Forecast Information Sharing and Tacit Collusion in Supply Chains

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Abstract

I study the ability of competing retailers to form a cartel by sharing information with their mutual manufacturer. In a market characterized by demand uncertainty, colluding retailers wish to share information about the potential market demand in order to coordinate on the optimal collusive retail price. Since direct information sharing between the retailers is viewed by antitrust authorities as a facilitating behavior and a possible signal for collusion, the retailers search for a mechanism to exchange information in an indirect manner. This paper offers such a mechanism: each retailer shares his private information with the mutual manufacturer and uses the wholesale price to infer the market condition and coordinate on the cartel price. Although a cartel at the retail level limits the manufacturer’s sold quantity, there are cases in which the manufacturer is better off receiving the retailers’ private information and thus facilitating the cartel formation.
People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices (The Wealth of Nations, Book I, Chapter X).

1 Introduction

Tacit collusion among rival firms is defined as implicit or unstated cooperation to make mutually beneficial pricing or production decisions (e.g. Tirole 1988, Carlton and Perloff 1999). In a collusion agreement, firms seek to establish and maintain a price which exceeds the price that would prevail in the absence of the collusive agreement. Collusion is considered by policy makers unwelcome, whether the goal is to maximize overall efficiency or consumer welfare. In order to fight collusion the Sherman Act was passed in 1890 in the U.S; The purpose of the act is to oppose the combination of entities that could potentially harm competition, such as monopolies or cartels. The European Community banns cartel by article 81.

Although economic theory has made progress in understanding the conditions that facilitate collusion, research did not provide clear rules which are verifiable in a court. Some economists (such as Kühn and Vives 1995, Kühn 2001 and Motta 2004) even claim that inferring collusion from market data is virtually impossible, as the relevant market data is not available for anti trust authorities. The difficulty of inferring collusion from market data can be learned from the opposed conclusions reached by Porter (1983) and Ellison (1994) about the U.S. railroad cartel in the 1880's. In another example, the European commission (1985) charged wood pulp producers with violation of Article 81(1) for colluding and exchanging price information. The European Court of Justice (1993) rejected the claim that parallel pricing is sufficient to infer collusion since there were alternative non-collusive explanations which were consistent with the data. Due to this difficulty of proving collusive behavior, competition authorities around the world have adopted the so-called parallelism plus rule. This policy allows prosecution of collusive behavior in cases where well-founded suspicion can be supported by hard evidence of facilitating practices. An example of such a facilitating practice is information sharing between competing firms.

For collusion to be sustainable, firms must coordinate and agree on what cooperative outcome above the competitive level to implement. Communication between firms can help firms solve their coordination problem. Due to the importance of information sharing in achieving collusion, Kühn (2001) suggested that although communication can increase welfare in static settings, due to the risk of potential welfare loss as a result of collusion, communication between rival firms about demand information "... can be considered
a restriction of competition if the commission can demonstrate that no significant efficiency gains can be expected from such a scheme. Other economists, such as Athey and Bagwell (2001), Athey, Bagwell and Sanchirico (2004) and Gerlach (2009) studied what are the implications of restricting communication between competing firms and to what extent firms can collude under settings of private information and no communication.

Since competing firms are aware of the fact that information sharing is perceived as a facilitating practice, they search for alternative mechanisms to share information and collude. Lee and Whang (2000) were the first to suggest that information sharing in supply chains can be subject to antitrust regulations. As they write:

"Suppose that two retailers regularly share with the supplier their demand projection for the next ten weeks. The projection by one retailer may implicitly signal the plan of a sales/promotion campaign in some future week. When this information is relayed to the other retailer through the supplier, it may be potentially used as a price fixing instrument between the two retailers. For example, the two retailers may take turns lowering the price by the use of forecast signals and avoid cut-throat price competition. This practice may be a subject of scrutiny by the antitrust authorities."

This paper rigorously model the idea of Lee and Whang (2000) that information sharing between a retailer and his manufacturer can be used as a signaling device to coordinate price fixing among competing retailers. In order to do so, I study a supply chain composed from \( n \) price setting retailers who source their product from a single manufacturer, who sets the wholesale price in a strategic manner. The market demand during each period is assumed to be fluctuating (in a similar manner to the seminal model of Rotemberg and Saloner 1986) and each retailer can observe a signal about the current market status. If the retailers are allowed to communicate, the efficient outcome, from the retailers perspective, is achieved by aggregating their private information and setting a monopoly price based on their cumulative knowledge. When the retailers cannot share information, due to antitrust issues, I study two alternative mechanisms that allow the retailers to collude. In the first, aligned with the traditional economics research about collusion with private information, each retailer sets a price based solely on his observed private information. I analyze the ability of the retailers to collude and show that a few obstacles arise when the retailers are unable to
communicate: since the retailers cannot coordinate the price, some retailers infer that demand is high while others think demand is actually low. This variability in the belief system of the retailers gives rise to a moral hazard problem - a retailer may prefer to set a low price even when observing that market demand is high. By setting a low price, a retailer secures some market share for low margins, which may be better than incurring the risk of setting a high retail price and losing all market share.

As an alternative mechanism for the retailers to collude, I study a scenario in which all the retailers share their private information with the manufacturer. Upon receiving the retailers’ information, the manufacturer sets her wholesale price. I demonstrate how the wholesale price aggregates the information the retailers need in order to coordinate on a monopoly outcome. The wholesale price solves the problem of coordination between the retailers, and allows the retailers to avoid the problems of price cutting and price wars which arise when retailers set prices based only on their own private information.

There exists a large body of research in operations management which analyzes the incentives of firms to share private demand information in supply chains. A few recent papers (Li 2002, Li and Zhang 2007, Anand and Goyal 2009) have demonstrated that in an environment of competing retailers and a mutual manufacturer, the retailers may be reluctant to share their private information with the manufacturer due to the risk that this information be leaked to their rivals. In this model, in which the retailers are looking for ways to share their private information, it is the leakage phenomena that provides the retailers an incentive to share information with the manufacturer. The retailers decide to share information with the manufacturer in anticipation that this information be later leaked to their competitors through the wholesale price. To the best of my knowledge, this is the first time a model of collusion is studied, in which the colluding firms are using their mutual manufacturer to share private information and coordinate the market price.

The rest of the paper is organized as follows. Section 2 discusses some of the relevant literature. Section 3 introduces the model, and section 4 analyzes the scope of collusion when retailers can exchange information. Section 5 studies the ability of the retailers to collude when they cannot exchange information at all, and section 6 studies the scope of collusion when the retailers exchange information with the manufacturer. Section 7 provides a comparison of the profit of the different firms in the supply chain under the different settings of collusion. I finally conclude and offer some future research directions in section 8.
2 Literature Review

This paper contributes to two streams of research: the first one studies the role of communication in cartel formation and the second studies the incentives for information sharing in decentralized supply chains.

Athey and Bagwell (2001) and Athey, Bagwell and Sanchirico (2004) studied in a series of papers the scope of collusion when firms have asymmetric cost information. They showed that if the cartel strategies have to be self-enforcing and firms are sufficiently impatient, they may not be able to sustain collusion without communication on costs. Cost communication would therefore be a facilitating device for collusion. Gerlach (2009) showed how communication can improve coordination by avoiding undercutting prices by poorly informed firms and hence allowing the firms to avoid costly price wars and adjust the prices better.

In a similar setting, Hanazono and Yang (2007) studied the possibility of competing firms to establish a cartel when they face fluctuating demand and each firm observes a signal about the market condition. Hanazano and Yang show that absent of communication the cartel must distort its price to eliminate incentives to deviate from the cartel scheme. At the extreme case, the cartel prefers to set a rigid price and ignore the private information. This work is different from the existing economic research by extending the environment in which the colluding retailers operate. The research cited above assumes that if the retailers cannot share information in a horizontal manner, they must collude using sophisticated signaling games. In contrast to this view, I assume that the retailers source their product from a mutual manufacturer and pose the question whether the retailers can collude using the manufacturer’s wholesale price as a coordination device.

A second effect of information sharing is related to the cartel’s monitoring problem. Since information exchange can help the colluding firms detect deviations from the collusive scheme, reducing demand uncertainty enlarges the scope of attainable collusive outcomes by increasing the efficiency of monitoring as analyzed by Green and Porter (1984) and Abreu, Pearce and Stachetti (1986); as the latter write: “The lack of transparency on prices and sales does not necessarily prevents collusion completely, but makes it both more difficult to sustain and more limited in scope”. In this work I show that sharing information with the manufacturer can also reduce the uncertainty in the market and thus increase the scope of collusion.

The second relevant stream of research to my work is that of information leakage in supply chains. Information leakage describes how information reaches unintended recipients in a competitive environment. Li (2002) was the first to study the incentives of firms to share information in a model with one supplier
and $n$ symmetric competing retailers. Li assumes that the retailers are engaged in a Cournot competition with uncertain demand and they have an opportunity to share their private demand information with the manufacturer. Li shows that those retailers who choose not to share their private information with the manufacturer can infer the information of those who did share their information by observing the wholesale price. Li calls this information leakage and concludes that no information is shared in equilibrium – a result which is undesired to the manufacturer. Zhang (2002) studies a similar model to that of Li (2002) and adds an analysis of Bertrand competition between the retailers. Li and Zhang (2007) explore the issue of information leakage in supply chains with one manufacturer and Bertrand competing firms. They show that when the retailers share their private information with the manufacturer, the retailers prefer to sign confidentiality agreements with the manufacturer. Such an agreement prevents the manufacturer from deliberately leaking the private information to the other retailers. Li and Zhang assume that the manufacturer sets a uniform wholesale price based on his available information and hence information can still be leaked indirectly through the wholesale price. They show how information confidentiality harms the manufacturer, while making the retailers better-off.

Other models that explore the incentives to share information when information can reach unintended recipients include for example Grossman and Stiglitz (1980), Singer (1999), Daughety and Reinganum (1994) and Anand and Goyal (2009).

Grossman and Stiglitz (1980) model the incentives for traders to acquire information under perfect competition. They show that the clearing price conveys information from informed traders to uninformed traders. As a result there is no pure strategy equilibrium with information acquisition. Singer (1999) describes a case of information exchange in the music industry. Sales data from Newbury Comics, a 20-store chain that sells records, was transmitted to SoundScan, a private company that tracks record sales and reports these to record labels, promoters and managers. This data was later leaked to stores like Wal-Mart and Kmart. Hayes (2004) reports that Wal-Mart announced that it will no longer share its sales data with outside companies such as Information Resources Inc. and ACNielsen, after this data was sold to other retailers. Anand and Goyal (2009) analyze the incentives of an incumbent retailer, facing possible entry to his market, to acquire new demand information when this information can be leaked to entrant by the mutual supplier. Anand and Goyal demonstrate how the manufacturer prefers to leak demand information to the entrant in order to promote competition at the retailers level. They further show that due to the risk of information leakage the incumbent may choose not to acquire new demand information. In a recent
paper, Kong, Rajagopalan and Zhang (2010) showed that the problem of information leakage that was highlighted by Anand and Goyal (2009) can be solved by using revenue sharing contracts between the incumbent retailer and the manufacturer.

Most of the models described above emphasize the negative effects of information leakage on the incentives of firms to share information in supply chains. In contrast with this view, I show that the incentives of the retailers to share their demand information with the manufacturer are driven by the retailers’ hope that their private information would be leaked to their competitors and enable them to coordinate on the monopoly price. In my model, the primary goal of the retailers is to share information at the horizontal level. When this option is not available to the retailers, they choose to share their private information with the manufacturer expecting this information to later reach their competing retailers.

3 The Model

3.1 The One Period Model

3.1.1 The Basic Model

Consider \( n \) retailers (I refer to each retailer as \( he \)) operate in a market, characterized by demand uncertainty. The retailers interact over infinite horizon and they source their product from a common manufacturer (\( she \)) for a cost of \( w_t \), which is determined at the beginning of each period by the manufacturer. The demand during each period is a function of the market price \( p \) (to be explained below) and an i.i.d random shock:

\[
Q(A_\theta, p) = (A_\theta - p)^+, \text{ where } \theta \in \{L, H\},
\]

with \( A_L < A_H \), and \( (A_\theta - p)^+ = \max(A_\theta - p, 0) \). The probability that demand is being in state \( \theta = H \) is given by \( \Pr(A_H) = \mu \in (0, 1) \). The profit of a monopoly at market condition \( A_\theta \) setting a retail price of \( p \) and having a constant marginal cost of \( w \) is given by

\[
\pi_R^m(A_\theta, p, w) \equiv Q(A_\theta, p)(p - w). \tag{1}
\]
I denote the price a monopoly would set at market state $A_i$ and having marginal cost of $w$ by $p^m(A_i, w) \equiv \arg \max_p \pi^m_R(A_i, p, w)$.

At the beginning of each selling period, each retailer has an opportunity to observe a signal $Y_i \in \{H, \phi\}$ about the market condition. The probability of observing the signal $Y_i = H$ is given by

$$\Pr(Y_i = H) = \begin{cases} \rho & \text{if } \theta = H; \\ 0 & \text{if } \theta = L, \end{cases}$$

and the probability of observing the uninformative signal $Y_i = \phi$ is given by

$$\Pr(Y_i = \phi) = \begin{cases} 1 - \rho & \text{if } \theta = H; \\ 1 & \text{if } \theta = L. \end{cases}$$

When the market condition is $\theta = H$, each retailer has a probability of $\rho$ to learn the true market condition and with the complement probability a retailer observes an uninformative signal $\phi$. When the market condition is $\theta = L$ a retailer can observe only the uninformative signal $\phi$. Upon observing the signal $Y_i$, each retailer updates in a Bayesian fashion the probability that market condition is $\theta = H$.

Following the information sharing stage (to be explained shortly), the manufacturer sets the wholesale price $w$ based on her available information. In the next stage, based on their information and the wholesale price, the retailers simultaneously set their retail prices $p_i$. After the retailers set their prices, the market price $\underline{p}(\underline{p}) = \min \{p_i\}$ is determined according to the lowest price quoted by the retailers, where $\underline{p} = (p_1, ..., p_n)$ denotes the vector of prices set by the retailers. The market share $m_i$ of retailer $i$ is given by

$$m_i(\underline{p}) = \begin{cases} 0 & \text{if } p_i > \underline{p}; \\ \frac{1}{k(\underline{p})} & \text{if } p_i = \underline{p}, \end{cases}$$

where $k(\underline{p})$ denotes the number of retailers setting the lowest price. I adopt the standard assumption that the retailers share the market equally if they set the same market price $\underline{p}$. The one period profit of retailer $i$ is given by

$$\pi_i(A_t, p_{it}, \underline{p}_{-it}, w_t) = Q(A_t, \underline{p}(\underline{p}))(p_i - w)m_i(\underline{p}).$$
Since the retailers have identical marginal costs the standard Bertrand game with homogenous products suggests that the stage game has a unique Nash equilibrium: all retailers charge their marginal costs regardless of the realized signals and earn zero profit in every period.

In some of the cases I analyze below in equilibrium each retailer can set a different price. In this case I use the superscript to denote the number of retailers setting a specific price. For example in the case \( \pi_i(A_t, p^1_k, p^{n-k}_2, w_i) \) \( k \) retailers in the market set the price \( p_1 \) and \( n - k \) retailers set the price \( p_2 \). I slightly abuse notation and do not use the superscript when it is clear how many retailers set the different prices in the market.

### 3.1.2 Information Structure

After observing their private signals, the retailers have an opportunity to share information. In order to focus on the effect of communication on the ability of the retailers to collude I analyze a few different settings of information sharing. In the first scenario, which serves as a benchmark, I assume that communication between the retailers is allowed and I analyze the scope of collusion when the retailers can exchange information. This scenario is denoted by \( S_1 \).

I then assume that the retailers cannot share information in a horizontal manner since such behavior results in the collusion being exposed by anti-trust authorities. I examine two alternative collusion mechanisms for the retailers: in the first (denoted by \( S_2 \)), the retailers collude without any information sharing; in the alternative option (denoted by \( S_3 \)) each retailer shares his private information in a vertical manner with the manufacturer alone. It is worth emphasis that in the last option the retailers share their private information with the manufacturer and each retailer is not exposed to the information shared between his competitors and the manufacturer.

When \( n \) retailers share their private information the posterior belief that the demand is high is given by:

\[
\mu(Y_n) = \Pr(A_H|Y_n) = \begin{cases} 
1 & \text{if there exists } i \text{ such that } Y_i = Y_H; \\
\frac{\mu(1-\rho)^n}{n(1-\rho)^n + 1-\mu} & \text{if } Y_i = Y_\phi \text{ for every } i,
\end{cases}
\]

where \( Y_n = (Y_1, Y_2, ..., Y_n) \) denotes the vector of \( n \) observed signals. I further summarize the information in the market using the information set \( I \). I denote by \( I_H \) the state in which at least one retailer observes an informative signal, and by \( I_\phi \) the state in which all retailers observe the non-informative signal.
<table>
<thead>
<tr>
<th>Setting</th>
<th>Information Available to the Retailers</th>
<th>Information Available to the Manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>$Y_n$</td>
<td>None</td>
</tr>
<tr>
<td>S2</td>
<td>$Y_i$</td>
<td>None</td>
</tr>
<tr>
<td>S3</td>
<td>$Y_i$ and $w(Y)$</td>
<td>$Y_n$</td>
</tr>
</tbody>
</table>

Table 1: Information available to the supply chain participants in the different settings

Table 1 summarizes the set of available information to the different parties in the supply chain under the different settings of information exchange.

In setting $S1$ and $S2$ the manufacturer does not have any information about the market demand, while in scenario $S3$ she receives messages from all the retailers. In scenario $S3$, although the retailers do not exchange information in a horizontal manner, they observe the wholesale price $w$ which may convey some information for the retailers about market demand.

### 3.2 The Repeated Game

I embed the single period model in an infinitely repeated game. The market demand in each period is independent of the market demand in any other period. In each period $t$, the complete history of interactions between the retailers and the manufacturer, $h_t = ((w_1, p_1), (w_2, p_2), \ldots, (w_{t-1}, p_{t-1}))$ is observable to both the retailers and the manufacturer. In addition, the complete history of shared information is available to the receivers of the information\(^1\). Let $H_t$ be the set of all possible period $t$ histories. A repeated game strategy $\sigma$, consists of a mapping from every possible history of actions, $h_t$, to the retailers’ and manufacturer one period strategy. The payoff to each retailer is a sum of the payoff during each period discounted using a common discount factor $\delta \in (0, 1)$, such that the discounted profit of the retailers is given by:

$$V_i = \sum_{t=1}^{\infty} \delta^{t-1} [\pi_i(A_t, p_{it}, p_{iti}, w_t)],$$  \hspace{1cm} (6)

and the manufacturer’s discounted profit starting from period 1 is:

$$V_M = \sum_{t=1}^{\infty} \delta^{t-1} [(A_t - p_{it})w_t].$$  \hspace{1cm} (7)

\(^1\)This implies that in scenario $S1$ the complete history of announcements is available to the retailers, and in scenario $S3$ the complete history of announcements is available to the manufacturer.
The history of the game up to date $t$ has no direct impact on either the payoffs or the feasible strategies from date $t$ onwards. The game beginning at date $t$ looks the same for all $t$, in the sense that the feasible strategies and the prospective payoffs that they induce are always the same. History matters only because the firms remember what has happened in the past and condition their current actions on previous behavior. Infinitely repeated games typically have many equilibria (Fudenberg and Tirole 1991). As a solution concept I use the public perfect equilibria ($PPE$). A strategy profile is a perfect public equilibrium if the strategy played by each player depends on the public information (and not on private information available to a specific player) and at each date $t$ and history $h_t$ the strategies are Nash equilibrium from that point on (i.e. no firm has a one shot profitable deviation at any point of time). I further limit my attention to symmetric $PPE$ ($SPPE$) of the retailers: the retailers pricing schedules are symmetric after any public history (Fudenberg et al., 1994). In a $SPPE$ after any history all retailers enjoy the same future rewards or suffer the same future punishments.

4 Collusion with Horizontal Information Sharing

As a benchmark, and in order to develop the intuition of the rest of the results, I analyze the collusive agreement between the retailers when they are able to exchange information. Although the manufacturer is aware of the information exchange between the retailers, she is not exposed to the shared information.

4.1 Retailers collusive price

When the retailers exchange information they are all in the same information set and need to determine for a given marginal cost $w$ and discount factor $\delta$ the set of collusive prices $p^{S1} = (p^{S1}_1, p^{S1}_H)$.

Let $V$ denote the discounted expected profit of the retailers starting from the current period. When the retailers are in state $I_H$ their profit is given by

$$V(I_H) = \pi(A_H, p_H, w) + \delta V.$$  \hspace{1cm} (8)

The discounted profit is composed from the cartel’s current period profit $\pi(A_H, p_H, w)$ and the discounted stream of future profits. When all retailers receive the non-informative signal (i.e. they are in the
information set \( I_\phi \) the retailers’ expected profit is given by:

\[
V(I_\phi) = \frac{(1 - \mu)}{(1 - \mu) + \mu(1 - \rho)^n} \pi(A_L, p_\phi, w) + \frac{\mu(1 - \rho)^n}{(1 - \mu) + \mu(1 - \rho)^n} \pi(A_H, p_\phi, w) + \delta V. \tag{9}
\]

In the information set \( I_H \), the retailers infer that demand is high and they set the price \( p_H \). In the information set \( I = \phi \) the retailers update in a Bayesian manner their belief about the state of demand and price the product to reflect the new updated belief.

To find the most collusive SPPE, I use the dynamic programming technique suggested by Abreu et al. (1986, 1990). Any SPPE of the repeated game can be decomposed into a pair of first period profit and a continuation value function. The continuation payoff \( V \) depends on the public history. In order for a vector of prices \( p(w, \delta) \) to qualify as a SPPE two conditions must hold: an individual firm should have no incentive to deviate from the current period collusive price and second all continuation values must be drawn from the set of \( SPPE \). Since the Nash equilibrium of the stage game is always a SPPE, the set of symmetric \( PPE \) is not empty and the lowest possible value \( V = 0 \) is obtained by playing the one period Nash Equilibrium.

In the case of complete information, each deviation from the collusive scheme is immediately detected and punished by the harshest possible punishment - playing repeatedly the one period Nash equilibrium and earning zero profit. Solving for (8) and (9) \( V \) can be expressed as:

\[
V = \frac{1}{(1 - \delta)} [\mu(1 - (1 - \rho)^n) \pi(A_H, p_H, w) + (1 - \mu) \pi(A_L, p_\phi, w) + \mu(1 - \rho)^n \pi(A_H, p_\phi, w)]
\]

The cartel’s price vector \((p_H, p_\phi)\) can be sustained if the following incentive constraints are satisfied:

\[
\pi(A_H, p_H, w) + \frac{\delta}{1 - \delta} V \geq \pi(A_H, \tilde{p}_i, p_H, w) \text{ for any } \tilde{p}_i \neq p_H \tag{IC-H}
\]

\[
E[\pi_i | I_\phi] + \frac{\delta}{1 - \delta} V \geq E_\theta[\pi(A_\theta, \tilde{p}_i, p_\phi, w) | I_\phi] \text{ for any } \tilde{p}_i \neq p_\phi \tag{IC-\phi}
\]

The constraint IC-H suggests that a retailer is better-off adhering to the cartel price at the information set \( I_H \). The LHS denotes the expected discounted profit for a retailer setting the collusive price, given that all other retailers adhere to the cartel rules as well. The RHS denotes the profit of deviating from the cartel scheme and setting the price \( \tilde{p}_i \). Naturally the best deviation is to slightly cut the cartel price and capture the entire market during the current period. Such a deviation triggers price war and results in zero profit.
at all future periods. The constraint IC-$\phi$ is similar and ensures that a retailer does not deviate in the
information set $I_\phi$.

For a given set $(w, \delta)$ the retailers maximizes their profit by solving for the following problem:

$$
\max_{p_H, p_\phi} V \quad (10)
\begin{align*}
s.t \quad & (IC - H) \text{ and } (IC - \phi).
\end{align*}
$$

Before characterizing the solution to (10) I introduce the following two definitions.

**Definition 1** $\delta(p^m, w)$ denotes the lowest discount factor for which the retailers can charge the monopoly
price during both possible demand states.

$\delta(p^m_H, w)$ denotes the lowest discount factor which enables the retailers to charge the monopoly prices
during both information sets for a given wholesale price $w$. Clearly, as the retailers become more patient
(measured by their discount factor) they are able to set higher prices, and for any discount factor $\delta(p^m, w) \leq
\delta < 1$, the retailers are able to set the monopoly price during both states of demand.

**Definition 2** Let $p(A_H, A_\phi, w)$ be the highest price set during periods of high demand such that $IC - H$ is
binding when $p_\phi = p^m(w)$

$p(A_H, A_\phi, w)$ denotes the highest price the cartel can set in order to satisfy the constraint $IC - H$
when the cartel sets the monopoly price $p_\phi = p^m(w)$ during periods in which all cartel members receive the
non-informative signal.

The next lemma characterizes the optimal collusive scheme as a function of the wholesale price $w$ and
the discount factor $\delta$.

**Lemma 1** The cartel price is characterized by:

$$(p_H, p_\phi) =
\begin{cases}
p^m(w) & \text{if } \delta \geq \delta(p^m, w); \\
p_\phi = p^m(A_\phi, w) \text{ and } p_H = p(A_H, A_\phi, w) & \text{if } \delta \in \left[\frac{n-1}{n}, \delta(p^m, w)\right).
\end{cases}$$
The result is based on Rotemberg and Saloner (1986) which studied the scope of collusion in markets with fluctuating demand. The result asserts that if the discount factor is high enough (i.e. \( \delta \geq \delta(p^m, w) \)), the retailers are able to set the monopoly price during each information set. However, if the discount factor is not high enough, measured by \( \delta \in \left[ \frac{n-1}{n}, \hat{\delta}(p^m, w) \right] \), the retailers cannot set the monopoly price during periods of high demand. During periods of high demand the temptation to deviate from the monopoly price increases. As a result, the retailers lower the price during periods of high demand in order to decrease the temptation to deviate from the collusive price during these periods. For a discount factor \( \delta < \frac{n-1}{n} \), no collusive outcome can be sustained.

4.2 Manufacturer’s wholesale price

When the retailers exchange information in a horizontal manner, the manufacturer is not exposed to this information. Denote by \( Q(A_\theta, p) \) the demand when the retailers set the price \( p \) and the market condition is \( A_\theta, \theta \in \{L,H\} \). The manufacturer’s wholesale price plays an important role in the ability of the retailers to collude. Note for example that by setting a wholesale price which exceeds the value \( A_L \frac{1-\mu}{(1-\mu) + \mu(1-\rho)^n} + A_H \frac{\mu(1-\rho)^n}{(1-\mu) + \mu(1-\rho)^n} \) the retailers do not sell during periods in which they all observe the non-informative signal. Therefore, in addition to the operational role of the wholesale price, the wholesale price also plays the important role of hindering or facilitating the ability of the retailers to collude. Since all periods are identical from the manufacturer’s perspective, maximizing her stream of discounted profit is equivalent to maximizing her one period profit. The manufacturer’s one-period problem is given by

\[
\pi_m^{S1}(\delta) = \max_w \left[ \mu(1 - (1 - \rho)^n)Q(A_H, p_H(w, \delta))w + \mu(1 - \rho)^nQ(A_H, p_\phi(w, \delta))w + (1 - \mu)Q(A_L, p_\phi(w, \delta))w \right]
\]

(11)

The first term denotes the manufacturer’s profit when demand is high and the retailers observe an informative signal. In this case the retailers set the price \( p_H \). When demand is also high but the retailers do not observe an informative signal they set the collusive price to be \( p_\phi \), and as a result the demand is given by \( Q(A_H, p_\phi) \). Finally, when the market condition is \( A_L \) and the retailers set the price \( p_\phi \), the manufacturer faces the demand \( Q(A_L, p_\phi) \). Denote by \( A_\mu \triangleq \mu A_H + (1 - \mu)A_L \) the manufacturer’s ex-ante belief about
the demand intercept. I further define the following threshold on the discount factor

\[ \delta^* = \frac{n - 1}{n - (1 - \mu(1 - (1 - \rho)^n))} \]

and use the following lemma:

**Lemma 2** For any discount factor \( \delta \geq \delta^* \), the retailers are able to set the monopoly price during both demand states regardless of the wholesale price.

If the retailers are patient enough (measured by the discount factor \( \delta \geq \delta^* \)), they can set the monopoly price for each demand state. For a discount factor higher than \( \delta^* \) the wholesale price does not play a role in deterring the ability or scope of collusion.

The next proposition provides some insight into the manufacturer’s pricing decision.

**Proposition 1** The manufacturer sets the following wholesale price:

(a) If \( \delta \leq \frac{n-1}{n} \), \( w = \frac{A_2}{2} \).
(b) If \( \delta \geq \delta^* \), \( w = \frac{A_2}{2} \).
(c) For intermediate levels of \( \delta \), \( w \geq \frac{A_2}{2} \).

When the retailers are impatient (measured by \( \delta < \frac{n-1}{n} \)) and they compete by setting prices, no collusive outcome can be sustained, and the intensity of the competition drives the retailers to equate the retail prices to the wholesale price \( w \). In this case, the manufacturer behaves as if she is a single decision maker that owns the supply chain and she sets the wholesale price to maximize the supply chain’s profit.

In the other extreme case, when the retailers are very patient (\( \delta \geq \delta^* \)) they can set the monopoly price during all selling periods. It is interesting to note, that at the extreme cases (for low or high discount values) the manufacturer pricing decision is identical and independent of the accuracy of the retailers’ signals.

In the intermediate case, when \( \frac{n-1}{n} < \delta < \delta^* \), the retailers can partially collude; the retailers can set the monopoly price during periods in which they observe the non-informative signal, but they cannot set the monopoly price during periods in which one of the retailers observe an informative signal about the high demand condition. In this case, the retailers must decrease their prices relative to monopoly price. The manufacturer enjoys the fact that the retailers cannot increase their prices, and consequently can raise the
wholesale price relative to the case in which the retailers can adopt the monopoly price. As the discount factor increases toward $\delta$ the retailers' price converges to the monopoly price, and the manufacturer is forced to decrease the wholesale price.

5 Collusion without Information Sharing

I now turn to the analysis of the case, in which the retailers cannot share their private demand information. As was discussed in the introduction, information sharing allows anti-trust authorities to infer that the retailers collude. As a result, the retailers seek for alternative mechanism that will allow them to collude without sharing information directly. In this section I study two options available for the retailers to collude without sharing information. In the first option, which I call the responsive pricing strategy, the retailers set different prices when they observe different signals (i.e. $p(Y_H) \neq p(Y_\phi)$). I show that this strategy introduces moral hazard problems: under certain conditions, a retailer who observes the informative signal might be tempted to set the price designed for retailers observing the non-informative signal in order to secure some market share. The second type of strategy discussed in this section is the rigid pricing strategy. Under this strategy all retailers set the same price, regardless of their observed private information. In this strategy there are no moral hazard issues, but the retailers do not take into account in their pricing decision the information which is available to them.

5.1 Responsive Pricing Scheme

I start analyzing the first option available to the retailers: setting a different price according to the observed signal. Denote the probability that, given high demand state, $k$ retailers observe the non-informative signal out of $n$ possible retailers by:

$$ P_k^n \triangleq \binom{n}{k} (1 - \rho)^k \rho^{n-k}. \quad (12) $$

The current period profit of a retailer observing the signal $Y_H$ is

$$ \Pi_i(p_H, Y_H) = \rho^{n-1} \pi(A_H, p_H, w), \quad (13) $$

and the current period profit of a retailer observing the non-informative signal $Y_\phi$ and setting the price $p_\phi$
\[ \Pi_i(p_\phi, Y_\phi) = \frac{1 - \mu}{1 - \mu + \mu(1 - \rho)} \pi(A_L, p_\phi, w) + \frac{\mu(1 - \rho)}{1 - \mu + \mu(1 - \rho)} \sum_{k=0}^{n-1} \rho_k^{n-1-k} \pi(A_H, p_{\phi, k+1}, p_{H, k-1}, w). \]  

Equation (13) denotes the current period profit of a retailer observing the informative signal. When a retailer observes the informative signal, he infers that demand in the current period is high. However, setting the collusive price \( p_H \) results in profit only if all other \( n - 1 \) retailers observe the informative signal as well. With probability \( 1 - \rho^{n-1} \) at least one retailer observes a non-informative signal. In this case, the retailer who observes the non-informative signal, sets the price \( p_\phi \) and leaves zero profit to all the retailers who observed the informative signal. Equation (14) denotes the profit of a retailer observing the non-informative signal; with probability \( \frac{1 - \mu}{1 - \mu + \mu(1 - \rho)} \) the demand during the current selling season is low, and hence all other retailers would observe the non-informative signal as well. With probability \( \frac{\mu(1 - \rho)}{1 - \mu + \mu(1 - \rho)} \) the demand is actually high, and the retailer shares the market with all other retailers observing the non-informative signal.

In order for the vector of prices \((p_\phi, p_H)\) to be sustainable in collusion, it must be immune to two types of deviations. First, a retailer can find it beneficial to choose an off-schedule deviation. In this case, a retailer can choose to set a price \( p \notin \{p_\phi, p_H\} \) which is different than the set of allowable prices by the cartel. Such a deviation is immediately detected by all other retailers and this deviation results in a price war in a similar manner to the analysis in the previous section. A second type of deviation is an on-schedule deviation. In this case, a retailer sets the price \( p \in \{p_H, p_\phi\} \) out of the set of allowable prices but not in a consistent manner with his observed signal. The set of collusive prices \((p_\phi, p_H)\) is sustainable if the following set of equations is satisfied:

\[ \Pi_i(p_H, Y_H) + \delta E[V(p_H)] \geq \Pi_i(\tilde{p}, Y_H) \text{ for every } \tilde{p} \notin \{p_H, p_\phi\}; \quad \text{(IC-off-H)} \]
\[ \Pi_i(p_\phi, Y_\phi) + \delta E[V(p_\phi)] \geq \Pi_i(\tilde{p}, Y_\phi) \text{ for every } \tilde{p} \notin \{p_H, p_\phi\}; \quad \text{(IC-off-\phi)} \]
\[ \Pi_i(p_H, Y_H) + \delta E[V(p_H)] \geq \Pi_i(p_\phi, Y_H) + \delta E[V(p_\phi)]; \quad \text{(IC-on-H)} \]
\[ \Pi_i(p_\phi, Y_\phi) + \delta E[V(p_\phi)] \geq \Pi_i(p_H, Y_\phi) + \delta E[V(p_H)]. \quad \text{(IC-on-\phi)} \]

Equation (IC-off-H) denotes the off schedule constraint for a retailer observing the informative signal. Note that by choosing a price \( \tilde{p} \notin \{p_H, p_\phi\} \), the subsequent profit of the retailer is zero. Equation (IC-off-\phi)
is the equivalent off schedule constraint when the retailer observes the non-informative signal. Constraints (IC-on-H) and (IC-on-\phi) denote the on-schedule constraints when the retailer observes the informative signal and the non-informative signal respectively. The above system of inequalities can be simplified using the following lemma:

**Lemma 3** The constraint IC – on – \phi is slack

The on-schedule constraint (IC-on-\phi) is slack. First, it is a non-profitable deviation for a retailer to set the price \(p_H\) when observing a non-informative signal. In order to see the intuition behind this result, note that when a retailer observes a non-informative signal his posterior belief is that market demand is high with probability \(\frac{\mu (1 - \rho)}{1 - \mu + \mu (1 - \rho)}\), and the collusive price (as I will show below) maximizes the cartel profit given that posterior belief. Furthermore, by setting a higher price than \(p_\phi\) a retailer receives some market share only if all the other retailers observe the informative signal. If the demand is actually low, or at least one additional retailer observes the non-informative signal a retailer setting the price \(p_H\) receives no market share. In addition there exists a strictly positive probability that the demand is low, and hence by setting the price \(p_H\) all other retailers can infer that the retailer has deviated and retaliate by initiating a price war.

If a retailer chooses an off-schedule deviation, it is immediately detected by all other retailers (by the definition of an off-schedule deviation), and it results in price war. Therefore, for a discount factor high enough, the off-schedule constraints and the constraint (IC-on-\phi) are not binding at the optimal cartel solution.

In the reminder of this section, I focus on the optimal cartel solution for firms which are patient enough. In this case, the cartel solves the following problem:

\[
\max_{p_H, p_\phi} \mu \left[ \rho^n n \pi(A_H, p_H^n, w) + \sum_{k=1}^{n} kP_k^n \pi(A_H, p_H^{n-k}, p_\phi^k, w) \right] + (1 - \mu) n \pi(A_L^n, p_\phi, w) + \delta E[V(p)]
\]

s.t

\[
IC - on - H.
\]

With probability \(\mu\) demand is high and given high demand, with probability \(\rho^n\) all the retailers observe the informative signal. In this case, the profit of the cartel is \(n \pi(A_H, p_H, w)\). However with prob-
ability $1 - \rho^n$ at least one retailer observes a non-informative signal, and the expected profit of the cartel is $\sum_{k=1}^{n} kP^n_k \pi(A_H, p_H, w)$. Note that the cartel profit does not depend on the number of retailers sharing a positive market share, but only on the lowest price quoted. As a result the expression $\sum_{k=1}^{n} kP^n_k \pi(A_H, p_H^{n-k}, p^*_\phi, w)$ is equivalent to $(1 - \rho^n) \pi(A_H, p_H, w)$. Finally, with probability $1 - \mu$, demand is low and all retailers observe the non-informative signal and set the price $p_\phi$.

The following proposition provides some insight into the optimal pricing strategy of the cartel.

**Proposition 2** (a) The non-constrained solution to problem (16) is given by the following set of equations:

$$\frac{\partial \pi(A_H, p_H, w)}{\partial p_H} = 0$$

$$\mu(1 - \rho^n) \frac{\partial \pi(A_H, p_\phi, w)}{\partial p_\phi} + (1 - \mu) \frac{\partial \pi(A_L, p_\phi, w)}{\partial p_\phi} = 0.$$

(b) This solution is feasible if

$$\rho^{n-1} \pi(A_H, p_H, w) \geq \sum_{k=1}^{n-1} P^{n-1}_k \pi(A_H, p_H^{n-1-k}, p^*_\phi, w)$$

(17)

Part (a) of the proposition characterizes the optimal collusion if we relax the constraint $IC - on - H$. Part (b) of the proposition states that this solution is feasible if the condition given in part (b) is satisfied. If this condition is not satisfied it means that a retailer observing the informative signal is better off mimicking a retailer observing the non-informative signal. However, if the difference between $p_H$ is $p^*_\phi$ is high enough (as given in (17)), a retailer prefers to set the price $p_H$ after observing the informative signal. I denote the vector of prices characterized by the first part of the above proposition (the unconstrained solution) by $(p^*_H, p^*_\phi)$.

When the condition in (17) is not met, the cartel must provide the proper incentive for a retailer observing the informative signal to set the price $p_H$. Two tools are available to the cartel to align the incentive of a retailer observing the informative signal: price distortion and price wars (in addition to price rigidity that will be discussed in the next sub-section).

Price distortion means that the cartel chooses a vector of prices which is different than $(p^*_H, p^*_\phi)$. By
choosing a price, for example, which is lower than $p^*_\phi$ the cartel reduces the incentive of a retailer observing an informative signal to mimic a retailer observing the non-informative one. The second tool available to the cartel is price war. In this case, when some members of the cartel set the price $p_\phi$ during periods of high demand, it can result in initiating a price war.

In order to understand the optimal choice of the cartel when the vector of prices $(p^*_H, p^*_\phi)$ is not sustainable I assume that after observing that the market demand is high and the market price was $p_\phi$ the cartel collapses with probability $\beta$ and the retailers set the retail price in a competitive manner. In this case, $IC - on - H$ is given by:

$$\rho^{n-1} [\pi(A_H, p_H, w) + \delta V] + (1 - \rho^{n-1})(1 - \beta) \delta V \geq \sum_{k=1}^{n-1} D^k_n \pi(A_H, p^{n-k}_H, p^*_\phi, w) + (1 - \beta) \delta V. \quad (18)$$

The LHS of this constraint denotes the profit of a retailer observing the informative signal. With probability $\rho^{n-1}$ all other retailers observe the informative signal as well, and they all share the market when setting the price $p_H$ and the cooperation continues to the next period with probability 1. However, with probability $(1 - \rho^{n-1})$ at least one retailer observes the non-informative signal, and hence a retailer observing the informative signal earns zero profit in the current period. Furthermore, the cooperation continues only with probability $(1 - \beta)$. The RHS of (18) denotes the profit of a retailer observing the informative signal but behaving as if he observed the non-informative signal. In this case, the deviating retailer shares the market with all other retailers observing the non-informative signal and the cooperation continues only with probability $(1 - \beta)$. When a retailer who observes a high signal decides to behave as if he observed the non-informative signal, he must take into account the increased probability of a price war and the fact that he sets a lower price compared with the price $p_H$.

The next proposition provides some insight into the cartel pricing decision when the vector of prices $(p^*_H, p^*_\phi)$ is not sustainable.

**Proposition 3** Assume $(p^*_H, p^*_\phi)$ is not sustainable in collusion. Then:

(a) $p_H = p^*_H$.
(b) $p_\phi < p^*_\phi$.

The proposition suggests that when the cartel cannot maintain the vector of prices $(p^*_H, p^*_\phi)$, it will choose to distort the prices when a retailer observes the non-informative signal, while maintaining the
price $p_H^*$ after observing the informative signal. This result is aligned with the results obtained by Gerlach (2009) who also studied the scope of collusion with private demand information. When the vector of prices $(p_H^*, p_φ^*)$ is not sustainable, it means that a retailer is better off setting the price $p_φ^*$ than setting the price $p_H^*$ and facing the risk that at least one retailer observes the non-informative signal. In this case, in order to eliminate the incentive to deviate, the cartel lowers the price a retailer sets during periods in which he observes the non-informative signal.

5.2 Rigid Pricing Scheme

As an alternative to the separating pricing scheme, the retailers can choose a rigid pricing. In this case, the retailers set the same price at all selling periods regardless of the observed signal. When the retailers set a rigid price, each deviation from this price is immediately observed and can be punished. For a high enough discount factor the rigid price is sustainable and hence the cartel can operate with a rigid pricing scheme over an infinite horizon.

The cartel chooses a rigid pricing scheme by solving for the problem

$$\max_p \mu (A_H - w)(p - w) + (1 - \mu) (A_L - w)(p - w).$$

The following lemma characterizes the pricing strategy and the per-period profit of the supply chain participants in the rigid pricing scheme.

**Lemma 4** When the retailers choose a rigid pricing scheme:

$$p = \frac{3}{4} A_μ; \quad w = \frac{1}{2} A_μ;$$

$$\pi_i = \frac{A_μ^2}{16(1-\sigma)}; \quad \pi_M = \frac{A_μ^2}{8(1-\sigma)}.$$

Since there are two possible ways for the retailers to collude without sharing any information, the natural question is what is the preferred way for the retailers to operate. The next proposition shows that there are cases in which the retailers choose the rigid pricing scheme over the separating one.

**Proposition 4** There exists a value $n$, such that for any $n \geq n$, the profit of the cartel is higher using a rigid pricing scheme than the cartel’s profit with a separating scheme.

As the number of retailers increases, the probability of a retailer, observing a high demand signal, to
capture some share of the market decreases, and hence the temptation of this retailer to deviate from the price $p_H$ increases. In order to satisfy the incentive compatibility constraint of this retailer, the cartel is forced to distort the price $p_\phi$, and perhaps be engaged also in price wars. The proposition suggests that when the number of cartel members is high, in order to satisfy the incentive compatibility constraints of the cartel members, the price distortion is so significant that it is better for the cartel to ignore the private information and instead of solving the coordination problem of the cartel members, the cartel is better off using a rigid pricing scheme.

6 Collusion with Vertical Information Sharing

In this section I study the scope of collusion when the retailers share their private information with the manufacturer. At the beginning of period $t$ each retailer $i$ shares with the manufacturer the message $Y_{it}$. By sharing their private information with the manufacturer, the model is switched from setting in which the retailers possess the superior information in the supply chain into a model in which the manufacturer has the cumulative knowledge about the observed signals. Such a setting raises a few questions: what are the incentives of the retailers to share their private information with the manufacturer and upon receiving the demand information, how does it affect the wholesale price. I focus on a mechanism that allows the retailers to infer the private information of the manufacturer based on the wholesale price. Although the retailers are not exposed to the private information of their competitors, they can infer what this information was based on observing the wholesale price $w$. The way each retailer interprets $I$ depends on the belief system of the retailers about the relationship between the wholesale price $w$ and the market condition. In the subsequent analysis I show that there exists a pair of prices $w_H$ and $w_\phi$ such that the retailers infer when observing $w_H$ that at least one retailer received an informative signal, and when observing $w_\phi$ they can infer that all the retailers received the non-informative signal. Furthermore, based on this belief system the manufacturer finds it in her best interest to set the price $w_H$ when receiving an informative signal and the price $w_\phi$ when receiving only non-informative signals.

Another point which is worth discussion is the incentives of the manufacturer to reveal the market condition via the wholesale price. Since the primary goal of this paper is to analyze the information flow between the manufacturer and her retailers and not the effect of repeated interaction, the focus in this section is on a mechanism that aligns the incentives of the manufacturer to share her private information in a truthful manner based on analysis of her profit during the current period, and not due to future discounted
profit and a threat of the retailers to punish her if a deviation is detected. Another way to think about this assumption is as if the manufacturer acts in a myopic manner, or alternatively, the retailers interact during each period with a different manufacturer.

Upon observing the posted wholesale price, the retailers establish the following belief about the set of messages observed by the manufacturer:

\[
\mu(w) = \Pr(A_H|w) = \begin{cases} 
1 & \text{if } w = w_H; \\
\frac{(1-\rho)^n}{\mu(1-\rho)^n + 1-\mu} & \text{if } w = w_\phi. 
\end{cases} 
\] (19)

In a separating equilibrium, upon observing the wholesale price the retailers are able to set the collusive price as if information was shared in a horizontal manner. In this case, for a high enough discount factor, the retailers set the price:

\[
p(w) = \begin{cases} 
\frac{A_H + w_H}{2} & \text{if } w = w_H; \\
\frac{(1-\rho)^n A_H + A_L(1-\mu)}{2(1-\rho)^n + 1-\mu} + \frac{w_\phi}{2} & \text{if } w = w_\phi.
\end{cases} 
\] (20)

Based on the wholesale price, the retailers are able to solve their coordination problem and they can infer in which information set they would have been if they were able to meet and exchange information in a horizontal manner.

For a discount factor high enough the retailers are able to set the monopoly price based on observing the wholesale price.

In order to provide the manufacturer with incentives to set a different wholesale price according the her observed set of signals, the manufacturer solves the following problem:

\[
\max_{w_H, w_\phi} E \left[ \Pi_M \right] 
\] (IC – M_{\phi H})

\[
E \left[ \Pi_M (A_i, p_\phi, w_\phi) | I_\phi \right] \geq E \left[ \Pi_M (A_i, p_{\phi H}, w_H) | I_\phi \right] 
\] (IC – M_{\phi H})

\[
E \left[ \Pi_M (A_H, p_{\phi H}, w_H) | I_H \right] \geq E \left[ \Pi_M (A_H, p_\phi, w_\phi) | I_H \right] 
\] (IC – M_{H \phi})

Constraint (IC – M_{H \phi}) ensures that upon observing an informative signal, the manufacturer prefers to set the price \( w_H \) and do not attempt to manipulate the retailers’ belief that she did not receive an informative signal. The second constraint, (IC – M_{\phi H}) is related to the case in which the manufacturer
does not receive an informative signal. In this case, this constraint ensures that the manufacturer prefers to set the price \( w_\phi \) and not the price designed for the information set in which she receives an informative signal. In this constraint, the expectation is taken over the demand state and it is conditioned on the fact that the manufacturer did not receive an informative signal.

Before characterizing a separating equilibrium when the retailers share information in a vertical manner, an additional definition is needed. Define \( \psi \) to be:

\[
\psi \triangleq \frac{A_H}{\mu(1-\rho)^n A_H + A_L (1-\mu)}. \tag{22}
\]

The parameter \( \psi \) is a proxy for demand uncertainty, and it provides a measure of the distance between the demand intercept during periods of high demand and the expected demand intercept upon receiving \( n \) non-informative signals.

The next proposition characterizes the separating equilibrium in the vertical information sharing scenario.

**Proposition 5** For a high enough discount factor, there exists an equilibrium in which:

(a) The manufacturer sets the wholesale price

\[
w_H = \begin{cases} 
\frac{A_H}{2} & \text{if she receives at least one informative signal;} \\
\frac{\mu(1-\rho)^n A_H + A_L (1-\mu)}{2\mu(1-\rho)^n + (1-\mu)} & \text{if } I_\phi \text{ and } \psi \geq 3; \\
\widehat{w}_\phi & \text{if } I_\phi \text{ and } \psi < 3
\end{cases}
\]

where

\[
\widehat{w}_\phi = \frac{2A_H - \frac{\mu(1-\rho)^n A_H + A_L (1-\mu)}{\mu(1-\rho)^n + (1-\mu)}}{3A_H^2 - 4A_H \frac{\mu(1-\rho)^n A_H + A_L (1-\mu)}{\mu(1-\rho)^n + (1-\mu)}} + \left( \frac{\mu(1-\rho)^n A_H + A_L (1-\mu)}{\mu(1-\rho)^n + (1-\mu)} \right)^2
\]
(b) The retailers set the cartel price:

\[ p_H = \frac{3}{4}A_H \quad \text{if } w = w_H \]

\[
p_{\phi} = \begin{cases} 
\frac{3 \mu(1-\rho)^nA_H + A_L(1-\mu)}{4} & \text{if } w = w_\phi \text{ and } \psi \geq 3; \\
\hat{p}_{\phi} & \text{if } w = w_\phi \text{ and } \psi < 3
\end{cases}
\]

where

\[
\hat{p}_{\phi} = \frac{\mu(1-\rho)^nA_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]} - \sqrt{\frac{3A_H^2 - 4A_H \frac{\mu(1-\rho)^nA_H + A_L(1-\mu)}{\mu(1-\rho)^n + (1-\mu)}}{4} + \left(\frac{\mu(1-\rho)^nA_H + A_L(1-\mu)}{\mu(1-\rho)^n + (1-\mu)}\right)^2}
\]

with the following belief system:

\[
\mu(w) = \Pr(A_H|w) = \begin{cases} 
1 & \text{if } w = w_H; \\
\frac{\mu(1-\rho)^n}{\mu(1-\rho)^n + 1-\mu} & \text{if } w = w_\phi.
\end{cases}
\]

In order to better understand the results of the proposition it is beneficial to compare them to a setting of complete information. In a complete information scenario, both the manufacturer and the retailers know if the current state is \( I_H \) or \( I_\phi \). In this case, the wholesale price has the operational role of maximizing the manufacturer’s profit. In a complete information setting the manufacturer would set the wholesale price to \( w_H = \frac{A_H}{2} \) during the state \( I_H \) and the wholesale price \( w_\phi = \frac{\mu(1-\rho)^nA_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]} \) during the state \( I_\phi \) (assuming a high enough discount factor).

However, under settings of asymmetric information, when the manufacturer has superior information, the wholesale price plays a dual role: the first role is the operational role of maximizing the profit of the manufacturer. The additional role, which is unique to a setting of asymmetric information, is to convey information about the demand state to the retailers. When \( \psi \) is high enough, i.e. there is a significant difference between the demand intercept during the state \( I_H \) and the expected demand intercept upon
observing only non-informative signals, the manufacturer is able to set the same wholesale price as in the complete information setting. In this case, the wholesale price $w_\phi = \frac{\mu(1-\rho)^nA_H+A_L(1-\mu)}{2[\mu(1-\rho)n+(1-\rho)]}$ conveys in a credible manner to the retailers that the manufacturer did not receive any informative signal. However, if $\psi$ is not high enough (to be more precise if $\psi < 3$) setting the complete information price during the state $I_\phi$ cannot convey in a credible manner to the retailers that indeed the manufacturer did not receive any informative signal. In this case, even if the manufacturer observed an informative signal, she has an incentive to set the price $w_\phi = \frac{\mu(1-\rho)^nA_H+A_L(1-\mu)}{2[\mu(1-\rho)n+(1-\rho)]}$ in order to induce the retailers to lower their retail prices. Anticipating such a behavior, the retailers would ignore the informative role of the wholesale price. Therefore, in order to achieve a separating equilibrium the manufacturer must design a price schedule that would convey to the retailers the true state of demand based on the wholesale price. The manufacturer achieves this goal by lowering her price during periods in which she does not receive any informative signal.

7 Comparison between Scenarios $S_2$ and $S_3$

Following the analysis of sections 5 and 6 which both allow the retailers to collude without direct information sharing, the natural question is what is the preferred mechanism from the retailers’ perspective? In this section I compare between the option of the retailers to collude without communication (setting $S_2$) and the option of the retailers to share information with the manufacturer and infer the market condition using the wholesale price. I start by examining some of the properties of each setting.

**Information Efficiency** - In setting $S_2$ with probability $\mu \sum_{k=1}^{n-1} P_k^n$ the cartel suffers from information inefficiency; although at least one member in the cartel observes an informative signal, there is also at least one retailer who observes the non-informative signal. In this case, the market price is set to $p^{S_2}_\phi$ instead of $p^{S_2}_H$. In contrast, in setting $S_3$, if at least one retailer observes an informative signal, all other retailers can infer that demand is high through the wholesale price. Clearly if the cartel adopts the rigid pricing scheme there is also information inefficiency since the cartel ignores the private information its members have. The difference in the information efficiency can also lead to implications regarding information acquisition. When the cartel members cannot use their information in an efficient manner, assuming information acquisition is costly, they may prefer not to acquire such information.

**Cartel Stability** - The second difference between scenario $S_2$ and scenario $S_3$ is the stability of the cartel. While, in scenario $S_3$ the cartel can operate over infinite horizon, in scenario $S_2$ (when the retailers choose a separating pricing scheme) the cartel eventually collapses since there exists a positive probability
Market Knowledge - The third interesting difference between the two settings is which firms in the supply chain have the superior information about the market demand. In scenario $S_2$ (when the retailers do not use price rigidity) each retailer has a private information and in order to satisfy the retailers’ incentive constraints, the cartel distorts (under certain conditions) the price in state $I_\phi$. However, in scenario $S_3$ the manufacturer has the superior information about the market demand and in a similar manner she needs to distort her price during the state $I_\phi$ in order to be able to credibly signal her observed information.

I now turn back to asking the question whether scenario $S_3$ is feasible. The retailers would prefer to share their private information with the manufacturer if their discounted profit in scenario $S_3$ is higher than their discounted profit in scenario $S_2$. In a similar manner, the manufacturer is willing to act as the information aggregator for the retailers if her discounted profit in scenario $S_3$ is higher than the one she can obtain in scenario $S_2$. Let $V_{R_2}$ and $V_{M_2}$ be the discounted profit of the typical retailer and the manufacturer respectively in setting $S_2$. Similarly, their discounted profit in setting $S_3$ is given by $V_{R_3}$ and $V_{M_3}$. The participants in the supply chain prefer colluding by using vertical information sharing if the following conditions hold:

$$
V_{R_3} \geq V_{R_2},
$$
$$
V_{M_3} \geq V_{M_2}.
$$

If both the manufacturer and the retailers prefer scenario $S_3$, one can assume that it would be the natural choice for the supply chain when the retailers try to collude without horizontal communication. The next proposition demonstrates that there are cases in which the constraints given in (23) are satisfied.

**Proposition 6** Assume the cartel is better off with the rigid pricing scheme in scenario $S_2$, and that $\psi > 3$. Then

$$
V_{R_3} \geq V_{R_2},
$$
$$
V_{M_3} \geq V_{M_2}.
$$

The proposition, which is an important result in this paper, suggests that there are cases in which the manufacturer agrees to receive the information sent by the retailers, and the retailers are better off sharing
their private information with the manufacturer. Proposition 4 demonstrates that there are cases, in which the cartel chooses in scenario $S2$ the rigid pricing scheme. In this case, the cartel sets the same price during all selling periods and the cartel can operate over an infinite horizon. When this is the case, the manufacturer knows that by refusing to share information she can expect to earn a profit of $\frac{A^2}{8}$ during all periods. By choosing not to share information, the manufacturer cannot increase the probability of price wars, since under the rigid pricing scheme the cartel is stable.

From the retailers’ perspective, choosing to share information enables them to benefit from the demand information. Choosing a rigid pricing scheme in scenario $S2$ solves the retailers' coordination problem. However, this problem is solved by ignoring the demand information available to the retailers. When the retailers share information with the manufacturer they are able to enjoy the value of better forecast of their demand. This proposition shows that indeed as Lee and Whang (2000) conjecture, firms can use vertical information sharing in order to facilitate collusion.

8 Conclusions

In this paper, I study the ability of a group of retailers to collude when they are endowed with private information about market demand. In order to establish a cartel, the retailers need to coordinate on the market price. Information sharing among competing firms is viewed by anti trust authorities as a facilitating practice and the retailers seek an alternative mechanism that would allow them to coordinate their strategies. In this paper I examine the retailers’ ability to share information with a mutual manufacturer to achieve this goal. The retailers share their private information with the manufacturer and use the wholesale price to determine the collusive market price. I show that there are cases in which establishing a cartel based on the information flow from the retailers to their mutual manufacturer results in higher profit to the retailers than any other traditional cartel formation mechanism. Furthermore, even if the manufacturer is aware of the fact that the retailers use her wholesale price in order to coordinate on a monopoly pricing scheme, there are cases in which it is beneficial for the manufacturer to agree to receive the retailers’ private information.

Information sharing has received considerable attention in operations management literature, and recent research has shown that in a complex environment in which there are multiple competing firms and one mutual manufacturer firms might be reluctant to share information since their private information can reach unintended recipients. In this paper, the retailers use the fact that their shared information can reach a third party in order to establish the cartel. Each retailer shares his private information with the manufacturer,
hoping that this information will be leaked to all the other competing retailers via the wholesale price. In this paper I highlight the positive effect, from the retailers perspective, of information leakage.

To the best of my knowledge this is the first paper that introduces a model in which retailers share information with the manufacturer in order to establish a cartel. There are a few interesting ways this research can be extended. First, I assume in my model that when information is shared between the retailers and between the retailers and the manufacturer, it is shared in a truthful manner. This assumption can be relaxed and it will introduce the question what is the incentive of the retailers of to share information truthfully, and how it affects the scope of collusion. Another possible research path can be to explore the incentives for information sharing when the manufacturer is part of cartel. In this paper I assume that the manufacturer sets a uniform wholesale price. One can consider different pricing schemes (quantity discount, two part-tariff etc) and examine how it affects the incentives of the retailers to share information with the manufacturer, and how it affects the scope of collusion.

References


Appendix

Proof of Lemma 1

Before proving lemma 1, it is helpful to use the following lemma.

Lemma A1: Assume $\delta \geq \frac{n-1}{n}$, then for $p^m(A_L, w)$, $IC - \phi$ is slack.

Proof: Assume for $\delta \geq \frac{n-1}{n}$ the retailers wish to set the price $p^m(A_L, w)$ during both periods of high demand and periods in which they received the non-informative signal. The IC constraints are given by:

$$\pi(A_H, p^m(A_L, w), w) + \frac{\delta}{1-\delta} V \geq \pi(A_H, \bar{p}_i, p_H, w) \text{ for any } \bar{p}_i \neq p^m(A_L, w);$$

$$E[\pi_i(p^m(A_L, w))|I_{\phi}] + \frac{\delta}{1-\delta} V \geq E[\pi(A_H, \bar{p}_i, p_{\phi}, w)|I_{\phi}] \text{ for any } \bar{p}_i \neq p^m(A_L, w).$$

Note that in both $IC - H$, and $IC - \phi$ the discounted future profit $\frac{\delta}{1-\delta} V$ is identical. However, for any given price $p$:

$$\pi(A_H, p^m(A_L, w), w) > E[\pi_i(p^m(A_L, w))|I_{\phi}],$$

the profit in a period with high demand is higher than the profit during periods with non-informative signal for the same price $p$. As a result, when the retailers can collude (i.e. $\delta \geq \frac{n-1}{n}$), they can always set the monopoly price during periods in which all retailers observe the non-informative signal.

We now return to the proof of lemma 1. By definition if $\delta \geq \tilde{\delta}(p^m, w)$ then the cartel can set the monopoly price during both information sets. This is clearly the price vector which maximizes the cartel profit. However, if $\delta \in \left[\frac{n-1}{n}, \tilde{\delta}(p^m, w)\right)$, the retailers can still collude, but they cannot set the monopoly price $p^m(A_H, w)$ during periods of high demand. The retailers must choose a different price than $p^m(A_H, w)$. By choosing a price $p > p^m(A_H, w)$ the retailers decrease their profit during periods of high demand and as a
consequence they increase the incentive to deviate from the cartel scheme. A retailer can still choose the price $p^m(A_H, w)$. However by choosing a price $p < p^m(A_H, w)$ the retailers decrease the incentive to deviate since any deviation rewards the deviating retailer with lower profit.

The last step is to show that there exists a price $p < p^m(A_H, w)$ which satisfies the IC constraints of the retailers during periods of high demand. Choose the price $p'$ such that

$$
\pi(A_H, p', w) = E[\pi_i(p^m(A_L, w))|I_\phi].
$$

When the retailers choose this price their profit during periods of high demand equals the profit during periods in which they receive the non-informative signal. Clearly $p' < p^m(A_H, w)$, and for this price vector both ICs constraints are identical and they are satisfied for any $\delta \geq \frac{n-1}{n}$.

**Proof of Lemma 2**

I show that for any wholesale price $w$, the retailers are able to set the monopoly price during periods in which they observe an informative signal. In the extreme case the manufacturer set the wholesale price $w$ so high that the retailers do not sell during periods in which they observe non-informative signals. In this case, the IC constraint during periods of high demand is

$$
\pi(A_H, p_H^m, w) + \frac{\delta}{1-\delta} \pi(A_H, p_H^m, w)\mu(1 - (1 - \rho)^n) \geq \pi(A_H, p_H^m - \varepsilon, w),
$$

(24)

which can simplified to the expression

$$
\delta \geq \frac{n-1}{n - (1 - \mu(1 - (1 - \rho)^n))} = \delta.
$$

Note that if the retailers are able to sell during periods in which they receive non-informative signals, the LHS of (24) is relaxed, and therefore the retailers are able to set the monopoly price during periods of high demand even for a lower discount factor. Also it is worth mentioning that based on Lemma 1 the retailers are always able to set the monopoly price during periods in which they observe non-informative signals, if the wholesale price allows them to make sales during these periods. Therefore, for any discount factor above $\delta$, regardless of the wholesale price, the retailers can set the monopoly price.
Proof of Proposition 1

(a) If $\delta < \frac{n-1}{n}$, the retailers are not able to collude. In this case, the retail price is equal to the wholesale price. The manufacturer solves the problem

$$\max_w [\mu (A_H - w) + (1 - \mu) (A_L - w)] w.$$ 

The solution to this problem is to set the wholesale price to $w = \frac{\mu A_H + (1 - \mu) A_L}{2}$.

(b) If $\delta$ is high enough, the retailers set the monopoly price during each information set. In this case $p_H = \frac{A_H + w}{2}$ and $p_\phi = \frac{\mu (1 - \rho)^n A_H + (1 - \mu) A_L}{2 [\mu (1 - \rho)^n + (1 - \mu)]} + \frac{w}{2}$. The sold quantity, from the manufacturer’s perspective is given by:

$$E[Q] = \mu (1 - (1 - \rho)^n) \left[ \frac{A_H - w}{2} \right]$$

$$+ \mu (1 - \rho)^n \left[ A_H - \frac{\mu (1 - \rho)^n A_H + (1 - \mu) A_L}{2 [\mu (1 - \rho)^n + (1 - \mu)]} - \frac{w}{2} \right]$$

$$+ (1 - \mu) \left[ A_L - \frac{\mu (1 - \rho)^n A_H + (1 - \mu) A_L}{2 [\mu (1 - \rho)^n + (1 - \mu)]} - \frac{w}{2} \right].$$

Simplifying the above expression the expected sold quantity is given by:

$$E[Q] = \frac{\mu A_H + (1 - \mu) A_L - w}{2}.$$ 

The manufacturer solves the problem

$$\max_w E[Q] w.$$ 

And the optimal wholesale price is given by $w = \frac{\mu A_H + (1 - \mu) A_L}{2}$.

(c) Consider the wholesale price $w = \frac{A_H}{2}$. I will show that for this wholesale price the FOC of the manufacturer’s optimization problem is positive, which implies that the optimal wholesale price must be higher. The manufacturer’s optimization problem can be expressed as:

$$\alpha \pi_M (A_H, p_H, w) + (1 - \alpha) \pi_M (A_\phi, p_\phi, w),$$
where \( A_\phi = \frac{\mu(1-\rho)^n A_H + (1-\mu) A_L}{\mu(1-\rho)^n + (1-\rho)} \) and \( p_\phi = \frac{\mu(1-\rho)^n A_H + (1-\mu) A_L}{2\mu(1-\rho)^n + (1-\rho)} + \frac{w}{2} \). The FOC of this problem is given by:

\[
\alpha \frac{\partial \pi_M(A_H, p_H, w)}{\partial w} + (1 - \alpha) \frac{\partial \pi_M(A_\phi, p_\phi, w)}{\partial w}.
\]

When \( p_H = \frac{A_H + w}{2} \) the FOC equals zero at the point \( w = \frac{A_H}{2} \), and the left term of the FOC is positive and the right term is negative. Furthermore the left-term is identical to the case in which \( \delta \) is high enough since the retailers set during periods of non-informative signal the price \( p^m(A, w) \). However, according to Lemma 1 \( p_H < p^m(A_H, w) \) which in turn implies that the right term is higher compared with the case in which the retailers set the price \( p^m(A_H, w) \). As a result the \( w > \frac{A_H}{2} \).

**Proof of Proposition 2**

(a) Assume the constraint in (16) is not binding. Then the FOC is given by:

\[
\frac{\partial \pi(A_H, p_H, w)}{\partial p_H} = 0 \quad \mu(1 - \rho^n) \frac{\partial \pi(A_H, p_\phi, w)}{\partial p_\phi} + (1 - \mu) \frac{\partial \pi(A_L, p_\phi, w)}{\partial p_\phi} = 0.
\]

The problem is concave, and hence the FOC are sufficient for optimality.

(b) A retailer who observes an informative signal and sets the price \( p_H \) earns an expected profit of \( \rho^{n-1} \pi(A_H, p_H, w) \).With probability \( \rho^{n-1} \) all other retailers observe the informative signal as well, and with probability \( (1 - \rho^{n-1}) \) at least one retailer observes the non-informative signal. In the latter case, a retailer setting the price \( p_H \) earns zero profit. A retailer who decides to deviate and set the price \( p_\phi \) earns the profit

\[
\sum_{k=0}^{n-1} p_k^{n-1} \frac{1}{k+1} \pi(A_H, p_\phi, w).
\]

Therefore, the cartel can implement the solution given in (a), if a retailer does not find it beneficial to deviate. This is given by the condition:

\[
\rho^{n-1} \pi(A_H, p_H, w) \geq \sum_{k=0}^{n-1} p_k^{n-1} \frac{1}{k+1} \pi(A_H, p_H, w).
\]
Proof of Proposition 3

When a retailer observes the informative signal his expected profit is given by

\[ \rho^{n-1} \pi(A_H, p_H, w) + \delta V + (1 - \rho^{n-1})(1 - \beta) \delta V. \]  

(25)

With probability \( \rho^{n-1} \) all other retailers observe the informative signal, and there is no price war. With probability \( (1 - \rho^{n-1}) \) at least one retailer observes the non-informative signal, and hence the cooperation continues with probability \((1 - \beta)\).

When a retailer observes the non-informative signal his expected profit is given by:

\[ \frac{(1 - \mu)}{1 - \mu + \mu(1 - \rho)} \pi(A_L, p_{\phi}, w) + \delta V + \frac{\mu(1 - \rho)}{1 - \mu + \mu(1 - \rho)} \sum_{k=0}^{n-1} P_k^{n-1} \pi(A_H, p^{k+1}_{\phi}, w) + (1 - \beta) \delta V. \]  

(26)

In the first case, demand is low, and hence all retailers set the price \( p_{\phi} \), and since demand is low cooperation continues in the next period. However with probability \( \frac{\mu(1 - \rho)}{1 - \mu + \mu(1 - \rho)} \) demand is high, and the retailer shares the market with all other retailers observing the non-informative signal. In this case, the cooperation continues only with probability \((1 - \beta)\).

Using equations (25) and (26) we can solve for \( V \):

\[
V = \mu \rho^n [\pi(A_H, p_{H}, w) + \delta V] + \mu \sum_{k=1}^{n} P_k^n \left[ \pi(A_H, p^{k}_{\phi}, w) + (1 - \beta) \delta V \right] + (1 - \mu) [\pi(A_L, p_{\phi}, w) + \delta V] \\
V = \frac{\mu \rho^n \pi(A_H, p_H, w) + \mu \sum_{k=1}^{n} P_k^n \pi(A_H, p^{k}_{\phi}, w) + (1 - \mu) \pi(A_L, p_{\phi}, w)}{1 - \delta (\mu \rho^n \beta + \mu(1 - \beta) + 1 - \mu)}
\]

The constraint \( IC - on - H \) is given by:

\[
\rho^{n-1} \pi(A_H, p_H, w) + \delta V + (1 - \rho^{n-1})(1 - \beta) \delta V \geq \sum_{k=0}^{n-1} P_k^{n-1} \pi(A_H, p^{k+1}_{\phi}, w) + (1 - \beta) \delta V. \]  

(27)

The LHS denotes the profit of a retailer who sets the price \( p_H \). With probability \( \rho^{n-1} \) all other retailers observe the informative signal, and they share the market and continue to cooperate. With probability \((1 - \rho^{n-1})\), at least one retailer observes the non-informative signal. In this case, a retailer observing the informative signal earns zero profit and the cooperation continues only with probability \((1 - \beta)\). The RHS denotes the profit of a retailer observing the informative signal, but deciding to set the price \( p_{\phi} \). In this
case, the retailer shares the market with all other retailers observing the non-informative signal, and the cooperation continues with probability \( (1 - \beta) \). One can write (27) as

\[
\rho^{n-1} \left[ \pi(A_H, p_H, w) \right] + \rho^{n-1} \beta \delta V \geq \sum_{k=0}^{n-1} P_k \rho^{n-1} \pi(A_H, p_{\phi}^{k+1}, w).
\]

The Lagrangian of the constrained problem is given by

\[
\mathcal{L}(p_H, p_\phi, \beta, \lambda) = \frac{\mu \rho^n \pi(A_H, p_H, w) + \mu \sum_{k=1}^{n} P_k \rho^n \pi(A_H, p_{\phi}^k, w) + (1 - \mu) \pi(A_L, p_\phi, w)}{1 - \delta (\mu \rho^n \beta + \mu (1 - \beta) + 1 - \mu)} + \lambda \left[ \rho^{n-1} \pi(A_H, p_H, w) + \rho^{n-1} \beta \delta V - \sum_{k=0}^{n-1} P_k \rho^{n-1} \pi(A_H, p_{\phi}^{k+1}, w) \right]
\]

Solving for the FOC with respect to \( p_H \) gives that:

\[
\frac{\partial \mathcal{L}}{\partial p_H} = \frac{\mu \rho^n}{1 - \delta (\mu \rho^n \beta + \mu (1 - \beta) + 1 - \mu)} \frac{\partial \pi(A_H, p_H, w)}{\partial p_H} + \lambda \rho^{n-1} \beta \delta \frac{\mu \rho^n}{1 - \delta (\mu \rho^n \beta + \mu (1 - \beta) + 1 - \mu)} \frac{\partial \pi(A_H, p_H, w)}{\partial p_H}.
\]

Note that at the point \( \frac{\partial \pi(A_H, p_H, w)}{\partial p_H} = 0 \) the FOC is also equal to zero. This proves the first part of the proposition, that the cartel sets the price for a retailer observing the informative signal at the point where \( \frac{\partial \pi(A_H, p_H, w)}{\partial p_H} = 0 \).

The FOC with respect to \( p_\phi \) gives that:

\[
\frac{\partial \mathcal{L}}{\partial p_\phi} = (1 + \lambda \rho^{n-1} \beta \delta) \frac{\mu \sum_{k=1}^{n} P_k \rho^n \frac{\partial \pi(A_H, p_{\phi}^k, w)}{\partial p_\phi} + (1 - \mu) \frac{\partial \pi(A_H, p_\phi, w)}{\partial p_\phi}}{1 - \delta (\mu \rho^n \beta + \mu (1 - \beta) + 1 - \mu)} - \lambda \sum_{k=0}^{n-1} P_k \rho^{n-1} \frac{\partial \pi(A_H, p_{\phi}^{k+1}, w)}{\partial p_\phi}.
\]

Note that at the point \( p_\phi^* \) (the price at the non-constrained problem) the first term is zero. The term \( \frac{\partial \pi(A_H, p_\phi, w)}{\partial p_\phi} |_{p_\phi = p_\phi^*} > 0 \), and hence the second term is negative. The value of \( \frac{\partial \mathcal{L}}{\partial p_\phi} |_{p_\phi = p_\phi^*} < 0 \), and hence at the constrained problem, the cartel reduces the price during periods in which a retailer observes a non-informative signal.

**Proof of Lemma 4**

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When the retailers choose a rigid pricing scheme, they choose the price $p$ by solving the problem

$$\max_p \mu(A_H - p)(p - w) + (1 - \mu)(A_L - p)(p - w).$$

The solution to this problem is $p = \frac{A_{\mu} + w}{2}$, and the expected sold quantity is $Q = \frac{A_{\mu} - w}{2}$.

The manufacturer solves the problem

$$\max_w E[Q]w = \frac{A_{\mu} - w}{2}.$$

The solution to the manufacturer’s problem is $w = \frac{A_{\mu}}{2}$, and the profits of the manufacturer and the retailers are as suggested in 4.

**Proof of Proposition 4**

At the separating pricing scheme $p_{\phi}$ is given as the solution to the following FOC:

$$\left[\mu(1 - p^n)\frac{\partial \pi(A_H, p_{\phi}, w)}{\partial p_{\phi}} + (1 - \mu)\frac{\partial \pi(A_L, p_{\phi}, w)}{\partial p_{\phi}}\right] = 0.$$ 

As the number of retailers increases we have that

$$\lim_{n \to \infty} p_{\phi} = p_{\mu},$$

assuming (17) holds. However, for $n$ high enough, (17) cannot hold, and hence as suggested by proposition 3, the cartel must lower the price during periods of non-informative demand information such that $p_{\phi} < p_{\mu}$, and perhaps also be engaged in price wars. Furthermore, when $n$ is high enough, the cartel is effectively setting the price $p_{\phi}$ during all periods of cooperation. Therefore, in this case the cartel is using a rigid pricing scheme, but must still satisfy the incentive constraints of the cartel members. Therefore, in this case it is better for the cartel to set the rigid pricing scheme and ignore the incentive constraints of the cartel members.

**Proof of Proposition 5**

The proof is very similar to that of Anand and Goyal (2009) proposition 1.
The optimization problem of the manufacturer when receiving only non-informative signals is:

\[
\max_{w_L} \frac{1 - \mu}{1 - \mu + \mu(1 - \rho)^n} \left( A_L - \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [1 - \mu + \mu(1 - \rho)^n]} - \frac{w_L}{2} \right) w_L \\
+ \frac{\mu(1 - \rho)^n}{1 - \mu + \mu(1 - \rho)^n} \left( A_H - \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [1 - \mu + \mu(1 - \rho)^n]} - \frac{w_L}{2} \right) w_L \\
s.t. \\
\left( A_H - \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [1 - \mu + \mu(1 - \rho)^n]} - \frac{w_L}{2} \right) w_L \leq \frac{A_H^2}{8}
\]

The Lagrangian for the above formulation is

\[
\mathcal{L}(w_L, \lambda) = \max_{w_L} \frac{1 - \mu}{1 - \mu + \mu(1 - \rho)^n} \left( A_L - \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [1 - \mu + \mu(1 - \rho)^n]} - \frac{w_L}{2} \right) w_L \\
+ \frac{\mu(1 - \rho)^n}{1 - \mu + \mu(1 - \rho)^n} \left( A_H - \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [1 - \mu + \mu(1 - \rho)^n]} - \frac{w_L}{2} \right) w_L \\
- \lambda \left[ \left( A_H - \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [1 - \mu + \mu(1 - \rho)^n]} - \frac{w_L}{2} \right) w_L - \frac{A_H^2}{8} \right]
\]

Define by \( A_\phi = \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [(1 - \mu + \mu(1 - \rho)^n]} \), which is the expected value of the demand intercept upon observing \( n \) non-informative signals.

The first-order conditions for the Lagrangian are:

\[
\frac{\partial \mathcal{L}}{\partial w_L} = A_\phi - w - \lambda \left( A_H - \frac{A_\phi}{2} - w \right) = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} = \left[ \left( A_H - \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [1 - \mu + \mu(1 - \rho)^n]} - \frac{w_L}{2} \right) w_L - \frac{A_H^2}{8} \right] \geq 0
\]

Solving for \( \lambda = 0 \), the unconstrained problem

\[
w_L = \frac{\mu(1 - \rho)^n A_H + (1 - \mu) A_L}{2 [1 - \mu + \mu(1 - \rho)^n]} = \frac{A_\phi}{2}
\]

This solution is feasible if

\[
\psi = \frac{A_H}{\mu(1 - \rho)^n A_H + (1 - \mu) A_L} \geq 3.
\]
We now turn to solve the case in which \( \lambda > 0 \). In this case

\[
wl = \frac{2A_H - A_\phi + \sqrt{3A_H^2 - 4A_H A_\phi + A_\phi^2}}{2}.
\]  

(28)

If \( \lambda > 0 \) it implies that for \( w_L = \frac{A_\phi}{2} \), a manufacturer who observes an informative signal prefers to set the low type. Note that the positive root in (28) is higher than \( \frac{A_\phi}{2} \), and hence it cannot be a candidate for a separating equilibrium. Therefore the only price that can support a separating equilibrium is the negative root; this is the price given in proposition 6.

**Proof of Proposition 6**

In scenario \( S_2 \) the profit of the retailers during each period is \( \Pi_i^{S_2} = \frac{A_2}{16} \). When they share information with the manufacturer their expected profit during each period is at least (if the manufacturer lowers the price during the periods with non-informative signals, the profit of the retailers is higher)

\[
\Pi_i^{S_3} = \mu \left[ (1 - (1 - \rho)^n) \frac{A_H^2}{16} \right] + \frac{(\mu(1-\rho)^n + (1-\mu)) [A_H \mu(1-\rho)^n + (1-\mu)A_L]^2}{16}.
\]

One can show that at the point \( \rho = 0 \), \( \Pi_i^{S_3} = \Pi_i^{S_2} \), and that \( \Pi_i^{S_3} \) is increasing in \( \rho \in (0, 1) \). Therefore, the retailers prefer to share information with the manufacturer over the rigid pricing scheme.

The manufacturer’s profit in scenario \( S_2 \) under the rigid pricing scheme is \( \Pi_M^{S_2} = \frac{A_2}{8} \), and in scenario \( S_3 \) it is

\[
\Pi_M^{S_3} = \mu \left[ (1 - (1 - \rho)^n) \frac{A_H^2}{8} \right] + \frac{(\mu(1-\rho)^n + (1-\mu)) [A_H \mu(1-\rho)^n + (1-\mu)A_L]^2}{8},
\]

if \( \psi \leq 3 \). Note that if \( \psi > 3 \), the profit of the manufacturer is lower, since she needs to lower the wholesale price during periods in which she receives only non-informative signals. Note that when \( \rho = 0 \), \( \Pi_M^{S_3} = \Pi_M^{S_2} \), and that \( \Pi_M^{S_3} \) is increasing in \( \rho \in (0, 1) \). Therefore, in this case the manufacturer is also better-off accepting information from the retailers.