WORKING PAPER

THE STRATEGIC ROLE OF BUSINESS INSURANCE

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Abstract

The use of business insurance has been traditionally studied in a single-firm setting. But in reality preventing operational accidents involves the (unobservable) efforts of multiple firms. We show that, in a multi-firm setting, insurance can be used strategically as a commitment mechanism to prevent excessive free-riding by other firms. In the presence of wealth imbalances, contracts alone leave wealth-constrained firms with inefficiently low incentives to exert effort (due to limited liability) and firms with sufficient wealth with excessive incentives. Insurance allows the latter to credibly commit to lower effort, thereby mitigating the incentives of the wealth-constrained firms to free ride. This finding shows that insurance can improve the efficiency of risk management efforts by decreasing free-riding problems.

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1 Introduction

Firms routinely face the possibility of causing operational failures. For example, a contaminated input may cause a product recall, an industrial accident may shut down production or a toxic spill may cause an environmental hazard. These events often lead to serious financial consequences that can threaten the survival of the firm. The likelihood of a failure, however, can be mitigated through costly effort such as operational maintenance or quality control.

Preventing a failure often involves the complementary efforts of multiple firms. As a case in point, consider oil drilling operations. The likelihood of an oil spill depends both on the care taken by the driller and the oil well cementer. The efforts of the two firms are partially substitutable because, for example, the oil driller can increase its drilling care to compensate for a poorly cemented oil well. However, the safety actions taken by the driller are more effective when the well is appropriately built and maintained by the cementing provider (and vice versa).

When risk management involves the efforts of multiple firms, a successful relationship requires proper coordination; "non-cooperative strategies in managing disruption risks are too costly, and leave synergies unexploited"(Kleindorfer and Saad, 2005). But coordinating reliability efforts is complicated by two factors. First, the efforts of the firms may not be observable (e.g. the level of care taken by a firm in operating its equipment). This will give rise to moral hazard problems. And second, it may be impossible to identify the root-cause of the operational failure, which will lead to ambiguity about the degree of responsibility of each party. This is a frequent problem in operations involving "interdependent" systems (Kim and Tomlin, 2013). For example, an executive from EcoMotors argues that in the event of a product defect leading to a recall, "assigning responsibility for warranties gets messy. Was it just a part that was not designed properly? Was it the environment that the part was in, which typically is controlled by the automaker?" (Automotive news, 2003). In other cases the failure automatically destroys all evidence about the root-cause, e.g. an explosion leading to a major fire (Okes, 2009).

Firms can deal with the above two problems through contractual clauses by allocating the financial burden of a failure, i.e. the *ex post liability*, to those parties best positioned to prevent it. These clauses take the form of performance-based penalties, liability-sharing agreements or quality warranties, and can achieve the right incentives for effort provision. Contractual tools are often used to apportion the potential costs of various types of failures, including product recalls arising from quality defects (Chao et al., 2009), oil spills (Hewitt, 2008) and the accidental release of other toxic materials (Gallagher, 2012).

But contractual agreements are not the only tool used by firms to allocate the financial liability of an operational failure. Firms also have the ability to transfer their liability away from the contractual relationship, to third parties. For example, a firm can purchase business insurance to provide coverage for losses arising from an operational failure. As a result, firms have two mechanisms to deal with failures: (i) contractual incentives, which allocate financial liability within the collaborative relationship and; (ii) insurance coverage, which transfers the liability away from this relationship.

Most firms rely on some form of insurance coverage as part of their overall operational strategy. In some cases, insurance offers coverage for losses arising from uncontrollable factors, e.g. a natural disaster or a terrorist attack. But in many other cases insurance offers coverage for events where either the operator or the supplier, or both, can potentially affect the likelihood of the outcome. For example, in the U.S. most insurance companies offer services like equipment breakdown insurance or boiler and machinery insurance to cover small losses arising from equipment failures. They also offer services like product recall insurance, nuclear liability insurance or oil-spill insurance to cover losses arising from more serious events. These services cover a variety of industries including light & heavy manufacturing, utilities, steel machinery, mining & minerals, chemical products, hi-tech companies, etc. (RSA Group, 2011).

When the firms can affect the likelihood of a failure, the use of insurance may lead to inefficiencies due to moral hazard. This is because insurance decreases overall incentives to ensure operational reliability, by leading to the "de-responsabilization of parties or agents" (Kogan and Tapiero, 2007). Why then do firms buy insurance coverage for these types of events? Would it not be more efficient to allocate all liability within the collaborative relationship, to those firms who are in the best position to prevent an operational failure?

One answer to the above question is that insurance coverage serves a liquidity-enabling role, by allowing wealth-constrained firms to avoid the possibility of bankruptcy or illiquidity due to an event causing large losses. For example, producers of nuclear energy can exchange the prohibitively large costs associated with a nuclear meltdown for a manageable insurance premium. Therefore, if all firms are wealth-constrained and the overall wealth of these firms cannot cover the potential impact of a failure, they may need to seek third-party insurance. Insurance can play a second role in the presence of risk aversion, which is to allow risk-averse firms to transfer risk to a (risk-neutral) insurer. In other words, insurance improves the allocative efficiency of risk within the economy.

Could insurance be optimally used for other purposes in a multi-firm setting? In this paper we show that insurance may also serve a strategic role in a collaborative relationship. Specifically, without insurance firms may excessively free-ride on the efforts of other firms. The purchase of insurance can serve as a mechanism that allows a firm to credibly commit not to increase its effort and, thereby, to mitigate the free-riding problem. In other words, firms can strategically use insurance as a commitment mechanism.

This strategic role arises in settings where the relationship is characterized by wealth imbalances. Consider a relationship where one of the firms has sufficiently large wealth to cover any potential losses arising from an operational failure, but the other firm has severe wealth constraints. In this case contractual agreements alone misallocate incentives within the relationship. In particular, contractual tools alone leave the wealth-constrained firm with inefficiently low incentives to exert effort because it is unable to take on the appropriate level of liability. Conversely, the "wealthy" firm ends up with excessively high incentives. Because effort is substitutable, any increase in the effort of the wealthy firm will further undercut the incentives of the wealth-constrained firm to exert effort. Insurance can play a strategic role by allowing the wealthy firm to credibly commit not to exert effort beyond some level and, in turn, the wealth-constrained firm has less incentive to free-ride. Insurance can therefore mitigate the distortion in effort provision and improve total welfare.

The above results are particularly relevant as risk management is becoming increasingly decentralized. For example, with the rising popularity of business services (e.g. maintenance and security outsourcing) larger corporations are looking to partner with small, specialized companies (Bank of Canada, 2013). Also, in emerging economies it is common to see large, wealthy multinationals partnering with small and medium enterprises (Etemad et al., 2001). This implies that today's contracting relationships are increasingly characterized by an uneven wealth distribution across firms. This uneven distribution of wealth leads to effort distortions in risk management and, in these cases, our results imply that managers can use insurance to mitigate the resulting inefficiencies.

We highlight a role for insurance that to the best of our knowledge is not widely understood or used in practice, and it is important for managers to take advantage of this tool. This paper therefore increases our understanding about the roles that insurance serves. It also shows that moving from a single- to a multi-firm setting has non-trivial implications for the role of insurance. In general, our results imply that insurance coverage and contractual incentives are not necessarily substitutes, but may complement each other.

2 Literature Review

This paper bridges two research streams: (i) contract theory and; (ii) risk management and insurance.

Contract Theory: We are interested in the sub-stream that focuses on contracts coordinating reliability investments. Kim et al. (2007) and Chu and Sappington (2010) are prominent examples of this literature. Specifically, we consider a context where the operational outcome can be influenced by the efforts of both the operator and the supplier, and also that these efforts are unobservable. This is a setting characterized by double-sided moral hazard, which is also studied by Roels et al. (2010) and Jain et al. (2013). Second, our model assumes that the root-cause of an operational failure cannot always be attributed to either party; a similar assumption is made by Saouma (2008) and Kim and Tomlin (2013).

Our model considers the case where firms have limited wealth in a principal-agent setting. This literature stems from Sappington (1983), who argues that "contracts in which the liability of one or more parties is explicitly limited are very common in practice." In the economics literature, this topic has been extensively explored; some of the most notable examples are Innes (1990), Holmstrom and Tirole (1997), Oyer (2000), Gromb and Martimort (2007) and Poblete and Spulber (2012). Some examples from the management literature include: Saouma (2008), who studies outsourcing relationships and warranties in a setting where the suppliers have limited wealth; DeVéricourt and Gromb (2014), who study capacity investments under limited liability and; Desiraju (2004), who studies intrabrand competition under the same assumption.

Risk management and insurance: The main goal of our paper is to contribute to a better understanding about the role of insurance in multi-firm settings. This literature is limited. In the operations management literature, Dong and Tomlin (2012) study the interplay between business insurance and inventory management, and find that insurance can increase the marginal value of inventory and the overall value of emergency sourcing. Dong et al. (2015) study the interplay between inventory, interruption insurance and preparedness actions in production chains. Unlike our paper, none of the above manuscripts study free-riding issues across decentralized agents. Therefore, the authors do not need to consider issues related to moral hazard, which are at the core of our model.

There is an extensive literature in economics that explores insurance in the presence of moral hazard. Winter (2000) summarizes this literature. Within this large literature, our model is most closely related to Tommasi and Weinschelbaum (2007) who study the robustness of principal-agent contracts to the introduction of third-party insurance. Our results differ from this paper in two key respects. First, while their analysis is driven by the assumption of risk aversion and single-sided moral hazard, we assume that the parties are risk-neutral and subject to double-sided moral hazard. Second, in their paper insurance opportunities are available for the agent, not for the principal; we assume that the principal has the ability to purchase insurance coverage. Due to this reason, they find that insurance decreases welfare while we show that insurance can improve total welfare in contractual relationships.

3 Model Preliminaries

3.1 Operational features

A risk-neutral operator (O) receives revenue π from operating a system that requires the technical expertise of a risk-neutral supplier (S). Consider the following examples: (i) an oil driller that delegates all cementing operations to an oil well cementer or; (ii) an equipment operator that outsources all supervision and maintenance tasks to a service supplier. The system is subject to unexpected operational failures, e.g. a biohazard spill leading to an environmental accident, or a defective product that must be recalled. These failures lead to financial losses for the operator from property damages, clean-up costs, production interruption, etc. Let $X \in \{0, x\}$ be a random variable representing the "failure costs" borne by the operator. If $X = 0$, operations have performed as planned, and the operator incurs no losses. If $X = x > 0$, an operational failure has occurred, and the operator incurs losses equalling this amount.[2]

3.1.1 Failure probability and reliability efforts:

To diminish the likelihood of a failure, the supplier and the operator can exert costly but unobservable effort. For example, the supplier may use better quality inputs or improve the design of its product or service. The operator can increase the level of care when performing operations, minimize the systems' exposure to strenuous conditions, hire skilled personnel, etc.

Let $e_S \geq 0$ and $e_O \geq 0$ denote the efforts of the supplier and the operator, respectively, where e_S and e_O are the dollar investments to improve operational reliability; e_S^* and e_O^* denote the optimal effort levels.

The probability of an operational failure is $F(e_S, e_O) = \frac{1}{(1+e_O)^{\beta}(1+e_S)^{1-\beta}}$, where parameter

[2] In §5 we consider an extension where the failure costs have a continuous support.

$\beta \in (0,1)$ represents the sensitivity of the failure probability to the operator's effort (relative to the supplier's). When β is low, reliability is more sensitive to the supplier's effort and less sensitive to the operator's effort, and vice versa. Note that $F(e_S, e_O)$ is an inverse Cobb-Douglas function, which reflects the idea that the efforts of the operator and the supplier are substitutable and also collaborative in nature. Effort is substitutable because the operator can exert higher effort to compensate for situations where the supplier exerts lower effort, and vice versa. However, it is collaborative because the effort of one firm is enhanced when the other firm exerts high effort, i.e. $F(e_S, e_O)$ has increasing differences in (e_S, e_O).[3] This assumption reflects the true nature of many, if not most, risk management relationships. Supply chain risk management, for example, is in essence a collaborative process due to the existence of synergies or complementarities in the specialized tasks of supply chain partners (Kleindorfer and Saad, 2005). In business service processes (e.g. maintenance and security outsourcing), the "joint-production character of the process" gives rise to complementarities in the efforts of the firms (Roels et al., 2010).[4]

3.2 Failure attribution

We focus on settings where the root-cause of an operational failure cannot be identified in a cost effective manner. This assumption is often the norm in practice and is studied in a subfield of reliability analysis called *dependent failure analysis*. From this literature we have identified three settings where this assumption holds and, hence, where our model is applicable:

1. Cascading failures: Large-scale operations often require the input of interdependent subsystems (e.g. power generation plants, oil and gas pumping, etc.), some of which need the managerial expertise of specialized suppliers. When there is a high level of interdependency, the subsystems are highly susceptible to experiencing a *cascading failure*, where the failure of one subsystem triggers the subsequent failure of other subsystems (Ericson, 2005). In these cases it is either impossible, or prohibitively costly, to identify the subsystem responsible for the failure. Kim and Tomlin (2013) study liability allocation rules across systems that are susceptible to experiencing cascading failures.

2. Destructive failures: In certain operational failures, evidence is damaged or destroyed following the event and, as Okes (2009) explains, this often makes it impossible to determine the root-cause of the problem. For example, following the explosion of a large wind turbine in An-

[3]While our results hold for a larger family of functions, $F(e_S, e_O)$ must have increasing differences in (e_S, e_O), i.e. the collaborative assumption is essential. A proof of this result is found in an extended version of this paper on the authors' website.

[4]Other processes characterized by collaborative efforts include joint ventures (Kellogg, 2011) and franchise relationships (Bhattacharyya and Lafontaine, 1995).

drossan, UK, "much of the evidence was burned, and Infinis [the wind farm operator] and Vestas [the supplier of wind turbines] disagree on which was the key initial cause of the destructive fire" (New Scientist, 2013).

3. Commingled and homogeneous goods: Whenever the efforts of the multiple firms contribute to the production of a good that is commingled, non-modular or homogeneous (e.g. chemical substances, food products), it is a challenge to determine the degree of responsibility of each party for the operational failure. For example, in 2007 a tainted food incident from ConAgra caused approximately 15,000 people in the U.S. to fall sick, but "ConAgra could not pinpoint which of the more than 25 ingredients in its pies was carrying salmonella" (New York Times 2009).[5] Also, in July 2013 an oil-cargo train derailment caused a massive explosion in Lac-Megantic, Quebec. To date, the authorities have not been able to determine if the explosion was caused by chemical contaminants in the oil (from a previous shipment), or because the oil itself contained high levels of flammable hydrogen sulphide gas (The Globe and Mail 2013).

3.3 Contracts

In the face of a potential failure, the operator needs to optimally apportion the liability for the accident. The operator can apportion some of the potential losses upstream to the supplier, through a contractual agreement. In addition, the operator can purchase insurance coverage to allocate some of the burden away from the relationship, to a third party insurer. We explain below the specifics of both agreements, (i) the procurement contract and (ii) the insurance policy.

3.3.1 Procurement contract:

The operator procures the services of the supplier by offering a 'take-it-or-leave-it' contract, $T(w, y, X)$. This contract consists of a fixed-fee payment, $w \geq 0$, and a liability-sharing (or penalty) rate, $y \in [0, 1]$. The penalty rate splits the cost of the failure between the contracting parties: the supplier pays yX, and the operator pays the remainder (recall that $X = x$ if there is an operational failure and $X = 0$ otherwise). The cash flows of the contract (from the operator to the supplier) are $T(w, y, X) = w - yX$, and the expected transfer payment is $E_X[T(w, y, X)|e_S, e_O] = w - yE[X|e_S, e_O]$, where $E[X|e_S, e_O] = xF(e_S, e_O)$.

This contractual form is widely observed in practice, where the supplier gets a fixed payment but a proportion of any liability is subtracted contingent on the realization of performance (Kim et al.,

[5] More generally, the FDA (2010) states that whenever there is a contaminated food product "traceability has proved particularly difficult because of the complexity of the distribution system and the practice within pack houses supplying the US market of co-mingling produce."

2007). By reviewing different procurement agreements, we found numerous examples of contracts that adopt this apportioning mechanism. For example, in a distribution agreement signed in 2001 between the technology companies Lucent Technologies and Agere Systems, the parties agreed to split any costs arising from product defects and accidental releases of contaminants; Lucent agreed to be liable for 86% of the costs, and Agere for the remaining share (article VI, clause i).

Note that sometimes the parties will first agree to do preliminary investigation to determine the root-cause of the failure, and the liability-sharing clause will only apply if the investigation is inconclusive.[6] While this behavior is certainly observed, we assume that the root-cause of a failure cannot be identified. But this assumption is not too restrictive because, as Moslelh et al. (1998) put it, "all too often investigations of failure occurrences... do not determine the root causes of failures."

3.3.2 Insurance Contract:

The operator has access to an insurance market to cover any losses arising from the operational failure.[7] The operator will thus negotiate an insurance policy with a third-party insurer. To model this relationship we reviewed numerous insurance policies and spoke with practitioners. We identified three key components (schedules) that characterize these policies: (i) a schedule of insured events and losses; (ii) a payments schedule and; (iii) a schedule of excepted causes.

1. Schedule of insured events and losses: This schedule defines those events for which the operator can obtain coverage (e.g. an oil spill or a product recall), and the types of losses covered by the policy. In our model we assume that both the operational failure and the failure costs are defined within this schedule.[8]

2. Payment Schedule: This schedule includes the cash flows of the policy. We consider an arrangement where the operator pays an *insurance premium* and, in exchange, obtains a *coverage* reimbursement. Let $v \in [0, 1]$ denote the *coverage* level, which is equal to the proportion of the failure costs reimbursed by the insurer in the event of an operational failure. This reimbursement amounts to vx.

[6] For example, in a chemical manufacturing contract between APP Pharmaceuticals and New Abraxis (2007, clause 12.9), the parties agreed to share the costs of a product recall only if the fault of the recall cannot be determined through preliminary tests performed by an independent FDA testing agency.

[7] Throughout the base model we assume that the operator has full control over the levels of insurance that are purchased. In reality, however, the supplier may have the option to purchase insurance to cover some of its losses. We analyze this case in Section 5.2.

[8] For example, in a product recall insurance policy from AIG Insurance Ltd. (2010), the policy defines an insured event as "any *Product Recall* or *Government Recall* resulting from any: (a) Defect; (b) Malicious Product Tampering; or (c) Product Extortion." The same policy defines the insured losses as "all reasonable and necessary (a) Insured's Product Recall Costs; (b) Business Interruption Costs; (c) Replacement Costs. . . "

The *insurance premium*, P, is an upfront payment made by the operator to the insurer. In practice, the premium is quoted as the sum of three factors: (i) the actuarially fair premium, (ii) an additive load, and; (iii) a proportional load. The actuarially fair premium represents the expected reimbursement to the operator, the additive load accounts for the fixed transaction costs of providing insurance and the proportional load accounts for costs that are proportional to the amount of coverage. Therefore, the premium is equal to

$$P\left(v|e_S,e_O\right)=\underbrace{vE\left[X|e_S,e_O\right]}_{\text{actuarially fair premium}}+\underbrace{l_{add}}_{\text{additive load}}+\underbrace{l_{prop}vE\left[X|e_S,e_O\right]}_{\text{proportional load}} \tag{3.1}$$

This type of pricing scheme has also been modeled in the literature, for example, by Patel et al. (2005) and Dong and Tomlin (2012). The cash flows of the insurance contract (from the operator to the insurance provider) are equal to $I\left(v,P,X\right)\equiv P-vX$, and the expected cash flows are $E_X\left[I\left(v,P,X\right)|e_S,e_O\right]=P-vE\left[X|e_S,e_O\right]$.

We make the following three assumptions. First, we assume that the insurer is knowledgeable about the product technology, i.e. the insurer is informed about the function $E\left[X|e_S,e_O\right]$, but cannot observe the effort provision of each firm. Second, we assume that the insurer can observe the procurement contract, (w,y), before setting P. Therefore, the insurance provider can infer the incentive compatible levels of effort, e_S^* and e_O^*, and price the premium in accordance with these levels. Both assumptions are standard in the insurance literature (see Winter 2000). Third, to focus on the intuition behind our results we let $l_{add}=l_{prop}=0$, i.e. the insurance premium is equal to the actuarially fair level of coverage. Our results do not depend on this assumption because both loads reflect transaction costs associated with purchasing insurance and, therefore, the only influence that these factors play in our model is to (uniformly) shift the results without adding any intuition.

Finally, note that the payment schedule of a policy generally includes more complicated payment arrangements, including deductible levels. In §5.3 we show that our results are robust to these complications.

3. Schedule of Excepted Causes: An insurance policy also defines a set of excepted causes for which the insurer is exempted from covering the insured. These exceptions generally include unacceptable actions from the insured party, or matters uninsured under the law.[9] We assume that

[9] In the *Product Recall Insurance* policy mentioned in Footnote 8, AIG defines a number of excepted causes, including "Uninsurable matters under the law", "Asbestos", "Intentional violation by the Insured of any governmental or regulatory requirements", etc.

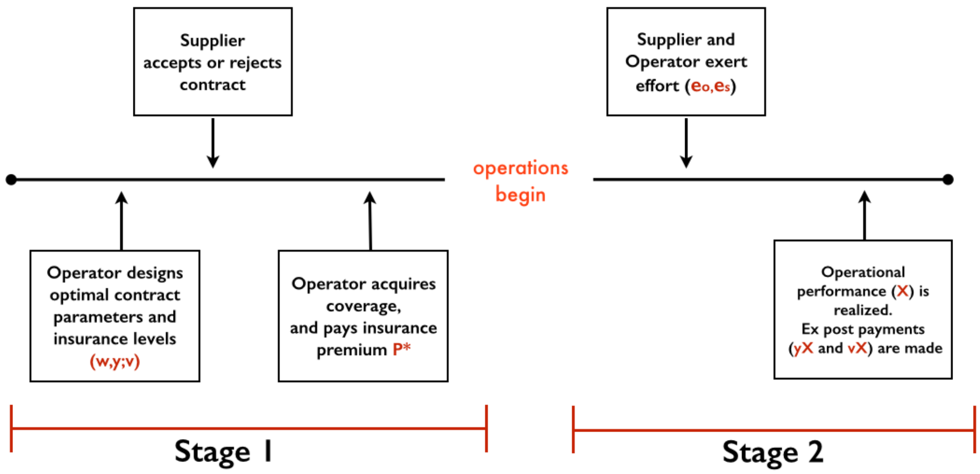


Figure 3.1: Model Timeline

the parties will not engage in any behavior that might lead to an excepted cause of a failure, e.g. unreasonable negligence, violations of the law, etc.

A possible misconception is that the insured party will not be able to obtain coverage unless the root-cause of the failure can be clearly identified. Note that under the law

> *The burden is on an insured to establish that the occurrence forming the basis of its claim is within the basic scope of insurance coverage. And, once an insured has made this showing, the burden is on the insurer to prove the claim is specifically excluded* (Supreme Court of California, 1989).

This means that to obtain coverage the policy holder only needs to show the existence of insured losses arising from an insured event. The insurer can *only* be exempted from making a reimbursement if it can convincingly show that the cause is excepted in the contract (e.g. if the insurer can show that the operator was unreasonably negligent, intentionally tampered with the product, etc.). And, as mentioned earlier, we abstract away from these issues.

3.4 Model Dynamics

In Figure 3.1 we illustrate the timeline of this model. In stage one the operator designs the procurement contract, $T(w, y, X)$, and chooses the level of insurance coverage, v. Next, the supplier accepts or rejects the deal offered by the operator. If the supplier accepts the arrangement, the operator purchases the insurance policy and makes the ex ante payments (i.e. the insurance premium and the fixed fee, P and w).

Operations begin in stage two. In this stage, the operator and the supplier exert effort levels e_O and e_S. Note that because we assume that the efforts of both parties are unobservable, the sequence in which effort is exerted will not affect the results analyzed below.[10]

After both parties have exerted effort, the operational performance, $X \in \{0, x\}$, is realized. The cash flows, yX and vX, are made at the end of the operational period.

3.5 Payoffs and wealth constraints

The supplier's and the operator's ex post profits are

$$\begin{aligned} \Pi_S (e_S, w, y, X) &= T (w, y, X) - e_S \\ \Pi_O (e_O, w, y, v, P, X) &= \pi - X - T (w, y, X) - I (v, P, X) - e_O \end{aligned}$$

Therefore, the expected profits for both firms are

$$\begin{aligned} E_X [\Pi_S (e_S, w, y, X) | e_O] &= E_X [T (w, y, X) | e_S, e_O] - e_S \\ E_X [\Pi_O (e_O, w, y, v, P, X) | e_S] &= \pi - E_X [X | e_S, e_O] - E_X [T (w, y, X) | e_S, e_O] - E_X [I (v, P, X) | e_S, e_O] - e_O. \end{aligned}$$

We focus on scenarios characterized by low-probability but high-impact failures (e.g. a product recall, a biohazard accident, etc.). In this context firms are often unable to sustain large financial losses. We assume that at the outset (i.e. prior to any contractual agreement) the operator has wealth $W_O \geq 0$ and the supplier has wealth $W_S \geq 0$. In other words, the optimal contracts must ensure that at the end of the game the supplier and the operator end up with non-negative wealth, i.e.

$$W_S + \Pi_S (e_S, w, y, x) \geq 0$$

and

$$W_O + \Pi_O (e_O, w, y, v, P, x) \geq 0$$

where $\Pi_O (e_O, w, y, v, P, x)$ and $\Pi_S (e_S, w, y, x)$ refer to the ex-post profits of the firms in the event of an operational failure (i.e. when $X = x$).

Firms generally write liability limits as a condition to enter into any contractual agreement;

[10] This is because, according to the *Principle of Interchange of Moves,* the order of play is "immaterial if one player does not have any information about the other player's action when making his choice." (Osborne and Rubinstein, 1994). In Section 5.1, however, we study a sequential game where the supplier is the first party to exert effort, and this effort is observable. Our results are robust to this setting.

these constraints have been thoroughly studied in the literature (see §2). As Sappington (1983) explains, these clauses can be established as bankruptcy or insolvency provisions, where no further liability can be imposed on the firm if it reaches a state of insolvency. The liability limits can also be explicitly stated as fixed dollar amounts.[11] Saouma (2008) also uses these constraints to study outsourcing relationships, by arguing that "excessive liability resulting from a single faulty product can drive even large suppliers into bankruptcy." Similarly, Desiraju (2004) argues that a principal reason these clauses arise is due to "equity considerations that mandate the guarantee of an appropriate level of well-being" for the various parties involved in a contractual relationship.

4 Analysis

In this section we study the interaction between liability-sharing through contracts and liability-sharing through insurance. We present a benchmark scenario in §4.1, by assuming that the operator and supplier are organized as a centralized entity. In §4.2 we consider the case where the firms operate as decentralized entities.

4.1 Benchmark Scenario: Centralized Setting

When the operator and the supplier are organized as a centralized entity, a contract between these parties is unnecessary. In this setting the parties seek to maximize the sum of their expected profits, $E_X\left[\Pi\left(e_S, e_O, v, P, X\right)\right]$, where $\Pi\left(e_S, e_O, v, P, X\right) = \Pi_S + \Pi_O = \pi - X - I\left(v, P, X\right) - e_S - e_O$.

In stage 1 the coordinated entity determines the optimal insurance level, $v^* \in [0, 1]$. In stage 2 the centralized entity determines the optimal effort levels, e_S^* and e_O^*, for a given v^*. Recall that at the end of the game the centralized firm must end up with non-negative wealth, W, where $W = W_O + W_S$. In other words, the centralized firm's problem is to find

$$\max_{v \in [0,1]} E_X\left[\Pi\left(e_S^*, e_O^*, v, P\left(v | e_S^*, e_O^*\right), X\right)\right]$$

subject to

$$\left(e_S^*, e_O^*\right) = \arg\max_{\left(e_S, e_O\right) \geq \mathbf{0}} \left\{E_X\left[\Pi\left(e_S, e_O, v, P, X\right)\right] \text{ subject to } \Pi\left(e_S, e_O, v, P, x\right) + W \geq 0\right\}$$

The following proposition characterizes the first best level of insurance coverage (all proofs are

[11] The following is a sample limited liability clause drawn from a contract for the Manufacturing of pharmaceutical inputs: *"Notwithstanding anything herein to the contrary, in no event will the GENERICO Indemnified Parties have any liability to NEW ALPHA or any of its Affiliates, or to any third party in connection with this Agreement, for monetary Damages in excess of $100 million in the aggregate"* (Manufacturing Agreement between New Abraxis Inc. (New Alpha) and APP Pharmaceuticals, LLC. (Generico), November 2007; Clause 12.9).

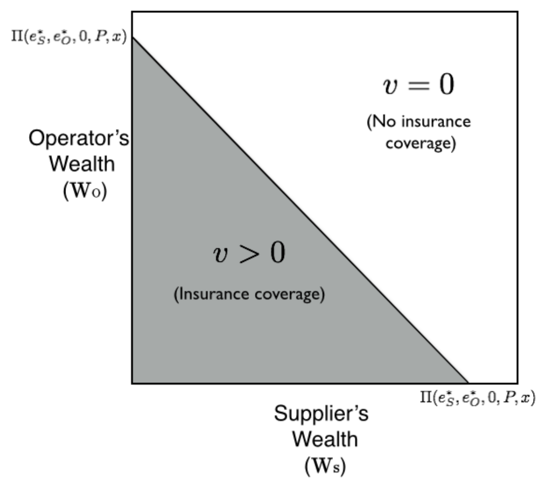


Figure 4.1: First best level of insurance.

provided in the appendix).

Proposition 1. *If* $W + \pi \geq x + \sqrt{\frac{x}{\beta^{\beta}(1-\beta)^{1-\beta}}}-2$*, the centralized entity purchases no insurance coverage, i.e.* $v^* = 0$*. If* $W + \pi < x + \sqrt{\frac{x}{\beta^{\beta}(1-\beta)^{1-\beta}}} - 2$*, the centralized entity purchases insurance coverage* $v^* > 0$*.*

We illustrate this proposition in Figure 4.1. The centralized firm will only need to purchase insurance if the wealth constraint of the centralized entity binds at optimum (i.e. if, given the realization of an operational accident, the potential losses without insurance exceed the entire wealth of the firms). In other words, in a centralized setting insurance only plays the liquidity-enabling role; insurance ensures the financial viability of the firm.

4.2 Decentralized setting

We now consider the case where the firms operate as decentralized entities. Recall that in stage 1 the operator seeks to maximize its own profits by determining the optimal operator-supplier contract, w and y, and the optimal level of insurance, v. If the supplier accepts the contractual arrangement, operations begin in stage 2. In this stage the operator and supplier independently set the profit-maximizing levels of effort, e_O and e_S, subject to the constraint that they must end up with non-negative wealth at the end of the game. In summary, the operator's problem is to solve

$$\max_{w \geq 0, y \in [0,1], v \in [0,1]} E_X \left[\Pi_O \left(e_O^*, w, y, v, P \left(v | e_S^*, e_O^* \right), X \right) | e_S^* \right]$$

subject to

$$E_X\left[\Pi_S\left(e_S^*, w, y, X\right)|e_O^*\right] \geq 0 \qquad (\mathcal{IR})$$

$$e_S^* \equiv \arg\max_{e_S \geq 0}\left\{E_X\left[\Pi_S\left(e_S, w, y, X\right)|e_O\right] \text{ subject to } W_S + \Pi_S\left(e_S, w, y, x\right) \geq 0\right\} \qquad (\mathcal{IC_S})$$

$$e_O^* \equiv \arg\max_{e_O \geq 0}\left\{E_X\left[\Pi_O\left(e_O, w, y, v, P, X\right)|e_S\right] \text{ subject to } W_O + \Pi_O\left(e_O, w, y, v, P, x\right) \geq 0\right\} \qquad (\mathcal{IC_O})$$

$\mathcal{IR}$ represents the individual rationality constraint for the supplier; the supplier must earn non-negative profits in expectation. The constraints $\mathcal{IC_S}$ and $\mathcal{IC_O}$ represent incentive compatibility constraints for the operator and the supplier, i.e. in stage 2 the firms choose the effort levels that maximize their private profits (under the constraint that they must end up with non-negative wealth).

To simplify our analysis we first present the results for three special cases:

- **Special case $\mathcal{UC}$ - unconstrained wealth:** we assume that $W_O = \infty$ and $W_S = \infty$.
- **Special case $\mathcal{SC}$ - Supplier with wealth constraints:** we assume that $W_O = \infty$ but $W_S < \infty$.
- **Special case $\mathcal{OC}$ - Operator with wealth constraints:** we assume that $W_O < \infty$ but $W_S = \infty$.

After analyzing these three special cases we present the general results, where $W_S \leq \infty$ and $W_O \leq \infty$.

4.2.1 Special case $\mathcal{UC}$: unconstrained wealth

Assume that both parties are financially unconstrained, i.e. $W_S = W_O = \infty$. In this case, the effort functions are not affected by the wealth constraints. We characterize the optimal contracts for region $\mathcal{UC}$, in Proposition 2.

Proposition 2. *Suppose $W_O = \infty$ and $W_S = \infty$. The optimal contracting parameters are:*

- $v^{\mathcal{UC}} = 0$

- $y^{\mathcal{UC}} = \begin{cases} 0.5 & \text{if } \beta = 0.5 \\ \frac{1-\beta^2-\sqrt{\beta(1-\beta)(1+\beta)(2-\beta)}}{1-2\beta} & \text{if } \beta \neq 0.5 \end{cases}$
- $w^{\mathcal{UC}} = \frac{(2-\beta)\sqrt{x}y^{\mathcal{UC}}}{\sqrt{(\beta(1-y^{\mathcal{UC}}))^{\beta}((1-\beta)y^{\mathcal{UC}})^{1-\beta}}} - 1$

In the absence of wealth constraints the operator does not purchase insurance coverage, i.e. $v^{\mathcal{UC}} = 0$. This is because insurance externalizes liability away from the contractual relationship and, therefore, reduces the incentives of each firm to invest in reliability. The operator thus finds it more profitable to allocate all financial liability between the two contracting parties, especially given that the overall wealth of the firms is enough to self-insure.

Since the failure probability depends both on the efforts of the operator and supplier, the operator chooses penalty $y^{\mathcal{UC}} \in (0, 1)$ to transfer some positive level of liability to each party; the operator internalizes a proportion equal to $1-y^{\mathcal{UC}}$, and the supplier internalizes a proportion equal to $y^{\mathcal{UC}}$. This penalty adjusts the effort levels so that the *marginal* effort of each firm is proportional to its influence in mitigating the failure probability.[12]

4.2.2 Special case $\mathcal{SC}$: supplier with wealth constraints

We next look at the case where the supplier has finite wealth, but the operator has no wealth constraints, i.e. $W_S < \infty$ and $W_O = \infty$. In Proposition 3 we present our main results for this case. This proposition shows the existence of two key wealth thresholds for the supplier, denoted by W_S^I and W_S^{II} (where $W_S^I > W_S^{II}$). The threshold W_S^I is such that when the supplier's wealth $W_S \geq W_S^I$, the supplier's wealth constraint does not bind. When this happens, the operator's problem reduces to the unconstrained case ($\mathcal{UC}$). If on the other hand $W_S^I > W_S$, the supplier's wealth constraint binds at optimum. This means that the supplier is unable to sustain large losses. Given this inability, the operator optimally reduces the penalty rate that the supplier bears. The operator, however, *only* finds it optimal to purchase insurance coverage when $W_S^{II} > W_S$.

Proposition 3. *Assume that* $W_S < \infty$ *and* $W_O = \infty$ *and define* $W_S^I \equiv \frac{xy^{\mathcal{UC}}(2-\beta)-\left(w^{\mathcal{UC}}+1\right)}{2-\beta}$ *and* $W_S^{II} \equiv x(1-\beta)(1-2m)$ *for some* m *satisfying* $\frac{((1-\beta)m)^{\beta-1}}{(\beta(1-m))^{\beta}} = x\left(\frac{m-(1-\beta)(1-2m)}{m}\right)^2$. *The optimal contracting parameters satisfy:*

[12] To see this we can verify that $e_O^* = \sqrt{\frac{x\left(\beta\left(1-y^{\mathcal{UC}}\right)\right)^{2-\beta}}{\left((1-\beta)y^{\mathcal{UC}}\right)^{1-\beta}}} - 1$ and $e_S^* = \sqrt{\frac{x\left((1-\beta)y^{\mathcal{UC}}\right)^{1+\beta}}{\left(\beta\left(1-y^{\mathcal{UC}}\right)\right)^{\beta}}} - 1$ and, hence, $\frac{\partial F\left(e_S^*,e_O^*\right)/\partial e_S}{\partial F\left(e_S^*,e_O^*\right)/\partial e_O} = \frac{1-\beta}{\beta}\left(\frac{e_O^*+1}{e_S^*+1}\right)$. This ratio is analogous to the one shown by Bhattacharyya and Lafontaine (1995) in their seminal paper on the nature of share contracts under double-sided moral hazard.

- $v^{\mathcal{SC}} = \begin{cases} 0 & \text{if } W_S \geq W_S^I \\ 0 & \text{if } W_S \in [W_S^{II}, W_S^I) \\ \max\left\{ \frac{(1-\beta)\left(1-2y^{\mathcal{SC}}\right)}{2-\beta}, \frac{x(1-\beta)\left(1-2y^{\mathcal{SC}}\right)-W_S}{x(1-\beta)+W_S/y^{\mathcal{SC}}} \right\} & \text{if } W_S < W_S^{II} \end{cases}$

- $y^{\mathcal{SC}} = \begin{cases} y^{\mathcal{UC}} & \text{if } W_S \geq W_S^I \\ \frac{W_S}{x(1-\Phi^{\mathcal{SC}})} & \text{if } W_S \in [W_S^{II}, W_S^I) \\ \max\left\{ \frac{W_S}{x(1-\Phi^{\mathcal{SC}})}, \frac{\Phi^{\mathcal{SC}}(1-\beta)}{2-\beta(1+\Phi^{\mathcal{SC}})} \right\} & \text{if } W_S < W_S^{II} \end{cases}$

- $w^{\mathcal{SC}} = \begin{cases} w^{\mathcal{UC}} & \text{if } W_S \geq W_S^I \\ xy^{\mathcal{SC}}\left(1+\Phi^{\mathcal{SC}}(1-\beta)\right) - 1 - W_S & \text{if } W_S \in [W_S^{II}, W_S^I) \\ xy^{\mathcal{SC}}\left(1+\Phi^{\mathcal{SC}}(1-\beta)\right) - 1 - W_S & \text{if } W_S < W_S^{II} \end{cases}$

where $\Phi^{\mathcal{SC}} \equiv \left[\sqrt{x\left(\beta\left(1-y^{\mathcal{SC}}-v^{\mathcal{SC}}\right)\right)^{\beta}\left(\left(1-\beta\right)y^{\mathcal{SC}}\right)^{1-\beta}}\right]^{-1}$.

We illustrate the above results in Figure 4.2. When $W_S \geq W_S^I$, the operator sets a penalty rate equal to $y^{\mathcal{UC}}$ and purchases no insurance coverage (as in the unconstrained case). If $W_S < W_S^I$ the supplier is unable to sustain large financial losses and, therefore, the operator has to decrease the penalty to $y^{\mathcal{SC}}<y^{\mathcal{UC}}$. When the operator decreases the penalty, it reduces the supplier's share of the liability. Note that this share is equal to $\left(y^{\mathcal{UC}} - y^{\mathcal{SC}}\right)X$.

The operator now needs to decide whether to: (i) absorb this share, or; (ii) transfer some of it away from the collaborative relationship by purchasing insurance coverage. If $W_S \in [W_S^{II}, W_S^I)$, the operator chooses to retain this share, i.e. to internalize the liability. However, if the wealth constraint of the supplier is very stringent, i.e. when $W_S < W_S^{II}$, the operator optimally transfers some of this share to the insurer by purchasing coverage.

It may seem optimal for a risk-neutral and wealth-unconstrained operator to always internalize the liability (instead of purchasing insurance). After all, when the operator internalizes the liability it has higher incentives to exert effort. On the other hand, when the operator purchases insurance, these incentives are reduced and the likelihood of an operational failure increases.

So why does the operator purchase insurance when the supplier's wealth constraint is very severe? The intuition is as follows. Because the efforts of the operator and supplier are partially substitutable, any increase in the effort of the operator has a negative externality on the incentives of the supplier. In other words, when the operator chooses to keep the extra share of the liability,

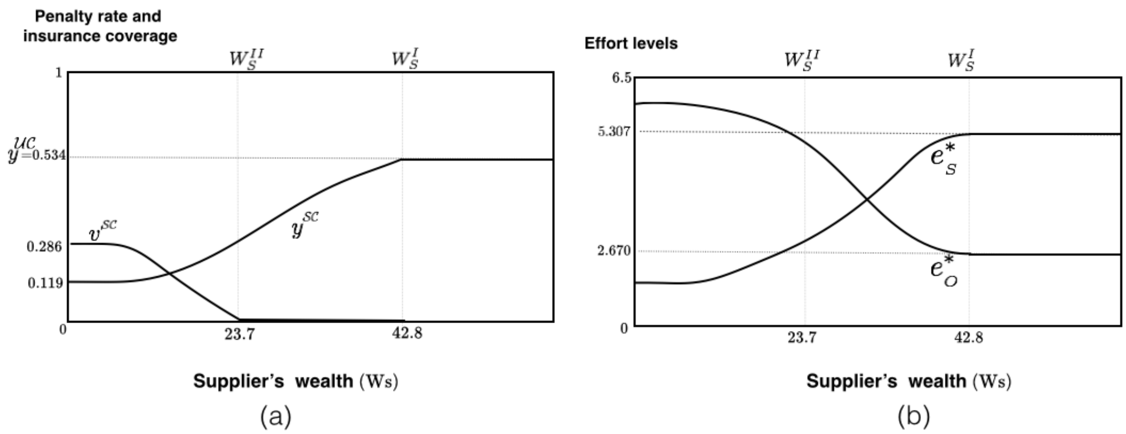


Figure 4.2: In this figure: $x = 100$, $\beta = 0.4$, $\pi = 45$ and $W_O = \infty$.

the supplier knows that the operator has higher incentives to increase effort (in stage 2). Therefore, the supplier has an incentive to free-ride on the efforts of its partner and, as a result, it further reduces its own effort levels. This implies that the increase in the operator's effort is partially offset by a reduction in the effort of the supplier. In other words, when the operator internalizes the extra share of the liability, this results in an effort distortion.

Consider instead what happens when the operator chooses to externalize the failure costs through insurance. In this case the overall incentives to exert effort are reduced. Hence, the operator faces the following trade-off: (i) the effort distortion that is caused by the free-riding problem and; (ii) the reduction in the overall incentives to exert effort that is caused by insurance. Our results show that when the supplier's wealth constraint is not stringent, i.e. when $W_S \in [W_S^{II}, W_S^{I})$, the effort distortion is preferred to the dampening in efforts. In other words, it is better to absorb all liability. When the supplier's wealth constraint is very stringent however, i.e. when W_S is less than the threshold W_S^{II}, it is better to dampen the effort incentives by purchasing insurance than to further distort the efforts of the firms (see Figure 3). The operator can achieve this by using insurance as a commitment not to increase effort. This effectively reduces the supplier's incentives to free-ride and, therefore, mitigates the effort distortion in the collaborative relationship.

Note that the operator may choose to buy insurance, not as a mechanism to ensure the financial viability of either party, but rather as a credible commitment mechanism not to increase effort. Insurance allows the operator to better coordinate effort along the collaborative relationship. This is what we refer to as the strategic role of business insurance. As we can see from Figure 4.3, the overall profits increase.

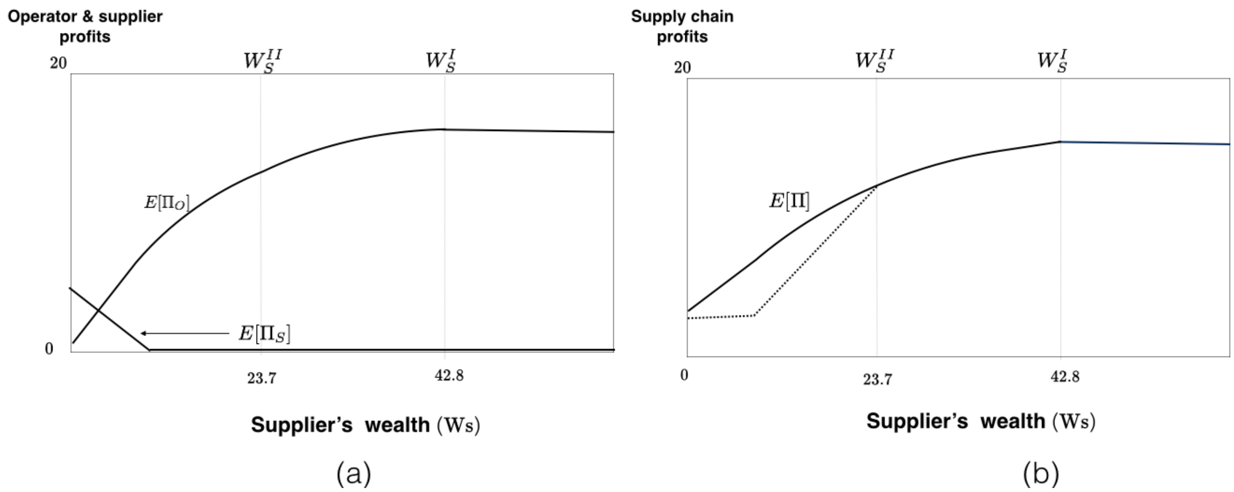


Figure 4.3: In this figure: $x = 100$, $\beta = 0.4$, $\pi = 45$ and $W_O = \infty$. The solid lines represent the equilibrium profits. The dashed line in (b) illustrates the expected profits in a setting where insurance is not available, i.e. where $v^* = 0$.

4.2.3 Special case $\mathcal{OC}$: operator with wealth constraints

We next move to the case where the operator has finite wealth, but the supplier has no wealth constraints, i.e. $W_O < \infty$ and $W_S = \infty$. Therefore, the supplier's wealth constraint never binds. In Proposition 4 we present our main results for case $\mathcal{OC}$, where we show that (as in case $\mathcal{SC}$) insurance may be purchased for strategic reasons. The intuition behind this result, however, is slightly different from the one presented in the previous case.

In the proposition below, we again show the existence of two wealth thresholds. This time the thresholds, W_O^I and W_O^{II}, are for the operator. The operator's wealth constraint binds if W_O is less than W_O^I and, in this situation, the operator optimally increases the penalty rate that the supplier bears. The operator purchases insurance only when $W_O < W_O^{II}$.

Proposition 4. *Assume that $W_O < \infty$ and $W_S = \infty$. Define $W_O^I \equiv x\Phi^{\mathcal{UC}}\left(2y^{\mathcal{UC}}(1-\beta)+\beta\right) + x\left(1-y^{\mathcal{UC}}\right) - 2 - \pi$ and $W_O^{II} \equiv \frac{x(2n(1-\beta)+\beta)}{\sqrt{x(y^{\mathcal{UC}}(1-\beta))^{1-\beta}(\beta(1-y^{\mathcal{UC}}))^{\beta}}} + x(1-n) - 2 - \pi$, where n satisfies $\frac{(n(1-\beta))^{\beta-1}}{x(\beta(1-n))^{\beta}} = \left(1+\beta\left(\frac{1-2n}{1-n}\right)\right)^2$. The optimal contracting parameters satisfy:*

- $$v^{\mathcal{OC}} = \begin{cases} 0 & \text{if } W_O \geq W_O^I, \\ 0 & \text{if } W_O \in [W_O^{II}, W_O^I) \\ \left(\frac{1}{1-\beta}\right)\left(\frac{y^2}{2} - \frac{(1-y)(W_O+2-xy)(1-\beta)-x}{x\beta}\right)^{\frac{1}{2}} - \beta + 2y\left(\frac{3}{4}+\beta\right) & \text{if } W_O < W_O^{II} \end{cases}$$

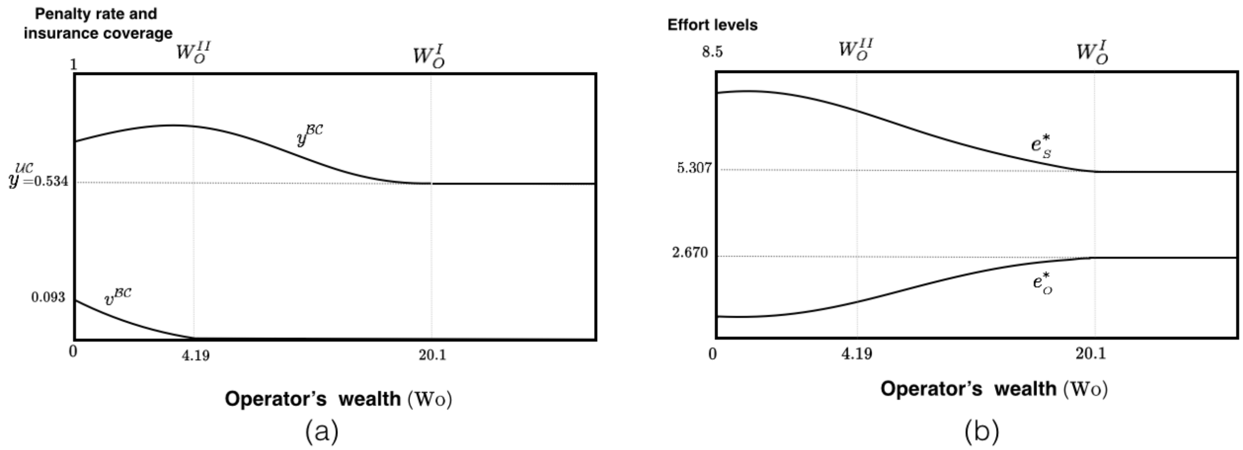


Figure 4.4: In this figure: $x = 100$, $\beta = 0.4$, $\pi = 45$ and $W_S = \infty$.

- $y^{\mathcal{OC}} = \begin{cases} y^{\mathcal{UC}} & \text{if } W_O \geq W_O^I \\ \Phi^{\mathcal{OC}}\left(2\left(1-\beta\right)+\beta\right)+1-\frac{2+W_O}{x} & \text{if } W_O \in [W_O^{II}, W_O^I) \\ \frac{\Phi^{\mathcal{OC}}-1-\beta\left(1-v^{\mathcal{OC}}\right)}{\Phi^{\mathcal{OC}}-2} & \text{if } W_O < W_O^{II} \end{cases}$

- $w^{\mathcal{OC}} = \begin{cases} w^{\mathcal{UC}} & \text{if } W_O \geq W_O^I \\ \left(2-\beta\right)xy^{\mathcal{OC}}\Phi^{\mathcal{OC}}-1 & \text{if } W_O \in [W_O^{II}, W_O^I) \\ \left(2-\beta\right)xy^{\mathcal{OC}}\Phi^{\mathcal{OC}}-1 & \text{if } W_O < W_O^{II} \end{cases}$

where $\Phi^{\mathcal{OC}} \equiv \left[\sqrt{x\left(\beta\left(1-y^{\mathcal{OC}}-v^{\mathcal{OC}}\right)\right)^{\beta}\left(\left(1-\beta\right)y^{\mathcal{OC}}\right)^{1-\beta}}\right]^{-1}.$

We illustrate Proposition 4 in Figure 4.4. When the operator's wealth constraint binds, the operator is unable to bear large financial losses. To satisfy this constraint, the operator needs to transfer away a higher share of the failure costs (relative to the unconstrained case $\mathcal{UC}$). The operator can transfer this share to the supplier, by increasing the supplier's penalty rate above $y^{\mathcal{UC}}$ or, alternatively, transfer this share away from the firms through insurance.

Similar to case $\mathcal{SC}$, when the supplier internalizes the extra share of the liability (due to a higher penalty rate) not only does the supplier have larger incentives to increase effort, but the operator also has a larger incentive to free-ride on the supplier's efforts. This implies that any increase in the efforts of the supplier is partially offset by a further decrease in the efforts of the operator. Therefore, the operator faces a trade-off between: (i) distorting the efforts of the firms by increasing the supplier's penalty rate, and exacerbating the free-riding problem and; (ii) dampening

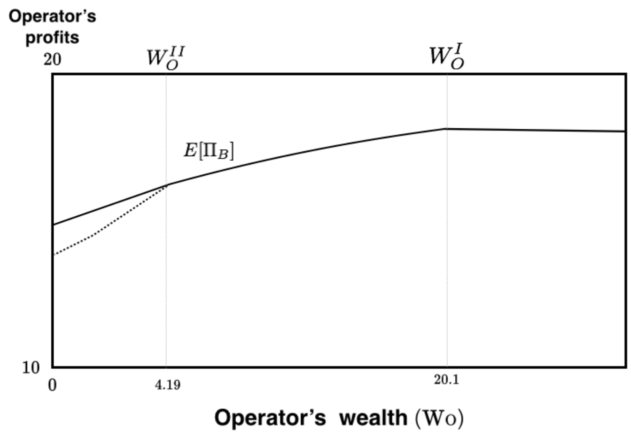


Figure 4.5: In this figure: $x = 100$, $\beta = 0.4$, $\pi = 45$ and $W_S = \infty$. Because $E[\Pi_S] = 0$, the operator's profits are also the overall profits. The dashed line illustrates the expected profits in a setting where insurance is not available, i.e. where $v^* = 0$.

the supplier's incentives by purchasing insurance. However, in case $\mathcal{OC}$ the operator is the party that free-rides on the effort of the supplier. This is unlike case $\mathcal{SC}$, where the supplier is the free-rider.

But if the operator is the one free-riding, why would it want to mitigate the free-riding problem? The intuition is as follows. Note that the operator coordinates the supplier's efforts through the penalty rate, y, and compensates these efforts through a fixed-fee transfer w. Therefore, by optimally choosing y and w, the operator extracts all rents from the supplier. When the free-riding problem becomes severe enough, the effort distortion becomes very large. Because of this inefficiency the operator is able to extract fewer rents from the supplier.

Due to the argument above, the operator faces the same trade-off as in case $\mathcal{SC}$. Our results show that when the operator's wealth constraint is not stringent, a distortion of the efforts is preferred to a dampening of the supplier's incentives. For this reason, the operator chooses to increase the penalty rate, instead of purchasing insurance. However, when $W_O < W_O^{II}$, a dampening in the supplier's incentives is preferred to an effort distortion (and the operator purchases insurance coverage). In Figure 4.5 we can observe that when $W_O < W_O^{II}$, overall welfare is improved through insurance.

4.2.4 The general case: supplier and operator with wealth constraints

We now assume that both the supplier and the operator are subject to wealth constraints, i.e. that $W_S \leq \infty$ and $W_O \leq \infty$. In Proposition 5 we show the existence of four regions. In the first region, none of the wealth constraints bind. This leads to the unconstrained case $\mathcal{UC}$. In the second and third regions, one of the wealth constraints is binding but the other is not. These lead to the cases $\mathcal{SC}$ and $\mathcal{OC}$.

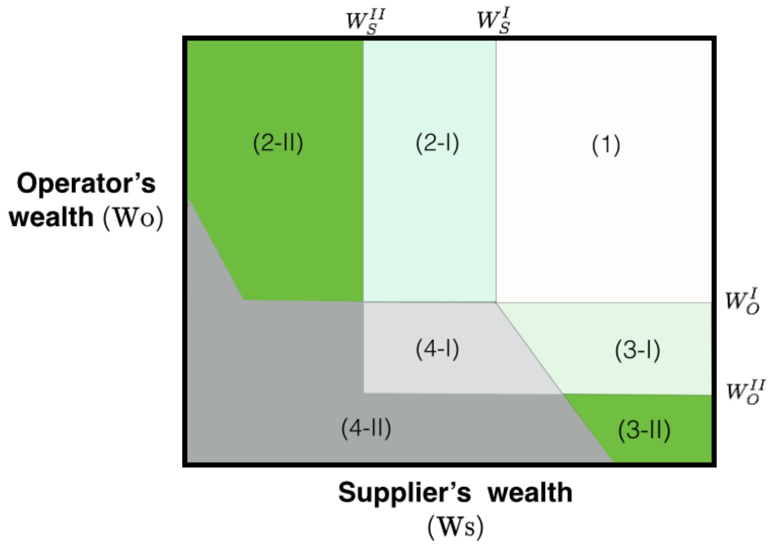
By looking at the general model, however, we must consider a new region. This is the region where both constraints bind at optimum. In this region the operator purchases insurance, not only for strategic reasons, but also to ensure the financial viability of the firms. In other words the operator's problem is only feasible with insurance. This is unlike the other regions, where the firms can run operations without insurance coverage.

Proposition 5. *Assume that* $W_S \leq \infty$ *and* $W_O \leq \infty$ *and define* $\tilde{W}(y,v) \equiv x\left(1+\frac{(\beta(1-y-v)+y(1-\beta))}{\sqrt{x((1-\beta)y)^{1-\beta}(\beta(1-y-v))^{\beta}}}\right)-\pi$. *The optimal contracting parameters satisfy:*

- $$v^* = \begin{cases} v^{\mathcal{UC}} & \text{if } W_S > W_S^I \text{ and } W_O > W_O^I \\ v^{\mathcal{SC}} & \text{if } W_S \leq W_S^I \text{and } W_O > \tilde{W}\left(y^{\mathcal{SC}}, v^{\mathcal{SC}}\right) - W_S \\ v^{\mathcal{OC}} & \text{if } W_O \leq W_O^I \text{and } W_S > \tilde{W}\left(y^{\mathcal{OC}}, v^{\mathcal{OC}}\right) - W_O \\ \frac{\Phi^* x(y^*(1-2\beta)+\beta)-W_S-W_O-\pi+x-2}{(1-\Phi^*)x+\beta x\Phi^*} & \text{otherwise} \end{cases}$$
- $$y^* = \begin{cases} y^{\mathcal{UC}} & \text{if } W_S > W_S^I \text{ and } W_O > W_O^I \\ y^{\mathcal{SC}} & \text{if } W_S \leq W_S^I \text{and } W_O > \tilde{W}\left(y^{\mathcal{SC}}, v^{\mathcal{SC}}\right) - W_S \\ y^{\mathcal{OC}} & \text{if } W_O \leq W_O^I \text{and } W_S > \tilde{W}\left(y^{\mathcal{OC}}, v^{\mathcal{OC}}\right) - W_O \\ \frac{W_S}{x(1-\Phi^*)} & \text{otherwise} \end{cases}$$
- $$w^* = \begin{cases} w^{\mathcal{UC}} & \text{if } W_S > W_S^I \text{ and } W_O > W_O^I \\ w^{\mathcal{SC}} & \text{if } W_S \leq W_S^I \text{and } W_O > \tilde{W}\left(y^{\mathcal{SC}}, v^{\mathcal{SC}}\right) - W_S \\ w^{\mathcal{OC}} & \text{if } W_O \leq W_O^I \text{and } W_S > \tilde{W}\left(y^{\mathcal{OC}}, v^{\mathcal{OC}}\right) - W_O \\ \Phi^* y^* x\left(2-\beta\right)-1 & \text{otherwise} \end{cases}$$

where $\Phi^* = \left[\sqrt{x\left(\beta\left(1-y^*-v^*\right)\right)^{\beta}\left(\left(1-\beta\right)y^*\right)^{1-\beta}}\right]^{-1}$

We illustrate these results in Figure 4.6. In region 1 the wealth constraints are non-binding. In region 2, only the wealth constraint of the supplier binds at optimum. Note that in sub-region 2-I, we have that $W_S \in [W_S^{II}, W_S^I)$. Therefore, by Proposition 3, the operator does not purchase insurance. Conversely, in sub-region 2-II we have that $W_S < W_S^{II}$ and the operator purchases insurance. In region 3, the supplier's wealth constraint is non-binding, but the operator's constraint is binding. In sub-region 3-I we have that $W_O \in [W_O^{II}, W_O^I)$ and, by Proposition 4, the operator does not purchase insurance. Conversely, in sub-region 3-II we have that $W_O < W_O^{II}$ and the



	No insurance	Insurance purchased (strategic role)	Insurance purchased (liquidity-enabling role)
No binding wealth constraints	(1)		
Binding wealth constraint for supplier	(2-I)	(2-II)	
Binding wealth constraint for operator	(3-I)	(3-II)	
Binding wealth constraint for both parties		(4-II)	(4-I,4-II)

Figure 4.6: Contracting regions for the general case.

operator purchases insurance. In region 4, insurance serves a liquidity-enabling role (note that in region 4-I, insurance serves a liquidity-enabling role, whereas in region 4-II insurance serves both the strategic and liquidity-enabling roles).

4.3 Summary of results

In §4.1 we show that a centralized entity purchases insurance as a way to ensure its financial viability (i.e. the liquidity-enabling role). In other words, insurance is only purchased if the potential costs of an operational failure exceed the wealth of the firms, $W = W_S + W_O$.

We show that the liquidity-enabling role also arises in a decentralized relationship (see region 4 in Figure 4.6). However, in a decentralized setting purchasing insurance is optimal even in situations where the overall wealth of the firms is enough to cover any ex post losses associated with an operational failure (see regions 2-II and 3-II). In these cases the traditional explanations for why firms should purchase insurance are absent. Our paper shows that in these cases insurance serves as a commitment mechanism to mitigate a free-riding problem. The free-riding problem arises when one of the firms has wealth constraints, but the other firm has sufficient wealth to sustain large losses.

5 Extensions

We discuss several extensions below but, due to space limitations, do not present all technical results in the document. A detailed presentation of the results discussed below can be found in an extended working paper version of this manuscript. This version is available in the authors' website.

5.1 Sequential effort

In the base model we assume that the efforts of the supplier and operator are unobservable and, as we explain in Section 3.4, we can treat these efforts as simultaneous - even if these efforts are sequentially exerted. This is by the *Principle of Interchange of Moves* (Osborne and Rubinstein, 1994).

But the assumption of unobservability does not always hold. Consider a supplier that is in charge of designing a piece equipment that is subsequently managed by the operator: the supplier will be the first party to exert effort, i.e. in the design of the equipment, and the operator will follow, i.e. by exerting effort when operating the equipment. The equipment operator may be able to infer the effort of the supplier by inspecting the quality of the equipment.[13] In these situations it is reasonable to assume a Stackelberg relationship in the exertion of the efforts.

We now study an extension where the supplier leads in the exertion of effort, and the operator follows after observing the effort of the supplier. We begin by presenting the incentive compatible levels of effort in Lemma 1.

Lemma 1. *Let e_O^* and e_S^* represent the optimal effort levels if the efforts are simultaneous and $e_{0,seq}^*$and $e_{S,seq}^*$ represent the optimal effort levels if these efforts are Stackelberg (and the supplier leads). We have that*

$$\frac{e_{O,seq}^* + 1}{e_{S,seq}^* + 1} = (1+\beta)\left(\frac{e_O^* + 1}{e_S^* + 1}\right) \tag{5.1}$$

The above lemma shows that when the supplier leads in the exertion of effort, there is an additional distortion in ratio of these efforts, favoring the supplier (by a degree of $1+\beta$). In other words, for

[13]Note that we are assuming that effort is observable *but* unverifiable in a court of law. This means that effort is not contractible. This is a commonly observed assumption (see page 38 in Tirole, 1988). If the efforts of the firms are verifiable, these efforts can be contracted and the model becomes equivalent to the first best benchmark.

given levels of y and v, the Stackelberg advantage allows the supplier to free-ride on the operator to a larger degree. We refer to this as a *Stackelberg distortion*.

So how does this assumption affect the results of the base model? We find numerically that if the supplier is the wealth-constrained party (i.e. case $\mathcal{SC}$), the levels of insurance are greater in a Stackelberg relationship. Conversely, if the operator is the wealth-constrained party (i.e. case $\mathcal{OC}$), the levels of insurance are higher when the efforts are simultaneously exerted.

This result is intuitive. Recall from the base model that, in case $\mathcal{SC}$, the wealth constraint of the supplier causes a distortion that *increase*s the ratio of efforts between the operator and the supplier. So when the relationship is Stackelberg, this distortion is multiplied by the Stackelberg distortion. The overall distortion thus is larger, and more insurance is needed to correct this problem.

Conversely, recall that in case $\mathcal{OC}$ the wealth constraint of the operator causes a *decrease* in the ratio of effort between the supplier and the operator. But when effort is sequential, the Stackelberg distortion moves in the opposite direction, and counteracts the distortion generated by the wealth constraints. This implies that less insurance is needed to correct this problem.

In all scenarios the overall profits decrease when the effort is sequentially exerted. This is because the supplier (who is the agent) gains an advantage on the operator.

5.2 Insurance opportunities for the supplier

In the base model we do not consider a scenario where the supplier is able to purchase insurance. This assumption allows us to focus on the coordinating role that insurance plays for business operators. But this assumption may not often hold in practice, as buying insurance is an option that is also available to other parties in the collaborative relationship.

To address this issue, we study a setting where both the supplier and the operator have the option of (independently) obtaining insurance coverage from third-parties.[14] In this extension we seek to address two questions: (i) when, and for what reasons, should the supplier buy insurance? and; (ii) how does this possibility affect the strategic role of insurance for the operator? The following proposition allows us to answer both questions.

Proposition 6. *If the supplier has the option of purchasing insurance coverage:*

1. *The supplier will choose coverage level* $v_S^* = \frac{\beta y}{1+\beta}$.

[14] Note that in cases where (i) multiple parties are involved in an operational failure, and (ii) the degree of fault is ambiguous, insurance companies have 'knock-for-knock' agreements, where the insurers agree to reimburse the losses for which their respective policy holders are responsible in their procurement agreements, regardless of fault (Lilleholt et al., 2012).

2. *Under coverage level v_S^*, the ratio of efforts between the operator and supplier will be identical to the ratio given in Lemma 1.*

The above proposition tells us that the supplier will purchase insurance regardless of whether the parties are wealth-constrained or not. The supplier will use insurance as a commitment not to exert effort, which will drive the operator to increase its own effort. The use of insurance by the supplier is not used to mitigate distortions in the efforts of the firms but, rather, to further distort these efforts in its favour. This is because the supplier (as the agent) maximizes its own profits and not the joint profits of the collaborative relationship. The supplier thus uses insurance as a device to increase its ability to free-ride at the expense of the operator.

Part 2 of the above proposition tells us that when the supplier buys insurance, the ratio of efforts is identical to the case where the efforts are sequential. In other words, the supplier gains a Stackelberg advantage on the operator by purchasing insurance. This is because insurance allows the supplier to commit to a lower level of effort. This means that the analysis of this extension is qualitatively similar to the one from §5.1. We find that the distortion generated by the supplier's insurance coverage increases the need for insurance for the operator in case $\mathcal{SC}$, but decreases it in case $\mathcal{OC}$. The reasoning is identical to the one presented in §5.1.

The overall profits decrease in all cases when the supplier has access to insurance coverage. This is because the supplier (i.e. the agent) gains a strategic advantage on the operator and worsens the free-riding problem in the collaborative relationship. For this reason, it may be reasonable for an operator to try to impose a condition that supplier not purchase insurance, which is consistent with the assumption made in the base model.

5.3 Stochastic Failure Costs

In this section we relax the assumption that the failure costs are ex ante known. Let $X \in \{0, [x_l, x_h]\}$ represent the failure costs. If $X = 0$, operations have run as planned and the operator does not incur any costs. If $X > 0$ a failure has occurred and the costs are equal to $x \in [x_l, x_h]$, where $0 < x_l < x_h$. Assume that X has a probability atom at 0. Specifically, $Pr\,(X = 0) = 1 - F\,(e_S, e_O)$ where $F\,(e_S, e_O)$ is defined as in the base model.

We can explore richer settings by looking at the case where the failure costs are stochastic. For example, tiered contracts (i.e. contracts where the penalty levels depend on the realized costs) are common in practice but not relevant in our main model. Second, under this assumption we can study the role of insurance deductibles in the insurance contract.

5.3.1 Tiered contracts:

Jain et al. (2013) show that, in the presence of financial constraints, contracts with tiered penalties are significantly more powerful in mitigating double-moral hazard. This is because the design of tiered penalties allows the operator to coordinate efforts without causing an excessive financial burden on the wealth-constrained firm. For example, if the supplier has wealth constraints, the operator can optimally increase the supplier's penalty for small failures, and decrease the penalty for large failures. The *expected* penalty for the supplier thus remains unchanged and, at the same time, the financial distress of the supplier is mitigated. For this reason tiered contracts are frequently used in practice.[15]

We assume that the operator designs two tiers, $y_1 \in [0,1]$ and $y_2 \in [0,1]$, and a tier threshold $x_T \in [x_l, x_h]$. When $X \leq x_T$, the operator penalizes the supplier with a penalty rate equal to $y_1 X$. When $X > x_T$, the size of the penalty is $y_2 X$. The cash flows of the procurement contract (from the operator to the supplier) are

$$T\left(w, y_1, y_2, x_T, X\right) = \begin{cases} w - Xy_1 & \text{if } X \leq x_T \\ w - Xy_2 & \text{if } X > x_T \end{cases}$$

We study this model by deriving some analytical results and by running several numerical simulations. We first do comparative statics by varying the wealth of the parties, under the assumption that the failure costs are uniformly distributed. A set of results is illustrated in Tables 1 and 2. In these tables we present the optimal parameters under a contractual structure involving tiered contracts and insurance $(x_T, y_1, y_2; v)$. For comparison purposes, we also present the optimal parameters under a structure involving a simple penalty rate and insurance $(y; v)$. In the right-most column of these tables we present $\Delta\Pi$, which shows how much the profits of the collaborative relationship increase when the operator uses tiered contracts. We obtain the following results.

First, we find the operator can efficiently coordinate efforts through tiered penalties and, at the same time, mitigate the impact of the wealth constraints (i.e. the free-riding problem). However, when the wealth of the operator, or the supplier, is small enough, a tiered contract is unable

[15] For example, in a joint operations agreement signed in 2001 between The Union Oil of California, the operator, and Ivanhoe Energy, the service supplier, the contractor uses tiered penalties for any costs incurred in the event of oil spills, blowouts, fires, etc. The operator will be compensated for:
(A) 5 % of total costs through $100,000; plus
(B) 3 % of total costs in excess of $100,000 but less than $1,000,000; plus
(C) 2 % of total costs in excess of $1,000,000. (Section III, clause 2)

W_O	x_T	y_1	y_2	v	y	v	$\Delta\Pi$
110	0	0.5	0.5	0	0.51	0	0
90	82.34	0.05	0.59	0	0.60	0	0.32
70	100	0.41	0.70	0	0.71	0	1.09
60	100	0.53	0.76	0	0.69	0.06	1.49
50	100	0.60	0.75	0.08	0.67	0.16	2.06
40	100	0.61	0.68	0.23	0.63	0.29	2.57

Table 1: We assume that $\pi = 10$, $W_S = \infty$, and $\beta = 0.5$.

W_S	x_T	y_1	y_2	v	y	v	$\Delta\Pi$
80	0	0.5	0.5	0	0.5	0	0
70	99.71	0.80	0.39	0	0.38	0	0.38
60	100	0.57	0.28	0	0.27	0	1.04
50	100	0.46	0.23	0	0.23	0.05	1.39
40	100	0.17	0.17	0.06	0.17	0.13	1.53
30	100	0.23	0.11	0.17	0.11	0.21	1.85

Table 2: We assume that $\pi = 10$, $W_O = \infty$, and $\beta = 0.5$.

x_l	x_h	x_T	y_1	y_2	v
25	175	93.73	0.59	0.73	0
50	150	95.48	0.63	0.75	0.05
75	125	98.63	0.68	0.75	0.10
100	100	100	0.76	0.76	0.14

Table 3: In this simulation, we let $\pi = 10, W_S = \infty, W_O = 50$ and $\beta = 0.5$.

x_l	x_h	x_T	y_1	y_2	v
25	175	93.72	0.25	0.13	0.07
50	150	95.54	0.25	0.16	0.11
75	125	98.13	0.25	0.20	0.14
100	100	100	0.25	0.25	0.17

Table 4: In this simulation, we let $\pi = 10, W_O = \infty, W_S = 20$ and $\beta = 0.5$.

to eliminate excessive free-riding among the contractual parties. In these cases the operator still purchases insurance coverage, but the amount of coverage is smaller.

Second, we study the sensitivity of our results to the variance of the failure costs.[16] We find that the role of insurance decreases when the variance of the failure cost increases (see Tables 3 and 4). This is because when the failure cost has a large variance, the operator has more flexibility to distribute the liability through tiers. As such, tiered penalties can be used more effectively when the failures cost has a large variance.

5.3.2 Insurance deductibles:

In this section we study the robustness of our model to more complex insurance contracts, by allowing the operator to not only choose the level of insurance coverage, v, but also a deductible level, $d \geq 0$. The insurance cash flows are thus equal to $I(v, d, P, X) = P - \max\{vX - d, 0\}$. If we assume that the conditional probability of X, given $X > 0$, is uniform, then the cash flows of the insurance contract are given by $E_X[I(v, d, P, X)|e_S, e_O] = P - F(e_S, e_O)\int_{x_l}^{x_h} \frac{\max\{vx-d,0\}}{x_h-x_l}dx$

We analyzed this model both analytically and numerically. We found that insurance deductibles are unnecessary when the supplier has binding wealth constraints, but the operator's wealth constraint is non-binding (i.e. case $\mathcal{SC}$). In this case, there always exists an optimal insurance contract where the deductible is equal to 0. When the wealth constraint of the operator is binding and the

[16]To this end, we symmetrically shift x_l and x_h in opposite directions, so that $x_l + x_h$ remains unchanged. For example, we look at a scenario where the expected failure costs are equal to 100, but we change the variance of the failure costs. To do this, we look at various cases: $x_l = x_h = 100$; $x_l = 75$ and $x_h = 125$; $x_l = 50$ and $x_h = 150$ and; $x_l = 25$ and $x_h = 175$.

W_O	y	v	d	y	v	$\Delta\Pi$
120	0.51	0	0	0.51	0	0
100	0.60	0	0	0.60	0	0.32
80	0.53	0.46	59.54	0.71	0	0.70
70	0.54	0.46	48.71	0.69	0.06	1.49
60	0.55	0.45	36.80	0.67	0.16	2.06
50	0.55	0.44	23.24	0.63	0.29	2.57

Table 5: In this simulation we look at the sensitivity of the optimal contract by varying the operator's wealth, W_O. We let $W_S = \infty$ and $\beta = 0.5$.

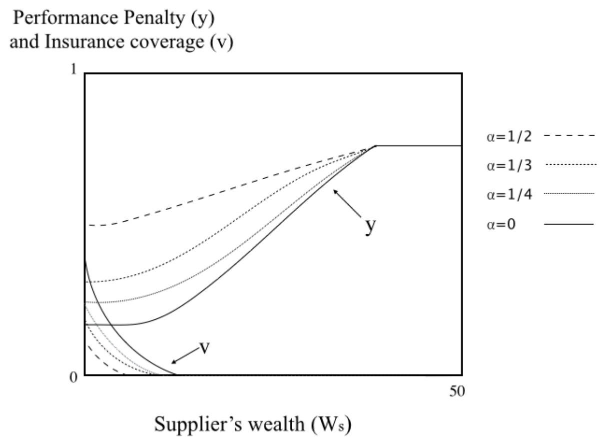


Figure 5.1: In this figure: $x_L = 0$, $x_H = 200$, $\beta = 0.5$, $\pi = 50$ and $W_O = \infty$.

wealth constraint of the supplier is non-binding (i.e. special case $\mathcal{OC}$), positive deductibles allow the operator to mitigate free-riding problem more efficiently. Specifically, we find that deductibles increase the range where insurance is strategically purchased. This is because a positive deductible allows the operator to seek high levels of insurance when the losses exceed its financial wealth. When these losses are small enough, however, the risk-neutral operator does not benefit from insurance. The operator thus uses a deductible to mitigate the impact of moral hazard, and seek high levels of insurance when needed (see Table 5). When the operator is wealth-unconstrained, and the supplier is wealth-constrained, this role is absent. Therefore, the operator only benefits from deductibles when its wealth constraints are binding.

5.4 Alternatives to the wealth constraints

The wealth constraints used in the base model assume that the parties will not enter the contractual relationship unless they are guaranteed to end up with non-negative wealth under all contingencies. These constraints are common in practice and have been extensively analyzed in the literature. In many cases, however, firms cannot entirely avoid the possibility of bankruptcy but this does not prevent them from engaging in operational activities.

Rather than avoiding the possibility of bankruptcy entirely, firms sometimes adopt other criteria to maximize their profits and, at the same time, minimize their exposure to insolvency risk. The literature has considered a number of alternative approaches to model this behavior. In this sub-section we consider two popular approaches: the financial distress approach and the cost of bankruptcy approach.

Financial Distress Approach: According to the financial distress approach, firms may be averse to a state of insolvency, but are willing to tolerate this possibility if the likelihood is sufficiently small. In other words, the firm will engage in a contractual relationship only if the probability of insolvency is below a *tolerance threshold* $\alpha \in [0,1]$. We write these constraints as follows:

$$\Pr\left[\Pi_O\left(e_O, w, y, v, P, x\right) + W_O \leq 0\right] \leq \alpha_O$$
$$\Pr\left[\Pi_S\left(e_S, w, y, x\right) + W_S \leq 0\right] \leq \alpha_S$$

When the levels of tolerance (α_O and α_S) are equal to zero, the financial distress constraints are equivalent to the wealth constraints. Through numerical results we show that when the tolerance to distress increases (i.e. when α_S and α_O increase), the levels of insurance are smaller (see Figure 5.1 for an illustration). This means that, while the wealth constraints can be seen as an extreme version of the financial distress approach, there are no qualitative changes in the results.

Cost of Bankruptcy: Both the wealth constraints and the financial distress constraints reflect the idea that firms only consider decisions where the risk of insolvency is either small or absent. Other papers consider financial distress by incorporating the costs of negative wealth into the objective function, rather than expressing them as constraints (See Greenwald and Stiglitz (1990), Swinney et al. (2011) and references therein). We explore this approach by assuming that the supplier and the operator maximize the utility functions

$$\begin{aligned} U_S\left(e_S, y, w, D_S, X\right) &\equiv \Pi_S\left(e_S, w, y, X\right) - D_S\psi_S\left(e_S, y, w, W_S, X\right) \\ U_O\left(e_O, w, y, v, P, D_O, X\right) &\equiv \Pi_O\left(e_O, w, y, v, P, X\right) - D_O\psi_O\left(e_O, w, y, v, P, W_O, X\right) \end{aligned}$$

where D_S and D_O are the (exogenously given) bankruptcy costs for each firm, and $\psi_S\left(e_O, w, y, W_S, X\right) \equiv \Pr\left\{\Pi_S\left(e_S, w, y, X\right) + W_S \leq 0\right\}$ and $\psi_O\left(e_O, w, y, v, P, W_O, X\right) \equiv \Pr\left\{\Pi_O\left(e_O, w, y, v, P, X\right) + W_O \leq 0\right\}$ are the bankruptcy probabilities.[17]

[17] U_O and U_S are known as *integrated objective functions.* This approach captures the idea that reaching a state of insolvency brings out non-trivial costs. For example, the firms may be forced to sell their illiquid assets at low

To analyze this setting, we consider a model where the operator seeks to find

$$\max_{w\geq 0, y\in[0,1], v\in[0,1]} E_X\left[U_O\left(e_O^*, w, y, v, P\left(v|e_S^*, e_O^*\right), D_O, X\right)|e_S^*\right]$$

subject to

$$\begin{aligned} & E_X\left[U_S\left(e_S^*, y, w, D_S, X\right)|e_O^*\right] \geq 0 \\ e_S^* &= \arg\max_{e_S\geq 0} E_X\left[U_S\left(e_S, y, w, D_S, X\right)|e_O^*\right] \\ e_O^* &= \arg\max_{e_O\geq 0} E_X\left[U_O\left(e_O, w, y, v, P, D_O, X\right)|e_S^*\right] \end{aligned}$$

We performed numerical simulations using this model and found that when the cost of bankruptcy is large for one party, but small for the other, the operator optimally buys insurance for its strategic value. Consider the case where $D_O = 0$. If D_S is large, the supplier will weigh in the costs of bankruptcy at the time of exerting effort. To mitigate this inefficiency, the operator optimally decreases the penalty rate, which decreases the probability of bankruptcy for the supplier. This will generate a distortion in the effort of both parties. When D_S is too large, the operator is forced to purchase insurance as a way to commit not to increase effort and, thereby, to decrease the distortion in the effort among the parties. The intuition is similar to the base model.

Insurance will be purchased if the cost of bankruptcy is high for the operator and low for the supplier, and vice versa. However, if the costs of bankruptcy are high for both parties, insurance will not be purchased. This is because the distortion in the efforts of the operator is counteracted by the distortion in the efforts of the supplier. In this case, the efficiency decreases, but the efforts are not distorted (i.e. there is no excessive free-riding problem).

6 Conclusion

In this paper we study a setting where the probability of an operational failure is a function of the interdependent (and collaborative) efforts of two firms, e.g. supply chain risk management, maintenance or security outsourcing. We show that in this context insurance has a strategic role. This happens when one of the firms has severe wealth constraints, but the other firm has sufficiently large wealth to cover potential losses. In this situation, contractual incentives alone leave the wealth-constrained firms with inefficiently low incentives to exert effort, and the "wealthy" firms with

prices to repay their debts. An insolvent firm will also have to pay for auditor and litigation fees in the event of filing bankruptcy.

excessively high incentives. Because effort is substitutable, the wealth-constrained firm (which is aware of this incentive distortion) excessively free-rides on the efforts of the wealthy firm. Insurance coverage can mitigate this problem by transferring the financial liability away to a third party insurer. Specifically, insurance allows the "wealthy" firm to credibly commit not to increase effort and this, in turn, decreases the incentives of the wealth-constrained firm to free-ride.

In this paper we demonstrate that business insurance may allow a firm to manage operational risk more efficiently. Our results are particularly relevant to situations where large firms contract with considerably smaller suppliers. In these scenarios the large firm would purchase business insurance, even if the firm is wealth-unconstrained and risk-neutral. By doing so the large firm can prevent the wealth-constrained supplier from excessively free-riding on its reliability efforts. A similar situation arises when small firms hire large suppliers. This implies that the availability of insurance has non-trivial implications in collaborative settings.

While our model is focused on wealth imbalances, our results could apply to broader contexts. As we show, wealth imbalances cause a misallocation of liability, which leads to the strategic role of insurance kicking in. However, it is not the wealth imbalances that directly cause insurance to acquire its strategic value, it is the misallocation of liability. If for any reason (beyond wealth imbalances) the parties are forced to misallocate the liability, insurance could be used as a strategic tool.[18] This means that wealth imbalances are a driver of the strategic role of insurance, but there may be a number of other drivers that are unrelated to the wealth of the parties.

The stylized model presented in this paper ignores some important operational features. For example, in our model the firms are able to decrease the likelihood of a failure, but not the magnitude of the failure costs (in the event of a failure). In the literature, the first type of effort is known as preventive effort, while the latter is known as contingency effort. If we relax this assumption, our main results would be affected if the contingency efforts are effective enough to decrease the failure costs to a point where the wealth constraints become non-binding. However, this is highly unlikely in many contexts, e.g. a nuclear meltdown, an oil spill or a product recall. In these contexts, an operational failure often causes financial distress or even bankruptcy. The effectiveness of contingency efforts may, therefore, be limited.

The contribution of this paper is twofold. First, this paper highlights a role for insurance that, to the best of our knowledge, is not widely applied and understood across the managerial and academic

[18] For example, there may be legal, contractual or other practical restrictions on the maximum liability that can be imposed on either party.

communities. Liability-sharing contracts are becoming increasingly popular in practice and have also gained attention in the academic literature. Given the recent shift in the contracting practices of firms towards liability-sharing, we believe that managers need to broaden their perspective about the role of insurance in multi-firm settings. In particular we show that insurance can help firms mitigate free-riding problems. To the academic community, this paper shows that translating our knowledge of insurance from a single- to a multi-firm setting carries non-trivial implications. More generally, these results contribute to bridging the contracting and risk management literatures.

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Appendix: Proofs

Proof of Proposition 1: The centralized entity seeks to maximize its expected profits, $E_X\left[\Pi\left(e_S^*, e_O^*, v, P, X\right)\right]$, subject to the effort levels of the parties, $\left(e_S^*, e_O^*\right) = \arg\max_{(e_S, e_O)\geq \mathbf{0}}\{E_X\left[\Pi\left(e_S, e_O, v, P, X\right)\right]: \Pi\left(e_S, e_O, v, P, x\right) + W \geq 0\}$.

Observe that for whenever $v > 1 - \frac{2+W+\pi}{x} + \left(x\left(1-v\right)\beta^{\beta}\left(1-\beta\right)^{1-\beta}\right)^{-1/2}$ the wealth constraint is non-binding and, as a result, the effort levels are $e_S^* = \sqrt{x\left(1-v\right)\left(1-\beta\right)^{1+\beta}\beta^{-\beta}} - 1$ and $e_O^* = \sqrt{x\left(1-v\right)\left(1-\beta\right)^{\beta-1}\beta^{2-\beta}} - 1$. At these effort levels the profits of the centralized entity are $E_X\left[\Pi\left(e_S^*, e_O^*, v, P, X\right)\right] = \pi - \left(\sqrt{\frac{x}{\beta^{\beta}\left(1-\beta\right)^{1-\beta}}}\right)\left(\sqrt{\frac{(2-v)^2}{(1-v)}}\right) + 2$. If, on the other hand, $v > 1 - \frac{2+W+\pi}{x} + \left(x\left(1-v\right)\beta^{\beta}\left(1-\beta\right)^{1-\beta}\right)^{-1/2}$, the wealth constraint is binding and the efforts of the firms satisfy $e_S^* = \left(1-\beta\right)\left(\pi - x\left(1-v\right) + W - F\left(e_S^*, e_O^*\right)vx + 2\right) - 1$ and $e_O^* = \beta\left(\pi - x\left(1-v\right) + W - F\left(e_S^*, e_O^*\right)vx + 2\right) - 1$. Evaluated at these effort levels, the firms earn profits $E_X\left[\Pi\left(e_S^*, e_O^*, v, P, X\right)\right] = x\left(1-v\right)\left(1 - F\left(e_S^*, e_O^*\right)\right) - W$.

Define $W^I \equiv x+\sqrt{\frac{x}{\beta^\beta(1-\beta)^{1-\beta}}}-2-\pi$. We can verify that if $W \geq W^I$ then the wealth constraint is non-binding for all $v \in [0,1]$. From here, we can directly observe that $\frac{\partial E_X\left[\Pi\left(e_S^*,e_O^*,v,P,X\right)\right]}{\partial v} = -\sqrt{\frac{x}{\beta^\beta(1-\beta)^{1-\beta}}}\left(\frac{v}{2(1-v)^{\frac{3}{2}}}\right) < 0$, which means that it is optimal to set $v^* = 0$.

Now suppose that $W < W^I$. In this case there exists $\varepsilon > 0$ such that if $v < \varepsilon$, then the wealth constraint is binding. To see why the insurance level is positive over this region, we can verify that $\frac{\partial E_X\left[\Pi\left(e_S^*,e_O^*,v,P,X\right)\right]}{\partial v}|_{v=0} = x\left(1-F\left(e_S^*,e_O^*\right)\right)\left(\frac{x-\beta^\beta(1-\beta)^{1-\beta}(\pi+W+2-x)^2}{(\pi+W+2-x)^2\beta^\beta(1-\beta)^{1-\beta}}\right) > 0$ (the inequality follows from the fact that $W < W^I$). Hence, it is optimal for the centralized entity to set $v > 0$. ■

Proof of Proposition 2: Suppose $W_S = W_O = \infty$. In this setting the operator seeks to find $\max_{w,y,v} E_X\left[\Pi_O\left(e_O^*, w, y, v, P\left(v|e_S^*, e_O^*\right), X\right)|e_S^*\right]$ subject to constraints $\mathcal{IR}$; $w \geq 0$; $y \in [0,1]$, $v \in [0,1]$, $e_O^* = \arg\max_{e_O \geq 0} E_X\left[\Pi_O\left(e_O, w, y, v, P, X\right)\right]$ and $e_S^* = \arg\max_{e_S \geq 0} E_X\left[\Pi_S\left(e_S, w, y, X\right)\right]$.
Let (w^*, y^*, v^*) be the maximizing values. To solve this problem first observe that the optimal effort levels are $e_S^*\left(y,v\right) = \Phi\left(y,v\right)(1-\beta)\,xy - 1$ and $e_O^*\left(y,v\right) = \Phi\left(y,v\right)\beta x\left(1-y-v\right) - 1$, where $\Phi\left(y,v\right) = F\left(e_S^*, e_O^*\right) = \left(x\beta\left(1-y-v\right)\right)^{\frac{-\beta}{2}}\left(x\left(1-\beta\right)y\right)^{\frac{\beta-1}{2}}$. Second, observe that $\mathcal{IR}$ binds at optimum. If this weren't true, then we would have that $w^* > xy^*\Phi\left(y,v\right) + e_S^*$. But this contradicts the fact that w^* is a maximizer, because the operator could always decrease the fixed fee by some $\varepsilon > 0$ small enough, which increases its profits without violating any constraint or affecting the efforts of the firms. Therefore the individual rationality constraint is binding, implying that $w^* = xy^*\Phi\left(y,v\right) + e_S^*$.

If we plug $w^* = xy^*\Phi\left(y,v\right) + e_S^*$ and the incentive compatibility constraints into the objective function, then the operator's problem is to find (y,v) to maximize $E\left[\Pi_O^*\left(y,v\right)\right] = \pi - x\Phi\left(y,v\right)\left(1+y\left(1-\beta\right)+\beta\left(1-y-v\right)\right)+2$ subject to $y \in [0,1]$ and $v \in [0,1-y]$. But note that $\lim_{y+v\to 1}\Pi_O^*\left(y,v\right) = \lim_{y\to 0}\Pi_O^*\left(y,v\right) = -\infty$. Hence, the optimal solution must be such that $y \in (0,1)$ and $v \in [0,1-y)$. Using this fact, we write the Lagrangian function of the operator's problem as $L\left(y,v\right) \equiv \pi - x\Phi\left(y,v\right)\left(1+y\left(1-\beta\right)+\beta\left(1-y-v\right)\right)+2+\lambda v$, where λ is the Lagrangian multiplier for the non-negativity constraint, $v \geq 0$. In this program, the KKT conditions are $-\frac{\partial\Phi(y,v)}{\partial y}\left(1+y\left(1-\beta\right)+\beta\left(1-y-v\right)\right) - \Phi\left(y,v\right)\left(1-2\beta\right) = 0$; $-x\frac{\partial\Phi(y,v)}{\partial v}\left(1+y\left(1-\beta\right)+\beta\left(1-y-v\right)\right) + x\Phi\beta + \lambda = 0$; $\lambda \geq 0$; $v \geq 0$ and $\lambda v = 0$. By noting that $\frac{\partial\Phi(y,v)}{\partial y} = \frac{\Phi}{2}\left(\frac{\beta}{1-v-y} - \frac{1-\beta}{y}\right)$ and $\frac{\partial\Phi(y,v)}{\partial v} = \frac{\Phi\beta}{2(1-v-y)}$, we can show that the KKT conditions are solved by setting $v^* = 0$ and $y^* = \begin{cases} 0.5 & \text{if } \beta = 0.5 \\ \frac{1-\beta^2-\sqrt{\beta(1-\beta)(1+\beta)(2-\beta)}}{1-2\beta} & \text{if } \beta \neq 0.5 \end{cases}$. ■

Proof of Proposition 3: Suppose $W_S < \infty$ and $W_O = \infty$ and define $\Phi(y,v) \equiv (x\beta(1-y-v))^{\frac{-\beta}{2}}(x(1-\beta)y)^{\frac{\beta-1}{2}}$. The Operator's problem is to find $\max_{v,y,w} E_X\left[\Pi_O\left(e_O^*, w, y, v, P\left(v|e_S^*, e_O^*\right), X|e_S^*\right)\right]$ subject to $\mathcal{IR}$, $e_S^* = \arg\max_{e_S}\{E_X\{\Pi_S(e_S, w, y, X)|e_O\}$ s.t. $\Pi_S(e_S,w,y,x)+W_S \geq 0\}$, $e_O^* = \arg\max\{E_X[\Pi_O(e_O,w,y,P,v)|e_S]\}$, $w \geq 0$, $v \in [0,1]$ and $y \in [0,1]$.

Because the supplier's effort is subject to the wealth constraint, then $(e_S^*, e_O^*) = \begin{cases}(\tilde{e}_S, \tilde{e}_0) & \text{if } W_S + \Pi_S(\tilde{e_S}, w, y, x) \geq 0 \\ (e_S^0, e_O^0) & \text{otherwise}\end{cases}$, where: $\tilde{e_S} = \Phi xy(1-\beta)-1$, $\tilde{e}_0 = \Phi x\beta(1-y-v)-1$, $e_S^0 = W_S + w - xy$ and $e_O^0 = \left[(x\beta(1-y-v))(1+W_S+w-xy)^{1-\beta}\right]^{\frac{1}{1+\beta}} - 1$. Without loss of optimality, we can restrict our attention to finding the optimum over the region where $W_S + \Pi_S(\tilde{e_S}, w, y, x) \geq 0$. To verify this, first suppose that the optimum is found over the region where $W_S + \Pi_S(\tilde{e_S}, w, y, x) < 0$ and that $\mathcal{IR}$ binds, i.e. $w^* = F(e_S^*, e_O^*)yx + e_S^*$. Observe that, over this region, the effort level of the supplier is $e_S^* = W_S + w - xy$. This implies that $w^*(y,v) = \left(\frac{xy}{xy-W_S}\right)^{\frac{1+\beta}{1-\beta}}((x\beta)(1-y-v))^{\frac{-\beta}{1-\beta}} - 1 + xy - W_S$. If we plug w^* and the incentive compatible efforts into the objective function, we have that $E_X\left[\Pi_O\left(e_O^0, w^*(y,v), y, P, v\right)|e_S^0\right] = \pi - \frac{x(((1-y)+(1-y-v)\beta))(W_S-xy)}{y} - w^0(y,v) + 1$. From here we can check that $\frac{\partial E_X\left[\Pi_O\left(e_O^0, w^0(y,v), y, P, v\right)|e_S^0\right]}{\partial y} = x\frac{(2+\beta)\left(xy^2-W_S\right)+vW_S(1+\beta)}{y^2} - \frac{\partial w^0(y,v)}{\partial y}$. We can further verify that this derivative is negative for any $y \geq y^0 \equiv \frac{W_S}{x(1-\Phi)}$. By plugging for any y^0 into the objective function, we can verify that the derivative is negative for any y satisfying $W_S + \Pi_S(\tilde{e_S}, w^*(y,v), y, x) < 0$. Therefore, it is optimal for the operator to decrease y to the point where $W_S + \Pi_S(\tilde{e_S}, w^*(y,v), y, x) = 0$. This directly implies that the optimum must be found over the region where $W_S + \Pi_S(\tilde{e_S}, w, y, x) \geq 0$. (The proof also holds for those cases where $\mathcal{IR}$ is non-binding, by noting that the derivative is steeper in this case).

To find the global optimum, we thus need to find $\max_{w,y,v}\{E_X[\Pi_O(\tilde{e}_O, w, y, P, v)|\tilde{e}_S]\}$ subject to $\mathcal{IR}$, $W_S + \Pi_S(\tilde{e}_S, w, y, v) \geq 0$ and $v \in [0,1]$ and $y \in [0,1]$.

Define W_S^I and W_S^{II} as in the proposition statement. If $W_S > W_S^I$, we can show that the optimal solution is identical to the solution in case $\mathcal{UC}$, i.e. $v^* = 0$, $w^* = w^{\mathcal{UC}}$ and $y^* = y^{\mathcal{UC}}$. To see this observe that the solution of case $\mathcal{UC}$ (which is a relaxation of case $\mathcal{SC}$) satisfies $W_S + \Pi_S\left(e_O^*\left(y^{\mathcal{UC}}, 0\right), w^{\mathcal{UC}}, y^{\mathcal{UC}}, 0, x\right) = W_S + \frac{xy^{\mathcal{UC}}(2-\beta)-\left(w^{\mathcal{UC}}+1\right)}{2-\beta}$; this implies that the solution of case $\mathcal{UC}$ satisfies the wealth constraint (and, hence, is the optimum solution) iff $W_S > W_S^I$.

Now, assume that $W_S \leq W_S^I$ or, alternatively, that the wealth constraint binds at optimum.

Because the wealth constraint is binding, we have that $w^* = yx + \tilde{e_S} - W_S$. If we plug w^* and the incentive compatible levels of effort into the objective function, we have that $E_X[\Pi_O(y,v)] = \pi - x\Phi(y,v)((1-y) + \beta(1-y-v) + (1-\beta)y) - xy + 2 + W_S$. Similarly, if we plug these values into the $\mathcal{IR}$ constraint, this constraint can be re-written as $xy(1-\Phi(y,v)) - W_S \geq 0$.

We can thus write this program via Lagrangian optimization, by setting the following Lagrangian function (with shadow prices μ and λ): $L(w,y,v,\lambda,\mu) \equiv \pi - x\Phi(y,v)((1-y) + \beta(1-y-v) + (1-\beta)y) - xy + 2 - W_S + \mu(xy(1-\Phi(y,v)) + W_S) + \lambda v$. The KKT conditions give us $-\frac{\partial \Phi}{\partial y}x(((1-y) + \beta(1-y-v) + (1-\beta)y)) + 2x\Phi\beta - x + \mu x(1-\Phi) - x\mu y\frac{\partial \Phi}{\partial y} = 0$, $-x\frac{\partial \Phi}{\partial v}((1-y) + \beta(1-y-v) + (1-\beta)y) + x\Phi\beta - \mu\left(xy\frac{\partial \Phi}{\partial v}\right) + \lambda = 0$, $\mu(xy(1-\Phi) - W_M) = 0$, $\lambda v = 0$, $\mu \geq 0$, $v \geq 0$ and $\lambda \geq 0$.

To solve the KKT conditions we note that $\frac{\partial \Phi(y,v)}{\partial y} = \frac{\Phi}{2}\left(\frac{\beta}{1-v-y} - \frac{1-\beta}{y}\right)$ and $\frac{\partial \Phi(y,v)}{\partial v} = \frac{\Phi\beta}{2(1-v-y)}$. By doing algebraic arrangements, we can show when $W_S \in (W_S^{II}, W_S^{I}]$, the KKT solutions can only be solved at a point where $\lambda > 0$ and $\mu > 0$, i.e. when both the $\mathcal{IR}$ and non-negativity constraints are binding. The optimal solution is thus given by $v = 0$ and $y^* = y^{\mathcal{SC}}$, where $y^{\mathcal{SC}}$solves $(1-\Phi(y,0)) = \frac{W_S}{xy}$.

We can also check that when $W_S \leq W_S^{II}$, the non-negativity constraint is non-binding, i.e. $\lambda = 0$ and $v > 0$. In this case the individual rationality constraint is binding and, at optimum $v^{\mathcal{SC}} = y^{\mathcal{SC}}\left(\frac{x\left(1-y^{\mathcal{SC}}\right)(1-\beta)}{W_S + xy^{\mathcal{SC}}(1-\beta)} - 1\right) > 0$ where $y^{\mathcal{SC}}$ satisfies $y^{\mathcal{SC}}\left(1-\Phi^{\mathcal{SC}}\right) = \frac{W_S}{x}$. Finally, if $W_S < xm\left(1 - \frac{m(2-\beta)}{1-\beta(1-m)}\right)$ for m satisfying $\frac{((1-\beta)m)^{\beta-1}}{(\beta(1-m))^{\beta}} = x\left(\frac{m-(1-\beta)(1-2m)}{m}\right)^2$, then the $\mathcal{IR}$ constraint is non-binding (i.e. $\mu = 0$). Therefore, the optimum is given by $v^{\mathcal{SC}} = 2\frac{(1-\beta)\left(1-2y^{\mathcal{SC}}\right)}{2-\beta} > 0$ and $y^{\mathcal{SC}} = \frac{\Phi^{\mathcal{SC}}(1-\beta)}{2-\beta(1+\Phi^{\mathcal{SC}})}$. ■

Proof of Proposition 4: Suppose $W_S = \infty$ and $W_O < \infty$ and define $\Phi(y,v) \equiv \left[\sqrt{x(\beta(1-y-v))^{\beta}((1-\beta)y)^{1-\beta}}\right]^{-1}$ and $\rho(W_O,y,v) = \left[(xy(1-\beta))^{1-\beta}(1+\pi+W_O - x(1-y-v) - w - xv\rho)^{\beta}\right]^{\frac{-1}{2-\beta}}$.

The Operator's problem is to solve $\max_{w,y,v} E_X[\Pi_O(e_O^*, w, y, v, P(v|e_S^*, e_O^*), X)|e_S^*]$ subject to $\mathcal{IR}$, $w \geq 0$, $v \in [0,1]$, $y \in [0,1]$, $e_S^* = \arg\max_{e_S}\{E_X\{\Pi_S(e_S, w, y, X)|e_O^*\}\}$ and $e_O^* = \arg\max_{e_O}\{E_X[\Pi_O(e_O, w, y, v, P, X)|e_S^*]$ s.t. $\Pi_O(e_O, w, y, v, P, x) + W_O \geq 0\}$.

To simplify this program we can make the same argument as in the proof of Proposition 2, to show that $y^* \in (0,1)$, $v^* \in [0, 1-y)$ and that $\mathcal{IR}$ binds at optimum (i.e. $w^* = yxF(e_S^*, e_O^*) + e_S^*$). Second, note that because the Operator's effort is subject to the wealth constraint, then

$(e_S^*, e_O^*) = \begin{cases} (\tilde{e}_S, \tilde{e_O}) & \text{if } \Pi_O(\tilde{e_O}, w, y, v, P, x) + W_O \geq 0 \\ (e_S^0, e_O^0) & \text{otherwise} \end{cases}$, where $\tilde{e}_S = \Phi x(1-\beta) y - 1$, $\tilde{e_O} = \Phi x \beta (1 - y - v) - 1$, $e_S^0 = (1-\beta) xy\rho(W_O, y, v) - 1$ and $e_O^0 = \beta(1-y-v)\rho(W_O, y, v) - 1$.

Without loss of optimality, we can restrict our attention to the region where $\Pi_O(\tilde{e_O}, w, y, v, P, x) + W_O \geq 0$. To see this, note that, in order to satisfy the condition $\Pi_O(\tilde{e_O}, w, y, v, P, x) + W_O < 0$, then we need $y > y^0$, where $y^0 \equiv \frac{x(v+\beta(1-v))\rho-(\pi+W_O-(1-v)x+2)}{x(1-2\rho(1-\beta))}$; in this case, the operator earns profits $E_X[\Pi_O(y,v)] = x(1-y-v)(1-\rho(W_O,y,v)) - W_O$. From here, we can observe that $\frac{\partial E_X[\Pi_O(y,v)]}{\partial y} = -x\left((1-v-y)\frac{\partial \rho}{\partial y} + (1-\rho)\right)$, where $\frac{\partial \rho}{\partial y} = \frac{\rho}{y}\left(\frac{x\beta(y+\rho v)-(e_O^0+1)}{x\beta\rho(v+y)-(2-\beta)(e_O^0+1)+\beta(e_S^0+1)} - 1\right)$ and, therefore, that $\frac{\partial E_X[\Pi_O(y,v)]}{\partial y} < 0$ for any $y > y^0$. In other words, it is optimal for the operator to decrease $y \rightarrow y^0$. Hence, the maximizing contract parameters must satisfy $W_O + \Pi_O(\tilde{e_O}, w^*, y^*, P(v^*|\tilde{e_S}, \tilde{e_O}), v^*, x) \geq 0$.

To find the global optimum, we thus need to find $\max_{y,v}\left\{E_X\left[\tilde{\Pi}_O(\tilde{e}_O, w^*, y, v, P(v|\tilde{e}_S, \tilde{e_O}), X)\,|\tilde{e}_S\right]\right\}$ subject to the wealth constraint of the operator $W_O + \Pi_O(\tilde{e}_O, w, y, v, P(v|\tilde{e}_S, \tilde{e_O}), x) \geq 0$ and the non-negativity constraint $v \geq 0$.

Define W_O^I and W_O^{II} as in the proposition statement. If $W_O > W_O^I$ we can check that the solution to case $\mathcal{UC}$ (which is a relaxation of case $\mathcal{OC}$) satisfies the wealth constraint. This implies that, if $W_O > W_O^I$, then $v^{\mathcal{OC}} = 0$ and $y^{\mathcal{OC}} = y^{\mathcal{UC}}$. Now, assume that $W_O \leq W_O^I$, i.e. that the wealth constraint binds at optimum. If we let λ be the shadow price of the non-negativity constraint of v, then the KKT conditions for this problem are

$$\lambda = \frac{\left(\frac{2(1-\beta)}{\beta}(1-y-v)^2\right)((1-y)(1-\Phi)+\beta(1-v-2y))}{((v-1)(v+2y-1)\beta^2+(v(3-y)-2(v^2+y^2)+y-1)\beta+(v^2-v(1+y)+2y(1-y)))\Phi-2y(1-v-y)};$$

$$W_O + \pi - x(1-v) - \Phi x v - \Phi x y(1-\beta) - \Phi x(1-y)\beta + 2 = 0;\ v \geq 0;\ \lambda \geq 0; \text{ and } \lambda v = 0.$$

By looking at these conditions we can show that whenever $W_O \in [W_O^{II}, W_O^I)$, then the KKT conditions are only solved by setting and $v = 0$ and $y = y^{\mathcal{OC}}$, where $y^{\mathcal{OC}}$ satisfies $2 + W_O = x\Phi^{\mathcal{OC}}(2(1-\beta)+\beta) + x\left(1-y^{\mathcal{OC}}\right)$. Finally, when $W_O \leq W_O^{II}$ the KKT solutions are solved by setting $v^{\mathcal{OC}} = \left(\frac{1}{1-\beta}\right)\left(\frac{y^2}{2} - \frac{(1-y)(W_O+2-xy)(1-\beta)-x}{x\beta}\right)^{\frac{1}{2}} - \beta + 2y\left(\frac{3}{4}+\beta\right)$ and $y^{\mathcal{OC}}$ satisfying $\Phi^{\mathcal{OC}} = \frac{\beta\left(1-v^{\mathcal{OC}}-y^{\mathcal{OC}}\right)}{1-y^{\mathcal{OC}}} + 1.$■

Proof of Proposition 5: Suppose that $W_S \leq \infty$ and $W_O \leq \infty$. In this case, the operator's problem is to find $\max_{w,y,v} E_X[\Pi_O(e_O^*, w, y, v, P(v|e_S^*, e_O^*), X)\,|e_S^*]$ subject to $\mathcal{IR}$, $\mathcal{IC_S}$,$\mathcal{IC_O}$, $y \in [0,1]$ and $v \in [0, 1-y]$. By Propositions 3 and 4, we know that if $W_S > W_S^I$ and $W_O > W_O^I$ then the wealth constraint of neither party binds. As a result, the operator's problem reduces to the one analyzed in case $\mathcal{UC}$.

Suppose that $W_S \leq W_S^I$. By Proposition 3, we know that the wealth constraint of the supplier is binding. Moreover, we can verify that, in this sub-region, the wealth constraint of the operator is binding if and only if $W_O < \tilde{W}\left(y^{\mathcal{SC}}, v^{\mathcal{SC}}\right) - W_S$. If this constraint is non-binding, the operator's problem is identical to the one analyzed in case $\mathcal{SC}$. If, however, $W_O \leq \tilde{W}\left(y^{\mathcal{SC}}, v^{\mathcal{SC}}\right) - W_S$, the solution to the operator's problem can be found by solving for the wealth constraints of the operator and the supplier. This means that the optimal solution must ensure that $W_O + W_S = \tilde{W}\left(y^*, v^*\right)$ and $2+\pi+W_O = x\left(1-y^*-v^*\right) + x\Phi\left(y^*, v^*\right)\left(2y^* + v^* + \beta\left(1-2y^*-v^*\right)\right)$. We can make the same analysis for the case where $W_O \leq W_O^I$ to derive the remaining results in the proposition.■

Proof of Lemma 1: Suppose that the efforts of the parties are Stackelberg, with the supplier leading. In stage 2 the operator seeks to find $e_{O,seq}^* = \arg\max_{e_O \geq 0}\left\{E_X\left[\Pi_O\left(e_O, w, y, v, P, X|e_S\right)\right] : W_O + \Pi_O\left(e_O, w, y, v, P, x\right) \geq 0\right\}$ and the supplier, in turn, finds $e_{S,seq}^* = \arg\max_{e_S \geq 0}\left\{E_X\left[\Pi_S\left(e_S, w, y, P, X|e_{O,seq}^*\right)\right] : W_S + \Pi_S\left(e_S, w, y, x\right) \geq 0\right\}$. Using the same argument as in the proofs of Propositions 3 and 4, we know that the optimal contracting parameters will ensure that $W_S + \Pi_S\left(e_S, w, y, x\right) \geq 0$ and $W_O + \Pi_O\left(\tilde{e}_O, w, y, v, P, x\right) \geq 0$, where $\tilde{e}_{S,seq} = E_X\left[\Pi_S\left(e_S, w, y, P, X|e_{O,seq}^*\right)\right] = \sqrt{\frac{x(\beta(1-y-v))^{2-\beta}}{\left(\left(\frac{1-\beta}{1+\beta}\right)y\right)^{1-\beta}}} - 1$ and $\tilde{e}_{O,seq} = \arg\max_{e_S \geq 0}\{E_X\left[\Pi_O\left(\tilde{e}_O, w, y, v, P, X|e_S\right)\right]\} = \left(\frac{x\beta(1-y-v)}{(e_S+1)^{1-\beta}}\right)^{\frac{1}{\beta+1}} - 1$. By plugging $e_{S,seq}^*$ into the operator's effort function, we have that $e_{O,seq}^* = \tilde{e}_{O,seq}\left(e_{S,seq}^*\right) = \sqrt{\frac{x(\beta(1-y-v))^{2-\beta}}{\left(\left(\frac{1-\beta}{1+\beta}\right)y\right)^{1-\beta}}} - 1$. Therefore, $\frac{e_{O,seq}^*+1}{e_{S,seq}^*+1} = \frac{(1+\beta)\beta}{(1-\beta)}\left(\frac{(1-y-v)}{y}\right)$.

Doing an analogous derivation as above, we can verify that the ratio of efforts obtained in the base model (when the efforts are simultaneous) is equal to $\frac{e_O^*+1}{e_S^*+1} = \frac{\beta}{(1-\beta)}\frac{(1-y-v)}{y}$. The result follows directly.■

Proof of Proposition 6: Suppose that both the supplier and the operator have the option of purchasing insurance coverage, v_S and v_O. In this model, the supplier's and the operator's ex post profits are $\Pi_S\left(e_S, w, y, v_S, P_S, X\right) = T\left(w, y, X\right) - I_S\left(v_S, P_S, X\right) - e_S$ and $\Pi_O\left(e_O, w, y, v_O, P_O, X\right) = \pi - X - T\left(w, y, X\right) - I_O\left(v_O, P_O, X\right) - e_O$, where $I_S\left(v_S, P_S, X\right)$, $I_O\left(v_O, P_O, X\right)$ and $T\left(w, y, X\right)$ are defined as in Section 3. By using the same argument as in the proofs of Propositions 3 and 4, the best response levels of the supplier and the operator are given by $\tilde{e_S} = \arg\max_{e_S \geq 0}\left\{w - \frac{(y-v_S)x}{(e_S+1)^{1-\beta}(e_O+1)^{\beta}} - e_S\right\} = \left(\frac{x(y-v_S)(1-\beta)}{(e_O+1)^{\beta}}\right)^{\frac{1}{2-\beta}} - 1$, and $\tilde{e_O}|_{e_S} = \arg\max_{e_O \geq 0}\left\{\pi - w - \frac{x(1-y-v_O)}{(e_S+1)^{1-\beta}(e_O+1)^{\beta}} - P - e_O\right\} = \left(\frac{x\beta(1-y-v_O)}{(e_S+1)^{1-\beta}}\right)^{\frac{1}{\beta+1}} - 1$. By solving for the correspondences of these functions, we get that $e_O^*\left(y, v_S, v_O\right) = \sqrt{\frac{x(\beta(1-y-v_O))^{2-\beta}}{((1-\beta)(y-v_S))^{1-\beta}}} - 1$ and $e_S^*\left(y, v_S, v_O\right) =$

$\sqrt{\frac{x((y-v_S)(1-\beta))^{1+\beta}}{(\beta(1-y-v_O))^\beta}} - 1$. Now, note that the optimal level of coverage for the supplier are given by $v_S^* = \arg\max_{v_S \geq 0} E_X \left[\Pi_S \left(e_S^*, w, y, v_S, P_S, X \right) | e_O^* \right] = w - \frac{(y-v_S)x}{\left(e_O^*+1\right)^\beta \left(e_S^*+1\right)^{1-\beta}} - P\left(v_S | e_O^*, e_S^*\right) - e_S^*$. By plugging the expression of e_S^* and e_O^* in the expected profits of the supplier, and by performing basic derivations, we can verify that $v_S^* = \frac{\beta y}{(1+\beta)}$. To prove part 2 of the proposition, we just need to evaluate the ratio of efforts at the optimal insurance level for the supplier, i.e. $\frac{e_O^*\left(y, \frac{\beta y}{1+\beta}, v_O\right)}{e_S^*\left(y, \frac{\beta y}{1+\beta}, v_O\right)}$. ■