Imperfect Competition in Selection Markets

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Abstract

Many standard intuitions about the distortions created by market power and selection are reversed when these forces co-exist. Adverse selection may be socially beneficial under monopoly, for example, and market power may be beneficial in the presence of advantageous selection. We develop a model of symmetric imperfect competition in selection markets that parameterizes the degree of both market power and selection. We derive basic comparative statics and illustrate them graphically to build intuition. We emphasize the relevance of the most counterintuitive effects with a calibrated model of the insurance market. We also apply our results to competition policy and show that in selection markets four core principles of the United States Horizontal Merger Guidelines are often reversed.

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JEL classifications: D42, D43, D82, I13, L10, L41

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1 Introduction

Adverse selection, or the tendency of the costliest consumers to also be most eager to enter the market, can limit the power of a monopolist to raise price, as higher prices entail facing costlier consumers. As a result, standard policies that aim to reduce the degree of adverse selection, such as risk adjustment or risk-based pricing in insurance markets, may allow firms to charge higher prices and thereby reduce consumer, or even social, surplus. Conversely, market power itself may help mitigate the tendency of advantageous selection, where the cheapest consumers are the most eager to enter the market, to create excessive supply as firms chase the most profitable, infra-marginal consumers. Thus, traditional competition policy that aims to reduce market power can lower social surplus in the presence of advantageous selection.

This paper studies imperfect competition in selection markets. Market power is perhaps the central topic in industrial organization, with a history tracing back to Cournot (1838). More recently, many leading scholars in the field have turned their attention to quantifying the welfare effects of selection (Einav, Finkelstein and Levin, 2010). Yet despite the striking contradictions of conventional wisdom when the two forces interact, we are not aware of a systematic investigation in the literature of the normative consequences of imperfect competition in selection markets. This paper provides such a treatment, derives from it several basic comparative statics and draws out from these several implications for competition and selection policy.

We begin by presenting a general model of symmetric imperfect competition in selection markets. Building on Weyl and Fabinger (2013) and Bresnahan (1989), we propose a model that nests standard micro-foundations of market structure including monopoly, perfect competition and versions of symmetric Cournot competition (with or without conjectural variations) and differentiated products Bertrand competition.

We first consider changes in selection that could arise from variation in the correlation between willingness to pay and costs across different markets. Adding selection to our model of symmetric imperfect competition requires strengthening the notion of symmetry in a way first proposed by Rochet and Stole (2002) and generalized by White and Weyl (2012) in the context of preferences for non-price product characteristics. In particular we assume that, at symmetric prices, all firms receive a representative sample of all consumers purchasing the product in terms of their cost, and that a firm cutting its price steals consumers with similarly representative distribution of costs from its competitors. This allows a simple parameterization of the “extent” of both market power and selection, each with a single parameter, $\theta$ and $\sigma$ respectively.

We use this model to derive comparative statics that sometimes match, and sometimes contradict, standard intuitions.

1. Under adverse selection, social surplus is (weakly) decreasing in market power. Adverse selection leads to undersupply and market power only worsens this problem.

2. Under advantageous selection, social surplus is inverted-U-shaped in market power. Advantageous selection leads to oversupply, thus market power is socially beneficial up to a point as it
offsets the natural tendency towards excessive supply.

3. Despite its direct costs, increasing the extent of adverse selection may benefit consumers, and even society, if market power and equilibrium quantity are both sufficiently high, as increased selection makes the marginal consumer less costly to serve, thereby lowering price and offsetting market power.

4. Conversely increasing advantageous selection is beneficial if the market is sufficiently competitive or quantity is sufficiently low, because increased selection lowers the cost of the marginal consumer and directly lowers firm costs by creating a better selection of consumers in the market.

We also consider changes to the degree of selection that could be brought about by risk adjustment, a policy that is increasingly used in health insurance markets (e.g., Brown et al., 2010). Reducing selection through risk adjustment impacts equilibrium price and quantity just as a reduction in correlation would. However, the effect on social surplus is different because implementing risk adjustment is not generically budget neutral for the organization implementing the risk adjustment scheme. We extend our results to this setting and obtain similar, but often stronger, counterintuitive findings about the interaction between market power and selection.

We illustrate the implications of our results by applying them to a canonical problem in competition policy: the evaluation of a merger of two firms. We show that several standard intuitions embodied in the latest revision of the United States Horizontal Merger Guidelines (United States Department of Justice and Federal Trade Commission, 2010) are partially or fully reversed in selection markets. “Upward Pricing Pressure” (UPP), which is a standard indicator of a prospective merger’s harm, can instead be generated by advantageous selection. The means that UPP can be large exactly in settings where there can be too much competition and additional market power can be socially beneficial. As another example, mergers between firms selling highly substitutable products, which are typically viewed as most harmful, should be interpreted positively in settings with advantageous selection. This is because advantageously selected markets with highly substitutable products are most likely to be those where industry supply exceeds the socially optimal level.

Beyond these theoretical points, we verify the practical relevance of our theoretical arguments in a calibrated model of a monopolized health insurance industry. The calibrated model of health insurance generates the counterintuitive result that reducing selection by reducing the correlation between cost and willingness-to-pay reduces quantity and harms consumers, while raising profits and social surplus. Doing the same through back-end risk adjustment has the same positive effects, but requires a subsidy from the risk adjuster making the increase in profits a transfer and thus risk adjustment socially harmful. Allowing firms to segment the market and implement risk-based pricing has quantitatively larger effects in reducing consumer surplus because it also allows price discrimination, though it increases social surplus. Providing improved information to consumers about their costs, which could be harmful because it creates adverse selection (Handel, 2012), actual benefits consumers, and even benefits society if true (rather than misperceived) risk is used to evaluate welfare.
Our paper is mostly closely related to Einav, Finkelstein and Cullen (2010) and Einav and Finkelstein (2011), who conduct a general analysis of a perfectly competitive selection markets that builds on the classical theory of a natural monopoly regulated to charge average cost prices (Dupuit, 1849; Hotelling, 1938). While this work has been influential, a constraint in applying their framework more broadly is that the assumption of perfect competition is questionable in many important selection markets. Perhaps because of this, existing work on imperfect competition in selection markets has typically taken an approach that relies more heavily on structural assumptions about firm and consumer behavior (e.g., Lustig, 2010; Starc, 2014).

We view our main contribution as providing a general understanding of the interaction between selection and imperfect competition. To do so, we extend the price-theoretic approach of Einav and Finkelstein, conveying our results when possible with simple graphs and verbal descriptions, with formal mathematical statements and proofs presented in the appendix. Our aim to build broader intuition and provide a foundation for empirical work in a range of socially important markets.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 presents the results. We provide an application to competition policy in Section 4.1 and an empirical application to health insurance markets in Section 4.2. Section 5 concludes.

2  Model

In this section, we describe a model of symmetric imperfect competition that nests monopoly, perfect competition and common models of imperfect competition including Cournot and differentiated products Bertrand competition. By placing these models in a common framework, we are able to develop results that are robust to the details of the industrial organization. Our model builds upon Weyl and Fabinger (2013), with modifications to allow for selection.

Consider an industry with symmetric firms that provide symmetric, though not necessarily identical, products. When firms produce symmetric quantities, prices are given by $P(q)$, where $q \in [0, 1]$ denotes the fraction of consumers served by the market. We do not specify the cardinality of the firms in the market to minimize the notational burden. Individuals who do not purchase the product from the industry receive no product, as in Einav and Finkelstein (2011).

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1This is an application of Marshall (1890)’s famous observation that competitive industries with economies or diseconomies of scale that are external to an individual firm’s production would operate identically to a monopolist regulated to charge a price at average cost.

2In their survey on empirical models of insurance markets, Einav, Finkelstein and Levin (2010) write that “there has been much less progress on empirical models of insurance market competition, or on empirical models of insurance contracting that incorporate realistic market frictions. One challenge is to develop an appropriate conceptual framework. Even in stylized models of insurance markets with asymmetric information, characterizing competitive equilibrium can be challenging, and the challenge is compounded if one wants to allow for realistic consumer heterogeneity and market imperfections.”

3Some consumers may favor one product over another, but there must be an equal number of consumers who have the symmetric opposite preference.

4Much of the literature considers a more general case when consumers choose between two products of different quality levels and most choose one of the two. This has been formulated in several ways, which have differing degrees of relationship to our model.

The first approach, proposed by Einav, Finkelstein and Cullen (2010), is to view the product in the market as the incre-
As in Einav and Finkelstein, and as described more formally by Veiga and Weyl (2014a), total costs for the industry are summarized by the aggregate cost function $C(q)$, given by the aggregation of the cost of all individuals served, and associated marginal cost function $MC(q) \equiv C'(q)$ and an average cost $AC(q) \equiv \frac{C(q)}{q}$. These may be increasing or decreasing in aggregate quantity depending on whether selection is respectively “advantageous” or “adverse.”\(^5\) We assume that firms have no internal economies or diseconomies of scale, and thus no fixed costs. At a symmetric equilibrium, firms supply segments of the market that are equivalent in terms of their distribution of costs and thus have average costs equal to $AC(q)$.

Industry profits are $qP(q) - C(q) = q \left[ P(q) - AC(q) \right]$. A competitive equilibrium requires that firms earn zero profits and is characterized by $P(q) = AC(q)$. A monopolist or collusive cartel chooses $q$ to maximize profit by equating marginal revenue to marginal cost:

$$P(q) + qP'(q) = MR(q) = MC(q).$$

Consumer surplus is $CS(q) = \int_0^q [P(x) - P(q)] \, dx$ and the marginal consumer surplus $MS(q) \equiv CS'(q) = -qP'(q)$. Social welfare is thus $CS(q) + qP(q) - C(q)$ and the first-order conditions for maximization of social welfare are

$$-qP'(q) + qP'(q) + P(q) - MC(q) = 0 \iff P(q) = MC(q).$$

Thus the socially optimal quantity (constrained as we are throughout the paper to uniform prices) is characterized by $P(q) = MC(q)$.

Panel (a) of Figure 1 shows the perfectly competitive equilibrium and monopoly and social optima in the case of “advantageous selection” where $AC'(q) > 0$ and the consumers with the highest willingness-to-pay are least costly. Panel (b) shows the same in the case of “adverse selection” where $AC'(q) < 0$ and the consumers with the highest willingness-to-pay are most costly.\(^6\)

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\(^5\)It is possible that these slopes have different signs over different ranges or that the two have slopes of different signs over a particular range. All of these cases do not fall cleanly into one category or the other and are not our focus in what follows. It would be interesting to extend our analysis to such cases.

\(^6\)We follow Einav and Finkelstein in defining the sign of selection in terms of the slope of the average cost curve as this determines the sign of the marginal distortion under perfect competition as $AC'(q) = \frac{MC(q) - AC(q)}{q}$. 

Figure 1: Panel (a) shows the perfectly competitive, monopoly, and socially optimal equilibria in the case of advantageous selection. Panel (b) shows these in the case of adverse selection.

2.1 Imperfect Competition ($\theta$)

We can nest the monopoly optimization and competitive equilibrium conditions into a common framework by introducing a parameter $\theta \in [0, 1]$. The parameter indexes the degree of competition in the market with $\theta = 0$ under perfect competition and $\theta = 1$ under monopoly. Equilibrium prices are given by

$$P(q) = \theta [MS(q) + MC(q)] + (1 - \theta)AC(q).$$  \hspace{1cm} (1)

Below we discuss how Equation 1 is a reduced-form representation of two canonical models of imperfect competition. Formal derivations of these representations appear in Appendix A.

1. Cournot: There are $n$ symmetric firms that each choose a quantity $q_i > 0$, taking the quantity chosen by other firms as given. Price is set by Walrasian auction to clear the market so that the price is $P(q)$ where $q = \sum q_i$. If we assume that each firm gets a random sample of all consumers who purchase the product, then the equilibrium is characterized by Equation 1 with $\theta \equiv \frac{1}{n}$. Intuitively, just as in the standard Cournot model, firms internalize their impacts on aggregate market conditions proportional to their market share ($\frac{1}{n}$ at equilibrium) and otherwise act as price- and average cost-takers. This model can easily be extended to incorporate conjectural variations; see Weyl and Fabinger (2013) for details.

2. Differentiated Product Bertrand: There are $n$ single-product firms selling symmetrically differentiated products. Each firm chooses a price $p_i$ taking as given the prices of all other firms.
Consumers have a type that determines their utility for each product and their cost to firms. The distribution of consumer types is symmetric in the interchange of any two products. In addition to these traditional assumptions of the symmetrically differentiated Bertrand model we add two additional assumptions proposed by White and Weyl (2012) that imply our representation is valid. First, the distribution of costs is orthogonal to the distribution of preferences across products given the highest utility a consumer can earn from any product. Second, the distribution of utility among the switching consumers that definitely will buy one product but are just indifferent between any two products is identical to that among the set of all consumers who are currently purchasing. These two assumptions imply that the average cost of consumers that switch between firms in response to a small price change is the same as the average cost among all participating consumers.\footnote{Even if this assumption fails, so long as average switching consumers have a cost that is strictly between that of average exiting consumers and average purchasing consumers most of our results are left unchanged. This property holds under a wide range of models; in a future draft we plan to include a calibrated example based on a standard structural model of demand in which this occurs, despite the exact assumptions underlying our differentiated products Bertrand model failing.}

In Appendix A we provide two micro-foundations for these assumptions. The first is a renormalized version of the Chen and Riordan (2007) “spokes” model that generalizes Hotelling (1929)’s linear city model in which the dimensions of consumer’s type other than her spatial position are orthogonal to her spatial position as in Rochet and Stole (2002). The second is a discrete choice, random utility model in the spirit of Anderson, de Palma and Thisse (1992) in which, rather than utility draws being independent across products as in Perloff and Salop (1985), the relative utility of different products is independent of the draw of the first-order statistic of utilities and the distribution of consumer costs is mean-independent of relative utilities conditional on the first-order statistic.

In this case, again, our representation is valid if $\theta \equiv 1 - D$ where the aggregate diversion ratio

\[ D \equiv -\frac{\sum_{j \neq i} \left( \frac{\partial Q_j}{\partial p_j} / \partial Q_i }{\partial Q_i} \right), \]

which is independent of the $i$ chosen at symmetric prices by symmetry. Note that, unlike in the previous case, $\theta$ will not be constant in this case; it will typically increase in price and thus decline in quantity (Weyl and Fabinger, 2013).

### 2.2 Selection ($\sigma$)

We model a change in the degree of selection as a flattening or steepening of the industry average costs curve, holding population average costs $AC(1)$ constant. Our approach contrasts with the approach taken by Starc (2014) in her analysis market power and selection in the Medigap supplemental insurance market. In her model, the mark-up of premiums over average costs depends on the demand elasticity and an elasticity that measures the degree of adverse selection. To calculate the counterfactual of eliminating selection, she sets this selection elasticity to zero, which results in consumers having constant marginal costs equal to average costs at the observed equilibrium. She finds that eliminating adverse selection in this manner raises prices and lowers consumer surplus.

In our framework, Starc (2014)’s approach to reducing adverse selection corresponds to a coun-
terclockwise rotation of the average cost curve around the equilibrium quantity—and a correspond-
ing shift of the marginal cost curve—until the point where marginal costs are constant and equal to
average costs at the observed equilibrium value. Since marginal costs are below average costs, this
causes marginal costs to rise, accounting for Starc’s finding that eliminating selection raises equilib-
rium prices. However, this rotation also raises population averages costs, because under adverse
selection equilibrium average costs are higher than average costs in the population. So while this
exercise provides an natural accounting decomposition for mark-ups over average costs, it does not
correspond to the standard ceteris paribus notion of changing selection (interpreted as the correla-
tion between willingness-to-pay and cost) holding other market primitives constant. In fact, because
this rotation leaves equilibrium average costs constant, it would leave prices constant under perfect
competition, in contrast to the common intuition (confirmed under our definition) that the presence
adverse selection causes prices to rise under perfect competition (Hendren, 2013).

We thus believe that a natural benchmark is to model changes in selection as a rotation of the
average cost curve holding population average costs AC(1) fixed. We operationalize this concept by
adding parameter σ to the model. This parameter indexes the degree of selection with σ = 0 rep-
resenting a situation in which costs are mean-independent of willingness to pay across individuals
with AC(q) = MC(q) = AC(1) and σ = 1 normalized to represent perfect correlation between costs
and willingness-to-pay as in the standard uni-dimensional model of heterogeneity in Akerlof (1970)
for adverse and de Meza and Webb (1987) for advantageous selection.

This parameterization maps to the type of regression approach taken to estimate the degree of
selection in the empirical selection literature. Building upon work by Chiappori and Salanié (2000),
a growing literature estimates the correlation between demand and marginal costs in range of selec-
tion markets (e.g., Finkelstein and Poterba, 2004; Bundorf, Levin and Mahoney, 2012). Consider a
standard econometric model of product choice:

$$v = \bar{\beta}_0 + \bar{\beta}_1(c - \mu_c) + \epsilon.$$  

Here willingness-to-pay $v$ depends linearly on expect costs $c$, which are distributed normally in the
population $c \sim \mathcal{N}(\mu_c, V_c)$, and a mean-zero idiosyncratic taste parameter $\epsilon$, which is independent
of costs and normally distributed $\epsilon \sim \mathcal{N}(0, V_\epsilon - \bar{\beta}_1^2 V_c)$. In this formulation, we parameterize
the variance of $v$ with $V_v$, rather than parameterizing the variance of $\epsilon$, so that the correlation between $c$ and $v$ may be adjusted holding fixed the marginal distribution of $v$. Similarly, we normalize $\bar{\beta}_0$ and $\bar{\beta}_1$ so that changing $\bar{\beta}_1$ does not impact the mean of the marginal distribution of $v$.

Consumers purchase the product if and only if their willingness to pay is greater than the price:

$$q = 1 \iff v > p \iff \bar{\beta}_0 + \bar{\beta}_1 c + \epsilon > p.$$  

If we divide through by the standard deviation of the taste parameter $\sqrt{V_\epsilon} = \sqrt{V_\epsilon - \bar{\beta}_1^2 V_c}$ and de-
finite $\beta_2 = 1/\sqrt{V_\epsilon - \bar{\beta}_1^2 V_c}$ and the coefficients $\beta_i = \beta_2 \bar{\beta}_i$ for $i = 0, 1$, the model can be estimated by a
Probit regression of product choice on expected costs and premiums, assuming we have a source of
exogenous variation in premiums:
\[
Pr(q = 1|c, p) = \Phi(\beta_0 + \beta_1c - \beta_2p),
\]
and the parameters \(\mu_c\) and \(V_c\) can be estimated directly from the data: \(\tilde{\beta}_1 = \beta_1/\beta_2\) and \(V_0 = 1/\beta_2^2 + \tilde{\beta}_1^2V_c\).

Using standard properties of the normal distribution, we show in the Appendix that these estimates imply that the industry marginal cost is
\[
\tilde{MC}(q) = \mathbb{E}[c|v = P(q)] = \tilde{\beta}_1\sqrt{\frac{V_c}{V_0}}\left[\sqrt{V_c}\Phi^{-1}(1 - q) \pm \mu_c\right] + \left(1 - |\tilde{\beta}_1|\sqrt{\frac{V_c}{V_0}}\right)\mu_c
\]
and average cost is
\[
\tilde{AC}(q) = \mathbb{E}[c|v \geq P(q)] = \tilde{\beta}_1\sqrt{\frac{V_c}{V_0}}\left[\sqrt{V_c}e^{-\frac{[\Phi^{-1}(1 - q) - q]^2}{2\sqrt{2\pi}q}} \pm \mu_c\right] + \left(1 - |\tilde{\beta}_1|\sqrt{\frac{V_c}{V_0}}\right)\mu_c
\]
where the \(\pm\) has the sign of \(\tilde{\beta}_1\). To fit our domain of \(\sigma \in (0,1)\), we define \(\sigma = \left|\tilde{\beta}_1\right|\sqrt{\frac{V_c}{V_0}}\), which is always between 0 and 1 because it is the absolute value of the correlation between \(v\) and \(c\). Then letting \(MC(q) \equiv \sqrt{V_c}\Phi^{-1}(1 - q) \pm \mu_c\) and

\[
AC(q) \equiv \sqrt{V_c}e^{-\frac{[\Phi^{-1}(1 - q) - q]^2}{2\sqrt{2\pi}q}} \pm \mu_c,
\]
we can write equilibrium conditions by replacing average costs with \(\sigma AC(q) + (1 - \sigma)AC(1)\) and marginal costs with \(\sigma MC(q) + (1 - \sigma)AC(1)\) in Equation 1. Collecting terms this yields
\[
P(q) = \theta MS(q) + \sigma\left[\theta MC(q) + (1 - \theta)AC(q)\right] + (1 - \sigma)AC(1).
\]
so that we have representation for price where \(\theta\) indexes the degree of market power and \(\sigma\) indexes the degree of selection in the market.

This precisely linear interpolation between \(AC(1)\) and \(AC(q)\) or \(MC(q)\) obviously relies on the precise joint normal structure of the example above. But much more generally reductions in parameterizations of the dependence (i.e. “correlation”) between cost and willingness-to-pay holding fixed population average cost will bring \(AC(q)\) and \(MC(q)\) towards \(AC(1)\) at each point, though not necessarily linearly or proportionally. Given that all of the results in the next section depend only on this property of moving towards \(AC(1)\) at each point and not on the linear structure, our results apply much more generally than this example. We provide an example with a very different statistical structure yielding the same results in our calibrated model of the health insurance market in Subsection 4.2.

However we maintain this linear form in what follows both for expositional simplicity and because it conveniently represents one of the most commonly policies used to correct the effects of
selection: risk adjustment. Medicare Advantage is a high-profile example. In the United States, elderly individuals with government health insurance can choose to opt out of the public Traditional Medicare (TM) program and purchase a private Medicare Advantage (MA) plan. For each enrollee, MA plans receive a payment from the government that is supposed to equal average costs under TM, partially risk adjusted to account for demographics and ex-ante health conditions.

We can use our framework with one additional modification to model changes in the degree of risk adjustment in this and other similar settings. Let \((1 - \sigma)\) indicate the fraction of the difference between expected average and population average costs that is compensated for by risk adjustment. The average risk adjustment payments in this setting are \(\text{ARA}(q) \equiv (1 - \sigma) [AC(q) - AC(1)]\) with \(\sigma = 0\) indicating a setting where firms are fully compensated for any differential selection they receive and \(\sigma = 1\) indicating a setting where firms receive no risk adjustment. Firms perceived average costs are the difference between their actual average costs and the average risk adjustment payments:

\[
\hat{AC}(q) = AC(q) - \text{ARA}(q) = \sigma AC(q) + (1 - \sigma) AC(1).
\]

Perceived industry marginal costs, as before, are the weighted average of marginal cost and \(AC(1)\):

\[
\hat{MC}(q) = \sigma MC(q) + (1 - \sigma) AC(1).
\]

The effects of risk adjustment on equilibrium price and quantity—and thus consumer and producer surplus—will be the same as a change in \(\sigma\) due to different correlations. However, the effect on social surplus will be different because implementing risk adjustment in this manner is not budget neutral. To reduce the degree of adverse selection, an exchange operator needs to make net payments to insurance plans, and will therefore run a deficit. To reduces the degree of advantageous selection, the exchange operator will run a surplus. In particular, social surplus depends only on whether quantity moves towards the socially optimal level under the original, non-risk adjusted demand and cost curves.

In the next section, we study equilibria characterized by Equation 2. To ensure a unique equilibrium exists, we impose global stability conditions that, while not necessary for our results, simplify the analysis. In particular we assume that \(P' < \min\{AC', MC', 0\}\) and \(MR' < \min\{MC', 0\}\). Under these conditions there is a unique equilibrium for a constant value of \(\theta\), the case we focus on below. While \(\theta\) is not constant in the Bertrand case, all of our results below can be extended to the case of non-constant \(\theta\) with appropriately generalized stability conditions at the cost of some notational complexity.

### 3 Results

In this section, we present results on the welfare effects of (i) market power in industries with selection and conversely (ii) selection in industries with market power. To do so, we build on the notation,
equilibrium and stability conditions of the previous section. To ease the exposition, all propositions are stated verbally. When possible, the results are illustrated graphically assuming linear demand and costs, and often focusing on the extreme cases of monopoly and perfect competition. Formal statements and proofs of all results appear in Appendix C.

3.1 Imperfect Competition

**Proposition 1.** Market power increases producer surplus and decreases consumer surplus

As firms gain market power, they increasingly internalize the impact of their output decisions on equilibrium price and quantity. This leads them to raise their price so long as price slopes downward more quickly than does average cost ($AC' > P'$), as implied by our stability assumptions. This internalization directly leads to higher producer surplus. The higher price that results reduces consumer surplus by the logic of the envelope condition.

**Proposition 2.** Under adverse selection, social surplus falls with market power. Anytime a market would collapse as a result of adverse selection no monopolist would choose to operate.

With perfect competition, adverse selection leads to too little equilibrium quantity, as shown in Panel (b) of Figure 1. Since market power reduces quantity, market power only further reduces social surplus. An implication is if the market collapses under perfect competition (Akerlof, 1970), and therefore the market generates no social surplus, then no amount of market power will restore the market and enable it to contribute to aggregate welfare (Dupuit, 1844).

Thus, at least under adverse selection, standard intuitions about the undesirability of market power are confirmed. However, while these results are in this sense unsurprising, they contrast with intuitions in the contract theory literature that market power may be beneficial under adverse selection. For example, Rothschild and Stiglitz (1976) argue that imperfect competition may be necessary to sustain the existence of markets under adverse selection when non-price product characteristics are endogenous, and Veiga and Weyl (2014b) show that imperfect competition can indeed restore the first-best, albeit in a stylized model. However, these analyses focus on the impacts of market power on product quality rather than on the fraction of individuals supplied. Our analysis indicates a trade-off between these quality benefits of market power and its quantity harms.\(^8\)

Under advantageous selection our analysis more directly contradicts conventional intuitions on the impact of market power.

**Proposition 3.** Under advantageous selection, social surplus is inverse-U shaped in market power. There is a socially-optimal degree of market power strictly between monopoly and perfect competition. Additional market power is socially beneficial below this level and socially harmful if it is above this level. The optimal degree of market power is increasing in the degree of advantageous selection.

\(^8\)In one extension of their baseline model, Veiga and Weyl consider a calibrated model that allows for both effects and find that an intermediate degree of market power is able to achieve welfare near the first-best and that even market power approaching monopoly leads to much higher welfare than does perfect competition. This suggests that, at least in some settings, the quality benefits may be more important than the quantity harms we emphasize here.
Figure 2: This figure shows that under advantageous selection, there is a socially optimal degree of market power strictly between monopoly and perfect competition. The monopoly optimum \((MR = MC)\) results in too little quantity, while perfect competition \((P = AC)\) results in too much. There is intermediate level of market power \(\theta^*\), leading to an equilibrium \(\theta^* MR + (1 - \theta^*)P = \theta^* MC + (1 - \theta^*)AC\), that results in the same equilibrium level of quantity as the socially optimum \((P = MC)\).
Under advantageous selection, perfect competition leads to excessive output because in an attempt to cream skim from their rivals, competitive firms attract higher marginal cost consumers into the market (de Meza and Webb, 1987). On the other hand, a monopolist, who internalizes the industry cost and revenue curves, will produce too little. As a result, there is an intermediate degree of market power that leads to the optimal quantity being produced.

Figure 2 shows this result graphically. The monopoly equilibrium, determined by \( MR = MC \), results in too little quantity. The perfectly competitive equilibrium, determined by \( P = AC \), results in too much. An intermediate level of market power \( \theta = \theta^* \), which leads to the equilibrium determined by \( \theta^*MR + (1 - \theta^*)P = \theta^*MC + (1 - \theta^*)AC \), results in the same equilibrium level of quantity as the equilibria achieved by setting \( P = MC \) and is therefore socially optimal. Because advantageous selection always pushes firms towards excessive production, the degree of market power required to offset this selection and restore optimality increases with the extent of advantageous selection.

Table 1 summarizes our results, with Panel A presenting the results on market power in selection markets, discussed above.

### 3.2 Selection

We begin our analysis of selection by considering the impact of changes in the degree of correlation between willingness to pay and costs. Because the degree of correlation is a property of a market, and not the result of a policy intervention, these results apply most directly to comparative statics across markets rather than the impacts of policy interventions. Our results are easiest to state verbally for the cases of monopoly and perfect competition. We thus confine our attention to these extreme cases. Results for intermediate cases are a natural interpolation between these extremes and are stated and proved in the formalization of these propositions in Appendix C.

**Proposition 4.** Under monopoly, reducing the degree of adverse selection raises profits but can raise or lower consumer surplus. Less adverse selection harms consumers when demand is high \( (q^* > \bar{q}) \). If demand is very high \( (q^* > \bar{q} > q) \), and the monopolist’s pass-through is bounded above zero, less adverse selection lowers both consumer and social surplus.

Figure 3 shows the effect of reducing the degree of adverse selection in the market. Panels in the left column show the scenario in which there is adverse selection and the average and marginal cost curves are downward sloping. Panels in the right column show the effect of reducing the degree of adverse selection, depicted by a counter-clockwise rotation of the average cost curve around the point \( AC(1) \) and a corresponding shift in the marginal cost curve. The resulting average cost and marginal costs curves are horizontal and have unchanged population average costs (\( AC(1) \) is the same). Panels

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9For example, Hendren (2013) compares outcomes in markets with different degrees of correlation under the assumption of perfect competition; our comparative statics with respect to \( \sigma \) would allow such analysis to be extended to imperfect competition. Hendren (2013)’s analysis focuses on markets with very low quantities where the impact of reducing selection is similar, according to our results, to that under perfect competition. However, our results in Section 4.2 suggest that the presence of a realistic degree of market power could substantially alter comparative statics results like his in markets where quantity is sufficiently high.
Figure 3: Reducing the degree of adverse selection in a market served by a monopoly provider. In panels (a) and (b) the equilibrium quantity is low and reducing adverse selection lowers price and raises quantity. In panels (c) and (d) the equilibrium quantity is high and reducing adverse selection increases price and lowers quantity.
in the top row show the effect of this shift when the equilibrium quantity is low \((q^* < \bar{q})\) and panels in the bottom row show the effect when the optimal quantity is high \((q^* > \bar{q})\).\(^{10}\)

When the equilibrium quantity is low, the reduction in selection lowers the cost of the average marginal consumer. This lowers the price and raises equilibrium quantity. When the equilibrium quantity is high, the reduction in selection raises the cost of the average marginal consumer, raising the price and lowering equilibrium quantity. In this setting with linear costs, reducing the degree of selection raises quantity whenever the optimal monopolist quantity is less than \(q = \frac{1}{2}\). More generally, reducing the degree of adverse selection reduces prices and increases quantity whenever the population average consumer has costs that are lower than the average marginal consumer at the optimal level of quantity.

By the envelope theorem, we can determine the effect of a reduction in adverse selection on a monopolist’s profits holding fixed the quantity the monopolist optimally chooses. Because a reduction in selection lowers average costs, as those participating in the market are selected adversely, producer surplus is necessarily increased. A reduction in the degree of adverse selection can lower welfare if the reduction in consumer surplus is large enough to offset the increase in firm profits. This only happens when optimal quantity is sufficiently high because in this case both the increase in marginal cost is large and the change in average cost is small as the firm’s average consumers are nearly representative of the whole population. The weight placed on the former effect relative to the latter effect in welfare terms is the monopolist’s pass-through rate, so it must be bounded above zero at high quantities for the result to hold.

When there is advantageous selection, the conditions under which a decrease in the degree of selection raises consumer surplus are reversed.

**Proposition 5.** Under monopoly, reducing the degree of advantageous selection lowers a monopolist’s profits but can raise or lower consumer surplus. Less advantageous selection benefits consumers when demand is high \((q^* > \bar{q})\). If demand is very high \((q^* > \bar{q} > \underline{q})\), and the monopolist’s pass-through is bounded away from zero, less advantageous selection raises both consumer and social surplus.

The graphs for this scenario are analogous to those for adverse selection and are shown in Appendix Figure A1. Reducing in the degree of advantageous selection rotates the average cost curve around AC(1) in a clockwise direction. When the optimal quantity is low \((q^* < \bar{q})\), this rotation increases the cost of the average marginal consumer, raising prices and lowering equilibrium quantity. When the optimal quantity is high \((q^* > \bar{q})\), the reduction in the degree of selection lowers the cost of the average marginal consumer, lowering prices and increasing quantity. A reduction in advantageous selection lowers industry profits by the same envelope logic discussed above. Reduced advantageous selection lowers welfare except when quantity is sufficiently high, in which case the increase in consumer surplus outweighs the decrease in firm profits.

Panels B and C of Table 1 summarize these results on the effects of selection in settings with

\(^{10}\)Of course, anything that impacts equilibrium quantities must do so by shifting the demand or supply curve. The necessary thresholds for these effects, \(q^*\) and \(\bar{q}\), can be defined as a function of the cost curves. We then interpret high and low quantities in terms of vertical shifts of the demand curve that thus vertically shift the marginal revenue curve without changing its shape.
Panel A: Greater Market Power

<table>
<thead>
<tr>
<th></th>
<th>Adverse Selection</th>
<th>Advantageous Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer Surplus</td>
<td>Higher</td>
<td>Higher</td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>Lower</td>
<td>Lower</td>
</tr>
<tr>
<td>Social Surplus</td>
<td>Lower</td>
<td>Inverse-U shaped</td>
</tr>
</tbody>
</table>

Panel B: Less Adverse Selection

<table>
<thead>
<tr>
<th></th>
<th>Perfect Competition</th>
<th>Monopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer Surplus</td>
<td>Always zero</td>
<td>Higher</td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>Higher</td>
<td>Lower ( q^* &gt; \frac{q}{2} )</td>
</tr>
<tr>
<td>Social Surplus</td>
<td>Higher</td>
<td>Lower ( q^* &gt; q &gt; \frac{q}{2} )</td>
</tr>
</tbody>
</table>

Panel C: Less Advantageous Selection

<table>
<thead>
<tr>
<th></th>
<th>Perfect Competition</th>
<th>Monopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer Surplus</td>
<td>Always zero</td>
<td>Lower</td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>Lower</td>
<td>Higher ( q^* &gt; \frac{q}{2} )</td>
</tr>
<tr>
<td>Social Surplus</td>
<td>Lower</td>
<td>Higher ( q^* &gt; q &gt; \frac{q}{2} )</td>
</tr>
</tbody>
</table>

**Table 1: Summary of Results**

market power. The results under adverse and advantageous selection can be thought about together by noticing that a reduction in the degree of selection lowers the cost heterogeneity in the population, moving individuals towards the population average cost. Because the monopolist internalizes the costs of the marginal consumer, reducing selection will reduce this marginal cost exactly when the marginal consumer is more costly than the population average consumer. Under adverse selection the marginal consumer has higher cost at lower quantity and under advantageous selection the marginal consumer has higher cost at higher quantity. Therefore benefits from reducing selection occur at low equilibrium quantities under adverse selection and high equilibrium quantities under advantageous selection.

**Proposition 6.** Under perfect competition, reducing the degree of adverse selection raises consumer surplus and is socially beneficial. Reducing the degree of advantageous selection lowers consumer surplus and is socially harmful. Producer surplus is always zero under perfect competition.

Under perfect competition, firms make no profits and thus the effect of selection on welfare is driven entirely by consumer surplus or equivalently prices. If consumers are adversely selected, consumers are always more costly than the population average, and therefore reducing the degree of selection always lowers average costs and therefore prices, making consumers and society better off. If consumers are advantageously selected, then by the same logic, reducing the degree of selection raises average costs and prices, and reduces consumer and social surplus.
3.3 Risk Adjustment

We next consider the impact of risk adjustment, which, as discussed in the previous section, has the same positive impacts as changing correlations but different normative implications.

**Proposition 7.** Under monopoly and assuming demand is strictly log-concave and satisfies a weak regularity condition, using risk adjustment to eliminate adverse selection has effects that are defined by the thresholds $q'$ and $q > q'$, where $q$ is defined exactly as in Proposition 4. The equilibrium quantity $q^*$ is defined as its value after risk adjustment.

1. If $q^* < q'$ then there is an interior optimal quantity of risk adjustment that achieves the socially optimal quantity. Social welfare is increasing in risk adjustment below this threshold and decreasing above it.

2. If $q' \leq q^* < q$ then welfare is monotonically increasing in risk adjustment and full risk adjustment achieves the socially optimal quantity if and only if $q^* = q'$.

3. If $q^* \geq q$ then risk adjustment is weakly socially harmful, and is strictly socially harmful if the inequality is strict.

If demand is not log-concave (or violates the regularity condition) it may be that $q' = 0$ so that behavior 1) above is irrelevant or that there are multiple thresholds between 1) and 2), but one or the other always occurs when $q^* < q'$.

Figure 4 graphically depicts these results for the different quantity ranges. The results are also summarized in Panel A of Figure 5. Social surplus depends on whether quantity is moved towards the socially optimal level under the original, non-risk adjusted demand and cost curves. Since monopoly results in too little quantity, risk adjustment that increases quantity is beneficial, so long as it does not increase quantity beyond the socially optimal level.

Panel (a) shows a setting where $q^* < q'$. In this case, risk adjustment is initially beneficial, but full risk adjustment reduces price below the original marginal cost, leading to socially excess quantity. Intuitively, this occurs at low quantity because this is where the monopoly distortion $MS(q^*)$ is smallest and where risk adjustment has biggest effect on reducing perceived marginal costs. Panel (b) shows a setting where $q' \leq q^* < q$ and where full risk adjustment is always beneficial but insufficient to achieve the social optimal level of quantity. Indeed, in this setting it would be optimal for the exchange operator to make excess transfers to the firms. Panel (c) shows a setting where $q^* \geq q$, and risk adjustment raises marginal costs perceived by the firm, lowering quantity and thereby reducing social welfare.

**Proposition 8.** Under monopoly and assuming that $MS' - MC'$ is globally signed, using risk adjustment to eliminate advantageous selection has effects that are defined by the thresholds $q$ and $q'' > q$, where $q$ is defined as in Proposition 5. The equilibrium quantity $q^*$ is defined as its value after risk adjustment.

1. If $q^* \leq q$ then risk adjustment is weakly harmful, and is strictly socially harmful if the inequality is strict.
Figure 4: The effect of full risk adjustment of adverse selection in a market served by a monopoly provider. Panel (a) shows full risk adjustment reducing price below the original marginal cost, leading to socially excess quantity. Panel (b) shows full risk adjustment as beneficial but insufficient to achieve the social optimal level of quantity. Panel (c) shows risk adjustment raising marginal costs perceived by the firm, lowering quantity and social welfare.
2. If \( q < q^* \leq q'' \) then welfare is monotonically increasing in risk adjustment and full risk adjustment achieves the socially optimal quantity if and only if \( q^* = q'' \).

3. If \( q^* > q'' \) then there is an interior optimal quantity of risk adjustment that achieves the socially optimal quantity. Social welfare is increasing in risk adjustment below this threshold and decreasing above it.

The threshold \( q'' \) may equal 1 in which case the last region irrelevant; this occurs if and only if \( MC(1) < AC(1) + MS(1) \). Again if \( MS' - MC' \) is not globally signed there may be back-and-forth between behaviors 2) and 3).

The results under advantageous selection are analogous to those under adverse selection with the regions reversed and are summarized in Panel B of Figure 5. When \( q^* \leq q \), quantity is below the socially optimal level and risk adjustment further reduces quantity. When \( q < q^* \leq q'' \), risk adjustment increases quantity but is insufficient to achieve the socially optimal level. When \( q^* > q'' \), there is an intermediate level of risk adjustment that increases quantity to the socially optimal level.

Under perfect competition, some risk adjustment is always beneficial although as before too much risk adjustment can sometimes be detrimental.

**Proposition 9.** Under perfect competition and either adverse or advantageous selection:

- If \( q^* < q \) then there is an interior optimal quantity of risk adjustment achieving the socially optimal quantity and social welfare increases in risk adjustment below and decreases in risk adjustment above
If \( q^* \geq \bar{q} \) then welfare is strictly increasing in the quantity of risk adjustment. Full risk adjustment achieves the socially optimal quantity if and only if \( q^* = \bar{q} \).

Risk adjustment, at least initially, moves average cost towards marginal cost and thus moves quantity towards the social optimum. However, when \( q^* < \bar{q} \) it may overshoot. Under adverse selection, this occurs because \( AC(1) \) is below \( MC(q^*) \). When \( q^* > \bar{q} \), even full risk adjustment is insufficient. Under adverse selection, this occurs because \( AC(1) \) is above \( MC(q^*) \). When \( q^* = \bar{q} \) then full risk adjustment exactly achieves the socially optimal quantity.

### 3.4 Other Forces Impacting Selection

Correlation and risk adjustment are only two of many forces that impact the extent of selection. Others commonly-discussed are changes in consumers’ knowledge of their own costs (Handel and Kolstad, 2013) and changes in the permitted extent of risk-based pricing (Finkelstein and Poterba, 2006). Unlike the micro-foundations above, these interventions will not only result in a change in the cost curves but will also shift the demand curves. In the first case this is because greater knowledge by consumers of their health risks will shift the distribution of willingness-to-pay for insurance, not only the correlation of this distribution with cost, and in the second case because characteristics that are used to price risk can also be used to price discriminate.

Because accounting for such effects requires a different analytical approach than the one we adopt here, we do not treat these forces generally. Instead we consider specific examples that illustrate possible and plausible cases. First in Appendix B we show these discrimination allowed by risk-based pricing can offset or even reverse the results we derived above about the effects of selection under market power. Second, in Subsection 4.2, we use our calibrated model of the insurance market to study the impact of these changes. We find that allowing for a price-discrimination effect or consumer misinformation effect actually strengthens our main results, especially our most counterintuitive result that eliminating adverse selection may harm consumers.\(^{11}\)

### 4 Applications

#### 4.1 Merger Analysis

In this subsection we discuss how the results we developed above should change approaches to a classic area of competition policy: the welfare evaluation of mergers. In particular, we examine a number of central principles principles articulated in the most recent revision of the United States Horizontal Merger Guidelines (United States Department of Justice and Federal Trade Commission, 2010) and show that many qualitative findings are altered or reversed in an industry with selection.

\(^{11}\)However, as discussed in Subsection 4.2, price discrimination will typically increase social welfare (Fabinger and Weyl, 2014) and thus will not tend to generate our counter-intuitive social surplus results if one accounts for the payments made by the government for risk adjustment. Poorly informed consumers may reinforce or mitigate this effect depending on how welfare is evaluated.
To facilitate the analysis, we focus on symmetrically differentiated Bertrand industry in which a potential merger changes the industry from a duopoly to a monopoly. This is not intended to be a realistic applied merger model, but simply to illustrate our argument in the cleanest and simplest case that has also been emphasized in previous theoretical merger analysis (Farrell and Shapiro, 1990; Werden, 1996; Farrell and Shapiro, 2010a).

1. **Price-raising incentives are harmful**: A basic principle of merger analysis is that the stronger are firms’ incentives to raise prices as a result of a merger, the more suspect antitrust authorities should be of the merger. However, to the extent that the incentive to raise prices is stronger because of selection, rather than because of demand-side substitution patterns, mergers are likely to be more **beneficial** the stronger the incentive to raise prices.

To see this, consider the “first-order” incentive of a firm to raise prices after a merger (Farrell and Shapiro, 2010a; Jaffe and Weyl, 2013), or “Upward Pricing Pressure” (UPP), measured by the externality a firm imposes on its rivals when it increases its sales by one (infinitesimal) unit. When a firm increases its sales by one (infinitesimal) unit, it diverts \( D \) units from its rivals, where \( D \) is the aggregate diversion ratio. In a market without selection, the markup associated with this unit is \( M = P - MC \) so that the sale exerts a negative externality on its rivals of \( DM = D(P - MC) \). In a market with selection, the marginal cost perceived by an individual firm is

\[
\sigma \left( D(q) AC(q) + [1 - D(q)] MC(q) \right) + (1 - \sigma) AC(1),
\]

so if we plug this measure of marginal costs into the standard UPP formula and drop arguments we get

\[
DM = D \left( P - \sigma [DAC + (1 - D) MC] \right).
\]

However, in selection markets, our assumption that switching consumers are representative of all consumers and have costs given by \( AC \) means that the incremental profit from this unit is \( P - \sigma AC - (1 - \sigma) AC \) and the sale creates a negative externality on rivals of \( D \left[ P - \sigma AC - (1 - \sigma) AC \right] \). As a result, the relevant UPP in selection markets is

\[
\text{UPP in Selection Markets} = D \left[ P - \sigma AC - (1 - \sigma) AC \right] =
\]

\[
= D \left( P - \sigma [DAC + (1 - D) MC] \right) + \sigma D \left( 1 - D \right) (MC - AC)
\]

\[
= \text{Standard UPP} + \sigma D \left( 1 - D \right) (MC - AC),
\]

which is the standard measure plus an additional term \( \sigma D \left( 1 - D \right) (MC - AC) \).

It is this additional term which reverses the standard logic that a greater incentive to increase prices makes a merger more suspect. To see this, note that increasing advantageous selection (raising \( \sigma \) when \( MC > AC \)) creates more upward pricing pressure, yet is precisely the setting where market power can be desirable because firms exert real externalities on other firms by skimming their inframarginal consumers. Conversely, greater adverse selection (raising \( \sigma \) when \( MC < AC \)) reduces upward pricing pressure but at the same time is the setting where market
power is most harmful because it further distorts the incentive to price above marginal cost which occurs even in a perfectly competitive market. Thus, to the extent that it is selection rather than changes in $D$ or $M$ that generate upward pricing pressure, a merger is actually most desirable when pricing pressure is large rather than small. For the rest of this subsection, we assume $\sigma = 1$ and drop the $q$ arguments to reduce notation.

2. *Competition-reduction is harmful:* A second principle of merger analysis is that when the services supplied by the merging firms are closer substitutes, antitrust authorities should be more suspect of the merger. However, in settings with advantageous selection, mergers between firms producing highly substitutable products are exactly the settings in which there may be too much competition and increases in market power may be more beneficial.

This point can be seen using the UPP framework discussed above. Standard analysis suggests that the larger is $D$ the more problematic a merger because it leads to a larger value of $UPP = D(P - MC)$. However, recall that $D = 1 - \theta$ and that under advantageous selection social surplus is inverse-U shaped in market power with an optimal level of $\theta = \theta^*$ strictly between 0 and 1. Thus if $D$ is sufficiently small, and as a result $\theta = 1 - D$ is larger than $\theta^*$, then the resulting merger will alway be harmful because it will further increase $\theta$ above its optimal level. And if $D$ is very large, and as a result $\theta = 1 - D$ is smaller than $\theta^*$, then the resulting merger may be preferable because it will reduce the externalities firms impose on each other in their efforts to cream skim the lowest cost customers. Thus, while under adverse selection the standard intuition is still valid, under advantageous selection it may be reversed: mergers may be socially beneficial (absent other efficiencies) if and only if $D$ is large enough.

3. *Marginal costs should be used to calculate mark-ups:* A third principle of merger analysis is that firm’s marginal rather than average cost should be used to assess their mark-ups in determining the incentive they will have to raise prices upon merging. In selection markets, recall that the valid upward pricing pressure we computed above is $D(P - AC)$ not $D (P - [DAC + (1 - D)MC])$. Thus, if we want to use the simple formula suggested by Farrell and Shapiro (2010a) to calculate upward pricing pressure, we should use *average cost not marginal cost* to calculate firms’ mark-ups.

Of course we have throughout ruled out non-linearities in cost at the firm level (non-additive-across-consumer cost structures); firm-level non-linearities from forces other than selection would still require attention to an adjusted notion of marginal cost. Nonetheless even in this case firm-level marginal costs would be inappropriate and some notion of average cost is likely to be more accurate in many cases.

4. *Demand data is preferable to administrative data:* As a result of the focus on marginal costs, demand-side data is often preferred to administrative data to evaluate the impact of a potential merger (Nevo, 2001). The reason is that marginal costs are hard to measure from firm administrative data (Laffont and Tirole, 1986). Therefore a standard approach to measuring marginal costs suggested by Rosse (1970) is to use demand-side data to estimate the firm’s mark-up and
recover marginal costs from first-order conditions. For example, Nevo (2001) backs out mark-ups from a structural model of pricing of cereals and uses these to conduct a merger analysis (Nevo, 2000).

However, in markets with selection, this approach identifies the mark-up in

\[ D (P - [DAC + (1 - D)MC]) \]

and not the relevant mark-up over average cost need need to calculate \( D(P - AC) \). Thus administrative data that reveals \( P \) and \( AC \) is not only sufficient to calculate valid upward pricing pressure in this context; it is necessary to do so and demand data will not suffice. Thus the administrative data obtained in many recent studies of selection markets (Einav, Finkelstein and Levin, 2010) are likely to prove particularly valuable for antitrust purposes, as well as the measurement of selection on which the literature has typically focused.

Our discussion above focuses on the lowest-hanging fruit that can most easily be derived from the simplest extension to the most canonical models. Many other standard antitrust intuitions, both within and beyond merger policy, should be reexamined in markets where selection is an important concern.

4.2 Health Insurance

In this subsection, we quantitatively examine the interaction of selection and market power with a calibrated model of health insurance coverage. We find that for standard parameters, we obtain the result that reducing adverse selection harms consumers, raises profits, and may harm aggregate social surplus depending on how selection is reduced.

4.2.1 Calibrated Model

Setup. There is market of potential consumers who decide whether to purchase an annual health insurance contract. We assume that consumers are expected utility maximizers with constant absolute risk aversion (CARA) preferences. Consumers are heterogeneous in their absolute risk aversion, denoted \( \alpha \), and their health-type, denoted \( \lambda \), which we assume are jointly log normally distributed according to

\[
\begin{align*}
\ln \alpha & \sim \mathcal{N} \left( \begin{bmatrix} \mu_\alpha \\ \mu_\lambda \end{bmatrix}, \begin{bmatrix} V_\alpha & \rho_{\alpha,\lambda} \sqrt{V_\alpha V_\lambda} \\ \rho_{\alpha,\lambda} \sqrt{V_\alpha V_\lambda} & \rho_{\alpha,\lambda} \sqrt{V_\lambda} \end{bmatrix} \right) \\
\ln \lambda & \sim \mathcal{N} \left( \begin{bmatrix} \mu_\alpha \\ \mu_\lambda \end{bmatrix}, \begin{bmatrix} V_\alpha & \rho_{\alpha,\lambda} \sqrt{V_\alpha V_\lambda} \\ \rho_{\alpha,\lambda} \sqrt{V_\alpha V_\lambda} & \rho_{\alpha,\lambda} \sqrt{V_\lambda} \end{bmatrix} \right)
\end{align*}
\]

Consumers with health-type \( \lambda \) are exposed to a distribution of shocks with realized values \( c \). We assume that consumers health type and health outcomes are jointly log normal distributed according
to the distribution
\[
\begin{align*}
\ln \lambda & \sim N \left( \left[ \begin{array}{c} \mu_\lambda \\ \mu_c \end{array} \right], \left[ \begin{array}{cc} \sqrt{V_\lambda} & \rho_{\lambda,c} \sqrt{V_\lambda V_c} \\ \rho_{\lambda,c} \sqrt{V_\lambda V_c} & \sqrt{V_c} \end{array} \right] \right) \\
\ln c & \sim N \left( \mu_c + \sqrt{\frac{V_c}{V_\lambda}} \rho_{\lambda,c} [\ln \lambda - \mu_\lambda], \sqrt{1 - \rho_{\lambda,c}^2} \sqrt{V_\lambda V_c} \right).
\end{align*}
\]

This implies that a consumer’s realized health risk, conditional on their health-type, is distributed according to
\[
\ln c | \ln \lambda \sim N \left( \mu_c + \sqrt{\frac{V_c}{V_\lambda}} \rho_{\lambda,c} [\ln \lambda - \mu_\lambda], \sqrt{1 - \rho_{\lambda,c}^2} \sqrt{V_\lambda V_c} \right).
\]

A health insurance contract is defined by an endogenous premium \( p \) and an exogenous cost-sharing function \( c_{OOP} = \kappa(c) \), which maps health realizations into out-of-pocket costs. We implicitly define a consumer’s willingness-to-pay \( v \) as the value that equates the expected utility with the insurance policy to the expected utility without insurance:
\[
E_c[u(-\kappa(c) - v)|\alpha, \lambda] = E_c[u(-c)|\alpha, \lambda].
\]

Consumers purchase the policy if and only if their willingness to pay is greater than the premium \( q = 1 \iff v \geq p \). The distribution of willingness-to-pay provides us with demand and marginal revenue curves for the industry according to the standard identities.\(^{12}\)

To emphasize the effects of market power and simplify the analysis, we assume that the industry is monopolized. Industry average costs are \( AC(q) = E[c - \kappa(c)|v \geq p] \) and marginal costs are \( MC(q) \equiv AC'(q)q + AC(q) \). As shown in Section 2, equilibrium price is determined by Equation 2:
\[
P(q) = MS(q) + \sigma MC(q) + (1 - \sigma) AC(1),
\]
where we normalize \( \sigma = 1 \) to the baseline degree of selection in the market coming directly from our calibration, which we now discuss.

**Calibration.** We calibrate the distributions of risk aversion using values from the literature and the distribution of health types and medical spending using values from the 2009 Medical Expenditure Panel Survey. Table 2 summarizes the exact calibrated variables. Below we discuss the calibrated values in more detail.

- **Risk aversion (\( \alpha \)).** We calibrate the distribution of absolute risk aversion to the values estimated by Handel, Hendel and Whinston (2013), which are estimated using over-time variation in the choice set of health insurance plans offered to employees at a large firm. These values are similar to those estimated by Cohen and Einav (2007). The mean value of \( \alpha = 0.000439 \) implies indifference between a 50-50 gamble for \( \{100, -96\} \) and \( 0 \) with certainty.

- **Realized costs (\( c \)).** We calibrate the distribution of realized medical costs \( c \) to match the pop-
ulation mean and standard deviation of medical spending for non-elderly individuals in the 2009 MEPS, excluding individuals with coverage from a public program such as Medicaid. The mean level of spending for this sample is $3,139 and the standard deviation is $10,126.

- **Health-type ($\lambda$).** To calibrate the degree of private information, we assume that consumers know of their future health costs is similar to that which can be predicted by standard risk adjustment software.\(^{13}\) The 2009 MEPS provides information on individual’s Relative Risk Scores, which is calculated using the Hierarchical Clinical Classification (HCC) model that is also used to risk adjust Medicare Advantage payments.

- **Correlation between risk aversion and health-type ($\rho_{\alpha,\lambda}$).** We assume that risk aversion and health risk are uncorrelated in the population. This is probably a reasonable assumption given the diverging estimates of the sign of this correlation in the literature.

- **Correlation between realized costs and health-type ($\rho_{\lambda,c}$).** Following our model, we estimate the correlation $\rho_{\lambda,c}$ with a regression of log realized health costs on the log Relative Risk Score, where both variables are normalized by subtracting the mean and dividing by the standard deviation. We estimate a coefficient of $\rho_{\lambda,c} = 0.498$. This estimate, combined with information on the mean and standard deviation of the Relative Risk Scores and realized costs, allows us to simulate the joint distributions of $\lambda$ and $c$.

- **Cost sharing ($\kappa(c)$).** We calibrate the structure of the insurance plan to cover approximately 60% of the costs of medical care for the population on average, assuming no moral hazard from the insurance contract.\(^{14}\) We use a plan with a $2,000 deductible, co-insurance of 10% above the deductible, and an out-of-pocket maximum of $8,000 per year.

### 4.2.2 Results

Figure 6 shows the calibrated model graphically. Panel (a) shows the baseline equilibrium with a monopolist health insurance provider and degree of selection that results from the calibration. Panel (b) shows the equilibrium from an alternative calibration where we keep the demand curve unchanged and reduce the variation of health type $\lambda$, holding constant population average costs under the insurance contract.\(^{15}\) Because demand is the same and there is less variation in costs, this exercise implements the same reduction in the degree of correlation between willingness-to-pay and costs that we explored theoretically. Panel (c) shows the equilibrium where an exchange operator implements perfect risk adjustment so that consumers have constant marginal costs equal to the population average.

---

\(^{13}\)This assumption follows standard practice in the literature (Handel, 2012; Handel, Hendel and Whinston, 2013) and is supported by the finding from Bundorf, Levin and Mahoney (2012) of little private information conditional on an industry standard measure of predicted health risk.

\(^{14}\)This is the level of generosity required by the “Bronze” plans available under the ACA.

\(^{15}\)Because of the non-linearity of the insurance contract, holding constant population average costs under the insurance contracts requires us to adjust the mean of population costs. We reduce the standard deviation of realized costs by 0.5, which results in an approximately 0.67 reduction in the standard deviation of the health-type parameter $\lambda$.  

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24
Figure 6: Panel (a) shows the baseline equilibrium. Panel (b) shows a scenario where the demand curve is unchanged but there is lower correlation between willingness-to-pay and marginal costs. Panel (c) shows an equilibrium with perfect risk adjustment so that marginal costs are constant in the population.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Absolute risk aversion</td>
<td>$4.39 \times 10^{-4}$</td>
<td>$6.63 \times 10^{-5}$</td>
<td>Estimates of absolute risk aversion from Table 3 of Handel, Hendel and Whinston (2013).</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Privately known health type</td>
<td>0.979</td>
<td>1.378</td>
<td>Values for Relative Risk Score (HCC, Private) in the 2009 MEPS.</td>
</tr>
<tr>
<td>$c$</td>
<td>Realized medical spending</td>
<td>$3,139$</td>
<td>$10,126$</td>
<td>Realized medical spending for non-elderly population without public insurance in 2009 MEPS.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Correlation of $\ln \lambda$ and $\ln c$</td>
<td>0.498</td>
<td></td>
<td>Estimated from a regression of normalized log realized medical spending on normalized log Relative Risk Scores in the 2009 MEPS.</td>
</tr>
</tbody>
</table>

Table 2: Calibration values for the health insurance model.

In the baseline scenario with no risk adjustment, premiums are $7,549 and 68.3% of the population has coverage. Because marginal costs are below average population costs at this equilibrium, reducing the degree of correlation increases the cost of the marginal consumer, raising premiums to $7,768 and reducing quantity by 2.1 percentage points. Eliminating selection by means of perfect risk adjustment further raises the price and reduces the quality provided by the market.

While these results are most stark for the case of a monopolist provider, the quantitative findings are robust to substantially less market power. In particular, we find that the reduction in correlation raises prices for all $\theta \geq 0.25$ and that perfect risk adjustment raises prices for all $\theta \geq 0.15$. To put these numbers in context, recall that for our Cournot microfoundation, $\theta = 1/n$, where $n$ is the number of firms. Thus, if we assume that conduct can be approximated by the Cournot model, this indicates that our qualitative results on correlations and risk adjustment hold when there are no more than 4 or 6 firms in the market, respectively.

Table 3 examines the normative implications of these counterfactuals. All values are presented as a percent of the first best total surplus under the baseline scenario, where the premium is determined by the intersection of willingness-to-pay and marginal costs curves. Under the baseline scenario, shown in the first column, market power reduces total surplus to 79.7% of the first best level. Producers capture two-thirds of this surplus, while consumers capture the remaining one-third. By raising prices, reduced correlations, shown in the second column, lowers consumer surplus by 2.7 percentage points of the total surplus at the social optimum. Profits increase by 5.6 percentage points due to the lower costs of providing coverage, more than offsetting the decline in consumer surplus.

---

Interpreted in terms of the differentiated Bertrand model this states that when the price of one insurer rises at least 25% or 15% of lost sales become uninsured.
Table 3: Welfare Effects of Reducing Adverse Selection

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Reduced Cost Heterogeneity</th>
<th>Perfect Risk Adjustment</th>
<th>Segmented Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Surplus</td>
<td>28.6%</td>
<td>25.9%</td>
<td>23.7%</td>
<td>21.6%</td>
</tr>
<tr>
<td>Producer Surplus</td>
<td>51.1%</td>
<td>56.7%</td>
<td>58.2%</td>
<td>68.5%</td>
</tr>
<tr>
<td>Exchange Operator Surplus</td>
<td>-8.1%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Surplus</td>
<td>79.7%</td>
<td>82.6%</td>
<td>73.8%</td>
<td>90.2%</td>
</tr>
</tbody>
</table>

and raising total surplus provided by the market. These results are consistent with Proposition 4 in the setting where optimal quantity takes a high, but not very high, value (i.e., $q > q^* > q$).

Full risk adjustment, shown in column 3, exacerbates the effects of reducing correlations on consumer and producer surplus. Relative to the baseline scenario, full risk adjustment reduces consumer surplus by 4.9 percentage points and increases profits by 7.1 percentage points of first best total surplus. Implementing full risk adjustment, however, requires the exchange operator to run a deficit equal to 8.1 percentage points of the optimized social surplus. Thus, the calibrated results indicate that risk adjustment has the counterintuitive effect of reducing surplus for consumers and surplus provided by the market, as described in Proposition 7 in settings where the optimal quantity is high (i.e., $q^* > q$).

Segmenting the market, shown in the fourth column, not only allows prices to reflect cost differences across consumers but also allows the monopolist to price-discriminate by charging different markups to different market segments. It thus does not correspond cleanly to our pure cost-side parameter $\sigma$. To implement segmentation we partition the distribution of $\lambda$ into quartiles and allowed the firm to charge a profit-maximizing price each market thus defined. Appendix Figure A2 shows plots which depict equilibrium price and quantity in each segment.

We find that the segmented markets have essentially no selection (a more-or-less flat cost curve) so that the results under segmentation reflect the elimination of selection as well as any price discriminatory effects. Segmentation reduces consumer surplus by 7.0 percentage points of the optimized total surplus, which is more than the decline under full risk adjustment. The reduced selection combined with the ability to price discriminate raises profits by 17.4 percentage points of the optimized total surplus. Total surplus from the market is 13 percentage points closer to the first best level. The incidence is significantly more skewed, with producers capturing more than three-quarters of the surplus generated by the market.

These findings on risk adjustment and risk-based pricing are far from universal. As discussed in Section 3 eliminating selection may raise or lower consumers and social surplus, and the same is famously true of the price discriminatory effects of market segmentation (Aguirre, Cowan and Vickers, 2010). However, in our calibrated model, (i) eliminating selection with risk adjustment and (ii) allowing price discrimination have similar qualitative effects: Both reduce consumer surplus while increasing firm profits, with an effect on total welfare that is determined by which effect is dominant.
Correlation Between Perceived and Actual Risk

<table>
<thead>
<tr>
<th></th>
<th>$\rho_{\lambda,\hat{\lambda}} = 1$</th>
<th>$\rho_{\lambda,\hat{\lambda}} = 0.5$</th>
<th>$\rho_{\lambda,\hat{\lambda}} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>7,410</td>
<td>7,705</td>
<td>7,747</td>
</tr>
<tr>
<td>Quantity</td>
<td>69.2%</td>
<td>66.4%</td>
<td>66.1%</td>
</tr>
<tr>
<td>Consumer Surplus*</td>
<td>Perceived Risk: 27.7%</td>
<td>25.0%</td>
<td>24.2%</td>
</tr>
<tr>
<td></td>
<td>Actual Risk: 27.7%</td>
<td>16.5%</td>
<td>10.5%</td>
</tr>
<tr>
<td>Producer Surplus*</td>
<td>51.1%</td>
<td>54.4%</td>
<td>58.5%</td>
</tr>
<tr>
<td>Total Surplus*</td>
<td>Perceived Risk: 78.8%</td>
<td>79.4%</td>
<td>82.7%</td>
</tr>
<tr>
<td></td>
<td>Actual Risk: 78.8%</td>
<td>70.9%</td>
<td>69.0%</td>
</tr>
</tbody>
</table>

Table 4: Effects of consumers misperceiving their cost risk.

We also use the model to examine the effect of change in correlations that would result from a change in consumer perceptions about the distribution of risk they face. For instance, an insurance-choice decision-aid might reduce misperceptions of costs and therefore increase the degree of selection in the market. We model this potential misperception by generating a perceived health-type $\hat{\lambda}$ that is jointly log-normally distributed with a consumer’s actual health-type $\lambda$ with correlation $\rho_{\lambda,\hat{\lambda}}$. We calculate equilibria where consumer’s willingness-to-pay is determined by perceived health type while costs are determined by the actual health type of the consumer. We calculate surplus under perceived demand, which might be relevant if consumers are never de-biased of their misperceptions, and under the actual demand curve. Appendix Figure A3 plots the equilibrium allocations generated by the perceived demand and marginal cost curves for different values of $\rho_{\lambda,\hat{\lambda}}$.

Table 4 shows the results of this exercise. The first column shows the setting where perceptions are fully accurate ($\rho_{\lambda,\hat{\lambda}} = 1$). The second column shows a setting where consumers have perceptions that are partially correlated with true health risk ($\rho_{\lambda,\hat{\lambda}} = 0.5$). The third column shows a setting where perceived health risk is completely uncorrelated with the truth ($\rho_{\lambda,\hat{\lambda}} = 0$). As above, the welfare values are shows as a percent of the optimized total surplus.

Reducing the correlation between perceived and actual health risk has the effect of decreasing the degree of selection in the market. Thus, similar to the results above, increased misperceptions raise price and reduce quantity in the market. Consumer surplus is reduced even under the demand curves that result from perceived risk. Increasing misperceptions raises producer surplus, suggesting that policy efforts to de-bias consumers through decision aids may be opposed by the insurance industry. Consumer surplus is even lower, and social surplus actually falls, under actual demand curves, since the misperceptions create an allocative inefficiency in who receives insurance coverage. This pushes against the argument, made in a perfectly competitive environment, that nudging (improving information) can hurt consumers by exacerbating the degree of selection (Handel, 2012).
5 Conclusion

This paper makes three contributions. First, we propose a simple but general model nesting a variety of forms of imperfect competition in selection markets. Second, we derive from this model several basic, and often counter-intuitive, comparative statics. Third, we show the empirical and policy relevance of these comparative statics by applying them to merger policy and a calibrated model of health insurance.

Our work here suggests several other directions for future research. We have shown calibrated and empirical examples where the counter-intuitive comparative statics we derived are relevant. However, it is not clear how prevalent such examples are or the breadth with which the issues we raise are first-order in determining optimal competition policy or selection policy. Further empirical research is important to investigate this.

We have also focused on a small number of policy instruments: merger policy, risk-rating and cost-based pricing. While these may be the most canonical policies for addressing selection and market power, many others, such as subsidies for group care and restraints on exclusive dealing, play an important role. Investigating the effect of market power on the first policy and selection on the second would be informative.

Finally our paper contributes to a growing literature that connects issues of contemporary interest, such as selection and imperfect competition, to classical price theory. While we primarily used this connection to draw out the implications of contemporary interest, our results also have implications for the classical theory of regulation of natural monopolies, as our monopoly and competition models correspond, respectively, to an unregulated monopoly and one bound to average cost pricing. In particular, to the best of our knowledge, the welfare implications of average cost pricing, when compared to unregulated monopoly, in a region of a monopoly’s cost curve where cost is increasing (corresponding to advantageous selection) have not been explored in previous literature. Exploring the relationship between these literatures further would be an interesting topic for future research.

References


Fabinger, Michal, and E. Glen Weyl. 2014. “Price Discrimination is Typically Efficient and Egalitarian.” This paper is under preparation. Contact Glen Weyl at weyl@uchicago.edu for notes.


Appendix

A Model

This appendix provides formal micro-foundations for the representations in the text.

A.1 Cournot model

Potential consumers of a homogeneous service are described by a multi-dimensional type $t = (t_1, \ldots, t_T)$ drawn from a smooth and non-atomic distribution function $f(t)$ with full support on a hyper-box $(\tilde{t}_1, \tilde{t}_1) \times \cdots (\tilde{t}_T, \tilde{t}_T) \subseteq \mathbb{R}^T$. Consumers receive a quasi-linear utility $u(t) - p$ if they purchase the service for price $p$. When the prevailing price is $p$, therefore, the set of consumers purchasing the service is $T(p) = \{ t : u(t) \geq p \}$ and the number of purchasers $Q(p) = \int_{T(p)} f(t)dt$. $T(p)$ is clearly decreasing in $p$ in the strong set order so that by our assumption of full support $Q(p)$ is strictly decreasing. Thus we can define the inverse demand function $P(q)$ as the inverse of $Q(p)$.

Each consumer also carries with her a cost of service, $c(t) > 0$ that must be incurred to supply the service to her by any supplier. Thus the average cost of all individuals served when the aggregate quantity is $q$ is

$$AC(q) = \frac{\int_{T(P(q))} c(t)f(t)dt}{Q(P(q))}.$$

There are $n$ firms that can each choose a quantity $q_i$ of the service to supply non-cooperatively. If $q = \sum q_i < 1$ then the prevailing market price is set by market clearing as $P(q)$. If $q > 1$ then price is 0. Clearly no equilibrium can involve $q > 1$ as all firms would make losses. Firms receive a uniform random sample of all customers who are in the market at the prevailing prices and thus earn profits $q_i [P(q) - AC(q)]$. Thus, to maximize profits non-cooperatively they must satisfy

$$P(q) - AC(q) + P'(q)q_i - \frac{MC(q)}{q} AC(q) q_i = 0.$$

At a symmetric equilibrium where $q_i = \frac{q}{n}$ for all $i$ this becomes

$$P(q) - \left(1 - \frac{1}{n}\right) AC(q) - \frac{MS(q)}{n} - \frac{MC(q)}{n} = 0$$

as claimed in the text.

A.2 Differentiated Bertrand model

There are $n$ firms $i = 1, \ldots, n$ each selling a single service. Consumers are described by two types, each possibly multidimensional, $(t, \epsilon)$. $t$ is drawn as in the Cournot case. $\epsilon$ consists of two components: $\epsilon = (l, e)$ where $l$ is an integer between 1 and $L$, with each value of $l$ having equal probability, and $e$ is drawn from a real hyper rectangle in $E$ dimensions. The distribution of $\epsilon$ is atomless, symmetric in all coordinates, independent of the value of $l$ and given by the distribution function $g$. The distributions of $t$ and $\epsilon$ are independent.

Consumers may consume at most a single service and receive a quasi-linear utility from consuming the service of firm $i$, $u_i(t, \epsilon) - p_i$, where $p_i$ is the price charged for service $i$. Let the first order statistic of utility $u^*(t, \epsilon) \equiv \max_i u_i(t, \epsilon)$. We assume (without loss of generality yet) that $u^*(t, \epsilon) = u^*(t)$;
that is that the value of the first-order statistic depends only on \( t \) and not on \( \epsilon \). Second, and this does entail a loss of generality, we make the following assumption.

**Assumption 1.** \( u_i = u^* \left( t \right) + \hat{u_i} \left( \epsilon \right) \) so that all valuations shift up uniformly with a shift in \( u^* \) induced by changes in \( t \).

This implies that the relative utility of services other than the one the individual most prefers, compared to that which she most prefers, are determined purely by \( \epsilon \) and not \( t \). Third we assume, with only a modest loss of generality, that \( u^* \left( t \right) \) is smooth in \( t \) and that \( \frac{\partial u^*}{\partial t} > k > 0 \) for some constant \( k \). This implies that raising \( t_T \) sufficiently causes \( u^* > u \) for any fixed \( u \) and lowering it sufficiently causes the reverse to be true.

Services are symmetrically differentiated in the sense that distribution of \( u \left( t, \epsilon \right) = \left( u_1 \left( t, \epsilon \right), \ldots , u_n \left( t, \epsilon \right) \right) \) induced by the distribution of \( \left( t, \epsilon \right) \) is symmetric in permutations of coordinates. The set of individuals purchasing service \( i \) is

\[
T_i \left( p \right) = \left\{ \left( t, \epsilon \right) : u_i \left( t, \epsilon \right) \geq p_i \wedge i \in \text{argmax}_i u_i \left( t, \epsilon \right) - p_i \right\}
\]

and the demand for good \( i \) is thus \( Q_i(p) = \int_{T_i(p)} f(t, \epsilon) d(t, \epsilon) \).

As in the Cournot example, the cost of serving a consumer depends on her type. However, we make the substantive assumption now that cost depends only on \( t \) and not on \( \epsilon \).

**Assumption 2.** The cost of serving a consumer of type \( t, \epsilon \) is \( c(t) \) and thus the total cost faced by firm \( i \) is \( C_i(p) = \int_{T_i(p)} c(t) f(t, \epsilon) d(t, \epsilon) \).

This assumption states that only the determinants of the highest possible utility a consumer can achieve, and not of her relative preferences across services, may directly determine her cost to firms. Given the independence of \( t \) and \( \epsilon \), this assumption implies a clean separation between determinants of relative “horizontal” preferences across services and “vertical” utility for the most preferred service that also determines the cost of service. Absent this assumption it is possible that the consumers that firms attract from their rivals when lowering their price are very different in terms of cost from the average consumers of the service more broadly.

Let \( 1 \equiv \left( 1, \ldots , 1 \right) \). Then by symmetry \( Q_i(p1) = Q_j(p1) \forall i, j \) and similarly for \( C_i \) and \( C_j \). Let the aggregate demand \( Q(p) = nQ_i(p1) \) for any \( i \) and similarly for aggregate cost. Then we define the inverse demand function \( P(q) \) as the inverse of the aggregate demand. Average cost is then \( AC(q) = \frac{C(P(q))}{q} \) and marginal cost \( MC(q) = C' \left( P(q) \right) P'(q) \).

We now describe two particular models satisfying these assumptions and show how they yield the reduced-form representation we use in the text. Any other micro-foundation of these assumptions should also yield our representation, but the notation required to encompass different cases is sufficiently abstract and not relevant enough to any results we derive. We thus omit it here and focus on specific micro-foundations.

First consider a random utility model in the spirit of Anderson, de Palma and Thisse (1992) proposed by White and Weyl (2012) in the context of heterogeneity of preferences for non-price product characteristics. \( L = n \) and the value of \( l \) represents which product is the individual’s favorite, \( e = \left( e_1, \ldots , e_E \right) \) and \( E \geq n - 1 \). We assume that

\[
u_i \left( u^* \left( t \right), l, \epsilon \right)\]

is increasing in \( e_i \) where \( i^* \) is \( i \) if \( i < l \) and is \( i - 1 \) if \( i > l \) and that it is constant in all other \( e_j \) where \( i \leq n - 1 \) and not \( i^* \). We also assume that \( u_i \) is smooth in its arguments other than \( l \), bounded and that and that \( \lim_{\epsilon_i \rightarrow \epsilon_i^*} u_i \left( u^* \left( t \right), l, \epsilon \right) = u^* \left( t \right) \) and \( \lim_{\epsilon_i \rightarrow \epsilon_i^*} u_i \left( u^* \left( t \right), l, \epsilon \right) = 0 \) for any value of the
other entries $u^*, l$ and $e_{-i^*}$, where $e_i$ and $\bar{e}_i$ are respectively the lowest and highest values of $e_i$. This implies that raising $e_i$ sufficiently for any $i$ while holding fixed the other components of $e$ makes (in the limit) service $i$ equally desirable to the most desirable service for the individual and lowering it makes it always uncompetitive with the best service regardless of the price differential.

An individual firm $i$’s profits are $p_i Q_i (p_1, \ldots, p_i, \ldots, p_n) - C_i (p_1, \ldots, p_i, \ldots, p_n)$. Thus the first-order condition for the optimization of any firm $i$ is

$$\frac{\partial Q_i}{\partial p_i} + Q_i = \frac{\partial C_i}{\partial p_i}. \quad (3)$$

Because price does not appear in the interior of the integrals defining $Q_i$ and $C_i$, the derivatives of these with respect to $p_i$ is, by the Leibniz Rule applied to multidimensional integrals (Veiga and Weyl, 2014a), given by the sum of the effects of the extensive margin effects from the change in the boundaries of integration. There are many such boundaries, so we use a shorthand notation for them. $\partial T^X_i (p) \equiv \{(t, e) \in T_i(p) : u^* (t) = p_i\}$ denotes the set of exiting consumers from product $i$ who are just indifferent between buying service $i$ and no service. $\partial T^S_i (p) \equiv \{(t, e) \in T_i(p) \cap T_j(p)\}$ denotes the set of switching consumers between services $i$ and $j$ who are just indifferent between the two services, but prefer purchasing one over purchasing nothing. To formally define the density of consumers on such boundaries it is useful to express the multidimensional integrals representing $Q_i$ and $C_i$ more explicitly.

At symmetric prices $p$, every individual $i$ with $t_T$ above this threshold buys from her most preferred services $l$ and any individual below this threshold buys no service. If a single price $p_i$ is elevated to $p_i + \delta$ then all individuals with $l \neq i$ continue to buy their preferred product as at symmetry. However, individuals with $l = i$ and $u^* (t) \in (p_i, p_i + \delta)$ will stop consuming any service and those with $l = i$ and $e_i$ sufficiently close to $\bar{e}_i$ will switch to purchasing service $j$. Let $t^*_j (p; t_T)$ be defined implicitly by $u^* (t_T, t^*_j (p; t_T)) = p$ and let $e^*_j (\Delta; u^* (t), e_{-\{1, \ldots, n-1\}})$ be implicitly defined for positive $\Delta$ by

$$u^* (t) - u_j \left( u^* (t), e^*_j (\Delta; u^* (t), e_{-\{1, \ldots, n-1\}}), e_{-\{1, \ldots, n-1\}} \right) = \Delta$$

where $e_{-\{1, \ldots, n-1\}}$ is all components of $e$ other than the first $n - 1$ and the dependence of $u_j$ on the other components of $e$ is dropped as these do not impact $u_j$.

Then when prices are symmetric except for price $p_i$ being above the other prices, we can write

$$Q_i (p, \ldots, p_i, \ldots, p) =$$

$$\frac{1}{n} \int_{t_T} \int_{e_{-\{1, \ldots, n-1\}}} \int_{t^*_j (p; t_T)} \int_{t_T} \int_{t_T} f(t) g(e) \, d(t, e)$$

and similarly

$$C_i (p, \ldots, p_i, \ldots, p) =$$

$$\frac{1}{n} \int_{t_T} \int_{e_{-\{1, \ldots, n-1\}}} \int_{t^*_j (p; t_T)} \int_{t_T} \int_{t_T} c(t) f(t) g(e) \, d(t, e).$$

To fill in the first-order condition (Equation 3), we need to differentiate these using the Leibniz rule.
\[- \frac{1}{n} \int_{\partial T_X^X(p1)} \frac{f(t-T_{ij}^T(p;\bar{t}-T))g(e)}{\partial u_i/\partial t_i(t-T_{ij}^T(p;\bar{t}-T))} d(t-T,e) + (n-1) \int_{\partial T_n^X(p1)} \frac{f(t)g(e-\bar{x})}{\partial u_i/\partial t_i(t,e-\bar{x})} d(t,e-j) \] (4)

for any \( j \neq i \) by symmetry and similarly

\[ \frac{\partial C_t}{\partial p_i}(p1) = \]

\[- \frac{1}{n} \int_{\partial T_X^X(p1)} \frac{c(t-T_{ij}^T(p;\bar{t}-T))f(t-T_{ij}^T(p;\bar{t}-T))g(e)}{\partial u_i/\partial t_i(t-T_{ij}^T(p;\bar{t}-T))} d(t-T,e) + (n-1) \int_{\partial T_n^X(p1)} \frac{c(t)f(t)g(e-\bar{x})}{\partial u_i/\partial t_i(t,e-\bar{x})} d(t,e-j) \] (5)

By contrast and following the same logic

\[ \frac{dQ_i}{dp}(p1) = -\frac{1}{n} \int_{\partial T_X^X(p1)} \frac{f(t-T_{ij}^T(p;\bar{t}-T))g(e)}{\partial u_i/\partial t_i(t-T_{ij}^T(p;\bar{t}-T))} d(t-T,e) \]

and

\[ \frac{dC_t}{dp}(p1) = \]

\[- \frac{1}{n} \int_{\partial T_X^X(p1)} \frac{c(t-T_{ij}^T(p;\bar{t}-T))f(t-T_{ij}^T(p;\bar{t}-T))g(e)}{\partial u_i/\partial t_i(t-T_{ij}^T(p;\bar{t}-T))} d(t-T,e) . \]

Thus by symmetry

\[ Q'(p) = -\int_{\partial T_X^X(p1)} \frac{f(t-T_{ij}^T(p;\bar{t}-T))g(e)}{\partial u_i/\partial t_i(t-T_{ij}^T(p;\bar{t}-T))} d(t-T,e) \]

and

\[ C'(p) = -\int_{\partial T_X^X(p1)} \frac{c(t-T_{ij}^T(p;\bar{t}-T))f(t-T_{ij}^T(p;\bar{t}-T))g(e)}{\partial u_i/\partial t_i(t-T_{ij}^T(p;\bar{t}-T))} d(t-T,e) , \]

so that

\[ MC(Q(p)) = \frac{\int_{\partial T_X^X(p1)} \frac{c(t-T_{ij}^T(p;\bar{t}-T))f(t-T_{ij}^T(p;\bar{t}-T))g(e)}{\partial u_i/\partial t_i(t-T_{ij}^T(p;\bar{t}-T))} d(t-T,e) \] \[ \int_{\partial T_X^X(p1)} \frac{c(t-T_{ij}^T(p;\bar{t}-T))f(t-T_{ij}^T(p;\bar{t}-T))g(e)}{\partial u_i/\partial t_i(t-T_{ij}^T(p;\bar{t}-T))} d(t-T,e) . \]

Furthermore

\[ \int_{\partial T_n^X(p1)} \frac{c(t)f(t)g(e-\bar{x})}{\partial u_i/\partial t_i(t,e-\bar{x})} d(t,e-j) = n \int_{T_0(p)} c(t)f(t)dt \int_{t-e-j} g(e-\bar{x}) d(t,e-j) = \]

\[ AC(Q(p))Q(p) \int_{t-e-j} g(e-\bar{x}) dt d(t,e-j) \equiv -AC(Q(p))s(p) \]

where \( s(p) \) is the density of consumers diverted to a rival from a small increase in one firms price starting from symmetric prices \( p \). Thus we can rewrite Expression 4 as

\[ \frac{Q'(p) - (n-1)s(p)}{n} = Q'(p) \frac{1}{n[1 - D(Q(p))]}, \]

where \( D(q) \equiv -\frac{(n-1)s(P(q))}{Q'(P(q))-(n-1)s(P(q))} \) is the aggregate diversion ratio (Farrell and Shapiro, 2010b), the fraction of consumers lost to a small increase in prices by a single first that go to rivals rather than the outside good. We can also rewrite expression 5 as

\[ Q'(p)MC(Q(p)) + \frac{D(Q(p))}{1-D(Q(p))} AC(Q(p)) = \]
Then Equation 3 becomes, at symmetric prices

\[
Q'(p) \frac{p}{n [1 - D(Q(p))]} + \frac{Q(p)}{n} = Q'(p) \frac{MC(Q(p)) + \frac{D(Q(p))}{1 - D(Q(p))} AC(Q(p))}{n} \implies
\]

that at any symmetric equilibrium

\[
\frac{P(q)}{1 - D(q)} - MS(q) = MC(q) + \frac{D(q)}{1 - D(q)} AC(q)
\]

because \( MS(q) = \frac{Q(p(q))}{Q'(P(q))} \). Letting \( \theta(q) = 1 - D(q) \) this becomes

\[
P(q) - \theta(q) MS(q) = \theta(q) MC(q) + [1 - \theta(q)] AC(q)
\]

as reported in the text.

A second model that delivers our form builds on the Chen and Riordan (2007) “spokes” extension of the Hotelling linear city model, combining it with modifications from Rochet and Stole (2002). There are \( n \) firms \( i = 1, \ldots, n \). For every pair of firms, \( (i, j) \) with \( i < j \) there is a line segment of unit length of potential consumers who will only consider purchasing either service \( i \) or service \( j \). Thus there are \( n(n - 1) \) such segments and we denote the segment \( (i, j) \) by the integer \( l = (j - i - 1) + (j \mod n) \).

\( e = (l,e) \) where \( l \) is the integer representing the line segment on which the consumer lives and \( e \in (0,1) \) is the distance of the consumer from \( i \) or \( 1 - e \) her distance from \( j \). In particular let \( i(l) \equiv \max_{l \in Z; \frac{l(n-1)}{2} < l} \frac{i(l) - 1}{2} \) and let \( j(l) \equiv l \mod n \); then \( e \) is the distance of the consumer from \( i(l) \). There are an equal number of consumers on each segment so \( \frac{2}{n(n - 1)} \) of the consumers are on each segment.

In addition to maintaining our assumptions about \( t \) and \( e \), we make two modifications to the set-up of Chen and Riordan:

1. We modify the exact form of consumer utility. In particular, \( u^*(t) \) is the utility a consumer earns from service \( i(l) \) if \( e \leq \frac{1}{2} \) and from good \( j(l) \) if \( e \geq \frac{1}{2} \) regardless of the other details of her position. This contrasts with the standard Chen and Riordan, and Hotelling (1929), model because it implies no transport cost to an individual’s most preferred service.

2. Consumers’ highest possible utility is not constant across consumers but instead follows a distribution \( u^*(t) \).

3. The gross utility a consumer derives from purchasing from \( j(l) \) if \( e < \frac{1}{2} \) is \( u^*(t) - (1 - 2e)t \),

where \( t \) is a transportation cost parameter absent in the Chen and Riordan model. If \( e > \frac{1}{2} \) the consumer derives gross utility of \( u^*(t) - (2e - 1)t \) from purchasing from \( i(l) \).

4. We allow arbitrary smooth and symmetric-about-\( \frac{1}{2} \) distributions of \( e \) on the unit interval, as long as this distribution is the same for all \( l \).

Calculations to derive the representation in the text are tedious and extremely similar to those in our modified Anderson, de Palma and Thisse model above. We therefore omit these calculations and simply explain why the results are the same. At symmetric prices, every consumer purchases from her most preferred firm, \( i(l) \) if \( e \leq \frac{1}{2} \) and \( j(l) \) if \( e \geq \frac{1}{2} \). All consumers with the same \( t \) make the same purchase decision at this price because only \( u^* \) impacts their total utility. Consumers with \( e = \frac{1}{2} \) are “switchers” between a pair of firms (if \( u^*(t) \geq p \)) and have the same distribution of \( t \) as all purchasers by the independence of \( e \) and \( t \). Thus switchers will be representative of the full
population of consumers and exiters everywhere will be on average identical. This is precisely what gave rise to our structure above.

B Example with Large Demand-Driven Effects of Risk-Based Pricing

One intervention commonly applied in selection markets is cost-based pricing. In Subsection 4.2 we showed that these discriminatory effects may reinforce the cost-based effects of selection. In this appendix we discuss how price discriminatory effects of cost-based pricing may instead reverse the results we established about the impact of changing the degree of selection in Section 3.

Proposition 5 states that increasing advantageous selection increases monopoly profits. However, clearly allowing cost-based pricing may never hurt a monopolist as she may maintain uniform pricing. It will generically aid the monopolist. Thus her gains from price discrimination swamp the effects we highlight.

To see that impacts on consumers may also be reversed by price discriminatory effects, consider our result (Proposition 4 and 5) that decreasing adverse and advantageous selection may both benefit consumers, depending on the equilibrium quantity. This may be true of cost-based pricing, but not in one simple, extreme case. Suppose that there is only a single dimension of heterogeneity determining both cost and valuation and that we move from uniform pricing to full cost-based pricing. This operates as perfect, first-degree price discrimination, extracting all surplus from consumers regardless of the equilibrium quantity and thus contradicting the natural extrapolation of our result.

Thus cost-based pricing cannot cleanly be interpreted as an example of increasing selection in our framework; price discrimination may be more important in some cases than are cost-based effects. However, in the leading counter-intuitive case we emphasize, the two effects reinforce one another to lower consumer surplus.

C Proofs

Throughout we assume that \( \theta, \sigma \in [0,1] \), that selection is either globally adverse or advantageous (either \( AC', MC' > 0 \) or \( AC', MC' < 0 \) for all \( q \)) and impose a global equilibrium stability condition: \( P' < \min\{AC', MC', 0\} \) and \( MR' < \min\{MC', 0\} \). Most of the results may be obtained absent these global monotonicity assumptions, but the additional expositional complexities add little insight. We also assume that \( \theta \) and \( \sigma \) are constant parameters, independent of \( q \); all results can be extended to the case when this fails, but again, the additional notation is cumbersome.

Lemma 1. Let \( F(q) \equiv P(q) - \sigma (\theta MC(q) + (1 - \theta) AC(q)) + (1 - \sigma) AC(1) - \theta MS(q) \). Then \( F' < 0 \).

Proof. The derivative of the expression is

\[
P' - \sigma \theta MC' - \sigma (1 - \theta) AC' - \theta MS' = \\
\sigma [\theta (MR' - MC') + (1 - \theta)(P' - AC')] + [1 - \sigma] [\theta MR' + (1 - \theta)P'] < 0.
\]

by our monotonicity assumptions. \( \square \)

Proposition (Formal) 1. For \( \theta \in (0,1) \), \( \frac{\partial q^*}{\partial \theta} \geq 0 \geq \frac{\partial CS}{\partial \theta} \), with strict inequality if \( q^* > 0 \).

Proof. By the implicit function theorem,

\[
P \frac{\partial q^*}{\partial \theta} - \sigma [MC (q^*) - AC (q^*)] - MS (q^*) = 0 \quad \Longrightarrow \quad \frac{\partial q^*}{\partial \theta} = \frac{MS (q^*) + \sigma [MC (q^*) - AC (q^*)]}{P}.
\]
Focusing on the numerator

\[ MS + MC - AC = -P'q + \sigma AC'q = q [\sigma (AC' - P') - (1 - \sigma)P'] > 0 \]

by our monotonicity assumptions. Thus by Lemma 1, \( \frac{\partial q^*}{\partial \sigma} < 0 \) if \( q \neq 0 \) and weakly if \( q = 0 \). This immediately implies that price rises in \( \theta \) by monotonicity and thus that \( CS \) falls. Producer surplus is

\[ PS(q) = q [P(q) - \sigma AC(q) - (1 - \sigma)AC(1)] \]

so

\[ PS'(q) = P(q) - \sigma AC(q) - (1 - \sigma)AC(1) + q [P'(q) - \sigma AC'(q)] = P(q) - MS(q) - \sigma MC(q) - (1 - \sigma)AC(1) < F'(q) \]
as \( MS + MC - AC, MS > 0 \) by the argument above so long as \( \theta < 1 \). Thus at \( q^* \) for any \( \theta < 1 \), \( PS' < 0 \).

**Proposition (Formal) 2.** If \( AC' < 0 \) and \( \theta \in (0, 1) \), \( \frac{\partial SS}{\partial \theta} \leq 0 \), strictly if \( q^* > 0 \).

*Proof.*** \( SS(q) = \int_0^q (P(q) - [\sigma MC(q) + (1 - \sigma)AC(1)]) dq \) so \( SS'(q) = P(q) - \sigma MC(q) - (1 - \sigma)AC(1) \). Thus

\[ SS'(q^*) = \sigma (1 - \theta) [AC(q^*) - MC(q^*)] + \theta MS(q^*) > 0 \]
because \( MS > 0 \) and \( AC'(q) = \frac{MC(q) - AC(q)}{q} \) < 0. Thus the result follows from the chain rule and the fact that \( \frac{\partial q^*}{\partial \sigma} < 0 \) as shown in the proof of the previous proposition. \( \square \)

**Proposition (Formal) 3.** If \( AC' > 0 \) and \( q^* > 0 \) for every \((\theta, \sigma) \in (0, 1)^2, \exists \theta^* \in (0, 1) \) such that \( \frac{\partial SS}{\partial \theta} > (\leq =)0 \) if \( \theta < (> =) \theta^* \). \( \frac{\partial q^*}{\partial \sigma} > 0 \) if \( \sigma \in (0, 1) \).

*Proof.*** By the logic of the previous proof, \( SS'(q) = P(q) - \sigma MC(q) - (1 - \sigma)AC(1) \) so

\[ SS''(q) = P'(q) - \sigma MC'(q) = (1 - \sigma)P'(q) + \sigma [P'(q) - MC'(q)] < 0. \]

Thus social surplus is concave in quantity. Quantity is below its optimal level at \( \theta = 1 \) by the standard monopoly argument and quantity is above its optimal level at \( \theta = 0 \) by the argument in the proof of the previous proposition. Thus the result follows from the fact, shown in the proof of Proposition 1, that \( \frac{\partial q^*}{\partial \sigma} < 0 \).

**Proposition (Formal) 4.** \( \frac{\partial q^*}{\partial \sigma} \) has the same sign as \( AC(1) - \theta MC(q^*) - (1 - \theta)AC(q^*) \). If \( AC' < 0 \) then providing a specific subsidy to the industry can only cause the sign of \( \frac{\partial q^*}{\partial \sigma} \) to move from being negative to being positive; a sufficiently large such subsidy guarantees this sign is positive. If \( \theta = 1 \) then \( \frac{\partial PS}{\partial \sigma} \) has the same sign as \( AC' \). Again if \( \theta = 1, AC' < 0 \) and if the pass-through rate, \( \rho(t) = \frac{\partial P(q^*)}{\partial t} > M > 0 \) for some \( M \) and all \( t \) such that \( q^* \in (0, 1) \) then \( \frac{\partial PS}{\partial \sigma} + \frac{\partial CS}{\partial \sigma} > 0 \) starting from a sufficiently large subsidy \(-t\) such that \( q^* < 1 \).

*Proof.*** With a specific tax (negative specific taxes are specific subsidies), the equilibrium condition is

\[ P(q) - \sigma (\theta MC(q) + (1 - \theta) AC(q)) + (1 - \sigma)AC(1) - \theta MS(q) = t = 0. \]

Thus by the Implicit Function Theorem

\[ F'(q^*) \frac{\partial q^*}{\partial \sigma} - [\theta MC(q^*) + (1 - \theta) AC(q^*) - AC(1)] = 0 \]

\( \implies \)
\[
\frac{\partial q^*}{\partial \sigma} = \frac{\theta MC(q^*) + (1 - \theta) AC(q^*) - AC(1)}{F'(q^*)}.
\]  

Because \( F' < 0 \) by Lemma 1, this has the same sign as \( AC(1) - \theta MC(q^*) - (1 - \theta) AC(q^*) \) regardless of the degree of tax or subsidy. By the same arguments as above, \( \frac{\partial q^*}{\partial \sigma} < 0 \). Thus if \( AC' > 0 \) the sign of this expression can only move, with an increase in tax, from being positive to being negative (from being negative to being positive).

Furthermore as \( t \) (a sufficiently large subsidy) becomes arbitrarily negative (a sufficiently large subsidy is given), \( q^* \rightarrow 1 \). Thus the denominator on the right-hand side of Equation 6 must approach \( \theta MC(1) - AC(1) \), which is negative if \( AC' < 0 \) and thus \( \frac{\partial q^*}{\partial \sigma} > 0 \) eventually.

As before \( PS(q) = q[P(q) - \sigma AC(q) - (1 - \sigma) AC(1)] \). When \( \theta = 1 \) profits are maximized over \( q \) so by the envelope theorem we can calculate \( \frac{\partial PS}{\partial \sigma} \) while holding fixed \( q^* \) yielding \( AC(1) - AC(q^*) \).

For \( q^* < 1 \) this clearly has the same sign as \( AC' \). By the envelope theorem for consumers and this result for producers

\[
\frac{\partial CS}{\partial \sigma} = -q^*P'(q^*) \frac{\partial q^*}{\partial \sigma} = -\rho(t)q^* [MC(q^*) - AC(1)],
\]

where the second equality uses the fact that when \( \theta = 1 \),

\[
P'(q^*) \frac{\partial q^*}{\partial \sigma} = P'(q^*) \frac{MC(q^*) - AC(1)}{F'(q^*)} = \rho(t) [MC(q^*) - AC(1)]
\]

as the tax enters linearly into the expression for \( F \). Thus

\[
\frac{\partial CS}{\partial \sigma} + \frac{\partial PS}{\partial \sigma} = -\rho(t)q^* [MC(q^*) - AC(1)] + AC(1) - AC(q^*).
\]

As \( t \) becomes sufficiently negative, \( q^* \rightarrow 1 \) so that the second term vanishes and the first term is bounded away from 0 as \( MC(q^*) - AC(1) \) grows in absolute value (becomes more negative) monotonically in \( q^* \) and \( \rho(t) > M > 0 \) by hypothesis.

Note that this result is formulated in terms of subsidies, but these are equivalent to upward demand shifts of we confine attention to in-market quantities and thus ignore the impact on the subsidy provider, as we do here.

**Proposition (Formal) 5.** If \( AC' > 0 \) then giving a specific subsidy to the industry can only cause the sign of \( \frac{\partial q^*}{\partial \sigma} \) to move from being positive to being negative; a sufficiently large such subsidy guarantees this sign is negative. If \( \theta = 1 \), \( AC' > 0 \) and if the pass-through rate, \( \rho(t) \equiv \frac{dP(q^*)}{dt} > M > 0 \) for some \( M \) and all \( t \) such that \( q^* \in (0, 1) \) then \( \frac{\partial PS}{\partial \sigma} + \frac{\partial CS}{\partial \sigma} < 0 \) starting from a sufficiently large subsidy \(-t\) such that \( q^* < 1 \).

**Proof.** This follows exactly from the logic of the proof of Proposition (Formal) 4.

**Proposition (Formal) 6.** If \( \theta = 0 \) then \( \frac{\partial SS}{\partial \sigma} \) has the same signs as \( AC' \).

**Proof.** At \( \theta = 0 \) there is no producer surplus so only the impact on consumer surplus is relevant. Because \( P(q) = \sigma AC(q) + (1 - \sigma) AC(1), \frac{dP(q^*)}{dq^*} \) has the same sign as \( AC(q) - AC(1) \) (given that \( P' > AC' \) by our stability assumptions) which is opposite to that of \( AC' \). By the envelope theorem, \( \frac{\partial CS}{\partial \sigma} = -q \). Thus the impact of \( \sigma \) on consumer and thus social surplus has the same sign as \( AC' \).

For the following results, \( \sigma \) represents risk-adjustment rather than correlation. Again we use a tax or subsidy to shift the demand curve and measure welfare now with respect to the primitive.
demand and supply curves, including the tax/subsidy, excluding any impacts of the tax/subsidy on the government budget and ignoring the risk adjustment (as this is just a transfer) except through its impacts on equilibrium quantity. We let \( q^* \) denote the socially optimal quantity. In what follows we treat the “specific tax” \( t \) a simply a uniform inverse demand/cost shifter and thus irrelevant to welfare quantities.

**Proposition (Formal) 7.** Let \( q_0 = q^* \) when \( \sigma = 0 \) (full risk-adjustment). If \( \theta = 1 \), \( AC' < 0 \) and \( MS' > MC' \) then there exist thresholds \( q' < q \) that are invariant to the level of a specific tax \( t \) such that

1. If \( q_0 < q' \) then \( \frac{\partial q^*}{\partial \sigma} < 0 \) and there exists \( \sigma^* \in (0, 1) \) such that at \( \sigma^* \), \( q^* = q^{**} \).
2. If \( q_0 = q' \) then \( \frac{\partial q^*}{\partial \sigma} < 0 \) and \( q^* = q^{**} \) when \( \sigma = 0 \).
3. If \( q' < q_0 < q \) then \( \frac{\partial q^*}{\partial \sigma} < 0 \) and \( q^* < q^{**} \) even when \( \sigma = 0 \).
4. If \( q_0 = q \) then \( \frac{\partial q^*}{\partial \sigma} = 0 \) and \( q^* < q^{**} \).
5. If \( q_0 > q \) then \( \frac{\partial q^*}{\partial \sigma} > 0 \) and \( q^* < q^{**} \)

\( q' > 0 \) if \( \lim_{q \to 0} P'(q)q = 0 \). \( q_0 \) ranges between 0 and 1 as a sufficiently large tax or subsidy is imposed.

The additional discussion about the direction of welfare in the text follows from this result and the observation that under our stability assumptions welfare is strictly concave in quantity.

This proposition imposes two additional conditions not discussed previously: that \( MS' > AC' \) and that \( \lim_{q \to 0} P'(q)q = 0 \). Log-concavity of direct demand is sufficient, but not necessary, for the first condition assuming that \( MC' < 0 \), as it implies \( MS' > 0 \) (Weyl and Fabinger, 2013) and thus clearly \( > MC' \). We have typically assumed that when \( AC' < 0, MC' < 0 \) as well. The second condition is neither necessary nor sufficient for log-concavity but is true of every log-concave demand function we are aware of, as shown by Fabinger and Weyl (2012). It also implies that demand is log-concave at sufficiently high prices as, letting \( \bar{P} = \lim_{q \to 0} P(q) \) and \( Q \) be the direct demand,

\[
\lim_{q \to 0} P'(q)q = 0 \iff \lim_{p \to \bar{p}} \frac{Q(p)}{Q'(p)}.
\]

Bulow and Pfleiderer (1983) show that the sign of the derivative of \( \frac{Q}{Q'} \) positive if and only if \( Q \) is locally log-concave; clearly \( Q' < 0 \) so \( \lim_{p \to \bar{p}} \frac{Q(p)}{Q'(p)} \) only if in the limit this quantity is increasing (towards 0). Thus it is closely connected to log-concavity and \( \lim_{q \to 0} P'(q)q = 0 \) are closely-allied concepts and thus we view the gap between them as being a “regularity” condition, as quoted in the text.

If the first condition fails it is possible that there are other points of switching between regimes 1) and 3) from the proposition; this does not lead to qualitatively different behavior, but would be more complex to state and thus we omitted discussing it in the text. If the second condition fails, then, as discussed in the text, it is possible (though not necessary) that even at very low \( q_0 \) full risk-adjustment is still insufficient.

**Proof.** First note that risk-adjustment payments are pure transfers and thus social surplus is invariant to them except in how they impact quantity. Second, note that their impact on quantity is precisely as in Proposition (formal) 4 as the equilibrium equations are identical to there. This establishes the claims about \( \frac{\partial q^*}{\partial \sigma} \). Point 5) follows because quantity is always too low when \( AC' > 0 \) and becomes
only lower with risk-adjustment; it may never cross \( q \) because there \( \frac{\partial q}{\partial \sigma} = 0 \). Point 4) follows directly from this observation: if \( q^* = q \) if \( \sigma = 1 \), \( q^* \) is invariant to \( \sigma \). All of this is, as claimed, invariant to the value of \( t \) by the same logic in the proof of Proposition (Formal) 4.

On the other hand when \( q^* < q \) if \( \sigma = 1 \) then \( \frac{\partial q}{\partial \sigma} < 0 \) but by the same logic \( q^* < q \) for all \( \sigma \in [0, 1] \). By the logic in the proof of Proposition (Formal) 3, social welfare is concave in quantity and quantity is too low when \( \sigma = 1 \). Thus either social surplus monotonically increases as \( q \) falls or social surplus reaches a peak and then declines beyond some point if \( q \) becomes too low. Which occurs is determined by the sign of \( SS'(q_0) \) as, by concavity and monotonicity of \( q^* \) in \( \sigma \), \( SS'(q_0) < SS'(q_0) \) for all \( \sigma > 0 \).

\[
SS'(q_0) = P(q_0) - MC(q_0) = MS(q_0) + AC(1) - MC(q_0).
\]

Thus if \( MS(q_0) > MC(q_0) - AC(1) \) then \( q^* \leq q_0 < q^{**} \) while if \( MS(q_0) = MC(q_0) - AC(1) \) then \( q^* = q_0 = q^{**} \) and if \( MS(q_0) < MC(q_0) - AC(1) \) then there is an interior optimal \( \sigma \) as there was an interior optimal \( \theta^* \) in Proposition (Formal) 3 and as described in point 1). Note that this is all invariant to \( t \) as this has no impact on either \( MS \) or \( AC(1) - MC \) as it shifts the latter two in parallel.

By definition of \( q \), \( q_0 < q \) implies that \( MC(q_0) > AC(1) \). Thus if \( \lim_{q \to 0} MS(q) = 0 \) then for sufficiently small \( q_0 \) the second case holds. Conversely \( MS(q_0) > 0 \) for all \( q_0 > 0 \) and as \( q_0 \to q \), again by definition of \( q \), \( MC(q_0) \to AC(1) \) and thus the first case holds.

\( q' \) is then simply defined as the threshold between these regimes, which exists by the assumption that \( MS' > MC' \) and thus \( MS' - MC' + AC(1) \) has a single crossing of 0 (from below to above). The range claim on \( q_0 \) as a function of \( t \) follows from the fact that \( F' < 0 \).

\[
\square
\]

**Proposition (Formal) 8.** Let \( q_0 \) be defined as in Proposition (Formal) 7. If \( \theta = 1 \), \( AC' > 0 \) and \( MC' - MS' \) is signed globally, there exist thresholds \( q'' > q \) that are invariant to the level of a specific tax \( t \), with \( q \) being identical to its value in Proposition (Formal) 7, such that

1. If \( q_0 < q \) then \( \frac{\partial q}{\partial \sigma} > 0 \) and \( q^* < q^{**} \) even at \( \sigma = 1 \).
2. If \( q_0 = q \) then \( \frac{\partial q}{\partial \sigma} = 0 \) and \( q^* < q^{**} \) for all \( \sigma \in [0, 1] \).
3. If \( q < q_0 < q'' \) then \( \frac{\partial q}{\partial \sigma} < 0 \) and \( q^* < q^{**} \) even at \( \sigma = 0 \).
4. If \( q_0 = q'' \) then \( \frac{\partial q}{\partial \sigma} < 0 \) and \( q^* = q^{**} \) when \( \sigma = 0 \).
5. If \( q_0 > q'' \) then \( \frac{\partial q}{\partial \sigma} < 0 \) and there exists a \( \sigma^* \in (0, 1) \) such that at \( \sigma^* \), \( q^* = q^{**} \).

\( q'' < 1 \) if and only if \( MC(1) - AC(1) < MS(1) \) and, as in Proposition (Formal) 7, adjusting \( t \) traces out the full possible range of \( q_0 \).

The additional conditions in this result have less intuitive content than those in the previous proposition. Again \( MC' - MS' \) being signed is necessary to ensure a simple structure on the regions of potential outcomes. A sufficient condition for this is log-convexity of demand as in this case \( MC' > 0 > MS' \), assuming that \( MC' \) has the same sign as \( AC' \). But \( MS' > MC' > 0 \) would also satisfy the condition and would have demand being very log-concave.

The second condition has little intuitive content, but is only possible in the case when \( MC' > MS' \). It states that the downward distortion from market power is smaller than the upward distortion from advantageous selection (that would occur under perfect competition) when equilibrium quantity is sufficiently high.
Proof. The proof follows precisely the logic of Proposition (Formal) 7, *mutatis mutandis* for the differences between the adverse and advantageous cases.

**Proposition (Formal) 9.** If \( \theta = 0 \) then \( \frac{\partial q}{\partial \sigma} \) has the same sign as \( -AC' \). If \( q_0 \leq \hat{q} \) then there exists a \( \sigma^* \in [0,1) \) such that at \( \sigma^* \), \( q^* = q^{**} \). If \( q_0 > \hat{q} \) then \( SS'(q^*) \frac{\partial q}{\partial \sigma} > 0 \) and \( q^* - q^{**} \) has the sign of \( AC' \) for \( \sigma \in [0,1] \).

Proof. The first claim follows directly from the logic of Proposition (Formal) 6 and the fact that the equilibrium conditions with \( \theta = 0 \) are the same for a given \( \sigma \) under the two models as above.

The second claim comes from a logic similar to the preceding two propositions. Social surplus is still concave for the same reasons. At \( \sigma = 1 \) it is always declining in \( \sigma \) because for \( AC' > 0 \) quantity is too high and for \( AC' < 0 \) quantity is too low. It is thus sufficient to verify whether this sign is maintained or not at \( q_0 \). We just consider one of the four cases; the other three are analogous.

Suppose that \( q_0 < \hat{q} \) and that \( AC' < 0 \). Then by definition of \( \hat{q} \), \( MC(q_0) > AC(1) \).

\[
SS'(q_0) = P(q_0) - MC(q_0) = AC(1) - MC(q_0) < 0,
\]

reversing the sign compared to \( SS'(q^*) \) when \( \sigma = 1 \) and implying an interior optimum by the reasoning in the proof of the previous two propositions. \( \square \)
Figure A1: Advantageous Selection under Monopoly

Note: Figure shows the effect of reducing the degree of advantageous selection in a market served a monopoly provider. Panels (a) and (b) consider a setting where the equilibrium quantity is low and reducing advantageous selection raises price and lowers quantity. Panels (c) and (d) consider a setting where the equilibrium quantity is high and reducing advantageous selection lowers price and increases quantity.
**Figure A2**: Risk-Based Pricing: Segmented Market

(a) First Risk Quartile

(b) Second Risk Quartile

(c) Third Risk Quartile

(d) Fourth Risk Quartile

**Note**: Figures shows the effects of risk-based pricing achieved by segmenting the market by quartiles of the risk-type parameter $\lambda$. The first risk quartile corresponds to the set of consumers with the lowest expected costs and the fourth risk quartile corresponds to the consumers with the highest expected costs in the market.
Figure A3: Misperceptions: Imperfect Correlation Between Perceived and Actual Risk

(a) Fully Accurate Perceptions: $\rho_{\lambda,\hat{\lambda}} = 1$

(b) Partial Misperceptions: $\rho_{\lambda,\hat{\lambda}} = 0.5$

(c) Full Misperceptions: $\rho_{\lambda,\hat{\lambda}} = 0$

Note: Figures show the effects of consumer misperceptions about health risk, modeled by allowing consumers perceived health type $\hat{\lambda}$ and actual health type $\lambda$ to jointly log-normally distribution with correlation parameter $\rho_{\lambda,\hat{\lambda}}$. 