Inclusive Innovation: Broader Market Coverage for Innovative Products with Deliberate Supply Chain Leadership

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Abstract

New technological and product innovations, including some life-saving ones, conventionally traverse a sequentially downward path of gradually lowering costs and prices, which limits their initial availability and affordability to the lower-end of the market. In this paper, we focus on the central question of how to achieve broader market coverage for innovative products, which we refer to as *inclusive innovation*. We unearth a new innovation investment degree of freedom in a multi-tiered supply chain that offers product development firms the ability to expand market coverage. Through an analytical model grounded in industrial practice, we show that deliberately choosing which firm in a multi-tiered supply chain invests in product quality improvement and acts as a leader by initiating contract offers can have a significant impact on the market coverage of the product. Our model deals with products that have non-linear development and production costs and a product lifecycle that is characterized initially by product innovation being the most dominant effect followed by a period of process innovation. We demonstrate that aligning product innovation investment decision making with price-quantity decision-making leads to greater total supply chain profits and market coverage. In addition, we are able to identify a sequence of deliberate leadership transfers by which contract leadership is shifted upstream during the product innovation phase and shifted back to downstream entities during the process innovation phase so as to maintain highest market coverage during the lifecycle of a product. We go beyond a normative prescription by identifying conditions under which tierwise rational leadership transfers occur and discuss how to align individually rational schedule of leadership transfers with an optimal schedule that results in the highest market coverage. These findings have subtle, but important, implications for firms launching innovative products and aspiring to expand market coverage. Specifically, to obtain broader market coverage for its innovations, innovating firms in a supply chain should finely tune the level of innovation investment, identity of the investor, and contract leadership. These results help to explain the evolution of industries such as automotives and personal computers and also offer opportunities for new industries and firms to expand both market coverage and profits.

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1 Introduction

Technological advances, based on Research & Development (R&D), lead to new product and process innovations that elevate the standard of living of societies and individuals over time. However, innovations in the past traversed a sequential path of gradually lowering costs and prices making them available/affordable to the high-end customers first and to the broader market only over a period of time (Bhattacharya et al. 2003). Even such important life-saving innovations such as passenger airbags and seat belts tended to be exclusive to luxury models and unavailable/unaffordable to the mass market for over a decade. For example, Mercedes-Benz first introduced airbags in its high-end S-Class models in the early 1980's, but it took a decade or more for the lower-end of the automotive market to avail of and benefit from these safety features.

As innovating organizations face intensifying competition, a major opportunity exists to create social surplus as well as profits and market value by advancing the availability of the innovative products to the mass-market. We refer to this approach as *inclusive innovation* in which new higher-performance quality products are made available and affordable to a broader swath of the market as opposed to being limited to a narrow segment of high-end customers. However, inclusive innovation has been difficult to achieve in the past due to a combination of higher costs (of development and production) and a desire by firms to extract the surplus generated by higher-end customers. This is the case for large established vertically-integrated monopolistic firms (with proprietary technology) that pursued development, production, and distribution in-house and whose profit-maximizing solution to introducing new higher-quality products is to cater to a narrow higher-end niche of the market (Moorthy and Png 1992).

Increasingly, however, more firms are focusing on specialized tasks (such as production or distribution) and turning to suppliers and partners outside of the firm for component and sub-system development. Such task unbundling has resulted in the emergence of multi-lateral supply chains that offer the benefits of specialization but suffering from coordination and agency issues/costs that must be carefully managed. These coordination and agency issues can further restrict market coverage and cause products to become even more exclusive (Villas-Boas 1998). However, new search and networking technologies have made customers much more aware of and clamor for innovative technological developments; therefore, it is becoming important for manufacturers to be more inclusive in such a democratizing market environment.

We propose and formalize the notion that a deliberate choice of product innovation investments by supply chain partners offers an opportunity to broaden market coverage and be more inclusive. This is particularly the case for complex products (such as next-generation automobiles and aircraft) whose performance quality improvement entails significant and non-linear development and production costs. Increasingly, software and hardware are integrated in cutting-edge products, such as those using wireless sensors and associated software to create *smart products*. Such products exhibit a hybrid cost structure with both fixed development costs and variable production costs that must be covered using investments to reach a certain concrete level of product quality.

One of our study companies that markets electric cars to US consumers offers a good illustration of the underlying issues considered in the paper. Electric cars present a cleaner emission-free alternative to conventional internal combustion engine-based automobiles, but they must achieve product innovation/performance quality improvement to overcome customer *range anxiety* arising from running out of battery power midway through a journey without access to a charging station. High quality electric cars that have a relatively long range are also quite expensive and are not affordable for broader segments of the market. A key limiting factor is the vehicle's battery, which is designed and manufactured by an upstream *Tier 2* supplier and then assembled into an electrical subsystem by a *Tier 1* supplier before being integrated and distributed by our focal *Tier 0* firm into a fully-functional electric vehicle. Our study company is acutely focused on maximizing the product's attractiveness as well as the available market for its products by making it more inclusive. The analysis presented in the paper offers such firms a new degree of freedom that not only entails higher-quality products and profits but also broader market coverage for such innovations.

Considering a three-tier supply chain developing and launching an innovative product using a simple wholesale price contract (to keep the focus on product innovation), we investigate who should initiate/lead the contract and who should be a primary investor in improving the product quality in order to get broader market coverage. We first derive a key result that demonstrates the criticality of aligning the innovation investment degree of freedom with the contract leadership; specifically, the supply chain tier that initiates the contract offers should be the dominant investor in product quality for broader market coverage as well as firms' profits. Next, we show that in the product innovation phase of the product lifecycle, the supply chain leadership should be gradually transferred upstream from the downstream tiers, whereas in the process innovation phase of the later part of the product lifecycle, a supply chain contract leadership with investment should be deliberately shifted back to downstream entities from the upstream tier. In addition, we also find that it is in the firms' best interest to hand over the leadership to the other tiers, consistent with the optimal leadership transfers for broader market coverage. When it is not in the firms' best interest to hand over the leadership, we discuss how to remedy this issue and to encourage voluntary leadership handovers in order to attain more inclusive innovation. We begin with a discussion of the literature related to our work.

2 Literature Review

There exists a substantial body of literature in the Economics and Management domains on the investments needed for innovation/R&D and the impact they engender in terms of social welfare and industry/firm profits. More than 50 years of Economics literature on R&D investments was initially reviewed by Griliches (1979) and most recently by Hall et al. (2009) – these papers vividly depict the challenges of measuring the returns to R&D and also show that investments are associated with strongly positive firm (private) and social returns. Nevertheless, these papers do not detail who should make these investments in a network of firms to achieve the best social/economic outcomes. The issues regarding the organization of R&D activity and contractual arrangements (such as the allocation of property rights) have also been studied by Economics researchers starting with the work of Aghion and Tirole (1994). Although they provide a good foundation for subsequent work, these papers do not deal with the issue of achieving broader (inclusive) market coverage for innovative products.

Recently, product innovation has been a topic of active research interest in the Management Science/Operations Management literature by Krishnan and Ulrich (2001), but most of the literature has tended to be single-firm-centric focusing on project scheduling and management. There is a small stream of work on the interactions between product and supply chain design decisions, (Ulrich and Ellison 2005, Grahovac and Parker 2003). However, this literature focuses only on how a single firm should make decisions involving its suppliers, rather than the interaction between the decisions of firms. Closer to our paper is the work of Bhaskaran and Krishnan (2009) who study the joint-development of products in a bi-lateral supply chain context; however, they focus on contractual arrangements between firms that extend beyond revenue sharing to include the sharing of development cost and work. In contrast, this paper focuses on understanding the linkage between the source of innovation investments and the ensuing market coverage in a trilateral supply chain.

Innovative products exhibiting strong integration of hardware and software components require the careful modeling of both variable and fixed costs. For example, one of our study companies marketing electric vehicles incurs both the development cost of attaining technical capacity to deliver vehicles with a specific battery range as well as the production cost associated with manufacturing and customer support. The effects of development cost in isolation have been studied extensively in the vertical product differentiation literature in Economics - specifically, Shaked and Sutton (1982) and Bonanno (1986) examine environments in which investment in quality is primarily associated with development costs. Alternatively, increased product quality may result in production costs that are convex in product quality as modeled by the classic papers of Mussa and Rosen (1978). We consider a setting where a product's development and production costs are both significant - modeling the non-linear form of both development and production costs is important and non-trivial. That is, it requires more than combining the two strands of literature and offers additional insights into how investor and leadership positions may have to change as costs decrease during the innovation process.

There exists a related stream of literature on products introduced sequentially to the market while experiencing rapid quality improvements, referred to as *rapid sequential innovation* (Dhebar 1994, Kornish 2001, Ramachandran and Krishnan 2008). However, this literature is primarily concerned with the purchase timing dilemmas of consumers when products improve in discounted terms, and the steps a monopolistic firm may take through pricing and product design to address such consumer concerns. This stream does not consider the context of a supply chain and does not focus specifically on broadening the market coverage.

Suppliers that invest in component technological innovation often wrestle with the issue of other supply chain participants not making mutually aligned decisions; as a result, they may under-invest in component technologies - downstream firms may sometimes decide to make the investment to foster innovation. Many papers in marketing and supply chain operations have studied the interaction between vertical firms and have proposed mechanisms to deal with price-quantity coordination problems (Jeuland and Shugan 1983, Lee and Staelin 1997, Cvsa and Gilbert 2002). Similar models have been used to analyze the effect of innovation by one of the firms on its channel partners. Gupta and Loulou (1998) study how interactions between firms in a channel affect innovation. Gilbert and Cvsa (2003) analyze the effect of strategic commitment to price by a supplier to stimulate downstream innovation in a supply chain. However, this stream of literature deals primarily with prices and quantities and ignores the decision-making about product quality, which constitutes the core of product innovation. Furthermore, complex contracts do not seem to be widely embraced by the industry due to the cost of implementing them (Arrow 1984). We address the misalignment/coordination problems in a way that keeps contracts simple - similar to the approach taken by Jerath (2007) for aligning marketing and operations efforts within a firm. The optimality of a contract leader position in a supply network has been examined by Majumder and Srinivasan (2008), who generalize the notion of double-marginalization from Spengler (1950) to show that contract leadership affects total supply chain profits. This literature provides us with a convenient solution approach to establish the equilibrium price and quantity resulting from a sequence of wholesale price contracts. However, our paper focuses on the identity of an investor, the choice of product quality, and the resulting profit and market coverage for complex products.

In this paper, we examine deliberate transfers of contract leadership as an additional degree of freedom that allows supply chains to adjust their leadership configuration at different stages of a product's lifecycle to achieve broader market coverage and higher total supply chain profits. A similar notion of profit improving dynamic adjustments in the individual firm's strategy and supply chain structure is examined by Druehl et al. (2009), identifying optimal time-pacing strategies for new product development, and Xiao and Xu (2012), using sequential royalty revisions to realign incentives between the innovator and the marketer of a new product. Furthermore, Rhee et al. (2012) study the patterns of high-end encroachment in which new products gradually become available to a broader share of the market driven by cost reductions and technology improvements.

There is also an emerging stream of literature studying the impact of the supply chain and organization structure on equilibrium outcomes in static environments. Bimpikis et al. (2014) use a supply network perspective to show the adverse effects of multi-sourcing in mitigating the aggregate disruption risk. Similar to the normative prescription of optimal contract leadership offered in our paper, Girotra et al. (2010) identify optimal organizational structure for the generation of new product ideas and Roels et al. (2010) study optimal contract types for delivering collaborative services. Our work, however, is focused on broader coverage for innovative products. We now turn to the discussion of the modeling framework before presenting and discussing the key results.

3 Model

We consider a three-tier supply chain that involves the development, production and distribution of an innovative product. In the three-tier supply chain, basic components are supplied from an upstream Tier 2 to an intermediate assembler, Tier 1, followed by final integration, marketing and distribution at downstream Tier 0. For example, in the electric car case mentioned above, the battery supplier represents the Tier 2 firm; the electric subsystem assembler would be a Tier 1 firm, and the manufacturer/OEM would be considered as a Tier 0 firm, which sells products directly to consumers whose total market size is N. Our stylized model and analysis methodology can be easily extended to more tiers and supply networks, and key qualitative insights are preserved under general supply networks.

The quality of the product to consumers is a function of its marketing, assembly and the quality of the components. For instance, in the case of an innovative product such as an electric car, the quality of a product to consumers is a function of its component performance (such as battery range), subsystem (drive train) capability, and the finished product quality and marketing (ease of use, safety, reliability, and design attractiveness). Each of these product features is associated with value added at one or multiple tier(s). We focus on the case in which a key bottleneck technology is at the components manufactured at Tier 2, which leads to the following assumption:

Assumption 1. *The product quality Θ is primarily associated with the performance of the Tier 2 component and the qualities of Tier 1 (assembly) and Tier 0 (marketing) are fixed.*

While our model can be extended to the case in which Tiers 1 and 0 qualities also affect the quality of the end product through the use of an appropriate (e.g., multiplicative or additive) quality functions, we aim to represent a simple, yet consistent, model with the motivating case of electric vehicles (EVs) and other industries, especially technology-driven industries, by focusing on Tier 2 quality; for example, one of the key hurdles for adoption of EVs under the current technology is consumer's range anxiety related to the quality of batteries, which is the quality of components produced at Tier 2. In addition, in the electronics industry, including personal computers and cell phones, one of the key components that determines the end product quality is the performance of central processing units or chips produced by a component supplier, who would be also a Tier 2 firm; in other words, we focus on the case in which a critical product feature limiting market penetration is associated with the quality of components supplied by an upstream tier. Our approach is similar to that of Altug and van Ryzin (2013), who also consider a problem in which consumer's willingness to pay is modeled as a function of the supplier's component quality, and the manufacturer/assembler does not contribute significant additional value in excess of what is derived from components themselves. Furthermore, our insights and results remain valid if a constraining product characteristic is associated with a different tier within the supply chain, as long as there exists a single primary bottleneck technology limiting the product's market penetration.

On the consumer demand side, we follow the traditional vertical differentiation model of quality evaluation (see, e.g., Mussa and Rosen 1978, Moorthy and Png 1992); specifically, given the product quality Θ, each consumer's type denoted as α is uniformly distributed on [0, 1], such that when a consumer of type α purchases a product with quality Θ at price p, her net utility is $U = \alpha\Theta^{\beta} - p$ with $\beta \leq 1$, which captures the saturation or decreasing returns to quality. A consumer's reservation utility when she purchased none is normalized to zero. Consequently, a product of quality Θ with price p is purchased by all consumer types with non-negative net utility, $\alpha \geq \underline{\alpha} = \frac{p}{\Theta^{\beta}}$. Here, $\underline{\alpha}$ corresponds to the marginal consumer who derives zero utility. Thus, depending on the product quality Θ, such a product exhibits market coverage $\rho(\Theta) = 1 - \underline{\alpha}$, and the total market demand becomes $N \cdot \rho(\Theta)$.

Definition 1. Consider two different cases of market coverage, Case A and Case B. Case A represents more inclusive innovation than Case B if the marginal consumer type $\underline{\alpha}$ for Case A is lower than that for Case B, i.e., the market coverage $\rho(\Theta)$ for Case A is higher than that of Case B.[1]

On the cost side, we consider both the production cost and the development cost of innovation to increase the product quality Θ. First, we assume the following form of production costs:

Assumption 2. For Tier 2 being a critical determinant of product quality, the production cost of delivering q_2 units of components with quality Θ is $C_2(q_2, \Theta) = K_2\Theta^{\delta_\theta} q_2^{\delta_q}$ with $\delta_\theta, \delta_q > 1$. For Tiers 0 and 1, the production costs are $C_0(q_0) = K_0 q_0^{\delta_q}$ and $C_1(q_1) = K_1 q_1^{\delta_q}$, respectively, for producing q_i units for $i = 0, 1$.

For Tier 2 that is associated with the key bottleneck technology, its production cost is increasing and convex in quality Θ as similarly modeled by Mussa and Rosen (1978) and Gal-Or (1983). Convexity in the quality parameter ($\delta_\theta > 1$) captures the diseconomies associated with the production of an increasingly higher quality product resulting from the use of more expensive raw materials or skilled labor. Moreover, consistent with the Operations/Supply Chain literature (exemplified by Majumder and Srinivasan 2008), we also focus on the production costs that are convex in quantity, i.e., $\delta_q > 1$, for instance, to incorporate capacity/resource constraints.[2]

Next, for the development costs, a firm in the supply chain invests in development and innovation to improve the product quality Θ. We assume the following form of the development cost:

Assumption 3. The development cost to attain the product quality Θ is of the form $D(\Theta) = \gamma\Theta^{\delta_D}$ with $\delta_D > 1$.

Throughout this paper, we consider product development and innovation investment as effort/investment in enhancing the product quality. In our development cost model, the marginal cost of improving the product quality Θ is increasing in Θ, i.e., $\delta_D > 1$, which is consistent with

[1] Market coverage is a natural measure for the inclusiveness of an innovative product. An alternative measure for social efficiency would be the consumer surplus $CS(\Theta)$, which can be written as $CS(\Theta) = \Theta^{\beta} \int_{\underline{\alpha}}^{1} (\alpha - \underline{\alpha})\, d\alpha = \frac{1}{2}\Theta^{\beta}\rho^2(\Theta)$. Unlike the market coverage, $CS(\Theta)$ captures the sum of the utilities of covered consumer types $\alpha \in [\underline{\alpha}, 1]$ in excess of the marginal type's utility $\Theta^{\beta}\underline{\alpha}$. Given the product development investment (or, equivalently, Θ), more inclusive innovation leads to a larger consumer surplus. Furthermore, since our focus is on inclusive innovation, market coverage directly captures the inclusiveness and, hence, is a more relevant metric for our purposes.

[2] Innovative products result in alternative uses of inputs to conventional products. Hence, the convexity in quantity results from expansion of demand for inputs beyond what can be accommodated by existing production capacity established with respect to the conventional uses of such inputs.

the literature (e.g., Jones and Mendelson 2011). The development cost can be also considered to be direct investment into a Tier 2 supplier to improve the corresponding component quality.

One of our research questions is who, or which tier firm, should invest in product development and innovation, i.e., in the improvement of product quality Θ. In order to examine this normative question, we study three cases in which each Tier i for $i \in \{0, 1, 2\}$ is an exclusive investor in product development within a supply chain to improve the quality of the product. Because we aim to establish the simplest link between the investor and the contract leader without the further complications to the model, we mostly focus on the case of a single dominant investor rather than joint investments by multiple firms, which has been already explored in previous literature including Bhaskaran and Krishnan (2009). Moreover, in practice, investment in new product development involves substantial fixed financing/transaction costs, which may curtail joint investments and lead to a dominant investment by a single firm within a supply chain consistent with our model. However, we do discuss what happens when firms jointly invest in product quality improvement in Section 4.3 and illustrate that our key results remain valid in this extension.

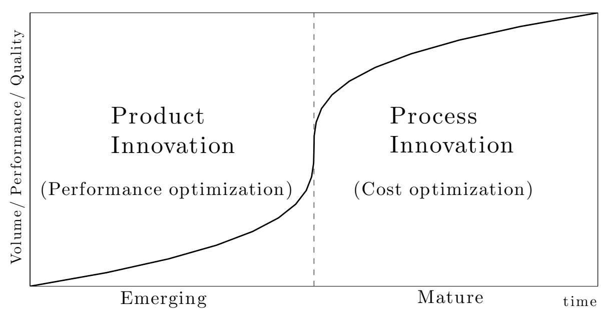


Figure 1: Product lifecycle

To examine how the identity and amount of innovation investment vary with the improvement in the product development and production costs, we adopt the product lifecycle perspective, as depicted in Figure 1. The inflection point of the well-studied S-curve motivates a division of the product lifecycle into two phases, which is consistent with the empirically demonstrated pattern of innovation established in Utterback and Abernathy (1975) - in which early in the product lifecycle, the industry is characterized by a rapid rate of product change and development cost reduction (performance optimization phase), followed by a second stage of process innovation leading to variable-cost reduction as the product matures. In our context, we translate this as follows: the initial product innovation phase is associated with fast-decreasing development costs (parameter γ), followed by process innovation characterized by an industry-wide decrease in production costs (captured by a reduction in component production cost parameter K_2. Large values of both γ and K_2 are indicative of the initial stage of product innovation, in which both development and production costs are high. However, the industry-wide focus on the improvement of product quality results in the reduction of development costs, i.e., γ is decreasing as the industry progresses through the product innovation phase. When the product enters maturity (in the process innovation stage depicted in Figure 1), the decreasing K_2 captures an improvement in the process productivity associated with the lowering production cost of the process innovation phase.

3.1 Decision Timing

At a given instance of the product lifecycle characterized by a pair of parameters (γ, K_2), the product quality Θ, quantity q and price p in a supply chain are determined in two stages. In the first stage, a Tier i investor for $i \in \{0, 1, 2\}$ determines her product development investment level to improve the product quality Θ. This first stage corresponds to the product development stage in which the investment leads to an innovative product of a fixed product quality. Once the product has been developed with the final product quality Θ, in the second stage, firms within a supply chain contract with each other, which then yields the equilibrium price and quantity. Specifically, for the contracts between tiers in a supply chain, we consider a simple widely-used wholesale pricing, and study who should initiate such a contract, i.e., who should be the contract leader.

We analyze this problem via backward induction, starting with the second stage in which the equilibrium price and quantity are determined given the product quality, followed by innovation/quality choices in the first stage (product development stage); therefore, we proceed with the investigation of equilibrium price and quantity choice in Section 3.2 followed by Section 3.3 focusing on investment in innovation and the resulting product quality.

3.2 Solution Concept: Equilibrium prices, quantities, and their dependence on quality

Once the product has been developed in the first stage, firms in a supply chain sequentially contract through a simple wholesale price agreement at stage 2, which determines the equilibrium price and quantity depending on the product quality Θ. In terms of the sequence of contracts, or who initiates the contracts, there are three cases, i.e., each tier l for $l \in \{0, 1, 2\}$ can initiate the contract. We call the supply chain tier that initiates the contract a contract leader. First, consider the case in which Tier 0 is a contract leader, as illustrated in Figure 2. In this case of Tier 0 contract leadership, Tier 0 initiates the wholesale price contract and offers the wholesale price ω_1 to Tier 1. Subsequently, Tier 1 then offers the wholesale price ω_2 to the Tier 2 supplier. Based on this wholesale price ω_2, the Tier 2 supplier then determines how much to sell to Tier 1 at this wholesale price, i.e., Tier 2 decides on the quantity q_2. Considering this maximum quantity q_2 bought from Tier 2 and the wholesale price ω_1 offered by Tier 0, Tier 1 now determines how much to sell to Tier 0, i.e., decides q_1. Finally, taking the maximum quantity q_1 procured from Tier 1, Tier 0 now determines the consumer price p and how much to sell to the consumers, i.e., q_0. Based on this consumer price p and the product quality Θ from the product development stage, the consumer market demand becomes $N \cdot (1 - \frac{p}{\Theta^\beta})$. The sales quantity of Tier 0 (q_0) is constrained by both the quantity supplied by Tier 1, q_1, and the consumer market demand $N \cdot (1 - \frac{p}{\Theta^\beta})$.

Equations (3.1) provide tierwise profit expressions, Π_2, Π_1 and Π_0, for Tier 2, Tier 1 and Tier 0, respectively, for a given product quality Θ in a supply chain with Tier 0 contract leadership, as

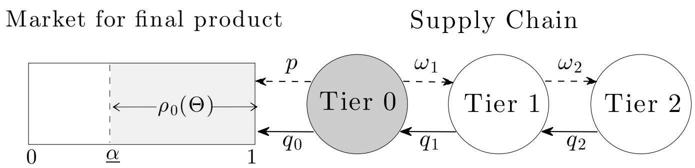


Figure 2: Price and quantity choice under Tier 0 contract leadership.

explained above. For example, Tier 1 obtains the revenue of $\omega_1 q_1$ by selling q_1 units at the unit wholesale price ω_1 to Tier 0, and it pays $\omega_2 q_2$ to Tier 2 to purchase q_2 units of the component at unit price ω_2. Furthermore, Tier 1 incurs an assembly/production cost of $K_1 q_1^{\delta_q}$, and its selling quantity q_1 to Tier 0 is constrained by q_2.

$$\begin{aligned}
\Pi_2(q_2;\omega_2) &= \omega_2 q_2 - K_2\Theta^{\delta_\theta} q_2^{\delta_q},\\
\Pi_1(q_1,\omega_2;\omega_1) &= \omega_1 q_1 - \omega_2 q_2 - K_1 q_1^{\delta_q} \quad \text{s.t. } q_1 \le q_2,\\
\Pi_0(p,q_0,\omega_1) &= pq_0 - \omega_1 q_1 - K_0 q_0^{\delta_q} \quad \text{s.t. } q_0 \le \min\left\{q_1,\, N(1-\frac{p}{\Theta^\beta})\right\}.
\end{aligned} \tag{3.1}$$

Within the second stage, given the product quality Θ, we analyze the equilibrium quantities and prices backwards, following the marginalization operation presented in Majumder and Srinivasan (2008). In this case, first, Tier 2 receiving a price offer ω_2 optimally responds with $q_2^*(\omega_2)$ by maximizing Π_2. When Tier 1 offers ω_2 to Tier 2, Tier 1 takes this optimal response $q_2^*(\omega_2)$ into consideration, and Tier 1 offers ω_2 so that $q_2^*(\omega_2) = q_1$. Technically, $q_2^*(\omega_2)$ constitutes an inverse factor demand for Tier 1, and Tier 1 firm replaces ω_2 in its profit function Π_1 using the inverse function of $q_2^*(\omega_2) = q_1$. In addition to the binding constraint $q_1 = q_2$ under optimality, Tier 1's profit function can be written as $\tilde{\Pi}_1(q_1;\omega_1) = \omega_1 q_1 - \tilde{C}_1(q_1,\Theta)$, where $\tilde{C}_1(q_1,\Theta) = \left(K_1 + \delta_q K_2 \Theta^{\delta_\theta}\right) q_1^{\delta_q}$. Note that Tier 1's profit $\tilde{\Pi}_1(q_1;\,\omega_1)$ takes a form similar to Tier 2's profit with a different modified production cost function. Applying the same procedure to Tier 0, $\Pi_0(p,\,q_0)$ can be written as

$$\tilde{\Pi}_0(p,\,q_0) = pq_0 - \tilde{C}_0(q_0,\Theta) \quad \text{s.t. } q_0 \le N(1-\frac{p}{\Theta^\beta}), \tag{3.2}$$

where $\tilde{C}_0(q_0,\Theta) = \left(K_0 + \delta_q K_1 + \delta_q^2 K_2 \Theta^{\delta_\theta}\right) q_0^{\delta_q}$. Again, $\tilde{\Pi}_0(p,\,q_0)$ takes a form similar to $\Pi_2(q_2;\,\omega_2)$. For Tier 0's case, it sets the consumer price p in addition to q_0. However, under optimality, it sets the consumer price p at the level at which its constraint is binding. To summarize, the tierwise profit expressions in (3.1) can be reduced to a single contract leader's problem in (3.2) by iterating the marginalization operation presented in Majumder and Srinivasan (2008). Once the problem has been reduced to a single contract leader's optimization problem, we maximize (3.2) with respect to p and q_0. After obtaining the optimal p^* and q_0^*, we subsequently arrive at the equilibrium prices and quantities ω_1, q_1, ω_2 and q_2, depending on the product quality Θ. The details of the analysis of the remaining cases, i.e., Tier 1 contract leadership and Tier 2 contract leadership, are presented in the Appendix. In this paper, we focus on the case of $\delta_q = 2$. However, our analysis and qualitative results can be generalized for any convex production cost, i.e., $\delta_q > 1$.

Lemma 1 establishes equilibrium prices and quantities in a supply chain with Tier l being a contract leader for $l \in \{0, 1, 2\}$.

Lemma 1. *A supply chain led by Tier l in equilibrium delivers $Q_l(\Theta)$ units of product at price $P_l(\Theta)$, depending on product quality Θ with*

$$P_l(\Theta) = \left(2 + 2\Theta^{-\beta}\Phi_l(\Theta)N\right)^{-1}\left(\Theta^{\beta} + 2\Phi_l(\Theta)N\right),$$
$$Q_l(\Theta) = N\left(2 + 2\Theta^{-\beta}\Phi_l(\Theta)N\right)^{-1},$$

where $\Phi_l(\Theta) = 2^l K_0 + 2^{|1-l|}K_1 + 2^{|2-l|}K_2\Theta^{\delta_\theta}$.

From Lemma 1, the product quality and contract leader location impact the equilibrium price and quantity outcomes. The dependence of equilibrium outcomes, $(Q_l(\Theta), P_l(\Theta))$, on leader location is driven by the *misalignment penalty* $\Phi_l(\Theta)$ that can be thought of as the severity of double marginalization associated with Tier l's contract leadership. The misalignment penalty $\Phi_l(\Theta)$ is the sum of the contract leader's direct production cost coefficient, i.e., K_0 for $l = 0$, and $K_2\Theta^{\delta_\theta}$ for $l = 2$, and the coefficients on the production costs of the other firms in the supply chain, weighted by the distance from the contract leader, i.e., $2K_1 + 4K_2\Theta^{\delta_\theta}$ for $l = 0$, and $4K_0 + 2K_1$ for $l = 2$. As a result, as $\Phi_l(\Theta)$ increases, the effective production cost increases, which decreases the equilibrium production quantity $Q_l(\Theta)$. Furthermore, leadership configurations with higher misalignment penalty values exhibit lower market coverage, since the market coverage corresponding to the contract leadership by Tier l, $\rho_l(\Theta)$, is a ratio of the equilibrium quantity sold in the market to the total market size, $\frac{Q_l(\Theta)}{N}$.

The notion of quality-driven misalignment is related to the difference in production costs between a single vertically integrated firm and a multi-tiered supply chain incurring additional agency costs. Hence, supply chains, regardless of the leader location $l \in \{0, 1, 2\}$, exhibit higher effective production costs relative to the vertically integrated case. However, some leader locations yield lower misalignment penalties resulting in lower effective production costs. The intrinsic magnitude of production cost coefficients $\{K_0, K_1, K_2\}$ and the product quality Θ from the development/investment decision determine the leader location with the lowest misalignment penalty. Thus, the supply chain leader located close to tiers with high production costs lowers $\Phi_l(\Theta)$ by reducing the effect of double marginalization captured by $2^{|j-l|}$, where $|j-l|$ is the distance between Tier j and the contract leader l. Note that Tier 2's production cost is increasing in Θ; hence, the contract leader location minimizing $\Phi_l(\Theta)$ gravitates upstream to Tier 2 as the product quality improves. This dependence of contract leader position on product quality has significant implications (as discussed in the subsequent section) and comes out of a model that jointly considers qualities, quantities, and prices.

		Contract Leader		
		$l = 0$	$l = 1$	$l = 2$
Investor	$i = 0$	$\boldsymbol{\Theta_{0\|0}}$	$\Theta_{1\|0}$	$\Theta_{2\|0}$
	$i = 1$	$\Theta_{0\|1}$	$\boldsymbol{\Theta_{1\|1}}$	$\Theta_{2\|1}$
	$i = 2$	$\Theta_{0\|2}$	$\Theta_{1\|2}$	$\boldsymbol{\Theta_{2\|2}}$

Table 1: All possible scenarios of contract leadership and investor.

3.3 Equilibrium Product Quality

Next, we consider the investment decisions in product development and innovation to improve the quality. From Lemma 1, there are three different supply chain leadership scenarios, depending on who the contract leader is. For each leadership scenario, there are three distinct possibilities depending on who invests in quality. As illustrated in Table 1 in a supply chain comprising three tiers, there are nine scenarios that differ by the identity of the investor and the identity of the contract leader.

Definition 2. Let $\Theta_{i|l}$ denote the equilibrium product quality resulting from Tier i being an investor in innovation under Tier l's contract leadership, i.e., $\Theta_{i|l} = \arg\max_{\Theta\geq 0} \{\Pi_i(P_l(\Theta), Q_l(\Theta)) - \gamma\Theta^{\delta_D}\}$.

For example, $\Theta_{1|0}$ represents the equilibrium product quality when Tier 0 is the contract leader and Tier 1 is the investor in innovation to improve the product quality. By substituting the equilibrium price and quantity from Lemma 1 into (3.1) and subtracting the development cost incurred by Tier i firm, we obtain tierwise profit expressions including the development cost, denoted as $\hat{\Pi}_i(\Theta_{i|0})$ for $i \in \{0, 1, 2\}$, given in (3.3) for the case of Tier 0's contract leadership, i.e., $l = 0$.

$$\begin{aligned}
\hat{\Pi}_2(\Theta_{2|0}) &= \max_{\Theta} \left\{ \left(K_2\Theta^{\delta_\theta}\right) Q_0^2(\Theta) - \gamma\Theta^{\delta_D} \right\}, \\
\hat{\Pi}_1(\Theta_{1|0}) &= \max_{\Theta} \left\{ \left(2K_2\Theta^{\delta_\theta} + K_1\right) Q_0^2(\Theta) - \gamma\Theta^{\delta_D} \right\}, \\
\hat{\Pi}_0(\Theta_{0|0}) &= \max_{\Theta} \left\{ \left(4K_2\Theta^{\delta_\theta} + 2K_1 + K_0 + \Theta^{\beta}N^{-1}\right) Q_0^2(\Theta) - \gamma\Theta^{\delta_D} \right\}.
\end{aligned} \tag{3.3}$$

Similar profit expressions for the remaining cases in which Tier 1 (or Tier 2) is a contract leader are provided in the Appendix. We now investigate the question of who should invest in product development and innovation to improve the product quality, given contract leadership:

Proposition 1. *When the production costs are moderately convex in quality, specifically, $\delta_\theta \in (1, 2\beta]$, and the development cost is not too low, i.e., $\gamma \geq \underline{\gamma}$,*[3]

(a) *A contract leader invests the most in innovation, yielding the highest product quality as well as the highest total supply chain profits;*

(b) *The case for the contract leader being an innovation investor also results in the largest market coverage, i.e., the most inclusive innovation.*

[3]The specific expression for $\underline{\gamma}$ is provided in the Appendix.

Proposition 1 establishes that the contract leader has the most interest in increasing the total quality of the product, which also generates the highest total supply chain profits, $\Pi_l^{\mathrm{T}}\left(\Theta_{l|l}\right) = \max_{i\in\{0,1,2\}}\left\{\left[\sum_{j=0}^{2}\hat{\Pi}_j(\Theta_{i|l})\right] - \gamma\Theta_{i|l}^{\delta_D}\right\}$. The supply chain contract leader gains the largest stake in the total supply chain profit compared to the other parties within the supply chain; hence, he has the greatest incentive to invest in product development and innovation to improve the product quality and the supply chain profits. Specifically, in the case of Tier 0 contract leadership, Tier 0's marginal revenue from investing in product quality is strictly greater than the marginal revenues faced by non-leader tiers investing in quality. As a result, from the supply chain perspective, part (a) of Proposition 1 suggests that it is desirable for the contract leader to be the investor in product development.

What is even more interesting is the mechanism that brings about the most inclusive innovation, i.e., the alignment of the contract leader to be the investor in product development leads to the broadest market coverage as well as largest supply chain profits. This finding suggests that by aligning the investor to be the contract leader, one can achieve more inclusive innovation without compromising the supply chain profits. Based on this result, we now focus our attention on three investment scenarios along the diagonal in Table 1, in which Tier i investor is also a supply chain contract leader.

Note that the analysis methodology presented in this Section is not restricted to a simple three tier supply chain. It can easily be extended to a complex multi-tier supply chain or supply networks (with the above result making the search for the investor linear in the size of the network). Furthermore, even under the complex supply network case, the contract leader will claim the largest share of the total supply chain profits, which then leads to our result: it is optimal for the supply chain contract leader to be the investor in product development for the greatest profits and market coverage.

4 Contract Leadership Identity and Handovers

Since we have established that the contract leader should be also an investor in product development, a natural question is then who should be the contract leader (or equivalently, the investor)? In other words, given that the off-diagonal cases in Table 1 are suboptimal for inclusive innovation as well as the supply chain profit perspective, among those three diagonal cases in which the contract leader is also the investor, which case leads to more inclusive innovation or greater supply chain profits? Section 4.1 answers this normative question, i.e., who should be the contract leader in a supply chain for more inclusive innovation and/or for greater supply chain profits? This analysis reveals that the identity of the contract leader depends on key development and product cost parameter settings and the contract leadership must be *handed over* to other tiers as the cost parameters change along the product lifecycle. Next, in Section 4.2, we then ask a question about voluntary leadership handovers; specifically, do the firms have incentives to hand over the contract leadership and/or to take the leadership, and if so, when? How do these voluntary handovers compare with the normative prescription in Section 4.1? Finally, we numerically investigate the case in which all firms can jointly invest to improve the component product quality in Section 4.3, and demonstrate that our key insights are preserved.

4.1 Normative Contract Leadership Handovers

In this section, we assume a normative perspective and answer the question of who should be the contract leader for (*i*) supply chain profit maximization and (*ii*) more inclusive innovation (broader market coverage), depending on the development cost γ and the key component production cost K_2. Equation (4.1) below provides a simplified expression of the contract leader's profit maximization problem:

$$\hat{\Pi}_{l|l} = \max_{\Theta} \left\{ \frac{1}{4}\Theta^{2\beta} \left(\Phi_l(\Theta) + \Theta^{\beta} N^{-1} \right)^{-1} - \gamma \Theta^{\delta_D} \right\}. \tag{4.1}$$

Recognize that the contract leader's revenue is decreasing in the component production cost K_2 as captured by the inverse dependence on the misalignment penalty $\Phi_l(\Theta)$. Furthermore, the rate at which higher values of K_2 or higher product quality levels reduce the contract leader's revenue is determined by the distance between the leader l and Tier 2. Intuitively, dependence on the distance between the contract leader and the investment target provides variation in the entity of optimal contract leader throughout a product's lifecycle where reduction in the development cost leads to higher product quality followed by a decreasing component production cost.

Proposition 2.

(a) *For moderately convex production costs at Tier 2, specifically,* $1 < \delta_\theta \leq 2\beta$*, and the production costs at Tiers 0 and 1, such that* $K_1 < K_0 < 4K_1$*, as the product development becomes less costly, i.e., as* γ *decreases, the contract leadership position yielding the highest market coverage and product quality shifts from the downstream to the upstream tiers, i.e., from Tier 0, to Tier 1, and then to Tier 2.*[4]

(b) *The leadership handover points yielding the highest product quality (or, equivalently, the highest supply chain profit) throughout product lifecycle occur earlier, i.e., at higher values of* γ*, relative to handovers yielding the highest market coverage.*

(c) *Supply chains with high upstream production costs, i.e., high values of* K_2*, exhibit earlier contract leadership handovers with respect to both product quality and market coverage.*

Part (a) of Proposition 2 shows that as γ decreases, the supply chain leadership should be deliberately transferred from the downstream to the upstream tier, i.e., from Tier 0, to Tier 1, and then to Tier 2, to generate the highest investment and product quality, which then leads to the largest supply chain profit as well as market coverage. Decreasing development costs in the emerging stage of a product's lifecycle yields rapid product innovation and, hence, increased product performance

[4]The condition $K_0 < 4K_1$ is a sufficient condition for the proof. We numerically illustrate that this result can hold in more general cases (see, e.g., Figures 4, 6, 7 and 8).

quality Θ. However, Tier 2 must bear the increase in the production cost associated with higher product quality.

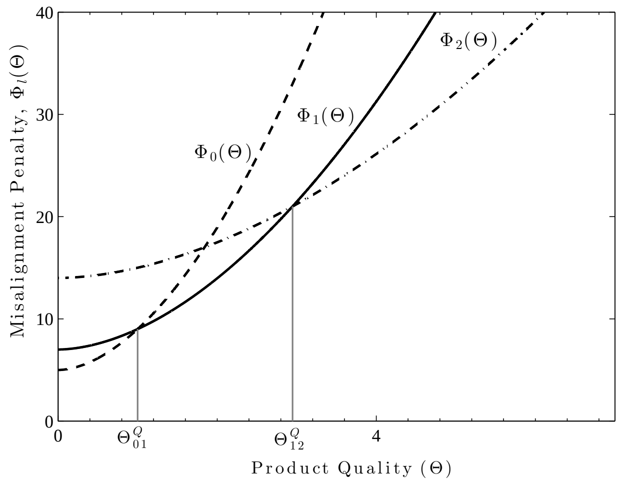


Figure 3: Misalignment penalty. Parameter values are $(K_0, K_1, K_2) = (3, 1, 1)$, $\delta_\theta = 2$, $\delta_D = 2$, $N = 1$ and $\beta = 1$.

Figure 3 depicts the misalignment penalty $\Phi_l(\Theta)$ increasing in Θ for tiers $l \in \{0, 1, 2\}$. Hence, as product quality Θ increases, the leader location minimizing the misalignment penalty gravitates upstream. From Figure 3, one can identify the existence of three distinct regions where the misalignment penalty $\Phi_l(\Theta)$ is the smallest under each corresponding leader location, $l = 0$, 1, 2. Any alteration in the relative levels of the misalignment penalty drives the shifts in the position of the contract leader yielding the highest total profits and market coverage.

Figure 4 illustrates the threshold of the γ value between Tier 0 and Tier 1 that generates the same market coverage. As illustrated in panel (c), at $\gamma = \gamma_{01}^{\rho}$, if Tier 0 is the contract leader, he makes an effort into attaining the product quality of Θ_0, whereas if Tier 1 is the contract leader, she puts effort into achieving the product quality of Θ_1. Moreover, at those quality levels of Θ_0 and Θ_1 under the corresponding contract leadership, the resulting market coverages are the same as those depicted in the lower part of panel (c). If $\gamma > \gamma_{01}^{\rho}$, Tier 0 contract leadership leads to broader market coverage; otherwise, Tier 1 contract leadership results in more coverage. Panel (d) illustrates how to find this threshold of γ; in this panel, the x-axis is the γ value for Tier 0 contract leadership and the y-axis is the corresponding γ value for Tier 1 contract leadership that leads to the same market coverage. For example, if $\gamma = \gamma_{01}^{\Theta}$, i.e., at the maximum γ in panel (d), as depicted in panel (a), the product quality becomes Θ_{01}^{MR} under Tier 0 contract leadership. In order to attain the same market coverage under Tier 1 contract leadership as illustrated in the lower part of panel (a), the product quality should be $\hat{\Theta}$, which is achieved at $\gamma = \hat{\gamma}_{01}^{\Theta}$ ($<\gamma_{01}^{\Theta}$) under Tier 1 contract leadership. In contrast, the opposite holds at the minimum $\gamma = \gamma_{01,0}^{Q}$ in panel (d), i.e., as depicted in panel (b), at $\gamma = \gamma_{01,0}^{Q}$, the product quality becomes Θ_{01}^{Q} under Tier 0 contract leadership. The same market coverage is achieved at $\gamma = \gamma_{01,1}^{Q}$ ($>\gamma_{01,0}^{Q}$) under Tier 1 contract leadership. Finally, we can prove that as in panel (d), there exists the unique $\gamma = \gamma_{01}^{\rho} \in (\gamma_{01,0}^{Q}, \gamma_{01}^{\Theta})$, such that Tier 0

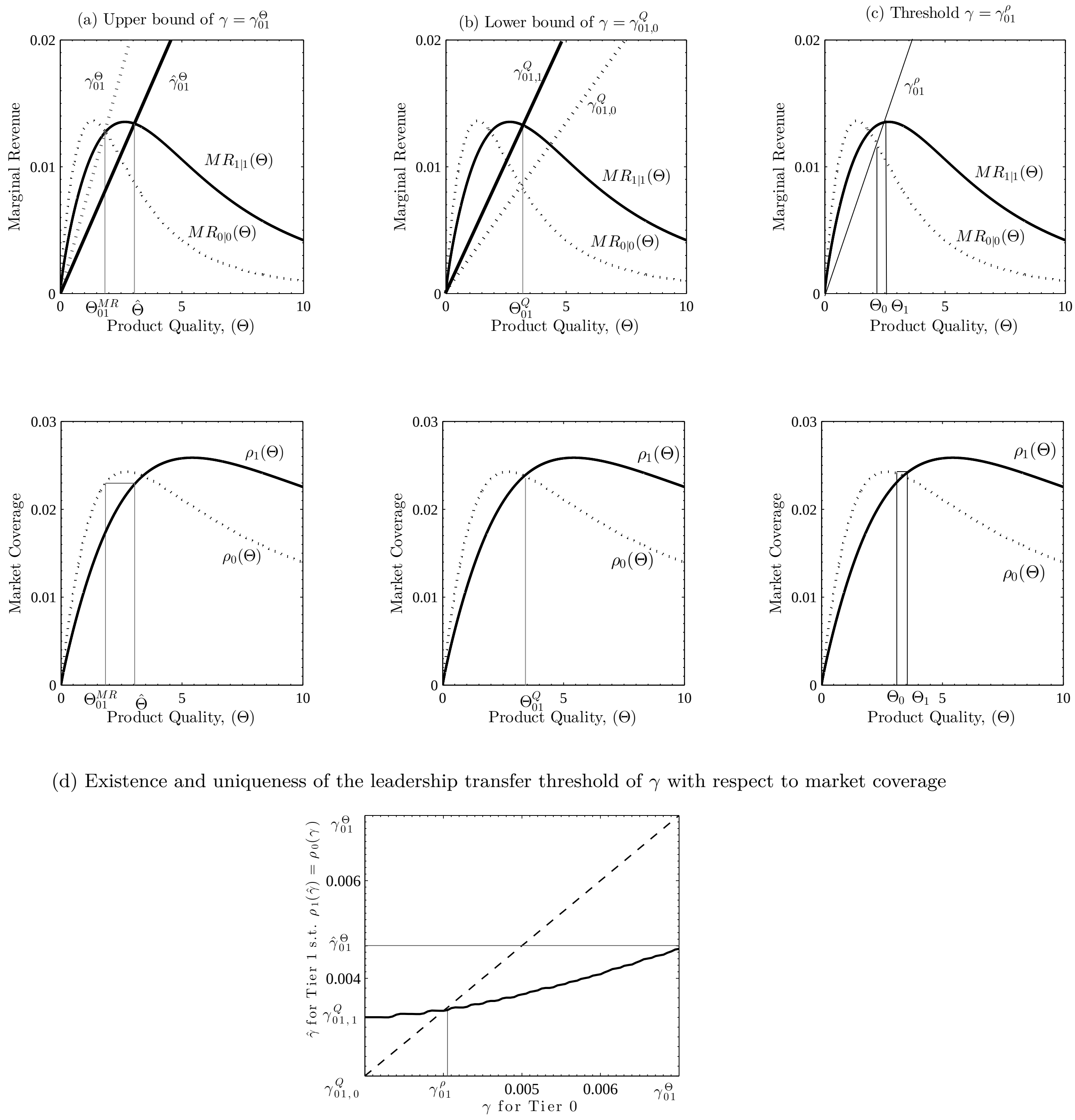
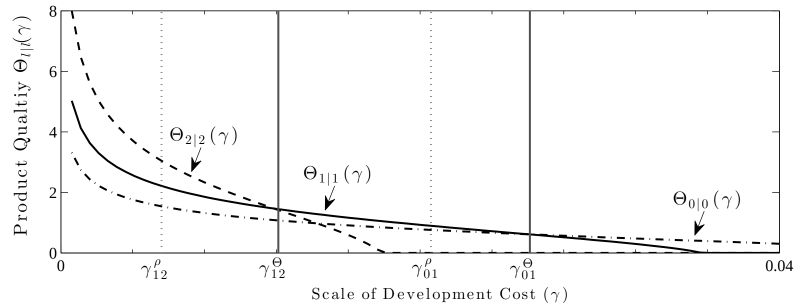
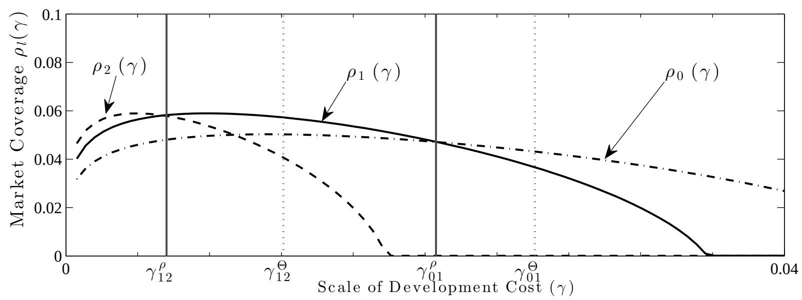


Figure 4: Illustration of the threshold for the contract leadership transfer between Tier 0 and Tier 1 to attain broader market coverage in part (a) of Proposition 2. Parameter values are $(K_0,\, K_1,\, K_2) = (10, 2, 1)$, $\delta_\theta = 1.7$, $\beta = 0.9$, and $N = 1$.

or Tier 1 contract leadership yields the same market coverage.



(a) Optimal Product Quality, $\Theta(\gamma)$.



(b) Market Coverage, $\rho_l(\gamma)$.

Figure 5: Threshold γ values for optimal contract leadership transfers for the highest product quality (and the total supply chain profit) in panel (a) and for inclusive innovation in panel (b). Parameter values are $(K_0, K_1, K_2) = (3, 1, 1)$, $\delta_\theta = 2$, $\delta_D = 2$, $N = 0.1$ and $\beta = 1$.

In addition, as stated in part (b) of Proposition 2 and illustrated in panel (d), this threshold $\gamma = \gamma_{01}^{\rho}$, that results in the same market coverage, is smaller than the threshold $\gamma = \gamma_{01}^{Q}$, which leads to the same product qualities as well as the same total supply chain profits for Tier 0 contract leadership and Tier 1 contract leadership. Figure 5 directly illustrates this comparison of the thresholds by indicating the locations of leadership handover points for the highest product quality, $\{\gamma_{12}^{\Theta}, \gamma_{01}^{\Theta}\}$ in panel (a), and for broader market coverage, $\{\gamma_{12}^{\rho}, \gamma_{01}^{\rho}\}$, in panel (b) with respect to the cost of product development γ. Note that with a liberal use of notation, we now consider $\Theta_{l|l}$ and ρ_l as functions of γ, i.e., $\Theta_{l|l}(\gamma)$ and $\rho_l(\gamma)$. Recognize that contract leadership transfers yielding the highest product quality (and consequently the highest supply chain profit) occur at larger values of γ, relative to contract leadership transfers yielding the highest market coverage, i.e., $\gamma_{01}^{\rho} < \gamma_{01}^{\Theta}$ and $\gamma_{12}^{\rho} < \gamma_{12}^{\Theta}$. Furthermore, recall from Figure 1 that during the product innovation phase, as time goes by, the magnitude of the development cost coefficient γ decreases. Therefore, the deliberate supply chain contract leadership transfers can benefit the supply chain as well as expand the market coverage. However, the leadership transfers with respect to the optimal supply chain profit occur at greater values of γ relative to the leadership transfer points allowing to maintain the highest possible market coverage.

Finally, in part (c) of Proposition 2, we establish the fact that products with higher upstream production costs, as indicated by large values of K_2, exhibit leadership shifts at relatively higher levels of γ for product quality and market coverage. In other words, part (c) demonstrates that contract leadership transfer points shift to lower values of γ as the upstream production cost, K_2, is decreasing, which implies that given a fixed γ, as K_2 decreases, downstream leadership transfers may become desirable from both the supply chain profit perspective and the inclusive innovation perspective.

Proposition 2 provides us with two mechanisms that drive leadership transfers and, hence, alteration of the entity of the optimal investor. For example, decreasing levels of γ result in leadership transfers in an upstream direction. A reduction in K_2 results in a shift in leadership transfer points in the direction of decreasing γ, which then leads to the reassignment of some constant development cost level in a downstream direction. This dependence of the entity of the optimal investor on the underlying choice of model parameters, (γ, K_2), leads us to examine comparative statics in the context of product and process innovation in Section 5.

4.2 Tierwise Rational Handovers

Thus far, we have taken a normative stance on who should be the contract leader in order to achieve the largest supply chain profit and/or to attain the broader market coverage, i.e., more inclusive innovation. The natural question is then whether each firm within a supply chain prefers (finds it individually rational) to transfer the contract leadership or assume the leadership. In this section, we question each tier's individual incentive to transfer the contract leadership. We demonstrate the existence of upstream voluntary contract leadership handovers during the product innovation phase:

Proposition 3. *Voluntary contract leadership handovers in an upstream direction, i.e., from Tier 0 to Tier 1, and Tier 1 to Tier 2, during the product innovation phase, i.e., as γ decreases, exist as both the production costs and the consumer's utility approach linearity in product quality.*

Figure 6 illustrates the voluntary handover of contract leadership from Tier 0 to Tier 1. Specifically, if $\gamma > \hat{\gamma}$ (in region (E)), Tier 0 prefers to be the contract leader investing in quality improvement, i.e., his profit is higher under his contract leadership than under the other tier's contract leadership. In addition, in this case, the other tiers also prefer Tier 0 to be the contract leader. Moreover, in this region (E), Tier 0 contract leadership also maximizes the total supply chain profits and leads to more inclusive innovation. If γ decreases below γ_{01}^{IR} (in region (A)), then Tier 0 is willing to hand over the contract leadership to Tier 1, and Tier 1 is also willing to take over the contract leadership and to be the investor. Furthermore, in this case, Tier 2 is also better off under Tier 1's contract leadership than under Tier 0's contract leadership. In addition, a voluntary handover of leadership in this region (A) occurs past the optimal handover point yielding the broadest market coverage, $\gamma_{01}^{\rho} \geq \gamma_{01}^{IR}$, so in this region (A), Tier 1 contract leadership yields the broadest market coverage as well as the largest total supply chain profits.

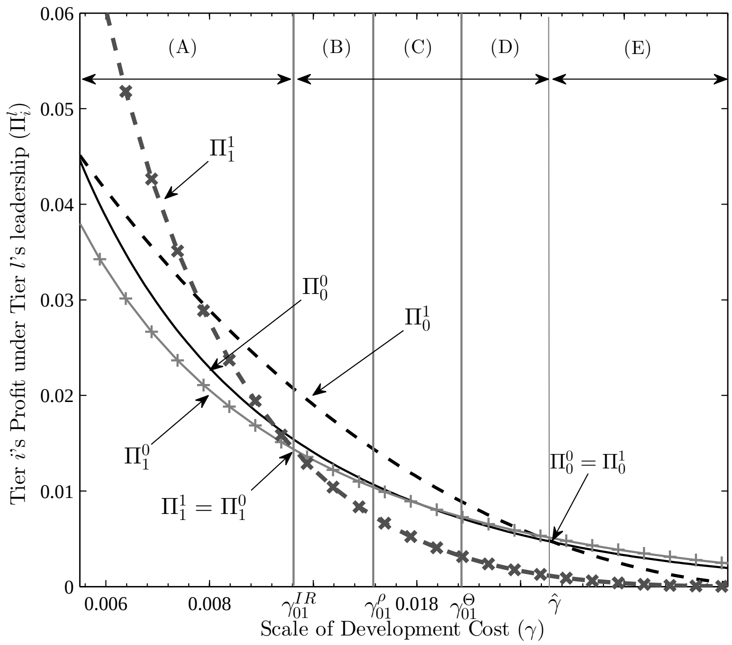


Figure 6: Illustration of a voluntary leadership handover from Tier 0 to Tier 1. Lines with "x" marks represent Tier 1's profits, and lines without marks represent Tier 0's profits. Solid lines represent profits under Tier 0 leadership and dotted lines represent profits under Tier 1 leadership. Parameter values are $(K_0,\, K_1,\, K_2) = (6, 1, 1)$, $\delta_\theta = 1.1$, $\beta = 1$, $N = 1$, and $\delta_D = 2$.

Denote Tier j's profit under Tier l's contract leadership as Π_j^l. Then, Tier l leader investing in product quality attains profits $\Pi_l^l(\gamma) = \left(\Phi_l(\gamma) + \Theta_{l|l}^{\beta}(\gamma)N^{-1}\right) Q_l^2(\gamma)$, which can be understood as the common revenue component $Q_l^2(\gamma)$ multiplied by the corresponding weight denoted as $g_{l|l}(\gamma) = \Phi_l(\gamma) + \Theta_{l|l}^{\beta}(\gamma)N^{-1}$. Once Tier 0 hands over leadership to Tier 1, his profit becomes $\Pi_0^1(\gamma) = K_0 Q_1^2(\gamma)$, which can be also understood as the multiplication of the common revenue component, $Q_1^2(\gamma)$ by the weight $g_{0|1}(\gamma) = K_0$. However, now Tier 0 does not incur the product development cost, $\gamma\Theta_{0|0}^{\delta_D}(\gamma)$. Similarly, under Tier 0's leadership, Tier 1's profit is, $\Pi_1^0(\gamma) = (K_1 + 2K_2\Theta_{0|0}^{\delta_\theta}(\gamma))Q_0^2(\gamma)$, which is again the multiplication of the weight $g_{1|0}(\gamma) = K_1 + 2K_2\Theta_{0|0}^{\delta_\theta}(\gamma)$ and the common revenue component $Q_0^2(\gamma)$. Figure 7 illustrates the behavior of the constituents of the profit expressions in Figure 6. Observe that at $\hat{\gamma}$ when Tier 0 would like to hand over leadership to Tier 1, the common revenue component under Tier 0's leadership is still larger than in the case of Tier 1's leadership, $Q_0^2(\hat{\gamma}) \geq Q_1^2(\hat{\gamma})$. Similarly, the weights associated with Tier 0 are such that $K_0 = g_{0|1}(\hat{\gamma}) \leq \left(\Phi_0(\hat{\gamma}) + \Theta_{0|0}^{\beta}(\hat{\gamma})N^{-1}\right) = g_{0|0}(\hat{\gamma})$, indicating that leadership should not be handed over. However, the benefit of handover for Tier 0 originates from avoiding the development cost, $\hat{\gamma}\Theta_{0|0}^{\delta_D}(\hat{\gamma})$. Moreover, Tier 1 will accept leadership only at γ_{01}^{IR}, when the revenue component $Q_1^2(\gamma)$ and the weighting factor, $g_{1|1}(\gamma)$ are sufficiently large to offset the cost of development incurred by the leader, $\gamma\Theta_{1|1}^{\delta_D}(\gamma)$, as illustrated in Figure 7.

We demonstrated that if γ is either higher than $\hat{\gamma}$ in region (E) in Figure 6 or lower than γ_{01}^{IR} in region (A), firms are willing to hand over the leadership. What happens at $\gamma \in \left(\gamma_{01}^{IR},\, \hat{\gamma}\,\right)$ in regions

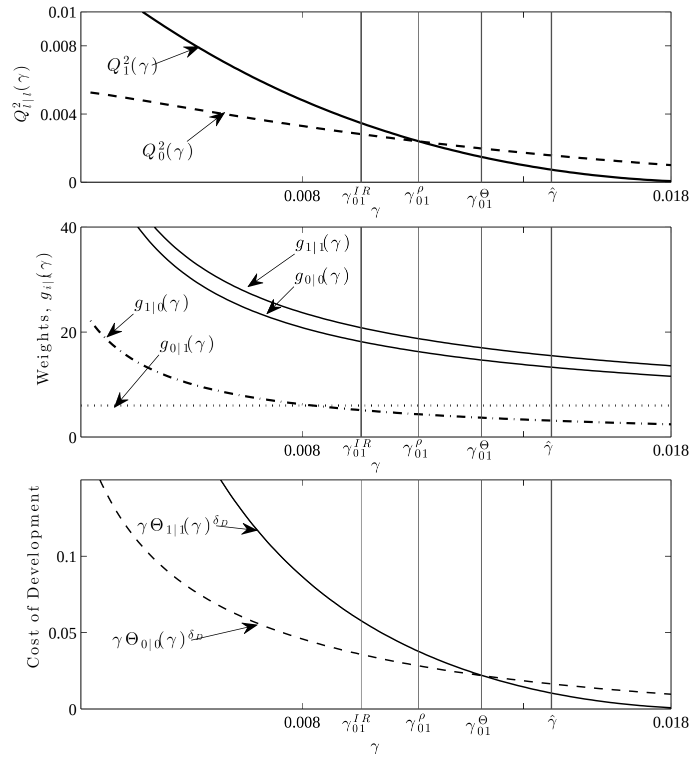


Figure 7: Components of profit expressions. Parameter values are the same as those in Figure 6.

(B), (C) and (D) in Figure 6? In these regions, the negative impact of the product development cost dominates the benefit of contract leadership. As a result, Tier 0 prefers to hand over the contract leadership to Tier 1. However, Tier 1 also prefers Tier 0 to remain the contract leader. That is, in these intermediate regions, no firm wants to be the leader. From the normative perspective that we investigated in Section 4.1, in region (B), it is better for Tier 1 to be the contract leader, whereas in region (D), Tier 0 leadership is optimal from both the supply chain perspective and the market coverage/inclusive innovation perspective. In region (C), it is better for Tier 1 to be the contract leader from the supply chain profit perspective, but it is better for Tier 0 to be the leader from the market coverage/ inclusive innovation perspective.

What can firms within a supply chain or policy makers do in the intermediate ranges, i.e., in regions (B), (C) and (D)? The policy maker may provide R&D tax credits for innovation investment to Tier 0 in regions (C) and (D) and to Tier 1 in region (B) to encourage more inclusive innovation with the deliberate contract leadership. From the supply chain perspective, Tier 1 may subsidize Tier 0's investment in $\gamma \in (\gamma_{01}^{T}, \hat{\gamma})$ as illustrated in Figure 8; note that in this region, Tier 1's gain from Tier 0 leadership compared to her own contract leadership is higher than Tier 0's loss from his own leadership, so Tier 1 has a financial incentive to subsidize Tier 0. Similarly, in $\gamma \in (\gamma_{01}^{IR}, \gamma_{01}^{T})$, Tier 0 has an incentive to subsidize Tier 1's contract leadership. In summary, either a tax credit from the policy maker's perspective or a development investment subsidy within a supply chain can help lead to the more desirable contract leadership pattern that yields higher coverage.

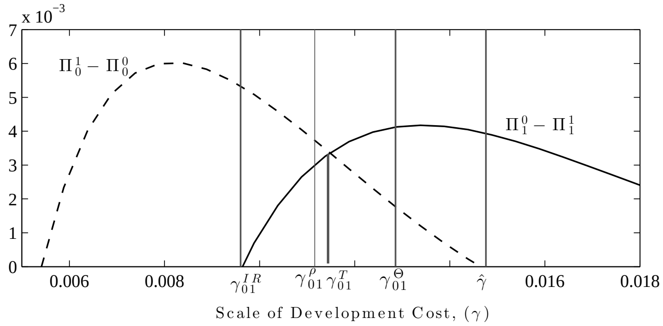
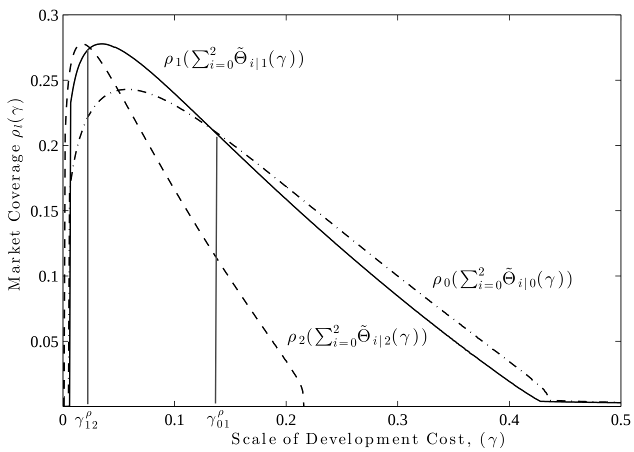
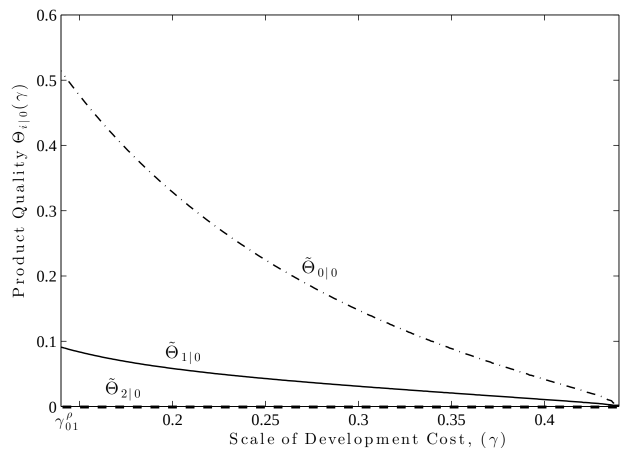


Figure 8: Transfer to reduce the size of the leadership handover gap. Parameter values are the same as those in Figure 6.



(a) Market coverage across different leader locations.



(b) Product qualities under Tier 0 leadership.

Figure 9: Joint Investment. Parameter values are $(K_0,\, K_1,\, K_2) = (0.3, 0.2, 0.1)$, $\delta_\theta = 2$, $\beta = 1$, $N = 1$, and $\delta_D = 2$.

4.3 Joint Investment

In this paper we have examined market coverage outcomes resulting from single investor scenarios in which Tier i investor under Tier l contract leadership is directly investing in product development, which yields final product quality and, hence, is the only investor. However, all tiers may invest jointly and the final product quality may become a function of these joint contributions. In order to understand whether our results remain valid under this joint investment case, we perform a numerical study of joint investment levels within an equilibrium framework where the Tier i^{th} choice of product quality under Tier l^{th} leadership, $\tilde{\Theta}_{i|l}\left(\gamma, \tilde{\Theta}_{-i|l}(\gamma)\right)$, depends on other investors' choices of product quality $\tilde{\Theta}_{-i|l}(\gamma)$. Furthermore, we let final product quality be additive in individual investment levels, such that $\Theta_l^{\text{joint}} = \sum_{i=0}^{2} \tilde{\Theta}_{i|l}$.

Panel (a) of Figure 9 plots the market coverage resulting from joint investments across different leader locations, $\rho_l\left(\Theta_l^{\text{joint}}\right)$ for $l = 0,\, 1,\, 2$. Recognize that our primary results remain valid under this joint investment case; specifically, there exist $\{\gamma_{12}^{\rho}, \gamma_{01}^{\rho}\}$, development cost levels at which handing over leadership across tiers allows to maintain the broadest market coverage. Under high development costs ($\gamma > \gamma_{01}^{\rho}$), Tier 0 contract leadership yields the most inclusive innovation, and under intermediate development costs ($\gamma \in (\gamma_{12}^{\rho}, \gamma_{01}^{\rho})$), Tier 1 contract leadership leads to the most inclusive innovation. Finally, under small γ ($< \gamma_{12}^{\rho}$), Tier 2 contract leadership is optimal from the market coverage perspective.

Panel (b) of Figure 9 examines tierwise investment in product development for $\gamma > \gamma_{01}^{\rho}$, where market coverage from joint investment is highest under Tier 0 leadership. This figure indicates that for $\gamma > \gamma_{01}^{\rho}$ product quality contributions by non-leader Tiers 1 and 2 are negligible relative to Tier 0 leader's investment in product development, $\tilde{\Theta}_{0|0}(\gamma)$. This illustration aligns with the intuition provided by Proposition 1, where the contract leader invests the most, justifying our focus on the cases in which the contract leader is a single dominant investor. Overall, this illustration extends the relevance of our normative recommendations about the contract leader's location yielding the highest market coverage, i.e., inclusive innovation.

5 Product Lifecycle and Contract Leadership Transfers for Broader Market Coverage

We now consider how the quality investment, market coverage, and leadership results of the previous section depend on the stage of a product's lifecycle. It is widely recognized that most innovative products go through two distinct phases of product and process innovation (Utterback and Abernathy 1975). The lifecycle usually commences with a product innovation phase of rapidly-rising development productivity/decreasing development cost as shown in Figure 1. However, as performance improvement/development productivity reaches maturity, opportunities for gains are associated with potential reductions in the production cost. Specifically, at the maturity stage, process innovation rather than continued product innovation dominates, and production costs decrease as a result of industry-wide improvement in manufacturing technology. In this section, we examine comparative statics in the context of these two phases of the product lifecycle.

Table 2 provides an environment in which product innovation is associated with a period of decreasing development cost γ, i.e., the first phase of the lifecycle with gradually decreasing levels of development cost, $\gamma_H > \gamma_M > \gamma_L$, for some fixed and initially high-scale upstream production costs, K_2^H. Afterward, a period of process innovation follows, which is modeled as a decrease in the scale of upstream production costs K_2, while the scale/value of development costs remains constant. That is, once the magnitude of development costs reaches its lower bound γ_L in $t = t_3$ at the maturity stage, process innovation proceeds with a decreasing sequence of upstream production costs, $K_2^H > K_2^M > K_2^L$. The notion of a product lifecycle here is independent of individual firms and instead is an industry-wide phenomenon. Hence, t_j represents an instance in a specific industry characterized by a pair of cost parameters $(\gamma(t_j), K_2(t_j))$.

In the results of Proposition 2, we see how the contract leadership offering the highest market coverage responds to decreasing product development costs, γ, and the scale of upstream production

t:	t_1	t_2	t_3	t_4	t_5
γ:	γ_H	γ_M	γ_L	γ_L	γ_L
K_2:	K_2^H	K_2^H	K_2^H	K_2^M	K_2^L

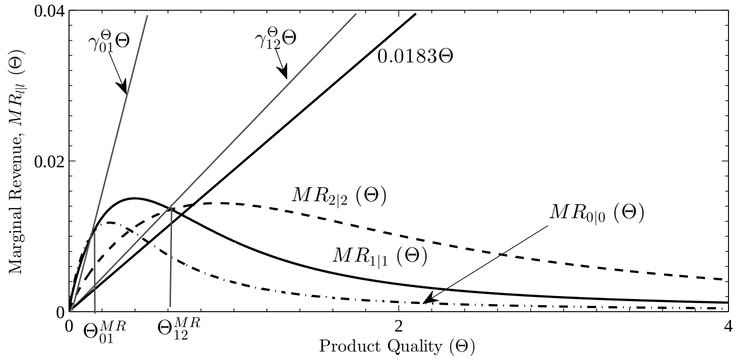
Table 2: The pairs (γ, K_2) along the product lifecycle. Note that $t = t_1, t_2, t_3$ correspond to the product innovation phase, which is followed by process innovation phase in $t = t_4, t_5$. Parameter values satisfy $t_1 < t_2 < t_3 < t_4 < t_5$, $\gamma_L < \gamma_M < \gamma_H$ and $K_2^L < K_2^M < K_2^H$.

costs, K_2. Thus, the examination of changes during a product lifecycle amounts to the application of Proposition 2 in the context of product and process innovation.

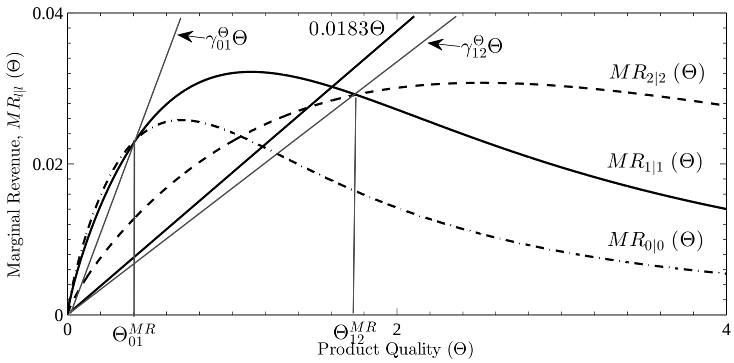
During product innovation phase that is associated with a reduction in γ, the contract leader location resulting in both highest product quality and market coverage shifts upstream as the magnitude of development cost γ decreases. Formally, the supply chain maximizes its profits and, hence, product quality, Θ, during product innovation by deliberately transferring leadership from Tier 0 to Tier 1 when the level of development cost shifts from the high, $\left[\gamma_{01}^{\Theta}, \infty\right)$, to the medium, $\left[\gamma_{12}^{\Theta}, \gamma_{01}^{\Theta}\right)$ cost regime. Finally, once the development cost reaches $\gamma_L < \gamma_{12}^{\Theta}$, the contract leadership should be further transferred to Tier 2. A similar argument applies to the case of market coverage when the contract leadership transfers occur at $\{\gamma_{01}^{\rho}, \gamma_{12}^{\rho}\}$. Thus, during the product innovation phase, it is optimal to gradually transfer leadership upstream and make the component suppliers play a leading role in making contract and investment decisions. This strategy parallels the experience of the personal computer industry in the 1990s when the leadership was transferred from downstream players like Apple and HP to upstream component suppliers such as Intel and Microsoft.

Now, consider the process innovation phase to see what happens to the contract leadership. The analysis of process innovation corresponds to the case of decreasing levels of K_2, given a fixed low γ. Part (c) of Proposition 2 states that supply chains with relatively lower upstream production costs exhibit optimal leadership handover points that are associated with relatively lower development cost levels. Therefore, as the process innovation phase unravels, the optimal supply chain leadership with respect to both product quality and market coverage is shifted downward. In other words, during the process innovation phase, the supply chain is better off with the leadership being gradually transferred downstream for maximum market coverage. The downstream leadership shift captured by our model is similar to the return to the leadership of downstream players like Apple in the computer supply chain during the 2000-2010.

We first illustrate the shifts in leaderships using the following parameter values before formally proving the existence of these shifts. Panels (a) and (b) of Figure 10 identify optimal product qualities, $\Theta_{l|l}(\gamma)$, via intersections of the marginal revenue of investment, $MR_{l|l}(\Theta)$, and the marginal development cost, $MC(\Theta) = \gamma\Theta$, for $\delta_D = 2$. Suppose that the development cost level is set to $\hat{\gamma} = 0.0183$. Panel (a) depicts the case of a high upstream production cost regime, $K_2 = 6$. In this regime, the optimal contract leader to achieve the largest supply chain profit is Tier 2 from



(a) High upstream production costs, $K_2 = 6$.



(b) Low upstream production costs, $K_2 = 1$.

Figure 10: Leadership transfers for decreasing production costs. Parameter values are $K_0 = 3$, $K_1 = 2$, $\delta_\theta = 1.7$, $\beta = 0.9$, $N = 1$ and $\delta_D = 2$.

$\hat{\gamma} = 0.0183 < \gamma_{12}^{\Theta}$. Panel (b) of Figure 10 considers a lower production cost regime, $K_2 = 1$ - where at the same development cost level $\hat{\gamma}$, the contract leader should be deliberately reassigned/transferred to Tier 1 in order to achieve the largest supply chain profit, since $\hat{\gamma} \in (\gamma_{12}^{\Theta}, \gamma_{01}^{\Theta})$ in this case. Thus, as the production cost parameter K_2 decreases, the contract leadership should be transferred in downstream direction, i.e., from Tier 2 to Tier 1.

A different way to illustrate a change in assigning the optimal leader location in a decreasing scale of production cost K_2 is provided in Figure 11. This figure enhances the understanding of the implications of the shift in handover points to a later time in the product lifecycle as a result of decreasing K_2, given a low fixed development cost level γ. The diagram in Figure 11 combines panels (a) and (b) of Figure 10 in its inner and outer quarter-circles, respectively. The division into sectors in the inner quarter-circle of the diagram corresponds to the development cost γ ranges under the high production cost regime in panel (a) of Figure 10. The outer quarter-circle indicates a downward shift of optimal leadership handover points resulting from lower production costs, $K_2 = 1$, as captured in panel (b) of Figure 10. Thus, movement along the marginal cost curve from the inner to the outer quarter-circle represents stages of process innovation, where despite a fixed development cost level $\hat{\gamma} = 0.0183$, it is optimal to hand over leadership from Tier 2 to Tier 1.

We present this sequence of smooth leadership handovers exhibiting a complete reversal towards

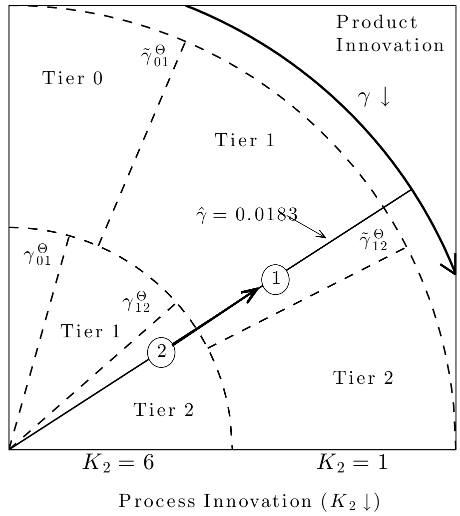


Figure 11: Mechanism for market coverage-maximizing leadership handovers in the downstream direction.

downstream in the process innovation phase in Table 2, and Proposition 4 formally establishes the existence of the parameter values $\{\gamma_H, \gamma_M, \gamma_L\}$ and $\left\{K_2^H, K_2^M, K_2^L\right\}$ mentioned in Table 2.

Proposition 4. *In supply chains exhibiting an initial asymmetry of production costs and process innovation resulting in a reduction of the scale of upstream production costs, i.e., K_2, there exists a sequence of deliberate leadership handovers, such that during product innovation phase (decrease in γ), the contract leadership is shifted upstream from Tier 0 to Tier 1 and then to Tier 2. This sequence is reversed during the process innovation phase in which the entity of an optimal leader with respect to both product quality and market coverage shifts downstream from Tier 2, to Tier 1, and then Tier 0.*

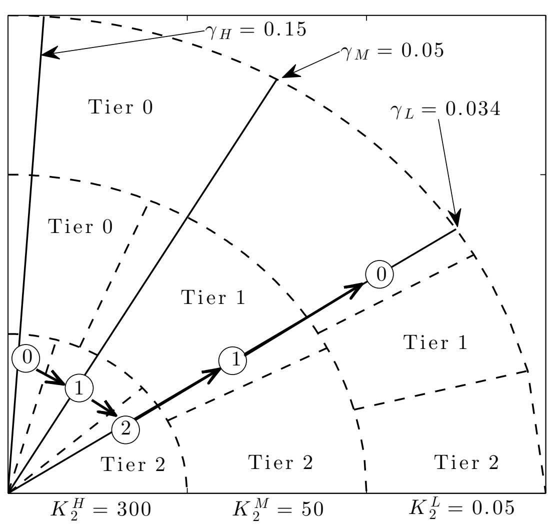


Figure 12: Leadership handovers during the product lifecycle

Figure 12 is a diagrammatic representation of the sequence of leadership handovers from Table 2. This diagram, similar to the one presented in Figure 11, captures the leadership handovers associated with product innovation via marginal costs $\{\gamma_H\Theta, \gamma_M\Theta, \gamma_L\Theta\}$ for $\delta_D = 2$ intersecting the high, medium and low development cost sectors within the inner quarter-circle associated with high upstream production costs, K_2. Thus, product innovation results in a shift of leadership to an upstream Tier 2. During the process innovation phase, handovers in the downstream direction are driven by decreasing K_2, such that the final marginal cost, $\gamma_L\Theta$, intersects the medium development cost range in the middle quarter-circle corresponding to K_2^M, and the high development cost range, i.e., Tier 0 range, of the outer quarter-circle associated with K_2^L. Thus, the sequence of optimal contract leaders exhibits a complete reversal back to Tier 0.

Examination of the comparative statics in a simple trilateral-supply chain yields a rich set of possible leadership handover sequences, depending on the choice of decreasing development costs and component production costs. Considering product and process innovation allows us to construct a leadership handover sequence that exhibits reversal/oscillation in the entity of the optimal leader with respect to both product quality (or total supply chain profit) and market coverage. Thus, this framework provides useful insights into the nature of the optimal leader location in the industries consisting of supply chains without a preexisting dominant contract leadership. Note that both in the initial stage of product innovation with a high development cost γ and the final stage of process innovation with a low production cost K_2, it is desirable for a Tier 0 firm to be the contract leader and the direct investor in a Tier 2 supplier, skipping the Tier 1 firm. It is interesting to notice that in our sequential contract stage, firms in the adjacent tiers contract with each other; however, in terms of investment, when a Tier 0 firm directly invests in a Tier 2 supplier skipping the Tier 1 assembler, it leads to higher supply chain profit as well as broader market coverage. As mentioned, these results mirror the development in industries such as the personal computer (PC) and cellphone industries. During the early stage product innovation phase of the PC industry in the 1990s, the leadership was gradually transferred from downstream players like IBM, Apple and HP to upstream component suppliers like Intel and Microsoft. Later, during the process innovation phase (a period of product maturity), we are seeing a return to the leadership of downstream players such as Apple and Lenovo. As the industry goes through a new S-curve of innovation, this oscillatory pattern may repeat itself explaining the swinging leadership and fortunes of companies during the lifecycle.

6 Conclusion and future research

In this paper, we have focused on the central question of how new innovative products can be made available and affordable to a broader market, making such innovations more inclusive. As discussed in the Introduction section, even important life-saving innovations such as automotive airbags have not been very inclusive in the past, missing the opportunity to improve both social welfare and the innovating firm's financial performance. Our key finding is that the locus of development investment decision making in a multi-tiered supply chain offers a new hitherto undiscovered degree of freedom to streamline decision making and improve the market coverage of innovative products.

In deriving the results, we considered products with substantial development and production costs, as is the case with new knowledge-intensive products - such as in life sciences and even software-enabled hard goods (for instance, next generation automobiles, smart phones, tablets, and machine tools). Unlike prior work in new product development modeling (which deals primarily with product quality) or supply chain management (where the focus is on quantity), we are able to jointly consider decisions made about product quality, price and quantity. Specifically, the choice of product quality Θ is modeled as an investment in innovation. The equilibrium price and quantity are established via a sequence of wholesale price offers using a backward-induction solution approach. Using the standard framework that all consumers who receive positive net utility purchase the product, we are able to show that deliberately choosing which firm in a multi-tiered supply chain invests in innovation and development to improve product quality Θ can have a significant impact on the market coverage of the product. This result is derived gradually, beginning with the notion of a supply chain leader (who initiates the sequence of wholesale price offers).

In conducting the analysis, we adopted the standard product lifecycle perspective; the early stages of product lifecycle are characterized by product innovation when the development productivity improves (the cost parameter γ decreases) substantially, whereas in the later stages in which the product enters maturity, process innovation dominates and the production cost K_2 decreases. We are able to formally show that (1) the supply chain leader invests the most relative to other tiers, leading to the largest total supply chain profits (for a substantial range of production costs), and (2) the supply chain leader should be deliberately transferred from the downstream to the upstream tier, i.e., from Tier 0, to Tier 1, and then to Tier 2, as the development cost γ decreases. In addition, we are able to construct a sequence of deliberate leadership handovers, such that during the product innovation stage, the leadership is optimally shifted upstream from Tier 0 to Tier 1 and then to Tier 2 whereas during the process innovation stage, the leadership should be shifted back downstream from Tier 2, to Tier 1, and then Tier 0 to optimize product quality and market coverage. Deeper analysis in Section 4.2 allows us to go beyond a normative prescription by identifying conditions under which tierwise rational leadership transfers occur and discuss how to align individually rational schedule of leadership transfers with an optimal schedule that results in the highest market coverage. We then translate these findings to the context of a product lifecycle in Section 5 and discover the interesting property of reversal/oscillation in the identity of the optimal leader for maximal market coverage.

Our results have important and subtle implications for firms such as the electric car maker (OEM) we discussed earlier in the paper. Specifically, the analysis presented in the paper suggests that to obtain broader market coverage for its innovations (a stated goal of the company), the OEM should initially drive investments in core technology (battery) innovation while gradually allocating a greater role to its suppliers as the product development becomes easier and costs decrease. Once the product is mature and the improvements have reduced production costs, it makes sense for the OEM to take greater control of the decision making with respect to innovation investments. By doing so, the firm is able to launch innovative products and ensure that more of the marketplace is able to enjoy them. As a result, the aggressive reduction of the development and production costs is naturally translated into more profitable, affordable, and higher-quality products. Our approach offers firms additional degrees of freedom, available in the context of easy-to-implement wholesale-price contracts, providing an alternative to more complex non-linear contracts in mitigating misalignment between vertically integrated and decentralized supply chain environments.

To keep a sharp focus on inclusive innovation and to manage complexity, we formulated a stylized analytical model with its own limitations. First, consistent with the electric car and electronics/computer industry examples discussed in the paper, we considered the innovation investment in a core component/technology, such as the electric car battery or the microprocessor, developed by a Tier 2 supplier. In other industries, the primary quality-enhancing investment can be associated with other tiers of the supply chain, e.g., the final product quality can be affected primarily by the design of the end product, which is related to the investment in Tier 0 manufacturer. However, our analysis remains valid as long as there exists a single primary investment target in a supply chain, regardless of its location, which determines the final product quality. Second, we investigated in this paper the case of a three-tier serial supply chain investing in an advanced/monopolistic technology. In reality, a multi-tiered supply chain can be a supply network with a complex network relationship and, potentially, other competing supply chains. We focused our analysis on the simple linear supply chain case to derive primary first-order insights on inclusive innovation, and our analysis can be a building block to examine more complicated cases including supply chain competition, such as that studied in Corbett and Karmarkar (2001) and Carr and Karmarkar (2005).

In closing, we believe that this paper represents an important first step on an issue of growing importance in today's digitally connected market environment, namely how innovative products can be made more inclusive or broadly affordable to a range of customers in a manner consistent with the profits of firms - by aligning actions and tapping previously unexplored degrees of freedom about investor identity in the industry supply chain.

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A Appendix: Proofs of Propositions and Lemmas

Proof of Lemma 1: Equilibrium price and quantity are obtained by reducing tierwise profit expressions into a single contract leader's profit expression by iteratively applying marginalization operation presented in Majumder and Srinivasan (2008). First, in the case of Tier 0 contract leadership, the leaf node Tier 2's profit expression can be written as

$$\Pi_2(q_2;\omega_2) = \omega_2 q_2 - \Theta^{\delta_\theta} K_2 q_2^2 \quad . \tag{A.1}$$

Tier 2 optimally responds to wholesale price offer of ω_2 by maximizing Π_2 above with respect to q_2 which leads to $q_2(\omega_2) = \omega_2 \left(2K_2\Theta^{\delta_\theta}\right)^{-1}$. Now, from Tier 1's perspective, taking Tier 2's optimal response $q_2(\omega_2)$ into account, Tier 1 faces the inverse factor demand, which is $\omega_2(q_2) = 2K_2\Theta^{\delta_\theta}q_2$. Furthermore, the quantity constraint in $q_1 \leq q_2$ in Eq. (3.1) optimally binds. Substituting $\omega_2(q_2)$ and the binding quantity constraint $q_1 = q_2$ into $\Pi_1(q_1, \omega_2; \omega_1)$ in Eq. (3.1), we obtain Tier 1's resulting profit expression as follows:

$$\tilde{\Pi}_1(q_1;\omega_1) = \omega_1 q_1 - \left(K_1 + 2\Theta^{\delta_\theta} K_2\right) q_1^2 \quad . \tag{A.2}$$

Similarly, Tier 0 faces the inverse factor demand of the form $\omega_1(q_1) = \left(2K_1 + 2^2\Theta^{\delta_\theta}K_2\right) q_1$. Substituting $\omega_1(q_1)$ into $\Pi_0(p, q_0, \omega_1)$ and using the optimally binding quantity constraint $q_0 = q_1$, we have

$$\tilde{\Pi}_0(p,q) = pq - \Phi_0(\Theta)\, q^{\delta_q} \quad \text{s.t. } N\left(1 - \frac{p}{\Theta^\beta}\right) \geq q \quad , \tag{A.3}$$

where $\Phi_0(\Theta) = K_0 + 2K_1 + 2^2\Theta^{\delta_\theta}K_2$. By optimizing (A.3) over p and q, it follows that

$$\begin{aligned} P_0(\Theta) &= \left(2 + 2\Theta^{-\beta}\Phi_0(\Theta)N\right)^{-1}\left(\Theta^\beta + 2\Phi_0(\Theta)N\right), \\ Q_0(\Theta) &= N\left(2 + 2\Theta^{-\beta}\Phi_0(\Theta)N\right)^{-1}, \end{aligned} \tag{A.4}$$

which completes the proof for the case of Tier 0 contract leadership. For the other remaining cases, by following the similar steps, we obtain the equilibrium outcomes as stated in Lemma 1. □

Proof of Proposition 1: Technically, we show that for all $\delta_\theta \in (1, 2\beta]$ and all $\gamma > \underline{\gamma}_l$ where $\underline{\gamma}$ is defined below conditional on leader location $l \in \{0,1,2\}$, $\Theta_{l|l} = \max_{i\in\{0,1,2\}}\left\{\Theta_{i|l}\right\}$ and $\Pi_l^{\mathrm{T}}\left(\Theta_{l|l}\right) = \max_{i\in\{0,1,2\}}\left\{\Pi_l^{\mathrm{T}}\left(\Theta_{i|l}\right)\right\}$.

$$\underline{\gamma}_l = \frac{\beta}{4\delta_D}\left(\tilde{K}\right)^{\frac{2\beta-\delta_D}{\delta_\theta}}\left(N^{-1}\tilde{K}^{\frac{\beta}{\delta_\theta}} + \tilde{K}\frac{\delta_\theta K_2}{\beta}\right)^{-1} \quad \text{for } \tilde{K} = \frac{\beta}{\delta_\theta - \beta}\frac{2^{|1-l|}K_1 + 2^l K_0}{2^{|2-l|}K_2}$$

First, we consider the case of Tier 2 contract leadership. Using the equilibrium price and quantity outcomes from Lemma 1, we obtain the resulting profit expressions for each firm as follows:

$$\hat{\Pi}_{i|2}(\Theta_{i|2}) = \max_{\Theta} \left\{ Q_2^2(\Theta) \left. \begin{cases} K_0 & \text{if } i=0 \\ 2K_0 + K_1 & \text{if } i=1 \\ \Phi_2(\Theta) + \Theta^{\beta} N^{-1} & \text{if } i=2 \end{cases} \right\} - \gamma \Theta^{\delta_D} \right\}. \tag{A.5}$$

In order to prove $\Theta_{2|2} = \max_{i\in\{0,1,2\}} \left\{\Theta_{i|2}\right\}$, it suffices to show that $MR_{2|2}(\Theta) > MR_{i|2}(\Theta)$ for $\forall\Theta \geq 0$ for $i = 0,\, 1$, where $MR_{i|2}(\Theta)$ corresponds to the marginal revenue for Tier i investor; specifically, they can be written as

$$\begin{aligned} MR_{0|2}(\Theta) =& Q_2^2(\Theta)\ \left(N^{-1}\Theta^{\beta} + \Phi_0(\Theta)\right)^{-1} \Theta^{-1} \left(\beta\left(2K_1 + 4K_0\right) - (\delta_\theta - \beta) K_2 \Theta^{\delta_\theta}\right) K_0, \\ MR_{1|2}(\Theta) =& Q_2^2(\Theta)\ \left(N^{-1}\Theta^{\beta} + \Phi_0(\Theta)\right)^{-1} \Theta^{-1} \left(\beta\left(2K_1 + 4K_0\right) - (\delta_\theta - \beta) K_2 \Theta^{\delta_\theta}\right) \left(2K_0 + K_1\right), \\ MR_{2|2}(\Theta) =& Q_2^2(\Theta)\, \Theta^{-1} \left(\left(2\beta - \delta_\theta\right) K_2 \Theta^{\delta_\theta} + \beta\Theta^{\beta} N^{-1} + 2\beta\left(4K_0 + 2K_1\right)\right). \end{aligned} \tag{A.6}$$

Note that $MR_{2|2}(\Theta) > MR_{0|2}(\Theta)$ is simplified to

$$\begin{aligned} &(\delta_\theta - 2\beta) K_2^2 \Theta^{2\delta_\theta - \beta} + \left(\left(2\delta_\theta - 14\beta\right) K_0 + \left(2\delta_\theta - 8\beta\right) K_1\right) K_2 \Theta^{\delta_\theta - \beta} + (\delta_\theta - 3\beta) K_2 N^{-1} \Theta^{\delta_\theta} \\ &\quad - \left[\beta\Theta^{\beta} N^{-2} + \left(2K_1 + 4K_0\right)\left(4K_1 + 6K_0\right) \beta\Theta^{-\beta} + 3\beta\left(2K_1 + 4K_0\right) N^{-1}\right] < 0, \end{aligned} \tag{A.7}$$

which holds for $\delta_\theta \leq \beta \cdot \min\left\{2, 3, 4 + \frac{6K_0}{2K_1 + 2K_0}\right\} = 2\beta$. Furthermore, $MR_{2|2}(\Theta) > MR_{1|2}(\Theta)$ is equivalent to

$$\begin{aligned} &(\delta_\theta - 2\beta) K_2^2 \Theta^{2\delta_\theta - \beta} + \left(\left(2\delta_\theta - 14\beta\right) K_0 + \left(2\delta_\theta - 8\beta\right) K_1\right) K_2 \Theta^{\delta_\theta - \beta} + (\delta_\theta - 3\beta) K_2 N^{-1} \Theta^{\delta_\theta} \\ &\quad - \left[\beta N^{-2} \Theta^{\beta} + 2\beta\left(4K_0 + 2K_1\right)\left(2K_1 + 3K_0\right) \Theta^{-\beta} + 3\beta\left(4K_0 + 2K_1\right) N^{-1}\right] < 0, \end{aligned} \tag{A.8}$$

which also holds for $\delta_\theta \leq \beta \cdot \min\left\{2, 3, \frac{7K_0 + 4K_1}{K_0 + K_1}\right\} = 2\beta$. As a result, $\Theta_{2|2} = \max_{i\in\{0,1,2\}} \left\{\Theta_{i|2}\right\}$ follows.

Next, we show that a contract leader's investment in product quality yields highest total supply chain profits. Denote the total marginal revenue of the supply chain under Tier 2's leadership as $MR_{T|2} = MR_{0|2} + MR_{1|2} + MR_{2|2}$. By comparing $MR_{T|2}(\Theta)$ and $MR_{2|2}(\Theta)$ and using algebra, we obtain $MR_{T|2}(\Theta) \geq MR_{2|2}(\Theta)$ for $\Theta \leq \left(\frac{\beta}{\delta_\theta - \beta} \frac{2K_1 + 4K_0}{K_2}\right)^{\frac{1}{\delta_\theta}}$. Furthermore, it follows that for all $\gamma \geq \underline{\gamma}$, $\Theta_{2|2} \leq \left(\frac{\beta}{\delta_\theta - \beta} \frac{2K_1 + 4K_0}{K_2}\right)^{\frac{1}{\delta_\theta}}$ holds. Consequently, it implies that for all $\gamma \geq \underline{\gamma}$, $\Theta_{2|2} \leq \Theta_{T|2}$, i.e., Tier 2 under-invests relative to the supply chain optimal investment level. In addition, $\Pi_2^T(\Theta)$ is quasi-concave. Hence, $\Pi_2^T(\Theta)$ is increasing in Θ for $\Theta \leq \Theta_{T|2}$. Therefore, Tier 2 contract leader's investment in product quality leads to the highest total supply chain profits.

It remains to show that contract leader's choice of product quality also yields highest market coverage. Technically we show that for $\gamma > \underline{\gamma}_l$ as specified below $\rho_l(\Theta_{l|l}(\gamma)) > \rho_l(\Theta_{i|l}(\gamma))$

$i \neq l,\ i,l \in \{0,1,2\}$. Since we have already proved that $\Theta_{l|l}(\gamma) = \max_{i\in\{0,1,2\}} \left\{\Theta_{i|l}\right\}$, it remains to show that that if $\gamma \geq \underline{\gamma}_l$ then $\left.\frac{\partial \rho_l(\Theta)}{\partial \Theta}\right|_{\Theta=\Theta_{l|l}(\gamma)} \geq 0$. By construction $\underline{\gamma}_l$ is such that $\Theta_{l|l}(\gamma) \leq \left(\frac{\beta}{\delta_\theta - \beta}\right)^{\frac{1}{\delta_\theta}} \left(\frac{2^{|1-l|}K_1 + 2^l K_0}{2^{|2-l|}K_2}\right)^{\frac{1}{\delta_\theta}} = \arg\max_{\Theta} \{\rho_l(\Theta)\}$ for $\gamma \geq \underline{\gamma}_l$. Therefore, given sufficiently high levels of development cost $\gamma \geq \underline{\gamma}_l$ for $l \in \{0,1,2\}$, contract leader's choice of product quality also yields highest market coverage in addition to highest product quality.

For the remaining cases of Tier 0 and Tier 1 contract leadership, we follow similar steps to establish that for all $\delta_\theta \in (1, 2\beta]$ and all $\gamma \geq \underline{\gamma}_l$ for $l \in \{0,1,2\}$, a contract leader's investment in product quality yields the highest product quality and the highest total supply chain profits. $\square$

Proof of Proposition 2: Technically, we prove that for all $\delta_\theta \in (1, 2\beta]$ and $4K_1 > K_0 > K_1$, there exist $\{\gamma_{01}^m, \gamma_{12}^m\}$ where $m = \rho$ for market coverage, and $m = \Theta$ for product quality, such that $\gamma_{01}^m > \gamma_{12}^m > \underline{\gamma}$ and the following holds: Given a partition $\{\Gamma_2^m, \Gamma_1^m, \Gamma_0^m\} = \left\{\left[\underline{\gamma}, \gamma_{12}^m\right), [\gamma_{12}^m, \gamma_{01}^m), [\gamma_{01}^m, \infty)\right\}$ for $m \in \{\Theta, \rho\}$,

(a.1) If $\gamma \in \Gamma_k^\Theta$ then $\Theta_{k|k}(\gamma) = \max_{l\in\{0,1,2\}} \left\{\Theta_{l|l}(\gamma)\right\}$ for $k \in \{0,1,2\}$;

(a.2) If $\gamma \in \Gamma_k^\rho$ then $\rho_k(\gamma) = \max_{l\in\{0,1,2\}} \{\rho_l(\gamma)\}$ for $k \in \{0,1,2\}$;

(b) $\gamma_{01}^\Theta > \gamma_{01}^\rho$ and $\gamma_{12}^\Theta > \gamma_{12}^\rho$;

(c) $\frac{\partial \gamma_{j,j+1}^\Theta(K_2)}{\partial K_2} > 0$;

(d) $\forall K_2 > 0\ \exists \triangle K_2$ such that $\gamma_{j,j+1}^\rho(K_2 + \triangle K_2) > \gamma_{j,j+1}^\rho(K_2)$.

First, for part ***(a.1)***, we prove the following claim.

Claim 1. There exists the unique value Θ_{01}^{MR} in $(0,\, \Theta_{01}^Q)$, where $\Theta_{01}^Q = \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{1}{\delta_\theta}}$, that solves $MR_{0|0}(\Theta) = MR_{1|1}(\Theta)$. In addition, there exists the unique value Θ_{12}^{MR} in $(\Theta_{01}^Q,\, \Theta_{12}^Q)$, where $\Theta_{12}^Q = \left(\frac{K_1 + 2K_0}{K_2}\right)^{\frac{1}{\delta_\theta}}$, that solves $MR_{1|1}(\Theta) = MR_{2|2}(\Theta)$.

Proof: The marginal revenue for a contract leader can be written as

$$MR_{l|l}(\Theta) = Q_l^2(\Theta)\Theta^{-1}\left(\left(2\beta - \delta_\theta\right)2^{|l-2|}K_2\Theta^{\delta_\theta} + \beta\Theta^\beta N^{-1} + 2\beta\left(2^l K_0 + 2^{|l-1|}K_1\right)\right), \tag{A.9}$$

where $Q_l^2(\Theta)$ is given in Lemma 1. From (A.9), recognize that Tier l's marginal revenue exhibits a particular form, $MR_{l|l}(\Theta) = Q_l^2(\Theta) g_l(\Theta)$. Therefore, Θ_{01}^{MR} solves $\frac{Q_0^2(\Theta_{01}^{MR})}{Q_1^2(\Theta_{01}^{MR})} = \frac{g_1(\Theta_{01}^{MR})}{g_0(\Theta_{01}^{MR})}$. Furthermore, note that $\frac{Q_0(\Theta_{01}^Q)}{Q_1(\Theta_{01}^Q)} = 1$ holds, and

$$\frac{g_1(\Theta_{01}^Q)}{g_0(\Theta_{01}^Q)} - 1 = \frac{(K_0 - K_1)}{\left(3\frac{\beta}{\delta_\theta} - 1\right) 2K_0 + 2K_1 + \frac{\beta}{\delta_\theta} \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}} N^{-1}} > 0, \tag{A.10}$$

i.e., $\frac{g_1(\Theta_{01}^Q)}{g_0(\Theta_{01}^Q)} > \frac{Q_0^2(\Theta_{01}^Q)}{Q_1^2(\Theta_{01}^Q)}$ also holds. In addition,

$$\frac{Q_0^2(0)}{Q_1^2(0)} = \left(2 - \frac{3K_1}{2K_1 + K_0}\right)^2 > \left(2 - \frac{3K_1}{2K_1 + K_0}\right) = \frac{g_1(0)}{g_0(0)}. \tag{A.11}$$

Next, we show that the difference of two objects, i.e., $\frac{Q_0^2}{Q_1^2} - \frac{g_1}{g_0}$, is decreasing in Θ for $\Theta \in (0, \Theta_{01}^Q)$, $\delta_\theta \in (1, 2\beta]$ and $4K_1 > K_0 > K_1$. Expression for $\frac{\partial}{\partial \Theta}\left(\frac{Q_0^2}{Q_1^2} - \frac{g_0}{g_1}\right)$ can be written as

$$\frac{A_0(\Theta)\frac{\delta_\theta - \beta}{N} 4K_2 \Theta^{\delta_\theta + \beta - 1} + A_1(\Theta) 12\delta_\theta K_0 K_2 \Theta^{\delta_\theta - 1} + A_2(\Theta) \frac{2\beta}{N}(K_0 - K_1)\Theta^{\beta - 1}}{\left(\Theta^\beta N^{-1} + \Phi_0(\Theta)\right)^2}, \tag{A.12}$$

where

$$\begin{aligned} A_0(\Theta) =& \frac{1-\alpha}{4} V - W, \quad A_1(\Theta) = \frac{1-\alpha}{2} V - W, \quad A_2(\Theta) = \frac{1}{4} V - W, \\ V(\Theta) =& \frac{\left(\Theta^\beta N^{-1} + \Phi_0(\Theta)\right)^2}{\left(\frac{1}{2}\Theta^\beta N^{-1} + (1-\alpha) 4K_2 \Theta^{\delta_\theta} + 2K_1 + K_0\right)^2}, \\ W(\Theta) =& \frac{\Theta^\beta N^{-1} + \Phi_1(\Theta)}{\Theta^\beta N^{-1} + \Phi_0(\Theta)}, \end{aligned} \tag{A.13}$$

and $\alpha = \frac{\delta_\theta}{2\beta}$. From (A.12), it follows that a sufficient condition for $\frac{\partial}{\partial \Theta}\left(\frac{Q_0^2}{Q_1^2} - \frac{g_0}{g_1}\right) < 0$ is $A_1(\Theta) = \max\{A_i(\Theta)\} < 0$ for $i = 0, 1, 2$. Furthermore, this sufficient condition $A_1(\Theta) < 0$ can be simplified to

$$\left(\Theta^\beta N^{-1} + \Phi_0(\Theta)\right)^3 < \left(\Theta^\beta N^{-1} + \Phi_1(\Theta) + H(\Theta)\right)^2 \left(\Theta^\beta N^{-1} + \Phi_1(\Theta)\right) \quad , \tag{A.14}$$

where

$$H(\Theta) = \left(2^{-\frac{1}{2}}\tilde{\alpha} - 1\right)\Theta^\beta N^{-1} + \left(2^{\frac{5}{2}}\tilde{\alpha}^{-1} - 2\right) K\Theta^{\delta_\theta} + \left(2^{\frac{3}{2}}\tilde{\alpha} - 1\right) K_1 + \left(2^{\frac{1}{2}}\tilde{\alpha} - 2\right) K_0 \quad , \tag{A.15}$$

and $\tilde{\alpha} = (1-\alpha)^{-\frac{1}{2}}$. Recognize that $H(\Theta) > 0$ for $K_0 < 4K_1$ and $\Theta < \Theta_{01}^Q$. Furthermore, since $\Phi_0(\Theta) < \Phi_1(\Theta)$ for $\Theta < \Theta_{01}^Q$, the sufficient condition $A_1(\Theta) < 0$ holds, i.e., $\frac{\partial}{\partial \Theta}\left(\frac{Q_0^2}{Q_1^2} - \frac{g_0}{g_1}\right) < 0$. Consequently, there exists the unique value Θ_{01}^{MR} in $(0, \Theta_{01}^Q)$ that solves $MR_{0|0}(\Theta) = MR_{1|1}(\Theta)$. Similarly, the unique existence of Θ_{12}^{MR} follows, which completes the proof of the claim. □

Now, denote γ_{01}^Θ as $MR_{1|1}\left(\Theta_{01}^{MR}\right)\delta_D^{-1}\left(\Theta_{01}^{MR}\right)^{1-\delta_D}$ and γ_{12}^Θ as $MR_{1|1}\left(\Theta_{12}^{MR}\right)\delta_D^{-1}\left(\Theta_{12}^{MR}\right)^{1-\delta_D}$. From the claim above, we have $\Theta_{01}^{MR} < \Theta_{12}^{MR}$. Furthermore, product quality level, Θ_{01}^{MR}, is indeed optimally chosen by Tier 0 and 1 if marginal cost of product development crosses $MR_{1|1}(\Theta)$ from below at Θ_{01}^{MR}. Marginal cost of development $\gamma_{01}^{\Theta}\delta_D\Theta^{\delta_D-1}$ for $\delta_D > 2$ is increasing by construction for all $\Theta \geq 0$. Tier 1 leader's marginal revenue is increasing for $\Theta \leq \left(\frac{\beta(K_1+2K_0)}{(\delta_\theta-\beta)2K_2}\right)^{\frac{1}{\delta_\theta}}$, such that $\Theta_{01}^{MR} \leq \Theta_{01}^{Q} < \left(\frac{\beta(K_1+2K_0)}{(\delta_\theta-\beta)2K_2}\right)^{\frac{1}{\delta_\theta}}$. Hence, $MR_{1|1}(\Theta)$ and marginal cost of development are both increasing on $[0, \Theta_{01}^{MR}]$. Moreover, $\lim_{\Theta\to 0} MR_{1|1}(\Theta) = 0$ and $\lim_{\Theta\to 0} \frac{\partial MR_{1|1}(\Theta)}{\partial\Theta} = \infty$. Thus, there exists $\bar{\epsilon} > 0$, such that for all $\varepsilon < \bar{\epsilon}$, $MR_{1|1}(\varepsilon) > \gamma_{01}^{\Theta}\delta_D\varepsilon^{\delta_D-1}$. Therefore, $\gamma_{01}^{\Theta}\delta_D\Theta^{\delta_D-1}$ crosses $MR_{1|1}(\Theta)$ from below, since marginal cost of development contains point $\left(\Theta_{01}^{MR}, MR_{1|1}(\Theta_{01}^{MR})\right)$. By submodularity of profit function for both Tiers 0 and 1, $\Pi_l(\Theta,\gamma)$ for $l = 0, 1$, it follows that optimal product quality is decreasing in γ, i.e., $\frac{\partial\Theta_{l|l}(\gamma)}{\partial\gamma} < 0$. Consequently, $\forall\gamma > \gamma_{01}^{\Theta}$, $\Theta_{l|l}(\gamma) < \Theta_{01}^{MR}$ for $l = 0, 1$. Further, given some $\gamma > \gamma_{01}^{\Theta}$, Tier 1 optimally invests $\Theta_{1|1}(\gamma)$ such that $MR_{1|1}(\Theta_{1|1}(\gamma)) = \gamma\delta_D\left(\Theta_{1|1}(\gamma)\right)^{\delta_D-1}$ and $MR_{0|0}(\Theta_{1|1}(\gamma)) > MR_{1|1}(\Theta_{1|1}(\gamma))$ resulting in $\Theta_{0|0}(\gamma) > \Theta_{1|1}(\gamma)$.

For the proof of ***(a.2)***, we first establish the existence of $\{\gamma_{01}^{\rho}, \gamma_{12}^{\rho}\}$ such that $\rho_j(\gamma_{j,j+1}^{\rho}) = \rho_{j+1}(\gamma_{j,j+1}^{\rho})$ for $j = 0, 1$. Figure 4 evaluating $MR_{l|l}(\Theta)$ and $\rho_l(\Theta)$ on $\Theta \in [0, 10]$ provides graphical illustration for the following proof: Recognize that $\rho_0(\gamma_{01}^{\Theta}) > \rho_1(\gamma_{01}^{\Theta})$ since at γ_{01}^{Θ} both Tiers 0 and 1 optimally choose Θ_{01}^{MR}, where $\Theta_{01}^{MR} \leq \Theta_{01}^{Q}$ and $\rho_0(\Theta) \geq \rho_1(\Theta)$ for $\Theta \leq \Theta_{01}^{Q}$. Construct $\hat{\gamma}_{01}^{\Theta}$ such that $\rho_0(\gamma_{01}^{\Theta}) = \rho_1(\hat{\gamma}_{01}^{\Theta})$, and let $\hat{\Theta}_{01}^{MR} = \Theta_{1|1}(\hat{\gamma}_{01}^{\Theta})$. Since $MR_{1|1}(\Theta) > MR_{0|0}(\Theta)$ for $\Theta > \Theta_{01}^{MR}$, it follows that $\hat{\gamma}_{01}^{\Theta} < \gamma_{01}^{\Theta}$. Hence, Tier 1 leadership yields the same coverage at lower development cost. Consider repeating this exercise of identifying value of $\hat{\gamma}$ at which market coverage under Tier 1's leadership, $\rho_1(\hat{\gamma})$, is equal to $\rho_0(\gamma)$ for values of γ increasing from $\gamma_{01,0}^{Q}$ to γ_{01}^{Θ}, where $\Theta_{0|0}(\gamma_{01,0}^{Q}) = \Theta_{01}^{Q}$. Similarly, define $\gamma_{01,1}^{Q}$ such that $\Theta_{1|1}(\gamma_{01,1}^{Q}) = \Theta_{01}^{Q}$. Recognize that $\gamma_{01,1}^{Q} > \gamma_{01,0}^{Q}$ since $MR_{1|1}(\Theta) > MR_{0|0}(\Theta)$ for $\Theta > \Theta_{01}^{MR}$. In this case, Tier 1's leadership yields the same market coverage, $\rho_0(\gamma_{01,0}^{Q})$, as observed under Tier 0's leadership. Mapping from values of $\gamma \in [\gamma_{01,0}^{Q}, \gamma_{01}^{\Theta}]$ to development cost $\hat{\gamma}$ such that $\rho_1(\hat{\gamma}) = \rho_0(\gamma)$ is continuous, since $MR_{1|1}(\Theta)$ is continuous and $\max_{\Theta>0}\{\rho_0(\Theta)\} < \max_{\Theta>0}\{\rho_1(\Theta)\}$ as indicated in (A.16), where $2K_1 + K_0 < K_1 + 2K_0$ since $K_0 > K_1$ is necessary for existence of handover between Tier 0 and 1.

$$\max_{\Theta>0}\{\rho_l(\Theta)\} = N\left(2 + 2\delta_\theta\left(2^{|2-l|}\beta^{-1}K_2\right)^{\frac{\beta}{\delta_\theta}}\left(\frac{2^{|1-l|}K_1 + 2^{|-l|}K_0}{\delta_\theta - \beta}\right)^{\frac{\delta_\theta-\beta}{\delta_\theta}} N\right)^{-1}. \tag{A.16}$$

By construction of this coverage under Tier 0's leadership is evaluated at increasing values of γ indicated by 45-degree line in panel (d) of Figure 4. Corresponding sequence of $\hat{\gamma}$ for Tier 1 yielding coverage $\rho_1(\hat{\gamma}) = \rho_0(\gamma)$ is strictly decreasing if $\arg\max_{\Theta>0}\{\rho_0(\Theta)\} \geq \Theta_{01}^{Q}$. If $\arg\max_{\Theta>0}\{\rho_0(\Theta)\} < \Theta_{01}^{Q}$, then sequence of $\hat{\gamma}$ is at first increasing for $\gamma \in [\gamma_{01,0}^{Q}, \gamma_0^{\max}]$ and then decreasing for $\gamma \in [\gamma_0^{\max}, \gamma_{01}^{\Theta}]$, where $\Theta_{0|0}(\gamma_0^{\max}) = \arg\max_{\Theta>0}\{\rho_0(\Theta)\}$. In both cases there exists a unique γ_{01}^{ρ} such that $\rho_0(\gamma_{01}^{\rho}) = \rho_1(\gamma_{01}^{\rho})$. Similarly one establishes existence of γ_{12}^{ρ}.

The proof of part ***(b)*** follows from the proof of parts ***(a.1)*** and ***(a.2)*** where $\gamma_{01}^{\Theta} > \gamma_{01}^{\rho}$ and $\gamma_{12}^{\Theta} > \gamma_{12}^{\rho}$.

For the proof of part ***(c)***, first, note that

$$\frac{\partial}{\partial K_2}\left(\frac{g_1(\Theta)}{g_0(\Theta)}\right) = -\frac{\left(6K_0 + \Theta^{\beta}N^{-1}\right)(2\beta - \delta_\theta)\,2\Theta^{\delta_\theta}}{\left(\left(2 - \frac{\delta_\theta}{\beta}\right)4K_2\Theta^{\delta_\theta} + \Theta^{\beta}N^{-1} + 2\left(K_0 + 2K_1\right)\right)^2} \quad . \tag{A.17}$$

Furthermore, we also obtain

$$\frac{\partial}{\partial K_2}\left(\frac{Q_0^2}{Q_1^2}\right) < -\frac{2\left((\delta_\theta - \beta)2K_2\Theta^{\delta_\theta} + \beta\left(K_0 - K_1\right)\right)\frac{\Theta^{\beta-1}}{N} + \left(\beta + (\delta_\theta - \beta)K_0\right)12K_2\Theta^{\delta_\theta - 1}}{\left(\Theta^{\beta}N^{-1} + \Phi_0(\Theta)\right)^2} \quad . \tag{A.18}$$

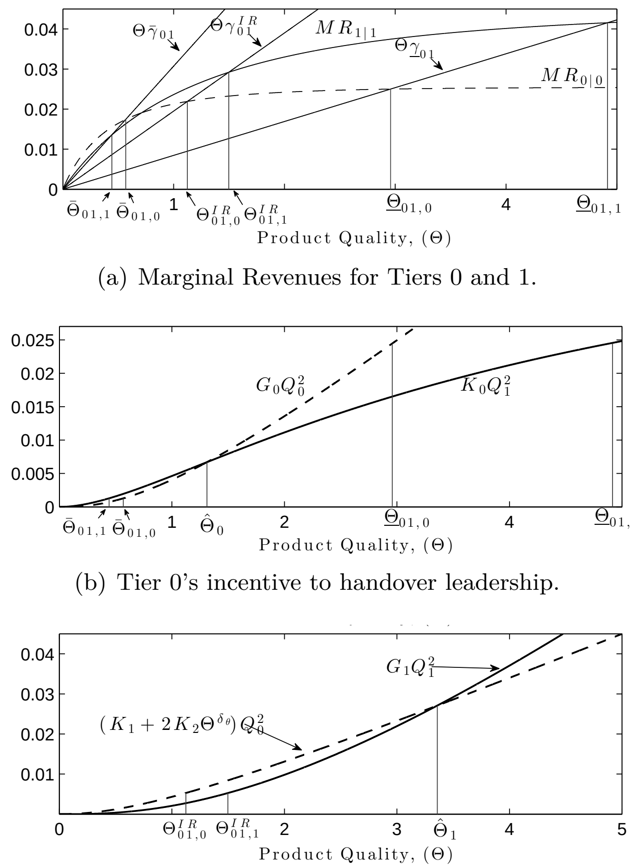
In addition, using the following inequality,

$$\begin{aligned}&\left(2(\delta_\theta - \beta)K_2\Theta^{-1} + \delta_\theta - 2\beta\right)2N^{-1}\Theta^{\delta_\theta + \beta} + 2\beta\left(K_0 - K_1\right)N^{-1}\Theta^{\beta-1} + \\ &\quad + \left(12\left(\beta + (\delta_\theta - \beta)K_0\right)K_2\Theta^{-1} + K_0\left(\delta_\theta - 2\beta\right)\right)12\Theta^{\delta_\theta} > 0,\end{aligned} \tag{A.19}$$

we obtain that the numerator of right-hand side expression in (A.18) is strictly greater than the numerator of (A.17). Moreover, the denominator of right-hand side expression in (A.18) is strictly smaller than the denominator of (A.17). As a result, $\frac{Q_0^2(\Theta)}{Q_1^2(\Theta)}$ decreases with respect to K_2 at a higher rate relative to $\frac{g_1(\Theta)}{g_0(\Theta)}$. Consequently, it follows that $\frac{\partial\Theta_{01}^{MR}(K_2)}{\partial K_2} < 0$, which in turn leads to $\frac{\partial\gamma_{01}^{\Theta}(K_2)}{\partial K_2} > 0$.

For the proof of part ***(d)***, first, recall, that we established existence of γ_{01}^{ρ} on an interval $(\gamma_{01,1}^{Q}, \gamma_{01}^{\Theta})$, where $\Theta_{01}^{Q} = \Theta_{1|1}(\gamma_{01,1}^{Q})$ and $\Theta_{01}^{MR} = \Theta_{1|1}(\gamma_{01}^{\Theta})$. Since $\Theta_{01}^{Q} = \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{1}{\delta_\theta}}$ it follows that $\frac{\partial\Theta_{01}^{Q}(K_2)}{\partial K_2} < 0$ which implies that $\frac{\partial\gamma_{01,1}^{Q}(K_2)}{\partial K_2} > 0$. Hence, γ_{01}^{ρ} is interior to the interval where both endpoints are strictly increasing in K_2. For every K_2 we construct $\triangle K_2 > 0$ such that $\gamma_{01}^{Q}(K_2 + \triangle K_2) = \gamma_{01}^{\Theta}(K_2)$, which guarantees that $\gamma_{01}^{\rho}(K_2 + \triangle K_2) > \gamma_{01}^{\rho}(K_2)$. Similar analysis applies to the case of handover between Tiers 1 and 2. $\square$

Proof of Proposition 3: Technically, we prove that $\exists\, \gamma_{j,j+1}^{\mathrm{IR}}$ such that $\Pi_{j|j+1}(\gamma_{j,j+1}^{\mathrm{IR}}) > \Pi_{j|j}(\gamma_{j,j+1}^{\mathrm{IR}})$ for $j = 0, 1$ and $\gamma_{01}^{\mathrm{IR}} > \gamma_{12}^{\mathrm{IR}}$. First, consider individual rationality constraints that must be satisfied for handover from Tier 0 to Tier 1 to occur. As indicated in (A.20), incumbent leader's profit must be higher under Tier 1's leadership and Tier 1's profit, when in a position of a leader, must exceed the one attained under Tier 0's leadership. Hence, we proceed to identify the highest, i.e., the first from product lifecycle perspective, value of development cost , $\gamma_{01}^{\mathrm{IR}}$, at which conditions $\left(\mathrm{IR}_{01}^{0}\right)$ and



(a) Marginal Revenues for Tiers 0 and 1.

(b) Tier 0's incentive to handover leadership.

(c) Tier 1's incentive to accept leadership.

Figure 13: Individual Rationality of handover from Tier 0 to Tier 1. Parameter values are $(K_0,\, K_1,\, K_2) = (6, 1, 2)$, $\delta_\theta = 1.01$, $\beta = 0.99$, $\delta_D = 2$, and $N = 1$.

$\left(\mathrm{IR}_{01}^{1}\right)$ are both satisfied, where

$$\begin{aligned} \left(\mathrm{IR}_{01}^{0}\right): \quad & \Pi_{0|0}(\gamma) \quad < \Pi_{0|1}(\gamma), \\ \left(\mathrm{IR}_{01}^{1}\right): \quad & \Pi_{1|0}(\gamma) \quad < \Pi_{1|1}(\gamma). \end{aligned} \tag{A.20}$$

Given profit expressions in (A.5) individual rationality constraints necessary for leadership handover from Tier 0 to 1 are reduced to inequalities in (A.21), where product qualities optimally selected by Tier 0 and 1 leaders are $\Theta_0 = \Theta_{0|0}(\gamma)$ and $\Theta_1 = \Theta_{1|1}(\gamma)$ for some γ

$$\begin{aligned} \left(\mathrm{IR}_{01}^{0}\right): \quad & G_0(\Theta_0)Q_0^2(\Theta_0) < K_0Q_1^2(\Theta_1), \\ \left(\mathrm{IR}_{01}^{1}\right): \quad & \left(K_1 + 2K_2\Theta_0^{\delta_\theta}\right) Q_0^2\left(\Theta_0\right) < G_1(\Theta_1)Q_1^2(\Theta_1), \end{aligned} \tag{A.21}$$

where

$$G_l(\Theta_l) = \left(1 - \frac{2\beta - \delta_\theta}{\delta_D}\right) 2^{|2-l|} K_2 \Theta_l^{\delta_\theta} + \left(1 - \frac{\beta}{\delta_D}\right) \Theta_l^{\beta} N^{-1} + \left(1 - \frac{2\beta}{\delta_D}\right) \left(2^{|0-l|} K_0 + 2^{|1-l|} K_1\right) \quad , \tag{A.22}$$

for $l = 0,\ 1,\ 2$.

Panels (b) and (c) of Figure 13 contain plots of objects on each side of $\left(\mathrm{IR}_{01}^{0}\right)$ and $\left(\mathrm{IR}_{01}^{1}\right)$ inequalities correspondingly. Recognize that right-hand sides of inequalities in (A.21) are functions of Tier 1 leader's choice of product quality, Θ_1, while left-hand sides depend only on Θ_0. This distinction between dependence on Θ_0 and Θ_1 is graphically indicated by the use of solid and dashed lines correspondingly. Further, panel (a) of Figure 13 identifies optimal product quality level $\Theta_{l|l}(\gamma)$ via intersection of marginal revenue of investment $MR_{l|l}(\Theta)$ and marginal cost of development, $\gamma\delta_D\Theta^{\delta_D - 1}$ for $l = 0, 1$.

To establish existence of $\gamma_{01}^{\text{offer}} = \max\left\{\gamma \mid \Pi_{0|0}(\gamma) < \Pi_{0|1}(\gamma)\right\}$ at which Tier 0 incumbent leader would prefer Tier 1 to become a contract leader instead, we prove the claim below:

Claim 2. If $\frac{\delta_D - \beta}{2N}\left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}} = (3\beta - \delta_\theta - \delta_D) K_0 + \delta_\theta K_1$ and $\beta \geq \frac{\delta_D}{3}$, then $\hat{\Theta}_0 = \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{1}{\delta_\theta}}$ such that $G_0(\hat{\Theta}_0) Q_0^2(\hat{\Theta}_0) = K_0 Q_1^2(\hat{\Theta}_0)$.

Proof: In Claim 1 we have established existence of $\Theta_{01}^{MR} \in \left(0, \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{1}{\delta_\theta}}\right)$ such that $MR_{0|0}(\Theta_{01}^{MR}) = MR_{1|1}(\Theta_{01}^{MR})$. Given $\frac{\delta_D - \beta}{2N}\left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}} = (3\beta - \delta_\theta - \delta_D) K_0 + \delta_\theta K_1$ we show that $\frac{K_0}{G_0(\Theta)}$ uniquely crosses $\frac{Q_0^2(\Theta)}{Q_1^2(\Theta)}$ at $\hat{\Theta}_0 = \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{1}{\delta_\theta}}$ when $\frac{\partial}{\partial\Theta}\left(\frac{K_0}{G_0(\Theta)}\right) < 0$, $\frac{K_0}{G_0(0)} > \frac{Q_0^2(0)}{Q_1^2(0)}$ and $\lim\limits_{\Theta\to\infty} \frac{K_0}{G_0(\Theta)} < \lim\limits_{\Theta\to\infty} \frac{Q_0^2(\Theta)}{Q_1^2(\Theta)}$ as indicated in (A.23) and (A.24) for $\beta \geq \frac{\delta_D}{3}$. Further, $\frac{\partial}{\partial\Theta}\left(\frac{K_0}{G_0(\Theta)}\right) < 0$ holds for all Θ since $G_0(\Theta)$ is strictly increasing in Θ, from (A.22). Figure 14 provides graphical intuition for the proof of this claim.

$$\frac{K_0}{G_0(0)} = \frac{K_0}{\left(1 - \frac{2\beta}{\delta_D}\right)(K_0 + 2K_1)} \geq \frac{1}{3\left(1 - \frac{2\beta}{\delta_D}\right)} \geq 1 \geq \left(2 - \frac{3K_1}{2K_1 + K_0}\right)^2 = \frac{Q_0^2(0)}{Q_1^2(0)} \tag{A.23}$$

$$\lim_{\Theta\to\infty} \frac{K_0}{G_0(\Theta)} = 0 < \frac{1}{4} = \lim_{\Theta\to\infty}\left(\frac{1 + 2K_2\Theta^{\delta_\theta - \beta} + \Theta^{-\beta}(K_1 + 2K_0)}{1 + 4K_2\Theta^{\delta_\theta - \beta} + \Theta^{-\beta}(2K_1 + K_0)}\right)^2 = \lim_{\Theta\to\infty} \frac{Q_0^2(\Theta)}{Q_1^2(\Theta)} \quad . \tag{A.24}$$

□

Existence of $\gamma_{01}^{\text{offer}}$ is established by performing the following exercise. For each $\gamma \in \left(\gamma_{01}^{\Theta}, \bar{\gamma}_0\right)$, we identify value $\hat{\gamma}$ such that $K_0 Q_1^2(\hat{\gamma}) = G_0(\gamma) Q_0^2(\gamma)$, where $\bar{\gamma}_0 = \lim\limits_{\Theta\to 0^+} \lim\limits_{\beta\to 1^-}\left(\frac{\partial MR_{0|0}(\Theta)}{\partial\Theta}\right) = \frac{N^2}{2K_1 + K_0}$. Value of γ_{01}^{Θ} is such that $\Theta_{0|0}(\gamma_{01}^{\Theta}) = \Theta_{1|1}(\gamma_{01}^{\Theta}) = \Theta_{01}^{MR}$ which was established in proof of Proposition 2 where Θ_{01}^{MR} exists on $\left(0, \left(\frac{K_0 - K_1}{2K_2}\right)^{\frac{1}{\delta_\theta}}\right)$. From Claim 2 it follows that $\Theta_{01}^{MR} \leq \hat{\Theta}_0$. Further, $G_0(\gamma_{01}^{\Theta}) Q_0^2(\gamma_{01}^{\Theta}) < K_0 Q_1^2(\gamma_{01}^{\Theta})$ and hence value of $\hat{\gamma}$ for which $G_0(\gamma_{01}^{\Theta}) Q_0^2(\gamma_{01}^{\Theta}) = K_0 Q_1^2(\hat{\gamma})$ is such that $\hat{\gamma} > \gamma_{01}^{\Theta}$. Now consider some value $\hat{\gamma}$ such that $G_0(\bar{\gamma}_0) Q_0^2(\bar{\gamma}_0) = K_0 Q_1^2(\hat{\gamma})$. The smallest such value

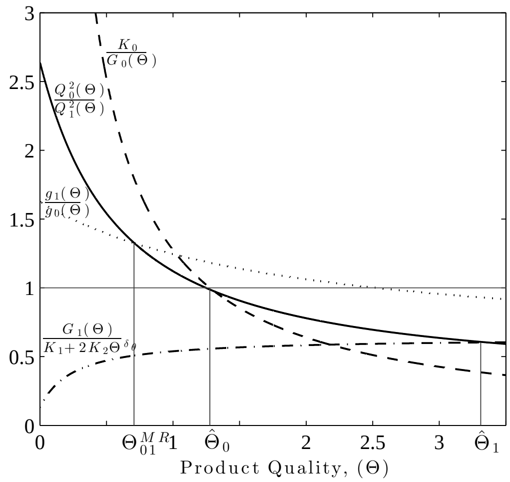


Figure 14: Reference points. Parameter values are the same as those in Figure 13.

is $\bar{\gamma}_1 = \lim\limits_{\Theta\to 0^+} \lim\limits_{\beta\to 1^-} \left(\frac{\partial MR_{1|1}(\Theta)}{\partial \Theta}\right) = \frac{N^2}{K_1+2K_0}$. Given that $K_0 > K_1$ it follows that $\bar{\gamma}_0 > \bar{\gamma}_1$. Mapping from $\gamma \in \left(\gamma_{01}^{\Theta}, \bar{\gamma}_0\right)$ to corresponding $\hat{\gamma}$ is continuous as resulting from continuity of $MR_{l|l}(\Theta)$ on $\Theta > 0$ for $l = 0, 1$. Hence, by Intermediate Value Theorem it follows that there exists $\gamma_{01}^{\text{offer}}$ such that for $\gamma \in \left(\gamma_{01}^{\Theta}, \gamma_{01}^{\text{offer}}\right)$, $G_0(\gamma)Q_0^2(\gamma) < K_0 Q_1^2(\gamma)$, meaning that Tier 0 is ready to handover leadership.

Similarly, we proceed to establish value of $\gamma_{01}^{\text{accept}} = \max\left\{\gamma \mid \Pi_{1|0}(\gamma) < \Pi_{1|1}(\gamma)\right\}$ by first proving the claim below:

Claim 3. If

$$\frac{\left(\lambda + 2 - \frac{6\beta}{\delta_D}\right) K_0 + (1-\lambda) K_1 + \left(1 - \frac{\beta}{\delta_D}\right)\left(\lambda \frac{K_0 - K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}} \frac{1}{N}}{\lambda K_0 + (1-\lambda) K_1} = \left(\frac{\left(\lambda \frac{K_0-K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}} \frac{1}{N} + (\lambda+2) K_0 - \frac{\delta_\theta \lambda}{2\beta} K_1}{\left(\lambda \frac{K_0-K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}} \frac{1}{N} + (1+2\lambda) K_0 - \frac{\delta_\theta \lambda}{\beta} K_1}\right)^2,$$

then $\hat{\Theta}_1 = \left(\lambda \frac{K_0-K_1}{2K_2}\right)^{\frac{1}{\delta_\theta}}$ such that $G_1(\hat{\Theta}_1) Q_1^2(\hat{\Theta}_1) = \left(K_1 + 2K_2 \hat{\Theta}_1^{\delta_\theta}\right) Q_0^2(\hat{\Theta}_1)$, where $\lambda = \frac{2\beta}{2\beta - \delta_\theta}$.

Proof: Condition above by construction allows one to establish that $G_1(\Theta)Q_1^2(\Theta)$ crosses $\left(K_1 + 2K_2\Theta^{\delta_\theta}\right) Q_0^2(\Theta)$ at $\left(\lambda \frac{K_0-K_1}{2K_2}\right)^{\frac{1}{\delta_\theta}}$. Further, $\hat{\Theta}_1$ is a unique point of intersection since $\frac{Q_0^2(\Theta)}{Q_1^2(\Theta)}$ is strictly decreasing in Θ as shown in the proof of Claim 2 and $\frac{G_1(\Theta)}{K_1+2K_2\Theta^{\delta_\theta}}$ is strictly increasing in Θ and approaching $\left(1 - \frac{2\beta-\delta_\theta}{\delta_D}\right)$ as $\Theta \to \infty$. Finally, note that at $\Theta = 0$, $\left.\frac{G_1(\Theta)}{K_1+2K_2\Theta^{\delta_\theta}}\right|_{\Theta=0} = \left(1 - \frac{2\beta}{\delta_D}\right)\left(\frac{2K_0}{K_1} + 1\right) < \frac{Q_0^2(0)}{Q_1^2(0)}$ when $\frac{K_0}{K_1} < \frac{\beta}{\delta_D - 2\beta}$, which completes the proof. $\square$

We proceed to show existence of γ_{01}^{accept} on $(\tilde{\gamma}, \bar{\gamma}_0)$ where $\hat{\Theta}_1 = \Theta_{0|0}(\tilde{\gamma})$. For each $\gamma \in (\tilde{\gamma}, \bar{\gamma}_0)$ we follow a similar steps of identifying value of $\hat{\gamma}$ such that $\left(K_1 + 2K_2\Theta_{0|0}^{\delta_\theta}(\gamma)\right) Q_0^2(\gamma) = G_1(\hat{\gamma})Q_1^2(\hat{\gamma})$. As was established in Claim 3, $\hat{\Theta}_1 \geq \left(\frac{K_0-K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}}$ and hence $\hat{\gamma} > \tilde{\gamma}$. Similarly, Tier 0 leader chooses not to invest and hence $\left(K_1 + 2K_2\Theta_{0|0}^{\delta_\theta}(\bar{\gamma}_0)\right) Q_0^2(\bar{\gamma}_0) = 0$. However, $\hat{\gamma}$ at which Tier 1 leader chooses not to invest is $\bar{\gamma}_1 < \bar{\gamma}_0$ as discussed in Claim 2. Therefore, by IVT there exists $\gamma_{01}^{\text{accept}}$ at which Tier 1 is ready to accept leadership from Tier 0. Recognize that $\lim_{\delta_\theta \to 1} \lim_{\beta \to 1} \left(\hat{\Theta}_1\right) = \hat{\Theta}_0$. Therefore, $\gamma_{01}^{IR} = \max\left\{\gamma_{01}^{\text{offer}}, \gamma_{01}^{\text{accept}}\right\}$. Similar analysis applies to the case of handover between Tie 1 and Tier 2. $\square$

Proof of Proposition 4: Technically, we show that there exists a range of upstream production cost coefficients, $[\tilde{K_2}, \sigma\tilde{K_2}]$ for $\sigma > 1$ and some relatively low development cost coefficient $\tilde{\gamma}$ such that $\Theta_{2|2}(\tilde{\gamma}, \sigma\tilde{K_2}) = \max_{l\in\{0,1,2\}}\left\{\Theta_{l|l}(\tilde{\gamma}, \sigma\tilde{K_2})\right\}$ and $\Theta_{0|0}(\tilde{\gamma}, \tilde{K_2}) = \max_{l\in\{0,1,2\}}\left\{\Theta_{l|l}(\tilde{\gamma}, \tilde{K_2})\right\}$, where with abuse of notation we indicate dependence of choice of optimal product quality $\Theta_{l|l}(\cdot,\cdot)$ on both development and production cost coefficients. Let $\tilde{\gamma} = MR_{1|1}\left(\Theta_{12,h}^{MR}\right)\delta_D^{-1}\left(\Theta_{12,h}^{MR}\right)^{1-\delta_D}$ under high upstream production cost regime $K_2 = \sigma\tilde{K_2}$ as indicated by subscript h. Hence, one needs to find σ such that $\tilde{\gamma} > MR_{1|1}\left(\Theta_{01,l}^{MR}\right)\delta_D^{-1}\left(\Theta_{01,l}^{MR}\right)^{1-\delta_D}$, where l indicates low production cost regime, $K_2 = \tilde{K_2}$. Recall that values of $\Theta_{01,l}^{MR}$ and $\Theta_{12,h}^{MR}$ are contained in the intervals $[0, \left(\frac{K_0-K_1}{2\tilde{K_2}}\right)^{\frac{1}{\delta_\theta}}]$ and $[\left(\frac{K_0-K_1}{2\sigma\tilde{K_2}}\right)^{\frac{1}{\delta_\theta}}, \left(\frac{K_1+2K_0}{\sigma\tilde{K_2}}\right)^{\frac{1}{\delta_\theta}}]$ correspondingly. Therefore, we construct below a sufficient condition for existence of reversal in leadership assignment by using right end-points of these intervals instead of $\Theta_{01,l}^{MR}$ and $\Theta_{12,h}^{MR}$.

$$\frac{\left(3-\frac{\delta_\theta}{2\beta}\right)K_0 + \frac{\delta_\theta}{2\beta}K_1 + \frac{1}{2N}\left(\frac{K_0-K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}}}{\left(3-\frac{\delta_\theta}{\beta}\right)(K_1+2K_0) + \frac{1}{2N}\left(\frac{K_1+2K_0}{\sigma K_2}\right)^{\frac{\beta}{\delta_\theta}}} < \left(\frac{\frac{1}{3N}\left(\frac{K_0-K_1}{2K_2}\right)^{\frac{\beta}{\delta_\theta}} + K_0}{\frac{1}{3N}\left(\frac{K_1+2K_0}{\sigma K_2}\right)^{\frac{\beta}{\delta_\theta}} + K_1 + 2K_0}\right)^2 \left(\frac{\sigma(K_0-K_1)}{2K_1+4K_0}\right)^{\delta_D - 2\beta} \quad . \square$$