Price-Quoting Strategies of a Tier-Two Supplier

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This paper studies the price-quoting strategies used by a tier-two supplier, whose tier-one customers compete for an OEM’s indivisible contract. At most one of the tier-two supplier’s quotes will ultimately result in downstream contracting and hence produce revenue for her. We characterize the tier-two supplier’s optimal price-quoting strategies and show that she will use one of two possible types of strategies, with her choice depending on the tier-one suppliers’ profit potentials: secure, whereby she will always have business; or risky, whereby she may not have business. Addressing potential fairness concerns, we also study price-quoting strategies in which all tier-one suppliers receive equal quotes. Finally, we show that a tier-two supplier’s optimal mechanism resembles auctioning a single quote among the tier-one suppliers. This paper can assist tier-two suppliers in their pricing decisions, and provides general insights into multi-tier supply chains’ pricing dynamics.

1. Introduction

Business-to-business transactions are important: According to Kshetri and Dholakia (2002), the global value of goods and services traded among businesses is estimated to exceed US$60 trillion annually. This important area is addressed by a growing operations management literature on procurement, including work on reverse, or procurement, auctions, which have become commonplace in practice (Jap 2007). In such studies the auctioneer (often an OEM) and the bidders (the OEM’s immediate suppliers, also called tier-one suppliers) are modeled as strategic decision makers, but the suppliers’ costs are typically assumed to be exogenous. However, in reality these suppliers’ costs are often influenced by many factors, including their own internal production costs, as well as their costs of sourcing from upstream suppliers. Although existing models capture cases where these upstream suppliers sell commodities and have little pricing power, when an upstream supplier is powerful she too should be considered as a strategic decision maker whose pricing decisions affect the downstream suppliers’ costs.
and thus the outcome of the OEM’s auction. To take a first step at analyzing this important but under-studied issue, in this paper we focus on the strategic pricing decisions of a tier-two supplier whose downstream customers (tier-one suppliers) will compete in an OEM’s reverse auction.

The authors became interested in this problem after encountering this type of setting in practice. We observed a situation in which a Fortune 50 OEM, who was considering whether it would be cost-effective to construct additional office space, ran a reverse auction to solicit bids from general contractors. For a particular key element of the construction, these general contractors were relying on the same specialized subcontractor who had quoted them prices. Obviously, the subcontractor faced a strategic problem of how to quote prices to the general contractors, but it was not clear to us what strategy should be employed, nor was there any existing research addressing this issue that we are aware of.

Several factors make understanding the tier-two supplier’s price-quoting strategy an interesting and difficult problem, and to examine it we employ a stylized model capturing the following salient features of the underlying supply-chain situation:

First, we initially assume that all the tier-one suppliers depend on one single tier-two supplier for a critical part of their product/service. In our example above the tier-two supplier had specialization in a particular type of construction. More generally, it is not rare for many tier-one suppliers to rely on a tier-two supplier that specializes in a particular type of component or service — for example, The Economist (2009) points out that 90% of the micro-motors used to adjust the rear-view mirrors in cars are made by Mabuchi, and TEL makes 80% of the etchers used in making LCD panels. Our initial assumption that all tier-one customers depend on the same tier-two supplier is a simplification of this reality, for expositional purposes. In §6 we will relax this simplifying assumption and show that the main insights from the base model still hold true.

Second, the tier-one customers are in competition for an indivisible contract from the OEM, modeled as an open-descending reverse auction held by the OEM. Naturally, a tier-one supplier will only execute a quote (purchase from the tier-two supplier at the quote price) if he has an order from the OEM. This alongside the first feature leads to the following implication in the supply chain: When her customers are competing for the same contract, the tier-two supplier can give each customer a quote, but anticipates that only one customer can possibly win the OEM’s contract and subsequently purchase from her at the quoted price.
Third, the tier-two supplier’s quotes directly affect her customers’ costs. We model this as the tier-one supplier’s costs for fulfilling the OEM’s contract consisting of the cost of inputs from the tier-two supplier plus their internal cost of processing the inputs into final products. The tier-two supplier has an incentive to quote high prices to maximize revenue, but of course the OEM would walk away and not award the contract if the tier-one suppliers’ costs end up being exorbitant. We capture the latter reality with a walk-away price, or reserve price, in the OEM’s auction.

Fourth, firms typically closely guard their cost information to protect profits, so in reality the tier-two supplier does not perfectly know her customers’ true internal processing costs. Of course, if the tier-two supplier somehow knew the tier-one suppliers’ processing costs, she could pick out the most cost-efficient supplier and give this supplier a price quote that consumes as much of his profit margin as possible, leaving only minimal profit for him. In contrast, when faced with uncertainty about the tier-one suppliers’ costs, the tier-two supplier benefits from giving quotes to multiple tier-one customers because doing so affords her multiple shots at the OEM’s contract. We model this using the canonical asymmetric information setting from economics, whereby each tier-one supplier knows exactly his own internal processing cost, but the tier-two supplier only has an estimate.

These factors conspire to make the tier-two supplier’s quoting problem interesting and difficult. The tier-two supplier has an incentive to offer quotes to multiple tier-one customers, but only has imperfect information about how aggressive she should be and which quote (if any) will ultimately bring her revenue. Particularly, she faces a complicating situation due to downstream competition — if she decides to tip the scales in favor of one customer (offers him a low quote that is likely to give him an overall cost advantage), this would come at the expense of making the other customers — those with higher quotes that would deliver more revenue to the tier-two supplier — less likely to win the auction. These tensions lead to several questions. Does the tier-two supplier always want to provide equal quotes to ex ante identical tier-one suppliers? In general, what does the tier-two supplier’s quoting strategy look like? We address these questions in §4, where we find that the tier-two supplier would indeed offer non-identical quotes, particularly when the value of the underlying OEM contract is high.

There are two implications in the forgoing discussion. The first implication is that the supplier may offer her customers different (unequal) quotes. There is evidence that this is done in practice and does survive legal challenges. These legal challenges attempt to
link unequal quotes to price discrimination, which the *Robinson-Patman Act* (RPA) forbids. In a recent example of this, Michael Foods (a manufacturer of egg and potato products) offered quotes for ingredients to two food service providers, Sodexo and Feesers, who then bid against each other for a food service contract at a downstream institution. Michael Foods offered Sodexo a quote lower than the quote it offered Feesers, and consequently Feesers sued alleging an RPA violation. While successful in a district court, this claim was overturned by the Third Circuit court (*Feesers, Inc. v. Michael Foods, Inc.*, Jan. 7, 2010), who cited RPA’s “two purchaser” requirement: For RPA to apply, at least two sales must be alleged to different purchasers at different prices, meaning mere *offers to sell* are not sufficient. Citing a series of similar court decisions in several competitive bidding settings, Stoll and Goldfein (2007) note that courts generally find in favor of suppliers who offer their customers different price quotes where only the winning bidder actually purchases from the supplier, because the two purchaser requirement is not met. Nonetheless, they also point out that in at least one instance, a circuit court has ignored the two purchaser requirement and applied the RPA in a competitive bidding setting. The preponderance of evidence suggests that unequal quotes generally survive legal challenges, but Stoll and Goldfein (2007) point out that the Supreme Court has yet to definitively rule on whether RPA can be applied to the competitive bidding setting, and is not likely to do so for at least several years.

The legal challenges make it clear that tier-one customers may find unequal quotes unfair, as a customer may resent being forced to accept a higher quote if others are given lower quotes for the exact same good. Thus a tier-two supplier might wish to avoid unequal quotes solely to preserve customer goodwill. Addressing this possibility, in §5.1 we extend our analysis to consider the case where the tier-two supplier always provides equal quotes to its customers, and study how the identical quote compares to non-identical quotes. We find that the restriction to identical quotes generally hurts the tier-two supplier, unless its customers are ex ante symmetric and the underlying value of the OEM’s contract is relatively low.

The second implication is that, the tier-two supplier quotes prices depending on her estimates of the tier-one suppliers’ costs. If the tier-two supplier can somehow learn her customers’ private cost information, she could potentially resolve her concern of getting too greedy with her quotes and accidentally ending up empty-handed. This is because better information can reduce/eliminate uncertainty over her customers’ costs, enabling her to better target quotes without pushing the tier-one suppliers’ costs past the OEM’s walk-away reserve price. Furthermore, better information may allow the tier-two supplier to identify
and back only the most efficient tier-one customer. While there is a clear benefit to soliciting information, the tier-two supplier cannot naively ask her customers for their costs — each tier-one supplier would tend to claim to have high internal processing cost, hoping that the tier-two supplier would quote him a lower price. These tensions lead to several questions: How should the tier-two supplier best solicit cost information from her customers? Does soliciting the cost information really resolve the aforementioned concerns? We explore these issues in §5.2, where we show that the tier-two supplier’s optimal mechanism resembles auctioning off a single quote among the tier-one customers. The tier-two supplier has her customers compete for the opportunity to receive a single quote before they bid for the OEM’s business, and in so doing the tier-two supplier increases the chance her quote results in revenue (leads to business with the OEM).

Before introducing modeling details in §3, in the next section we briefly review related literature.

2. Literature Review

Our paper deals with procurement and sourcing. There is a long line of research on this topic. A major branch of procurement literature our paper fits into deals with competitive bidding. In these settings multiple suppliers compete in an auction for a contract from a buyer (see Elmaghraby (2000) for a survey on this topic; recent examples in the operations management literature include Chen et al. (2005), Chen et al. (2008), Li and Scheller-Wolf (2010), etc.). In this vast literature the bidding is virtually always analyzed at just a single supply-chain interface, namely the multiple suppliers seeking the contract and the buyer offering the contract are the only players considered. In our paper, we expand upon this scope by also considering the actions of a tier-two supplier who is situated one tier below the interface at which the auction for the contract occurs. To the best of our knowledge, the only other paper to consider competitive bidding within a multiple-tier supply chain is Lovejoy (2010). Lovejoy considers a supply chain-formation problem in a supply chain of multiple tiers, from a commodity market to a final buyer (similar to the OEM in our paper). In each tier there may be one or more potential suppliers, but only one will emerge as active. It is assumed that the supply cost of the commodity market and the final buyer’s purchase price are exogenous, and the costs of the suppliers at all tiers are publicly known. Lovejoy defines the “balanced principal solution” as a prediction of the supply chain formation and
profit distribution outcome, whereby each supplier bids a price to each buyer and vice versa, then the mutually preferred pair form a supply chain and share the margin. A key difference between Lovejoy (2010) and our paper is that the former assumes all supply chain members have complete and perfect cost information of all other parties, while imperfect information and the resulting information asymmetry and uncertainty play a pivotal role in our paper.

A major part of our model and results revolve around the possibility of the tier-two supplier quoting different prices to her customers. Thus we also want to compare our findings to the vast literature on price discrimination (Stole (2007) offers an excellent review). In this literature, price discrimination generally occurs either between imperfect substitute goods, or between separated markets/segments. When there is perfect competition (all customers have full access to perfect substitute goods), no price difference is sustainable. Therefore, the source of price discrimination in the price discrimination literature is imperfect competition. In our setting, however, price discrimination arises despite the fact that the tier-one suppliers’ offerings are perfect substitutes, and the OEM is free to choose any one supplier. This is because the source of our price discrimination is asymmetric information (tier-one suppliers are privileged with better information about their costs than the tier-two supplier). With asymmetric information, the tier-two supplier quotes different prices in order to manage the trade-off of her potential revenue versus the risk of not having an order. Thus, while similar in appearance to the traditional price discrimination which arises to take advantage of imperfect competition, the price discrimination in our setting arises for a totally different reason, namely to manage uncertainty caused by asymmetric information.

More generally, the main novelty of our paper is that it adopts the perspective of a tier-two supplier. While there have been many studies of buyers’ procurement auctions and suppliers’ bidding strategies therein, ours is the first to study the price-quoting decisions of a supplier whose customers will be competing with each other in a downstream auction.

3. Base Model

3.1 Supply Chain Structure

We model a three-layer supply chain. The top layer is an original equipment manufacturer (OEM) who wishes to auction off an indivisible contract for the provision of goods or services. We refer to the suppliers who compete in the OEM’s sourcing auction as tier-one suppliers. For expositional purposes we assume that there are two tier-one suppliers competing for the
OEM’s business, and denote them as $TO_1$ and $TO_2$ ($TO$ stands for tier-one). The tier-one suppliers do not supply the good or service entirely by themselves; they require inputs from an outside source before they can produce the good or supply the service requested by the OEM. We model this outside source as a tier-two supplier, $TT$ (standing for tier-two), who supplies $TO_1$ and $TO_2$ with a critical component or service. This stylized model captures a variety of situations, ranging from manufacturing (where $TT$ supplies a critical component) to food service (where $TT$ Michael Foods provides inputs to downstream $TOS$ Sodexo and Feesers). For consistency we will refer to $TT$ as providing a component used in production by $TOS$. The focus of our paper is on how $TT$ should price her component for customers $TO_1$ and $TO_2$. (A more general model with any number of $TOS$ and also participants in the OEM’s auction who do not depend on $TT$ for a component is examined in §6.)

3.2 OEM’s Auction

The OEM uses an open-descending auction with a reserve price. Such an auction is simple to describe to bidders and is widely used in practice (Jap 2007): The auction begins at the publicly announced reserve price $r$, and participants alternately bid the price down, until no one is willing to bid any lower. The last remaining bidder wins the contract and is paid the auction’s ending price. If no one is willing to match the starting price $r$, no contract is awarded. The reserve price sets a ceiling on the amount the OEM is willing to pay for the contract, capturing the OEM’s alternatives to contracting with a supplier. For example, when purchasing goods, the OEM would forgo the purchase if exorbitant procurement costs would make acquiring the goods unprofitable.

During the auction, tier-one suppliers $TO_1$, $TO_2$ compete on contract price. $TO_i$’s cost to fulfill the OEM’s contract is composed of two parts $x_i + y_i$, where $x_i$ is the cost of purchasing the component produced by $TT$, and $y_i$ is the cost of processing the component into the final product, shipping it to the OEM, etc. For simplicity we refer to $y_i$ as processing cost. We assume the $TOS$ are rational and seek to maximize their expected profits, as is standard in the auction-theoretic literature. If $TO_i$ wins the OEM’s auction at price $p$, his profit equals $p - x_i - y_i$; if he loses the auction his profit is zero. Note that in the OEM’s auction, each $TO_i$ finds it a dominant strategy to lower his bid until either he wins the auction, or the price drops below his total cost $x_i + y_i$. Therefore, $TO_1$ will win the contract if and only if $x_1 + y_1 < x_2 + y_2$ and $x_1 + y_1 < r$. Similarly, $TO_2$ will win the contract if and only if
$x_2 + y_2 < x_1 + y_1$ and $x_2 + y_2 < r$. When $x_i + y_i > r$, $i = 1, 2$, the OEM will not award the contract.

### 3.3 TT’s Problem

Like the TOs, TT is assumed to be a rational expected profit maximizer. The goal of our paper is to study TT’s price-quoting strategy that maximizes her expected profit. Let $s_i$ denote TT’s cost of supplying $TO_i$ with the component needed for the OEM’s contract, and normalize the components’ alternative value for TT to be zero. We allow $s_1 \neq s_2$ even though the components delivered to each $TO_i$ are identical, to address the possibly heterogeneous additional costs associated with each $TO_i$ such as shipping costs. $TO_i$’s processing cost $y_i$ is his private information, and is a realization of random variable $Y_i$ which has a commonly known, positive and finite pdf over a closed interval. Assume the $Y_i$’s are independently distributed. The sequence of events begins with each $TO_i$ soliciting from TT a quote $x_i$ for a supply of the component. $TO_i$ has the power to decide whether to execute the quote he receives, and obviously would only want to do so if he wins the OEM’s contract. With their respective quotes in hand, $TO_1$ and $TO_2$ then compete in the OEM’s auction. If $TO_i$ wins the auction, TT will incur $s_i$ to supply the component at the quoted price $x_i$; if neither $TO_i$ is able to meet reserve price $r$, no deliveries or payments are made.

In what follows, for convenience we define $TO_i$’s realized base margin to be $z_i = r - y_i$, which is distributed according to $Z_i = r - Y_i$. To ensure the typical auction-theoretic property that $Z_i$’s failure rate is increasing (IFR), we assume that the $Z_i$’s (or equivalently, $Y_i$’s) have log-concave probability densities.\(^1\) The realized base margin $z_i$ is the highest revenue that TT can possibly achieve by selling to $TO_i$.

Thus far we have not imposed any assumptions on how TT will deal with the TOs regarding supplying the component. A common and easily implemented approach is that TT simply quotes a fixed price to each $TO_i$. Of course, in doing so TT would strategically account for the distributions of the base margins $Z_i$, $i = 1, 2$. We refer to this approach as Quoting Prices (QP). Note that we did not rule out the possibility that TT provides different price quotes to its customers. As mentioned in §1, this is indeed done in practice. While the $TO_i$’s

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\(^1\)Corollary 3 in Bagnoli and Bergstrom (2005) ensures that assuming $Z_i$’s having log-concave probability densities leads to IFR of $Z_i$’s. Corollary 5 in the same paper shows that assuming $Z_i$’s having log-concave probability densities is equivalent to assuming so for $Y_i$’s. Finally, the same paper points out that many common distributions have log-concave probability densities, including uniform, normal, logistic, and exponential distributions.
may perceive dissimilar quotes as unfair, the preponderance of court decisions suggests little
if any legal ground for opposing such practices. Written formally, TT’s problem is

$$\max \ P(x_1, x_2),$$

where

$$P(x_1, x_2) = \sum_{i=1,2} (x_i - s_i) \Pr\{TO_i \text{ wins the contract}\},$$

$$= \sum_{i=1,2} (x_i - s_i) \Pr(x_i + Y_i < r, \ x_i + Y_i < x_j + Y_j, \ j \neq i),$$

$$= \sum_{i=1,2} (x_i - s_i) \Pr(Z_i > x_i, \ Z_i - x_i > Z_j - x_j, \ j \neq i).$$

We call a pair of quotes \((x_1, x_2)\) a quoting strategy and denote the pair as a whole by
\(X = (x_1, x_2)\). In particular, the optimal quoting strategy, namely the maximizing solution
to problem (1), is denoted by \(X^* = (x^*_1, x^*_2)\). It is clear that by a simple change of variables
argument\(^2\) we can transform the model into an equivalent one with \(s_i = 0\). Therefore,
without loss of generality, for concision we will hereafter assume \(s_i = 0\). In subsequent
sections we will study two other quoting approaches, but to avoid confusion we delay their
formalization until we are ready to discuss them in §5.

4. Quoting Prices

Quoting prices to two competing tier-one suppliers can lead to quite complex trade-offs.
For example, with two tier-one suppliers, TT can provide different prices to the two TOs.
Suppose \(Z_i\) is uniformly distributed over \([a_i, a_i + 2]\), and \(a_1\) and \(a_2\) take the value of either 2
or 9 in the four possible combinations in Figure 1. At points A and D where \((a_1, a_2) = (2, 9)\)
and \((9, 2)\), since \(TO_1\) and \(TO_2\) have very dissimilar base margin distributions, one would
expect that TT will quote different prices to them. However, given that the two TOs are
ex ante symmetric at points B and C where \((a_1, a_2) = (2, 2)\) and \((9, 9)\), would TT ever want
to quote different prices? The answer is not immediately clear. These questions will be
explored in this section.

4.1 Intuition

Before analytically examining TT’s optimal quotes, we would like to discuss the subtleties
involved using a simple example. Suppose two tier-one suppliers \(TO_1\) and \(TO_2\) are present

\(^2\) Define \(\tilde{x}_i \doteq x_i - s_i, \tilde{Y}_i \doteq Y_i + s_i, \tilde{y}_i \doteq y_i + s_i\). Consequently, \(\tilde{Z}_i = Z_i - s_i\) and \(\tilde{s}_i = 0\).
for quotes. Assume both $Z_i$’s have uniform distribution over $[2, 4]$. Since $TT$’s quotes can be different for $TO_1$ and $TO_2$, for illustrative purposes we fix the quote to $TO_2$ at $x_2 = 2.5$ and examine what happens as we increase $x_1$, the quote to $TO_1$. As seen in Figure 2, the trade-offs are quite complex. As $x_1$ increases, the dark-colored lines represent $TO_1$’s chance of winning the OEM’s contract and the payoff that $TT$ will receive if $TO_1$ wins, and light-colored lines represent the same for $TO_2$. Naturally, the payoff from $TO_1$, $x_1$, increases, and $TO_1$’s chance of winning decreases. However, there is more: As $x_1$ increases, even though $x_2$ is fixed, the chance that $TO_2$ wins increases because $TO_1$ becomes less competitive. $TT$’s profit is greatly affected by the competition amongst $TT$’s own customers. For example, suppose $TT$ quotes $x_1 = 3.6$ to $TO_1$. Although $x_1$ is much higher than $x_2$ so $TT$ prefers $TO_1$ to win the contract, in reality $TO_1$’s chance of winning is only about 7%, while $TO_2$’s chance of winning is much higher at over 70%. This results in an interesting effect for $TT$: Whichever $TO$ she prefers to win, because she quotes a high price to him, actually competes unfavorably against the other $TO$ for the very same reason! Such subtleties make it difficult to intuitively see when, if ever, and why, a high-low quote combination for ex ante symmetric $TO$s as in our example may be a good idea. To answer this question, we need to resort to
4.2 Analytical Results

Recall that $TT$’s problem is described by (1) and (2), and we assume $s_i = 0$ without loss of generality. First we provide a preliminary result related to the support of $Z_i$, denoted by $[a_i, a_i + h_i]$. Proof of this and subsequent results are provided in the Appendix.

**Proposition 1** *It is never optimal to quote $x_i$ out of the support of base margin $Z_i$, $i = 1, 2$.***

This is an intuitive result considering that $TT$ will not give up guaranteed profit potential by quoting below $a_i$, but neither would $TO_i$ execute a quote yielding him negative “profit” as would happen when $TT$ quotes above $a_i + h_i$. Because the optimal $x_i$ always stays in $[a_i, a_i + h_i]$, we only need to characterize $x_i$ in this interval. We next identify a special category of quoting strategies.

**Definition 1** *A quoting strategy $X_s = \{x_1^s, x_2^s\}$ is said to be secure if $x_i = a_i$ for some $i$. A quoting strategy $X_r = \{x_1^r, x_2^r\}$ is said to be risky if $x_i > a_i$, $i = 1, 2$.***
We call a strategy secure when with this strategy at least one TO$_i$ will certainly meet the reserve price $r$, and thus TT can secure her business. With a secure strategy, $\Pr\{\text{both TOs losing}\} = 0$. On the other hand, with a risky strategy we have $\Pr\{\text{both TOs losing}\} > 0$, namely there is positive probability that TT will not transact.

Intuitively, when using a secure strategy, TT only needs to secure the business with one TO$_i$. The next proposition formalizes this idea and characterizes the optimal secure strategy, namely the one that generates the highest expected profit for TT among all secure strategies.

**Proposition 2** An optimal secure strategy $X^{**}$ will have only one $x^{**}_i = a_i$. For the other $j \neq i$ it must be true that $x^{**}_j > a_j$. In addition, either $x^{**}_j > a_i$, or $x^{**}_j = a_j + h_j$.

The intuition behind Proposition 2 is straightforward. When quoting $x^{**}_i = a_i$ to TO$_i$, TT is already guaranteed payoff $a_i$ even before quoting to TO$_j$. Therefore, it would only make sense to provide a quote to TO$_j$ that can possibly generate higher payoff to TT, i.e., $x^{**}_j > a_i$. The only exception is when $a_i > a_j + h_j$ and no meaningful quote to TO$_j$ ($a_j \leq x_j < a_j + h_j$) can match $a_i$; in this case TT’s best strategy is to ensure TO$_j$ never meets the reserve price by quoting $x^{**}_j = a_j + h_j$ to him.

Developing upon this intuition, we show a special property of the optimal secure strategy. Assume $Z_i$ is replaced by $\hat{Z}_i = Z_i + a$ in (2), $i = 1, 2$ (i.e., shift both $Z_i$’s by $a$). We denote the optimal secure strategy for problem (1) by $X^{**} = \{x^{**}_1, x^{**}_2\}$, and denote the optimal secure strategy after shifting $Z_i$’s by $\hat{X}^{**} = \{\hat{x}^{**}_1, \hat{x}^{**}_2\}$.

**Proposition 3** For all $a$, $\hat{x}^{**}_i = x^{**}_i + a$, $i = 1, 2$. In other words, the optimal secure strategy remains fixed relative to the support of $Z_i$’s as both $Z_i$’s are shifted by the same amount.

According to Proposition 2, with a secure strategy where $x^{**}_i = a_i$, TT locks in a guaranteed payoff $a_i$, then gambles with TO$_j$ for additional profit, or eliminates TO$_j$ if he is too inefficient. When quoting for TO$_j$, TT only cares about balancing the additional profit $x^{**}_j - x^{**}_i$, and the chance of getting it. Neither is affected when both $Z_i$’s are shifted by the same amount. Thus the optimal secure strategy $X^{**}$ remains fixed relative to the support of $Z_i$’s, invariant in $a$.

A secure strategy guarantees TT at least a minimal payoff. The following theorem, which studies how TT’s quoting strategy changes as both TOs’ base margins increase, characterizes when TT finds it optimal to use a secure strategy, and when she instead finds it optimal to use a risky strategy.
Theorem 1 Replace $Z_i$ by $\hat{Z}_i = Z_i + a$ in (2) and consider the resulting optimal strategy. There exists a threshold $T_{\text{sec}} < \infty$ such that when $a < T_{\text{sec}}$ the optimal strategy is risky and when $a > T_{\text{sec}}$ the optimal strategy is secure.

This theorem establishes a threshold in $a$ for the transition of the optimal strategy between secure and risky. The result is important because it describes how the tier-two supplier changes her behavior as she faces tier-one suppliers with higher base margins (e.g., because the tier-one suppliers become more efficient and cut their costs, or because the OEM announces a higher reserve price). As TOs’ base margins increase, TT will eventually want to use a secure strategy. This makes intuitive sense: The business becomes more and more lucrative, and TT wants to make sure she at least obtains the business. On the other hand, if the base margins are fairly low, losing the business is not as damaging, and TT may want to take a risk and gamble with higher quotes to the TOs. Doing so may lead to loss of business, but can provide higher payoffs if business is won.

We continue to use the numerical example in Figure 1 to demonstrate this behavior (see Figure 3). Recall that $Z_i \sim U[a_i, a_i + 2]$. We plot TT’s strategy (risky or secure) as $a_1$ and $a_2$ take values from 0 to 10. Theorem 1 indicates that starting anywhere in Figure 3, moving northeast along a 45° line will eventually lead to using a secure strategy (and never risky again).

We are now ready to answer an important question asked at the beginning of §4 about Figure 1, namely how TT would quote when facing ex ante identical TOs. Theorem 1 finds that with ex ante identical TOs, at sufficiently high $a_i$ (e.g., point B $(a_1, a_2) = (9, 9)$), TT will use a secure quoting strategy, which (by Proposition 2) is always asymmetric. We want to point out that, although TT provides asymmetric quotes at point B $(9, 9)$ as well as points A $(2, 9)$ and C $(9, 2)$, the reasons are different. Quoting asymmetric prices at points A $(2, 9)$ and D $(9, 2)$ is expected as the TOs are ex ante asymmetric. At point B $(9, 9)$, although the TOs are ex ante identical, TT views them differently: She uses one to lock in the business, and gambles with the other. It is the different roles TT wants them to play that leads to asymmetric quotes to symmetric TOs.

The different roles TT has her customers play in a secure strategy lead to the following interesting dynamic: TT puts up with the fact that the TO with the lower quote is most likely to win in the OEM’s auction, leaving less chance for TT to get the higher quote. Despite this fact, Theorem 1 shows that TT will desire to treat her two customers differently.
Figure 3: Regions of Risky and Secure Strategies, where $Z_i \sim U[a_i, a_i + 2]$.

and induce this situation when the profit potentials are high enough and locking in a sizeable profit is paramount. Finally, we also want to point out that when using a secure strategy, $TT$ may still get the higher quote so there is usually still uncertainty in $TT$’s payoff.

On the other hand, when a secure strategy is not as attractive, we may expect that $TT$ would want to treat her two customers more equally. Indeed, the next theorem shows that when the two $TO$s are ex ante symmetric, if the profit potential is sufficiently low, the optimal strategy will be risky and symmetric, and thus provides both customers with equal quotes.

**Theorem 2** Assume $Z_1$ and $Z_2$ are i.i.d., replace $Z_i$ by $\hat{Z}_i = Z_i + a$ in (2), $i = 1, 2$, and consider the optimal quotes $(x^*_1, x^*_2)$. There exists some $T_{sym} \leq T_{sec}$ such that when $a < T_{sym}$ the optimal strategy is risky and symmetric.

With Theorem 2 we can finish answering the question raised earlier about Figure 1, namely how $TT$ would quote when facing ex ante identical $TO$s: With ex ante identical $TO$s, at sufficiently low $a_i$ (e.g., point C (2, 2)), $TT$ will quote equal prices. The reason is that, when the profit potential is sufficiently low, $TT$ does not care simply about securing
the minimal possible profit, but seeks to maximize her expected profit in light of the risk of getting nothing. This is achieved with a symmetric quoting strategy.

In conclusion, TT’s price-quoting strategy is greatly affected by the profit potentials of the TOs. When the profit potentials are high, TT gives one TO a low quote and the other TO a higher quote, using the TOS for different strategic purposes. This is the consequence of TT’s desire to lock in a sure-fire payoff, but this results in even ex ante symmetric customers being treated differently. On the other hand, when the tier-one profit potentials are low, TT uses a different approach that does not guarantee her a payoff, but takes the risk of not getting any business in exchange for potentially higher payoffs. Since neither TO is given a sure-fire low quote, there is no clear advantage granted to either customer. In such a case, TT treats her customers more equally as they serve similar strategic purposes for her. A numerical example of the optimal price quotes will be later provided in Figure 4.

5. Alternative Approaches

Thus far we have studied how TT should best provide quotes to her customers (the TOS). In doing so we allowed TT to provide different quotes to different TOS. However, there can be situations where the TOS would perceive different quotes as being unfair. Such a concern may prompt TT to restrict herself to only offering identical quotes to her customers (imposing the constraint $x_1 = x_2$). We refer to this approach as Quoting Equal Prices (QEP). Intuitively, this case is more likely to occur when TT does not have much supply chain power.

At the other extreme, if TT is unconcerned about fairness issues, and she has the power to set forth any rules of her choosing in supplying the component to the TOS, she could use an Optimal Mechanism (OM) to maximize her expected profit. As optimality rather than simplicity is the primary concern, an optimal mechanism may transcend quoting prices only based on priors about TOS’ costs, and involve more elaborate procedures such as soliciting cost information from the TOS. In this section we study these two alternative approaches.
5.1 Quoting Equal Prices

With the QEP approach, TT’s problem is written formally as

$$\max \ P(x)$$

where

$$P(x) = x \sum_{i=1,2} \Pr\{TO_i \text{ wins the contract}\},$$

$$= x \Pr(x < \max\{Z_1, Z_2\}).$$

The QEP approach is the same as QP except for the constraint to provide equal quotes to the TOs. Having learned that the optimal QP quotes can be different even to ex ante symmetric TOs, it is natural to ask in what way the constraint of using equal quotes affects TT’s strategies. We take a number of steps to answer this question.

First, as a natural extension of Proposition 1, we have the following preliminary result.

**Proposition 4** Suppose TO_i’s base margin Z_i has support [a_i, a_i + h_i]. Then it is never optimal to quote x out of [max{a_1, a_2}, max{a_1 + h_1, a_2 + h_2}].

Then we show the optimal quote is described by the following condition.

**Proposition 5** The optimal QEP quote $x^*$ must satisfy

$$x^* = \frac{1-F_1(x^*) F_2(x^*)}{f_1(x^*) F_2(x^*) + F_1(x^*) f_2(x^*)}.$$  \hspace{1cm} (5)

Furthermore, when $Z_1$ and $Z_2$ are i.i.d. with cdf $F$ and pdf $f$, the unique optimal QEP quote $x^*$ is determined by

$$x^* = \frac{1-F(x^*)^2}{2F(x^*) f(x^*)}.$$  \hspace{1cm} (6)

Proposition 5 allows us to characterize the behavior of the optimal QEP quote with symmetric TOs.

**Theorem 3** Assume $Z_1$ and $Z_2$ are i.i.d. with support [0, h], replace $Z_i$ by $\hat{Z}_i = Z_i + a$ in (4), $i = 1, 2$ and consider the optimal QEP quote $x^*$. As $a$ increases, $x^*$ must be greater than $a$, but $x^* - a$ will asymptotically converge to 0. In other words, facing symmetric TOs, the optimal QEP quote asymptotically converges to, but never reaches, a secure quote.

**Corollary 1** Assume $Z_1$ and $Z_2$ are i.i.d., replace $Z_i$ by $\hat{Z}_i = Z_i + a$ in (2) and (4), $i = 1, 2$, and consider the optimal QP quotes $(x_1^*, x_2^*)$ and the optimal QEP quote $x^*$. When $a$ is sufficiently small, we have $x^* = x_1^* = x_2^*$, and when $a$ is sufficiently large, we have $x_1^* < x^* < x_2^*$ (assume without loss of generality that $x_1^* \leq x_2^*$).
Figure 4: Optimal QP and QEP Strategies, with Symmetric Uniform Costs.

Comparing Theorem 3 to Theorem 1 reveals an important insight: While the optimal QP strategy may be risky or secure (Theorem 1), the optimal QEP strategy with ex ante symmetric TOs is always risky (Theorem 3). This distinction makes clear the significance of the capability to discriminate, as the usage of secure strategies depends on it. In addition, Corollary 1 establishes how the optimal QEP quote compares to the optimal QP quotes. As illustrated in Figure 4, which continues the numerical example of Figure 1 \((Z_i \sim U[a, a+2])\), when the profit potential is low \((a is small)\) the optimal QEP quote coincides with the equal optimal QP quotes, and when the profit potential is high \((a is large)\) the optimal QEP quote lies between the unequal optimal QP quotes. This addresses the question posed at the beginning of the subsection: With symmetric TOs, the constraint of using equal quotes has a negative impact on TT’s expected profit only when the TOs’ profit potentials are high.

As an aside, Figure 4 also illustrates several results from Section 4; in this example we have \(T_{sym} = T_{sec} \approx 6.4\) (Theorems 1 and 2), and \(x_1^*\) and \(x_2^*\) move in parallel as \(a\) increases once the optimal strategy becomes secure at \(a = T_{sec}\) (Proposition 3).
5.2 Optimal Mechanism

In §4 we established how TT should optimally quote prices to the TOs based on her priors about TOs’ processing costs. These quotes, however, could potentially lead to non-transaction (if she quotes above both TOs’ profit margins), or transaction with money left on the table (if she quotes too low). TT may mitigate such concerns if she could find ways of soliciting TOs’ private cost information. However, soliciting accurate information is not easy. For example, if TT simply asks the TOs “what is the highest quote you can accept (what is your base margin $z_i$)”, each TO will claim to have a very low base margin in the hopes of receiving a lower quote from TT. The asymmetry of information clearly puts TT at a disadvantage. On the other hand, although both TOs are potential customers of TT, eventually at most one TO will get the OEM’s contract. Therefore TT might counter against the TOs’ incentives to underreport their base margins by playing one TO against another when she determines their quotes.

There are of course infinitely many ways in which TT could solicit, and predicate her actions upon, information from the TOs. The challenge is to find the way that generates the greatest expected profit for TT. To tackle this challenge, we utilize optimal mechanism design theory. A generic mechanism can be described as a set of “rules” that operate on cost signals provided by the TOs. We will formalize this below, but before doing so we make an observation. Canonical mechanism design analysis, Myerson (1981), designs a seller’s optimal mechanism where the mechanism is comprised of allocation and payment rules, and after the seller receives cost signals the mechanism determines who receives the item and who pays how much. In our setting, however, TT does not have the power to allocate the OEM’s contract; the decision of which TO (if any) gets the contract is determined by the OEM’s auction. Instead of directly controlling the contract allocation, TT can indirectly affect the contract allocation via the quotes she provides the TOs. To capture this fact, in our setting we replace allocation rules with quoting rules, which alongside the (upfront) payment rules form our mechanism. The resulting mechanism subsumes a range of possibilities, from pure non-contingent upfront purchasing as in Myerson (1981) (positive upfront payments and zero quotes), to payments and purchasing purely contingent upon the OEM’s auction (zero upfront payments and positive quotes), and any mixture in-between.

Having described that our mechanism involves cost reports, upfront payments, and
quotes, we now formalize these concepts. TT announces a mechanism — a set of rules

\[ \{p_1(\hat{z}_1, \hat{z}_2), p_2(\hat{z}_1, \hat{z}_2), x_1(\hat{z}_1, \hat{z}_2), x_2(\hat{z}_1, \hat{z}_2)\} \]

that map the TOs reported base margins \( \hat{z}_i \) into upfront payments \( p_i \geq 0 \) and quotes \( x_i \geq 0 \) to TO, \( i = 1, 2 \). Each TO chooses whether to participate, and if so, reports his base margin as \( \hat{z}_i \) (not necessarily equal to his true base margin \( z_i \)). With the TOs’ reports in hand, TT then announces the payment \( p_i(\hat{z}_1, \hat{z}_2) \) that TO must pay TT upfront (before participating in the OEM’s auction), and quotes the price \( x_i(\hat{z}_1, \hat{z}_2) \) that TO must pay TT if he chooses to order the component from TT (of course, a rational TO would order only if he wins the OEM’s contract). In a sense, the upfront payment can be interpreted as a fee that a TO pays for the right to purchase the component if he wins the OEM’s contract. Thus, this setup is similar to the capacity reservation and execution type of supply contracts, which are prevalent in practice. The TOs’ strategic behaviors in the mechanism are predicted by the Bayesian-Nash equilibrium concept, and TT’s expected profit is derived from the TOs’ equilibrium strategies.

The goal of mechanism design is to find the mechanism which maximizes TT’s expected profit. By the revelation principle, it suffices to search for the optimal mechanism only among direct-revelation mechanisms, namely those in which each TO willingly reports his true base margin in equilibrium. Consequently, TT’s (simplified) mechanism design problem is

\[
\max_{p_1(\cdot,\cdot),p_2(\cdot,\cdot),x_1(\cdot,\cdot),x_2(\cdot,\cdot)} \sum_{i=1,2} [p_i(\hat{z}_1, \hat{z}_2) + x_i(\hat{z}_1, \hat{z}_2) \Pr\{\text{TO}_i \text{ wins the contract}\}] \tag{5}
\]

s.t. \( v_i(z_i, z_i) \geq 0, \tag{6} \)

\( v_i(z_i, \hat{z}_i) \geq v_i(z_i, z_i), \forall z_i, \hat{z}_i, \tag{7} \)

where

\( v_i(z_i, \hat{z}_i) = E_{Z_j}[z_i-x_i(\hat{z}_i, Z_j) - (Z_j-x_j(\hat{z}_i, Z_j))^+] \Pr\{Z_j-x_j(\hat{z}_i, Z_j) < z_i-x_i(\hat{z}_i, Z_j)\} - p_i(\hat{z}_i, Z_j)] \)

is TO’s expected profit under mechanism \( \{p_1(\hat{z}_1, \hat{z}_2), p_2(\hat{z}_1, \hat{z}_2), x_1(\hat{z}_1, \hat{z}_2), x_2(\hat{z}_1, \hat{z}_2)\} \), given that TO’s true base margin is \( z_i \), he reports base margin \( \hat{z}_i \), and TO, \( j \neq i \) reports truthfully \( \hat{z}_j = z_j \). In the formulation, (6) and (7) ensure, respectively, that TT’s mechanism is indeed individually rational (each TO, willingly participates in the mechanism) and incentive compatible (each TO finds truthfulness optimal). With the range of search significantly
reduced by the revelation principle, we can now characterize the optimal mechanism. Define the virtual base margin as \( \psi_i(z_i) = z_i - \frac{1-F_i(z_i)}{f_i(z_i)} \), where \( F_i \) and \( f_i \) are the cdf and pdf, respectively, of TO\(_i\)’s base margin \( Z_i = r - Y_i \).3 The following theorem describes the optimal mechanism.

**Theorem 4** An optimal mechanism \( \{p^*_i(z_1, z_2), p^*_2(z_1, z_2), x^*_i(z_1, z_2), x^*_2(z_1, z_2)\} \) is as follows:\(^4\) If \( \psi_i(z_i) > \max\{\psi_j(z_j), 0\} \), then \( p^*_i + x^*_i = \max\{\psi_i^{-1}(\psi_j(z_j)), \psi_i^{-1}(0)\} \), \( p^*_j = 0 \), and \( x^*_j > z_j \); if \( \max\{\psi_1(z_1), \psi_2(z_2)\} < 0 \) then \( p^*_i = 0 \) and \( x^*_i > z_i \), \( i = 1, 2 \).

Theorem 4 reveals three key aspects of the optimal mechanism. First, TT chooses to “back” at most one (or possibly neither) TO\(_i\) in the OEM’s auction, meaning she provides him with a quote low enough to guarantee that this TO\(_i\) can meet the OEM’s reserve price. Any TO that is not “backed” instead receives a very high quote that effectively prices them out of the running in the OEM’s auction. Note that how much TT can get a TO to pay her depends on how much payment this TO can expect to get from the OEM. Once the TOs reveal their true costs, TT can control which TO\(_i\) will win the OEM’s contract. Eliminating the other TO\(_j\) will maximize how much TO\(_i\) can get from the OEM, and in return, how much TT can get from TO\(_i\).

Second, the optimal mechanism can be implemented by pure quotes, wherein the upfront payments are always zero. Notice that after soliciting cost information, TT knows which TO is going to win the contract. Because the outcome of the OEM’s auction is no longer uncertain, contingent payments are interchangeable with upfront payments; in particular, the optimal mechanism can be achieved with quotes only, a favorable result considering the simplicity of using just quotes. However, we will need both upfront payments and price quotes to describe the optimal mechanism when we extend our model to more general settings in §6.

Our third observation is based on the fact that under the optimal mechanism, TT’s equilibrium realized profit equals \( \max\{\psi_1(z_1), \psi_2(z_2), 0\} \), where \( \psi_i(z_i) = z_i - \frac{1-F_i(z_i)}{f_i(z_i)} \) is TT’s profit if she “backs” TO\(_i\). Note that TT does not take all of TO\(_i\)’s base margin \( z_i \), but leaves him a profit of \( \frac{1-F_i(z_i)}{f_i(z_i)} \), which increases in \( z_i \). This setup is necessary to counter the TOs’ natural tendency to underreport their base margins. It explains our third observation: When

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3 Since by assumption \( Z_i \)’s probability density is log-concave, the results of Bagnoli and Bergstrom (2005) can be used to show that \( \psi_i(\cdot) \) must be an increasing function.

4 For readability we suppress the arguments \((\hat{z}_1, \hat{z}_2)\) when writing \( p^* \) and \( x^* \).
TT compares the TOs to determine which one to back, the comparison is done on virtual base margin rather than actual base margin. Of course, as is well known in the mechanism design literature, with ex ante symmetric tier-one suppliers, the one who reports the highest base margin will be chosen, while when the tier-one suppliers are ex ante asymmetric, this may not always be the case.

The distinction between the optimal mechanism and the QP approach is evident. With the QP approach, TT is willing to quote prices to both TOs, thus effectively taking two shots at success in the OEM’s auction. Doing so is optimal for TT because at the time she provides quotes she is uncertain about which TO will succeed. However, when TT can make her quotes contingent upon information reported by her customers, she uses this opportunity to identify which TO is more profitable to back exclusively in the OEM’s auction. This confirms our speculation made at the outset of this subsection that soliciting information could be superior to passively accommodating uncertainty.

Although Theorem 4 found the optimal mechanism, we have thus far been silent on how this mechanism might be implemented by TT. The optimal mechanism can actually be implemented as an auction. Instead of directly providing quotes to both TOs, TT has the TOs bid against each other to receive a quote. When the TOs are ex ante symmetric ($Z_1$ and $Z_2$ have identical distributions), the optimal mechanism can be implemented as the following open-ascending auction. Before the auction, TT sets the reserve price at $\psi_1^{-1}(0) = \psi_2^{-1}(0)$, and announces that only the TO willing to accept the highest price will receive a quote. Starting from the reserve price, the TOs take turns bidding higher prices they are willing to accept until one drops out, at which point the remaining TO is the auction winner and is provided a quote from TT equal to his final bid (the auction’s ending price). If no TO is willing to match the reserve price, TT will not provide any quotes. When the TOs are ex ante asymmetric, the optimal mechanism can be implemented similarly, but with one key change: TT uses a biasing rule when comparing bids to decide the auction winner, and when computing the winner’s quote price. This biasing rule intensifies competition by favoring the weaker TO (the one with a lower ex ante base margin). (For brevity we omit the details of running a biased auction; interested readers are referred to Duenyas et al. (2010) which describes running such auctions in detail.)

A key takeaway from this subsection is that the optimal mechanism resembles an auction for a quote. In practice, auctions for quotes are not commonplace; what is more common (as is seen in the court cases referenced in §1) is for the tier-two supplier to simply quote prices
to their customers. Thus, for now the analyses of Sections 4 and 5.1 appear to more closely resemble how tier-two suppliers act in practice. However, since auctions for firm contracts are becoming prevalent in supply chains, it is plausible that an auction for a quote could be deployed by a tier-two supplier, especially as we have shown it to be the optimal mechanism. Also, although a tier-two supplier using an auction to select an exclusive customer is seldom observed, it is not uncommon to see a supplier causes a bidding war among its customers who want to buy its whole business to obtain exclusive access to an important technology it possesses. While the scenario of buyers bidding for the whole business of the supplier is different from our setting, the motivation behind it is actually very similar, namely the supplier can achieve maximal value by making the buyers compete for exclusive access to the important technology.

6. Extensions

Thus far in this paper, to make the presentation simple, we assumed that only two TOs participate in the OEM’s auction, and both depend on TT for the crucial component. Of course, in many cases in industry there are more than two TOs competing in the OEM’s auction. Furthermore, it is possible that while some TOs rely on components from TT, others (e.g., a TO with its own internal component production capability) might not. This section examines such possibilities.

The base model we introduced in §3 has a three-tier structure: TT quotes prices for the component to TO1 and TO2, and TO1 and TO2 compete for the OEM’s contract. In this section’s more general model, the basic structure remains unchanged, but instead of two TOs, we assume there are n TOs (TOi, i = 1, ..., n) who request price quotes from TT before they compete for the OEM’s contract. In addition, we also assume there are m outside competitors CPj, j = 1, ..., m participating alongside the n TOs in the OEM’s auction, where the CPs do not require inputs of the component from TT. (When n = 2 and m = 0, the general model reduces to the base model.) Exactly as in Section 3, TOi’s cost to fulfill the OEM’s contract is xi + yi where xi is the cost of purchasing the component from TT, and yi is the cost of processing the component into the final product; yi is TOi’s private information but it is common knowledge that yi is a realization of random variable Yi. In contrast, CPj’s total cost cj includes its cost of the component and cost of processing it into the final product. cj is a realization of random variable Cj whose distribution is common
knowledge, and to ensure typical auction-theoretic properties (IFR), we assume that \( C_j \) has a positive, log-concave probability density; see Footnote 1. We assume all \( Y_i \) and \( C_j \) are independently distributed. Again, the OEM’s reserve price, announced prior to the auction, is denoted by \( r \). Note that under this setting, if a TO\(_i\) is to win the OEM’s contract, it means \( x_i + y_i < x_j + y_j, \forall j \neq i \) (TO\(_i\) has the lowest total cost among TOs), \( x_i + y_i < r \) (TO\(_i\) can meet the reserve price), and \( x_i + y_i < c_j, \forall j \) (TO\(_i\) has lower total cost than all CPs).

We find it convenient to define \( R = \min\{r, C_1, \ldots, C_m\} \). \( R \) represents a “sufficient statistic” capturing the OEM’s reserve price and the outside competitors that all TOs face in the auction. Namely, to win the OEM’s contract, TO\(_i\)’s total cost must be the lowest among all TOs and lower than \( R \). Thus, TO\(_i\)’s base margin in the general model is given by \( R - Y_i \), replacing \( r - Y_i \) in the base model. With TO\(_i\)’s base margin defined as \( Z_i = R - Y_i \) and with \( Z_i \)’s support denoted by \([a_i, a_i + h_i]\), we now extend our analytical approach for the base model. A critical, complicating factor of course is that \( R \) is a random variable, whereas \( r \) was a constant.

As before, we consider three different approaches: Quoting Prices (QP), Quoting Equal Prices (QEP), and Optimal Mechanism (OM).

### 6.1 Quoting Prices

TT’s problem is

\[
\max P(x_1, \ldots, x_n) \\
\text{where } P(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i \Pr\{\text{TO}_i \text{ wins the contract} \} \\
= \sum_{i=1}^{n} x_i \Pr(Z_i \geq x_i, Z_i - x_i > Z_j - x_j, j \neq i). \quad (8)
\]

Despite the increased complexity in TT’s problem, we nonetheless show that many key results from §4 remain.

**Theorem 5** Propositions 1, 2, 3 and Theorem 1 still hold true for the general model.

Therefore, our main insight holds true in the general model: TT uses two different types of strategies, risky or secure, and the TOs’ profit potentials determine which will be used — a risky strategy when the profit potentials are low, and a secure strategy when they are high.
6.2 Quoting Equal Prices

Now we impose the identical quotes restriction. Denote $TT$’s (identical) quote to all $TO_i$ by $x$. $TT$’s problem is

$$\max \ P(x)$$

where $P(x) = x \Pr\{\text{Some } TO_i \text{ wins the contract}\} = x \Pr(x < \max_{i=1,\ldots,n} \{Z_i\})$.

Section 5.1’s main results continue to hold:

**Theorem 6** Propositions 4 and 5, and Theorem 3, hold for the general model, where we make the following changes to the results’ statements to accommodate the general model: In Proposition 5 the condition $x^* = \frac{1-F_1(x^*)F_2(x^*)}{f_1(x^*)F_2(x^*)+F_1(x^*)f_2(x^*)}$ is replaced by $x^* = \frac{1-F(x^*)}{f(x^*)}$ where $F(x)$ and $f(x)$ are, respectively, the cdf and pdf of the random variable $\max_{i=1,\ldots,n} \{Z_i\}$; and in Proposition 5 and Theorem 3 the statement “$Z_1$ and $Z_2$ are i.i.d.” is replaced by “the $Y_i$s are i.i.d. and $R$ has a log-concave pdf”.

Therefore, when using the identical quotes approach, the insight that secure strategies are never optimal is preserved with the general model.

6.3 Optimal Mechanism

With the general model, a mechanism can be described with a set of rules

$$\{p_1(\hat{y}_1, \ldots, \hat{y}_n), \ldots, p_n(\hat{y}_1, \ldots, \hat{y}_n), x_1(\hat{y}_1, \ldots, \hat{y}_n), \ldots, x_n(\hat{y}_1, \ldots, \hat{y}_n)\}$$

where $p_i \geq 0$ is the transfer payment $TO_i$ must pay $TT$ upfront (before participating in the OEM’s auction), and $x_i \geq 0$ is $TT$’s price quote to $TO_i$, which he pays if and only if he places an order for the component. Both $p_i$ and $x_i$ are functions of all $TO_i$’s reported processing costs $\hat{y}_1, \ldots, \hat{y}_n$ (where $\hat{y}_i$ does not necessarily equal $TO_i$’s real cost $y_i$). Let the upper bound of $R$’s support be denoted by $u$, let $Y_i$’s cdf and pdf be denoted by $G_i$ and pdf $g_i$, respectively, and define virtual cost function $\omega_i(y) = y + \frac{G_i(y)}{g_i(y)}$.

5In §5.2, because $r$ is known there is no difference between asking $TO_i$ to report base margin $\hat{z}_i$ or cost $\hat{y}_i$. In the general model however, since the $TO_i$s do not know $R$ before the auction, it is only possible to ask them to report their costs as we do here.

6Since we assume $Y_i$ has log-concave probability density, due to Bagnoli and Bergstrom (2005) we know $\omega_i(\cdot)$ must be an increasing function.
Theorem 7  An optimal mechanism \( \{ p^*_1(\hat{y}_1, \ldots, \hat{y}_n), \ldots, p^*_n(\hat{y}_1, \ldots, \hat{y}_n), x^*_1(\hat{y}_1, \ldots, \hat{y}_n), \ldots, x^*_n(\hat{y}_1, \ldots, \hat{y}_n) \} \) is as follows\(^7\): If \( \omega_i(\hat{y}_i) \leq \min_{j \neq i} \{ \omega_j(\hat{y}_j), u \} \) then

\[
p^*_i = E_R[1_{\{\omega_i(\hat{y}_i) < R \}}(R - \omega_i(\hat{y}_i) + \hat{y}_i - \omega_i^{-1}(\min_{j \neq i} \{ \omega_j(\hat{y}_j), u \}))) \quad \text{and} \quad x^*_i = \frac{G_i(\hat{y}_i)}{g_i(\hat{y}_i)},
\]

and \( \forall j \neq i, \ p^*_j = 0 \) and \( x^*_j > u - \hat{y}_i \); if \( \min_i \{ \omega_i(\hat{y}_i) \} > u \), then \( p^*_i = 0 \) and \( x^*_i > u - \hat{y}_i \), \( \forall i \).

The optimal mechanism for the general model is a generalization of that for the base model. The main difference is that price quotes (which are paid only upon ordering) and upfront payments are required in the general model and cannot be interchanged, whereas they are interchangeable in the base model. What leads to this distinction is that in the base model the reserve price \( r \) is a constant, while its counterpart in the general model \( R \) is a random variable. With an uncertain \( R \), the outcome of the OEM’s auction is no longer fully predictable even when TT has all TOs’ cost information, and thus contingent and upfront payments are not interchangeable. Nonetheless, the optimal mechanism’s basic structure remains: TT backs only one TO, and the optimal mechanism can still be implemented via an auction-type approach in a similar spirit to that described in §5.2 (details are omitted for brevity).

7. Concluding Discussion

In this paper we consider a tier-two supplier’s optimal price-quoting strategy in the following model: The tier-two supplier gives price quotes to her customers, who are tier-one suppliers competing for an OEM’s contract. Because (i) the tier-two supplier’s price quote will only realize as revenue when her customer places an order, and (ii) at most one of the competing tier-one suppliers will get the contract and hence need to place an order, this situation has an interesting and complicating implication: Even if the tier-two supplier can give price quotes to multiple customers, eventually at most one quote can results in revenue, and this may or may not be the quote that the tier-two supplier would most like to fulfill, i.e., the most lucrative quote. To the best of our knowledge, despite the widespread existence of multi-tier supply chains, our paper is the first to identify and study the tier-two supplier’s price-quoting problem. By studying decision-making at the second tier of the supply chain, we complement the extant procurement literature that focuses primarily on transactions within a single supply-chain interface (e.g., a buyer and her immediate suppliers).

\(^7\)For readability we suppress the arguments \((\hat{y}_1, \ldots, \hat{y}_n)\) when writing \( p^* \) and \( x^* \).
In analyzing the problem, we identify two types of strategies deployed by the tier-two supplier: secure, where at least one of the tier-two supplier’s price quotes will be exercised (i.e., the customer will place an order); and risky, where there is positive probability that none of the quotes will be exercised. We find that the optimal strategy is risky when the tier-one suppliers’ profit potentials are low, and secure when the profit potentials are high. With a secure strategy, the tier-two supplier uses her customers for different strategic purposes (even when they are ex ante symmetric): She provides one customer a low quote to guarantee winning the business, and gambles with the others’ quotes to obtain possible high profits, but at lower probabilities.

In analyzing the tier-two supplier’s problem we allow her to strategically provide non-identical quotes to her customers. Non-identical quotes for the same item are seen in practice (cf. paragraph 9, §1), and our work has the potential to inform how to make the best price-quoting decisions and why. However, there is also evidence from practice that tier-one suppliers might perceive non-identical quotes as unfair. This motivates us to also study the tier-two supplier’s strategy when she constrains herself to provide equal quotes to her customers. We find that with this constraint, the tier-two supplier facing ex ante identical tier-one customers will never use a secure strategy, and that such a constraint negatively affects her expected profit only when her customers’ profit potentials are high.

Finally, while the results above are motivated by the existing practice of tier-two suppliers offering price quotes to their customers, it is of interest to ask what among all mechanisms would be theoretically optimal for the tier-two supplier. To answer this question, we employ mechanism design theory. The identified optimal mechanism has the following structure: The tier-two supplier solicits cost information from her customers, and with this information she strategically backs only one of the tier-one suppliers. This structure enables the optimal mechanism to be implemented by auctioning off a single quote among the tier-one suppliers. Although to our knowledge auctions for price quotes are not commonly deployed by tier-two suppliers, our analysis points to clear benefits of using an auction in such a scenario, and thus has the potential to inspire the use of such approaches in practice.

This paper should be of interest to tier-two suppliers seeking to make better pricing decisions. We also hope that the paper will spur further research into pricing decisions at various tiers of the supply chain, an important but under-studied area of procurement.
References


Appendix

Note. Although Propositions 1 through 4 and Theorem 1 are stated for two TOs with independent Z_i’s, their proofs here are provided for any number of TOs and without assuming Z_i’s are independent. This will facilitate proofs of Theorems 5 and 6.

Proof of Proposition 1. Recall that Z_i has positive pdf over [a_i, a_i + h_i]. We take the example of TO_1 and show that any quoting strategy (x_1, ..., x_n) with x_1 > a_1 + h_1 or x_1 < a_1 is weakly dominated by a quoting strategy with x_1 ∈ [a_1, a_1 + h_1].

First we show quoting strategy (x_1, ..., x_n) with x_1 > a_1 + h_1 is weakly dominated by (a_1 + h_1, x_2, ..., x_n). This is straightforward: The former strategy renders TO_1 unable to meet the OEM’s reserve price, and so does the latter. Therefore, they lead to the same level of expected profit for TT.

Next we show that, for any i, any strategy with x_i < a_i is weakly dominated by one where x_i ≥ a_i. Assuming without loss of generality a_i − x_i ≥ a_i − x_i, ∀i and a_i − x_i = t > 0, it suffices to show that quoting strategy (x_1, ..., x_n) is weakly dominated by (a_1, x_2 + t, ..., x_n + t), called the revised strategy.

Revised strategy (a_1, x_2 + t, ..., x_n + t) is generated by increasing each quote in (x_1, ..., x_n) by t = a_1 − x_1. Notice that doing so will not affect the TOs’ sample path (realizations of Z_i’s) cost comparison in the OEM’s auction; namely, the most cost-efficient TO will remain so after the change. On the other hand, notice that at least one TO can meet the OEM’s reserve price with both strategies (x_1, ..., x_n) and (a_1, x_2 + t, ..., x_n + t), meaning the most cost-efficient TO will always meet the reserve price. As a result, the revised strategy does not change the winning TO. However, because quotes to all TOs increased by t, whichever TO wins will, under the revised strategy, yield an additional profit of t for TT, meaning TT’s expected profit with (a_1, x_2 + t, ..., x_n + t) is higher than (x_1, ..., x_n) by t > 0.

Proof of Proposition 2. We begin by stating a simple fact useful in showing the proposition: TT’s profit is improved by increasing the lowest quote(s) of a secure strategy until either reaching the next lowest quote, or reaching the point beyond which the strategy would become risky. This is easy to see by a sample-path argument. When increasing the lowest quote(s), the sample paths where these TOs would win the OEM’s contract either remain so, or become ones where the other TOs would win, both of which mean higher profit for TT. (Note that the key assumption that the quoting strategy remains secure ensures these sample-paths do not become ones where no TO could meet the reserve price.)

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Without loss of generality, we consider a secure strategy \( X^s = (x_1^s, ..., x_n^s) \) where \( x_i^s = a_1 \geq a_i \) for all \( i \) such that \( x_i^s = a_i \). We take three steps to show the proposition. First, repeatedly using the above argument, we can improve TT’s expected profit by increasing any \( x_j^s < a_1 \) to \( a_1 \) (the strategy remains secure because we still have \( x_1^s = a_1 \)). Denote the resulting strategy by \( X'^{ss} = (x_1'^{ss}, ..., x_n'^{ss}) \). Now, \( x_1'^{ss} = a_1 \) is the lowest quote and is a secure quote. Without loss of generality, assume \( a_1 = x_1'^{ss} = ... = x_j'^{ss} < x_{j+1}'^{ss} \leq ... \leq x_n'^{ss} \). Second, simultaneously increasing \( x_i'^{ss} \), \( i = 2, ..., j \) to \( x_{j+1}'^{ss} \) improves TT’s expected profit. We use a sample-path argument to show this. When increasing \( x_i'^{ss} \), \( i = 2, ..., j \) to \( x_{j+1}'^{ss} \), the sample paths where these TOs would win the OEM’s contract become ones where TO1 wins, or one of TOk, \( k \geq 2 \) wins. The former sample paths do not affect TT’s profit, and the latter sample paths will improve TT’s profit. Denote the resulting secure strategy by \( X'^{ss} = (x_1'^{ss}, ..., x_n'^{ss}) \), where \( x_1'^{ss} = a_1 \), and \( x_i'^{ss} > \max\{a_1, a_i\}, i = 2, ..., n \). Finally, if \( a_i + h_i < a_1 \), then we can decrease \( x_i'^{ss} \) to \( a_i + h_i \) without reducing TT’s expected profit — this is part of Proposition 1’s proof.

As shown in the above three steps, any secure strategy \( X^s \) is dominated by another secure strategy that satisfies the conditions in Proposition 2. Therefore these conditions are necessary for optimality.

**Proof of Proposition 3.** For any secure strategy \( X^s \), define \( \hat{X}^s \) by \( \hat{x}_i^s = x_i^s + a, \forall i \). Since

\[
P(\hat{X}^s) = a + a_1 + \sum_{i=2}^{n} (\hat{x}_i^s - a - a_1) \Pr\{TO_i \text{ winning}\} = a + a_1 + \sum_{i=2}^{n} (x_i^s - a_1) \Pr\{TO_i \text{ winning}\},
\]

(A-1)

the optimal secure strategy \( \hat{X}^{ss} \) that maximizes \( P(\hat{X}^{ss}) \) is obviously invariant in \( a \) and is characterized by \( \hat{x}_i^{ss} = x_i^{ss} + a, \forall i \).

**Proof of Theorem 1.** First we prove a lemma that will be used in this proof.

**Lemma A-1** Fix a secure quoting strategy \( X^s \) and a risky quoting strategy \( X^r \). Replace \( Z_i \) by \( \tilde{Z}_i = Z_i + a \) in (2) and consider the secure strategy \( \tilde{X}^s = X^s + a = (x_1^s + a, ..., x_n^s + a) \) and the risky strategy \( \tilde{X}^r = X^r + a = (x_1^r + a, ..., x_n^r + a) \). The derivatives of TT’s expected profit \( P(\tilde{X}^s) \) and \( P(\tilde{X}^r) \) with respect to \( a \) are constants and satisfy

\[
\frac{d}{da} P(\tilde{X}^s) \equiv 1, \quad \frac{d}{da} P(\tilde{X}^r) \equiv 1 - \Pr\{\text{All TOs losing}\} < 1.
\]

**Proof of Lemma A-1.** First consider the secure strategy. Assume \( X^s \) has \( x_1^s = a_1 \). Since the expected profit can be written as in (A-1), it is immediately seen that \( \frac{d}{da} P(\tilde{X}^s) \equiv 1 \).
Next consider the risky strategy. Because

$$P(\hat{X}^r) = \sum_{i=1}^{n} (a + x_i^r) \Pr\{TO_i \text{ winning}\},$$

we know $\frac{d}{da} P(\hat{X}^r) \equiv 1 - \Pr\{\text{All TOs losing}\} < 1$.

Take the optimal secure strategy $X^{ss}$ and any risky strategy $X^r$. Replace $Z_i$ by $\hat{Z}_i \equiv Z_i + a$ in (2), and consider the optimal secure strategy $\hat{X}^{ss} \equiv X^{ss} + a$ and the risky strategy $\hat{X}^r \equiv X^r + a$. Due to Lemma A-1, $\frac{d}{da} P(\hat{X}^{ss}) - \frac{d}{da} P(\hat{X}^r) > 0$ is a constant. As a result, there exists a finite $a_X^r$ such that $P(\hat{X}^{ss}) > P(\hat{X}^r)$ when $a > a_X^r$. Define $T_{sec} = \sup\{a_X^r\}$ for all $X^r$. Then when $a > T_{sec}$, the optimal secure strategy $\hat{X}^{ss}$ generates higher expected profit for $TT$ than all risky strategies, thus is the optimal strategy.

Next we show $T_{sec}$ is finite. Denote $\hat{Z}_i$’s pdf by $\hat{f}_i$ and recall that $\hat{f}_i$ is positive and finite over $[a + a_i, a + a_i + h]$. Consider a family of strategies $\hat{X}^\delta \equiv (\hat{x}_1 = a + a_1, \hat{x}_2(a + a_1 + \delta)^*, ..., \hat{x}_n(a + a_1 + \delta)^*)$ where $\hat{x}_i(a + a_1 + \delta)^*$ is the optimal $\hat{x}_i$ given $\hat{x}_1 = a + a_1 + \delta$. We first show that there exists $T_1 < \infty, \varepsilon > 0$, such that $a > T_1$ and $0 < \delta < \varepsilon$ imply that $TT$ prefers using $\hat{X}^0$ to using $\hat{X}^\delta$, that is, $P(\hat{X}^0) \geq P(\hat{X}^\delta)$. Since by definition $P(\hat{X}^0) \geq P(a + a_1, \hat{x}_2(a + a_1 + \delta)^*, ..., \hat{x}_n(a + a_1 + \delta)^*)$, it suffices to show $P(a + a_1, \hat{x}_2(a + a_1 + \delta)^*, ..., \hat{x}_n(a + a_1 + \delta)^*) - P(\hat{X}^\delta) \geq 0$. Notice that $\hat{X}^\delta$ can be thought of as strategy $(\hat{x}_1 = a + a_1, \hat{x}_2(a + a_1 + \delta)^*, ..., \hat{x}_n(a + a_1 + \delta)^*)$ with the quote $\hat{x}_1 = a + a_1$ increased by $\delta$. For small $\delta$, increasing $\delta$ has three effects on $TT$’s profit. First, with some positive probability $p_0 > 0$ invariant in $a$, $TT$ may no longer have business because no $TO$ meets the reserve price, which leads to a loss of at least $a + a_1$. Second, $TT$ will receive $\delta$ additional profit when $TO_1$ wins the contract, whose probability is at most $1 - \delta \hat{f}_1(a + a_1) + o(\delta)$. Third, the chance that another $TO_i$ wins may increase by at most $\delta \hat{f}_1(a + a_1) + o(\delta)$, which brings in an increased profit of at most $\max_{2 \leq i \leq n} \{a + a_i + h_i - a - a_1\}$. Combining the three effects, we can upper bound the profit impact of increasing $\hat{x}_1$ by $\delta$:

$$P(a + a_1, \hat{x}_2(a + a_1 + \delta)^*, ..., \hat{x}_n(a + a_1 + \delta)^*) - P(\hat{X}^\delta) \geq (a + a_1)p_0 - \delta(1 - \delta \hat{f}_1(a + a_1))$$

$$- \max_{2 \leq i \leq n} \{a_i + h_i - a_1\} \delta \hat{f}_1(a + a_1) + o(\delta).$$

It is obvious that there exist $T_1 < \infty, \varepsilon > 0$ such that when $a > T_1$ and $0 < \delta < \varepsilon$, the right hand side is positive, and thus $TT$ prefers using $\hat{X}^0$ to using $\hat{X}^\delta$. In other words, for $a$ greater than a finite $T_1$, a secure strategy is preferred to all risky strategies that have $a + a_1 < \hat{x}_1 < a + a_1 + \varepsilon$. Repeating the proof for all $\hat{x}_i$, we know that for $a$ greater than a
finite $T_1$, the optimal strategy among all strategies that have $\hat{x}_i < a + a_i + \varepsilon$ for some $i$ must be secure.

Next we show that there exists a $T_2 < \infty$ such that for $a$ greater than $T_2$ the optimal strategy is secure among all strategies that have $\hat{x}_i > a + a_i + \varepsilon$ for all $i$. To show this, assume $\hat{X}^*$ is the optimal strategy among those that satisfy $\hat{x}_i > a + a_i + \varepsilon$ for all $i$. For all $a$, $TT$'s profit with $\hat{X}^*$ is upper-bounded by $(a + \max_i \{a_i + h_i\})(1 - \Pr\{\text{all TOs losing}|\hat{x}_i = a + a_i + \varepsilon\})$. In comparison, $TT$'s profit with any secure strategies is lower-bounded by $a + \min_i \{a_i\}$. Obviously, $\Pr\{\text{all TOs losing}|\hat{x}_i = a + a_i + \varepsilon\}$ is a positive constant invariant in $a$. Therefore, there exists some finite $T_2$ such that as $a > T_2$, $\hat{X}^*$ is dominated by a secure strategy.

Combining the above results, we know $T_{sec} \leq \max\{T_1, T_2\} < \infty$, namely $T_{sec}$ is finite. \hfill \blacksquare

**Proof of Theorem 2.** Denote the i.i.d. $Z_i$'s (common) cdf by $F$. Fix a quoting strategy $X = (x_1, x_2)$, and consider the strategy $\hat{X} = (\hat{x}_1, \hat{x}_2) = (x_1 + a, x_2 + a)$. $TT$'s expected profit under strategy $\hat{X}$ equals

$$P(\hat{x}_1, \hat{x}_2) = (a + x_1) \int_{x_1}^{\infty} F(z_1 - x_1 + x_2) dF(z_1) + (a + x_2) \int_{x_2}^{\infty} F(z_2 - x_2 + x_1) dF(z_2),$$

and

$$\frac{\partial P(\hat{x}_1, \hat{x}_2)}{\partial x_1} - \frac{\partial P(\hat{x}_1, \hat{x}_2)}{\partial x_2}$$

$$= 2(x_2 - x_1) \int_{0}^{\infty} f(x_1 + t)f(x_2 + t) dt + (a + x_2)f(x_2)F(x_1) - (a + x_1)f(x_1)F(x_2)$$

$$- \int_{0}^{\infty} (f(x_1 + t)F(x_2 + t) - F(x_1 + t)f(x_2 + t)) dt.$$

Furthermore,

$$\frac{d}{da} \left[ \frac{\partial P(\hat{x}_1, \hat{x}_2)}{\partial x_1} - \frac{\partial P(\hat{x}_1, \hat{x}_2)}{\partial x_2} \right] = f(x_2)F(x_1) - f(x_1)F(x_2). \quad (A-2)$$

Notice that $1/\sqrt{2}$ times (A-2) equals the constant rate of change of $P(\hat{x}_1, \hat{x}_2)$'s $(1, -1)$ directional derivative in $a$. Obviously, (A-2) is anti-symmetric in $x_1$ and $x_2$, meaning switching the indices will change its sign. Therefore, if we can show for $x_1 < x_2$ it is negative, then when $a$ is sufficiently small (possibly negative), $P(\hat{x}_1, \hat{x}_2)$'s $(1, -1)$ directional derivative will be positive when $x_1 < x_2$, and the $(-1, 1)$ directional derivative will be positive when $x_1 > x_2$, implying the optimal quotes must be symmetric. Below we show (A-2) is indeed negative for $x_1 < x_2$. Observe that

$$(A-2) < 0 \text{ for } x_1 < x_2 \iff f(x)/F(x) \text{ is decreasing in } x,$$
which is in turn implied by the assumption that $Y_i$ has log-concave pdf.

$T_{sym} \leq T_{sec}$ is implied by Proposition 2, which requires an optimal secure strategy to never be symmetric (implying that an optimal strategy that is symmetric must be risky and can only occur with $a \leq T_{sec}$).

**Proof of Proposition 4.** Quoting $x < \max_i \{a_i\}$ is strictly dominated by quoting $\max_i \{a_i\}$, because $\max_i \{a_i\}$ is a higher quote and yet guarantees at least one TO will meet reserve price $r$. Quoting $x > \max_i \{a_i + h_i\}$ is weakly dominated by quoting $\max_i \{a_i + h_i\}$, because both quotes guarantee $T_{O_i}$ cannot meet the reserve price.

**Proof of Proposition 5.** The necessary condition is a first-order condition. To show the condition is also sufficient when the $Z_i$'s are i.i.d., we only need to show $\max \{Z_1, Z_2\} = r - \min \{Y_1, Y_2\}$ is IFR, so that its failure rate function $\frac{2F(x)f(x)}{1-F(x)^2}$ is increasing. Since $Y_i$ is assumed to have a log-concave density, by the results of Huang and Ghosh (1982), $\min \{Y_1, Y_2\}$ also has a log-concave density; then by the results of Bagnoli and Bergstrom (2005), $\max \{Z_1, Z_2\} = r - \min \{Y_1, Y_2\}$ is IFR.

**Proof of Theorem 3.** Denote the cdf and pdf of $\hat{Z}_i$ by $\hat{F}$, $\hat{f}$, respectively. Notice that $\hat{Z}_i$ has support $[a, a+h]$. From Proposition 5 we know the optimal QEP quote $x^*$ is characterized by $x^* = 1 - \hat{F}^2(x^*)$. Since as $x \to a$, $\frac{1-\hat{F}^2(x)}{\hat{F}(x)f(x)} \to \infty > x$, we know $x^* > a$ must always be true.

To show the optimal quote converges to a secure quote $x^* = a$, we take any $\varepsilon > 0$ and consider a quote $x = a + \eta$ where $\eta > \varepsilon$. TT's expected profit from this quote is upper-bounded by $(a + h)(1 - \hat{F}^2(a + \varepsilon))$, which is obviously dominated by $a$, TT's profit from quoting $x = a$, when $a > h(1 - \hat{F}^2(a + \varepsilon))/\hat{F}^2(a + \varepsilon)$. Therefore, for any $\varepsilon > 0$, quoting $x > a + \varepsilon$ cannot be optimal when $a$ is sufficiently large, meaning $x^*$ asymptotically converges to a secure quote $x = a$.

**Proof of Corollary 1.** When $a$ is sufficiently small, we know the optimal QP quotes are symmetric (Theorem 2), which must also be the optimal QEP quote. When $a$ is sufficiently large, we know that $x^*_1 = a$ and $x^*_2 - a$ is invariant in $a$ (Theorem 1, Proposition 3), but $x^*$ asymptotically converges to $a$ (Theorem 3), so it must be true that $x^*_1 < x^* < x^*_2$.

**Proof of Theorem 4.** Obviously, the mechanism design problem in Theorem 4 is a special case of the problem in Theorem 7, with $R \equiv r$ and $n = 2$. Assuming Theorem 7 is true (the proof is provided later), we only need to show the mechanism of Theorem 7 reduces to the mechanism of Theorem 4.

Now assume $R \equiv r$ and $n = 2$. By definition we have $z_i + y_i = Z_i + Y_i \equiv r$, $i = 1, 2,$
and \( u \equiv r \). Considering the relationship between \( Z_i \) and \( Y_i \), it is not difficult to verify the following useful equation: \( \psi_i(z_i) + \omega_i(y_i) = r \). Theorem 7 states that the supplier \( i \) (if any) satisfying \( \omega_i(\tilde{y}_i) \leq \min_{j \neq i} \{ \omega_j(\tilde{y}_j), u \} \) is charged upfront payment \( p_i^* = E_R[1_{\{\omega_i(\tilde{y}_i) < R\}}(R - \omega_i(\tilde{y}_i) + \tilde{y}_i - \omega_i^{-1}(\min_{j \neq i} \{ \omega_j(\tilde{y}_j), u \}))] \) and quoted price \( x_i^* = \frac{G_i(\tilde{y}_i)}{g_i(\tilde{y}_i)} \). Notice that when \( R \equiv r \) and \( n = 2 \),

\[
\omega_i(\tilde{y}_i) \leq \min_{j \neq i} \{ \omega_j(\tilde{y}_j), u \} \Leftrightarrow \psi_i(\tilde{z}_i) \geq \max \{ \psi_j(\tilde{z}_j), 0 \},
\]

and for supplier \( i \) that satisfies the above condition,

\[
p_i^* = E_R[1_{\{\omega_i(\tilde{y}_i) < R\}}(R - \omega_i(\tilde{y}_i) + \tilde{y}_i - \omega_i^{-1}(\min_{j \neq i} \{ \omega_j(\tilde{y}_j), u \}))] = \psi_i^{-1}(\max \{ \psi_j(\tilde{z}_j), 0 \}) - \frac{1 - F_i(\tilde{z}_i)}{f_i(\tilde{z}_i)}
\]

\[
x_i^* = \frac{G_i(\tilde{y}_i)}{g_i(\tilde{y}_i)} = \omega_i(\tilde{y}_i) - \tilde{y}_i = r - \psi_i(\tilde{z}_i) - r + \tilde{z}_i = \frac{1 - F_i(\tilde{z}_i)}{f_i(\tilde{z}_i)}.
\]

Furthermore, since \( R \equiv r \) and \( TT \) backs only one \( TO_i \) in the auction, we know the OEM will pay \( r \) to \( TO_i \) for the contract and \( TO_i \) will transfer a total amount of \( p_i + x_i \) to \( TT \), implying \( r \geq p_i + x_i \). Alternatively, any upfront payment \( p'_i \) \( \geq 0 \) and price quote \( x'_i \) \( \geq 0 \) satisfying \( p'_i + x'_i = p_i + x_i \) would ensure that \( TO_i \) meets the reserve price (because \( r \geq p_i + x_i \)), and transfer the same amount to \( TT \). Therefore, we know that any \( p_i^* \geq 0 \) and \( x_i^* \geq 0 \) satisfying \( p_i^* + x_i^* = \max \{ \psi_i^{-1}(\psi_j(\tilde{z}_j)), \psi_i^{-1}(0) \} \) form an optimal mechanism. This concludes the proof.

**Proof of Theorem 5.** The proofs of Propositions 1 through Proposition 3 and Theorem 1 were provided for the generalized model (notice that none of the proofs require the \( Z_i \)'s to be independently distributed).

**Proof of Theorem 6.** The proof of Proposition 4 was provided for the generalized model.

To show Proposition 5 extends to the general model, we first need to show \( \max_i \{ Z_i \} = R - \min_i \{ Y_i \} \) is IFR. Since \( Y_i \) and \( R \) are assumed to have log-concave density, by the results of Huang and Ghosh (1982), \( \min_i \{ Y_i \} \) also has log-concave densities; then by the results of Bagnoli and Bergstrom (2005) and Theorem 3.2 in Barlow, Marshall and Proschan (1963), \( \max_i \{ Z_i \} = R - \min_i \{ Y_i \} \) is IFR.

To show the next step, we denote the cdf, pdf and support of \( R \) by \( W, w \) and \([R, \overline{R}]\), and those of \( Y = \min_i \{ Y_i \} \) by \( G, g, \) and \([Y, \overline{Y}]\), respectively. We need to show \( \max_i \{ Z_i \} = R - Y \) has zero probability density at left end point of its support: \( f(R - \overline{Y}) = 0 \) (so \( x^* = \frac{1 - F(x^*)}{f(x^*)} \) cannot hold at this point). By definition,

\[
F(z) = \int_{\overline{Y}} g(y)W(y + z)dy, \quad f(z) = F'(z) = \int_{\overline{Y}} g(y)w(y + z)dy.
\]
Therefore, \( f(R - \overline{Y}) = \int_{Y}^{\overline{Y}} g(y) w(y + R - \overline{Y}) dy \). Notice that \( \forall y \in [Y, \overline{Y}], y + R - \overline{Y} \leq R \), and consequently \( f(R - \overline{Y}) = 0 \). This concludes the extension of the proof.

**Proof of Theorem 7.** For concision we use subscript \( _i \) to denote subscript \( j, \forall j \neq i \). Suppose \( Z_i \) has support \([a_i, b_i]\). For \( TO_i \) with cost \( y_i \) and report \( \tilde{y}_i \), if we assume all other \( TO_j \)'s report their true costs \( \tilde{y}_j = y_j \), then \( TO_i \)'s expected profit is

\[
v_i(\tilde{y}_i, y_i) = E_{R,Y_{-i}}[(\min\{R, Y_{-i} + x_{-i}(\tilde{y}_i, Y_{-i})\} - y_i - x_i(\tilde{y}_i, Y_{-i})^+ - p_i(\tilde{y}_i, Y_{-i})],
\]

\[
\frac{\partial v_i}{\partial \tilde{y}_i} = -E_{R,Y_{-i}}[\mathbf{1}_{\{\min\{R,Y_{-i} + x_{-i}(\tilde{y}_i, Y_{-i})\} - y_i - x_i(\tilde{y}_i, Y_{-i}) > 0\}}].
\]

A direct-revelation mechanism must satisfy \( \max_{\tilde{y}_i} v_i(\tilde{y}_i, y_i) = v_i(y_i, y_i) \). By the envelope theorem:

\[
v_i(y_i, y_i) = v_i(b_i, b_i) + \int_{y_i}^{b_i} E_{R,Y_{-i}}[\mathbf{1}_{\{\min\{R,Y_{-i} + x_{-i}(z, Y_{-i})\} - z - x_i(z, Y_{-i}) > 0\}}] dz,
\]

and

\[
E_{Y_{-i}}[p_i(y_i, Y_{-i})] = E_{R,Y_{-i}}[(\min\{R, Y_{-i} + x_{-i}(y_i, Y_{-i})\} - y_i - x_i(y_i, Y_{-i})^+)] - v_i(y_i, y_i). \quad (A-3)
\]

So \( TT \)'s expected profit equals

\[
P = \sum_{i=1}^{n} E_Y E_{R,Y_{-i}}[p_i(Y_i, Y_{-i}) + x_i(Y_i, Y_{-i}) \mathbf{1}_{\{\min\{R,Y_{-i} + x_{-i}(Y_i, Y_{-i})\} - Y_i - x_i(Y_i, Y_{-i}) > 0\}}]
\]

\[
= \sum_{i=1}^{n} \left\{ \int_{a_i}^{b_i} g_i(z) E_{R,Y_{-i}}[(\min\{R, Y_{-i} + x_{-i}(z, Y_{-i})\} - z - x_i(z, Y_{-i})^+)]
\right.
\]

\[
- \int_{z}^{b_i} E_{R,Y_{-i}}[\mathbf{1}_{\{\min\{R,Y_{-i} + x_{-i}(w, Y_{-i})\} - w - x_i(w, Y_{-i}) > 0\}}] dw
\]

\[
+ E_{R,Y_{-i}}[(x_i(z, Y_{-i})) \mathbf{1}_{\{\min\{R,Y_{-i} + x_{-i}(z, Y_{-i})\} - z - x_i(z, Y_{-i}) > 0\}}] dz - v_i(b_i, b_i) \right\}.
\]

Notice that

\[
\int_{a_i}^{b_i} g_i(z) \int_{z}^{b_i} E_{R,Y_{-i}}[\mathbf{1}_{\{\min\{R,Y_{-i} + x_{-i}(w, Y_{-i})\} - w - x_i(w, Y_{-i}) > 0\}}] dw dz
\]

\[
= \int_{a_i}^{b_i} \int_{a_i}^{w} g_i(z) E_{R,Y_{-i}}[\mathbf{1}_{\{\min\{R,Y_{-i} + x_{-i}(w, Y_{-i})\} - w - x_i(w,Y_{-i}) > 0\}}] dz dw
\]

\[
= \int_{a_i}^{b_i} G_i(w) E_{R,Y_{-i}}[\mathbf{1}_{\{\min\{R,Y_{-i} + x_{-i}(w, Y_{-i})\} - w - x_i(w,Y_{-i}) > 0\}}] dw.
\]

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Plugging it back in, we have

\[
P = \sum_{i=1}^{n} \left\{ \int_{a_i}^{b_i} g_i(z) [E_{R,Y_i}[(\min\{ R, Y_i - x_i(z, Y_i) \} - z - x_i(z, Y_i))]^+ \\
- \frac{G_i(z)}{g_i(z)} 1_{\{ \min\{ R, Y_i - x_i(z, Y_i) \} - z - x_i(z, Y_i) > 0 \}} \\
+ x_i(z, Y_i) 1_{\{ \min\{ R, Y_i - x_i(z, Y_i) \} - z - x_i(z, Y_i) > 0 \}}] dz - v_i(b_i, b_i) \right\}
\]

\[
= \sum_{i=1}^{n} \left\{ \int_{a_i}^{b_i} g_i(z) \left[ E_{R,Y_i} \left[ 1_{\{ \min\{ R, Y_i + x_i(z, Y_i) \} - z - x_i(z, Y_i) \}} - \frac{G_i(z)}{g_i(z)} + x_i(z, Y_i) \right] \right] dz - v_i(b_i, b_i) \right\}
\]

\[
= E_{R,Y} \left[ \sum_{i=1}^{n} \left\{ 1_{\{ \min\{ R, Y_i + x_i(Y_i, Y_i) \} - Y_i - x_i(Y_i, Y_i) > 0 \}} - Y_i - \frac{G_i(Y_i)}{g_i(Y_i)} - v_i(b_i, b_i) \right\} \right].
\]

To maximize \( P \), we set \( v_i(b_i, b_i) = 0 \). Observe that \( \min\{ R, y_i - x_i(y_i, y_i) \} - y_i - x_i(y_i, y_i) > 0 \) can be true for at most one \( i \). Thus, if \( \min\{ R, y_i - x_i(y_i, y_i) \} - y_i - \frac{G_i(y_i)}{g_i(y_i)} \) takes value \( (R - y_i - \frac{G_i(y_i)}{g_i(y_i)})^+ \) whenever \( y_i - \frac{G_i(y_i)}{g_i(y_i)} < y_j - \frac{G_i(y_j)}{g_i(y_j)}, \forall j \neq i \), then the sum must be maximized. This is achieved by setting \( x_i(y_i, y_i) = \frac{G_i(y_i)}{g_i(y_i)} \) for the TO\( i \) with the smallest \( \omega_i(y_i) \) if \( \omega_i(y_i) < u \) (otherwise \( x_i(y_i, y_i) > u - y_i \), and \( x_i(y_i, y_i) > u - y_i \) (eliminate all other TOs with forbiddingly high quotes). This is the optimal quoting rule. With the optimal quotes, TT’s expected profit can be written as

\[
P = E_{R,Y}[(R - \min\{ \omega_i(Y_i) \} )^+].
\]

Now consider the payment rule. The envelope theorem implies (A-3). With the optimal quotes, (A-3) can be rewritten as

\[
E_{Y_i}[p_i(y_i, Y_i)] = E_{R,Y_i} \left[ 1_{\{ \omega_i(y_i) < \min\{ R, \omega_i(Y_i) \} \}} (R - \omega_i(y_i)) - \int_{y_i}^{b_i} 1_{\{ \omega_i(z) < \min\{ R, \omega_i(Y_i) \} \}} dz \right].
\]

(A-4)

Define the following payment rule:

\[
p_i(y_i, y_{-i}) = E_R \left[ 1_{\{ \omega_i(y_i) < \min\{ R, \omega_i(y_{-i}) \} \}} (R - \omega_i(y_i)) - \int_{y_i}^{b_i} 1_{\{ \omega_i(z) < \min\{ R, \omega_i(y_{-i}) \} \}} dz \right].
\]

Obviously, this payment rule ensures that (A-4) always holds. Assuming \( \omega_i(y_i) < \omega_j(y_j), \forall j \neq i \)
$i$, this payment rule is equivalent to

$$p_j(y_i, y_{-i}) = 0, \ j \neq i,$$

$$p_i(y_i, y_{-i}) = E_R[(R - \omega_i(y_i))^+ - \int_{y_i}^{\omega_i^{-1}(\min\{\omega_{-i}(y_{-i})\})} \Pr(R > \omega_i(z))dz]$$

$$= E_R[1_{(\omega_i(y_i) < R)}(R - \omega_i(y_i) + y_i - \omega_i^{-1}(\min\{\omega_{-i}(y_{-i})\}))].$$

This is the optimal payment rule, and coupled with the optimal quote rule we have an optimal mechanism.

Finally, we must confirm incentive compatibility and individual rationality of the above mechanism. Under this mechanism, $TO_i$’s expected profit, reporting cost $\tilde{y}_i$, equals

$$v_i(\tilde{y}_i, y_i) = E_{R,Y_i}[1_{\{\omega_i(\tilde{y}_i) < \min\{R, \omega_{-i}(Y_{-i})\}\}}[(R - \omega_i(\tilde{y}_i) + \tilde{y}_i - y_i)^+$$

$$- (R - \omega_i(\tilde{y}_i) + \tilde{y}_i - \omega_i^{-1}(\min\{\omega_{-i}(Y_{-i})\}))].$$

When $\tilde{y}_i = y_i$, obviously $v_i(\tilde{y}_i, y_i) \geq 0$, so the participation constraint is satisfied.

When $\tilde{y}_i > y_i$, on the sample paths of $\{\omega_i(\tilde{y}_i) < R\}$, we must have $R - \omega_i(\tilde{y}_i) + \tilde{y}_i - y_i > 0$, and

$$(R - \omega_i(\tilde{y}_i) + \tilde{y}_i - y_i)^+ - (R - \omega_i(\tilde{y}_i) + \tilde{y}_i - \omega_i^{-1}(\min\{\omega_{-i}(Y_{-i})\}))$$

$$= \omega_i^{-1}(\min\{\omega_{-i}(Y_{-i})\}) - y_i > 0$$

does not depend on $\tilde{y}_i$. However, by reporting higher than true cost $\tilde{y}_i > y_i$, $TO_i$ receives this positive profit on fewer sample paths. Thus $TO_i$ prefers reporting truthfully, $\tilde{y}_i = y_i$.

When $\tilde{y}_i < y_i$, on the sample paths of $\{\omega_i(\tilde{y}_i) < \omega_i(y_i) < \min\{\omega_{-i}(Y_{-i})\}\}$, since $R - \omega_i(\tilde{y}_i) + \tilde{y}_i - y_i < R - \omega_i(y_i)$, $TO_i$’s profit is no higher than reporting truthfully. On the sample paths of $\{\omega_i(\tilde{y}_i) < \min\{\omega_{-i}(Y_{-i})\} \omega_i(y_i)\}$, when $R - \omega_i(\tilde{y}_i) + \tilde{y}_i - y_i > 0$, the profit is $\omega_i^{-1}(\min\{\omega_{-i}(Y_{-i})\}) - y_i < 0$; when $R - \omega_i(\tilde{y}_i) + \tilde{y}_i - y_i < 0$, the expected profit is $-E_R[R - \omega_i(\tilde{y}_i) + \tilde{y}_i - \omega_i^{-1}(\min\{\omega_{-i}(Y_{-i})\})] = -\int_{\tilde{y}_i}^{\omega_i^{-1}(\min\{\omega_{-i}(Y_{-i})\})} \Pr(R > \omega_i(z))dz < 0$. In both cases, these sample paths would have reduced $TO_i$’s expected profit. Thus $TO_i$ again prefers reporting truthfully, $\tilde{y}_i = y_i$.

The above analysis shows that $v_i(\tilde{y}_i, y_i) \leq v_i(y_i, y_i)$ is always true, so the incentive constraints are satisfied. This concludes the proof.