Demand Dynamics in the Seasonal Goods Industry: An Empirical Analysis

Gonca P. Soysal, University of Texas at Dallas, gonca.soysal@utdallas.edu
Lakshman Krishnamurthi, Northwestern University, laksh@kellogg.northwestern.edu

Abstract

This study develops and estimates a dynamic model of consumer choice behavior in markets for seasonal goods where products are sold over a finite season and availability is limited. In these markets, retailers often use dynamic markdown policies in which an initial retail price is announced at the beginning of the season and the price is subsequently marked down as the season progresses. Strategic consumers face a tradeoff between purchasing early in the season when prices are higher but goods are available and purchasing later when prices are lower but the stock-out risk is higher. If the good starts providing utility as soon as it is purchased (e.g., apparel), consumers purchasing earlier in the season can also get more use from the product compared to those purchasing later.

Our structural model incorporates three features essential to modeling the demand for seasonal goods: changing prices, limited availability and possible dependence of total consumption utility on the time of purchase within a finite season. In this model, heterogeneous consumers have expectations about future prices and product availability and strategically time their purchases. We estimate the model using aggregate sales and inventory data from a fashion goods retailer.

The results indicate that, in the fashion goods context, ignoring consumers’ expectations about future availability or the change in total consumption utility over the season can lead to biased demand estimates. We find that strategic consumers delay their purchases to take advantage of markdowns and that these strategic delays hurt the retailer’s revenues. Retailer revenues facing strategic consumers are 11% lower than they would have been facing myopic consumers. Limited availability on the other hand reduces the extent of strategic delays by motivating consumers to purchase earlier. We find that the impact of strategic delays on retailer revenues would have been as high as 30% if there were no stock-out risk. By means of counterfactual experiments, we show that the highest retailer profits are achieved by offering small markdowns early in the season. On the other hand, given current markdown percentages, the retailer can improve profits by carrying less stock as consumers accelerate purchases and purchase at higher prices when they anticipate scarcity in future periods. As long as the reduction in availability is not great, the profit gain from earlier higher priced sales can overcome the loss due to the reduction in overall sales.
1 Introduction

The goal of this paper is to develop and estimate a dynamic model of consumer choice behavior in markets for seasonal goods\(^1\) where products are sold over a finite season, and availability is limited.

Some examples of seasonal goods are fashion apparel, holiday merchandise and concert and airline tickets. The empirical context for our analysis is the fashion apparel market. Seasonal goods exhibit unique demand characteristics when compared to consumer packaged goods or durable goods. First, there is a well defined, finite selling horizon; goods are introduced into the market, sold over a (usually short) season and are discontinued. Second, products provide utility either over a finite season (e.g., fashion goods or holiday merchandise) or as a lump sum all at once (e.g., airline or concert tickets). Also, in the case where the product provides utility over a finite season, consumers’ total utility from consumption could depend on the time of purchase within the finite season. For example, a consumer in the market for a swimsuit would get more use out of the product if she purchases it earlier in the season.

These demand characteristics coupled with two important supply side considerations create challenges for the seasonal goods retailer in pricing and inventory management. First, replenishment lead times are much longer in some seasonal goods industries (e.g., fashion apparel), compared to the length of the selling season. This limits the retailer’s opportunity to replenish the inventory during the season (capacity is fixed well in advance for other seasonal goods like airline, concert tickets or hotel rooms as well and can only be increased at a substantially high marginal cost). Second, end of season salvage value is very low, i.e., products are perishable. So, the seasonal goods retailer faces the challenge of maximizing profits from ordering a fixed amount of inventory before the selling season and selling it over a finite horizon.

After setting an initial stock level, seasonal goods retailers often resort to inter-temporal (dynamic) pricing policies and prices for seasonal goods exhibit substantial variation within the season. In the fashion goods industry for example, it is common practice to employ markdown pricing. Every new product line is introduced at a “retail price” and prices are marked down a number of times until the inventory is cleared or the selling season ends. In the rest of the paper, we will talk about the fashion apparel market but our methodology and results would extend to any market where the selling horizon is well defined and availability is limited.

\(^1\) Also called short life cycle goods or perishable goods
Inter-temporal pricing can help the retailer in two ways. First, it enables the retailer to segment the market and take advantage of differences in consumers’ willingness to pay and willingness to wait for the products. If some consumers prefer to buy early in the season at the expense of paying higher prices while others choose to wait to purchase at lower prices, the retailer can serve the first group at higher prices and can lower the price to serve the second group once the first group purchases and exits the market. Second, it helps the retailer to respond to a decrease in his option value from holding onto inventory as the season progresses. If the retailer has overstocked in the initial period for example, he can reduce prices later in the season in order to clear shelves before the end of the season (i.e., before the inventory perishes).

Recent years have witnessed a significant increase in the percentage of retail products sold on sale and in the magnitude of markdowns. Surveys conducted by the National Retail Federation (1998) indicate that markdowns as a percentage of sales have risen from 6% in 1967 to 20% by 1998 for department stores, and from 10% to 28% for specialty stores. The same survey also reports that more than 72% of fashion products sold in 1998 were sold at a discount. Both an explosion in the variety of products offered and diversification in consumer tastes are believed to be driving factors behind this increase (Fisher et al., 1994, Pashigian, 1988).

Advances in information technology and marketing research have increased the ability of retailers to collect and analyze consumer data, making it easier for them to employ complex pricing strategies. On the other hand, consumers have been getting increasingly more sophisticated as well. In making a purchase, a strategic consumer weighs the benefits of purchasing today against the benefits of waiting and purchasing in the future at a lower price. As a recent Wall Street Journal (2002) article reports, it is now possible for the consumers to “crack the retailer’s pricing code” and “not pay retail.”

In contrast to durable goods markets where prices also decline over a product’s lifecycle, the game between the retailers and the consumers has an interesting dimension in the fashion goods market (and in all markets where capacity is fixed in the short term). As supply is limited, a consumer cannot wait for a sale without taking into account the risk of stock-out. So, consumers need to tradeoff decreasing prices against the possibility of not being able to find the product later in the season. The retailer also faces a tradeoff. Having too much in stock might increase his chances of meeting the demand but limiting the stock on the other hand might motivate strategic consumers to buy earlier at higher prices. Zara, a large Spanish producer and
retailer of fashion goods, for example, is well known for its success in implementing a deliberate limited stock strategy.

This study develops a structural model of the dynamic decision process on the consumer (demand) side and uses the estimates from this model to investigate retailer pricing and inventory policies. We model strategic consumer behavior where consumers form expectations about future prices and availability and take stock-out risk into account when timing their purchases. Our contribution can be summarized under three headings. First, we empirically investigate the impact of consumers’ product availability expectations on purchase behavior. An important strategic consideration is that consumers may accelerate purchases (even at the expense of paying higher prices) if they anticipate that the product may be unavailable in the future periods. We are not aware of prior empirical work that models consumers’ expectations about product availability. Previous work on the role of price expectations in consumer purchase behavior assumes unlimited capacity, so that consumers would have the option of buying the product again next period with certainty if they choose to delay their purchases. (Erdem et al. 2003, Song and Chintagunta, 2003, Nair, 2007). Through a counterfactual experiment we empirically illustrate that anticipated scarcity can lead to purchase acceleration and higher retailer profits. Second, we assume a finite horizon and explicitly model the dependence of the total utility consumers get from the consumption of a product on the time of purchase. Previous work on durable goods adoption assumes an infinite horizon and that consumers’ total discounted utility from the product does not depend on time of adoption (Song and Chintagunta, 2003, Nair, 2007). Consumer uncertainty about future product availability and change in total consumption utility over time are two factors that rationalize early purchases despite declining prices over a short season. Ignoring these factors would result in biased demand estimates in the fashion goods context. Third, we add to the vast theoretical literature on revenue management by providing empirical evidence on seasonal goods demand and pricing using a realistic demand model and counterfactual experiments.

Our demand model enables a seasonal goods retailer to decompose the effects of different factors that contribute to change in demand over time. Understanding the separate effects of decreasing total consumption utility, increasing stock-out risk, changing market size and market composition on demand, and market responsiveness is important as each of these factors has different implications for the retailer’s pricing and inventory strategy. Our structural approach allows us to obtain behavioral predictions that are invariant to the effects of policy changes and
allows us to simulate various pricing/stocking policies and study their profit implications. A seasonal goods retailer can use our demand model in jointly determining optimal initial inventory levels, retail prices and timing and depth of markdowns.

Our focus is on permanent markdowns as opposed to temporary promotions (e.g., Labor Day sale) where prices are reduced for a limited time and go back up again. In our model, we assume that there is a separate market for each product. Every period (week), each consumer in the market for a specific product decides to buy the product or wait until the next period. Consumers are strategic and heterogeneous with respect to their response parameters. Consumers have expectations regarding the likelihood of future states like prices and availability and choose to buy the product and exit the market if the expected discounted sum of utilities from buying in that period exceeds that from waiting.

The data used to estimate our model comes from a specialty apparel retailer that sells private label fashions. Aggregate weekly sales, inventory and cost data are available for 105 SKUs (which we aggregate into 61 products as discussed in the data section) from the women’s coats category, for a period of two years. Each product is introduced, sold over a finite season and is discontinued. The season length varies from 11 to 30 weeks. There is significant variation in sales and prices across products. Each product is marked down at least once during its season.

To preview our results, we note that our model produces a good fit to the data. Our estimates imply that the market is composed of two distinct consumer segments. The first segment consists of low price sensitivity consumers who account for 86% of the total market at the start of the season. The second segment consists of high price sensitivity consumers who start purchasing late in the season and account for an important percentage of the end-of-season sales.

We compare our model to a benchmark model which reflects the current state of the art. The benchmark model, similar to Nair (2007), accounts for heterogeneity in response parameters and for the price expectations of strategic consumers but does not take consumers’ availability expectations into account and assumes that the total consumption utility does not depend on time of purchase. This model produces unreasonable demand estimates (e.g., the price sensitivity estimate is positive for both segments) and the likelihood ratio test shows that our model outperforms the benchmark model.

We find that price elasticities for both segments decrease over time and pricing decisions in the early periods are very critical since early markdowns can significantly increase sales but might affect total revenue very negatively if not timed optimally. Given the estimated demand
model, we run three counterfactual experiments to investigate different elements of a retailer’s markdown and inventory policy. The first experiment provides insights about the retailer’s tradeoff between the timing and depth of markdowns. This experiment shows that, under a uniform single markdown policy, the highest retailer profits are achieved by offering early and small markdowns. Late markdowns do not have very favorable profit outcomes and early and deep markdowns are very detrimental to retailer profits. The second experiment examines how limiting availability impacts the retailer’s performance and shows that given the current markdown percentages and timing, the retailer can improve his profits by carrying less stock. We find that a slight decrease in the initial stock offered can encourage strategic consumers to purchase earlier at higher prices due to increased stock-out risk. Despite the negative effect on the total quantity sold, sales at higher prices compensate for lost sales. In the third experiment, we show that strategic consumers who delay their purchases to take advantage of lower prices contribute to a 11% reduction in retailer’s revenue. However, decreasing product availability reduces consumers’ incentive to wait for lower prices. We show that if consumers had not taken stock-out risk into account when timing their purchases, strategic delays would have been more pronounced and the loss in revenue due to strategic behavior would have been larger (30%).

The rest of the paper is organized as follows. Section 2 introduces the related literature. Section 3 presents the model and Section 4 outlines the estimation strategy. Section 5 introduces the data used in the empirical application. Section 6 presents demand estimates and the counterfactual experiments and discusses pricing and inventory management implications. Section 7 concludes with a discussion of the results and future directions.

2 Related Literature

Our study is closely related to three main streams of literature. First is the economics literature on inter-temporal price discrimination. Second is the economics and operations literature on revenue management and third is the recent marketing literature on structural models of strategic consumer behavior. We will briefly discuss each research stream highlighting ideas and articles closely related to our study.

Interest in inter-temporal demand considerations and strategic consumer behavior in the economics literature started in the area of durable goods monopoly pricing. (Coase, 1972, Stokey, 1979, Besanko and Winston, 1990). This literature is rich in theoretical work
investigating demand side considerations but does not study supply side considerations like limited availability.

Clearance sales have received some attention in the economics literature. Pashigian (1988) provides empirical evidence on sales offered by department stores. He offers the growing importance of “fashion” (variety) as an explanation for the changes in markdowns over time and between merchandise groups. Pashigian and Bowen (1991) provide further empirical evidence and offer demand uncertainty and price discrimination as two alternative hypotheses to explain the observed pricing practices.

The second research stream of interest is the vast operations literature on revenue management. The reader is referred to Elmaghraby and Keskinocak (2003) for an extensive review. Gallego and van Ryzin (1994), Bitran and Mondschein (1997) and Bitran et al. (1998) all study analytical dynamic pricing models where a retailer sells a fixed inventory over a finite horizon. These studies derive two structural properties of the optimal policy that state that at any given time the optimal price decreases in the number of items left, and for any given number of items the optimal price decreases over time. Research in revenue management has largely focused on analytical models incorporating supply side considerations. Strategic consumer behavior has been considered only very recently in the literature on seasonal goods pricing (Su, 2007, Aviv and Pazgal, 2008, Dasu and Tong, 2008, Zhou et al., 2006). These models show analytically that heterogeneity in willingness to pay, change in consumer utility over time and limited and uncertain product availability may all be factors that can rationalize early purchasing by some consumers when prices are expected to fall. In this study, we empirically model and investigate the potential role of these factors on consumer purchase decisions. Liu and van Ryzin (2008) using a stylized two period model show that under certain conditions, it might be profitable for a monopolist seller facing strategic consumers to create rationing risk by intentionally under-stocking products. We provide empirical evidence for a seller’s incentive to deliberately limit stock to induce earlier purchases at higher prices. This literature, however, lacks empirical research with realistic demand models. To our knowledge, Heching et al. (2002) is the only recent empirical study in this area. They estimate a simple demand model using data from a specialty apparel retailer and obtain estimates of revenues under various pricing policies. The analysis, consistent with our findings, suggests that the firm would have increased its revenue if it had smaller mark-downs earlier in the season.
There has been some recent theoretical (McAfee and te Velde, 2006) and empirical (Sweeting, 2008) work investigating dynamic pricing of seasonal goods in the economics literature. Sweeting (2008) in the context of major league baseball ticket prices shows that significant decline in ticket prices as the game approaches is mostly due to declining option values of the sellers rather than changes in elasticity of demand. Sweeting shows that many cases of observed early buying can be rationalized by uncertainty about future prices and availability as well as return to market costs. In the case of baseball tickets (and airline tickets, concert tickets, etc.) consumption utility from the product is received as a lump sum all at once and does not depend on the time of purchase. On the other hand, when goods start to provide utility as soon as they are purchased like in the case of fashion goods (and holiday merchandise, etc.) there is another interesting dynamic. Waiting decreases the lifetime utility from the product and the likelihood of purchase. We show that decrease in lifetime utility over time is another important dynamic that rationalizes early purchases in the presence of decreasing prices over a short season. It would be possible for future researchers to incorporate return to market costs into our model as another factor that explains early purchases to study goods that have the dynamics captured by Sweeting (2008) as well as the dependence of total consumption utility on the time of purchase. There might also be other costs involved in waiting to buy, as pointed out by Sweeting (2008). One such cost could be complementary investments like booking hotels or transportation tickets for a sporting event, finding complements for apparel items, e.g., a matching shirt for a pant. Consistent with this, we should observe people living further from the stadium and people who have hard to find sizes purchasing earlier. This effect and its interaction with uncertainty about future availability can also be incorporated into the model.

There has been an increasing interest in studying the impact of product availability and stock-outs on consumer demand. Gönül and Srinivasan (1996) and Erdem et. al (2003) model household product availability and consumers' consideration of in-house inventory and stock-out risk in a utility maximization framework. Bruno and Vilcassim (2008) incorporate information on store product availability in a structural demand model and show that neglecting the effects of stock-outs leads to biases in the demand estimates. Musalem et al. (2010) develop a structural demand model using store level sales and inventory data to endogenously capture the effect of stock-outs and provide insights on the lost sales and financial consequences due to stock-outs. We add to this literature by explicitly modeling forward looking consumers' consideration of expected future stock-outs in making purchase decisions. In the presence of scarcity risk in
future periods, we show that consumers would be willing to purchase earlier even at the expense of paying higher prices. This finding highlights the importance of product availability as a strategic decision variable for the retailer. Impact of product availability on demand is not limited to lost sales or product substitution due to stock-outs. We also show that limited availability does not always have to have negative financial consequences.

The third literature stream is the recent economics and marketing literature on structural models of strategic consumer behavior. A large number of articles have studied dynamic models of consumer decision making where there is uncertainty about product quality, future prices, promotions or product introductions. Erdem and Keane (1996) present a structural dynamic choice model where “forward looking” consumers are uncertain about attributes of a set of brands, and learn about these brands through advertising exposure and usage experience. They find that the forward-looking model fits the data statistically better than myopic models. Erdem et al. (2003) and Hendel and Nevo (2005) study demand models for frequently purchased, storable goods that are subject to stochastic price fluctuations. Both studies show that price expectations have important effects on demand elasticities, and long-run cross price elasticities (allowing for the effect of price-cut on future expected prices) are much larger than the short-run cross-price elasticities (holding expectations fixed). All these models are constructed for frequently purchased consumer goods and are estimated on scanner panel data.

High-tech durables markets are similar to the seasonal goods markets in the sense that prices exhibit a declining pattern over the life-cycle of a product creating an incentive for consumers to delay purchases and repeat purchases are rare. Our model is similar to discrete choice models of durable goods adoption developed in the high-tech durables context. Melnikov (2000) models strategic consumers’ adoption behavior using data from the computer printer market but does not allow for consumer heterogeneity. Song and Chintagunta (2003) analyze the impact of price expectations on the diffusion patterns of new high-tech products using aggregate data but do not allow for econometric errors in the demand function. Our model is closest to Nair (2007) who empirically estimates a dynamic structural pricing model in the video game industry using aggregate data, allowing for consumer heterogeneity and econometric error terms but does not allow for availability considerations or the dependence of the total utility from consumption on the adoption timing. We not only apply the durable goods adoption model to the seasonal goods markets but also extend this model to allow for consumer uncertainty about future availability and to account for change in total consumption utility over the season.
Our study lies on the interface of the recent economics and marketing literature on structural dynamic discrete choice demand models and the vast operations literature on revenue management.

3 Model

3.1 Overview

We develop a dynamic structural model of demand in markets for seasonal goods where consumers are strategic and heterogeneous. We treat each product as a separate market in our model. Every period, a consumer in the market for a specific product decides to buy the item and exit the market or wait until the next period and make a decision again. Consumers are strategic and choose to buy the product if the expected discounted sum of utilities from buying in that period exceeds that from waiting. Consumers also have the option of not buying the item at the end of the season. We assume that consumers do not buy multiples of the same product (i.e., our model is an incidence model, not a volume model). When calculating expected future utilities, consumers take into account their expectations about future prices as well as their expectations about future availability (i.e., stock-out risk).

Our model captures three important characteristics of seasonal goods demand. First, consumers’ responsiveness to prices and other marketing variables change through the season due to the decrease in number of time periods left for consumption and increase in stock-out risk. Second, since the product is a durable, consumers purchasing the product exit the market and the potential market for a specific item shrinks through the season. Third, the composition of the market changes over time (as long as there is heterogeneity in the consumer population). For example, if consumers have different price sensitivities and face declining prices, less price sensitive consumers will purchase the product in the earlier periods and exit the market and the proportion of more price sensitive consumers in the remaining market will increase over time.

Capturing the effect of variation in total utility from consumption over the finite horizon as well as consumers’ consideration of future stock-out risk is important. An empirical regularity in the data is that, except for a brief period early in the season, sales for a specific product decline over time at a given price. Sales increase in the periods where prices are marked down but decrease immediately after the markdown period. Decreasing total utility from consumption and increasing stock-out risk both reduce a consumer’s incentive to wait and contribute to the decrease in sales over time. Estimates from a demand model would be biased if one does not
account for these two effects. Ignoring these effects would result in a downward bias in the price sensitivity parameter and/or an upward bias in the markdown sensitivity parameter.

It is also important to account for strategic consumer behavior. A number of studies have found that, in the CPG industry, consumers form expectations about product quality (Erdem and Keane, 1996), coupon availability (Gönül and Srinivasan, 1996) and future prices (Erdem, Imai and Keane, 2003). These studies have shown that strategic models fit the data better than myopic models. In the fashion apparel market, consumers have even higher incentives to behave strategically in timing their purchases as they face significant reduction in prices over a short season and are also subject to availability risk. The retailer needs to account for strategic behavior as it affects the shape of the aggregate sales curve and induces price dynamics in the market (Song and Chintagunta, 2003). Facing strategic consumers, the retailer needs to take inter-temporal substitution into account since a price reduction today will influence sales in future periods. Besanko and Winston (1990) showed that reduction in profit by assuming that the consumers are myopic when in fact they are strategic could be rather significant for a retailer.

In addition to modeling strategic behavior, it is important to account for consumer heterogeneity. Allowing for heterogeneity provides a flexible pattern for the aggregate sales curve (Song and Chintagunta, 2003). The retailer needs to understand and account for heterogeneity in the consumer population as this understanding enables the retailer to take advantage of differences in the population by adjusting prices dynamically through the sales season. We incorporate consumer heterogeneity through an aggregate analog to the latent class models used with household purchase data (Kamakura and Russel, 1989). We assume that each consumer belongs to one of a finite number of segments and each segment is characterized by its own response parameters.

Finally, our structural approach allows us to obtain behavioral predictions that are invariant to the effects of policy changes and allow us to simulate various pricing/stocking policies and study their profit implications. The interested reader is referred to Chintagunta et al. (2005) for a discussion comparing structural and reduced form modeling approaches.

### 3.2 The Utility Specification

A consumer $i$’s conditional indirect utility from purchasing product $j$ in period $t$ is defined as:

$$U_{ijt} = \alpha_j + \sum_{t=1}^{T} \gamma^{t-t_0} c + \beta_p \ p_j + \beta_m \ d_j + \beta_{it} \ s_i + \xi_j + \epsilon_{ijt}$$

(1)
Where $p_{jt}$ is the price of product $j$ in period $t$, $d_{jt}$ is the markdown dummy and $s_t$ is a seasonal dummy. $\alpha_j$ is the intrinsic preference for product $j$, $c$ is the per period consumption utility, $T_j$ is the length of the sales season for product $j$ and $\gamma$ is the discount factor. $\beta_{ip}$ is price sensitivity, $\beta_{im}$ is markdown sensitivity and $\beta_{is}$ is the seasonality parameter. $\bar{z}_{jt}$ is a product and time specific demand shock and $\varepsilon_{ijt}$ is a mean-zero stochastic term.

$U_{ijt}$ is defined as the total utility a consumer gets from purchasing the product and includes not only the instantaneous (current period) utility but also the discounted sum of all future utilities the consumer will get from using this product within the finite season. As we discussed earlier, the total utility a consumer gets from a product changes over the season depending on the time of purchase. A consumer in the market for a winter coat for example, would be able to wear it for a longer period if she purchases the coat earlier in the season.

In the utility specification, $\alpha_j$ controls for taste for product $j$ relative to the outside option that is constant over time and $c$ is the per-period consumption utility. $\sum_{t'=0}^{T_j} \gamma^{t'-t} c$ represents the discounted present value of the total utility from consumption of the product through the finite season if a purchase takes place in period $t$. We assume that the product provides utility only over the finite season and that there is no salvage value. We further assume that per-period utility from consumption ($c$) is constant over the season. The change in the remaining number of periods that the product can provide utility over the season allows us to capture the change in total consumption utility as the season advances. As we discuss later in Section 4.3, both $\alpha_j$ and $c$ are assumed to be common across segments (to reduce the number of non-linear parameters to a manageable number) but since the price parameter is heterogeneous, it allows us to capture the differences in willingness to pay across segments. We also assume that $c$ is constant across products since $\alpha_j$ controls for cross-product differences. The discount factor is also assumed to be common across segments following the literature.

The markdown dummy, $d_{jt}$, is set to 0 in the earlier periods when the product is sold at retail (full) price and is set to 1 as soon as the product is marked down and stays at 1 until the end of the season. This variable is included to capture the “mere markdown” effect; the possibility that consumers might get extra (or less) utility from purchasing on-sale. This parameter also captures any merchandising effort used to support markdown products, that is uniform across
products. In our empirical application, markdown products are moved to a special section in the store, and if this treatment makes these products more noticeable, this effect will be captured by the markdown dummy. The seasonal dummy, $s_t$, is included to capture the possibility that utility from a product might be higher (lower) during peak or low seasonal periods. A close examination of the seasonality patterns in the data reveals a strong demand peak in the six week “holiday shopping period” that starts after Thanksgiving and ends after Christmas. A regression of overall sales on relative prices and a set of dummies for all possible seasonal periods (e.g., Mother’s day, Labor day, holiday shopping period) reveals that the holiday shopping period is the only period that has a significant effect on the overall demand. In our application, $s_t$ is set to 1 for the holiday shopping period and 0 for all other periods.

Note that the $\xi_{jt}$ is a product and time specific demand shock and $\epsilon_{ijt}$ is a mean-zero stochastic term, observed by the consumers but not by the econometrician. $\xi_{jt}$ controls for any additional product and time specific factors consumers observe and take into account in making a purchase decision but the econometrician does not observe. In the fashion apparel context, $\xi_{jt}$’s could correspond to demand shifters like a specific product appearing in an advertisement or a T.V. show in a specific week. $\xi_{jt}$’s also serve as the econometric error term in the estimation of demand. The instantaneous utility from not buying product j in period t is normalized to $\epsilon_{ij0t}$: $U_{ij0t} = \epsilon_{ij0t}$.

### 3.3 Availability

In a limited stock environment, a consumer is likely to face a stock-out in any period. In a consumer packaged goods context and a multi-store environment, Bruno and Vilkassim (2006) operationalize availability as the probability of finding the product in a store in a given shopping trip. For the purposes of this study we resort to a similar definition and define availability of a specific item in a time period as the probability that a consumer visiting a store in that period finds the item in stock.

It is well accepted that consumers value high availability. However, accounting for the effect of availability on demand is a challenge. To account for the effect of availability on individual consumer purchase decisions in a multi-store retail environment, one would need real-time data on individual consumer store visits, purchases and real-time inventory data at the store
and SKU level. Practitioners and researchers though, typically only have access to data where sales and inventory information is aggregated across time and/or stores. The data set we use in our empirical application comes from a multi-store retailer. In our main data set, sales and inventory information is aggregated across stores and total sales and opening inventory levels for each (active) SKU is reported for 104 weeks.

In the absence of detailed real-time data, “retail distribution” has been used as a proxy for availability (Bruno and Vlccassim, 2006). Retail distribution is defined, in its simplest form, as “the number of outlets carrying a product (has the product in stock) as a percentage of total outlets.” In this study, we follow a similar approach and use retail distribution as our availability measure. In order to calculate retail distribution, we supplement our main data set with weekly individual store level inventory data over the 104 week period. In the supplementary data set, we have store level inventory data for 57 of the 105 SKUs in our main data set.

Calculation of retail distribution levels is straightforward for these 57 SKUs. Define $D_{jt}$ as the store level inventory vector for product $j$ and week $t$ as $D_{jt} = (i_{j1t}, i_{j2t}, \ldots, i_{jSt})$ where $i_{jt}$ represents the inventory level for product $j$ in store $s$ in period $t$. Next, define $I_{jst} \in \{0,1\}$ as the indicator of the event “item $j$ is in stock at store $s$ in period $t$”. For each product and time period, we need to set the indicator $I_{jst}$ to 1 for those stores that have positive inventory of product $j$ and to 0 for other stores. The retail distribution, $\lambda_{jt}$, then is computed by summing $I_{jst}$ across all stores and dividing by the total number of stores $S$; i.e., $\lambda_{jt} = \sum_{s} I_{jst} / S$. So if the retailer has 100 stores and 65 stores have positive inventory of product $j$ in period $t$, $I_{jst}$ is set to 1 for these 65 stores and to 0 for all other stores and retail distribution is computed as $\lambda_{jt} = 65/100 = 0.65$.

At the data preparation stage, we calculate retail distribution levels observed in the store level data for each product-week combination for these 57 SKUs. For the remaining 48 SKUs in our sample we observe aggregate weekly inventory levels but not the store level weekly inventory levels. $Inv_{jt}$ corresponds to the aggregate inventory level for product $j$ and period $t$, i.e., $Inv_{jt} = \sum_{s} I_{jst}$. In order to predict the retail distribution levels for these 48 SKUs, we use the relation $\lambda_{jt} = a_1 \ln(Inv_{jt}) + a_2 Inv_{jt} + a_3 Inv_{jt}^2 + \epsilon_{jt}$. Since we observe both $Inv_{jt}$ and $\lambda_{jt}$ for the 57 SKUs, we estimate the specified relation between retail distribution, $\lambda_{jt}$, and aggregate
inventory, $Inv_{jt}$, with OLS using data for these SKUs ($R^2 = 0.98$, $N = 1556$). Then we use the parameter estimates to predict the retail distribution levels ($\lambda_{jt}$) for each period for the 48 SKUs for which we observe only the aggregate inventory levels ($Inv_{jt}$).

3.4 Expectations

Consistent with the majority of studies in the dynamic choice models literature (e.g., Song and Chintagunta, 2003, Erdem, Imai and Keane, 2003) we assume that consumers have rational expectations about the future values of state variables. Rational expectations assumptions have been questioned by Manski (2004) because these assumptions may be intrinsically implausible in some contexts. It would be ideal to collect data on individual consumers’ expectations and incorporate this information into the dynamic choice model. Erdem et al. (2005), for example, relax the rational expectations assumption by using survey data on self-reported consumer price expectations. We do not, however, have access to similar data on consumer expectations. In the absence of such data, we use the rational expectations assumption to provide a reasonable approximation to consumer expectations.

It is well accepted that consumers rely on past experience and other signals to predict future states of the world like future prices and availability. In modeling consumers’ price and availability expectations, we assume that consumers observe the current price and availability every period and compute expected future prices and availability relying on past experience.

3.4.1 Price Process

In order to approximate consumers’ expectations of future prices, we need to specify a realistic price process that captures key characteristics of seasonal goods pricing: (1) each item is introduced at a “retail” (full) price, (2) prices are constant for several weeks followed by permanent markdowns, and (3) the probability and the magnitude of markdowns depend on the retail price, time in the season and whether the product has been previously marked down. One important observation is that products with higher initial prices are typically marked down earlier and deeper. Also, the time between the introduction of the product and the announcement of the first markdown is typically longer than the time between two successive markdowns and the first markdown is typically deeper than the later markdowns in dollar terms.

To capture these important characteristics, we specify a multivariate jump process (see Erdem et al. 2003 for a similar specification in the CPG context). We assume that price for
product $j$ that was introduced in period 1 is marked down in period $t$ with probability $\varphi_j$ and stays constant from period $(t-1)$ to period $t$ with probability $\varphi_{0j} = (1 - \varphi_j)$. If we define $P_{jt}$ as price of product $j$ in period $t$, and $MD_{jt}$ as markdown depth for product $j$ in period $t$, we have:

$$P_{jt} = P_{j,t-1} - MD_{jt} \text{ with probability } \varphi_j \text{ and } P_{jt} = P_{j,t-1} \text{ with probability } \varphi_{0j}.$$  

Define $P_{j1}$ as the retail (initial) price of product $j$ and $time_{jt}$ as the number of weeks since the introduction of the product (time in the season). To accommodate the possibility that the price process may differ before and after the first markdown, we specify and estimate the markdown probability process separately for these two cases. We specify the markdown probability conditional on no previous markdowns as:

$$P_{jt} = P_{j,t-1} - MD_{jt} \text{ with probability } \varphi_j \text{ and } P_{jt} = P_{j,t-1} \text{ with probability } \varphi_{0j}.$$  

$$P_{jt} = P_{j,t-1} - MD_{jt} \text{ with probability } \varphi_j \text{ and } P_{jt} = P_{j,t-1} \text{ with probability } \varphi_{0j}.$$  

Define $P_{jt}$ as the retail (initial) price of product $j$ and $time_{jt}$ as the number of weeks since the introduction of the product (time in the season). To accommodate the possibility that the price process may differ before and after the first markdown, we specify and estimate the markdown probability process separately for these two cases. We specify the markdown probability conditional on no previous markdowns as:

$$(\varphi_j \mid P_{j,t-1} = P_{jt}) = \frac{\exp[a_0 + a_1P_{jt} + a_2time_{jt}]}{1 + \exp[a_0 + a_1P_{jt} + a_2time_{jt}]} \quad t=2,\ldots,T \quad (2)$$  

And specify the markdown probability conditional on a previous markdown as:

$$(\varphi_j \mid P_{j,t-1} < P_{jt}) = \frac{\exp[b_0 + b_1P_{jt} + b_2time_{jt}]}{1 + \exp[b_0 + b_1P_{jt} + b_2time_{jt}]} \quad t=3,\ldots,T \quad (3)$$  

In case of a markdown for product $j$ in period $t$, the markdown depth (in log-dollars) is specified as a function of log-retail price and a markdown dummy variable $mdum_{jt}$ (that is set to 1 if the product was previously marked down and to 0 otherwise) as follows:

$$\ln(MD_{jt}) = \theta_0 + \theta_1 \ln(P_{jt}) + \theta_2 mdum_{jt} + \varepsilon_j \quad (4)$$  

Where the price shocks are assumed to be distributed i.i.d normal across products and time periods, $\varepsilon_j \sim N(0,\sigma_{MD}^2)$.

We tested numerous alternative specifications for markdown probabilities and markdown depth (e.g., inclusion of the current period price instead of the retail price, time since last price change instead of time since introduction, etc.), none of which improved the predictive power significantly. We believe our specifications capture important characteristics of the seasonal goods price process in a parsimonious way.

### 3.4.2 Availability Process

In order to approximate consumers’ expectations of future product availability, we specify the following linear process that links current availability to past availability, product’s retail price and time in the season:
\[ \lambda_{jt+1} = \gamma_0 \lambda_{jt} + \gamma_1 P_{jt} + \gamma_2 \text{time}_{jt} + e_{jt} \quad \text{where} \quad e_{jt} \sim N(\mu_{jt}, \sigma_{jt}^2) \]  

(5)

where \( \lambda_{jt} \) is the availability of item \( j \) at time \( t \), \( P_{jt} \) is the retail (initial) price for item \( j \) and \( \text{time}_{jt} \) is the number of periods since the beginning of the season for item \( j \) at time \( t \). \(^2\)

The price and availability process parameters are estimated in a first stage using data from our full data set and are reported in the results section. In the demand estimation stage, we assume that consumers know and use these parameters to form their estimates of prices and availability for future periods. A similar strategy has been employed to model consumers’ price expectations in a CPG context by Erdem, Imai and Keane (2003). Note that the specified processes are adaptive. In other words, consumers are assumed to observe actual realizations of prices and availability every period and update their expectations for future periods.

### 3.4.3 Evolution of States

The consumer’s optimal purchase timing problem can be described by the solution to a finite horizon dynamic programming problem with time, price, availability, markdown status, seasonality, demand shocks and the error terms as state variables. We define the state vector, \( S_t \), as a vector of all these variables that influence a consumer’s purchase decision at time \( t \).

We assume that the unobservable (by the retailer) error terms \(( \varepsilon_{jt}, \varepsilon_{j0t} )\) evolve independently from the other state variables. Partitioning \( S_t \) into \( X_t \) and \( \varepsilon_t = (\varepsilon_{jt}, \varepsilon_{j0t}) \) where \( X_t \) represents all state variables, except the unobservable error terms, the transition probabilities have the following form:

\[ P(S_{t+1} \mid S_t) = P(X_{t+1}, \varepsilon_{t+1} \mid X_t, \varepsilon_t) = P(X_{t+1} \mid X_t) P(\varepsilon_{t+1} \mid \varepsilon_t) \]

This is the well known conditional independence assumption widely used in the literature (Rust, 1994). We further assume that the unobservable error terms are i.i.d. extreme value distributed. Among the state variables time and seasonality evolve naturally. The price process we specified in section 3.4.1 describes the evolution of prices and the markdown status and the availability process we defined in section 3.4.2 describes the evolution of availability. Every period, consumers observe actual realizations of price and availability for the specific product.

---

\(^2\)Note that the availability measure lies between 0 and 1 but we chose a linear specification with a normal error term. In our empirical application, since availability is closely related to last period’s availability and price, this model works very well prediction wise and the fitted \( \hat{\lambda} \)’s stay between 0 and 1. In an alternative specification, one can transform the dependent variable to \( \ln((1 - \hat{\lambda}_{jt}) / \hat{\lambda}_{jt}) \). This way, \( \hat{\lambda}_{jt} = 1/(\exp(Z) + 1) \) where \( Z = \gamma_0 + \gamma_1 \hat{\lambda}_{jt-1} + \gamma_2 P_{jt} + \gamma_3 \text{time}_{jt} + e_{jt} \) and is constrained to be between 0 and 1 where \( e_{jt} \sim N(0, \sigma_{jt}^2) \). The \( \hat{\lambda} \)’s from the alternative and the original specifications are highly correlated (0.96). We decided to retain the linear model for its simplicity and smaller standard error.
and update their expectations for future period prices (markdowns) and availability levels. Consumers also observe time, seasonality, demand shocks and error terms before making a purchase decision. The retailer on the other hand is assumed to observe time, prices, availability, markdown status, seasonality, and the product and time specific demand shocks ($\xi_{jt}$’s) but not the error terms ($\epsilon_{jt}$) before making pricing decisions.

### 3.5 Consumer’s Decision Rule and Dynamic Optimization Problem

$U_{ijt}(S_t)$ represents the utility consumer $i$ gets from purchasing item $j$ in period $t$ when the state of the world is $S_t$ and $U_{ij0}(S_t)$ is the instantaneous utility from the “no-purchase” option under the same conditions. Then, the value of buying product $j$ at time $t$ for a strategic consumer $i$ is given by:

$$V_{ijt}(S_t) = U_{ijt}(S_t)$$

The value of the “no-purchase” option (waiting) at time $t$ for a strategic consumer $i$ is the value from delaying the purchase. The value of the “no-purchase” option in period $t$ is modeled as the sum of (a) the discounted expected value that a consumer can get at time $t+1$ and (b) the instantaneous utility the consumer can get from the “no-purchase” option. With the “no-purchase option” the consumer gets to choose again next period between purchasing and waiting. Therefore, her expected next period value is the maximum of the value from choosing to wait and the value from choosing to buy. One important point to note here is that the consumer will make this choice only if the product is available next period. If the product is not available, she will get zero utility. So, a strategic consumer $i$ calculates the expected value from waiting in period $t$, taking expected availability and prices into account as follows:

$$V_{ijt}(S_t) = \gamma E\left[\lambda_{j,t+1} \max\{ V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})\} + (1-\lambda_{j,t+1}) \times 0 \left| S_t \right\} + U_{ij0}(S_t) \right]$$

Where $\gamma$ is the discount factor and $\lambda_{j,t+1}$ is availability in period $t+1$. If we normalize $U_{ij0}(S_t)$ to $\epsilon_{ij0t}$, the expression simplifies as follows:

$$V_{ijt}(S_t) = \gamma E\left[\lambda_{j,t+1} \max\{ V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})\} \left| S_t \right\} + \epsilon_{ij0t} \right]$$

(6)

The individual consumer’s decision rule is such that consumer $i$ buys item $j$ and exits the market in period $t$ only if her value from buying in period $t$ exceeds her value from waiting and she had chosen to wait in all previous periods:
\[ V_{ij} \geq V_{ij0t} \quad \text{and} \quad V_{ijt} < V_{ij0t} \quad \text{for all} \quad \tau < t \]

On the other hand, the consumer does not buy product \( j \) in period \( t \) and stays in the market if her value from waiting in period \( t \) exceeds that from buying and she had chosen to wait in all previous periods:

\[ V_{ij} < V_{ij0t} \quad \text{and} \quad V_{ijt} < V_{ij0t} \quad \text{for all} \quad \tau < t \]

4 Estimation

4.1 Overview

We have described the consumer’s decision process in the model section. In this section we describe the estimation of the model parameters. We start by discussing the computation of the unconditional purchase probabilities under distributional assumptions about the stochastic term, \( \varepsilon_{ijt} \). Then we discuss the computation of the market shares by aggregating these probabilities across heterogeneous consumers for each product and time period. And finally, we present the MLE estimation strategy we use in our dynamic setting to estimate the model parameters. This estimation strategy allows efficient estimation of a large number of parameters.

4.2 Calculation of the Purchase Probabilities

Recall the specification of the value function for the purchase and no-purchase options respectively;

\[
V_{ij}(S_t) = \alpha_j + \sum_{t=1}^{T_j} \gamma^{-t} c + \beta_{ij} p + \beta_{im} d + \beta_{ia} s + \varepsilon_{ijt} \quad (7)
\]

\[
V_{ij0t}(S_t) = \gamma E[\max\{V_{ij1t}(S_{i1}), V_{ij02t}(S_{i2})\} | S_i] + \varepsilon_{ij0t} \quad (8)
\]

The expectation in (8) is taken with respect to the distribution of future variables unknown to the consumer conditional on the current information.

Remember that we have assumed that the unobservable error terms \( (\varepsilon_{ijt}, \varepsilon_{ij0t}) \) evolve independently from the other state variables. We further assume that the unobservable error terms are i.i.d. extreme value distributed.

We define \( W_{ij} \) and \( W_{ij0t} \) as the observable (by the retailer) part of the value functions for the purchase and no-purchase options respectively.

\[ V_{ij0t} = W_{ij0t} + \varepsilon_{ij0t} \quad \text{and} \quad V_{ijt} = W_{ijt} + \varepsilon_{ijt} \]

We can write down \( W_{ij} \) and \( W_{ij0t} \) as a function of state variables as follows:

\[
W_{ij}(S_t) = \alpha_j + \sum_{t=1}^{T_j} \gamma^{-t} c + \beta_{ij} p + \beta_{im} d + \beta_{ia} s + \varepsilon_{ij} \quad (9)
\]
\[ W_{jt}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{g,t+1}(S_{t+1}), V_{g0,t+1}(S_{t+1})\} | S_t] \]  

(10)

Note that in equation (10), following Rust (1987), calculation of the expectation with respect to the distribution of future variables unknown to the consumer can be simplified. The integration with respect to the extreme value errors can be done analytically and \( W_{jt}(S_t) \) can be expressed by the following equation:

\[ W_{jt}(S_t) = \gamma \int D_{j,t+1} \ln\{\exp[W_{jt,t+1}(S_{t+1})] + \exp[W_{jt0,t+1}(S_{t+1})]\} dF(S_{t+1} | S_t) \]  

(11)

Equations (8)-(11) define \( W_{jt} \) and \( W_{jt0} \) as a function of state variables, respectively. Since we have assumed that the unobserved error terms in equations (7) and (8) follow an i.i.d. extreme value distribution, the individual level unconditional purchase probabilities, have the following logit form:

\[ P_{jt} = \frac{\exp(W_{jt})}{\exp(W_{jt}) + \exp(W_{jt0})} \]  

(12)

The aggregate purchase probability (market share) for each product \( j \) and period \( t \) is calculated by integrating \( P_{jt} \) over the consumer heterogeneity distribution. Before specifying the aggregate purchase probabilities, we will discuss our approach in modeling consumer heterogeneity and the evolution of heterogeneity.

4.3 Consumer Heterogeneity and Market Shares

We model consumer heterogeneity using a random coefficients approach. We use a discrete approximation to the parameter distribution and our method is an aggregate analog of the latent-class models widely used for individual level data (Kamakura and Russel, 1989). We assume there are \( K \) segments in the population and consumers in segment \( k \) (\( k=1,\ldots,K \)) share the common parameters \( \beta_k \) where \( \beta_k \) is a vector consisting of the price sensitivity parameter \( (\beta_{kp}) \), the markdown sensitivity parameter \( (\beta_{km}) \) and the seasonality parameter \( (\beta_{ks}) \); that is, \( \beta_k = (\beta_{kp}, \beta_{km}, \beta_{ks}) \). These parameters are common across all products. The initial size of segment \( k \), i.e., proportion of consumers who belong to segment \( k \) in the potential market, is represented by \( \pi_k \) and \( \sum_{k=1}^{K} \pi_k = 1 \). As segments are allowed to be heterogeneous in their response parameters, they would exhibit different adoption patterns and segment sizes would change over time within the season. Segments with lower price sensitivities would adopt earlier.
and the proportion of consumers belonging to these segments in the remaining market would fall over time. Notice that we have assumed that intrinsic preferences and per-period consumption utilities are homogenous across segments in order to reduce the number of non-linear parameters to a manageable number but heterogeneity in the price parameter allows us to capture the differences in willingness to pay across segments.

Let \( M_{j0} \) be equal to the market size for product \( j \), i.e., the number of potential consumers that are in the market for product \( j \). Define \( N_{jkt} \) to be the number of remaining consumers from segment \( k \) in the market for product \( j \) at any period \( t \). \( N_{jkt} \) is determined by the total market size \( M_{j0} \), segment size \( \pi_k \) and the proportion of consumers from segment \( k \) who have not bought the item at any period before \( t \). Then, the evolution of \( N_{jkt} \) in the market is given by:

\[
N_{jkt} = M_{j0} \pi_k \prod_{l=1}^{t-1} (1 - P_{jkt})
\]

If we define \( \theta_{jkt} \) as the size of segment \( k \) in the market for product \( j \) at time period \( t \), \( \theta_{jkt} \) can be calculated as follows:

\[
\theta_{jkt} = \frac{N_{jkt}}{\sum_{m=1}^{K} N_{mjt}} = \frac{\pi_k \prod_{l=1}^{t-1} (1 - P_{jkt})}{\sum_{m=1}^{K} \pi_m \prod_{l=1}^{t-1} (1 - P_{mjt})}
\]

Aggregating over the heterogeneity distribution, market share for product \( j \) at time \( t \), \( MS_{jt} \) can be calculated as follows:

\[
MS_{jt} = \sum_{k=1}^{K} \theta_{jkt} P_{jkt} = \sum_{k=1}^{K} \theta_{jkt} \frac{\exp(W_{jkt})}{\exp(W_{jkt} + W_{jkt})}
\]

Now that we have explained the calculation of market shares, we will next discuss the strategy employed to estimate the model parameters.

### 4.4 Estimation Strategy

The model parameters to be estimated consist of product fixed effects represented by the vector \( \alpha \), \( \alpha = (\alpha_1, \ldots, \alpha_J) \), the per period consumption parameter \( c \), segment specific parameters represented by the vector \( \beta \), \( \beta = (\beta_1, \ldots, \beta_K) \) where \( \beta_k = (\beta_{kp}, \beta_{km}, \beta_{kl}) \), standard deviation of the mean zero normally distributed demand shocks \( \sigma_i \), and the initial segment sizes \( \pi = (\pi_2, \ldots, \pi_K) \). Call the set of all model parameters \( \Omega \) where \( \Omega = (\alpha, c, \beta, \pi, \sigma_i) \).
We do not estimate the discount factor, \( \gamma \). It has been noted that it is extremely difficult to identify the discount factor in dynamic decision models (Gowrisankaran and Rysman, 2007, Magnac and Thesmar, 2002) and it is common practice to set the discount factor to a reasonable pre-determined value. Following Gowrisankaran and Rysman (2007) and Song and Chintagunta (2003) who set the monthly discount factor to 0.99, we set the weekly discount factor to 0.9975 which is equivalent to a monthly discount of 0.99. Define \( X_{jt} = (p_{jt}, d_{jt}, s_{jt}) \) as the set of covariates. Let \( \beta_{jt} = \alpha_j + \sum_{i=1}^{T_j} \gamma^{t-j} c + X_{jt} \beta_t + \xi_{jt} \) denote segment 1’s mean utility for product \( j \) at time \( t \). Also, let \( \overline{\beta}_k = (\beta_k - \beta_1) \) denote segment \( k \)’s parameter difference relative to segment 1 for \( k=2,...,K \). Using this notation, we can now rewrite the share equation (13) as:

\[
MS_{jt} = \theta_{1jt} \frac{\exp(\delta_{jt})}{\exp(W_{jt}) + \exp(\delta_{jt})} + \sum_{k=2}^{K} \theta_{kjt} \frac{\exp(X_{jt} \overline{\beta}_k + \delta_{jt})}{\exp(W_{jt}) + \exp(X_{jt} \overline{\beta}_k + \delta_{jt})} \tag{14}
\]

\( W_{kj0t} \), the observable part of the value from waiting for segment \( k \), product \( j \) and time period \( t \), is a function of \( (\delta_{j,t+1},...,\delta_{jt}) \), observed covariates and model parameters.

### 4.4.1 Calculation of Value from Waiting

Next, we will discuss how one can compute value from waiting in period \( t \), \( W_{kj0t} \) starting from period \( T_j \) and working backwards for \( t=T_j-1,T_j-2,...,1 \) and \( k=2,...,K \). Note that \( \overline{\beta}_1 = 0 \) for segment 1. Consumers are uncertain about the evolution of prices and availability levels through the season conditional on the current information. Therefore, integration will be performed over the distribution of future prices and availability given the current state of the world. We can re-write equation (11) as:

\[
W_{kj0t} = \gamma \int \lambda_{j,t+1} \ln(\exp[\delta_{j,t+1} + X_{j,t+1} \overline{\beta}_k]) + \exp[W_{kj0,t+1}(S_{t+1})])dF(S_{t+1} | S_t) \tag{15}
\]

The value from waiting is calculated by simulated integration of (15). Recall that we have earlier defined the process consumers use to form their expectations about future prices and availability given the current state of the world.

In order to integrate over price expectations, we simulate price paths conditional on the current state. Let us first define an expected markdown path for product \( j \) in period \( t \) as \( \Delta_{jt} = (Y_{t+1}, Y_{t+2},...,Y_T) \) where \( Y_i \) equals 1 if a new markdown takes place in period \( t \) and 0 otherwise. If we are 2 periods away from the last period for example, there are four expected
markdown paths. We might face another markdown the next period (1,0) or in the last period (0,1), we might face one markdown in each period (1,1), or we might not face a markdown at all (0,0). Suppose there are M such possible paths, $\Delta^m_{jt}$, m=1,…M, in period $t$. Using equation (2) and equation (3) we can calculate the probability associated with each path conditional on the current state of the world, $P(\Delta^m_{jt} | S_t)$. Given all possible future markdown paths, we draw N random vectors, $E_n$, n=1…N, each consisting of $(T_j-t)$ error terms corresponding to equation (4) that defines markdown depth for a possible markdown in each period until the end of the season. $E_n = (\varepsilon^n_{j,t+1}, \varepsilon^n_{j,t+2}, \ldots, \varepsilon^n_{j,T})$. Error terms in these random vectors are i.i.d. normal with mean zero and variance $\sigma^2_{MD}$. As mentioned in Section 3.4.1, $\sigma^2_{MD}$ is estimated from the data together with other coefficients in equation (4) in the first stage. For each random vector, there is a corresponding markdown depth vector, $MD^n_{jt}(S_t) = (MD^n_{j,t+1}, MD^n_{j,t+2}, \ldots, MD^n_{j,T})$. The combination of a possible markdown path and a simulated vector of markdown depths correspond to a simulated price path.

In order to integrate over availability expectations on the other hand, we simulate availability paths conditional on the current state. For each time period $t$ and product $j$, we draw L random vectors consisting of $(T_j-t)$ error terms, $\Psi_l = (\varepsilon^l_{j,t}, \varepsilon^l_{j,t+1}, \ldots, \varepsilon^l_{j,T})$. Error terms in these random vectors are i.i.d. normal with mean zero and variance $\sigma^2_\lambda$. As mentioned in Section 3.4.2, $\sigma^2_\lambda$ is estimated from data together with other coefficients in equation (5) in the first stage. For each random vector, there is a corresponding future availability vector $\lambda^l_{jt}(S_t) = (\lambda^l_{j,t+1}, \lambda^l_{j,t+2}, \ldots, \lambda^l_{j,T})$. $W_{kj0}$ is calculated by averaging the value of the integrand over N random availability vectors, L random markdown depth vectors and M markdown paths in two steps. In the first step, we calculate the value from waiting corresponding to an availability vector and a markdown depth vector as can be seen in equation (16) below. In order to calculate value from waiting in any period $t$, we make use of the finite horizon nature of the problem, start at the final period, period $T_j$, and calculate value from waiting at each time period backwards recursively. Value from waiting in the last period is normalized to zero, $W^{lam}_{kj0T_j} = 0$.

---

3 In order to reduce the dimensionality of the problem, we assume a maximum of 2 markdowns for each product during a season. This is a reasonable assumption because 99.4% of all sale revenue comes from sales before the 3rd markdown.
In the second step, we compute value from waiting for each product and time period by averaging the values from the first step for the L markdown depth and N availability vectors.

\[ W_{kj0}(S_t) = \frac{1}{L} \sum_{l=1}^{L} \sum_{n=1}^{N} W_{kj0}(S_t) \]  

(17)

### 4.4.2 Estimation Algorithm

Now that we have completed the discussion of how we compute the market shares for each product and each period, we will next discuss the estimation algorithm. Our estimation algorithm nests a finite horizon dynamic programming solution to the optimal stopping problem within a Berry (1994) style fixed point calculation for demand estimation. We assume time and product specific demand shocks are distributed i.i.d. normal, \( \xi_{jt} \sim N(0, \sigma^2_\xi) \).

The estimation algorithm can be summarized in four levels of loops:

1. In the outermost loop, we search over non-linear parameter values \( \bar{\beta}_k \), parameter differences of other segments relative to segment 1 and \( \pi_k \), relative segment sizes, \( k=(2,\ldots,K) \). The objective function we optimize over is explained in the third step.

2. In the second loop, given a value of the unknown non-linear parameters \( \bar{\beta}_k \) and \( \pi_k \), mean valuations, \( \delta_{jt} \), that equate the observed market shares to the predicted market shares (these come from the fourth loop) are computed by inverting (14). Inversion is made using the contraction-mapping algorithm suggested by Berry (1994) since the function cannot be inverted analytically. Output of this loop is the corresponding mean valuations, \( \delta_{jt} \) given a fixed value of the non-linear parameters.

3. In the third loop, given mean valuations \( \delta_{jt} \), linear parameters (response parameters of the first segment, product fixed effects and per period consumption parameter), \( (\beta_1, \alpha_j, c) \), are estimated and the implied demand shocks, \( \xi_{jt} \)'s, are calculated using the equality \( \delta_{jt} = \alpha_j + \sum_{t=1}^{T} \gamma^{t-1} c + X_{jt} \beta_t + \xi_{jt} \). Then we maximize the likelihood for the observed data generated by the demand distribution implied by the distribution of the demand.
shocks, $\xi_j$. Outputs of this loop are $(\beta, \alpha_j, c)$ and the objective function value given a fixed value of non-linear parameters and the corresponding mean valuations.

4. In the inner most (fourth) loop, given a value of the non-linear parameters and the corresponding mean valuations, $\delta_j$, the predicted market shares are calculated by solving for the consumers’ dynamic optimization problem. As discussed in section 4.4.1 this step requires the calculation of value from waiting for each segment for each remaining time period starting from the last period and working backwards. Given the value from waiting in each time period, calculation of predicted market shares follows the discussion in section 4.3. Output of this loop is the predicted market shares given a fixed value of the non-linear parameters and the corresponding mean valuation.

Our estimation strategy is similar to those used in the recent literature for estimating aggregate discrete choice demand models of differentiated goods. We include dynamics by nesting the innermost loop that is used for solving the consumers’ dynamic optimization problem in a finite horizon setting. We also allow for a discrete heterogeneity distribution. Interested readers are referred to Berry, Levinsohn and Pakes (1995) and Nevo (2000) for a more detailed discussion of a related estimation strategy using the GMM framework in a static setting.

4.5 Market Size

In order to estimate demand, we need to have a measure of the initial (potential) market size for each SKU. Knowledge of the initial market size allows us to calculate the observed market share of purchasers (and non-purchasers) using sales data every time period. The previous literature has reported two important considerations in defining the initial market size (Nevo, 2000). The first consideration is to allow for a nonzero share of the outside good and the second consideration is to check the sensitivity of the results to the market definition. Unlike the retailer, who has information on the market potential of each product, the researcher does not have data on market potential and needs to infer the size of the potential market from the data.

---

4 The likelihood as a function of demand parameters is: $l(\beta) = \prod_{t=1}^{T} f_\xi(D_t^{-1}(q_t; \Omega)) J$ where $D_t$ is the aggregate demand function, $f_\xi$ is the pdf for $\xi$ and $J$ is the Jacobian. $\|J\| = \|D_t^{-1}(q_t; \Omega)/\partial q_j\| = \|\partial \xi_j / \partial q_j\|$. We can define $G = \sum_{k=1}^{K} N_{kj} P_{kj}(q_t; \Omega) - q_j = 0$ and calculate the Jacobian, using the implicit function theorem $\|J\| = -(\partial G / \partial q_j)^T (\partial G / \partial \xi_j) = 1/\sum_{k=1}^{K} N_{kj} P_{kj}(q_t; \Omega) \left[ 1 - P_{kj}(q_t; \Omega) \right]$. The derivation follows a similar derivation by Nair(2007).
Previous studies define the market size by choosing a variable to which the market size is proportional, and choosing the proportionality factor. Nevo (2000) calculates the size of the market for ready-to-eat cereal to be one serving of cereal per capita per day. Bresnahan et al. (1997) define the market for computers to be the total number of office-based employees.

In our case, since the retailer places his order for a specific product before the product is introduced, this order size provides us information on the retailer’s beliefs about the sales potential for that product. We chose the order size as the variable to which the market potential is proportional to and selected a proportionality factor of 1.25 to allow for a nonzero share of the outside good. Our demand estimates are not sensitive to the proportionality factor. When we increase the proportionality factor, the only change that we observe is an increase in the size of the bargain hunter (more price sensitive) segment and the increase in the size of this segment comes from additional customers who do not purchase the product by the end of the season.

4.6 Identification

Our model is a dynamic logit model estimated with aggregate data using the BLP (Berry, Levinsohn and Pakes, 1995) contraction mapping approach combined with MLE. The reader is referred to Nevo (2000) for a detailed discussion of the identification of the random coefficients multinomial logit model in a static setting and to Gowrisankaran and Rysman (2007) for the identification of the same model in a dynamic setting. Since we use a binomial logit specification, the identification of our model directly follows that of Nair (2007) who discusses identification of the binomial logit model in a dynamic setting.

We can summarize the identification of our model parameters under three headings. Product fixed effects are identified from the variation in mean level of sales across different coats. Price, per period consumption, markdown, and seasonality parameters are identified from the within coat variation in these characteristics over the coat’s season. The change in market share of product \( j \) associated with a change in a characteristic of \( j \) (e.g., price) over time identifies the (first segment’s) parameter associated with that characteristic. Heterogeneity on the other hand, is identified from the deviations from the standard logit implied elasticities. Without heterogeneity, our model is a standard binomial logit model which implies own elasticities are proportional to the outside good’s market share. If our consumer population was homogenous, i.e., we had only one segment, when the elasticity varies over time, it would do so in a way that it
stays proportional to the outside good’s market share\(^5\). A number of studies have also provided simulation based evidence on identification of heterogeneity from aggregate data with logit demand models in static and dynamic settings (Chintagunta, 1999, Song and Chintagunta, 2003).

We find that a 2-segment specification fits the data best. This results in an estimation of a total of 70 parameters. Remember that we represent the set of all model parameters with \(\Omega\), where \(\Omega = (\alpha, c, \beta, \pi, \sigma_\varepsilon)\). First, we have a set of 61 fixed effect parameters, one for each product (the \(a_j\)). Second, we have the per period consumption parameter, c. Third, we have \(\beta\). We estimate a price sensitivity parameter, a markdown sensitivity and a seasonality parameter for each segment, adding up to 6 parameters. And last, we estimate a segment size parameter which informs us about the relative size of the first segment in the initial potential market, \(\pi\), and the standard deviation of the mean zero normal distributed demand shocks, \(\sigma_\varepsilon\).

### 4.7 Competition and Endogeneity

In this section, we discuss three key modeling assumptions and address potential concerns related to these assumptions. The first assumption concerns cross-demand effects from products within the same category, the second one relates to the impact of competition from retailers selling similar products and the third relates to the concern for price endogeneity.

This study treats each product as a separate market and does not consider demand effects across different coats. This assumption is motivated by the characteristics of “fashion” categories. We expect substitution effects to be small in the women’s coats category and in all categories on the “fashion” end of the spectrum. In contrast to categories closer to the “staples” end of the spectrum (e.g., men’s white dress shirts), fashion categories are associated with greater use of colors, prints and unique designs (Pashigian, 1988) and different products within these categories often serve unique tastes. Despite these facts, there is still concern for competition between different colors and or models of coats within a store. In order to address these concerns, we resort to aggregating across similar SKUs that are likely to compete with each other. To be more specific, we aggregate SKUs for different colors of the same style and SKUs

---

\(^5\) Consider two periods \(t_1\) and \(t_2\), with price levels \(p_1\) and \(p_2\), and a 1% change in price in both periods holding everything else constant. If our consumer population was homogenous, these changes would have resulted in percentage changes in demand proportional to the outside good’s market share. Let the outside good’s market share in period \(t_1\) and \(t_2\) be \(OS_1\) and \(OS_2\) and % change in demand be \(PCD_1\) and \(PCD_2\). We should have the equality \(PCD_1/PCD_2 = (p_1*OS_1)/(p_2*OS_2)\). But instead, if we observe a change in demand different than the change implied by the logit elasticity, this would indicate the existence of a second consumer segment that is different than the first in price sensitivity and the extent of the deviation helps to identify the extent of the difference in the two segments’ price sensitivities. See Nair (2007) for a similar example on how rate of change in market shares helps identify the relative sizes of the segments.
for different styles of coats that are sold at the same price level within a season where appropriate. Details of this product aggregation is provided in the data section. This aggregation across similar SKUs allows for substitution between close alternatives. Aggregation reduces the number of products analyzed from 105 to 61. After aggregation, we have on average 9 products sold together in the same season and there is substantial variation in these products in retail price levels (e.g. a $99 coat, a $199 coat and a $269 coat) and in markdown price levels.

Our model also does not explicitly account for the effect of competitors’ (other coat sellers’) within-season pricing decisions on demand. One big limitation we face is data availability. While company level data on sales and prices is more readily available, data from multiple competitors is hard to obtain not only for academicians but also for practitioners. There are no data vendors who track clothing sales such as Nielsen or IRI for grocery products or IMS for pharmaceutical products. Not being able to account for the impact of inter-store competition in markdown prices on demand is a limitation of this study. If competitors respond to markdowns with price decreases on similar products, normalizing utility from the outside option to zero over the season might lead to underestimation of the price sensitivity parameter and/or the markdown sensitivity parameter depending on the timing of the competitive markdowns. Within the context of this study, though, we don’t expect competitive effects to be very pronounced. While retail prices and temporary promotions (e.g., Mother’s day sale) are set before the season starts taking competitors’ strategy into account, decisions regarding the timing and depth of markdowns are often operational made dynamically during the short season, taking into account inventory and time left until the end of the season (Bitran and Mondschein, 1997). Also, since different products are discounted at different points in time, markdowns are rarely advertised (Bitran and Mondschein, 1997) and competitive reactions seem less likely for unadvertised markdowns (Smith and Achabal, 1998) especially for fashion products that are fairly unique.

There might also be concern for potential endogeneity between product and time specific demand shocks and prices. This concern would arise if the retailer observes the product and time specific demand shocks (\( \xi_{jt} \))’s and take them into consideration in his pricing decisions. In order to address this concern, we first estimated our model in a GMM framework using average men’s

---

6 This is because the observed demand increase in response to a markdown would be smaller in the presence of competitive markdowns. If we fail to account for competitive markdowns when they are present, we would attribute the small demand response to low price sensitivity and/or markdown sensitivity.
coats prices from the same period as instruments for women’s coats prices. The issue here is that men's coat prices may not be very appropriate instruments since men’s coat prices and women's coat prices are likely to be subject to some common demand factors. We were unable to find data on other suitable instruments. Next, we estimated our model removing the instruments and using MLE. Using the instrument produced an incorrect negative sign for the markdown sensitivity parameter in one of the segments. Removing the instrument corrected the estimate. The model implications, however, are very similar. So we decided not to use the instruments.

5 The Data

Main distribution channels for the apparel industry in the US are specialty apparel stores and department stores. Specialty apparel retailers sell their own brands through multiple stores across the country mostly located within shopping malls. In 2007, specialty store sales for clothing and clothing accessories added up to $225 billion and department store sales accounted for $210 billion. In the market for apparel, specialty retailers have been gaining market share from the department stores. Some examples of prominent specialty apparel retailers are Polo Ralph Lauren, Liz Claiborne, Ann Taylor, Chico’s, Limited, GAP, Banana Republic, Old Navy, Gymboree and the Children’s Place. In addition to a brand name to communicate with customers, specialty apparel retailers have the advantages of specialization and closer customer contact. Specialization results in more effective targeting and positioning while close contact with the consumers enables the retailer to respond faster to consumer needs. Specialty apparel retailers also often offer superior service levels compared to department stores.

The data used in our analysis comes from a leading specialty apparel retailer that sells its own private label fashion apparel across hundreds of stores throughout the US. The data consists of weekly sales, revenue and starting inventory levels as well as unit acquisition costs at the SKU level. The data cover a two year period, including the years 2003 and 2004.

We estimate the model using data from the “Women’s Coats” category. We believe this category is suitable for the purposes of this study as the products in this category are sold typically over a season, it is a high involvement category, consumer purchase frequency is low, and repeat purchases from the same consumer especially for the same product are unlikely.

We excluded data for about 30 SKUs for which we do not observe the whole sales cycle and ended up with 105 SKUs from this category. Most coat styles are offered in a number of different colors (2-4 colors). Different colors of the same style are separate SKUs but share the
same retail price and are put on shelves within a few weeks of each other. Also, markdowns for different colors are usually bunched together, i.e. they are marked down at the same period and the markdown magnitude is the same. This is almost always true for the first markdown. So price differences across colors are negligible over the season in most cases. Further, there are also some instances where two different styles are introduced within a few weeks of each other, at the same retail price level and are marked down to the same price. Since it is likely that different colors of the same style and similar styles (offered at the same price level and marked down together) compete with each other, we decided to aggregate across such similar SKUs where appropriate. This aggregation allows us lessen the concern for intra-store competition in our model. In order to calculate the price of the aggregate product, we average prices across all aggregated SKUs. In order to calculate the initial market size of the aggregate product, we add the initial market sizes of the aggregated SKUs (as explained in section 4.5, we basically multiply the retailer's initial order size by the proportionality factor 1.25 to calculate the market size for each SKU). Size of the remaining potential market is calculated each period by subtracting the sum of sales for all aggregated SKUs from the aggregate remaining potential market size at the end of the previous period. In order to calculate the aggregate availability level on the other hand, we take the maximum of availability levels across the aggregated SKUs. The reason is that not all colors are ordered in equal amounts but the colors with less stock are distributed to the stores that carry the more popular colors. So, the distribution of the aggregate SKU is successfully captured by the highest distribution observed across SKUs. There might be some exceptions to it at the very end of the sales period when only very few items are left in the stores but sales at the end of the season are very small so it should not impact our results. This aggregation reduces the number of products analyzed from 105 to 61. After aggregation, we have on average 9 products sold together in the same season (reduced from 15) and there is substantial variation in these products in retail price levels (e.g. a $99 coat, a $199 coat and a $269 coat) and in markdown price levels.

We observe each product through its season (lifecycle). Products were introduced and discontinued at different times during the 2 year observation period. So, different products have different seasons and different season lengths. The season length varies from 11 to 30 weeks.

---

7 We thank an anonymous reviewer for this suggestion of aggregating across SKUs to handle intra-store competition.
with a median season length of 19 weeks. Each observation corresponds to a product-week combination and we have a total of 1178 observations.

One important observation is that there is significant variation in total sales across products as well as the number and the depth of markdowns the product faces during its sales cycle. Total sales range from 834 units to 43,222 units and the median is 8166 units. Retail (initial) price ranges from $100 to $350 and the median retail price is $200. In our sample, all products face at least 1 markdown, the maximum number of markdowns is 5, and the median number of markdowns is 3. Average first markdown is 38% of the retail price and the average second markdown is an additional 21% of the retail price. Table 1 summarizes total revenue and quantity sold at relative price points (price as a percentage of the retail price). We can see that in our sample, only 43% of the quantity sold and 57% of the revenue from sales was at full (retail) price over the two year period. 45% of the quantity sold and 36% of the revenue correspond to sales that took place when the price relative to retail price was in the range 40 to 80%.

<table>
<thead>
<tr>
<th>Relative Price</th>
<th>Revenue</th>
<th>Quantity Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td>80% - 99%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>60% - 79%</td>
<td>16%</td>
<td>16%</td>
</tr>
<tr>
<td>40% - 59%</td>
<td>20%</td>
<td>29%</td>
</tr>
<tr>
<td>&lt;40%</td>
<td>5%</td>
<td>9%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 1: Total Revenue and Quantity Sold at Different Relative Price Points

Looking at the prices and sales over time one can easily see important patterns. Figure 1 plots unit sales and prices over time for a sample product. This product consists of a single SKU and is offered for sale in period 1 at a retail price of $200. Sales start high but fall down quickly until the first markdown. The first markdown occurs in period 12 and is around 60% and sales make a significant jump at that period, increasing 15-fold compared to the period before the markdown. Following the first markdown, sales fall down even more quickly until the second markdown in period 19. The second markdown is around an extra 15% of the retail price and this time sales increase by 80%. After the second markdown, sales decrease quickly for a few periods and then die slowly.

We observe significant response to price changes in the early periods but sales drop very quickly over time at a given price. The drop in sales over time at a given price can be the result of decrease in useful life of the product (i.e., consumers prefer to purchase earlier than later at a given price) and/or shrinkage in the potential market size as well as limited availability. The
spikes on the other hand could be the result of dramatic promotion response, strategic consumer waiting or capturing different segments of customers. Let us focus on the period before the first markdown. There might be at least two different explanations behind the sales decrease before the first markdown. One explanation might be that strategic consumers are familiar with the retailer’s discounting pattern and are delaying their purchases to take advantage of lower prices. This is similar to the pre-promotion dip documented in the marketing literature in the CPG context (e.g., Van Heerde et al., 2000). Another explanation might be that the retailer faces different consumer segments with different levels of price sensitivities. This first drop in sales might mean that the retailer has only a small segment of low price sensitivity consumers and this segment is saturated early in the season and the retailer needs to lower prices to capture demand from more price sensitive consumers. This emphasizes the importance of a model like ours, since understanding the reasons behind these patterns of observed demand is very important for the retailer’s policy. So, the retailer wants to know: Do consumers strategically wait for markdowns? To what extent does strategic waiting explain the observed demand peak? And, can limiting availability help to dampen the effect of strategic waiting by creating urgency in consumers?

![Sales and Prices Over Time](image)

**Figure 1:** Prices and Sales for a Sample Product
6  Empirical Results

6.1  Parameter Estimates

a.  Price Process Parameter Estimates

Table 2a reports the maximum likelihood estimates of the parameters of the markdown probability model (conditional on no previous markdown) specified in equation (2).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>a0</td>
<td>-6.2187</td>
<td>0.5809</td>
</tr>
<tr>
<td>Retail Price</td>
<td>a1</td>
<td>0.0035</td>
<td>0.0020</td>
</tr>
<tr>
<td>Time in Season (Week)</td>
<td>a2</td>
<td>0.4315</td>
<td>0.0398</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>1054</td>
<td>Log Likelihood</td>
<td>-256.4</td>
</tr>
</tbody>
</table>

Table 2a: Markdown Probability Model (Cond. on no previous MD) Parameter Estimates

Table 2b on the other hand reports the markdown probability model parameters (conditional on a previous markdown) specified in equation (3). The estimates indicate that probability of markdown conditional on a previous markdown is positively related to the product’s retail price. Probability of markdown is also positively related to the time in the season (number of weeks since product j has been introduced) both conditional on no previous markdown and conditional on a previous markdown.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>b0</td>
<td>-3.2683</td>
<td>0.6151</td>
</tr>
<tr>
<td>Retail Price</td>
<td>b1</td>
<td>0.0039</td>
<td>0.0017</td>
</tr>
<tr>
<td>Time in Season (Week)</td>
<td>b2</td>
<td>0.0590</td>
<td>0.0300</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>799</td>
<td>Log Likelihood</td>
<td>-347.7</td>
</tr>
</tbody>
</table>

Table 2b: Markdown Probability Process (Conditional on previous MD) Parameter Estimates

Table 2c reports the OLS estimates of the parameters of the markdown depth model specified in equation (4). The estimates indicate that the natural logarithm of markdown depth is positively related to the natural logarithm of the product’s retail price and negatively related to the markdown dummy. Products with higher retail prices face deeper markdowns compared to products with lower retail prices and first markdowns are deeper than later markdowns.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>θ₀</td>
<td>-0.6102</td>
<td>0.4554</td>
</tr>
<tr>
<td>ln(Retail Price)</td>
<td>θ₁</td>
<td>0.9201</td>
<td>0.8868</td>
</tr>
<tr>
<td>Markdown Dummy</td>
<td>θ₂</td>
<td>-0.7225</td>
<td>0.6566</td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td>0.535</td>
<td># of Observ</td>
</tr>
<tr>
<td>MSE</td>
<td></td>
<td>0.164</td>
<td></td>
</tr>
</tbody>
</table>

Table 2c: Markdown Depth Process Parameter Estimates
b. Availability Process Parameter Estimates

Table 3 reports the OLS estimates of the parameters of the availability expectations process specified in equation (5). The estimates indicate that availability in the next period is closely related to the availability in the current period. The retail price parameter is significant and has a positive sign, meaning next period availability is high when the retail price is high, and vice versa, i.e., products with higher retail prices are sold at a slower rate. Time in season (number of periods since the beginning of the season) is significant and has a negative sign, meaning availability falls over time.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Period Availability</td>
<td>$y_0$</td>
<td>0.9360</td>
<td>0.0078</td>
</tr>
<tr>
<td>Retail Price</td>
<td>$y_1$</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td>Time in Season</td>
<td>$y_2$</td>
<td>-0.0039</td>
<td>0.0003</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>0.947</td>
<td></td>
</tr>
<tr>
<td>MSE</td>
<td></td>
<td>0.012</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Availability Expectations Process Parameter Estimates

c. Demand Model Parameter Estimates

Table 4 reports the MLE estimates of demand model parameters for a two segment specification. Following Besanko et al. (2003), we determine the number of segments by adding segments until one of the segment size parameter estimates is not statistically different from zero. The estimates for the three segment specification yield an insignificant segment size parameter for the third segment. So, the data identifies two segments. As discussed in the estimation section, we estimate the demand parameters for the first segment and deviations of the second segment’s parameters from those of the first segment. Segment 2 parameters reported in Table 4 are calculated using these estimates and the standard errors are adjusted accordingly. We also estimate 61 product fixed effects which are not reported here. Product fixed effect estimates lie in the range (-6.4, -3.3) with an average of -5.1. Our demand estimates reflect a larger, less price sensitive segment, Segment 1, and a smaller, more price sensitive segment, Segment 2. We call the first segment, the fashion sensitive segment and the second segment, the bargain hunter segment. Price sensitivity parameters for both segments have the expected negative sign (not significant for Segment 1). Segment 1 corresponds to 86% of the total potential market at the beginning of the season and the estimated price sensitivity parameter for this segment is -0.001. The estimated price sensitivity parameter for Segment 2 is -0.029. Mere markdown effect (markdown sensitivity) is positive and significant for both Segment 1 and Segment 2 (1.956 and 1.816 respectively). The seasonality parameter is positive and significant for both segments.
(1.171 for Segment 1 and 1.113 for Segment 2). This indicates that both segments get extra utility from purchases in the 6 week holiday shopping period.

<table>
<thead>
<tr>
<th>Parameter Estimates</th>
<th>Segment 1</th>
<th>Segment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>S.o.</td>
</tr>
<tr>
<td>Price Sensitivity</td>
<td>-0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Per-Period Consumption Utility</td>
<td>0.149</td>
<td>0.008</td>
</tr>
<tr>
<td>Markdown Sensitivity</td>
<td>1.966</td>
<td>0.126</td>
</tr>
<tr>
<td>Seasonality</td>
<td>1.171</td>
<td>0.077</td>
</tr>
<tr>
<td>Segment Size Parameter (ln((1-B)/B)</td>
<td>-1.619</td>
<td>0.035</td>
</tr>
<tr>
<td>Standard Deviation of Demand Shocks</td>
<td>0.993</td>
<td>0.035</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-5186.4</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Demand Estimates

* We also estimate 61 product fix effects that lie in the range of (-6.4, -3.3) with an average of -5.1.

We have seen that the fashion sensitive segment accounts for a significant portion of the potential market at the beginning of the season. As consumers in this segment represent a relatively less price sensitive group, they make purchases and exit the market early in the season. As a result, relative sizes of the two segments in the remaining potential market change dramatically over time. Averaging across all products, size of the fashion sensitive segment reduces from 86% in Period 1 to 52% in Period 25.

In order to understand the purchase behavior of the two segments, we investigate the simulated sales for a sample product. Figure 2 represents the sales simulated using the demand estimates from our model, for both segments across time for the sample product in Figure 2. Fashion sensitive segment (Segment 1) consumers start purchasing early in the season and some consumers of this segment take advantage of the early markdowns. The bargain hunter segment (Segment 2) consumers on the other hand, start purchasing later in the season. Simulated sales for the two segments show that although Segment 2 customers account for an important portion of sales at the end of the season and are important in clearing the shelves of the retailer for the next season, they do not start buying until late in the season.

![Figure 2: Simulated Sales for a Sample Product](image-url)
6.2 Model Comparison

In order to demonstrate the importance of accounting for consumers’ availability expectations and the change in total utility from consumption over time, we present estimates from a benchmark model and compare our model to this model. The benchmark model is the state of the art model in the current literature. Similar to Nair (2007), it accounts for forward looking behavior (where consumers have expectations about future prices) and consumer heterogeneity in response parameters but does not account for consumers’ availability expectations and does not consider the change in total utility from consumption over the season due to the reduction in the product’s remaining useful life. Our model on the other hand, takes both of these considerations into account.

In the benchmark model, consumers have expectations about future prices and take these expectations into account when making a purchase decision but they do not take the stock-out risk into account. In estimating this model, $\hat{\lambda}_{j,t+1}$ in equation (8) is set to 1 for all products and time periods. The benchmark model also assumes that total utility from consumption of the product does not depend on the time of purchase so the per period consumption utility, $c_j$, is set to 0 for all segments and time periods and total discounted consumption utility is captured by the product fixed effect, $\alpha_j$. Table 5 presents the parameter estimates from this restricted model. This model produces positive and significant price coefficients for both segments. This is because the model ignores the impact of availability expectations and decreasing total consumption utility over time and attributes consumers’ incentive to accelerate purchases (to avoid stock-outs and to get the most use out of the product) to lower price sensitivity.

<table>
<thead>
<tr>
<th>Parameter Estimates</th>
<th>Segment 1</th>
<th>Segment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>I.e.</td>
</tr>
<tr>
<td>Price Sensitivity</td>
<td>0.024</td>
<td>0.002</td>
</tr>
<tr>
<td>Markdown Sensitivity</td>
<td>1.230</td>
<td>0.164</td>
</tr>
<tr>
<td>Seasonality</td>
<td>0.657</td>
<td>0.091</td>
</tr>
<tr>
<td>Segment Size Parameter ($\ln(1-5%)$)</td>
<td>-1.825</td>
<td>0.001</td>
</tr>
<tr>
<td>Standard Deviation of Demand Shock</td>
<td>0.929</td>
<td>0.059</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-7048.5</td>
<td># of Observations: 1178</td>
</tr>
</tbody>
</table>

Table 5: Demand Estimates for the Benchmark Model

A likelihood ratio test comparing the benchmark model log likelihood of -7048.5 to our model’s log likelihood of -5186.4 (Table 4) produces a chi-square test statistic of 3724.2 which is significant.
6.3 Price Elasticities of Demand

We calculate price elasticities by first simulating the predicted sales using the observed prices, discounting the prices in a specific period by a small amount for each product, simulating the predicted sales once again for the new prices and computing the change in sales. In order to isolate the price effects, we hold consumers’ availability expectations constant throughout the simulations and exclude the mere markdown effect since the price decrease in our simulations is temporary. Throughout the season, the retailer faces a less elastic demand curve from Segment 1 consumers (fashion sensitive segment, mean price elasticity of -0.07, across all products and time-periods) than from Segment 2 consumers (bargain hunter segment, mean price elasticity of -3.48). Price elasticities of demand indicate that demand from both segments becomes less responsive to price changes throughout the season. This clearly indicates that correct timing of the markdowns is very critical for the retailer. Earlier markdowns can significantly accelerate demand as demand from either segment is more sensitive to price changes in the earlier periods. On the other hand, a higher than necessary early markdown can have very negative profit implications.

Although current period price elasticities for both segments decrease (in absolute value) over time, the overall price elasticity increases over time because the relative size of the bargain hunter segment in the remaining potential market increases over time. However, because the bargain hunter segment accounts for only a small portion of the sales, the overall price elasticity is small. A number of recent studies have documented impact of changing market composition on demand and overall price elasticity (Ching, 2000, Nair, 2007).

Price elasticities exhibit significant variation across products and time periods. The most price elastic product has an average price elasticity of -0.25 over its life-cycle, while the least price elastic product has an average price elasticity of -0.09 over its life-cycle.

![Figure 3: Distribution of Overall Price Elasticities Across Coats Over Time](image-url)
7 Counterfactuals

An important strength of a structural demand model is that we can forecast how consumer behavior will change in response to fundamental changes in pricing and inventory management policy. In this section we investigate effects of two such policy changes through policy experiments and conduct a third experiment to quantify the impact of strategic consumer behavior on retailer revenues. The first experiment investigates the retailer’s tradeoff between the timing and depth of markdowns through a uniform single markdown policy. This experiment shows that the highest retailer profits are achieved by small markdowns offered early in the season. The second experiment studies the effect of limiting availability throughout the season and shows that, counter to intuition, the retailer can improve his profits by stocking less. The third experiment quantifies the impact of strategic consumer behavior on retailer revenues. This experiment shows that the retailer’s revenue is much lower under strategic consumer behavior than it would have been under myopic consumer behavior and that limited availability considerably helps to dampen the effect of strategic behavior.

7.1 Uniform Single Markdown Policy

In this experiment we investigate the retailer’s tradeoff between the timing and depth of markdowns when setting a uniform single markdown across all products. We focus on a single markdown because 82% of all sales at markdown prices take place at the first markdown price for the retailer in our application. In this policy, the seller changes the price only once during the season and sets the same percentage markdown for all products. In order to investigate the sales and revenue impacts of different uniform single markdown policies, we keep the initial prices and initial inventories fixed and vary the timing and depth of the markdown. We vary the depth of the markdown between 5% and 50% in 5% increments and vary the timing of the markdown between periods 1 and 25 in 1 period increments. For each depth-timing combination, we simulate sales and calculate resulting total revenue. Note that revenue is the relevant metric here since the entire inventory is purchased at the beginning of the season and salvage value is zero. Table 6 summarizes revenue outcomes of each time-depth combination. Due to confidentiality concerns, results are rescaled so that the maximum table entry is 100. After calculating and normalizing the total revenues, we divide the table entries into 4 regions where Region 1 corresponds to the top (fourth) quartile of all table entries; Region 4 corresponds to the bottom (first) quartile and so on. Each region is represented by a different color.
Results indicate that early and deep markdowns as well as late markdowns (Region 4) result in the lowest revenues. Since the market consists of mostly lower price sensitivity customers, marking down the prices too early and too deep does not have a big impact on sales but the revenue loss due to sales at lower prices is significant. And since the consumers’ interest in the product reduces significantly by the end of the season (due to diminished opportunity to use the product), very late markdowns also don’t produce favorable revenue outcomes. Early and small markdowns (Region 1) have the most favorable revenue outcomes. The retailer in our empirical application, under a uniform single markdown strategy, should offer markdowns earlier in the season before consumers lose interest in the product. Smaller markdowns result in higher revenues but if the business conditions and rules indicate a certain markdown percentage, markdown time should be set carefully since marking down too early as well as marking down too late can result in lower revenues.

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Markdown Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>20%</td>
</tr>
<tr>
<td>5%</td>
<td>25%</td>
</tr>
<tr>
<td>10%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Table 6: Revenues from a Uniform Single Markdown Policy

7.2 Change in Availability

In this section, we investigate the results of a change in the inventory policy. The initial stock ordered by the retailer at the beginning of the season is an important part of the retailer’s pricing and stocking strategy. In this experiment, we simulate sales for both segments using 5% to 30% reduction in the initial stock offered, varying the reduction in 5% increments. In doing the simulations, we hold the pricing strategy constant and allow for consumers’ availability expectations to adjust in accordance with the change in the initial period availability and the changes in sales in all periods. Table 7 reports the resulting change in sales, retailer revenue and retailer profits relative to the original policy. Note that the retailer profit is the relevant measure.
here since the policy change involves a reduction in the initial stock ordered and changes the total cost of acquisition.

Results show that although reducing availability has a negative effect on the total quantity sold, a 5% decrease in the initial stock offered can improve retailer’s profits. A slight decrease in the availability increases stock-out risk and motivates consumers to buy earlier at higher prices and the profit gain from earlier sales overcomes the reduction due to lost sales. Reducing availability further on the other hand, results in lower profits (compared to 5%) and a reduction of more than 15% would result in lower profits compared to the current situation.

<table>
<thead>
<tr>
<th>Change Compared to Current Policy</th>
<th>Change in Total Sales</th>
<th>Change in Total Revenues</th>
<th>Change in Total Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limit Initial Stock by 5%</td>
<td>-2.30%</td>
<td>0.68%</td>
<td>5.50%</td>
</tr>
<tr>
<td>Limit Initial Stock by 10%</td>
<td>-7.25%</td>
<td>-2.36%</td>
<td>4.38%</td>
</tr>
<tr>
<td>Limit Initial Stock by 15%</td>
<td>-12.39%</td>
<td>-6.88%</td>
<td>2.21%</td>
</tr>
<tr>
<td>Limit Initial Stock by 20%</td>
<td>-17.54%</td>
<td>-9.69%</td>
<td>-0.58%</td>
</tr>
<tr>
<td>Limit Initial Stock by 25%</td>
<td>-22.69%</td>
<td>-13.79%</td>
<td>-3.90%</td>
</tr>
</tbody>
</table>

Table 7: Effects of a Change in the Initial Stock

7.3 Impact of Strategic Consumer Behavior on Retailer Revenue

We have seen that although the fashion sensitive segment accounts for a large portion of the potential market, almost half of the sales take place at markdown prices. Under the current pricing and inventory policy, some fashion sensitive consumers strategically delay their purchases to take advantage of lower price. But how significant are these strategic purchase delays and what is the impact of strategic consumer behavior on retailer revenue?

In order to quantify this effect, we keep the retailer’s current price schedule and initial stock levels for all the products, use the demand estimates from our model and simulate sales and resulting revenues assuming consumers are myopic. Myopic consumers maximize immediate utility and buy when purchase utility exceeds utility from the outside option (which is normalized to 0). Under myopic consumer behavior, we observe earlier sales at higher prices and the resulting total revenue when compared to retailer’s current revenue helps us quantify the effect of strategic consumer behavior on the retailer’s revenues. As can be seen in Figure 4, under strategic consumer behavior retailer’s revenue is 10.8% lower than it would have been under myopic consumer behavior.

Another interesting question is whether limited product availability is helpful in dampening the effect of strategic consumer behavior on retailer revenue. As we have discussed before, limited product availability within the season creates stock-out risk that is increasing over
time and reduces the strategic consumers’ incentive to delay their purchases and wait for lower prices. In order to quantify the extent to which limited availability (stock-out risk) dampens the impact of strategic behavior on retailer revenue, we once again keep the retailer’s initial stock levels and pricing schedule fixed, use the demand estimates and simulate sales under strategic consumer behavior but assume that consumers do not discount future utilities due to stock-out risk. As there is no future stock-out risk, consumers are more likely to wait for lower prices and we observe further delays in purchases which results in lower retailer revenues. As summarized in Figure 4, this analysis shows that if the availability was not limited, strategic consumer behavior would have resulted in a 30% (10.8%+19.2%) reduction in retailer revenue and stock-out risk considerably helps to dampen the effect of strategic behavior on retailer revenue.

Figure 4: Impact of Strategic Behavior and Limited Availability on Retailer Revenue

8 Discussion

In this paper, we estimate a dynamic structural model of consumer choice behavior in a market for seasonal goods. Our model accounts for features essential to modeling seasonal goods demand: decreasing prices, limited availability and change in total utility from consumption through the season. In our model, heterogeneous consumers have expectations about future prices and availability levels and strategically time their purchases. The results indicate that ignoring the consumers’ expectations about future product availability and change in total utility from consumption can result in incorrect demand estimates. We find that a model that ignores these two effects produces positive price sensitivity parameters for both segments.

Our analysis shows that the retailer in the empirical application faces a large, less price sensitive segment and a much smaller, more price sensitive segment. Although the more price sensitive segment is essential in clearing up the excess inventory at the end of the season, their share of total sales and revenue is quite small. Calculated price elasticities suggest that
responsiveness of demand to price changes decreases for both segments over time. This finding highlights the importance of price changes in the earlier periods on the retailer’s sales and revenues. Through counterfactual experiments we show that the highest retailer profits are achieved by offering small and early markdowns. Late markdowns do not have very favorable profit outcomes and early and deep markdowns are very detrimental to retailer profits. Given the current markdown schedule on the other hand, the retailer can improve his profits by carrying less stock. When the retailer limits the initial stock, increase in stock-out risk in later periods forces the customers to buy earlier at higher prices. As long as the reduction in availability is not large, profit gain from earlier sales can overcome the loss due to the reduction in overall sales. We also show that strategic consumers who delay their purchases to take advantage of lower prices contribute to a 11% reduction in retailer’s revenue. However, increasing stock-out risk later in the season motivates consumers to purchase earlier at higher prices. We show that if the consumers had not taken stock-out risk into account when timing their purchases, strategic delays would have been more pronounced and the loss in revenue due to strategic behavior would have been much larger (30%).

This study contributes to the current literature on both methodological and substantive grounds. With regard to the methodological dimension, we develop an estimable structural model of strategic consumer choice in the presence of stock-out risk that accounts for change in total consumption utility depending on the time of purchase within a finite season. With regard to the substantive dimension, we demonstrate that the limited availability and the change in total utility from consumption over the season can affect the aggregate sales curve, and show that our model can effectively explain interesting regularities in the data like big sales spikes at the markdown periods and rapid decrease in sales over time at a given price. An important empirical finding is that anticipated scarcity leads to purchase acceleration and under limited supply, a retailer might benefit from reducing availability. Our demand model enables the retailer to understand the different factors resulting in change in demand over time: change in total utility from consumption, reduced availability over time, shrinking potential market and changing consumer mix over time. Accounting for each of these factors separately gives the retailer the opportunity to set optimal initial stock levels and dynamically set optimal prices over the course of the season for different products.

In this study we use counterfactual experiments to investigate the performance implications of changes in the retailer’s pricing and inventory policy rather than solving the
retailer’s optimization problem. One possible extension of this study is formulating and solving the retailer’s optimal dynamic pricing and initial inventory level determination problem. This would require incorporating retailer’s uncertainty about demand before the season starts and learning about demand as the season progresses into the dynamic pricing problem. Modeling retailer’s demand uncertainty and demand discovery though is beyond the scope of this study and we leave the modeling of the retailer’s optimal pricing problem for future research.

Allowing for within and cross category demand effects between products is another important extension to this study. We expect substitution effects to be small in the category we study, women’s coats. This category has a high fashion element and products are fairly unique and serve different tastes. These effects, however are important for a seasonal goods retailer in pricing products from complementary categories (e.g., shirts and ties) or substitute products in categories with lower fashion element (e.g., men’s dress shirts). On the other hand, accounting for these effects at the SKU level in the fashion apparel context brings a computational challenge due to the large number of SKUs simultaneously offered for sale. In this study, we resort to combining similar SKUs where appropriate to lessen the concern for substitution effects and leave the structural treatment of substitution across SKUs to future research.

We have taken the initial steps in developing a realistic demand model for seasonal goods products accounting for limited availability and change in total consumption utility over time as well as strategic consumer behavior and consumer heterogeneity. Future research can benefit from richer data on consumer expectations, availability and competitors’ prices.
REFERENCES


