be in my case excessive. An appropriate daily level of aggregation that retains the pricing patterns results in mismeasured market shares, that are frequently equal to zero or are even undefined if no firms sells during a particular day. Instead of applying BLP directly, this paper considers an alternative GMM estimator in which demand and supply is estimated jointly using a nested fixed point approach reminiscent to Rust (1987). In the inner loop, I solve the dynamic system of demand and supply into its reduced form for a selected set of equilibrium outcome variables of interest conditional on all observed and unobserved states. In the intermediate loop, I integrate out all unobserved states including unobserved demand and supply shocks.[3] In the outer loop, I match the moments from the model to the corresponding moments from the data. Such approach requires me to solve a dynamic stochastic game with large state space, including unobserved states. To lower the computation burden, I use Gauss-Hermite quadrature to numerically integrate out unobserved demand and supply shocks. To further reduce the state space, I use cubic interpolation to interpolate firms' value functions over capacity states. Finally, I use *Julia*'s parallel computation on a multi-processor server.

## *1.1. Related Literature*

This paper is related to several streams of literature. Firstly, this paper contributes to the understanding of airlines' dynamic price competition. Airline industry is in essence an oligopoly market. Its price competitions have received great attention from the policy makers. Surprisingly, not many structural papers have examined airlines' price competition. As a notable exception, Berry and Jia (2010) treat airline price price competition as a static problem under BLP framework. Recently, Lazarev (2013) and Williams (2013) extend airline demand estimations into monopoly dynamic pricing frameworks. The current paper is the first structural paper on oligopoly airline dynamic pricing and it provides a useful picture of demand and supply dynamics underlying oligopoly airline markets.

Most of other empirical work on airline price competition are based on reduced-form analysis. Among these, an important stream of research investigates how competition affects

[3] The possibility of unobserved demand and supply shocks makes two-step methods, such as Bajari et al. (2007), hard to apply. To see that note that Bajari et al. (2007)'s first-stage estimator would be a hedonic price regression, which is known to fail if unobserved demand characteristics are present.

airlines' price discrimination/dispersion. In a seminal work, Borenstein and Rose (1994) show a striking result that airfare dispersion increases with competition. In a following work, Gerardi and Shapiro (2009) find the opposite effect.[4] The current paper studies this via counterfactual experiments and highlights the mechanism behind. The results show that competition increases price dispersion due to the time-varying brand preferences (Borenstein (1985)). In particular, competition lowers early market prices much more than it does to late markets' prices. As an interesting new result to this literature, one airline even increases its prices in late market after a competitor enters the market. The intuition follows Rosenthal (1980). When the airline is a monopoly, it lowers its price to cover the whole market. However, when a competitor enters, rather than maintaining a low price and subsiding its loyal segment, the airline gives up the non-loyal segment and raises price to sell only to its loyal segment.[5]

Secondly, this paper adds to a small body of empirical literature on competitive price discrimination, for instance, Besanko et al. (2003), Villas-Boas (2009), Hendel and Nevo (2013), etc.[6] In the current paper, consumers can differ in both their valuations and their brand preferences. The former determine the industry-demand elasticities, i.e., tendency to drop out the market; whereas the later determine the cross-demand elasticities, i.e., tendency to switch between competitors. Without capacity constraint, the ability to price discriminate may increase or decrease airlines' profits and social welfare depending on the ratio of the two (Holmes (1989) and Corts (1998)). This paper shows empirically that early-arriving consumers' cross-demand elasticities are low relative to their industry elasticities, thus airlines' price discrimination is more motivated by their collective incentive to sell to more travelers and fill in the capacity instead of private incentives to undercut and steal business from each other. Therefore, oligopoly airlines' price discrimination benefits airlines and increases social welfare.

Thirdly, this paper relates to the literature on dynamic competition with capacity constraint. IO theorists have long recognized that quantities are limited and capacity constraint has important implications on firms' strategic interactions (Edgeworth (1925)). Related to the current work, theorists have looked at multistage price and quantity games (Kreps and Scheinkman (1983) and Davidson and Deneckere (1986)) and repeated pricing games where firms face capacity constraints and demand uncertainty (Staiger and Wolak (1992)). Re-

[4] See also Stavins (2001), Gaggero and Piga (2011),Siegert and Ulbricht (2015), Chandraa and Ledermana (2016) etc. Outside the airline industry, empirical work has documented that competition is associated with an increased curvature in the price schedule under second-degree price discrimination, see for instance, Busse and Rysman (2001), Borzekowski et al. (2009),Courty and Pagliero (2012), etc.

[5] See Iyer and Pazgal (2003) for a related discussion.

[6] See Stole (2007) for a summary.

cent empirical work have studied capacity constrained dynamic auction game (Jofre-Bonet and Pesendorfer (2003) and Jeziorski and Krasnokutskaya (2016)) and demand fluctuation in concrete industry (Collard-Wexler (2013)). Finally, this paper is related to the revenue management literature. The literature has traditionally focused on monopoly cases. As a notable exception, Gallego and Hu (2014) theoretically analyze a revenue management game similar to the current one.

The plan of this paper is as the following: Section 2 presents the empirical setting with some reduced form evidence that motivates the structural model. Section 3 sets up the model. Section 4 discusses the empirical strategies and identifications. Section 5 shows estimation results. Section 6 performs counterfactual analysis. Section 7 concludes.

# 2. Empirical Setting

## *2.1. Airline Industry*

Airline industry is of significant importance in modern economy. In terms of volume, it contributes to $1.5 trillion U.S. economic activity and helps create more than 10 million U.S. jobs.[7] In addition to its massive economic scale, airlines are also arguably the most sophisticated practitioners of dynamic pricing. It has thus received tremendous attention from both policy makers and the academia.

Airlines pioneered in the development of revenue management. In 1970, British Airways offered "early bird" discounts to consumers who bought tickets at least four months in advance. In 1977 shortly before the deregulation of U.S. airline industry, American Airlines initiated an inventory-based pricing system called Dynamic Inventory Optimization and Maintenance Optimizer. As the inventor of the first large-scale dynamic pricing system, American gained great competitive advantage against their competitors. According to American Airlines' own report, it generated 1.4 billion in additional incremental revenue over a three-year period starting around 1988 (Smith et al.). This system also contributed to the bankruptcy of People Express. The CEO of People Express Donald Burr had famously made the following comments.

[7] http://airlines.org/industry/

> "We were a vibrant, profitable company from 1981 to 1985, and then we tipped right over into losing 50 million a month. We were still the same company. What changed was Americans ability to do widespread Yield Management in every one of our markets... [If I were to do it again,] the number one priority on my list every day would be to see that my people got the best information technology tools. In my view, thats what drives airline revenues today more than any other factor– more than service, more than planes, more than routes."
>
> — Donald Burr, CEO of People Express

After years of intensive development under the rapidly growing information technology and computational capability, today the practice of RM in the airline industry is both prevalent and mature. Revenue management has been viewed as critical to running a modern airline profitably. It has also spread to many other industries such as energy, hospitality, entertainment, broadcasting, transportation, etc. Nowadays, the revenue management market is estimated to grow from $9.27 billion in 2015 to $21.92 billion by 2020, at a compound annual growth rate of 18.8 % during the forecast period.[8]

Airline industry's invention of dynamic pricing system is largely driven by the nature of its supply and demand. Within a reasonable time horizon, airlines are capacity constrained. Given availability, it is believed that the marginal cost of selling one more seat is lower than the marginal consumers' willingness to pay. Since both the number and the valuation of the potential consumers are unknown, aggregate demand is uncertain under a price posting mechanism. The perishability of seats makes airlines pricing problem a dynamic one. On top of the operational marginal cost, airlines need to take into account the option value of a seat. By selling a seat today, airlines lose the possible revenue of selling it tomorrow. On the other hand, if a seat is not sold before its departure date, it lose all value. Thus, given their remaining capacity, the current demand state and the expected future demand, airlines rationally adjust their prices in every period.

Firstly, as perhaps the most well-known industry regularities, airfares are on average more expensive closer to departure date. This is viewed as evidence of intertemporal price discrimination. Generally, leisure consumers with higher price elasticities arrive early, whereas business travelers who are less price sensitive arrive late. Airlines thus are able to screen consumers based on the time of their arrivals. Therefore, with the option of price discrimination, airlines overcharge high valuation consumers thus sell less to them.

Secondly, airlines do not commit to any price path for most of their flights. They frequently adjust prices based on real-time factors in supply and demand. As a revenue manager from one major airline company commented *"Fares are so dynamic since they are based on*

[8] http://www.marketsandmarkets.com/Market-Reports/revenue-management-market-264806846.html

*market conditions and the actual number of passengers who are currently booked on a specific flight that they can change rapidly at any time."*[9] Alderighi et al. (2012) and Escobari (2012) show evidence on airlines' scarcity-based pricing. I also provide discussions in the appendix.

These dynamic pricing systems clearly benefit firms in a monopoly setting. However, its impact under a competitive setting is unclear. On one hand as discussed earlier, it increases airlines' abilities to exploit consumer surplus. On the other hand, it could have intensified airlines' competitions against each other. To mitigate competition, airlines invented loyalty-inducing marketing devices– frequent flyer program (FFP). In 1981, American Airlines introduced the first FFP. Soon after 1986, FFP has spread to all major airlines. It is believed that FFP reduces travelers' cross price elasticities, by encouraging them to buy tickets from a single airline. It increases brand loyalty and switching cost (Borenstein (1992)). In 2002, Norway banned domestic FFP in order to promote competition among its airlines, but lifted the ban later in 2013.[10] On one hand, consumers are complaining about airlines' monopoly powers. On the other hand, airlines are complaining about intensive price competition. In light of this, the degree of differentiation in the airline industry is an important questions that remains to be quantified. The current paper fills in this gap by estimating an equilibrium model of airlines' dynamic price competition in differentiated product markets. I discuss the data and research design in the remaining of this section.

## *2.2. Data Sources*

The classic data set for flight tickets is the 'Airline Origin and Destination Survey" from U.S. Department of Transportation (DB1B). This data reports a 10% random sample of all domestic airline tickets at quarter level. It has been widely used in the airline literature. Unfortunately, this data set is aggregated to quarter level, and it does not report either the pricing date or the purchase date of a ticket. As a result, it does not have enough intertemporal variations to estimate dynamic pricing. Recently, researchers have obtained higher frequency prices and sales data. Lazarev (2013) use a high frequency data set of daily-flight-level prices to approximate transaction prices. Escobari (2012) uses a dynamic panel of seat maps obtained from the internet to approximate daily sales.

The current paper collects a similar data set. Although using posted daily-flight-level prices to approximate transaction prices has been widely accepted, using seat map to approximate inventories is a very recent invention in the empirical airline literature. The biggest concern is that if a consumer did not select a seat at the time of purchase, the data would fail

[9] http://www.foxnews.com/travel/2011/12/08/confessions-airline-revenue-manager.html
[10] http://www.aftenposten.no

to detect that a seat had been filled.[11] To address this concern, Williams (2013) is the first to provide a very careful analysis on the accuracy of the seat map. By comparing collected seat map data with airlines' reported loading factor, he finds that seat maps understate reported load factor by an average of 2.3% at the flight level, with a range of 0-4%. The aggregate error at monthly level is 0.81%. The size of the measurement error seems acceptable. I note that if necessary the current GMM procedure can account for random measurement errors. In the following, I focus on discussing my data collection process.

I define a market as a tuple $\{$departure date, pricing date, origin, destination$\}$. This implicitly assumes that consumers do not substitute across departure dates. This assumption avoids the complication of multi-product pricing. Then I collect a high frequency panel data of daily prices and inventories. Everyday, I manually search for all flights that depart within 100 days in the future. I recover the prices and the remaining seats from the source code of the airlines' official websites. Figure 1 shows an example from one of the airlines. I then discuss my route selection procedure.

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▶<div class="seat seat-core right regularSeat occupiedSeat" title="07C" style="left:189px;top:335px;">…</div>
▶<div class="seat seat-core left regularSeat occupiedSeat" title="07D" style="left:274px;top:335px;">…</div>
▶<div class="seat seat-core mid regularSeat availableSeat" style="left:317px;top:335px;" id="plane_seat_7E" title="07E">
…</div>
▶<div class="seat seat-core right regularSeat occupiedSeat" title="07F" style="left:360px;top:335px;">…</div>
▶<div class="seat seat-core left regularSeat occupiedSeat" title="08A" style="left:103px;top:377px;">…</div>
▶<div class="seat seat-core mid regularSeat availableSeat" style="left:146px;top:377px;" id="plane_seat_8B" title="08B">
…</div>
▶<div class="seat seat-core right regularSeat occupiedSeat" title="08C" style="left:189px;top:377px;">…</div>
▶<div class="seat seat-core left regularSeat occupiedSeat" title="08D" style="left:274px;top:377px;">…</div>
▶<div class="seat seat-core mid regularSeat occupiedSeat" title="08E" style="left:317px;top:377px;">…</div>
▶<div class="seat seat-core right regularSeat occupiedSeat" title="08F" style="left:360px;top:377px;">…</div>
▶<div class="seat seat-core left regularSeat availableSeatNonReclining availableSeat" style="left:103px;top:419px;" id=
"plane_seat_9A" title="09A">…</div>
▼<div class="seat seat-core mid regularSeat availableSeat" style="left:146px;top:419px;" id="plane_seat_9B" title="09B">
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Fig. 1. Source code from an airline's official website

I adopt a systematic route selection procedure. I analyze the DB1B data in the first quarter of 2014 and obtain summary statistics for all U.S. domestic airline markets. Firstly, I exclude all routes where consumers cannot reserve a seat for free at the time of purchase. This is because I will use seat-map to approximate inventories. I exclude all routes where Southwest and Sprint operate. Southwest does not allow any advance seat assignment. Some low cost airlines like Spirit charge consumers for seat reservations. Secondly, I only include routes that roundtrip tickets are priced twice as much as an oneway ticket. This excludes most of American Airlines's routes and many of Delta's. By doing so, the pricing and purchasing behavior of a round trip tickets can be approximately viewed as two oneway tickets. This criteria helps simplify the model and avoid extra complications. Thirdly, I rank the routes by proportions of the travelers that choose non-connecting and non-stop

[11] Additional measurement errors may result from seat blocking and ticket cancellation. I observe the former but not the latter. Strategic seat blocking is not modeled. The possibility of ticket cancellation is also ignored.

flights. I choose only the routes where these proportions are high (79%-95%). When a seat is reserved, it is thus more likely that a nonstop ticket is purchased. Finally, I only include markets with (i) two operating airlines and (ii) on average fewer than two daily flights each firm. As I will adopt a full solution method when estimating the model these reduce the computational burden while keeping the key ingredient of competition.

Meanwhile, I also collected an additional dataset from Google Flight API. I search for more than 15,000 randomly generated pairs of domestic airports. It covers more than 50% of all possible pairs out of 237 major U.S. airports. I use this data set to update the DB1B since it may be out of date. It helps me identify the number of running carriers in all the 15,000 routes. This selection procedure restricts my attention to a final sample of 10 directional routes: Seattle-Tucson, San Diego-Boston, New York-Sarasota, and New York-Aguadilla and New York-San Antonio. In particular, the sample was chosen to contain a route that was expecting an exit event, namely Seattle-Tucson. The operating airlines in the sample include JetBlue, Delta, Alaska and United. The data records all nonstop flights in these routes for a period of six months.

Table 1 reports the summary statistic of the data. The data covers $225,704$ observations, daily-flight-level prices and sales, for $4,550$ flights up to 100 days before its departure date. On average, a flight sells 0.91 seat each day. From day $t+1$ to day $t$, airlines increase price 15% of the time and decrease price 9% of the time. An average flight sells 45 seats since 100 days before departure. Flight-level Gini coefficient in the bottom row captures the intertemporal price dispersion for a given flight. The mean of the flight-level Gini coefficient in my data equals Siegert and Ulbricht (2015), although they study European airline industry whereas I look at the U.S. one. A flight-level Gini coefficient of 0.12 means that an expected absolute difference is 24% between two randomly selected prices for the same flight priced at two different dates.

Figure 2 shows the average path of prices and loading factor by number of days to departure. Loading factor increases relatively smoothly over time. Price increases as it gets closer to departure day. Noticeably, price jumps up at certain threshold such as 4 days, 1 week, 2 weeks, 3 weeks, etc. Prices are relative stationary more than one month before departure date. For later analysis, I will focus on seven weeks (49 days) before departure. The price path looks similar to existing literature. However I note that it is steeper than Williams (2013). If one is willing to extrapolate from his monopoly markets to the current duopoly market, this suggest that competition increase the slope price path. Siegert and Ulbricht (2015) find that the rate at which prices increase over time decreases in competition.

Sales seemed smooth over time. More than seventy percent of the seats are available 100 days before departure. On average, airlines fill in more than 50% of their capacities within

Table 1: Data Summary

| Statistic | Mean | Median | St. Dev. |
|---|---|---|---|
| **Market level** | | | |
| N=225,704 | | | |
| Price ($) | 249.53 | 227.10 | 115.35 |
| Number of remaining seats | 61.52 | 58 | 34.86 |
| Days to departure | 49.17 | 49 | 28.06 |
| Number of seats sold | 0.91 | 0 | 1.70 |
| Probability of price increase | 0.15 | 0 | 0.36 |
| Probability of price decrease | 0.09 | 0 | 0.29 |
| Price change ($) | +3.41 | 0 | 43.22 |
| **Flight level** | | | |
| N=4,550 | | | |
| Average sales | 45.22 | 40 | 33.77 |
| Load factor | 0.83 | 0.86 | 0.16 |
| Gini coefficient | 0.12 | 0.12 | 0.08 |

this time window. In summary, the general patterns of data is very consistent with existing literatures. As we move to the next section, I will discuss the research design of the paper.

Capacity constraint is important in the airline industry. Generally, equilibrium prices are higher when capacities are more constrained (Osborne and Pitchik (1986)). Figure 3 demonstrates this using price and capacity variation for flights between New York and Aguadilla. Each dot is a mean price for a flight on a departure date, and the solid line plots industry total capacity. Before May 4th, JetBlue used an Airbus A320 with 150 seats. They temporarily switched to an Airbus A321 with 190 seats during May 5th to June 15th. Meanwhile on July 1st, UA switched from an Airbus A320 with 138 economy seats to a Boeing 737-700 Micronetia with 106 economy seats. Other fluctuations are because UA sometimes changed their planes conditional on weekdays. I normalize price by its airline-day-to-departure mean because of a missing data issue. My data collection started at Feb-15th and was interrupted for 3 weeks from June-6th to June-27th. I note that this missing data is not random, for instance for a flight departure at June 26th, I only observed its prices more than 3 weeks from its departure. The normalization should take out intertemporal variations in prices. I note that the graph is similar without normalization.

It suggests that mean prices are negatively correlated with industry capacity. Since the endogeneity of capacity is likely to reverse this correlation, this graph provides causal

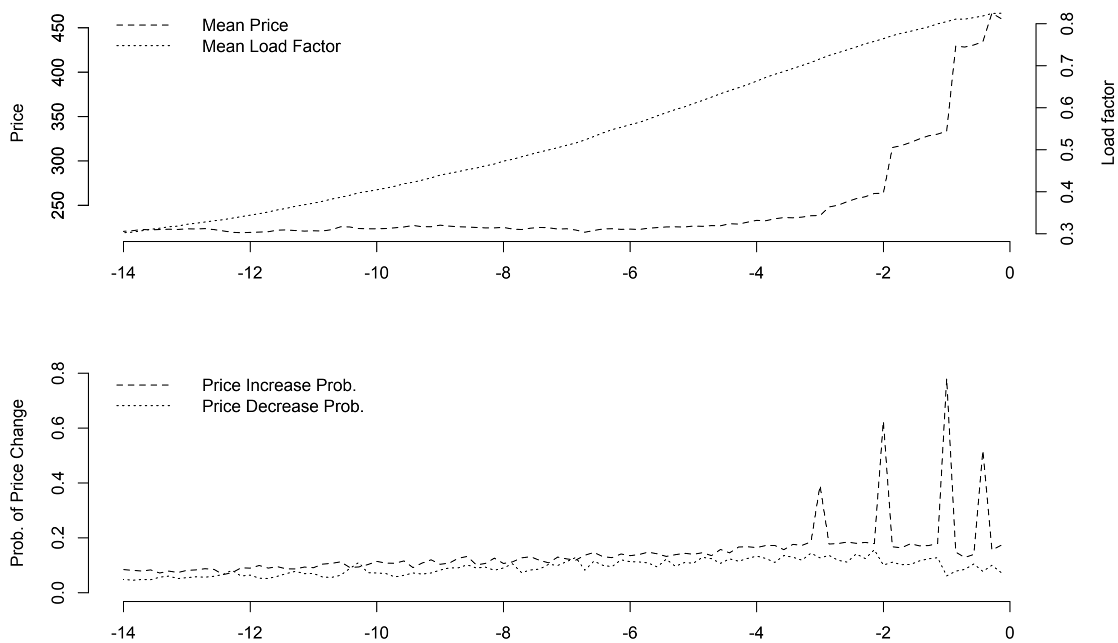



Fig. 2. Average path for prices and sales (N=225,704)

evidence on the role of capacity constraint.

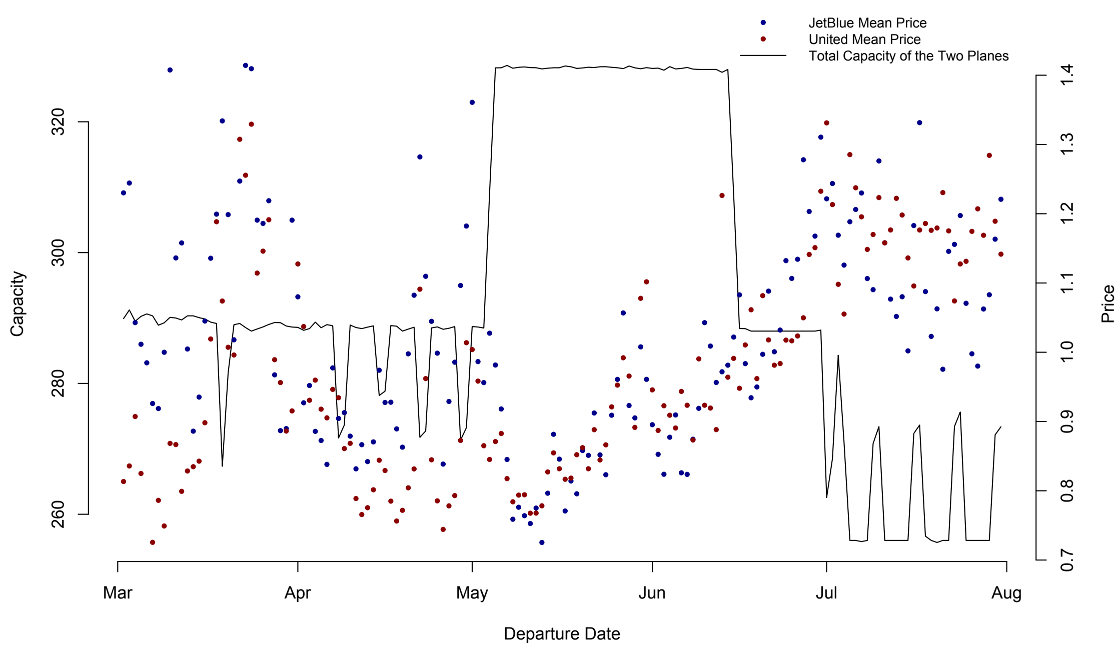



Fig. 3. When airlines change planes (New York-Aguadilla)

### *2.3. Research Design*

A key challenge in demand estimation is to find exogenous variation that can identify preference on endogenous variable, i.e. price. This is especially hard when estimating demand in the airline industry. Aside from well-known limitations, the typical BLP instruments are static. Thus it will necessarily miss the dynamic aspect of the demand and supply system (Berry and Jia (2010)). An alternative that may come to one's mind is daily prices of jet fuel. Although daily oil prices have high frequency and are readily available, it is unclear that how these costs enter airlines' pricing equations. Realtime price adjustments on oil cost, even exist, are likely to be weak for a small sample. Moreover airline is doing fuel hedging. It is even possible that an increase in oil price will reduce airlines' cost.[12]

To overcome this, I leverage the exit event that I included in my data collection. Before Mar 31 2015, both Alaska and Delta offer direct flights between Seattle and Tucson. Alaska has served this route for many years, whereas Delta entered the market only since 2013 and offers a small flight operated by a regional airline, SkyWest Airlines. Delta stops its service after Mar 31 2015. Thus this route changed from duopoly to a monopoly market.

Firstly, I assume that within a small enough time window, exit is uncorrelated with demand shocks in a similar way as in the regression discontinuity design. Specifically, I will only use variations 15 days before and after Delta's exit. The exit was scheduled one year before it actually took place. Thus, unless Delta was able to predict aggregate market shocks within a 15-day window 365 days beforehand, the exact timing of the exit should be exogenous.

Even if the exit is uncorrelated with such route-specific shocks, the exit may, however, still be correlated with aggregate demand trend. To control for the demand trend before and after the exit, I supply the market with exit (a treated market) with a control market. The control market is a duopoly that had the closet average price path to the treated market before the exit event. Specifically, I define a set of treatment conditions $\mathcal{C}$ based on a two-by-two design:

$$\begin{aligned}\mathcal{C} = \Big\{ &\text{exit}= 1, \text{before}= 1\text{: exit route 15 days before exit;}\\ &\text{exit}= 1, \text{before}= 0\text{: exit route 15 days after exit.;}\\ &\text{exit}= 0, \text{before}= 1\text{: control route 15 days before exit.;}\\ &\text{exit}= 0, \text{before}= 0\text{: control route 15 days after exit.}\Big\}\end{aligned}$$

Alaska Airline operates in both the treated and the control market before and after the

[12]https://www.wsj.com/articles/airlines-pull-back-on-hedging-fuel-costs-1458514901

exit event. Figure 4 show the price paths of Alaska in the exit route and in the control route before and after the exit event. It only uses data 15 days before and after the exit event. Before Delta's exit, Alaska's price paths are very similar in the exit route and in the control route. However, Alaska raised its prices significantly in the exit route after the exit event. Interestingly, most of the price increases happened 2-5 weeks before the departure date. This may suggest that competition increases price discrimination. For example, competition is the strongest 2-5 weeks before departure. However, a scarcity-based pricing theory could also explain this.

As shown later in Section 4.3, I will use these market conditions as instruments to generate moment conditions. I make similar assumptions as the reduced-form difference-in-difference models. By assuming parallel trend, I am making both weaker and stronger assumptions. If the parallel trend assumption is valid, this structure will automatically control for omitted variables. My assumption is weaker in the sense that I allow for aggregate common utility shock between before exit and after exit. Thus, the notion that before/after is exogenous sounds more appealing, and I am relaxing the regression discontinuity assumptions. However, if the common trend assumption is misspecified, I will be worse off just like other difference-in-difference research designs. Fortunately, I will allow most parameters to be route specific. This bias is, in a less rigorous sense, bounded by the number of parameters I will jointly estimate.

# 3. Model

The model can be seen as building a dynamic revenue management problem (Gallego and Van Ryzin (1994) ) into the static equilibrium model of supply and demand in differentiated product markets (Berry et al. (1995)).

## *3.1. Demand*

In the following set up, one may think of the products as all flights, for instance, from San Francisco to Boston on Nov $22^{nd}$. The set up shall be applicable to other settings such as hospitality, entertainment, broadcasting, transportation, etc.

Let $t = 1, 2, .., T$ be the selling periods. Thus time is discrete and $T$ is the deadline of selling, after which the product is assumed to have zero value. Each consumer arriving at period $t$ chooses from a choice set $\mathcal{J}_t = \{0, 1.., J_t\}$. $\mathcal{J}_t$ includes all the flights in a specific directional route and on a specific departure date available at time $t$. If a consumer does not choose any flight, then she chooses the outside option 0. The outside option is a reduced

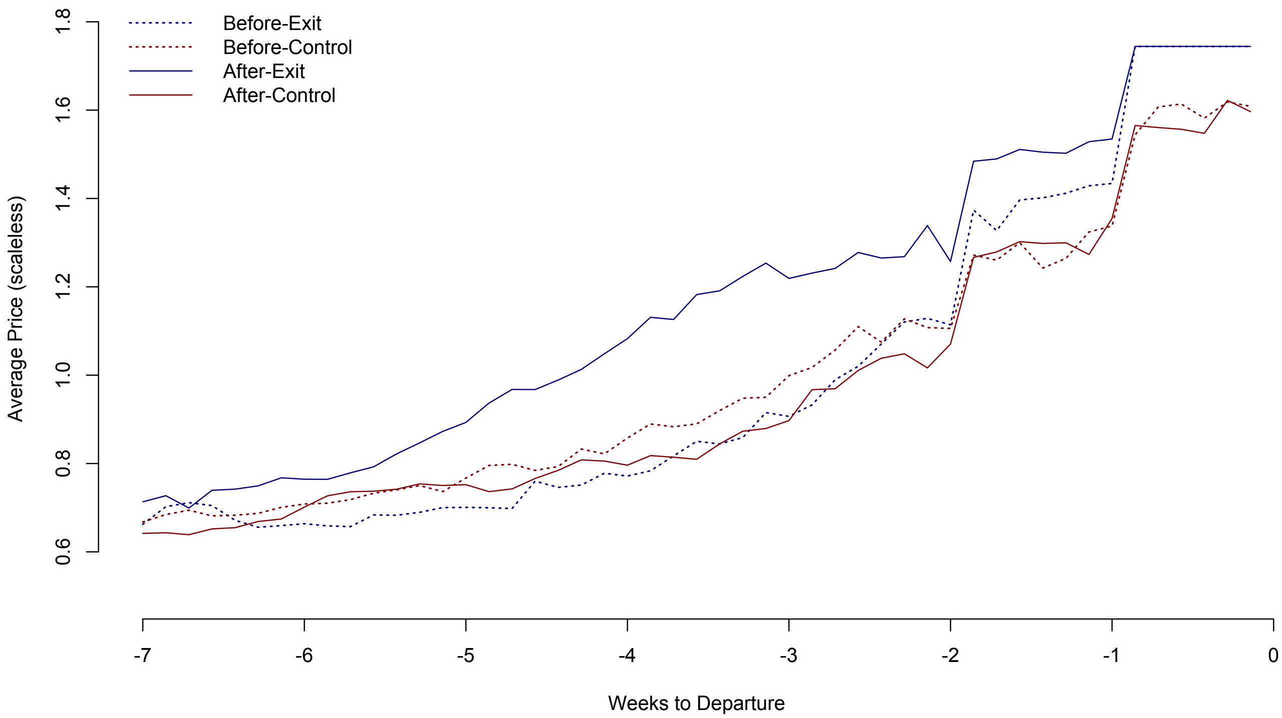



Fig. 4. Alaska's price paths under the four conditions

form way of capturing all other possible alternatives. I assume that consumers live for only one period and will not wait. Not surprisingly, doing this simplifies the model and allows me to focus on the supply side. In fact, this assumption is driven by the data. Given the equilibrium price patterns in Figure 2, a forward-looking consumers would have behaved in a similar way as a myopic consumer. Therefore, forward-looking behavior is hardly identified. However, to the extent that strategic consumers with evolving demand uncertainty can be interesting, I note that the current set up can be extended to incorporate strategic consumers similar to Goettler and Gordon (2011). In that set up, one can allow consumers to learn their travel plans gradually (Dana (1998)). However, I leave this to future research.

Let $M(t)$ be the random variable for the number of consumers arriving at time $t$. I assume that $M(t)$ follows a Poisson stochastic process and the Poisson parameter takes some functional form $\lambda(t; \gamma_{arrival})$:

$$M(t) \ \sim \text{Pois}\left(\lambda(t; \gamma_{arrival})\right) \tag{1}$$

Where $\gamma_{arrival}$ is the parameters on arrival process to be estimated.

Modeling aggregate consumer arrival as an exogenous Poisson process is intuitive. The Poisson distribution is a classic modeling choice for arrival process in many fields, including economics and operation research (McGill and Van Ryzin (1999)). As opposed to Gershkov et al. (2016), I assume this arrival process is known to the airlines. Importantly, the exact number of arriving consumers in any period is unknown to airlines.

I model consumer choice process as generated by a discrete choice model. A consumer $i$ arriving at time $t$ is endowed with preference $\left\{\alpha_{it}, \beta_{it}, \{\varepsilon_{ijt}\}^{J_t}\right\}$, where $\beta_{it}$ measures her preference over product characteristic $X_j$ and $\alpha_{it}$ denotes her price coefficient and $\varepsilon_{ijt}$ is her idiosyncratic choice-specific preference shock. I assume that $\varepsilon_{ijt}$ follows Type-I extreme value and that $\{\alpha_{it}, \beta_{it}\}$ is randomly drawn from some exougenous distribution $F(\alpha_{it}, \beta_{it}; t, \gamma_{type})$, where $\gamma_{type}$ is the parameters on random utilities to be estimated. Note this characterization permits a demand structure that is both heterogeneous within period and non-stationary across time. It can thus be used to study intertemporal price discrimination.

To account for price endogeneity, I allow for product-specific demand shock $\xi_{jt}$ that is observed by market participants but not by researchers. The importance of controlling for unobserved product characteristic has been highlighted in previous literature. Airlines are likely to gather informations for realtime demand. As a result, observed prices are likely to be correlated with these informations. Unfortunately, these informations are not observed by researchers. Thus without an explicit treatment, price elasticities will likely be biased downwards (in absolute value).

Thus, I obtain the classic BLP utility specification that combines consumer heterogeneity through latent taste shocks and endogeneity through product-specific demand shocks:

$$U_{ijt}\big(X, \boldsymbol{p}, \alpha, \beta, \xi, \varepsilon\big) = X_j\beta_{it} + \alpha_{it}p_{jt} + \xi_{jt} + \varepsilon_{ijt} \tag{2}$$

Without loss of generality, let $U_{i0t} = \varepsilon_{i0t}$. I make a simplifying assumption that consumers simply choose the product that maximizes her utility. If the product is sold out during a selling period, then a simple lottery is used to decide who get the remaining seats. However, consumers do not consider this possibility when make purchases. Finally, I obtain the familiar expression for market share:

$$s_{jt}\big(X, \boldsymbol{p}, \xi; \gamma_{type}\big) = \int_{\alpha_{it},\beta_{it}} \frac{\exp\big(X_j\beta_{it} + \alpha_{it}p_{jt} + \xi_{jt}\big)}{1 + \sum_{j'=1}^{|\mathcal{J}_t|} \exp\big(X_{j'}\beta_{it} + \alpha_{it}p_{j't} + \xi_{j't}\big)} \times dF(\alpha_{it}, \beta_{it}; t, \gamma_{type}) \tag{3}$$

Let $\gamma = (\gamma_{type}, \gamma_{arrival})$ that puts together all demand parameters. Let $Q_{jt}\big(X, \boldsymbol{p}, \xi; \gamma\big)$ be the sales for firm $j$ at time $t$. Proposition 1 states that multinomial choice process conditional on a Poisson arrival process is mathematically equivalent to mutually independent Poisson choice processes.

**Proposition 1.** *We must have:*

$$Q_{jt}\big(X, \boldsymbol{p}, \xi; \gamma\big) \sim \text{Pois}\left(\lambda(t; \gamma_{arrival}) \times s_{jt}\big(X, \boldsymbol{p}, \xi; \gamma_{type}\big)\right) \tag{4}$$

$$Q_{jt}\big(X, \boldsymbol{p}, \xi; \gamma\big) \perp\!\!\!\perp Q_{j't}\big(X, \boldsymbol{p}, \xi; \gamma\big) \tag{5}$$

This equivalence simplifies our model. To understand what it actually says, consider the following thought experiment. If the number of arriving consumers follows Pois(4). A arriving consumer have 50% chance of choosing firm A from a choice set of firm A and firm B. Without any further information, one can correctly predict the expected sales of firm A is 2. However, if now firm B's sales is known to be 4, should one update her belief about firm A's sales? Proposition 1 says no. Since the sales of the two firms are independent. This result simply comes from the curious mathematical property of Poisson-Multinomial distribution.

### *3.2. Supply*

The supply model extends the BLP static pricing into a dynamic competition. For purpose of this paper, I assume that each firm carries only one product. The presence of capacity constraint forces airlines' pricing game to be dynamic. Firstly same as in single agent dynamic programming problem, prices have to satisfy intertemporal optimality condition reflecting the shadow price of capacity. In addition, airlines also rationally expect that todays' pricing strategies will affect all players' sales today and thus affect all players' capacities tomorrow. Since in any period airlines do not observe how many consumers arrive, nor do airlines know the valuation of these consumers, today's sales and tomorrow's capacities follow random distributions. Therefore, each player's "marginal cost" (the shadow price of capacity) changes constantly. As a result, profit maximization dictates airlines to adjust prices dynamically on time as well as own and others' inventories.

In the beginning of each period, firms observe each others' capacities **c** and the remaining time $t$,. Each period, there are firm-specific random draws on aggregate preference shocks $\xi_t$ and an industry-specific draw on marginal cost $\omega_t$. These shocks put together real time demand and supply information known to firms but not researchers. Finally, firms' payoff relevant state variables are summarized as $\{t, X, \mathbf{c}, \xi_{\mathbf{t}}, \omega_{\mathbf{t}}\}$. Firms know the distributions of the Poisson arrival process as well as the distribution of consumers random utilities. They do not know the exact number of consumers, nor do they know exact types and preferences of these consumers. Firms simultaneously choose prices that maximize their own expected payoffs. The optimal prices account for both the expected current period payoffs and the expected future payoffs. In the end of the period, demand is realized and capacities are filled. Then the game proceeds to the next period until $t = T$. After the final period, all remaining capacities have zero value. I discuss the details of this set up in the remaining of this section.

Let marginal cost be:

$$mc_{jt} = w_j \eta + \omega_t \tag{6}$$

Where $\omega_t$ is a common cost shock, $w_j$ is covariates for firm $j$'s marginal cost, and $\eta$ is the cost parameter to be estimated. Cost informations are common knowledge.

Recall that $\boldsymbol{c}_t$ is the remaining capacity vector at time $t$. Under capacity constraint, sales for firm $j$ thus follows a truncated Poisson:

$$Q^*_{jt}\big(X, \boldsymbol{p}, \boldsymbol{c}, \xi; \gamma\big) \;\sim \text{Pois}\left(\lambda(t; \gamma_{arrival}) \times s_{jt}\big(X, \boldsymbol{p}, \xi; \gamma_{type}\big) \,\middle|\, Q^*_{jt} \leq c_{jt}\right) \tag{7}$$

Let $\theta$ be the combined vector of demand and supply parameters for the structural model.

In each period, the expected static payoff is given by:

$$\Pi_j^t\big(p_j\big|\boldsymbol{p}_{-j},X,\boldsymbol{c},\xi,\omega;\theta\big)=\mathbb{E}_{Q_{jt}^*}\Big[Q_{jt}^*\big(X,\boldsymbol{p},\boldsymbol{c},\xi;\gamma\big)\times\big(p_j-mc_{jt}\big)\Big] \tag{8}$$

For simplicity, I omit dependence on $X$ and $\theta$ hereafter. Let $\boldsymbol{Q}_t^*$ be the $J\times 1$ random vector for sales. Firm $j$'s expected total revenue is a discounted sum of all future revenues, which depends on all players' prices, the realized states and demand and supply shocks:

$$\mathbb{E}\Big[\sum_{t=1}^{T}\int_{\xi}\int_{\omega}\Pi_j^t\big(\boldsymbol{p},\boldsymbol{c},\xi,\omega\big)\Big|\boldsymbol{c}_1\Big] \tag{9}$$

Firm $j$'s Bellman equation:

$$\begin{aligned}V_j^t\big(\boldsymbol{c},\xi,\omega\big|\boldsymbol{p}_{-j}\big)=\max_{p_j}&\Bigg\{\Pi_j^t\big(p_j\big|\boldsymbol{p}_{-j},\boldsymbol{c},\xi,\omega\big)+\\ \delta\int_{\xi'}\int_{\omega'}\mathbb{E}_{\boldsymbol{Q}_t^*}&\Big[V_j^{t+1}\big(\boldsymbol{c}',\xi',\omega'\big|\boldsymbol{p}'\big)\Big]\times dG(\xi')dH(\omega')\Bigg\}\end{aligned} \tag{10}$$

I make the following assumptions and comment on them in the remaining of this section.

1. Model
   (a) All error terms are *iid* in all their respective subscripts and uncorrelated with state variables.
2. Empirical
   (a) No overselling.
   (b) Remaining capacities are common knowledge.

The conditional independence assumptions are standard and pragmatic in the context of dynamic models(Rust (1987)). Yet assuming unobserved product-specific demand shocks are iid across time seems restrictive comparing to static demand estimations. For instance, BLP places little distributional assumptions on $\xi$.

The paper has made strong informational assumptions. Since capacity is payoff relevant, the competitive incentive in theory will possibly drive airlines to keep track of all the available capacities in the market. Indeed, most of the recent a few competitive RM papers that the author has seen have maintained this informational assumption, for instance Levin et al. (2009), Gallego and Hu (2014), etc. I note that airlines indeed have invested great resources setting optimal prices based on seat availability, but I admit that the exact algorithms and the degree of sophistication are largely unknown to the public.

In addition, I have assumed away learning of demand. I think of this as a limitation to this paper. Especially, given some of the very recent theoretical papers, such as Gershkov et al. (2016). Modeling airlines' learning of demand is very interesting, and I leave this to future work.

Finally, recent theoretical papers such as Board and Skrzypacz (2016) and Gershkov et al. (2016) have applied dynamic mechanism design for monopoly revenue management with strategic consumers. Their set ups are related to this current paper. This current paper only looks at simultaneous price posting.

### *3.3. Equilibrium*

I look at Markov perfect Nash equilibrium for this dynamic stochastic game.

Unfortunately, proving the uniqueness of this game is not easy. Note that the game has a finite horizon, therefore one may attempt to argue backwards by showing uniqueness for each static subgame. Caplin and Nalebuff (1991) has established a set of sufficient conditions on existence and uniqueness of price competition with differentiated products. Unfortunately, the results do not easily generalize to BLP setting. In a recent paper, Pierson et al. (2013) provide another set of sufficient conditions for uniqueness of the pricing game under mixed multinomial logit demand. Pierson et al. (2013)'s proof mostly relies on restricting market concentrations and/or price spaces. To the best of my knowledge, there is no more general results.[13]

Suppose the static pricing game is indeed unique, the next question is whether the uniqueness remains under a dynamic setting. Now the cost function needs to account for the option value of a sale and is thus more complicated. Without charactering the structure of the value functions, it is unclear how the standard techniques can possibly work. Although it may not be so hard to prove structures of monopoly dynamic programming problems (Gallego and Van Ryzin (1994)), it is significantly harder to do so in dynamic games. In fact, the structure of a RM game with stochastic demand is largely unknown (Lin and Sibdari (2009) and Gallego and Hu (2014)).

When the competition is mild enough, the equilibrium will be unique. In practice, my numerical algorithm works well. It is robust to different initial values. I will discuss the numerical algorithm with more details in the appendix.

[13]Gallego et al 2004, 2006.

# 4. Estimation

There are two major challenges in the estimation. The first one is how to incorporate unobserved product specific errors in small "non-invertible" market (I will be more precise later); and the second is how to keep the estimation computationally feasible. I adopt a nested fixed point approach. In the first step, I solve the system of demand and supply into its reduced form for all market outcomes conditional on all observed and unobserved states. In the second step, I integrate out all unobserved states. In the final step, I match model predicted outcomes and empirical outcomes. I adopt a GMM estimator and interact the predicted errors with a set of instruments. I discuss identification in the end of this section.

## *4.1. Econometric Specification*

Firstly, I use a third order polynomial to approximate the Poisson aggregate arrival rate:

$$\lambda(t;\gamma_{arrival}) = \gamma^{(1)}_{arrival} + \gamma^{(2)}_{arrival} \times t + \gamma^{(3)}_{arrival} \times t^2 + \gamma^{(4)}_{arrival} \times t^3 \tag{11}$$

In estimation, I approximate the random coefficient demand model with discrete types. This is typical in economic literature. Examples include, for instance, Berry and Jia (2010) on airline demand, Besanko et al. (2003) on price discrimination. It captures the correlation of tastes but remains computationally cheap under certain circumstances. Firstly, I allow for two vertical types of consumers $\{H, L\}$ differ in their price sensitivities, $\alpha^H$ for high type and $\alpha^L$ low type. This is a parsimonious way of modeling traveler types as business travelers and leisure travelers.

I allow their arrival process to be correlated with time. The probability that a consumer arrive at time $t$ is low type is the following:

$$P^L(t;\gamma_{type}) = \frac{1}{1 + \exp\left[\gamma^{(1)}_{type} + \gamma^{(2)}_{type} \times t + \gamma^{(3)}_{type} \times \mathbb{1}\{t \leq 14\} + \gamma^{(4)}_{type} \times \mathbb{1}\{t \leq 7\}\right]} \tag{12}$$

It follows that

$$P^H(t;\gamma_{type}) = 1 - P^L(t;\gamma_{type}) \tag{13}$$

Note that this parametric assumption is pragmatic in several ways. Firstly, the logit transformation bounds probabilities in $[0, 1]$. Secondly, it helps explain discrete price jumps at two weeks and one week before departure date.

Within each vertical type of consumers, I further allow for two segments with possibly different horizontal brand preferences. In fact, I can not identify brand preference from product preference, so I do not distinguish them in the paper. Let $\gamma^H_{type}$ be the proportion of high type consumers that relatively prefer firm 1 and $\gamma^L_{type}$ be the proportion of low type consumers that relatively prefer firm 1. Thus I have $2 \times 2$ discrete segments. Let $\beta^{H_1}$ be a $2 \times 1$ vector for brand preference of firm-1-leaning high type consumers. Note that $\beta^{H_1}$ has two elements, one for each firm. Define $\beta^{H_2}$, $\beta^{L_1}$, and $\beta^{L_2}$ accordingly.

Therefore, I can write out the probability distribution on the four discrete types of consumers:

$$F(\alpha_{it}, \beta_{it}; t, \gamma_{type}) = \begin{cases} P^H(t; \gamma_{type}) \times \gamma^H_{type} & \text{if } \alpha_{it} = \alpha^H, \beta_{it} = \beta^{H_1} \\ P^H(t; \gamma_{type}) \times \left[1 - \gamma^H_{type}\right] & \text{if } \alpha_{it} = \alpha^H, \beta_{it} = \beta^{H_2} \\ P^L(t; \gamma_{type}) \times \gamma^L_{type} & \text{if } \alpha_{it} = \alpha^L, \beta_{it} = \beta^{L_1} \\ P^L(t; \gamma_{type}) \times \left[1 - \gamma^L_{type}\right] & \text{if } \alpha_{it} = \alpha^L, \beta_{it} = \beta^{L_2} \end{cases}. \tag{14}$$

## *4.2. Endogeneity*

Price endogeneity in simultaneous demand and supply system has been studied for decades in economics. Ignoring the endogeneity problem will cause biased estimates on price coefficient (Villas-Boas and Winer (1999)), since prices are often strategically chosen in response to demand errors unobserved by researchers, which violates identification conditions. If demand function is aggregated from discrete choice and thus is nonlinear, a simple IV regression is not immediately applicable. The classic solution to this was developed in Berry (1994) and Berry et al. (1995).[14] The endogenous component in demand is assumed to be captured by an addictive product shock $\xi_j$ observed by market players but not by researchers. The proposed solution is to linearize the demand equation and invert out the unobserved error $\xi_j$. Once back to a linear setting, all the remaining work is to find appropriate instrument variables.

Unfortunately, Berry et al. (1995)'s inversion method put restriction on market size: it has to go to infinity at certain speed such that observed market shares (1) can approximate choice probabilities and (2) are bounded away from zero. Neither of this holds in the current setting. In each market (itinerary×departure-date×pricing-date) I observe on average fewer than two prices and two sales. This type of data are a deviation from the standard market level data, but is very common in dynamic pricing setting (see Sweeting (2015) for event ticket). Outside of the dynamic pricing context, Goolsbee and Petrin (2004) looked at survey data with small (but non-zero) market size in cable market. They confirmed the importance of allowing unobserved demand shock and proposed method for dealing with the

[14]Control function is another approach, Petrin and Train (2010).

measurement error in market shares.

I adopt a "reduced form method" discussed in Berry (1994). Let $\mathcal{I}$ be all the relevant states for the joint system of demand and supply. Let $\mathcal{I} = \{\mathcal{I}_o, \mathcal{I}_u\}$ such that $\mathcal{I}_o$ is observed in the data while $\mathcal{I}_u$ is not. So $\mathcal{I}_u = \{\xi, \omega\}$. I first solve the game and translate the structured system into a reduced form. Let $\Psi(\mathcal{I}, \vartheta)$ be such an operator that takes structural parameters $\vartheta$ as well as all payoff relevant states $\mathcal{I}$ and returns a vector of market outcome (prices, quantities, prices interacting with quantities, etc). Then I integrate out unobserved shocks $\mathcal{I}_u$ and obtain the expected market equilibrium variables $\boldsymbol{\psi}$ conditional on only observables $\mathcal{I}_o$ and the structural parameters $\vartheta$. Let $\vartheta = \{\vartheta_o, \vartheta_u\}$ be the structural parameters on observables and unobservables respectively. Note that the conditional independence assumption implies that $\Phi(\mathcal{I}_u|\mathcal{I}_o, \theta_u) = \Phi(\mathcal{I}_u|\theta_u)$. Therefore:

$$\boldsymbol{\psi}(\mathcal{I}_o|\vartheta) = \int_{\mathcal{I}_u} \Psi(\mathcal{I}|\vartheta_o) d\Phi(\mathcal{I}_u|\theta_u) \tag{15}$$

Note that $\Psi$ takes $\xi$ and $\omega$ as its arguments and jointly solves demand and supply. It implicitly accounts for the dependence of prices on unobserved demand error $\xi$. This is different from integrating out $\xi$ separately for the demand equation while using observed prices. The latter approach, as pointed out by Berry (1994), is not consistent since it assumes that price does not response to $\xi$. In practice, I solve for the following equilibrium outcomes into reduced form:

$$\boldsymbol{\psi} := \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \\ \mathbf{p} \otimes \mathbf{s} \end{bmatrix} \tag{16}$$

Where $\mathbf{p}$ is a $2 \times 1$ vector of prices and $\mathbf{s}$ is a $2 \times 1$ vector of sales.

The drawbacks of the reduced form method are discussed in Berry (1994). Firstly, BLP does not impose any distributional assumption on $\xi$ and $\omega$. In practice, I assume that $\xi$ and $\omega$ are independent normal and use numerical integration method to calculate $\boldsymbol{\psi}(\mathcal{I}, \vartheta)$. In addition, as in other full solution method, stronger assumption is needed to address potential multiple equilibria.[15]

## *4.3. Instruments*

I adopt a method of moments estimator by minimizing the differences between the expected market outcomes $\boldsymbol{\psi}(\theta)$ and the observed market outcomes $\boldsymbol{\psi}_t$. I match a set of

[15] Villas-Boas (2007) provides a more general perspective.

observed moments with corresponding predicted moments jointly for the system of supply and demand. The prediction errors by construction has mean zero. Now I discuss the selection of instrumental variable. I consider the following $\mathbf{z_b}$ as the baseline instruments:

$$\mathbf{z_b} = \begin{bmatrix} \text{vec}\left(\mathbf{x} \otimes \mathbf{w}\right) \\ \text{vec}\left(\mathbf{c} \otimes \mathbf{w}\right) \end{bmatrix} \tag{17}$$

Where $\mathbf{x}$ denote exogenous covariates in consumers' utility function and $\mathbf{c}$ is a vector of remaining capacities. In practice, I let $\mathbf{x}$ include a $2 \times 1$ vector of 1 and weekend indicator. $\mathbf{w}$ is a vector of dummies for weeks to departure. The reduced form evidence show that there is rich cross-time dynamics. Moreover, these dynamics can be captured well for weekly time windows.

To implement the Diff-in-Diff design, I use the treatment conditions $\mathbf{z_c}$ as an additional set of instruments. Thus $\mathbf{z_c}$ includes interactions between routes and exit. This creates exogenous variations. I thus obtain the instruments variable $\mathbf{z}$:

$$\mathbf{z} = \text{vec}\left(\mathbf{z_b} \otimes \mathbf{z_c}\right) \tag{18}$$

The moment restriction is:

$$\boldsymbol{g}\left(\vartheta_0\right) = \mathbb{E}\left[\left(\boldsymbol{\psi}(\theta_0) - \boldsymbol{\psi}_t\right)\Big|\mathbf{z_t}\right] = \mathbf{0} \tag{19}$$

In the end, I have 469 moments and $|\vartheta| = 38$ parameters. I use a two-step generalized method of moments and assume that necessary regularity condition holds for the GMM. As a first step, I estimate the model with each moment weighted by its empirical counterpart. This estimator is consistent but not efficient. With the first-step estimates $\hat{\vartheta}_1$, I can calculate the estimator for the optimal weighting matrix $\hat{W}_T(\hat{\vartheta}_1)$. With this weighting matrix, I update the estimators and calculate the standard error using the asymptotic variance matrix for the two-step feasible GMM estimator.

$$\hat{\vartheta} = \underset{\vartheta \in \Theta}{\text{argmin}} \left[\hat{\boldsymbol{g}}_T(\vartheta)\right]' \hat{W}_T(\hat{\vartheta}_1) \left[\hat{\boldsymbol{g}}_T(\vartheta)\right] \tag{20}$$

## *4.4. Identification*

The parameters are jointly identified from the demand and supply of the model. Price variations before and after the exit identify consumers' price coefficients. In addition, since demand follows random Poisson process, and randomly realized demand creates variation in firms' optimal prices. This variations in prices will also help identifies price coefficient. The mixture of consumer types are identified from violations of the iid properties of simple logit demand. Temporal preference heterogeneity is identified by the temporal variations of prices and quantities.

In many applications, market size $M$ is directly observed. For instance, researchers set $M$ for automobile and TV cable equal to the number of households in the whole population (Berry et al. (1995), Goolsbee and Petrin (2004)). When there is information in a number of markets, $M$ can be parameterized as depending on market level data (Berry (1990), Berry and Jia (2010), Lazarev (2013). In our setting, the number of potential air travelers is crucial given the constraints on capacities, and it affects our inference on market competition and welfare. Unfortunately, it is unobserved by the researcher. In a related dynamic setting, Nair (2007) looked at intertemporal price discrimination in monopoly video-game markets. Similarly, he does not assume the market size to be the whole population (all users for the game platform), otherwise the relative small sales would imply that the game has almost zero market share at any period.

The current identification on market size comes from two sources. First, the dependence of firms' optimal prices on remaining capacities help identify market size. In addition, the current difference-in-difference research design also helps. The intuition is the following: changes of relative sales to change of relative prices across the two periods uniquely identifies price coefficient. Knowing this, I should know the change of utilities for product 1 across the two periods. I also observe the change of shares for product 1 across two periods, since it equals to the change of its sales under the assumption that market size is constant. Thus I already know (1) the change of utility for product 1 (after identifying price coefficient); (2) the change of utility for outside option (equals zero by assumption); (3) the change of shares for good 1 (observed from sales); and I can infer the change of shares for outside option. Since I observe the absolute change of sales for outside good (from the absolute change of sales for inside goods), I identify market size. [16]

# 5. Results

## 5.1. Estimates

In this section, I discuss the estimates from the structural model. Figure 5 shows the fitted errors in percentage for all the 469 moments. The mean fitting error in percentage is 10.5%. This suggests that the data can be reasonably well explained by the model set

[16] The appendix gives a closed form solution for the identification when demand is simple logit.

up. Figure 6 compares the model predicted price paths with their empirical counter parts conditional on the market conditions. It suggests that the model not only captures the upward-slopping trends for prices but also the relative price levels between firms. Similarly, Figure 7 shows that the model also does well in predicting the absolute magnitudes, relative magnitudes, and general trends in sales.

The estimates are very precise with a few exceptions. The data indeed gives enough variations to identify the structural parameters. Low type consumers' price coefficients are about three times bigger (in absolute value) than the high types. Alaska seems to be the bigger player in the exit market but the smaller one in the control market. This is consistent with the observation that Alaska is the incumbent in the exit market but the entry firm in the control market. Estimates on the ratio of each player's loyal segment seems to be consistent with their relative sales. Estimates on the strengths of consumers' brand preference seems to be reflected by firms' relative average price levels.

The estimates confirm that there is great heterogeneity in consumers' relative brand preferences. Generally, high type consumers have stronger brand preference than low type consumers. In the exit route, Alaska is the incumbent firm. An average Alaska-leaning high type consumers would accept \$393 to be indifferent between Alaska and Delta. An average Delta-leaning high type consumers would accept \$839 to be indifferent between Alaska and Delta. These estimates suggest that high type travelers' brand preferences are very strong. The competition in the late market is very weak and the late market is close to monopoly. The late market in the control route follows a similar pattern. An average Alaska-leaning high type consumers would accept \$226 to be indifferent between Alaska and JetBlue. An average JetBlue-leaning high type consumers would accept \$769 to be indifferent between Alaska and JetBlue. Note that JetBlue is the incumbent firm in this market.

The competitions in early markets are much stronger. The relative brand preference for leisure travelers range from \$8 to \$195. In the control route, most leisure travelers prefer or weakly prefers the incumbent firm JetBlue. In fact, the Alaska-leaning low type travelers are almost indifferent between Alaska and JetBlue. This suggests that Alaska would have to price aggressively to attract leisure consumers in the early markets.

Figure 8 plots the estimated Poisson arrival rates and the shares of high value consumers in the exit market (the upper one) and in the control market (the lower one). The estimated arrival patterns are very similar for the two markets. Each day, about 3-6 travelers arrive in each of the two markets. The number of arriving consumers is lowest 2-3 weeks before departure. It then increases to its highest level just one week before departure.

The estimated ratio of high type consumers is increasing with time. This result is consistent with prior belief. In particular, in the exit market, the ratio of business travelers starts from close to zero percent seven weeks before departure. It increases gradually to around 40 percent one week before departure. In the last week, about 80 percent of the arriving travelers are business travelers. Meanwhile, in the control market, the ratio of business travelers start at around 20 percent. It then gradually increases to around 50 percent just one week before departure. In the last week, the ratio of business travelers are close to 100%.

Figure 9 summarizes the probability distribution of consumers' brand preferences conditional on the arrival time. In the exit market, the incumbent firm Alaska has bigger loyal segment for both business travelers and leisure travelers. 90 percent of high type consumers prefer Alaska to Delta, whereas 80 percent of low type consumers prefer Delta to Alaska. In the control market, 25 percent of high type consumers prefer the incumbent firm JetBlue. This is consistent with the prior that Alaska may have stronger market power in the business traveler market. The incumbent firm JetBlue has very strong loyal consumers in the low end market. Firstly, around 50 percent of leisure travelers strongly prefer JetBlue by as much as $200. Secondly, the other 50 percent of leisure consumers are more or less indifferent between Alaska and JetBlue.

Table 2: Consumers' preferences in the exit route

| | Estimates | |
|---|---|---|
| | High type | Low type |
| Price coefficients | −0.553<br>(0.003) | −1.510<br>(0.015) |
| Alaska's consumers' preference to | | |
| Alaska | 2.452<br>(0.048) | 1.576<br>(0.046) |
| Delta | 0.028<br>(0.062) | 0.005<br>(0.027) |
| Delta's consumers' preference to | | |
| Alaska | 0.314<br>(0.036) | 0.140<br>(0.029) |
| Delta | 4.951<br>(0.021) | 2.037<br>(0.033) |

Note: Price coefficients on $100.

Table 3: Consumers' preferences in the control route

| | Estimates | |
|---|---|---|
| | High type | Low type |
| Price coefficients | −0.499 | −1.452 |
| | (0.003) | (0.019) |
| Alaska's consumers' preference to | | |
| Alaska | 1.782 | 1.043 |
| | (0.040) | (0.043) |
| Jetblue | 0.652 | 0.928 |
| | (0.024) | (0.037) |
| Jetblue's consumers' preference to | | |
| Alaska | 0.609 | 0.399 |
| | (0.166) | (0.181) |
| Jetblue | 4.444 | 3.225 |
| | (0.199) | (0.206) |

Note: Price coefficients on $100.

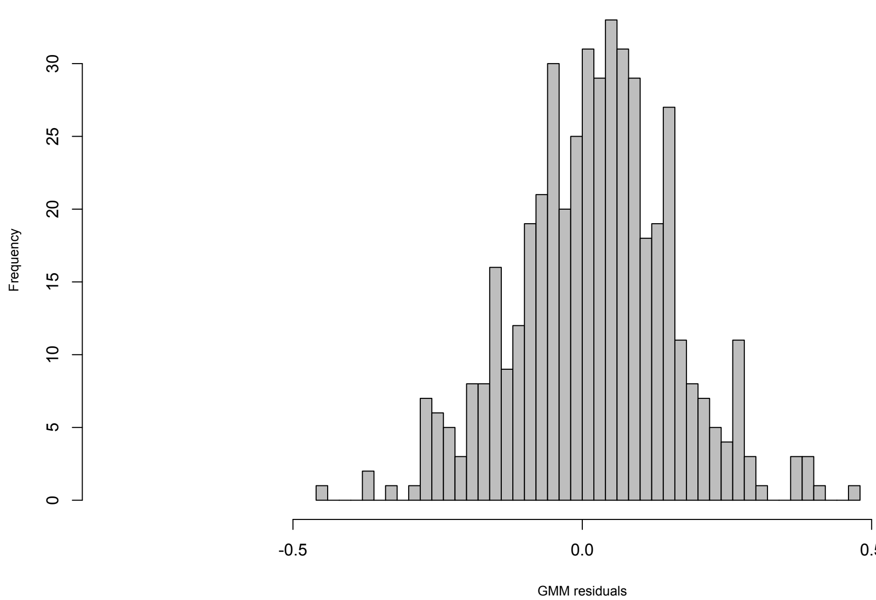



Fig. 5. Fitted error for all (469) moments

Table 4: Consumer arrival and segmentation

| | Estimates | |
|---|---|---|
| | Exit route | Control route |
| **Probability of preferring Alaska** | | |
| High type | 0.900 | 0.757 |
| | (0.002) | (0.008) |
| Low type | 0.806 | 0.494 |
| | (0.011) | (0.020) |
| **Poisson arrival** | | |
| $\gamma^1_{arrival}$ | 4.292 | 3.738 |
| | (0.062) | (0.053) |
| $\gamma^2_{arrival}$ | 0.401 | 1.143 |
| | (0.026) | (0.012) |
| $\gamma^3_{arrival}$ | $-0.378$ | $-0.548$ |
| | (0.004) | (0.004) |
| $\gamma^4_{arrival}$ | 0.053 | 0.066 |
| | (6e-4) | (6e-4) |
| **Probability on types** | | |
| $\gamma^1_{type}$ | -2.876 | -1.612 |
| | (0.031) | (0.043) |
| $\gamma^2_{type}$ | 0.397 | 0.270 |
| | (0.007) | (0.009) |
| $\gamma^3_{type}$ | 0.005 | 0.001 |
| | (0.052) | (0.059) |
| $\gamma^4_{type}$ | 2.045 | 5.571 |
| | (0.371) | (12.366) |

Note: $\gamma^3_{type}$ seems useless.

Table 5: Other common parameters

| | Estimates |
|---|---|
| Preference shock | |
| After dummy | 0.0446 |
| | (0.006) |
| Weekend dummy | -0.047 |
| | (0.006) |
| Unobserved shock | |
| $\sigma_\omega$ | 15.084 |
| | (1.710) |
| $\sigma_\xi$ | 0.324 |
| | (0.128) |
| Cost parameter | |
| $mc$ | 28.022 |
| | (1.686) |

Note: Weekend dummy$<0$...

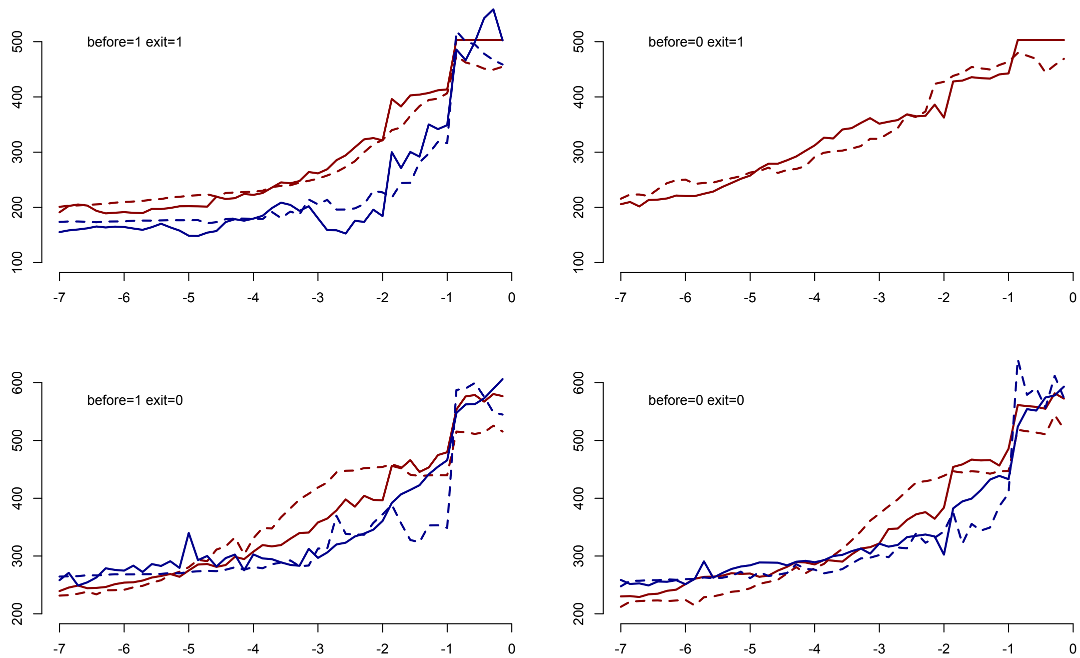



Fig. 6. Fitted price path

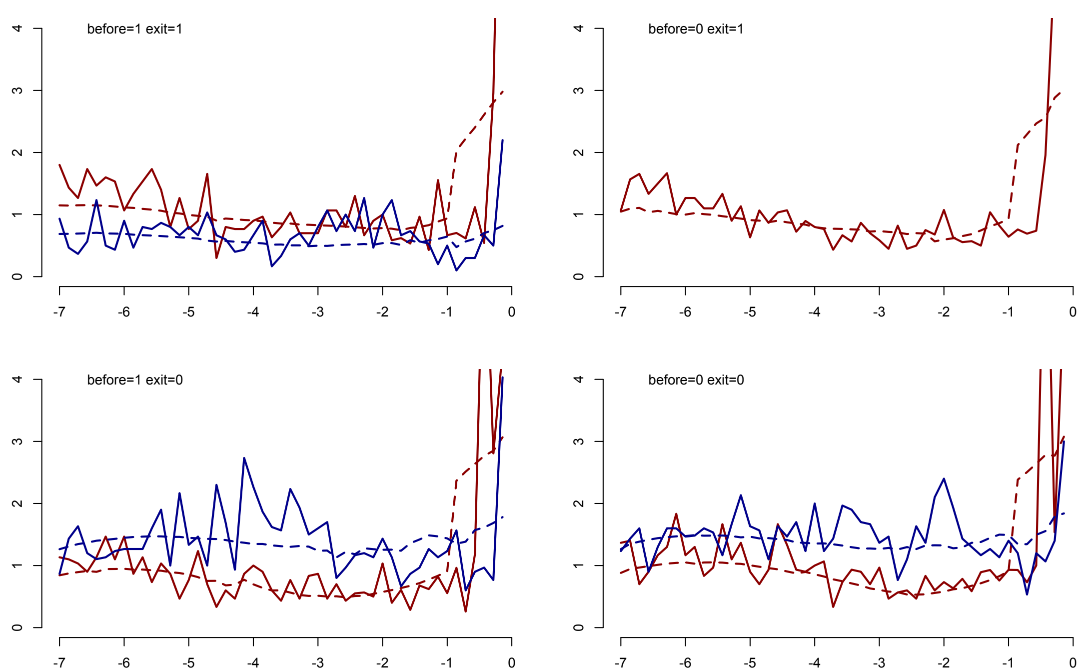
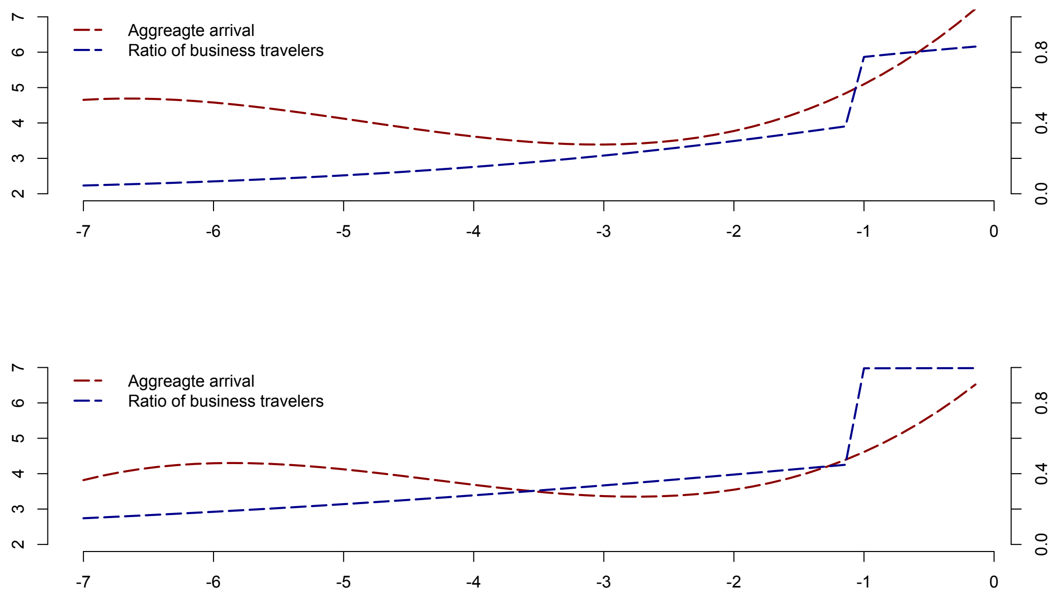
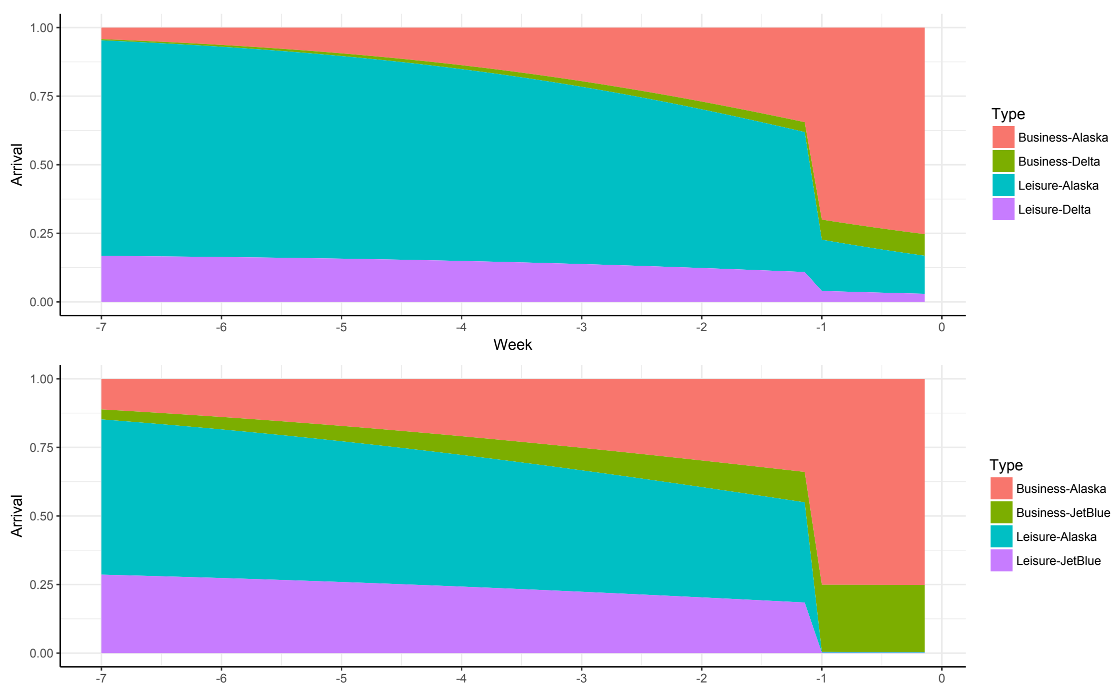



Fig. 7. Fitted sales path




Note: The graph above is for exit route, while the one below is for control route.

Fig. 8. Estimated aggregate arrival parameters




Note: The graph above is for exit route, while the one below is for control route. Business-1 means Alaska-leaning business travelers. Leisure-1 means Alaska-leaning leisure travelers. Y-axis is the probability distribution for each type.

Fig. 9. Estimated arrival by type of consumers

## *5.2. Capacity Constraint*

As noted earlier, capacities affects airlines price competition. Leaders in the industry have repeatedly complaining about how overcapacity has intensified airline price competition and reduce industry revenues. Here I highlight the impact of capacity constraint on firms' profits. Although a monopoly firm's profit always increases with its capacity, this is no longer the case for duopoly firms playing a noncooperative game. To illustrate this idea, I use the obtained estimates to calculate firms' expected revenues for different capacities at time 0. As an example, Figure 10 plots JetBlue's expected profit at period 0 as a function of its own capacity and its competitor's capacity. Not surprisingly, JetBlue's expected profit decreases with its competitor's capacity. However, when JetBlue's capacities are higher than 80, its expected revenue actually decrease.

Indeed, Figure 11 shows that industry revenue decrease 5% when capacity grows from (50,50) to (100,100). The magnitude of this is considerable given that the industry total profit is only 3-5 %.

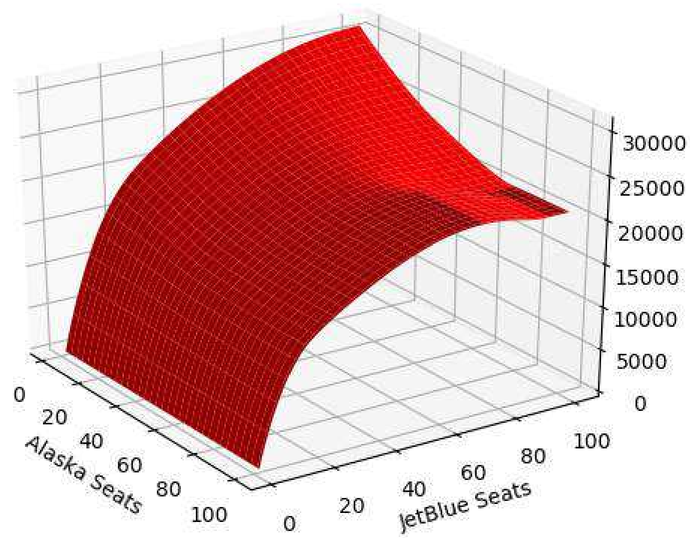



Fig. 10. JetBlue's expected profit conditional on initial capacities

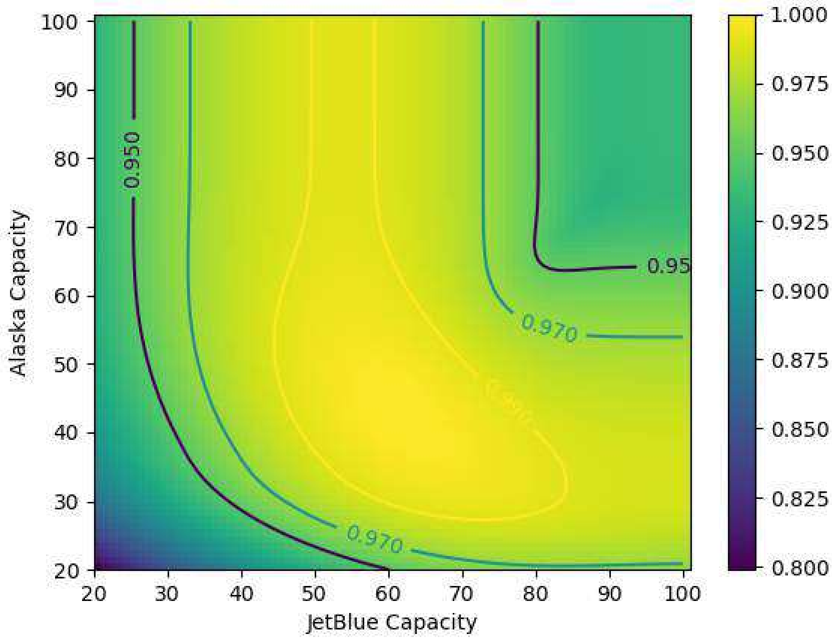



Fig. 11. Industry expected total revenue on initial capacities

# 6. Counterfactuals

I perform counterfactual simulations on one of the estimated route. Here I assume that the estimated arrival patterns do not change when airlines' pricing policies change. This is harmless under my model assumption of myopic consumers with exougenous arrivals. However, there will be potential consequences if the arrival pattern is an endogenous equilibrium outcome. I use the empirical capacity distribution at time 0 (49 days before departure). By doing so, I ignore the fact that the change of policies can affect the capacity distribution 49 days before departure. For this route, I have 237 observations on initial capacities, which include initial capacities that are not in the original estimation sample. For each scenario considered below (including the estimated one), I start from each of the 237 initial capacities and simulate the paths 100,00 times. In all counterfactuals, I fix unobserved demand and supply errors at zero. This reduces computations significantly.

## 6.1. *Price Commitment*

### 6.1.1. *Single Price*

Firms can only charge a single price. Their prices are mutual best responses conditional on their expectations of all possible realized demand paths.

$$p_j^* = \text{argmax}_{p_j \in \mathcal{R}^+} \int_{\boldsymbol{c}_0} \mathbb{E}_{Q^*}\left[\sum_{t=0}^{T} \Pi_j^t\big(p_j, p_{-j}^*, \boldsymbol{c}\big)\Big|\boldsymbol{c}_0\right], \quad j = 1, 2. \tag{21}$$

This removes all dynamic pricing. Figure 12 shows the optimal constant prices for the two firms. Not surprisingly, each firm chooses a price that lies in between its highest and lowest price. Firms differentiate themselves more by focusing on different vertical segments. In particular, JetBlue focus on early markets by charging a low price whereas Alaska focus on late markets by charging a high price. Figure 13 shows that JetBlue has more than 15% chance to sell out before departure.

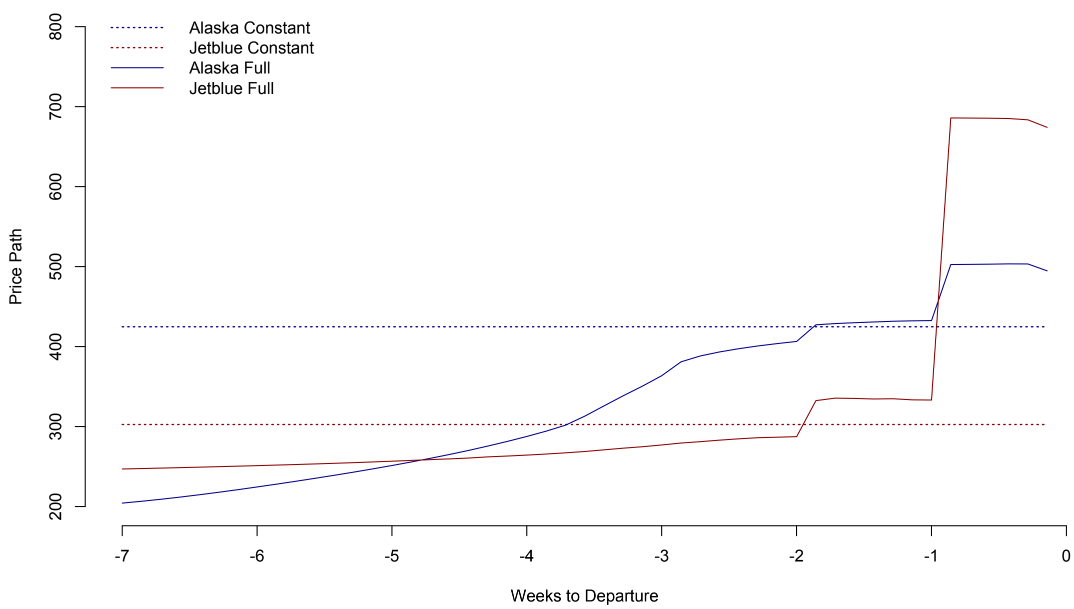



Fig. 12. Constant price paths

Alaska-leaning consumers overall surfer from constant pricing. The welfare of Alaska-leaning low type consumers decrease from \$1290.6 to \$402.5 per flight. The welfare of Alaska-leaning high type consumers increases only marginally, from \$15283.2 to \$15381.3 per flight. Alaska-leaning high type consumers benefit marginally from a slightly lower

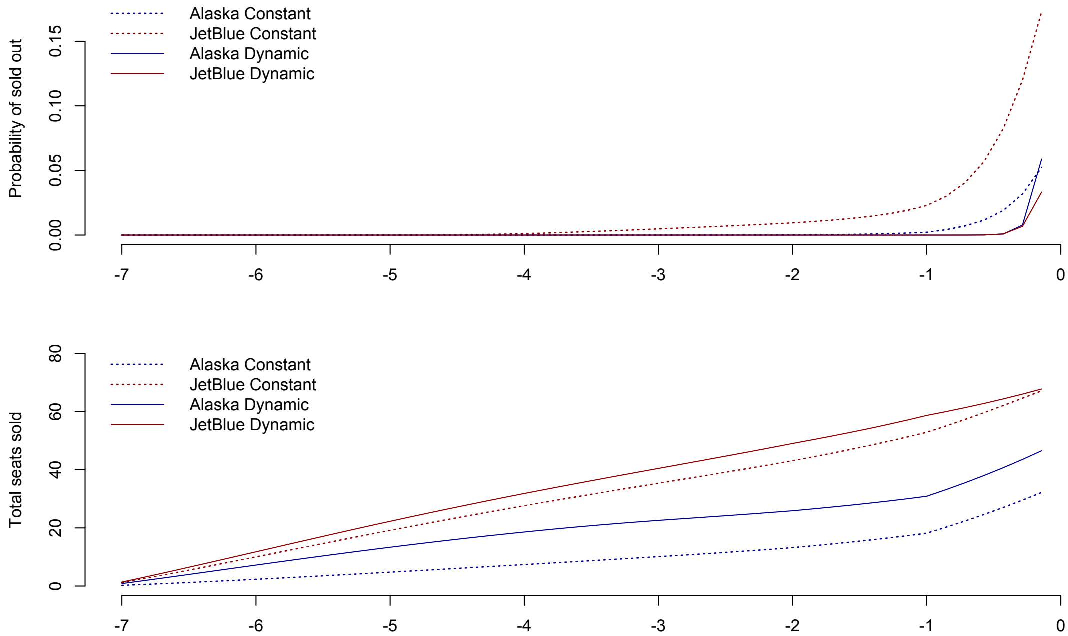



Fig. 13. Sales paths under constant pricing v.s. dynamic pricing

price. The overall welfare for Alaska-leaning consumers decreases by 4.7%.

On the other hand, the overall welfare for JetBlue-leaning consumers increases by 6.8%. High type JetBlue-leaning consumers' welfare increases from \$12002.6 to \$14612.1 per flight. Low type JetBlue-leaning consumers' welfare decrease from \$4460.6 to \$3071.9. The overall welfare for JetBlue-leaning consumers increases by 10.82%.

Table 6: Committing on a constant price on firms' profits

| | Constant Price | Dynamic Pricing | Change |
|---|---|---|---|
| Alaska's Profits | \$12416.49 | \$14543.35 | -\$2126.86 (-14.62%) |
| JetBlue's Profits | \$17692.29 | \$19299.86 | -\$1607.56 (-8.32%) |
| Consumer Surplus | \$ 33456.21 | \$33036.49 | +\$419.72 (+1.27%) |
| Total Welfare | \$63564.99 | \$66879.71 | -\$3314.72 (-4.96%) |

Table 7: Constant pricing on consumer welfare by segment

| Welfare Change | High Type | Low Type |
|---|---|---|
| Alaska-leaning | +\$98.08 (+0.6%) | -\$888.01 (-68.8%) |
| JetBlue-leaning | +\$2609.52 (+21.7%) | -\$1399.86 (-31.4%) |

Overall, constant pricing has mixed effect on consumers. Particularly, total consumer surplus increases marginally by +\$419.72 per flight or +1.3%. At the same time, both firms' revenues decrease considerably. JetBlue's revenue decreases by \$2126.86 per flight, which amounts to 14.62% of its total profits. Alaska's revenue decreases by \$1607.57 per flight, which amounts to 8.32% of its total profits. The magnitude is significant comparing to the magnitudes of airlines' margins. According to IATA, airlines' expected margin per traveler is \$7.54 per traveler in 2017, and 2017 is expected to be "another strong year".[17] Thus, my estimates suggest that airline would go bankrupted under constant pricing regime.

Overall, constant pricing decreases total welfare by 4.96%, or \$3314.72.

#### *6.1.2. Constant Price Path*

This counterfactual considers a scenario when both firms commit to a price path. At period 0, each firm submits a price policy conditional on time only. At each period, each firm chooses the price specified in its price policy for any level of positive capacity.

This counterfactual thus shuts down firms' ability to smooth the impact of demand fluctuations on capacity utilizations.

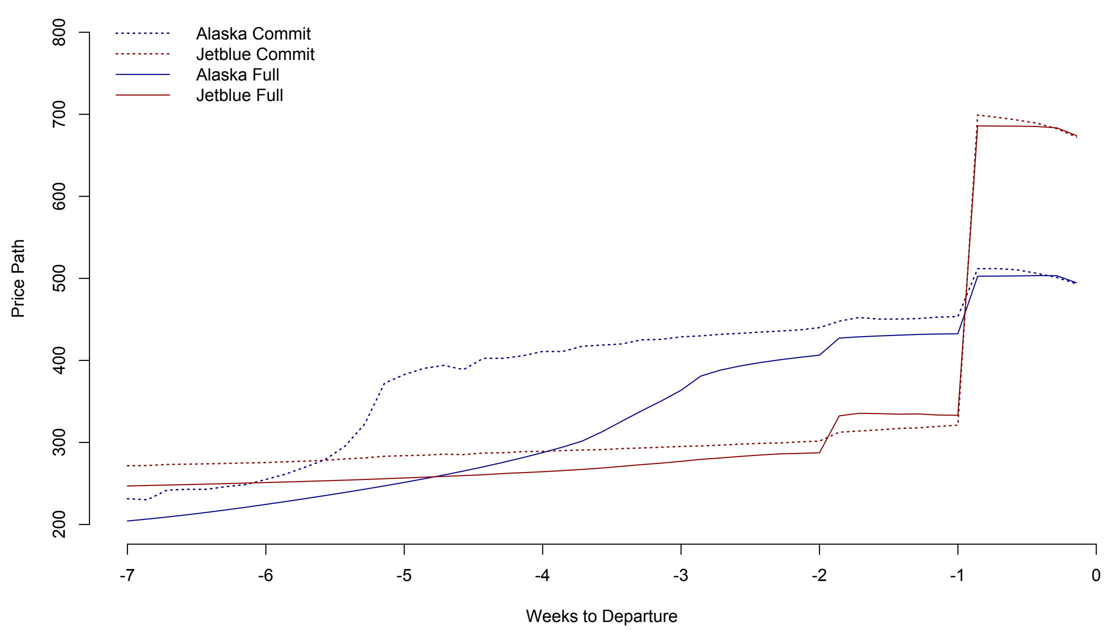



Fig. 14. Comitted price paths

Figure 14 shows the committed price paths. When firms cannot adjust prices conditional on capacities, the price levels start higher in the early periods.

[17]http://www.iata.org/pressroom/pr/Pages/2016-12-08-01.aspx

Committing to a price path has only small effects on firms' profits. Alaska's profit decreases by $297.79 or 2.05%, whileJetBlue's profit increases by $81.05 or 0.42%. In fact, industry total revenue change only $216.74 or 0.60%.

However, it decreases consumers welfare for all types regardless of their vertical or horizontal preferences. When firms commit to a fixed price path, they can no longer adjust their prices conditional on remaining capacity. Since demand uncertainty can add up across all periods, the variance of remaining capacity is very big. Therefore, capacity starts binding more often. Firms start raise prices in early periods, which hurt early consumers. Although later periods prices do not increase by much, the probability of selling out increases because of the variance of remaining capacities. This hurt late consumers.

Table 8: Committing on a price path on firms' profits

| | Constant Price Path | Dynamic Pricing | Change |
|---|---|---|---|
| Alaska's Profits | $14245.56 | $14543.35 | -$297.79 (-2.05%) |
| JetBlue's Profits | $19380.92 | $19299.86 | +$81.05 (+0.42%) |
| Consumer Surplus | $ 29469.04 | $33036.49 | -$3567.45 (-10.80%) |
| Total Welfare | $63095.51 | $66879.71 | -$3784.19 (+5.66%) |

Table 9: Committing on a price path on consumer welfare by segment

| Welfare Change | High Type | Low Type |
|---|---|---|
| Alaska-leaning | -$1601.18 (-10.5%) | -$617.18 (-47.8%) |
| JetBlue-leaning | -$426.94 (-3.6%) | -$922.15 (-20.7%) |

## *6.2. Homogenous Consumer Arrival*

In this counterfactual, I consider a world in which consumers arrive homogeneously across time. Therefore, firms are unable to price discriminate across time. In particular, I consider two cases: (i) each firm chooses an optimal constant price and commit to it until the end of the game; (ii) firms can set optimal price conditional on remaining capacities. By comparing the two scenarios, I single out the effect of revenue management in airlines' price competition in absence of price discrimination.

Table 12 shows that scarcity pricing increases total welfare by 2.3%. It has small impact on firms' profits. Figure 16 shows the price paths under the two conditions. Alaska's dynamic prices are below its constant prices. JetBlue's dynamic prices are above its constant price for the first two weeks or so but go below the constant price afterwards.

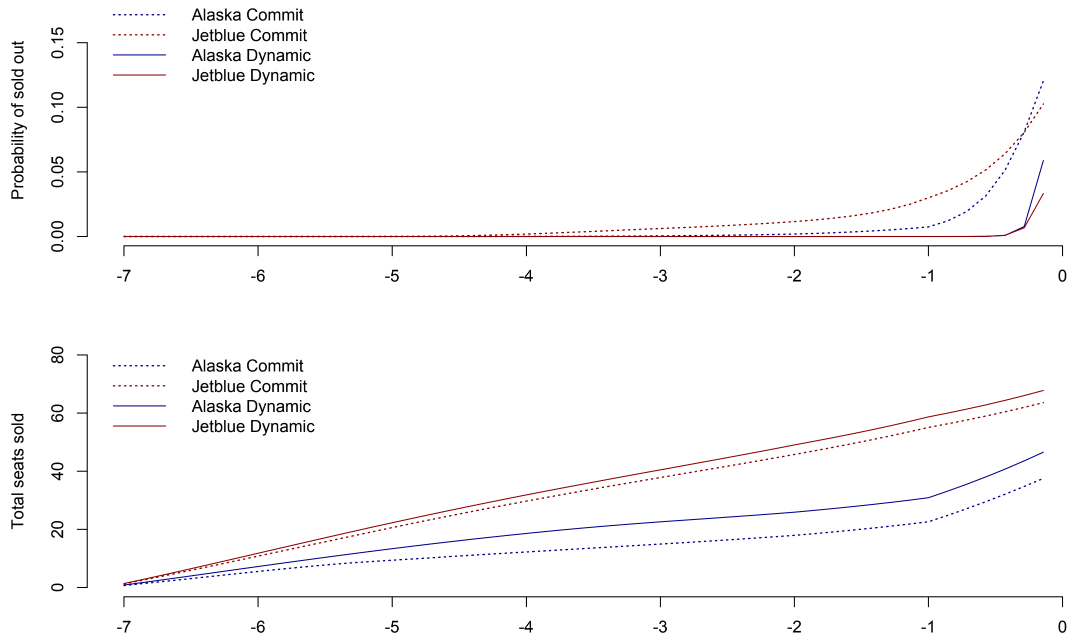



Fig. 15. Sales paths under price commitment v.s. dynamic pricing

Table 10: Canstant pricing v.s. scarcity pricing under homogenous consumer arrival

| | Constant Pricing | Scarcity Pricing | Change |
|---|---|---|---|
| Alaska's Profits | \$12415.40 | \$12371.16 | -\$42.24 (-0.3%) |
| JetBlue's Profits | \$17648.98 | \$17764.96 | \$115.98 (+0.7%) |
| Consumer Surplus | \$ 33913.65 | \$35373.86 | \$1460 (+4.13%) |
| Total Welfare | \$63978.03 | \$65509.98 | \$1531.94 (+2.3%) |

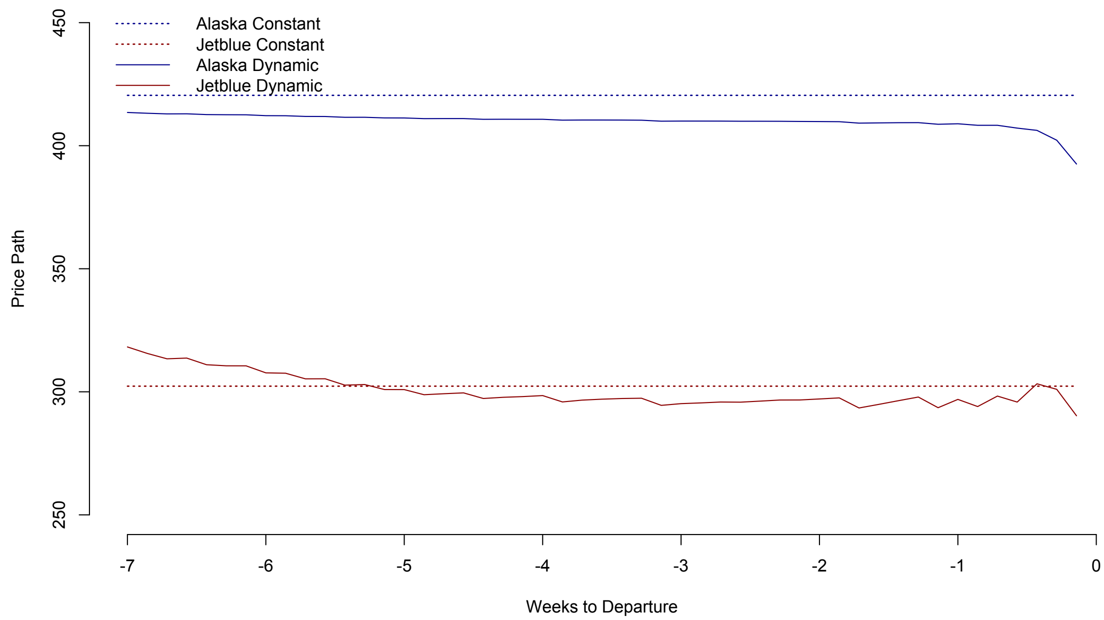



Fig. 16. Canstant price v.s. scarcity price under homogenous consumer arrival

## 6.3. *Remove Capacity Constraint*

In this counterfactual, I consider a world in which firms have no capacity constraint. In this situation, there is no need for firms to smooth demand fluctuations any more. In particular, I consider two cases: (i) each firm chooses an optimal constant price and commit to it until the end of the game; (ii) firms can set optimal price conditional on time to departure. By comparing the two scenarios, I single out the effect of price discrimination in airlines' price competition in absence of revenue management.

Table 11: Price discrimination on consumer welfare (without capacity constraint)

| Welfare Change | High Type | Low Type |
|---|---|---|
| Alaska-leaning | -\$248.40 (-1.5%) | +\$1176.72 (+196.1%) |
| JetBlue-leaning | -\$3354.65 (-27.0%) | +\$1182.00 (+29.9%) |

## 6.4. *The Effects of Competition*

In this counterfactual, I consider the effect of competition. To do so, I simulate each firm's price path when the competitor exits the market. Figure 18 compares monopoly price

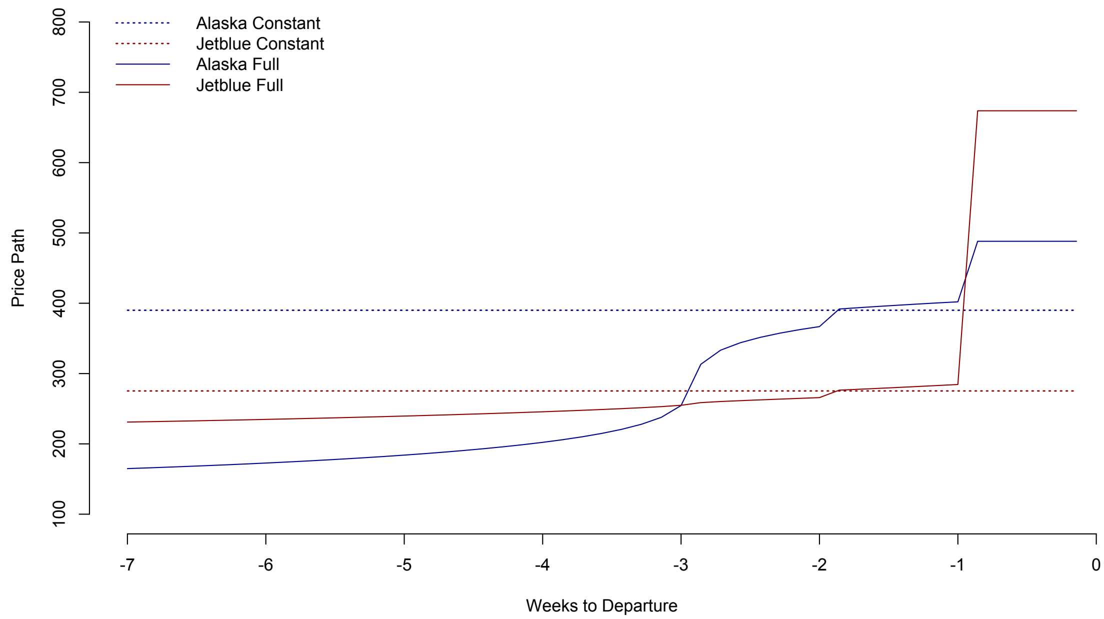



Fig. 17. Constant price v.s. discriminatory price without capacity constraint

Table 12: Price discrimination on firms' profits (without capacity constraint)

| | Constant Pricing | Discriminatory Pricing | Change |
|---|---|---|---|
| Alaska's Profits | $12112.68 | $14535.21 | +$2422.5 (+20.0%) |
| JetBlue's Profits | $17900.73 | $18922.65 | +$1021.92 (+5.7%) |
| Consumer Surplus | $ 37406.16 | $36161.83 | -$1460 (-3.3%) |
| Total Welfare | $67419.57 | $69619.69 | $1531.94 (+3.3%) |

path with duopoly price path. It shows that competition lowers early market prices more than it affects late market prices.

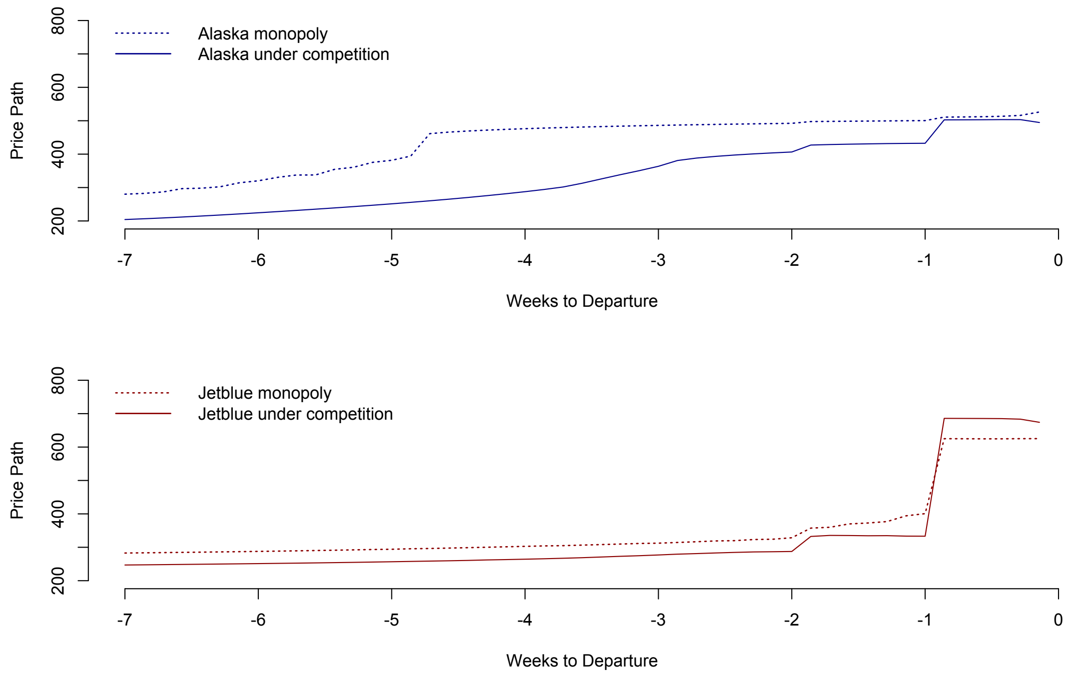



Fig. 18. Monopoly vs duopoly price path

After its competitor exits the market, the firm becomes a monopoly. Its prices in early market starts higher. Alaska raises its price by $100 seven weeks before departure. At around 4-5 weeks before departure, Alaska already raises its prices close to the duopoly late-market prices. This is because without a competitor, Alaska starts to sell to high type consumers much earlier. This is not optimal if JetBlue is still in the market. Meanwhile in the late market Alaska does not raise its prices by much after JetBlue exits the market. This is because the original late market prices at close to departure is already close to the monopoly optimal prices.

This result is consistent with Borenstein and Rose (1994). Competition is likely to increase the spread between early market prices and late market prices. Since competition lowers the early market prices much more than it does to late market. Interestingly, when Alaska exits the market, JetBlue even lower its prices in the late market. This is a result explained in Rosenthal (1980). When its competitor Alaska enters the market, JetBlue increases its late-market prices because it becomes optimal for them to focus on extracting surplus from its own high type consumers. As a result, JetBlue gives up the Alaska-leaning high type consumers and raise its prices.

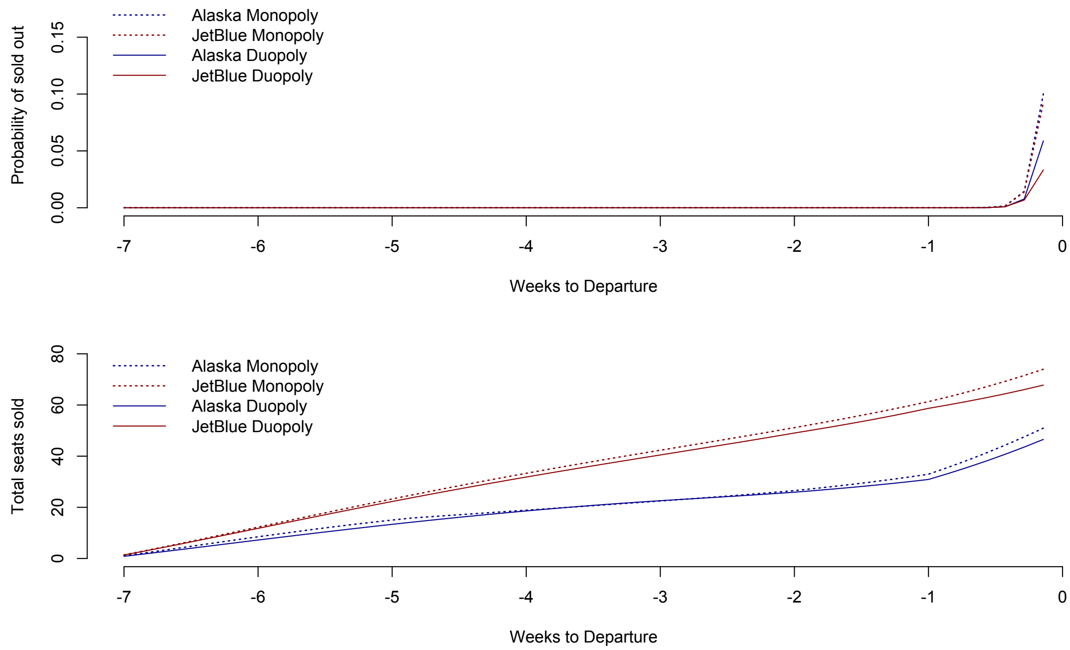



Fig. 19. Sales paths under monopoly v.s. under competition

# 7. Discussion

In summary, this paper attempts to empirically investigate the social and private incentives of dynamic price competition in the oligopoly airline markets. The pricing strategies being considered include price discrimination and pricing on demand uncertainty. In doing so, I first build an dynamic oligopoly model of differentiated products where firms face demand uncertainty, capacity constraint and selling deadline. To facilitate price discrimination, the demand model incorporates heterogenous and non-stationary consumer preferences. The baseline supply model allows firms to compete on time and remaining capacities of each other. To estimate the model, I collect a data set of daily-flight level prices and inventories for duopoly airline markets. I leverage price variations 15 days before and after an exit event to identify price sensitivities. To control for potential time trend, I select a control route that ex-ante has similar price paths. The inference strategy thus relies on a Difference-in-Difference research design. The estimation then concludes with a customized GMM estimator that accounts for unobserved product characteristic in small "non-invertiable" markets. Through counterfactuals, the paper shows how changing airlines strategy sets will affect competitive outcomes such as consumer surplus and firms' own profits, highlighting the motivations and impacts of these strategic toolkits.

# 8. Appendix

## 8.1. Proofs

### 8.1.1. Example: the elasticity ratio condition

The following toy model summarizes the welfare and profit effects of price discrimination in the context of airline price discrimination.

**Example 1.** *Consider that United and Delta compete for travelers from San Francisco to Boston. For a given flight, consumers can be segmented into two markets, i.e. late market and early market. For now, (1) ignore capacity constraint; (2) assume that each market is endowed with simple logit demand; (3) firms are symmetric. Late consumers are less price sensitive relative to early consumers. The impacts of price discrimination depend on the elasticity ratio.*[18] *As shown in Figure 20, the signs of price discrimination on both total industry profit and social welfare are not certain. When the early market's cross elasticity is high, price discrimination is more driven by excessive private incentives of airlines to undercut each other. This undercutting decreases both social welfare and their own profits. On the other hand, when the early market's industry elasticity is high, price discrimination is more driven by airlines' collective incentive. It is thus more likely to increase profits.*

### 8.1.2. Proposition 1

*Proof.* We have:

$$
\begin{aligned}
f^{(1)}\Big(Y_{-1} = y_{-1}, ..., Y_J = y_J\Big) &= P\Big(Y_{-1} = y_{-1}, ..., Y_J = y_J, X = \sum_{j\in J} y_j\Big) \\
&= P\Big(Y_{-1} = y_{-1}, ..., Y_J = y_J \mid X = \sum_{j\in J} y_j\Big) \times P\Big(X = \sum_{j\in J} y_j\Big) \\
&= \left(\frac{(\sum_{i\in J} y_j)!}{y_{-1}! \cdots y_J!} s_{-1}^{y_{-1}} \cdots s_J^{y_J}\right) \times \frac{e^{-\lambda}\lambda^{(\sum_{j\in J} y_j)}}{(\sum_{j\in J} y_j)!} \\
&= \frac{e^{-\lambda s_{-1}}(\lambda s_{-1})^{y_{-1}}}{y_{-1}!} \times \cdots \times \frac{e^{-\lambda s_J}(\lambda s_J)^{y_J}}{y_J!} \\
&= f(y_{-1}) \times \cdots \times f(y_J) \\
&= f^{(2)}\Big(Y_{-1} = y_{-1}, ..., Y_J = y_J\Big)
\end{aligned}
$$

□

[18] In this example, the price coefficients are −0.005 and −0.015 in late and early market respectively. Late market is twice bigger than early market. Late market preference intercept is fixed at zero.

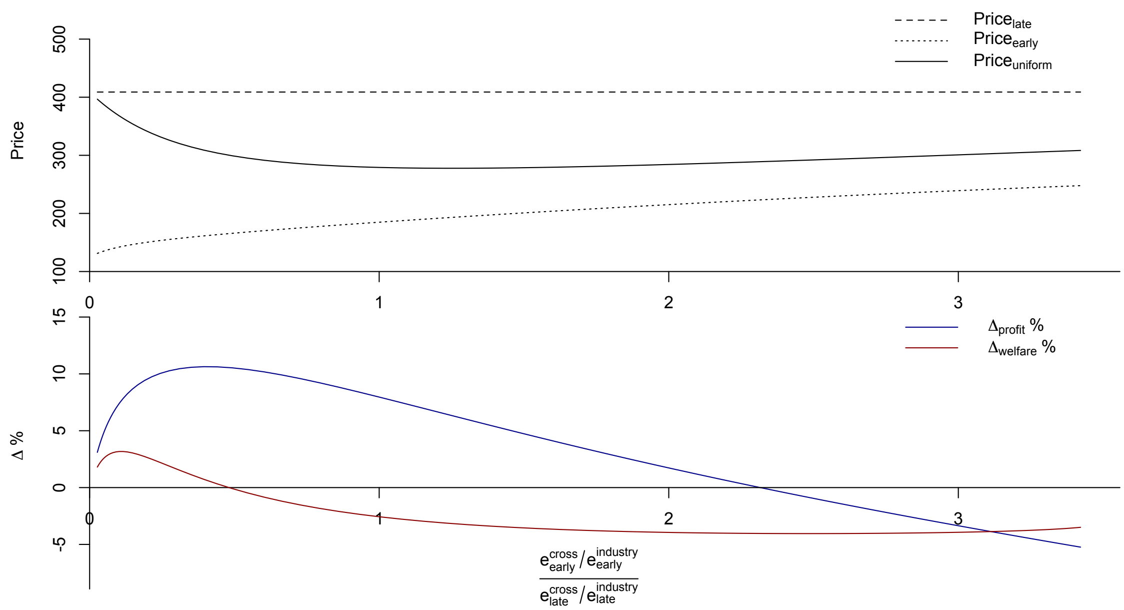



Fig. 20. The impact of price discrimination v.s. the elasticity ratio

*8.1.3. Market size identification*

We discuss in a more mathematical way with the following simple example. Consider a static duopoly pricing game with simple logit demand. Suppose we observe two realizations of market outcomes under an exogenous supply shock, $\{price_1^A, price_2^A, sale_1^A, sale_2^A\}$ and $\{price_1^B, price_2^B, sale_1^B, sale_2^B\}$. One can identify four demand parameters $\{\alpha, \delta_1, \delta_2, M\}$, i.e. price coefficient, two product intercepts and market size. Without loss of generality, we normalize the outside option's mean utility to zero.

- $\alpha = \frac{\log\left(\frac{sale_1^A/sale_2^A}{sale_1^B/sale_2^B}\right)}{(price_1^A - price_2^A) - (price_1^B - price_2^B)}$.

$$\begin{aligned}
\frac{sale_1^A/sale_2^A}{sale_1^B/sale_2^B} &= \frac{share_1^A/share_2^A}{share_1^B/share_2^B} \\
&= \frac{\frac{\exp\left(\alpha \times price_1^A + \delta_1\right)}{\exp\left(\alpha \times price_2^A + \delta_2\right)}}{\frac{\exp\left(\alpha \times price_1^B + \delta_1\right)}{\exp\left(\alpha \times price_2^B + \delta_2\right)}} \\
&= \exp\left(\alpha \times \left[(price_1^A - price_2^A) - (price_1^B - price_2^B)\right]\right)
\end{aligned}$$

- $\delta_1 - \delta_2 = \log\left(\frac{sale_1^A}{sale_2^A}\right) - \alpha \times (price_1^A - price_2^A)$.

$$\begin{aligned}\frac{sale_1^A}{sale_2^A} &= \frac{\exp\left(\alpha \times price_1^A + \delta_1\right)}{\exp\left(\alpha \times price_2^A + \delta_2\right)} \\ &= \exp\left[\alpha \times (price_1^A - price_2^A) + (\delta_1 - \delta_2)\right]\end{aligned}$$

- $M = sale_1^B + sale_2^B + \frac{(sale_1^B + sale_2^B) - (sale_1^A + sale_2^A)}{\frac{sale_1^A}{sale_1^B} \times \exp\left[\alpha \times (price_1^B - price_1^A)\right] - 1}$.

  Under condition $A$, outside option has "sales" $sale_0^A$:

$$\begin{aligned}sale_0^A &= share_0^A \times M \\ &= share_0^A \times \frac{sale_1^A}{share_1^A} \\ &= sale_1^A \times \frac{share_0^A}{share_1^A} \\ &= sale_1^A \times \frac{1}{\exp(\alpha \times price_1^A + \delta_1)}\end{aligned}$$

  Under condition $B$, outside option has "sales" $sale_0^B$:

$$\begin{aligned}sale_0^B &= share_0^B \times M \\ &= share_0^B \times \frac{sale_1^B}{share_1^B} \\ &= sale_1^B \times \frac{share_0^B}{share_1^B} \\ &= sale_1^B \times \frac{1}{\exp(\alpha \times price_1^B + \delta_1)}\end{aligned}$$

  Divide both sides of the two equations:

$$\frac{sale_0^A}{sale_0^B} = \frac{sale_1^A}{sale_1^B} \times \exp\left[\alpha \times (price_1^B - price_1^A)\right] \tag{22}$$

  Note that

$$\frac{sale_0^A}{sale_0^B} = \frac{M - sale_1^A - sale_2^A}{M - sale_1^B - sale_2^B} \tag{23}$$

  Combine the two equations we must have:

$$\frac{M - sale_1^A - sale_2^A}{M - sale_1^B - sale_2^B} = \frac{sale_1^A}{sale_1^B} \times \exp\left[\alpha \times (price_1^B - price_1^A)\right] \tag{24}$$

  Note this is one linear equation wit one unknown. We can solve

$$M = sale_1^B + sale_2^B + \frac{sale_1^B \times \left[(sale_1^B + sale_2^B) - (sale_1^A + sale_2^A)\right]}{sale_1^A \times \exp\left[\alpha \times (price_1^B - price_1^A)\right] - sale_1^B} \tag{25}$$

- It is easy to see that $\delta_1 - 0 = \log\left[\frac{sales_1^A}{M - sales_1^A - sales_2^A}\right] - \alpha \times p_1^A$.

## 8.2. Data

In this section, I use the whole dataset to show some stylized facts in the airline industry.[19] On the demand side, I use simple reduced-form regression analysis to confirm that sales are less elastic as it gets closer to the departure date. Moreover, my regression results suggest that early demand also has higher cross price elasticity. These motivate my later structural assumption on demand. On the supply side, I show evidence that supports my anecdotal discussion on airline pricing practice. Specifically, my regression results are consistent with stochastic pricing based on realized demand. It suggests that firm is more likely to increase its price in response to its own past sales as well as its competitor's sale. These motivate my later structural assumption on supply.

### 8.2.1. Demand Elasticities

On the demand side I consider separate regressions for each firm $j = 1, 2$:

$$\begin{aligned}\text{logit}\left[\mathbb{1}\{\text{Sale}^j_{i,d,t} > 0\}\right] = a_1 \times \log\left(\text{Price}^1_{i,d,t}\right) + a_2 \times \log\left(\text{Price}^2_{i,d,t}\right) \\ + \text{Flight}_{i,d} + \text{Trend}_{i,t}\end{aligned}$$

Since daily sales are very small with an average of 0.91, I adopt a logit specification with binary response. The subscripts $i$ denotes itinerary (or directional route), $d$ denotes departure date, and $t$ denotes days to departure. The upper-script $1, 2$ denote firms' identities. $\text{Sale}^j_{i,d,t}$ is the observed sale(s) for firm $j$'s flight in itinerary $i$, $t$ days before the departure date $d$. For this reduced form demand regression, I control for endogeneity in a pragmatic way by including more control variables. Specifically, $\text{Flight}_{i,d}$ is itinerary interacted with departure date, and it controls for flight-specific effect. $\text{Trend}_{i,t}$ is itinerary interacted with days to departure, and it captures itinerary-specific trend.

I split the data into two parts by the number of days to departure. For each half, I do two independent logit regressions for the two firms. I pool across all itineraries, so I need to decide how to match the asymmetric firms across the itineraries. I simply denote the bigger firm in each itinerary as firm 1.

Table 13 showed the results for these regressions. The results suggest that five to seven weeks before departure, demand are more elastic. Cross-elasticities are significantly greater than zero five to seven weeks before departure, and the coefficients are no longer significant for consumers arrive less than three weeks before departure.

### 8.2.2. Stochastic Pricing

On the supply side I would like to show some suggestive evidence that firms are pricing based on scarcity. One straightforward way is to regress prices on capacities, and show that flights sold better are priced higher. However, I am concerned about the endogeneity of capacities. Instead I consider the following regressions

[19] My goal is to show some general demand and supply patterns to motivate our structural assumptions, and I will discuss the formal research design in the following section.

Table 13: Sales on Prices

| | Logit | | | |
|---|---|---|---|---|
| | $\mathbb{1}\{\text{Sale}^1 > 0\}$ | | $\mathbb{1}\{\text{Sale}^2 > 0\}$ | |
| Week to departure | $5-7$ | $1-3$ | $5-7$ | $1-3$ |
| log(Price$^1$) | $-3.638^{***}$ | $-2.125^{***}$ | $0.509^{***}$ | $-0.054$ |
| | (0.153) | (0.108) | (0.144) | (0.106) |
| log(Price$^2$) | $0.295^{**}$ | 0.091 | $-3.827^{***}$ | $-1.202^{***}$ |
| | (0.120) | (0.071) | (0.151) | (0.081) |
| Controls | | | | |
| Flight | Yes | Yes | Yes | Yes |
| Trend | Yes | Yes | Yes | Yes |
| Observations | 18,451 | 17,320 | 18,451 | 17,320 |
| Log Likelihood | $-10{,}765$ | $-10{,}450$ | $-10{,}302$ | $-9{,}230$ |
| Akaike Inf. Crit. | 24,373 | 23,755 | 23,447 | 21,315 |

Note: S.E clustered at flight level. $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01

for each firm $j = 1, 2$:

$$\begin{aligned}\text{Price}^j_{i,d,t} = b_1 \times \mathbb{1}\{\text{Sale}^1_{i,d,t-1} > 0\} + b_2 \times \mathbb{1}\{\text{Sale}^2_{i,d,t-1} > 0\} \\ + \text{Price}^1_{i,d,t-1} + \text{Price}^2_{i,d,t-1} \\ + \text{Flight}_{i,d} + \text{Trend}_{i,t} + \epsilon_{i,d,t}\end{aligned}$$

Here I do a fixed effect regression, with $\text{Flight}_{i,d}$ capturing itinerary×departure date fixed effect and $\text{Trend}_{i,t}$ capturing itinerary× days to departure fixed effect. I pool across all itineraries and run the regression for each of the two firms which I labeled in a same way as before.

Table 14 shows results of these regressions. The result is mostly consistent with stochastic pricing. Though one of the coefficient is not significant. Note that I control for $\boldsymbol{p}_{t-1}$ to control for endogeneity. I try to make a point that an airline increases its price if it sells or if its competitor sells. I note that ruling out serial correlation of unobserved errors is very challenging. A sufficient condition is that airlines observed all $\xi_{t-1}$ when setting prices.

Table 14: Pricing on Realized Demand

| | Linear | |
|---|---|---|
| | $\text{Price}^1_t$ | $\text{Price}^2_t$ |
| $\mathbb{1}\{\text{Sale}^1_{t-1} > 0\}$ | $2.614^{***}$ | 0.587 |
| | (0.431) | (0.517) |
| $\mathbb{1}\{\text{Sale}^2_{t-1} > 0\}$ | $1.080^{***}$ | $4.120^{***}$ |
| | (0.396) | (0.597) |
| Controls | | |
| $\text{Price}^1_{t-1}$ | $0.664^{***}$ | $0.060^{***}$ |
| | (0.015) | (0.008) |
| $\text{Price}^2_{t-1}$ | $0.040^{***}$ | $0.644^{***}$ |
| | (0.005) | (0.016) |
| Fixed Effects | | |
| Flight | Yes | Yes |
| Trend | Yes | Yes |
| Observations | 47,931 | 47,931 |
| $R^2$ | 0.912 | 0.861 |
| Adjusted $R^2$ | 0.908 | 0.854 |

Note: S.E. clustered at flight level. $^*p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

# References

Alderighi, M., Nicolini, M., Piga, C. A., 2012. Combined effects of load factors and booking time on fares: Insights from the yield management of a low-cost airline .

Bajari, P., Benkard, C. L., Levin, J., 2007. Estimating dynamic models of imperfect competition. Econometrica 75, 1331–1370.

Berry, S., Jia, P., 2010. Tracing the woes: An empirical analysis of the airline industry. American Economic Journal: Microeconomics 2, 1–43.

Berry, S., Levinsohn, J., Pakes, A., 1995. Automobile prices in market equilibrium. Econometrica: Journal of the Econometric Society pp. 841–890.

Berry, S. T., 1990. Airport presence as product differentiation. The American Economic Review 80, 394–399.

Berry, S. T., 1994. Estimating discrete-choice models of product differentiation. The RAND Journal of Economics pp. 242–262.

Besanko, D., Dubé, J.-P., Gupta, S., 2003. Competitive price discrimination strategies in a vertical channel using aggregate retail data. Management Science 49, 1121–1138.

Board, S., Skrzypacz, A., 2016. Revenue management with forward-looking buyers. Journal of Political Economy 124, 1046–1087.

Borenstein, S., 1985. Price discrimination in free-entry markets. The RAND Journal of Economics pp. 380–397.

Borenstein, S., 1992. The evolution of us airline competition. The Journal of Economic Perspectives 6, 45–73.

Borenstein, S., Rose, N. L., 1994. Competition and price dispersion in the us airline industry. Journal of Political Economy 102, 653–683.

Borzekowski, R., Thomadsen, R., Taragin, C., 2009. Competition and price discrimination in the market for mailing lists. QME 7, 147–179.

Busse, M. R., Rysman, M., 2001. Competition and price discrimination in yellow pages advertising .

Cabral, L. M., Villas-Boas, M., 2005. Bertrand supertraps. Management Science 51, 599–613.

Caplin, A., Nalebuff, B., 1991. Aggregation and imperfect competition: On the existence of equilibrium. Econometrica: Journal of the Econometric Society pp. 25–59.

Chandraa, A., Ledermana, M., 2016. Revisiting the relationship between competition and price discrimination. Tech. rep., Working paper, Rotman School of Management.

Collard-Wexler, A., 2013. Demand fluctuations in the ready-mix concrete industry. Econometrica 81, 1003–1037.

Corts, K. S., 1998. Third-degree price discrimination in oligopoly: all-out competition and strategic commitment. The RAND Journal of Economics pp. 306–323.

Courty, P., Pagliero, M., 2012. The impact of price discrimination on revenue: Evidence from the concert industry. Review of Economics and Statistics 94, 359–369.

Dana, Jr, J. D., 1998. Advance-purchase discounts and price discrimination in competitive markets. Journal of Political Economy 106, 395–422.

Davidson, C., Deneckere, R., 1986. Long-run competition in capacity, short-run competition in price, and the cournot model. The Rand Journal of Economics pp. 404–415.

Edgeworth, F. Y., 1925. The pure theory of monopoly. Papers relating to political economy 1, 111–142.

Ericson, R., Pakes, A., 1995. Markov-perfect industry dynamics: A framework for empirical work. The Review of Economic Studies 62, 53–82.

Escobari, D., 2012. Dynamic pricing, advance sales and aggregate demand learning in airlines. The Journal of Industrial Economics 60, 697–724.

Gaggero, A. A., Piga, C. A., 2011. Airline market power and intertemporal price dispersion. The Journal of Industrial Economics 59, 552–577.

Gallego, G., Hu, M., 2014. Dynamic pricing of perishable assets under competition. Management Science 60, 1241–1259.

Gallego, G., Van Ryzin, G., 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons. Management science 40, 999–1020.

Gerardi, K. S., Shapiro, A. H., 2009. Does competition reduce price dispersion? new evidence from the airline industry. Journal of Political Economy 117, 1–37.

Gershkov, A., Moldovanu, B., Strack, P., 2016. Revenue maximizing mechanisms with strategic customers and unknown, markovian demand. Management Science .

Goettler, R. L., Gordon, B. R., 2011. Does amd spur intel to innovate more? Journal of Political Economy 119, 1141–1200.

Goolsbee, A., Petrin, A., 2004. The consumer gains from direct broadcast satellites and the competition with cable tv. Econometrica 72, 351–381.

Hendel, I., Nevo, A., 2013. Intertemporal price discrimination in storable goods markets. The American Economic Review 103, 2722–2751.

Holmes, T. J., 1989. The effects of third-degree price discrimination in oligopoly. The American Economic Review 79, 244–250.

Iyer, G., Pazgal, A., 2003. Internet shopping agents: Virtual co-location and competition. Marketing Science 22, 85–106.

Jeziorski, P., Krasnokutskaya, E., 2016. Dynamic auction environment with subcontracting. The RAND Journal of Economics 47, 751–791.

Jofre-Bonet, M., Pesendorfer, M., 2003. Estimation of a dynamic auction game. Econometrica 71, 1443–1489.

Kreps, D. M., Scheinkman, J. A., 1983. Quantity precommitment and bertrand competition yield cournot outcomes. The Bell Journal of Economics pp. 326–337.

Lazarev, J., 2013. The welfare effects of intertemporal price discrimination: an empirical analysis of airline pricing in us monopoly markets. New York University .

Levin, Y., McGill, J., Nediak, M., 2009. Dynamic pricing in the presence of strategic consumers and oligopolistic competition. Management science 55, 32–46.

Lin, K. Y., Sibdari, S. Y., 2009. Dynamic price competition with discrete customer choices. European Journal of Operational Research 197, 969–980.

Maskin, E., Tirole, J., 1988. A theory of dynamic oligopoly, i: Overview and quantity competition with large fixed costs. Econometrica: Journal of the Econometric Society pp. 549–569.

McGill, J. I., Van Ryzin, G. J., 1999. Revenue management: Research overview and prospects. Transportation science 33, 233–256.

Nair, H., 2007. Intertemporal price discrimination with forward-looking consumers: Application to the us market for console video-games. Quantitative Marketing and Economics 5, 239–292.

Osborne, M. J., Pitchik, C., 1986. Price competition in a capacity-constrained duopoly. Journal of Economic Theory 38, 238–260.

Petrin, A., Train, K., 2010. A control function approach to endogeneity in consumer choice models. Journal of marketing research 47, 3–13.

Pierson, M. P., Allon, G., Federgruen, A., 2013. Price competition under mixed multinomial logit demand functions .

Rosenthal, R. W., 1980. A model in which an increase in the number of sellers leads to a higher price. Econometrica: Journal of the Econometric Society pp. 1575–1579.

Rust, J., 1987. Optimal replacement of gmc bus engines: An empirical model of harold zurcher. Econometrica: Journal of the Econometric Society pp. 999–1033.

Siegert, C., Ulbricht, R., 2015. Dynamic oligopoly pricing: Evidence from the airline industry .

Staiger, R. W., Wolak, F. A., 1992. Collusive pricing with capacity constraints in the presence of demand uncertainty. The RAND Journal of Economics pp. 203–220.

Stavins, J., 2001. Price discrimination in the airline market: The effect of market concentration. Review of Economics and Statistics 83, 200–202.

Stole, L. A., 2007. Price discrimination and competition. Handbook of industrial organization 3, 2221–2299.

Sweeting, A., 2015. A model of non-stationary dynamic price competition with an application to platform design .

Villas-Boas, J. M., 2007. A note on limited versus full information estimation in non-linear models. work-ing paper, Haas School of Business, University of California, Berkeley .

Villas-Boas, J. M., Winer, R. S., 1999. Endogeneity in brand choice models. Management science 45, 1324–1338.

Villas-Boas, S. B., 2009. An empirical investigation of the welfare effects of banning wholesale price discrimination. The RAND Journal of Economics 40, 20–46.

Weintraub, G. Y., Benkard, C. L., Jeziorski, P., Van Roy, B., 2008. Nonstationary oblivious equilibrium. Manuscript, Columbia University pp. 1375–1411.

Williams, K. R., 2013. Dynamic airline pricing and seat availability. Department of Economics, University of Minnesota (Working Paper) .