Dynamic versus Static Pricing  
in the Presence of Strategic Consumers

Gérard P. Cachon • Pnina Feldman

Department of Operations and Information Management,  
The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA  
cachon@wharton.upenn.edu • pninaf@wharton.upenn.edu

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The stochastic nature of demand suggests that firms can benefit from applying dynamic pricing strategies, where pricing decisions are postponed until information about demand is revealed. Many service providers, however, announce prices in advance and do not frequently adjust them as a response to market conditions (i.e., static pricing). This may seem suboptimal when demand is high and the firm can support a higher price. Yet, the firm may actually be better off with static pricing when consumers are strategic and consider whether to visit based on the firm’s pricing strategy. With static pricing, consumers face a rationing risk (they may not obtain the unit) whereas with dynamic pricing, consumers face a price risk (they may need to pay a high price) and it may be better for a firm to impose a rationing risk on its customers, especially when consumers’ valuations are dispersed. The problem with dynamic pricing is the firm’s inability to commit to always leave consumers with positive surplus. This provides an explanation for why firms commit to low base prices and may be willing to dynamically lower their prices but are averse to raising their prices, even when heavy demand suggests they should. In addition, we show that the advantage of static pricing relative to dynamic pricing can be substantially larger than the advantage of dynamic pricing over static pricing. Finally, the relative attractiveness of static pricing may be further improved by offering reservations or providing availability guarantees. We conclude that static pricing can be better than dynamic pricing and even better is a strategy that marks down the price if necessary and never marks up the price.

1 Introduction

Companies can benefit from dynamic pricing, because of the stochastic nature of demand. By pricing dynamically, the firm delays its pricing decisions until after market conditions are revealed and can adjust prices accordingly. This seems to suggest that a firm which implements dynamic pricing will earn higher revenues than a firm that commits to a static price before learning the realization of demand.

Despite the apparent advantages, many firms do not adjust prices to respond to market conditions. Examples include: movie theaters charge a fixed price, regardless of whether the movie turned out to be a hit or a flop; restaurants do not adjust their menu prices depending on whether
it is a busy or a slow night; and sports teams keep the prices for games fixed, regardless on how well the team is performing, who the opponent is, or if the weather on a particular game day turns out to be bad.\footnote{A few exceptions exist. The San Francisco Giants of the MLB and the Dallas Stars of the NHL are experimenting with dynamic pricing techniques using a software developed by the Austin-based start-up, QCue (Branch 2009). It has been reported that for Giants’ tickets, “the price change will most likely be 25 cents to $1” (Muret 2008), where tickets range from $8 to $41. These price changes do not appear very significant and it is not yet clear how using the software affects these teams’ revenues.} Note that we are not claiming that firms do not try to take advantage of consumer heterogeneity by segmenting the market (charging different prices to different types of consumers). Movie theaters do charge lower prices for matinee shows, on weekdays and to students or senior citizens. Restaurants have different menus for lunch and dinner and the Philadelphia Phillies charge different prices to different seats and before/after Labor Day. These pricing decisions, however, are usually made before demand is realized and are presumably known to consumers in advance.

In the paper we are interested in explaining the following phenomenon: why is it that firms do not often adjust prices to respond to changing demand conditions? The academic literature has provided several alternative explanations to this phenomenon (which is sometimes referred to as \textit{price stickiness} or \textit{price rigidity}). Probably the most common explanation used by macroeconomists to explain price rigidity is the idea that firms incur fixed \textit{menu costs} to change prices (Mankiw 1985). If a firm incurs a fixed cost each time it changes prices, it may lead to fewer price changes than optimal. Menu costs were originally thought of as the physical costs for changing prices, such as the cost to reprint restaurant menus, printing new catalogs, attaching new tags to merchandise, etcetera. While these costs certainly exist, it is hard to believe that they are high enough to fully explain price rigidity. And, in fact, a number of empirical studies document that these costs are low (see e.g., Blinder \textit{et al.} 1998; Zbaracki \textit{et al.} 2004). In addition to physical menu costs, authors have also identified managerial costs (information gathering and decisions-making) and customer costs (communication and negotiation of new prices) as costs of price adjustments. In a study done on a large industrial firm, Zbaracki \textit{et al.} (2004) report that “the managerial costs are more than six times, and the customer costs of price adjustment are more than twenty times, the physical costs associated with changing prices”. However, as the authors note, these costs are “a characteristic of primarily a multi-product producer. They would either be non-existent or would, at most, be trivial in a setting of a single product producer”. Thus, we believe that the costs for price adjustment cannot fully explain price rigidity, especially not when it comes to examples such as restaurants, movie theaters and baseball teams.

An alternative explanation to sticky prices considers the psychology of consumers. Consumers dislike price changes (Hall and Hitch 1939) especially if they perceive the changes to be “unfair”.
Kahneman et al. (1986) found that potential consumers perceive increases in prices as a response to changes in demand as unfair (as opposed to, for example, price increases that are due to higher production costs). Therefore, firms keep their prices constant so as not to antagonize their consumers (Blinder et al. 1998).

While we think that firms do (or should) consider the psychology of consumers when they make pricing decisions, we question whether this can be the whole story. Or, can there be a different explanation? In other words, can charging a static price dominate dynamic pricing in the absence of menu costs, when the firm can costlessly obtain full information about demand and without accounting for consumer psychology, but rather assuming that consumers are rational? In this paper, we try to provide an answer to this question. We compare between dynamic and static pricing in the presence of strategic consumers. The firm decides on a pricing strategy. It either waits until potential demand is realized to decide on a price (“dynamic pricing”) or it charges a fixed price, which it commits to before observing demand (“static pricing”). Consumers in our model are strategic. They are aware of the firm’s pricing strategy and rationally choose whether to visit the firm. Visiting the firm, however, is costly to consumers: consumer incurs a physical cost to visit the firm (e.g., driving to the ball park or calling the restaurant to make reservations) and an opportunity cost, which is the outside option the consumer forgoes when visiting the firm. Therefore, a consumer chooses whether or not to visit the firm and this decision depends on the firm’s pricing scheme and on his expectation of the visiting decisions of other consumers. Under static pricing, consumers know what price they are going to be charged, but do not know for sure whether the product will be available, i.e., consumers face a rationing risk. Under dynamic pricing, consumers do not know how much they will have to pay for the product, i.e., they face a price risk (in addition to a possible rationing risk). Thus, the problem of comparing between static and dynamic pricing reduces to determining which type of risk or (combination of risks) is better to impose on consumers.

In addition to comparing between static and dynamic pricing, we consider a general set of pricing schemes in which the firm commits to a set of possible prices to charge prior to the realization of demand, of which it optimally chooses one price once demand is realized. Static and dynamic pricing are examples of pricing schemes which belong to this set. We characterize this set and find properties of pricing schemes which dominate both static and dynamic pricing within this set. This helps to further explain the problem with dynamic pricing.

The remaining of the paper is organized as follows. Section 2 sets up the preliminaries of the model. In Section 3 we review the relevant literature. Section 4 analyzes the static pricing scheme.
and Section 5 analyzes the dynamic pricing scheme, followed by a comparison of the two schemes in Section 6. Section 7 analyzes a set of general pricing schemes for which dynamic and static pricing are special cases and characterizes properties of this set, which are illustrative for understanding why dynamic pricing may not perform optimally. Section 8 concludes.

2 Model Description

On the demand side, there are two types of consumers. There is a potential number of \( X \) high value consumers, where \( X \) is a non-negative random variable with cumulative distribution function \( F(\cdot) \), pdf \( f(\cdot) \), complimentary cdf \( F(\cdot) = 1 - F(\cdot) \) and mean \( \mu = \mathbb{E}[X] \). The high value consumers are infinitesimal. They have value \( v_h \) and incur cost \( c < v_h \) to visit the firm. For example, consumers must drive down to the ball park, walk to the movie theater or call the restaurant to ask whether there is an available table. All of these actions involve some cost on the side of the consumers. The visit cost may not be a physical cost. It can also be interpreted as a mental cost to consider an alternative or an opportunity cost—the cost of forgoing an outside option when choosing to consider visiting the firm. Each high value consumer must decide whether to visit the firm or not. High-type consumers know the firm’s pricing strategy and choose whether or not to visit the firm. We analyze the high-type consumers’ purchasing decision in mixed strategies. We let \( \gamma \in [0, 1] \) be the probability that a high-type consumer visits the firm.

In addition to the high value consumers, there is also a sufficient demand of low value consumers, with value \( v_l \). We assume that these consumers do not incur a visit cost to the firm. This implies that the firm can always sell its entire capacity by charging \( v_l \). Another way to interpret \( v_l \) is as the amount which is equivalent to the maximum willingness to pay which guarantees the firm can sell its entire capacity regardless of market conditions. Table 1 summarizes the consumer types.

On the supply side, a single firm has \( k \) units of capacity and needs to determine how to price to maximize expected revenues. Consider the following definition of a pricing strategy. Let \( \mathcal{A} \subseteq \mathbb{R}^+ \) be a set of prices that the firm commits to prior to observing demand and from which the firm chooses the actual price after observing the demand realization. Recall that the high-type demand

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<tr>
<th>Segment</th>
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<th>Travel cost</th>
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<tr>
<td>High type (strategic)</td>
<td>( X \sim F(\cdot) )</td>
<td>( v_h )</td>
<td>( c )</td>
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<td>Low type (myopic)</td>
<td>( \infty )</td>
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distribution has support on $\mathcal{R}^+$ and the fraction of high-type consumers who visit, $\gamma$, has support on $[0, 1]$.

**Definition 1** A pricing strategy is composed of a set $A$ and a function $\Lambda(x, \gamma)$, where $A \subseteq \mathcal{R}^+$ is a set of preannounced prices and $\Lambda(x, \gamma) : \mathcal{R}^+ \times [0, 1] \to A$ maps every pair $(x, \gamma)$ of realized demand and the equilibrium fraction of consumers who visit to a price $p^*$ in the set of allowable prices $A$. $p^*$ is a revenue maximizing price, i.e., $p^* = \arg \max_{p \in A} \{ R(p, x, \gamma) \}$, where

$$R(p, x, \gamma) = \begin{cases} pk & p \leq v_l \\
 p \cdot \min \{ \gamma x, k \}, & if \ v_l < p \leq v_h \\
 0 & p > v_h \end{cases}$$

Definition 1 includes the set of all possible contingent pricing strategies under which the firm first commits to a set of allowable prices before the realization of demand and then it *optimally* chooses one of these prices, after observing the realized demand state. The assumption that a firm commits not to set a price $p \notin A$ is reasonable in many settings. This is especially true if consumers have repeated interactions with the firm and it fits well with the examples provided in the introduction. While we do not model the game as repeated, one can easily modify the analysis and find conditions for which committing to a static price is credible when the game is played repeatedly over an infinite horizon. (Such an analysis is performed by Liu and van Ryzin 2008.)

We combine the firm’s choice of a pricing strategy with the high-type consumer purchasing behavior and find the equilibrium of the game. The sequence of events is as follows: the firm announces a set of possible prices, $A$. Then, the event date approaches, potential demand of high type consumers, $X$, is realized and they decide whether or not to visit the firm. The firm observes the realization of demand, $x$, and sets a price $p^* \in A$ to maximize revenues. High type consumers who visited the firm, $\gamma x$, and low type consumers decide whether to purchase and try to purchase. Finally, the event date occurs and any leftover units are wasted.

Among all possible choices of $A$, in the first part of the paper, we analyze and compare between two special cases, which we refer to as *static* and *dynamic pricing*. Under static pricing, $A = \{ p_s \} \in \mathcal{R}^+$ and $\Lambda(x, \gamma) : \mathcal{R}^+ \times [0, 1] \to \{ p_s \}$, where $p_s$ is the price the firm commits to in advance and will be characterized in Section 4. Observe that because the firm committed to a single price, that is the price that will be charged regardless of the realization of demand. That is, with static pricing, the firm announces a single price $p_s$ before observing the realization of demand and consumers know this price before visiting the firm. This is equivalent to announcing prices to baseball games before the beginning of the season, committing to prices of movie tickets by posting them online in advance, or mailing detailed restaurant menus to consumers. Under dynamic pricing, the firm
commits to \( A = \mathbb{R}^+ \) and \( \Lambda(x, \gamma) : \mathbb{R}^+ \times [0, 1] \to \mathbb{R}^+ \), i.e., the firm essentially does not put any restriction on prices before observing the demand realization and is thus free to adjust the price to the realization of demand. In what follows, we describe the two pricing strategies, characterize the equilibrium joining behavior and compare expected revenues. We analyze both pricing schemes (Sections 4 and 5) and compare between them (Section 6). We find that static pricing can perform better than dynamic pricing despite its inability to adjust prices to respond to market conditions. To understand why dynamic pricing fails, in Section 7 we analyze the general pricing scheme, for which static and dynamic pricing are special cases.

3 Related Literature

There is a vast literature on dynamic pricing. This literature typically models a firm that sells to consumers whose valuations for the product are either heterogeneous or unknown. The firm is able to change prices over time and is concerned with determining what price it should charge at a given moment. By changing pricing dynamically, the firm can inter-temporally price discriminate between consumers (“price skimming”) and take advantage of the heterogeneity in consumer valuations. Our model is not concerned with price skimming. We assume that the firm sets a single price under both static and dynamic pricing. Instead, in our model of dynamic pricing, the firm uses information about demand to adjust the price.

Most research on dynamic pricing that takes strategic consumer behavior into account either models the consumer’s decision of when to purchase or the decision of what to purchase. We consider the question of whether to consider purchasing. We postulate that if customers incur a hassle cost, different pricing schemes can result in different effective demands, which can influence the benefit of setting a particular pricing scheme. More specifically, we find that announcing a static price before observing the realization of demand, makes consumers face a rationing risk, if they decide to visit the firm: they do not know whether the product will be available. However, setting the price dynamically, after observing demand, makes consumers face a price risk when they visit the firm: they do not know what price will be charged. This paper is concerned with comparing the two types of risks and establishing which risk is better to impose on consumers. Several papers have considered how the two types of risk affect the consumer trade-off between buying early at a high price or waiting for a markdown. In Liu and van Ryzin (2008) and Su and Zhang (2008), the firm commits to regular and discounted prices, but consumers face a rationing risk: consumers who decide to wait for the markdown may find the product out-of-stock. In Cachon
and Swinney (2009), consumers who decide to wait for a markdown face a price risk, because they are uncertain of how deep it will be. None of these papers, however, compare between the two types of risks.

In this paper, we compare between dynamic and static pricing and find that committing to a static price before learning the realization of demand can improve the firm’s revenues (relative to pricing dynamically). This is related to the literature on flexibility and price postponement (e.g., Van Mieghem and Dada 1999). Those models do not account for strategic consumer behavior and are concerned with the added value of ex-post pricing: when consumers are not strategic, the monopolist always achieves higher revenues with dynamic pricing, because it can use the information about demand to adjust prices. Instead, we find that when consumers are strategic and incur a cost to visit the firm, committing to a price before observing demand may improve the firm’s revenues by increasing expected sales.

Papers that compare between pricing dynamically and committing to prices when consumers are strategic include Besanko and Winston (1990), Dasu and Tong (2006), Cachon and Swinney (2008) and Aviv and Pazgal (2008). In all four papers, consumers decide whether to purchase at the full price or to wait for a future markdown. In a model with no capacity constraints, Besanko and Winston (1990) find that the firm benefits from committing to a price, because such commitment mitigates strategic waiting—if consumers believe that the price is not going to decline, they will not wait. Dasu and Tong (2006) compare between a posted pricing scheme and a contingent scheme when capacity is fixed, and thus consumers may face a rationing risk by waiting (under both pricing schemes). While by committing to a price path, the firm forgoes the flexibility of dynamic pricing, it also eliminates some options for the consumers. Overall, they find that when consumers are strategic neither scheme dominates, but that the difference between the two is small. Aviv and Pazgal (2008) consider a model in which arrivals are stochastic and valuations are decreasing in time. They show that committing to a price path can be better for the firm if the optimal price path is such that the discounted price is not much lower than the full price, so that strategic waiting is greatly reduced. In Cachon and Swinney (2008), price commitment results in charging a single price (no markdown) and thus eliminates strategic waiting, but also the opportunity to salvage inventory by selling to low value consumers. They find that dynamic pricing performs generally better than static pricing. Our model differs from these papers in that it does not involve strategic waiting or price segmentation (we limit both price schemes to a single price). Static pricing, in our paper, does not eliminate strategic behavior. Consumers decide whether to visit the firm under both pricing schemes taking into account the price/rationing risk which is imposed on them.
Other papers that compare between different pricing schemes when consumers are strategic include single versus priority pricing (Harris and Raviv 1981), subscription versus per-use pricing (Barro and Romer 1987; Cachon and Feldman 2009), and markdown regimes with and without reservations (Elmaghraby et al. 2006). In all of these papers the firm does not have perfect information about demand until after selecting the pricing strategy, whereas in our paper, we compare between setting a price either before or after demand is realized.

The results in our paper rely on the assumption that consumers incur a cost to visit the firm. This cost affects consumers demand in equilibrium under the two pricing schemes and is the reason that the comparison between the two schemes is not obvious. (Without visit costs, all consumers will attempt to obtain the product under both pricing schemes and dynamic pricing will dominate static pricing.) There are a few other papers in operations management that also assume that consumers incur a travel cost to visit the firm or, equivalently, have a non-negative outside option. None of these papers, however, compare between dynamic and static pricing. Dana and Petruzzi (2001) explore the stocking decisions of a firm in the presence of strategic consumers who anticipate the probability to obtain the product. They show that a firm which internalizes the effect that expected availability has on demand, holds more inventories. In our model, the capacity level is fixed and the firm faces a pricing problem. Su and Zhang (2009) analyze the effect of product availability on consumer behavior and firm’s profits using a rational expectations (RE) equilibrium in which the firm cannot commit to availability. They compare the RE equilibrium to the equilibrium obtained where the firm is able to credibly commit to the information provided and show that the commitment equilibrium improves profits. Su and Zhang (2009) also show that providing availability guarantees to compensate consumers in case of a stock-out can enhance profits. The firm, in our model, has a fixed capacity which we assume to be verifiable. In addition, we assume that the firm can credibly commit to charge one price out of a preannounced set of prices. We view posted prices as a type of implicit contract between the firm and consumers. Finally, two recent papers examine restaurant reservations. Alexandrov and Lariviere (2008) explore whether a restaurant should allow reservations. They find that reservations may be recommended because they lead to increased sales: when there is aggregate demand uncertainty, reservations encourage more consumers to join in slow nights, because they are guaranteed a seat (while fewer consumers join on busy nights, because consumers with reservations may become “no shows”); when an evening has peak and off-peak periods, reservations provide information to consumers and shift demand to off-peak periods. Cil and Lariviere (2007) investigate how many seats to allow for reservations when a firm sells to two segments. Both papers assume that prices are fixed, whereas in our model,
the firm controls demand by adjusting the price. We describe the effects of allowing reservations
and providing availability guarantees on the comparison between static and dynamic pricing in
Section 6.

4 Static Pricing Strategy

With a static price strategy, the firm preannounces a single price \( A = \{p\} \in \mathcal{R}^+ \) before observing
demand, which it will set regardless of demand conditions. Therefore, consumers know what price
will be charged before deciding whether or not to visit the firm. In this section we characterize
\( p_s \).

Let \( p \) be the price the firm announces. High value consumers know that if they visit the firm
and are able to obtain the unit, they get \( v_h - p - c \). If, however, they visit the firm and cannot
obtain the unit, they get utility \( -c \). Thus, the value of visiting the firm depends on the chance of
obtaining a unit. Let \( \phi \) be the high value customer’s expectation for the probability of getting a
unit. The customer is indifferent between visiting the firm and the outside option, if

\[
\phi (v_h - p) = c. \tag{1}
\]

We search for a mixed strategy equilibrium and let \( \gamma \) be the probability that a high type customer
attaches to visiting the firm. Because consumers are infinitesimal, in equilibrium, \( \gamma \) determines the
fraction of high type consumers who visit the firm, and \( \phi \) is the customer belief of the probability
of obtaining a unit if he decides to visit and is a function of the fraction \( \gamma \) of high value consumers
visiting the firm.

In determining how the expectation of \( \phi \) is set, we assume that the belief about the probability is
consistent with the equilibrium outcome. That is, the consumer’s expectation about the probability
to obtain the unit is rational and coincides with the actual probability that a high type consumer
obtains the unit, if he visits the firm. To calculate \( \phi \), we must specify a rationing rule that defines
how the units are allocated amongst high type consumers. We make two assumptions regarding
the allocation of units. First, we assume that high value consumers can obtain the unit before
low value consumers. This assumption is reasonable if high value consumers are more eager to get
the unit compared to the low value consumers. As such, they should have a higher probability for
obtaining the product. (See Su and Zhang (2008) and Tereyağoğlu and Veeraraghavan (2009) for
a similar assumption.) This assumption simplifies the analysis, but does not drive the results. In
fact, as we will later show, any other allocation may only strengthen our main result. (See Cachon
and Swinney 2009 for a different allocation assumption.) Second, we assume that if high value
demand exceeds supply, the units are allocated randomly among high value consumers. Given this allocation rule, the probability that a high value consumer obtains a unit is equivalent to the fill rate conditional on the high value consumer being in the market. That is, we require that in equilibrium

\[ \phi = \frac{S_{X}(k)}{\gamma \mu} = \frac{S(k/\gamma)}{\mu}, \]  

(2)

where \( S_{D}(q) = \mathbb{E}_{D}[\min\{D,q\}] \) is the sales function. We write \( S(\cdot) \) as shorthand for \( S_{X}(\cdot) \).

Observe that the expression for the fill rate given in (2) is different from the unconditional expression for the fill rate, which does not consider the presence of a particular consumer in the market (see Deneckere and Peck (1995) and Dana (2001) for a complete derivation of the conditional fill rate).

For consumers to calculate the probability according to (2), they need to know the capacity level and the distribution of demand and have correct beliefs about the fraction of consumers who visit the firm (which, as we will see, is a function of the price the firm announced and can be inferred rationally given the other information).\(^2\)

Observe that under static pricing, consumers know the price they will be charged before visiting the firm, but they are unsure whether the unit will be available—thus, under static pricing consumers face a rationing risk when they visit the firm.

After characterizing \( \phi \), we may now turn to determine the fraction of high type consumers who visit the firm in equilibrium. Depending on the static price, \( p \), there are two possibilities to consider. If the static price is small enough, all high type consumers visit the firm, i.e., \( \gamma = 1 \). That occurs if

\[ \frac{S(k)}{\mu}(v_h - p) \geq c \]

or

\[ p \leq v_h - \frac{\mu c}{S(k)} = \overline{p}. \]

If \( p > \overline{p} \), high type consumers will choose \( \gamma < 1 \), which satisfies

\[ S(k/\gamma) = \frac{\mu c}{v_h - p}. \]

(3)

Observe that for every price \( p \) there exists a unique \( \gamma(p) \) that satisfies (3). In addition \( \gamma(p) \) is decreasing in \( p \). Therefore, finding the optimal static price is equivalent to “choosing” the fraction

\(^2\)Alternatively, we can use the rational expectations equilibrium concept, which does not put restrictions on information available to consumers. All it requires is that the consumer expectation with respect to the availability probability, \( \phi \), is consistent with the actual probability. Whether we assume that consumers know all required information or not, the resulting equilibria will be identical. Thus, for the rest of the paper we will not worry about the amount of information consumers have.
of high value consumers that visit the firm. The firm’s revenue is

$$R_h = S_{\gamma X}(k)p$$

$$= \gamma S(k/\gamma) \left( v_h - \frac{\mu c}{S(k/\gamma)} \right).$$

The next lemma finds the equilibrium fraction of consumers who visit the firm under static pricing, $\gamma_s$.

Lemma 1 The firm’s revenue function under static pricing, $R_h$, is quasi-concave. To maximize $R_h$, the firm sets a price, $p^h$, such that (i) if $v_h \int_0^k x f(x) dx \geq \mu c$, $\gamma_s = 1$ and $p^h = \bar{p}$; or (ii) otherwise, $\gamma_s$ is the unique solution to

$$v_h \int_0^{k/\gamma_s} x f(x) dx = \mu c$$

and $p^h = v_h - \frac{\mu c}{S(k/\gamma_s)}$.

Lemma 1 determines the optimal fraction of high value consumers who visit the firm in equilibrium. Note that $v_l$ does not factor into this outcome. If the cost to visit the firm, $c$, is low enough all customers visit the firm. In these cases, it is optimal to price at $\bar{p}$ so that all consumers visit the firm. Otherwise, only a fraction of these consumers visit and this fraction is given by the solution to (5). Furthermore, if the solution is interior, the optimal revenue from selling to high value consumers is $R_h^* = kv_h F(k/\gamma_s)$. Finally, note that the firm can either set a price $p > v_l$ and sell to a fraction of high value consumers (following Lemma 1) or ignore the high type consumers and sell its entire capacity at a price $v_l$ (which is possible, because there is a sufficient number of low value consumers). If it decides to do that, it obtains a revenue of $v_l k$. Thus, $v_l k$ is a lower bound on the firm’s revenue. The firm’s optimal price, $p_s = \max \{p^h, v_l\}$. It is the price that achieves the maximum revenue between selling only to a fraction of high type consumers (charging the price $p^h$ of Lemma 1) or setting a low price, $v_l$, and serving both consumer types. We denote the optimal revenue under static pricing by $R_s^*$. The function $R_s^*$ is piecewise linear in $v_l$. It is constant for low values of $v_l$, because the fraction of high-type consumers who visit the firm does not depend on $v_l$. However, when $v_l$ is high enough, the firm is better off charging $v_l$, selling its entire capacity and obtaining $v_l k$. Hence, for high $v_l$ values, $R_s^*(v_l)$ increases linearly with a slope of $k$. 

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5 Dynamic Pricing Strategy

With dynamic pricing, the firm preannounces the set $A = \mathcal{R}^+$ from which it will choose the optimal price after observing the realization of high type demand. The firm, is therefore, essentially free to set any price to adjust to demand conditions. After observing the realization of demand, the firm has two non-dominated options. As with static pricing, the firm can price at $p = v_l$ and earn revenue $v_l k$. Alternatively, it can price at $p = v_h$ and earn revenue $v_h \gamma x$, where $\gamma$ is the fraction of consumers who visit the firm and $x$ is the realization of demand. Note that once $\gamma x$ high-type consumers visit, setting any other price but $v_l$ or $v_h$ is suboptimal. Consequently, the firm will price at $v_l$ when

$$x \leq \frac{v_l k}{v_h \gamma},$$

which has probability $F(v_l k/(v_h \gamma))$.

Observe that high value consumers only earn positive utility if the price is $v_l$ and they are able to obtain the unit. In all other cases, consumers get zero surplus. Thus, to find the high-type consumer’s surplus from visiting the firm, we let $\psi$ be the high-type consumer’s expectation for the probability that the firm charges $v_l$ and he is able to get the unit. A high value consumer is indifferent towards visiting the firm if

$$\psi (v_h - v_l) = c.$$

As in the discussion of Section 4, in equilibrium, the belief about the probability $\psi$ has to be consistent with the actual probability. According to the rationing rule defined above, a high type customer is always served before all low type consumers. Because $v_l$ is charged only when $\gamma x \leq \frac{v_l}{v_h} k < k$ (from (6)), high type consumers are guaranteed to get the unit at a low price. Thus, under dynamic pricing consumers do not face a rationing risk if the price is $v_l$ (they may face a rationing risk if the price if $v_h$, but in this case, their surplus is zero, so it does not affect the expected surplus). Consumers do face a price risk—high value consumers know that if they visit the firm, they will be able to obtain the unit if the price is low, but they do not know what price will be charged. Note that without making this priority assumption, high-type consumers may not be guaranteed to obtain a unit at a low price, and thus they may face a rationing risk in addition to facing a price risk. Under a different allocation rule, a lower fraction of high-type consumers will visit the firm. Thus, when making this assumption, the firm’s revenue under dynamic pricing is an upper bound.

The actual probability of a high type consumer to obtain a unit at a price of $v_l$ therefore becomes
the probability that the firm charges that price, conditional on the high type consumer being in the market. Because the market size, \( X \), is uncertain, conditional on his presence in the market, a high type consumer’s demand density is \( xf(x) / \mu \) (following Deneckere and Peck 1995). Therefore, this consumer anticipates that the probability that the price is \( v_l \) is

\[
\psi = \frac{\int_{0}^{\frac{v_l}{v_h}} x f(x)}{\mu}.
\]

Note that \( \psi \leq F(v_l k/(v_h \gamma)) \): if a high type customer is in the market, the probability that the demand is low (which implies that the price charged is \( v_l \)) is lower than the unconditional probability.

If \( v_l < v_h - c \), then there exists some \( \gamma \) that satisfies (8). If \( v_l \geq v_h - c \), then \( \gamma = 0 \) is the optimal strategy for consumers. \( v_l \) is the minimum price that the firm would charge. If the benefit that a high type customer can gain is lower than the minimum price, he will never visit the firm.) In the rest of this paper we concentrate on the interesting case and assume that \( v_l < v_h - c \). Let \( \gamma_d \) be the fraction of high-type consumers who visit the firm in equilibrium under dynamic pricing. The following lemma characterizes \( \gamma_d \).

**Lemma 2** The fraction of high-type consumers who visit the firm in equilibrium, \( \gamma_d \), is unique. Furthermore, (i) \( \gamma_d = 1 \), if

\[
\int_{0}^{\frac{v_l}{v_h}} x f(x) \geq \frac{\mu c}{v_h - v_l};
\]

and (ii) otherwise, \( \gamma_d \) is the solution to

\[
(v_h - v_l) \int_{0}^{\frac{v_l}{v_h}} \gamma_d x f(x) = \mu c.
\]

Lemma 2 determines the fraction of high-type consumers who visit the firm under dynamic pricing. Observe that while under static pricing consumers face a rationing risk–high value consumers know what price they will be charged–but are unsure whether they will be able to obtain a unit, under dynamic pricing consumers face a *price risk*. High value consumers know that if they visit the firm, they will be able to get the unit at \( v_l \) (they do not face a rationing risk), but they do not know at what price.

Observe, that while the value of \( v_l \) did not factor into the solution of \( \gamma_s \), it definitely affects the fraction of high-type consumers who visit the firm under dynamic pricing. The next result establishes the shape of \( \gamma_d (v_l) \) in equilibrium and the equilibrium values of \( \gamma_d \) in the boundaries of \( v_l \).

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Lemma 3 If \( F(\cdot) \) is an increasing generalized failure rate (IGFR) distribution, the fraction of consumers who visit the firm in equilibrium under dynamic pricing, \( \gamma_d(v_l) \), is quasi-concave. Furthermore, the following limits hold: (i) \( \lim_{v_l \to 0} \gamma_d(v_l) = 0 \); and (ii) \( \lim_{v_l \to v_h - c} \gamma_d(v_l) = 0 \).

Lemma 3 shows that when \( v_l \) is either very low or very high, high-type consumers do not visit the firm under dynamic pricing. If \( v_l \to v_h - c \), consumers know that whether the price charged is \( v_l \) or \( v_h \), they will obtain no utility from the product, and therefore they decide not to visit. When \( v_l \to 0 \) high-type consumers can potentially obtain the highest surplus. However, in this case, consumers anticipate that the probability that the firm will set the price at \( v_l \) is negligible. Hence, they decide not to visit the firm.

The firm’s revenue with dynamic pricing is

\[
R_d^* = F\left( \frac{v_l}{v_h} \frac{k}{\gamma_d} \right) v_l k + v_h \gamma_d \int \frac{k}{\gamma_d} x f(x) \, dx + F\left( \frac{k}{\gamma_d} \right) v_h k
\]

\[
= F\left( \frac{v_l}{v_h} \frac{k}{\gamma_d} \right) v_l k + v_h \gamma_d \left( \frac{k}{\gamma_d} F\left( \frac{k}{\gamma_d} \right) - \frac{v_l}{v_h} \frac{k}{\gamma_d} F\left( \frac{v_l}{v_h} \frac{k}{\gamma_d} \right) - \int \frac{k}{\gamma_d} F(x) \, dx \right) + F\left( \frac{k}{\gamma_d} \right) v_h k
\]

\[
= v_h k - v_h \gamma_d \int \frac{k}{\gamma_d} F(x) \, dx
\]

\[
= v_h \gamma_d \left( \frac{k}{\gamma_d} - \int_0^\frac{k}{\gamma_d} F(x) \, dx + \int_0^\frac{v_l}{v_h} \frac{k}{\gamma_d} \int_0^\frac{v_l}{v_h} \frac{k}{\gamma_d} F(x) \, dx - \int_0^\frac{v_l}{v_h} \frac{k}{\gamma_d} F(x) \, dx + \int_0^\frac{v_l}{v_h} \frac{k}{\gamma_d} \right)
\]

\[
= v_l k + v_h \gamma_d \left( S\left( \frac{k}{\gamma_d} \right) - S\left( \frac{v_l}{v_h} \frac{k}{\gamma_d} \right) \right),
\]

where \( \gamma_d \) is characterized in Lemma 2. Observe that with dynamic pricing, the firm does not have control over the fraction of high-type consumers who visit the firm, \( \gamma_d \). While under static pricing, the firm can change the price and influence \( \gamma_s \), under dynamic pricing consumers know that the firm will charge either \( v_l \) or \( v_h \) to optimally adjust to demand conditions. Because consumers anticipate that the firm will optimally respond to the realization of demand, the firm loses control over \( \gamma_d \). The next lemma characterizes the revenue function at the boundaries of \( v_l \).

Lemma 4 The following limits hold: (i) \( \lim_{v_l \to 0} R_d = 0 \); and (ii) \( \lim_{v_l \to v_h - c} R_d = (v_h - c) k \).

6 Comparison

In this section we compare between static and dynamic pricing. Because when pricing dynamically the firm is able to adjust the price according to demand conditions, it may seem that dynamic pricing should always perform better than static pricing. However, in this section we show that this may
not always be the case and that a static pricing scheme can actually be better. Comparing between
these two pricing schemes amounts to determining which risk is better to impose on consumers—is it
to impose a rationing risk or a price risk on consumers? The next theorem compares
between the fraction of consumers who visit the firm under both pricing schemes.

**Theorem 1** The fraction of consumers who visit the firm under dynamic pricing is lower than
under static pricing—i.e., $\gamma_d(v_l) \leq \gamma_s$ for every $v_l$.

According to Theorem 1 the fraction of consumers who visit the firm under static pricing is
always greater than the fraction of consumers who visit the firm under dynamic pricing. Thus, the
price risk faced by consumers is larger than the rationing risk. If consumers incur a cost to visit
the firm, fewer consumers will visit the firm if they know that the firm prices dynamically than if
they anticipate a fixed price. We show that this result does not hold for every $v_l$ when high-type
consumers have heterogenous values. This, however, will not qualitatively affect our main result.

To understand the difference between the visiting behavior in equilibrium under the two pricing
schemes, observe that, in equilibrium, the fraction of consumers who visit the firm under static
pricing is given by (5) and can be written as

$$v_h \cdot \frac{\int_0^{v_h} x f(x) \, dx}{\mu} = c,$$

which can be interpreted in terms of a price risk as well. The left-hand side of (10) is equivalent
to the utility of a high-type consumer when the firm sets a price at $v_h$ if the effective demand, $\gamma_a x$, is
higher than capacity and a price of 0, otherwise. The firm’s revenue, $kv_h F(k/\gamma_s)$, can be easily
re-interpreted according to these prices as well. With dynamic pricing, the fraction of consumers
who visit the firm is given by (9) and can be written as

$$(v_h - v_l) \frac{\int_0^{v_h} x f(x) \, dx}{\mu} = c.$$

Comparing between the price risk faced by consumers under both pricing schemes we observe that
the price risk faced under dynamic pricing is higher. To see this, assume that the fraction of
consumers who visit the firm under both pricing schemes is the same—that is, fix $\gamma$—and look at the
left-hand sides of conditions (10) and (11). If the effective demand is the same under both pricing
schemes, with dynamic pricing consumers benefit less if the firm sets a low price (because $v_l \geq 0$)
and the probability of getting a low price is lower (because $v_l < v_h$). Both of these effects imply
that with static pricing more consumers visit the firm in equilibrium.
It is also illustrative to establish how the change in $k$ and $c$ affect the consumers’ visiting behavior. This is summarized in the next Theorem.

**Theorem 2** The following hold: (i) $d \gamma_s / dc \leq 0$ and $d \gamma_d(v_l) / dc \leq 0 \forall v_l$; and (ii) $d \gamma_s / dk \geq 0$ and $d \gamma_d(v_l) / dk \geq 0 \forall v_l$.

Theorem 2 determines that as the cost to visit the firm decreases, more consumers visit the firm under both pricing schemes. This is intuitive–when $c = 0$, all consumers visit the firm no matter what the pricing scheme is. Furthermore, as the capacity level increases, more consumers visit the firm under both pricing schemes. With static pricing, more capacity corresponds to a higher product availability–i.e., a lower rationing risk. With dynamic pricing, consumers are guaranteed to obtain the unit. However, a higher capacity level implies that there is a higher probability that the firm sets $p = v_l$ – i.e., a lower price risk.

Figure 1 illustrates the fraction of high-type consumers who visit the firms under static pricing ($\gamma_s$) and under dynamic pricing ($\gamma_d$) as a function of $v_l$ for $X \sim Gamma(1, 1)$, $v_h = 1$, and under different values of capacity, $k$ and visit costs $c$. Observe first that the fraction of consumers who visit the firm under static pricing is always greater than the fraction under dynamic pricing (Theorem 1), that the fraction of consumers who visit under static pricing is independent of $v_l$ and that no consumers visit when $v_l$ is either high or low with dynamic pricing (Lemma 3). Furthermore, observe that the fraction of consumers who visit the firm increases with the capacity level and decreases with the cost of visit for both pricing schemes (Theorem 2).

Next, we compare between the revenue functions under both pricing schemes. The following result establishes that static pricing can perform better than dynamic pricing if the value of $v_l$ is low enough.

**Corollary 1** There exists a $\tilde{v}_l$, such that $R^*_s(v_l) > R^*_d(v_l)$ for all $v_l < \tilde{v}_l$. Further, $R^*_s(v_h - c) = R^*_d(v_h - c) = (v_h - c) k$.

It is difficult to provide additional comparative results analytically, because the function $\gamma_d(v_l)$ is not monotone in $v_l$ and because usual machinery (such as the Envelope Theorem) cannot be applied on the dynamic pricing revenue function, $R^*_d$, which was not obtained through optimization, but rather is a consequence of equilibrium behavior. Still, we are able to show additional results numerically. We conducted a numerical study and summarize the results in the remaining of this section. (Details of the numerical study are outlined in the technical appendix.) First, we observe numerically that $\tilde{v}_l$ is unique, so that $R^*_s(v_l) > R^*_d(v_l)$ for all $v_l < \tilde{v}_l$ and $R^*_s(v_l) \leq R^*_d(v_l)$ for all
Figure 1. Fraction of high-type consumers who visit the firm under static pricing ($\gamma_s$) and dynamic pricing ($\gamma_d$) as a function of $v_l$ for $X \sim \text{Gamma}(1, 1)$, $v_h = 1$, and different values of capacity, $k$ and visit costs, $c$.

$v_l \geq v_h$. Figure 2 illustrates the revenue functions under both pricing schemes as a function of $v_l$. With static pricing, $R^s$ is first constant—for low values of $v_l$ consumer visiting behavior and the fixed price do not depend on $v_l$. Then, it linearly increases with $v_l$—when $v_l$ is high enough the firm is better off by charging $v_l$ and selling its entire capacity. With dynamic pricing, we observe that $R^d$ increases monotonically with $v_l$. Note that under dynamic pricing, an increase of $v_l$ affects the firm in two opposite ways. First, similarly to static pricing, as $v_l$ increases, the firm has more incentive to set a price of $v_l$ and sell its entire capacity. But also, as $v_l$ increases, fewer high-type consumers visit the firm and in fact, in the limit, when $v_l \to v_h - c$, $\gamma_d \to 0$ (Lemma 3). Combining these two effects, we observe that the first effect dominates.

Comparing between the two revenue functions, Corollary 1 shows that when $v_l \to v_h - c$ both pricing schemes yield the same revenue. This is not surprising—for this value, the firm charges $v_l$ and sells its capacity under both pricing schemes. We also find that for relatively low values of $v_l$, static pricing performs better than dynamic pricing. This is because when $v_l$ is low, high-type consumers anticipate that the probability that the firm will charge $v_l$ is low and therefore they decide not to visit the firm. This does not happen under static pricing, where the firm commits to a fixed price, which consumers are aware of. From a managerial perspective, this seems to suggest that firms who sell products for which consumer values are variable, need not concern themselves with dynamic pricing: high heterogeneity in valuations favors static pricing. Observe also that
the advantage of $R^*_s$ over $R^*_d$ (when $v_l < \tilde{v}_l$) is more significant than the advantage of $R^*_d$ over $R^*_s$ (when $v_l > \tilde{v}_l$). Thus, the revenue loss of a firm which commits to a static price whenever dynamic pricing is better, is relatively modest. Taking into account the additional menu costs of changing prices and consumers’ aversion to price changes strengthens this point ever further.

The main result, which states that static pricing can perform better than dynamic pricing for low enough values of $v_l$ is robust. Our base models assumes that high-type consumers are homogeneous in visit costs and in values. We find that relaxing these assumptions to allow for heterogeneity in visit costs or heterogeneity in high-type values does not alter this result. Assuming that visit costs differ among consumers leads to a threshold equilibrium of purchasing behavior (instead of a mixed strategy one)– high-type consumers with low visit costs, visit the firm and those with high visit costs do not. All of the results follow through. When the high-type values are heterogeneous, it may be that more high-type consumers visit under dynamic pricing (Theorem 1 need not hold), but static pricing still performs better than dynamic pricing when $v_l$ is low enough. We provide a complete analysis of the heterogeneous cases in the technical appendix.

We are also interested in answering how different selling strategies affect the comparison between the two pricing schemes. In particular, we investigate how providing availability guarantees and allowing consumers to make reservations influence dynamic and static pricing. When the product requested by consumers is out of stock, firms may offer some compensation to consumers who tried to purchase, but found the unit to be unavailable. Firms can, for example, hand out coupons for future purchases, offer the next best alternative or ship the requested unit at a later time. Alternatively, many service providers allow consumers to make reservations. Assuming that there is no overbooking, a customer that is able to reserve is guaranteed to obtain the unit, if he visits the firm. Both selling strategies provide consumers with a guarantee to get the unit or a compensation
if they cannot get the unit. Thus, they reduce consumers’ rationing risk. Thus, these selling strategies are likely to improve static pricing, but are unlikely to influence dynamic pricing, because under dynamic pricing high type consumers do not face a rationing risk. Thus, a firm which prices dynamically need not offer availability guarantees or the ability to reserve. This conclusion is, of course, an abstraction. There are many alternative reasons why a firm may want to offer coupons to consumers—e.g., it may give them an incentive to visit the firm again.

Finally, we investigate how the value \( \tilde{v}_l \) changes with respect to changes in the parameters. The change in this value determines the effect of the parameters on the range in which static pricing dominates dynamic pricing.

**Effect of capacity.** The level of capacity affects consumer visiting behavior under both pricing schemes (Theorem 2). With static pricing, as \( k \) decreases, the probability to obtain the unit decreases. That is, consumers face a higher rationing risk. With dynamic pricing, consumers do not face a rationing risk. However, as \( k \) decreases, the probability that the firm charges \( v_l \) decreases. That is, consumers face a higher price risk. Under both pricing schemes, the decrease of \( k \) negatively affects consumers visiting behavior, which influences revenues. We observe that \( \tilde{v}_l \) increases when the level of capacity decreases implying that the price risk effect is stronger than the rationing risk.

**Effect of visit cost.** The next lemma demonstrates results in the limits of \( c \).

**Lemma 5** The following hold: (i) When \( c = 0 \), \( \gamma_s = \gamma_d = 1 \) and \( R^*_s(v_l) \leq R^*_d(v_l) \) \( \forall v_l \); and (ii) \( \lim_{c \to v_h} \gamma_s = \lim_{c \to v_h} \gamma_d = 0 \) and \( R^*_s(v_l) = R^*_d(v_l) = v_l k \) \( \forall v_l \).

Lemma 5 shows that when the cost to visit the firm is either negligible or very high, dynamic pricing dominates static pricing. When \( c \to 0 \), all high-type consumers visit the firm regardless of the pricing strategy. In this case, dynamic pricing will naturally always perform better. When \( c \to v_h \), the visit cost is so high that high-type consumers do not visit the firm. Thus, under both pricing schemes the firm is better off charging \( v_l \) and selling all its capacity. Finally, we observe that as the visit cost, \( c \), increases, \( \tilde{v}_l \) first increases and then decreases. Therefore, the range of \( v_l \) for which static pricing dominates is the largest for intermediate values of \( c \). Figure 3 illustrates this, by plotting \( \tilde{v}_l / (v_h - c) \) as a function of \( c \) for different capacity levels, where \( X \sim \text{Gamma} (1,1) \) and \( v_h = 1 \). Note that the value of \( \tilde{v}_l / (v_h - c) \) measures the fraction below which static pricing

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3 Recall that in our base model, we assumed an efficient allocation of capacity, which led all high value strategic consumers to receive the unit under dynamic pricing. A different allocation may result in consumers facing a rationing risk as well, implying that it would be beneficial for the firm to offer compensation even under dynamic pricing. Note, however, that any allocation other than an efficient one will lead to lower revenues for the firm—a customer should prefer to get the original requested unit rather than the firm’s compensation.
performs better than dynamic pricing. Each line represents the value of $\tilde{v}_l/(v_h - c)$ for a different capacity level. For example, when $k = 2$ and $c = 0.2$, static pricing is strictly better than dynamic pricing in 30% of the $v_l$ parameter range. Observe also that as the capacity level decreases, the range for which static pricing dominates, increases, as was previously discussed.

**Effect of coefficient of variation.** Assuming that the number of high-type consumers is either $X \sim \text{Gamma}(\alpha, \beta)$ ($\alpha > 0$) or $X \sim \text{Pareto}(\alpha)$ ($\alpha > 2$) provides a simple way to numerically test how a change in the coefficient of variation ($CV$) affects $\tilde{v}_l$. The coefficient of variation is defined as $CV = \sigma/\mu$, where $\sigma$ is the standard deviation of $X$. For the Gamma distribution the coefficient of variation is simply $CV = \alpha^{-\frac{1}{2}}$ and for the Pareto distribution $CV = (\alpha (\alpha - 2))^{-\frac{1}{2}}$. Thus, for both distributions, $CV$ decreases with the parameter $\alpha$. We observe that $\tilde{v}_l$ increases as $CV$ decreases ($\alpha$ increases). As the coefficient of variation decreases, consumers face a lower rationing risk and therefore, static pricing should perform better.

### 7 Understanding the Results: Generalization

In the previous section, we demonstrated that static pricing can perform better than dynamic pricing when $v_l$ is low relative to $v_h$. But what exactly is the problem with the dynamic pricing strategy? Is it the fact that under dynamic pricing the firm may raise the price to $v_h$, the fact that it may reduce the price to $v_l$, or a combination of the two? To answer this question, we refer back to the general set of pricing strategies defined above (Definition 1) and of which static and dynamic pricing are special cases. The next result is a property of that set.
Theorem 3. For every \( \mathcal{A} \), there exists a subset \( \mathcal{B} \subseteq \mathcal{A} \) with \( |\mathcal{B}| \leq 2 \) such that

\[
\max_{p \in \mathcal{A}} \{ R(p,x,\gamma) \} = \max_{p \in \mathcal{B}} \{ R(p,x,\gamma) \} .
\]

Furthermore, \( \mathcal{B} = \{ p_l, p_h \} \), where \( p_l = \sup_{p \in \mathcal{B}} \{ p \leq v_l \} \) and \( p_h = \sup_{p \in \mathcal{A}} \{ p \leq v_h \} \).

Theorem 3 demonstrates that within the general set of pricing strategies, it is sufficient for the firm to consider only pricing strategies in which the firm commits to at most two prices before demand is realized. To explain, note that there are two types of consumers to cater to and that we restrict the set of pricing strategies to schemes in which the firm chooses the optimal price after observing the realization of demand. This implies that among all prices that the firm preannounced, only one of at most two prices will be chosen, \( p_l \) (the highest preannounced price that low-type consumers will buy at) or \( p_h \) (the highest preannounced price that low-type consumers will buy at). High-type consumers rationally anticipate this and thus, their equilibrium joining behavior under set \( \mathcal{A} \) is equivalent to their equilibrium joining behavior under set \( \mathcal{B} \). As an example, observe that the dynamic pricing strategy with \( \mathcal{A} = \mathbb{R}^+ \) is equivalent to a pricing strategy in which the firm commits to \( \mathcal{B} = \{ v_l, v_h \} \).

Therefore, for the rest of this section we can restrict attention to the subset of the pricing schemes given in Definition 1, in which the firm preannounces at most two prices. Denote the allowable prices under static and dynamic pricing by \( \mathcal{B}_s = \{ p_s \} \) and \( \mathcal{B}_d = \{ v_l, v_h \} \), respectively.

Theorem 4. The following properties hold:

1. Announcing \( \mathcal{B}_s \) is dominated by \( \mathcal{B} = \{ v_l, p_s \} \).

2. If \( R_d \) is increasing in \( v_l \), then announcing \( \{ v_l, p_h \} \) dominates \( \{ p_l, p_h \} \) \( \forall p_l \leq v_l \).

The first statement of Theorem 4 implies that static pricing is always dominated by a strategy in which the firm announces \( \{ v_l, p_s \} \) and then chooses one of these prices once demand is observed. Because \( p_s \geq v_l \), the two strategies only differ when \( p_s > v_l \). In this case, by announcing \( \{ v_l, p_s \} \), more consumers visit the firm (because they anticipate that they may be charged \( v_l \)) and in addition the firm gains the capability to choose the better price to respond to demand conditions. Thus, in this sense, when the firm can reduce the price, dynamic pricing actually works better for the consumers and the firm. This suggests that the problem with dynamic pricing is not the fact that it may drop the price \( v_l \).

The second statement of Theorem 4 shows that the firm can never improve its revenue by preannouncing a low price which is lower than \( v_l \). A sufficient condition for this result to hold is
that $R_d$ is increasing. While we were unable to prove that $R_d$ is increasing, our numerical analysis suggests that this is the case for many demand distributions. Finally, we make the following conjecture, which we observe numerically.

**Conjecture 1** If $\gamma_d < 1$, there exists a price $p_h < v_h$ such that announcing $\{v_l, p_h\}$ strictly dominates $B_d$.

Clearly, if all high-type consumers join under the dynamic pricing scheme ($\gamma_d = 1$), the firm cannot do better than announcing $B_d$. Combining all three results, we conclude that the problem with dynamic pricing is the fact that the firm may charge a low price, $v_l$, but it is the fact that the firm may raise the price to $v_h$ and leave high type consumers with zero surplus. A better pricing strategy is one in which the firm commits to leave consumers with a positive surplus in all demand states. Such a policy can be implemented by committing to a “low base price” (i.e., lower than $v_h$) and only marking down. This result provides a possible explanation to the asymmetric price adjustments phenomenon, which is both empirically observed and theoretically assumed (e.g., Aviv and Pazgal 2008; Liu and Van Ryzin 2008; Su and Zhang 2008) according to which firms announce a base price which is lower than the highest valuation and are willing to mark down from that price but they will not mark up.

8 Conclusion

In this paper, we explain why a firm may prefer static pricing over dynamic pricing when consumers are strategic and decide whether to consider to purchase based on the firm’s chosen pricing strategy. By charging a static price a firm imposes a rationing risk on consumers whereas a firm that changes prices dynamically imposes a price risk on consumers. Imposing a rationing risk on consumers can dominate, especially when consumers’ valuations for the product are highly variable and the advantage of static pricing over dynamic pricing can be substantially larger than the advantage of dynamic pricing over static pricing. Offering availability guarantees to compensate consumers for stock-outs or allowing reservations may serve to benefit static pricing even further. We find that the problem with dynamic pricing is that the firm may charge a high price that leaves consumers with zero surplus, so the firm can improve its revenues by implementing a pricing strategy which leaves consumers with a positive surplus in all states of demand. Overall, we conclude that even though dynamic pricing responds better to demand conditions, charging a static price can be the preferable pricing strategy when consumers are strategic and even better can be a pricing strategy
in which the firm charges a base price (which is lower than the highest valuation) and it commits not to raise it, but can potentially decrease it.

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References


Dasu, S., C., Tong. 2006. Dynamic pricing when consumers are strategic: analysis of a posted
pricing scheme. Working paper: University of Southern California, Los Angeles.


Appendix: Proofs

Proof of Lemma 1. First, note that the expected sales function is given by

\[ S(k) = \int_0^{k/\gamma} xf(x) \, dx + \frac{k}{\gamma} F\left(\frac{k}{\gamma}\right) \]

and that

\[ S'(k) = \frac{dS(k)}{d\gamma} = -\frac{k}{\gamma^2} F\left(\frac{k}{\gamma}\right) \]

Differentiating \( R^h_s(\gamma) \) with respect to \( \gamma \), we get:

\[
\zeta_s(\gamma) = \frac{dR^h_s(\gamma)}{d\gamma} = v_h(S(k/\gamma) + \gamma S'(k/\gamma)) - \mu c
\]

\[ = v_h \int_0^{k/\gamma} xf(x) \, dx - \mu c. \]

\( R^h_s(\gamma) \) is concave because \( \zeta_s(\gamma) \) is decreasing in \( \gamma \). The optimal \( \gamma_s \) may be 1 (a corner solution) if \( \zeta_s(1) \geq 0 \) (result (i)) or interior, in which case solving the first-order condition \( \zeta_s(\gamma) = 0 \) gets the result (ii). Note that \( \gamma_s \neq 0 \), because we assume that \( v_h > c \). \( \blacksquare \)

Proof of Lemma 2. Under dynamic pricing, the indifferent consumer solves

\[ \frac{v_l x}{\mu} \int_0^{v_l/\gamma_d} xf(x) \, dx = (v_h - v_l) c. \]  

(12)

As the left-hand-side (LHS) strictly decreases with \( \gamma \) and the right-hand-side (RHS) is constant, there either exists a unique \( \gamma \in [0,1] \) which solves (12), or, if \((v_h - v_l) \int_0^{v_l/\gamma_d} xf(x) \, dx > \mu c\), there does not exist a \( \gamma \) which solves (12), in which case \( \gamma_d = 1 \). \( \blacksquare \)

Proof of Lemma 3. Limit calculations: (i) Let \( h'(x) = xf(x) \) so that \( h(\xi) = \int_0^\xi xf(x) \, dx \).

Therefore, from the Fundamental Theorem of Calculus, \( \int_0^{v_l/\gamma_d} xf(x) \, dx = h\left(\frac{v_l}{v_h \gamma_d}\right) - h(0) = h\left(\frac{v_l}{v_h \gamma_d}\right) \) (because \( h(0) = 0 \)). Note that \( h'(\xi) > 0 \) and thus invertible. From (12), we can write

\[ h\left(\frac{v_l}{v_h \gamma_d}\right) = \frac{\mu c}{(v_h - v_l)} \]

and

\[ h^{-1}\left(\frac{\mu c}{v_h - v_l}\right) = \frac{v_l}{v_h \gamma_d}. \]

Rearranging, we get:

\[ \gamma_d = \frac{v_l}{h^{-1}\left(\frac{\mu c}{v_h - v_l}\right)} \cdot \]

\( h^{-1}\left(\frac{\mu c}{v_h - v_l}\right) > 0 \), since \( \mu c/(v_h - v_l) > 0 \), \( h(0) = 0 \) and \( h'(x) > 0 \). Thus, taking the limit, we get \( \lim_{v_l \to 0} \gamma_d = 0 \). (ii) Rearranging (12) and letting \( v_l \to v_h - c \), we get that for (12) to hold, we must have \( \lim_{v_l \to v_h - c} \int_0^{v_l/\gamma_d(v_l)} xf(x) \, dx = \mu \), which implies that \( \lim_{v_l \to v_h - c} \gamma(v_l) = 0 \).
To show that $d(v_l)$ is quasi-concave, write:

$$F = (v_h - v_l) \int_0^{v_l} x f(x) \, dx - \mu c.$$  

Note that if condition (12) holds, $F = 0$. Differentiating $F$ and applying the Implicit Function Theorem, we get:

$$\frac{\partial F}{\partial v_l} = \left( \frac{v_h - v_l}{v_l} \right) \left( \frac{v_l}{v_h \gamma_d} \right)^2 f \left( \frac{v_l}{v_h \gamma_d} \right) - \int_0^{v_l} \frac{v_l}{v_h \gamma_d} x f(x) \, dx,$$

$$\frac{\partial F}{\partial \gamma_d} = - \left( \frac{v_h - v_l}{\gamma_d} \right) \left( \frac{v_l}{v_h \gamma_d} \right)^2 f \left( \frac{v_l}{v_h \gamma_d} \right)$$

and

$$\frac{\partial \gamma_d}{\partial v_l} = \frac{\gamma_d}{v_l} \left( 1 - \frac{v_l}{v_h - v_l} \frac{f(y)}{y f(y)} \right)$$

implying that we must have that $s > v_l$.

Proof of Theorem 2. Let $\mathcal{F}_s \equiv v_h \int_0^{K_s} x f(x) \, dx - \mu c$ and $\mathcal{F}_d \equiv (v_h - v_l) \int_0^{v_h \gamma_d} x f(x) \, dx - \mu c$. 

Proof of Lemma 4. The results immediately follow from the limits of Lemma 3. 

Proof of Theorem 1. To establish the result, assume first that $\gamma_s$ and $\gamma_d$ are interior. Denote the LHS of (5) and the LHS of (9) by $\tau_s(\gamma)$ and $\tau_d(\gamma)$, respectively. Observe that $\tau_s(\gamma) > \tau_d(\gamma)$ $\forall \gamma$. Furthermore, $\tau_s'(\gamma) < 0$ and $\tau_d'(\gamma) < 0$. Since the RHS of both conditions is the same, the result follows. Also note that the conditions for boundary solutions are such that

$$v_h \int_0^k x f(x) \, dx > (v_h - v_l) \int_0^{v_h \gamma_d} x f(x) \, dx,$$

implying that we must have that $\gamma_s \geq \gamma_d(v_l) \forall v_l$. 

Proof of Theorem 2. Let $\mathcal{F}_s \equiv v_h \int_0^{K_s} x f(x) \, dx - \mu c$ and $\mathcal{F}_d \equiv (v_h - v_l) \int_0^{v_h \gamma_d} x f(x) \, dx - \mu c$. 

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Observe that $F_s = 0$ and $F_d = 0$. Differentiation yields:

$$\frac{\partial F_s}{\partial \gamma_s} = -\frac{v_h}{\gamma_s} \left( \frac{k}{\gamma_s} \right)^2 f \left( \frac{k}{\gamma_s} \right) \leq 0$$

and

$$\frac{\partial F_d}{\partial \gamma_d} = -\frac{v_h - vl}{\gamma_d} \left( \frac{vl \cdot k}{v_h \cdot \gamma_d} \right)^2 f \left( \frac{vl \cdot k}{v_h \cdot \gamma_d} \right) \leq 0.$$

(i) Differentiating with respect to $c$:

$$\frac{\partial F_s}{\partial c} = \frac{\partial F_d}{\partial c} = -\mu < 0.$$

Applying the Implicit Function Theorem, we get the desired results.

(ii) Differentiating with respect to $k$:

$$\frac{\partial F_s}{\partial k} = kv_h \left( \frac{1}{\gamma_d} \right)^2 f \left( \frac{k}{\gamma_s} \right) \geq 0$$

and

$$\frac{\partial F_d}{\partial k} = k \left( v_h - vl \right) \left( \frac{vl}{\gamma_d \cdot v_h} \right)^2 f \left( \frac{vl \cdot k}{v_h \cdot \gamma_d} \right) \geq 0.$$

Applying the Implicit Function Theorem, we get the desired results. 

Proof of Lemma 5. (i) When $c = 0$, condition (i) of Lemma 1 and condition (i) of Lemma 2 hold, and therefore $\gamma_s = \gamma_d = 1$. Furthermore, $R^*_s (vl) = \max \{ vlk, v_h S (k) \}$ and $R^*_d (vl) = vlk + v_h S \left( \frac{vl}{v_h} \right)$. Suppose first that $vlk \geq v_h S (k)$. Then, $R^*_s (vl) = vlk$ and $R^*_d (vl) \geq R^*_s (vl)$. Otherwise, suppose that $vlk < v_h S (k)$. Then, $R^*_s (vl) = v_h S (k)$ and $R^*_d (vl) = vlk + R^*_s (vl) - v_h S \left( \frac{vl}{v_h} \right)$. $R^*_d (vl) \geq R^*_s (vl)$, because $\frac{vl}{v_h} k \geq S \left( \frac{vl}{v_h} \right)$ (from the definition of the expected sales function). (ii) Following the same steps of Lemma 3, we get that

$$\gamma_s = \frac{k}{h^{-1} \left( \frac{\mu}{v_h} \right)}.$$

Therefore, the $\lim_{c \to v_h} \gamma_s$ exists and is unique. To find the limit, observe that, when $c \to v_h$, $\lim_{c \to v_h} c\mu/v_h = \mu$ and we must have

$$\int_0^{\gamma_s} x f (x) dx = \mu,$$

which implies that $\lim_{c \to v_h} \frac{k}{\gamma_s} = \infty$ or $\lim_{c \to v_h} \gamma_s = 0$. Furthermore, since $\gamma_s \geq \gamma_d (vl) \forall v_l$ and $\gamma_d \in [0, 1]$, $\lim_{c \to v_h} \gamma_d (vl) = 0$. For the revenues, when $\gamma_s = 0$, $R^*_s (vl) = \max \{ vlk, 0 \} = vlk$ and when $\gamma_d = 0$, $R^*_d (vl) = vlk$. 

Proof of Theorem 3. Suppose that the firm preannounced a set of prices $\mathcal{A}$ and that based
on this set, $\gamma x$ high-type consumers visit the firm. Partition the set $\mathcal{A}$ to two disjoint sets, $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$, such that $\mathcal{A}_1 = \{ p \in \mathcal{A} | p \leq v_l \}$ and $\mathcal{A}_2 = \{ p \in \mathcal{A} | p > v_l \}$. Given that $\gamma x$ high-type consumers visited, the firm can choose to serve only high-type consumers, by choosing a price $p \in \mathcal{A}_2$ (if exists) or to serve both consumer types, by choosing a price $p \in \mathcal{A}_1$ (if exists). Suppose there exist two prices, $p^1 \in \mathcal{A}_2$ and $p^2 \in \mathcal{A}_2$, where $p^1 > p^2$. Because the choice of a price among $\mathcal{A}_2$ will not affect $\gamma$, setting $p^1$ strictly dominates $p^2$. Similarly, suppose there exist two prices, $p^3 \in \mathcal{A}_1$ and $p^4 \in \mathcal{A}_1$, where $p^3 > p^4$. Because the firm is guaranteed to sell $k$ units by choosing any price among $\mathcal{A}_1$, setting $p^3$ strictly dominates $p^4$. ■

**Proof of Theorem 4.** For the two general prices $(p_l, p_h)$, such that $p_l \leq p_h$, $p_l \leq v_l$ and $p_h \leq v_h$ the revenue function is given by

$$R(p_l, p_h) = p_l k + p_h \gamma \left( S \left( \frac{k}{\gamma} \right) - S \left( \frac{p_l k}{p_h \gamma} \right) \right),$$

where $\gamma$ is given by

$$\frac{v_h - p_h}{\mu} S \left( \frac{k}{\gamma} \right) + \frac{p_h - p_l}{\mu} \int_0^{\frac{p_l}{p_h \gamma}} x f(x) = c.$$  (14)

(1) $p_s = \max \{ v_l, p^h \}$. If $p_s = v_l$, then $\mathcal{B}_s = \mathcal{B}$. If $p_s = p^h$, then (14) implies that $\gamma_s \leq \gamma$ and $R(v_l, p_s) \geq R_s$, because $\gamma_s \leq \gamma$ and because $\max_{p \in \mathcal{B}} \{ R(p, x, \gamma) \} \geq \max_{p \in \mathcal{B}'} \{ R(p, x, \gamma) \}$ if $\mathcal{B}' \subseteq \mathcal{B}$;

(2) First note that from the assumption that $R_d$ is increasing in $v_l$, we get that

$$\frac{dR_d}{dv_l} = \frac{\partial R_d}{\partial v_l} + \frac{\partial R_d}{\partial \gamma_d} \frac{\partial \gamma_d}{\partial v_l} = k F(y) + v_h \int_y^{v_l} x f(x) dx \cdot \gamma_d \left( \frac{1}{v_l} - \frac{c \mu}{(v_h - v_l)^2 y^2 f(y)} \right) \geq 0,$$

where $y = \frac{v_l k}{v_h \gamma_d}$. To prove the property, we need to show that $dR(p_l, p_h)/dp_l \geq 0$. Let $z = \frac{p_l k}{p_h \gamma}$. Differentiating, we get:

$$\frac{\partial R(p_l, p_h)}{\partial p_l} = k F(z),$$

$$\frac{\partial R(p_l, p_h)}{\partial \gamma} = p_h \int_z^{v_l} x f(x) dx$$

and from the Implicit Function Theorem,

$$\frac{\partial \gamma}{\partial p_l} = \gamma \cdot \frac{(p_h - p_l) \frac{z}{p_l} f(z) - \int_0^z x f(x) dx}{(p_h - p_l) \frac{z}{p_l} f(z) + (v_h - p_l) k \frac{F(k)}{\gamma}}.$$

As $\frac{\partial R(p_l, p_h)}{\partial p_l} \geq 0$ and $\frac{\partial R(p_l, p_h)}{\partial \gamma} \geq 0$, the result follows immediately if $\frac{\partial \gamma}{\partial p_l} \geq 0$. It remains to show
that \( dR(p_l, p_h) / dp_l \geq 0 \) if \( \frac{\partial \gamma}{\partial p_l} < 0 \). Note that because \( (v_h - p_h) \frac{k}{\gamma} F \left( \frac{k}{\gamma} \right) \geq 0 \), when \( \frac{\partial \gamma}{\partial p_l} < 0 \),

\[
\frac{\partial \gamma}{\partial p_l} \geq \gamma \left( \frac{1}{p_l} - \frac{\int_0^z x f(x) \, dx}{(p_h - p_l) z^2 f(z)} \right)
= \gamma \left( \frac{1}{p_l} - \frac{c \mu - (v_h - p_h) S \left( \frac{k}{\gamma} \right)}{(v_h - v_l)^2 z^2 f(z)} \right)
\geq \gamma \left( \frac{1}{p_l} - \frac{c \mu}{(v_h - v_l)^2 z^2 f(z)} \right).
\]

Note that the last term is equivalent to the derivative \( \frac{\partial \gamma}{\partial p_l} \) of dynamic pricing in (15), where \( v_l = p_l \) and \( v_h = p_h \). Thus, if \( R_d \) is increasing, it must be that \( \frac{\partial R(p_l, p_h)}{\partial p_l} \geq 0 \). ■