The Relational Advantages of Intermediation

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THE RELATIONAL ADVANTAGES OF INTERMEDIATION

ABSTRACT. This paper provides a novel explanation for the use of supply chain intermediaries such as Li & Fung Ltd.. We find that even in the absence of the well-known transactional and informational advantages of mediation, intermediaries improve supply chain performance. In particular, intermediaries facilitate responsive adaptation of the buyers’ supplier base to their changing needs while simultaneously ensuring that suppliers behave as if they had long-term sourcing commitments from buying firms. In the face of changing buyer needs, an intermediary that sources on behalf of multiple buyers can responsively change the composition of future business committed to a supplier such that a sufficient level of business comes from the buyer(s) that most prefer this supplier. On the other hand, direct buyers that source only for themselves must provide all their committed business to a supplier from their own sourcing needs, even if they no longer prefer this supplier. Unlike existing theories of intermediation, our theory better explains the observed phenomenon that while transactional barriers and information asymmetries have steadily decreased, the use of intermediaries has soared, even among large companies such as Walmart.

1. INTRODUCTION

This paper is inspired by the phenomenal growth of supply chain intermediaries that source products or services on behalf of other firms. These often completely take over the sourcing function– they select, verify and approve suppliers, they allocate business between different suppliers, and manage the relationship with each supplier, including provision of incentives for investments, performance and compliance.

A notable sourcing intermediary is Li & Fung Ltd., which provides sourcing services to major brands and retailers worldwide, including Walmart, Target, Zara and Levis. Li & Fung has grown at a compounded annual rate of 23% for the last 14 years to achieve annual sales of over HK$ 120 Billion. While best known for sourcing apparel and toys from the low-cost economies of Asia, the group today operates in an expanding range of categories. It is present in over 40 economies across North America, Europe and Asia, with a global sourcing network of nearly 15,000 international suppliers, as well as thousands of buyers. It has abilities to provide both low-cost and quick, responsive sourcing. Yet, Li & Fung does not own any means of production or transport, nor is it
in the business of directly retailing the vast majority of the products it sources. It provides only an interface between multiple buyers and suppliers (McFarlan et al. (2007)).

The benefits and costs that intermediaries bring to supply chains have long been studied by scholars in Finance, Economics and Supply Chain Management (cf. Wu (2004) for a comprehensive summary). Two main benefits are identified to justify the existence of intermediaries: transactional and informational benefits. Transactional benefits include the ability of intermediaries to provide immediacy by holding inventory or reserving capacity, and the benefits that arise out of the reduced costs of trade. Intermediaries that aggregate demand can use their scale for better utilization of facilities, amortization of fixed costs, reduction in the costs of searching and matching. Transactional benefits are most salient for smaller firms that do not individually possess the scale to justify fixed investments, and when the institutional barriers to trade are high. A second class of benefits arises from the informational role that intermediaries play. An intermediary's exposure to and better ability to synthesize dispersed information allows it to reduce information asymmetries, ensure better price discovery, and provide superior administration of contractual coordination mechanisms. Both these gains increase the efficiency of a supply chain, and the intermediary can appropriate some of these gains while sharing the rest with its supply chain partners. On the other hand, an additional tier in a supply chain is known to increase incentive misalignment, which can lead to insufficient stocking levels, poor information sharing and insufficient investments (Cachon and Lariviere (2005)).

Interestingly, with advancements in communication technologies and reductions in barriers to trade, many scholars have predicted a "flat world", in which global economic integration and democratizing technologies would render both the informational and transactional roles of intermediaries irrelevant. In particular, scholars have long hypothesized that one of the major business impacts of the internet would be the dis-intermediation of traditional entities (Wigand and Benjamin (1995); Friedman (2007)). Online platforms such as Alibaba.com have indeed rendered the traditional price discovery and matching roles of intermediaries irrelevant. The growth of intermediaries in the face of changes brought about by the internet and economic integration suggests that the conventional view on the advantages of intermediation may be incomplete.

Further, it is instructive to examine the firms that have decided to move away from direct sourcing to mediated sourcing. In January 2010, Walmart Inc. decided to enter into an open-ended sourcing arrangement with Li & Fung Ltd. (Cheng (2010)). The agreement delegated the sourcing of certain Walmart products to Li & Fung, which was expected to bring revenues in excess of US$2 Billion to Li & Fung. Many of Li & Fung's clients are similar large firms, such as Target, Gap, Benetton, etc. Existing theory on the role of intermediaries based on scale and informational advantages
seem less credible in explaining the move of big firms to adopt mediated sourcing. In particular, firms like Walmart arguably have more scale, similar market access, and local information than the intermediaries that they hire.\(^1\) An anecdotal analysis of the reasons provided by firms for employing sourcing intermediaries highlights two key themes. First, the ability of firms like Li & Fung to ensure better supplier collaboration, investments and compliance with quality, social and environmental norms is highlighted. Supplier investments in capacity and in ensuring compliance are cited as major business risks that are alleviated by intermediation. Second, it is argued that mediated sourcing allows firms to be more responsive in adapting their supplier base in the face of changes in the business environment such as supply chain disruptions brought about by adverse natural events, political upheaval, and volatility in the trade environment (energy costs, exchange rates, tariffs, etc.) (Fung et al. (2008); Loveman and O’Connell (1995); McFarlan et al. (2007)).

This paper provides a new, previously unidentified advantage of sourcing through intermediaries. We develop a stylized model to compare direct and mediated sourcing. Our model captures two key features of the sourcing environment: the fact that buyer’s preferences over suppliers change over time as the business environment changes, and the presence of incomplete contracts due to non-verifiability/non-contractability of supplier investments in capacity, quality or compliance with social, environmental norms, limited legal liabilities, etc. (Aghion and Holden (2011)).

Our analysis illustrates that an intermediary that pools the sourcing needs of different buyers is better than individual direct buyers at incentivizing beneficial supplier behavior and at responsively adjusting the buyers’ supplier base. With incomplete contracts that typify the sourcing of all but the simplest commodities, suppliers are typically incentivized by committing to provide future business contingent on performance. However, with changing preferences over suppliers, meeting these commitments may require sourcing from less-preferred suppliers. An intermediary that sources on behalf of multiple buyers breaks this trade-off by exploiting differences between different buyers’ preferences over suppliers. An intermediary can responsively change the composition of the committed business such that the level of business required to ensure desired supplier behavior comes as much as possible from the buyer(s) that most prefer this supplier. On the other hand, direct buyers, which source only for themselves, must provision all the committed business from their own sourcing needs, irrespective of what their preferences over suppliers may be. Sourcing for multiple buyers provides intermediaries with a certain flexibility in meeting the commitment to provide future business to a supplier—the flexibility of choosing which buyer to match to which supplier.

\(^1\)In 2011, Walmart’s annual revenues were US$421.85 billion, compared to Li & Fung’s US$15.96 Billion. Walmart also operates over 189 super-centers in China and employs over 50,000 local employees, making it one of the larger organized hypermarket chains in China. Source: Li & Fung Annual Report, 2010. Walmart 10-K filing, Q1 2011.
We demonstrate the existence and operation of this effect in a model with two buyers, two suppliers and an intermediary that allows for any generic game-theoretic interactions between buyers, suppliers and intermediaries that contribute to contractual incompleteness. We allow buyer preferences over suppliers to vary in an arbitrary, stochastic, non-stationary, heterogeneous fashion. Our analysis illustrates that the key to the existence of the highlighted advantage is a difference in buyer preferences over suppliers, at any given time. This difference could arise out of stochastic preferences over suppliers of ex-ante identical buyers or deterministic but non-stationary preferences of heterogeneous buyers.

Our analysis of mediated sourcing makes three key contributions: First, we provide a new explanation for the existence of intermediaries and their rapid growth. Second, to the best of our knowledge, this is the first paper in the supply chain literature that provides a generic, rigorous and highly adaptable foundation for analyzing incomplete contracts in a three-tier, multi-buyer, multi-supplier repeated-sourcing setting. Third, it contributes to the sourcing and procurement literature by bringing together the largely parallel literatures on operational flexibility (cf. Goyal and Netessine (2011)) and relational contracts (cf. Taylor and Plambeck (2007a)). Our analysis captures the changing preferences over suppliers, central to the operational flexibility literature and the incomplete contractability that drives results in the relational contracting literature. It illustrates the trade-off between the opposite sourcing strategies prescribed in the two streams and demonstrates how mediated sourcing breaks the trade-off.

2. Literature Review

Strategies for sourcing have been a central focus of recent research in supply chain management. Work on flexible sourcing to manage changing sourcing needs, and relational contracts to deal with contractual incompleteness are most relevant to our study.

Studies on Flexible Sourcing. Flexible sourcing or responsively sourcing from multiple suppliers has been suggested as a strategy to deal with the changing business environment. Kouvelis et al. (2004) demonstrates the exposure of global sourcing firms to risks arising out of subsidized financing, tariffs, regional trade rules and taxation. Allon and Van Mieghem (2010) and Lu and Van Mieghem (2009) study the choice between sole and dual sourcing strategies and consider the influence of changing logistics costs and trade barriers. Finally, Tomlin (2006) and Chod et al. (2010) examine the value of these flexible sourcing strategies under different contingencies. In line with this literature, our model allows for buyers to have changing preferences over suppliers and is agnostic to the source of these changing preferences, thus allowing us to address each of the reasons highlighted above.
Studies on Relational Contracting. This literature addresses the inefficiencies that arise due to the profit-relevant non-contractible actions of sourcing partners. This has been a central focus of micro-economics research for over three decades (cf. Aghion and Holden (2011) for a summary), and there is a growing body of operations literature that highlights the use of relational contracts as a remedy to these inefficiencies. Taylor and Plambeck (2007a,b) study settings where price and capacity are non-contractible. Debo and Sun (2004) study a setup where inventory levels are non-contractible. Plambeck and Taylor (2006) study joint production with unobservable utility-relevant actions. Ren et al. (2010) consider forecast sharing by a buyer in a setup where he has an incentive to inflate the forecasts. In each of these studies, building long-term relationships is presented as a mechanism for providing inter-temporal incentives that mitigate myopic opportunistic behavior. In line with this literature, the transaction step game of our model (introduced in Section 3.1) captures these non-contractible aspects of sourcing interactions. As in our treatment of changing buyer preferences over suppliers, rather than model any of the specific non-contractible actions studied in this literature, we consider a generic game that captures the key elements of each of the above settings.

Trade-off between Flexible Sourcing and Relational Contracting. Flexible sourcing and relational contracting are competing strategies. Tunca and Zenios (2006) consider the trade-off between relational contracts and flexible procurement auctions in a setting with multiple buyers and sellers. Swinney and Netessine (2009) look at the same trade-off when there is a possibility of supplier bankruptcy or default. Li and Debo (2005, 2009) illustrate the long-term shortcomings and benefits of committing to source from a single supplier when future sourcing options may change. Our study continues in the tradition of examining the trade-off between relational contracts and flexible sourcing, and we demonstrate the utility of mediated sourcing in relieving this trade-off. Sourcing intermediaries have never before been studied in this context.

3. Model Setup, Direct and Mediated Sourcing

3.1. Model Preliminaries. Consider two buyers, \( b_1 \) and \( b_2 \), that repeatedly source products or services available from two potential suppliers, \( s_1 \) and \( s_2 \). Each supplier has ample capacity and the capability to meet the sourcing needs of one or both buyers. We model the repeated trade between these buyers and suppliers as an infinitely repeated game— in each stage game, both buyers source the product. Buyers and suppliers discount future profits with a discount factor \( \delta \), which captures the time value of money and the probability of exit from the market. The sourcing exercise itself proceeds in three steps (Figure 3.1). First is the Information Gathering step, where the differences in the costs of sourcing from the two suppliers are revealed. Second is the Supplier Selection step,


where each buyer’s business is distributed amongst the two suppliers. Finally, the product or service is actually sourced in the Transaction step. These three steps constitute the stage game that is repeated in every period \( t \in \{0, 1, 2, \ldots\} \). We describe the three steps in detail below.

**The Information Gathering Step.** In this step, buyers acquire information about the prices, capabilities and performance of different suppliers to ascertain the advantage of one supplier over the other. This advantage could arise out of a match between the buyers’ product specifications and the suppliers’ idiosyncratic capabilities, or differences in exchange rates, transportation or telecommunication costs, cross-border tariffs, pass-through input costs, etc. To capture the dynamic business environment and the evolution of the buyers’ business, we allow this relative advantage to change stochastically from one sourcing period to another. In particular, at time \( t \), the profit of buyer \( i, i \in \{1, 2\} \) if he sources from supplier 1, includes an additive component, \( X^i_t \), the relative advantage of supplier 1 in supplying buyer \( i \), that is publicly drawn from a probability distribution function that has both positive and negative support and can be asymmetric. \( F^t \) denotes this joint bi-variate distribution of the relative advantage that supplier 1 has in supplying buyer 1 and 2. \( F^t_1 \) and \( F^t_2 \) are the partial densities. All else being equal, if the realization of \( X^i_t \) is positive, buyer \( i \)'s profits will be higher if he sources from supplier 1 than from supplier 2, and supplier 1 is the current preferred or, taking a total cost of ownership view, the “lower-cost” supplier. Note here that we make no assumptions on the stationarity of the buyers’ preferences over suppliers, nor do we assume that the buyers are symmetric. Our setup allows heterogeneous buyers’ preferences over suppliers to randomly and systematically vary over time, in both their direction and intensity, in an arbitrary fashion.

**The Supplier Selection Step.** In this stage, the sourcing business is allocated between the two suppliers. To facilitate clear illustration, we assume that the two buyer’s sourcing needs are comparable in dollar value, and without loss of generality, we normalize that value to one unit.\(^2\)

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\(^2\)A simple extension allows us to consider buyers with different sourcing budgets. All effects presented below continue to hold.
The Transaction Step. The actual sourcing of the product or service takes place in this step. Both the buyer and supplier can now undertake some actions that influence the profits of their sourcing partner. On the supplier side, these could include operational actions such as efforts in ensuring quality, timely delivery, conforming to technical and labor standards, following environmental and social norms, maintaining confidentiality of proprietary information, providing prompt after-sales service and support, etc. On the buyer’s side, these could include accurate sharing of demand information, timely payments, access to new business opportunities and capital, cross-investments, access to capital, training, technology transfer, recommendations, rewards, sanctions, etc.

We model all buyer-supplier interactions in the transaction step as a completely general finite two-player game that can capture any economic interactions during the sourcing stage between the buyer and supplier, including those mentioned above. We denote the extensive form of this generic game by \( \Gamma \). In game \( \Gamma \), the set of buyer and supplier feasible actions are denoted as \( A_b, A_s \subset \mathbb{R}^n \). The set of feasible action profiles is then given by \( A = A_s \times A_b \). Each element of set \( A \), \( a \), describes the actions undertaken by the two players in this game. On completion of game \( \Gamma \), the action profile \( a \) is perfectly and publicly observable. Buyer and supplier profits are given by general profit functions \( u_b, u_s: A \rightarrow \mathbb{R} \). We denote the Nash equilibrium of game \( \Gamma \) as \( a^N \in A \), associated with actions corresponding to “opportunistic behavior”, and we assume that it is unique and the payoff associated with it is inefficient. In particular, there exists an efficient outcome \( a^C \in A \), associated with “cooperative behavior”, that makes each player better off.

The above setup allows any number of sequential or simultaneous buyer or supplier actions, and the profits can be any arbitrary function of these actions. We consider situations where self-interested behavior and the consequent Nash equilibrium outcome are inefficient. The classic prisoner’s dilemma type game is a simple example of the game, \( \Gamma \). In the sourcing context, game \( \Gamma \) captures situations where incomplete contracts and incentive misalignment lead to a departure from first-best behavior. This departure could arise on account of poor performance on unobservable quality dimensions and the accompanying low prices (Tunca and Zenios (2006)), insufficient investments in unverifiable capacity (Taylor and Plambeck (2007b)), inefficiencies due to limited information sharing (Ren et al. (2010)), etc. Additionally, our setup also captures some key decentralization issues from the service outsourcing literature related to service quality, capacity building, utilization, etc. (Ren and Zhang (2009); Roels et al. (2010)).

Alternate Supply Chain Structures. We model and compare two alternate sourcing structures:
(1) Direct Sourcing: Each buyer sources directly from the suppliers. The buyers allocate business between suppliers, and each buyer acts independently in the transaction step.

(2) Mediated Sourcing: Both buyers source through a third party, the Intermediary. The intermediary allocates business and acts for both buyers in the transaction step.

Finally, note that in our setup there are no fixed investments, fixed order costs or other scale advantages, nor are there any information asymmetries or any benefits from information aggregation. Thus, the previously documented transactional and informational advantages of mediation do not exist in our setup. Based on existing theory, mediated sourcing should offer no advantage over direct sourcing. In fact, in the presence of incentive misalignment, one would a priori expect vertical integration and reduction of number of tiers to be superior due to limited incentive misalignment. In the next sections, we describe the game in each of the two setups, and we compare the equilibrium outcomes in Section 4. A formal, technical description of the games and the equilibria is provided in the Appendix.

### 3.2. Direct Sourcing.

In direct sourcing, the buyers act independently, and their choices can be analyzed in two identical but distinct games. We analyze buyer $i$’s game next.

#### The Stage Game at Time $t$

Figure 3.2 illustrates the stage game played between buyer $i$, and suppliers 1 and 2. First, the random cost advantage of supplier 1 in supplying buyer $i$, $X_i \sim F_i(x)$, is drawn. Next, buyer $i$ sources a fraction $\theta_i \in \{X_i\} \to [0,1]$, from supplier 1 and $1-\theta_i (X_i)$ from supplier 2. Finally, buyer $i$ and each of the suppliers play the transaction step subgame $\Gamma$. We denote the game involving buyer $i$ and supplier $j$ as $\Gamma_{ij}$, and the actions in this game are denoted as $a_{ij} \in A$. The stage game payoffs are:

$$u_{b_i} = \theta_i (X_i) \cdot (u_b(a_{i1}) + X_i) + (1-\theta_i (X_i)) \cdot u_b(a_{i2}),$$

$$u_{s_1} = \theta_i (X_i) \cdot u_s(a_{i1}), \quad u_{s_2} = (1-\theta_i (X_i)) \cdot u_s(a_{i2}).$$
The action profile $\alpha_{d_1}^*$, which prescribes setting $\theta_i^t(X_i^t)$ as $\tilde{\theta}_i^t \equiv I(X_i^t \geq 0)$ followed by actions $a_{i1}^t = a_{i2}^t = a^N$, is a subgame-perfect equilibrium of the direct sourcing stage game, where $I(\cdot)$, the indicator function, is 1 when the condition is satisfied (Appendix, Lemma 3).

**The Repeated Game.** In the repeated game, the stage game is played in each period $t \in \{0, 1, 2, \ldots\}$.

**Potential Equilibrium Strategies.** In each of the two transaction step games the players may play the cooperative or the Nash actions. Specifically, three kinds of behavior may arise in equilibrium: 1) the buyer and both suppliers always play the Nash actions, or 2) the buyer and one supplier (supplier 1 or supplier 2) play the cooperative actions in the transaction games that involve them, while the buyer and the other supplier play Nash actions in the transaction step game; or 3) the buyer and both suppliers always play the cooperative action. We call these the direct transactional $(d,T)$, the direct single relationship $(d,s_1$ or $d,s_2)$ and the direct dual relationship $(d,d)$ sourcing strategies, respectively. Note that in all three of these strategies, at any time, the buyer can choose freely to source from both the suppliers or just one of them. The difference lies in the choice of suppliers with which the buyer decides to play the cooperative outcome, or the supplier(s) with whom the buyer enters into a so-called long-term relationship (Taylor and Plambeck (2007b)).

Formally, $\forall k \in \{T, s_1, s_2, d\}$, strategy $\sigma^{d,k}(\theta_i)$, where $\theta_i$ is the sequence of the allocation functions $\theta_i \equiv \{\theta_i^t(X_i^t), t \geq 0\}$, prescribes the following play: if in all past play, the outcomes of the selection and transaction step actions prescribed below were observed, continue to play the corresponding selection and transaction step actions; else play action $\alpha_{d_1}^*$ (the stage game equilibrium) forever.\(^3\)

**Selection Step Actions:** At time $t$, the amount sourced from supplier 1 (the sourcing fraction) is given by the $t^{th}$ element of the sequence $\theta_i$, $\theta_i^t(X_i^t)$.

**Transaction Step Actions:** The prescribed actions are $(a^N, a^N)$ for strategy $d_iT$, $(a^C, a^N)$ for strategy $d_is_1$, $(a^N, a^C)$ for strategy $d_is_2$, and $(a^C, a^C)$ for strategy $d_id$, where the first element denotes the actions in the transaction game with supplier 1, and the second with supplier 2.

The present value of the expected normalized profit of player $n$, $n \in \{s_1, s_2, b_i\}$, under strategy $\sigma$ is given by

$$U_n(\sigma) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t E \left[ u_n^t(X_1^t, \theta_i^t(\sigma), a_{i1}^t(\sigma), a_{i2}^t(\sigma)) \right]. \quad (3.1)$$

Further, define operator $\varepsilon^t(\mathbf{u}_n) \equiv (1 - \delta) \sum_{t=0}^{\infty} \delta^{t-1} E \left[ u_n^t \right]$, where $\mathbf{u}_n \equiv \{u_n^t, t \geq 0\}$ and the expectation is taken over each $X_1^t$ and $X_2^t$ using $F^t$. Given a payoff stream, $\mathbf{u}_n$, the operator, $\varepsilon^t(\mathbf{u}_n)$,

\(^3\)These and all the other strategies proposed in this paper are Nash reversion trigger strategies, that is, on observation of a deviation from the equilibrium path, the Nash outcome is played in all future periods.
denotes the normalized expected present value of this payoff stream starting from period $t$. Applying Equation 3.1 to the four potential equilibrium strategies described above gives us the expected normalized discounted profits earned by following each of the strategies.

The buyers’ profits from any strategy depend on the degree of relational sourcing and the allocation of business amongst suppliers. In particular, all else being equal, the strategies with more relationships (dual $\rightarrow$ single $\rightarrow$ transactional) and strategies in which $\theta_i$ is chosen "responsively", i.e. after observing each $X_i$, element, $\theta_i$ is chosen to maximize the current period payoff, provide the highest profit. For dual relationships, this responsive $\theta_i$ is $\tilde{\theta}_i \equiv \left\{ \tilde{\theta}_i^t, t \geq 0 \right\}$, which dictates always sourcing everything from the lower-cost supplier. However, the ability to sustain the above strategy profiles as subgame-perfect equilibria of the repeated game depends on the incentives for the buyers and the suppliers to deviate from the strategy. The next Lemma provides restrictions on $\theta_i$ that ensure that the strategy is an equilibrium.

**Lemma 1. Equilibrium Outcomes of the Direct Sourcing Game.**

1. The strategy profile $\sigma^{d_i T} (\tilde{\theta}_i)$ is the only transactional subgame-perfect equilibrium of the repeated direct sourcing game.

2. The strategy profile $\sigma^{d_i k} (\theta_i)$ is a subgame-perfect equilibrium of the repeated direct sourcing game if and only if, for all $t \geq 0$ and all $X_i^t$, the difference between each player $i$’s, $n \in \{s_1, s_2, b_i\}$, expected normalized continuation profit from this strategy, $U_n \left( \sigma^{d_i k} (\theta_i) \right)$, exceeds profit from the above transactional equilibrium, $U_n \left( \sigma^{d_i T} (\tilde{\theta}_i) \right)$, by at least the values provided in the table below.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Buyer $i$</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^{d_i 1} (\theta_i)$</td>
<td>$\frac{1-\delta}{\delta} \max \left{ \bar{\theta}_i^t G_b, \left( \tilde{\theta}_i - \theta_i^t (X_i^t) \right) X_i^t - \eta \theta_i (X_i^t) \right}$</td>
<td>$\frac{1-\delta}{\delta} G_s \bar{\theta}_i^t$</td>
<td>$\frac{1-\delta}{\delta} G_s (1 - \bar{\theta}_i^t)$</td>
</tr>
<tr>
<td>$\sigma^{d_i 2} (\theta_i)$</td>
<td>$\frac{1-\delta}{\delta} \max \left{ (1 - \bar{\theta}_i^t) G_b, \left( \tilde{\theta}_i - \theta_i^t (X_i^t) \right) X_i^t - \eta (1 - \theta_i^t (X_i^t)) \right}$</td>
<td>$\frac{1-\delta}{\delta} G_s (1 - \bar{\theta}_i^t)$</td>
<td>$\frac{1-\delta}{\delta} G_s (1 - \bar{\theta}_i^t)$</td>
</tr>
<tr>
<td>$\sigma^{d_i d} (\theta_i)$</td>
<td>$\frac{1-\delta}{\delta} \max \left{ G_b, \left( \tilde{\theta}_i - \theta_i^t (X_i^t) \right) X_i^t - \eta \right}$</td>
<td>$\frac{1-\delta}{\delta} G_s \bar{\theta}_i^t$</td>
<td>$\frac{1-\delta}{\delta} G_s (1 - \bar{\theta}_i^t)$</td>
</tr>
</tbody>
</table>

$\bar{\theta}_i^t \equiv \max_{X_i^t} \theta_i^t (X_i^t)$, $\bar{\theta}_i^t \equiv \min_{X_i^t} \theta_i^t (X_i^t)$ are the maximum and minimum amount of business allocated to supplier 1 in any state; $G_s$ and $G_b$ denote the gain from the most profitable deviations of the supplier and buyer in the subgame $\Gamma$. This is the difference between the profit of the best-response action to the cooperative actions of the other player in game $\Gamma$, and the profit of the cooperative action. $\eta_b \equiv u_b (a^c) - u_b (a^s)$, $\eta_s \equiv u_s (a^c) - u_s (a^s)$ are the buyer’s and supplier’s gain from cooperation.

**Proof.** The formal proof is provided in the Appendix (Page 27), and the intuition follows. In direct transactional sourcing, player actions do not influence subsequent stages of the game. Thus, the stage game equilibrium, played in every period, is the subgame-perfect equilibrium of the repeated direct sourcing game. Sustaining the latter three relational strategy profiles as equilibrium outcomes
requires that the immediate gains from the most profitable deviation should be smaller than the loss in the continuation benefits. The loss in continuation benefits is given by the difference in the profits earned by following the relational strategy and the profits from the transactional sourcing strategy (recall that on observation of any deviation, all players resort to following the transactional strategy). The expressions in Part 2 of the theorem capture the immediate gains from the most profitable deviation. The most profitable deviation arises when the maximum amount of business is transacted with cooperative behavior ($\tilde{\theta}_1^t$ for supplier 1 and $1 - \tilde{\theta}_2^t$ for supplier 2). Further, for the buyer there is a deviation possible both in the selection step and in the transaction step. The more profitable deviation of these two defines the immediate gain of deviation for the buyer.

Recall that the buyer’s profits are highest in the dual relationship strategy with responsive allocation, $\theta_i = \tilde{\theta}_i$. However, to sustain any relationship and allocation in equilibrium, the buyer must restrict the allocation as per the conditions in Lemma 1, departing from the responsive allocation. This tension between responsive allocation and the provision of the incentives to sustain relationships (cooperative outcomes) is a key characteristic of sourcing that our model is designed to capture.

This tension is captured in Figure 3.3. For any given discount factor, the figure illustrates the highest equilibrium profits that can be achieved.\textsuperscript{4} Formally, as is typical in repeated game analysis and in statements of Folk Theorems (Mailath and Samuelson (2006)), this figure illustrates the achievable payoff region of the buyer as a function of the discount factor. For any $\delta$, this is:

\[
\max_{k \in \{T, s_1, s_2, d\}} \max_{\theta_i} U_b \left( \sigma_{d,k}^i (\theta_i) \right),
\]

s.t. strategy $\sigma_{d,k}^i (\theta_i)$ is an equilibrium.

\textsuperscript{4}As is typical in repeated games, we express our equilibrium conditions in terms of the discount factor. However, these conditions can equally be interpreted as conditions on all exogenous parameters: the distribution, $F^t (x)$, the general profit functions, $u_b$ and $u_s$, the gains from deviation $G_b$ and $G_s$, and the benefits from cooperation $\eta_b, \eta_s$. 

**Figure 3.3.** Buyer i’s Achievable Payoff Region
where the optimization is over all equilibrium strategies. The first maximum refers to the kind of strategy and the second to the allocation function sequence in that strategy. Two characteristics of the equilibrium conditions in Lemma 1 help us to understand this achievable payoff region. First, the equilibrium conditions for dual relationship strategies are more restrictive than the conditions for single relationship strategies (in dual relationship, sufficient incentives need to be provided to the two suppliers, while in single relationship only to one). Second, for both dual or single relationships, the trade-off between responsive allocation and the provision of sufficient business to maintain relationship(s) is more restrictive as the discount factor is smaller and the suppliers value future business less, thus requiring larger and larger departures from the responsive allocation to sustain relationships.

For the highest values of \( \delta \), region \((v)\) in Figure 3.3, future business is valued highly by suppliers and the buyer can potentially maintain relationships with both suppliers while also allocating business responsively. Put differently, in this region, \( \delta \) is high enough that the equilibrium conditions for even dual relationships are not binding. However, as \( \delta \) gets smaller, the conditions become binding, and the buyer must now sacrifice the responsive allocation to maintain the two relationships and this decreases his profits (region \((iv)\)). Next, at some point, the equilibrium conditions become so tight that no allocation can satisfy the dual relationship equilibria conditions, but single relationship equilibria may be sustained, first with responsive allocation and then potentially with a non-responsive or, restricted allocation (regions \((iii)\) and \((ii)\)).\(^5\) Eventually, only transactional sourcing can be sustained as an equilibria, region \((i)\). In subsequent sections, we will illustrate how the tradeoffs shown in Figure 3.3 change with mediated sourcing.

3.3. Mediated Sourcing. With mediated sourcing, both buyers delegate their supplier selection and their transaction step actions to a third party, the intermediary. The intermediary chooses the supplier for each buyer and acts on behalf of buyers in the transaction step. In lieu of the sourcing services provided by the intermediary, the buyers pay the intermediary an agreed upon commission. Specifically, the intermediary gets a fraction, \( \beta \), of the total buyer-side profits. This fraction \( \beta \) could arise as a function of a bargaining process prior to signing up for the intermediary’s services or by any other mechanism that divides the total profits generated.

In the setup described above, buyers do not have any profit-relevant actions after they have signed up for the intermediary’s services. As such, they are no longer relevant players in the mediated sourcing game. In essence, the mediated sourcing game follows along exactly the same lines as the

\(^5\)Note here that in the illustration, we show the single relationship equilibria with one supplier. With heterogeneous suppliers it is possible that we may have two single relationship regimes, one for each of the suppliers.
two direct sourcing games, except that the actions of the two individual buyers are now taken by
one intermediary. In all other respects, the two described structures are identical.

The Stage Game at Time $t$. First, in the information gathering step, the differences in sourcing
from different suppliers are revealed, i.e. $X^t_1$ and $X^t_2$ are drawn from joint distribution $F^t(X^t_1, X^t_2)$. Next, the intermediary allocates a fraction $\nu^t_i : \{X^t_1, X^t_2\} \rightarrow [0,1]$ of buyer $i$’s sourcing business to supplier 1, $i \in \{1, 2\}$. Note that the allocations $\nu^t_i (X^t_1, X^t_2)$ correspond to the allocations $\theta^t_i (X^t_i)$ from direct sourcing, but now the allocations are a function of the relative cost advantage of supplier 1 in supplying both buyer 1 and 2, $X^t_1$ and $X^t_2$. Put differently, the intermediary takes into account both buyers’ preferences for a supplier in the sourcing decision. We denote the allocations of buyer 1 and buyer 2’s business to supplier 1, $(\nu^t_1, \nu^t_2)$ as $\nu^t$, and the total business to supplier 1, $\nu^t_1 + \nu^t_2$, is denoted by $\langle \nu^t \rangle$.

Finally, actual sourcing takes place in the transaction step, and the intermediary and the suppliers play transaction games $\Gamma$. The games are identical to the ones that buyers play in direct sourcing, except that buyers are replaced by the intermediary. We denote the game between the intermediary and supplier $j$ as $\Gamma_{ij}$ and the actions in this game as $a^t_{ij}$. Finally, the suppliers, the buyers and the intermediary earn their profits. The profits are given as

$$u^t_i = \beta \sum_{i=1}^2 (\nu^t_i (X^t_1, X^t_2) \cdot (u_b(a^t_{i1}) + X^t_i) + (1 - \nu^t_i (X^t_1, X^t_2)) \cdot u_b(a^t_{i2})),$$

$$u^t_{s1} = \langle \nu^t (X^t_1, X^t_2) \rangle \cdot u_s(a^t_{i1}), \quad u^t_{s2} = (2 - \langle \nu^t (X^t_1, X^t_2) \rangle) \cdot u_s(a^t_{i2}).$$

The action profile $\alpha^*_m$, that prescribes $\nu^t = \nu^t \equiv \langle \theta^t_1, \theta^t_2 \rangle$ followed by actions $a^t_{i1} = a^t_{i2} = a^N$ is a subgame-perfect equilibrium of the mediated sourcing stage game (Appendix, Lemma 4).

The Repeated Game. In the repeated game, the stage game is played in each period $t \in \{0, 1, 2, \ldots\}$.

Potential Equilibrium Strategies. Supplier selection is now a function of the realization of both the relative cost advantages, $X^t_1$ and $X^t_2$. With respect to transaction step actions, the choices follow along the same lines as those in direct sourcing. Specifically, the intermediary and the chosen supplier(s) may play Nash actions in all games, or the intermediary and one supplier may play cooperative actions in transaction games that involve them and Nash actions in the transaction games that involve the other supplier, or the intermediary and each supplier may always play the cooperative action. We call these the mediated transactional ($mT$), single relationship ($ms_1$ or $ms_2$), and dual relationship ($md$) sourcing strategies, respectively.

---

$\nu^t_1, \nu^t_2, \langle \nu^t \rangle$ and $\nu^t$ are all functions of $X^t_1$ and $X^t_2$, but we often suppress the arguments in subsequent discussion.
Lemma 2. Strategy profile $\nu$ As before, the profits are highest with dual relationship strategies, when given by

$$U_n(\sigma) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t E [u_n^t(\sigma)].$$

As before, the profits are highest with dual relationship strategies, when $\nu$ is chosen responsively, $\nu = \tilde{\nu} \equiv \{\tilde{\nu}^t(X_1^t, X_2^t), t \geq 0\}$. Next we provide the necessary and sufficient conditions to sustain a strategy profile $\sigma^{mk}(\nu)$ as an equilibrium.

**Equilibrium Outcomes of the Mediated Sourcing Game**

1. The strategy profile $\sigma^{mT}(\tilde{\nu})$ is the only transactional subgame-perfect equilibrium of the mediated sourcing game.

2. The strategy profile $\sigma^{mk}(\nu)$ is a subgame-perfect equilibrium of the repeated mediated sourcing game if and only if, for all $t \geq 0$ and for all $X_1^t$ and $X_2^t$, the difference between each player's $n$'s, $n \in \{s_1, s_2, I\}$, expected normalized continuation profit from this strategy, $U_n(\sigma^{mk}(\nu))$, exceeds profit from the above transactional equilibrium, $U_n(\sigma^{mT}(\tilde{\nu}))$, by at least the values provided in the table below.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Intermediary</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^{m_1}(\nu)$</td>
<td>$\frac{1 - \delta}{\delta} \max \left{ \langle \tilde{\nu}^t \rangle G_{\nu}, \sum_{i=1}^{2} \left( \bar{\theta}^i - \nu^i \right) X_1^i - \eta_i \nu^i \right}$</td>
<td>$\frac{1 - \delta}{\delta} G_s(\langle \tilde{\nu}^t \rangle)$</td>
<td>$\frac{1 - \delta}{\delta} G_s(\langle \tilde{\nu}^t \rangle)$</td>
</tr>
<tr>
<td>$\sigma^{m_2}(\nu)$</td>
<td>$\frac{1 - \delta}{\delta} \max \left{ (2 - \langle \tilde{\nu}^t \rangle) G_{\nu}, \sum_{i=1}^{2} \left( \bar{\theta}^i - \nu^i \right) X_1^i - \eta_i (1 - \nu^i) \right}$</td>
<td>$\frac{1 - \delta}{\delta} G_s(2 - \langle \tilde{\nu}^t \rangle)$</td>
<td>$\frac{1 - \delta}{\delta} G_s(2 - \langle \tilde{\nu}^t \rangle)$</td>
</tr>
<tr>
<td>$\sigma^{md}(\nu)$</td>
<td>$\frac{1 - \delta}{\delta} \max \left{ 2G_{\nu}, \sum_{i=1}^{2} \left( \bar{\theta}^i - \nu^i \right) X_1^i - \eta_i \right}$</td>
<td>$\frac{1 - \delta}{\delta} G_s(\langle \tilde{\nu}^t \rangle)$</td>
<td>$\frac{1 - \delta}{\delta} G_s(2 - \langle \tilde{\nu}^t \rangle)$</td>
</tr>
</tbody>
</table>

$\langle \tilde{\nu}^t \rangle \equiv \max_{X_1^t, X_2^t} \langle \tilde{\nu}^t \rangle$ and $\langle \tilde{\nu}^t \rangle \equiv \min_{X_1^t, X_2^t} \langle \tilde{\nu}^t \rangle$ are the maximum and minimum amount of business allocated to supplier 1 in any state. $G_s$ ($G_b$) denotes the gain from the most profitable deviation (defined as before).
Proof. A formal proof is provided in the Appendix (Page 29). Note here that the equilibria that can be sustained in mediated sourcing (and all future results) do not depend on the specific split of profits, $\beta$.

Like the direct buyers, the intermediary acting on behalf of the two buyers in mediated sourcing faces a trade-off. Profits are increased by establishing relationships and responsive allocation, but the intermediary may need to restrict his business allocation to sustain relationship(s) in equilibrium (Lemma 2). Further, as before, dual relationships are harder to sustain than single relationships, and all relationships are harder with lower values of the discount factor. Thus, the achievable payoff has a similar shape to the one illustrated for direct sourcing in Figure 3.3. However, there is one difference between this trade-off for mediated sourcing and direct sourcing. Rather than an individual buyer sourcing for himself, the intermediary is now sourcing on behalf of both buyers. This implies that the intermediary’s allocation of business to the two suppliers is based on business accruing from the two buyers and his total costs are a function of both $X_t^1$ and $X_t^2$, i.e. the relative cost difference between suppliers in supplying both buyers 1 and 2. In the next section, we will see how this drives the advantages and disadvantages of mediated sourcing.

4. The “Benefits” of Intermediation

Consider the total buyer-side surplus or the “sourcing profits”, $\pi$: in the case of direct sourcing, this is the sum of the two buyers’ profits. In the case of mediated sourcing, it is the sum of the buyers’ and the intermediary’s profits. If the buyer-side surplus is higher for the mediated sourcing strategy, then there exists a surplus division factor $\beta$ such that both buyers and the intermediary are better off under mediated sourcing. Thus, to compare direct and mediated sourcing it is sufficient to compare the respective achievable sourcing profits. For each set of parameter values, the supply chain structure (direct or mediated sourcing) that achieves the higher sourcing profits is the preferred supply chain structure. Note that using sourcing profits for comparing strategies also brings scale parity between direct and mediated sourcing— in both cases, we are comparing the profits from sourcing two units from the suppliers.

Recall that the achievable sourcing profit regions were obtained by choosing the highest profit strategy that is also an equilibrium for a given set of parameter values. For both direct and mediated sourcing, the strategy space can be characterized by the type of relationship(s) (transactional, single or dual relationship ($k \in \{T, s_1, s_2, d\}$) and the allocation of business between suppliers (choice of $\theta_i/\nu$). Thus, to find the highest profit strategy that is an equilibrium, we need to consider the
choice of relationship type and the choice of business allocation. To build our intuition, we first consider the highest equilibrium sourcing profit for a given type of relationship.

**Definition.** ∀δ, i and k, define \( \pi^{d,k}(\delta) = \max_{\theta} \pi^{d,k}(\theta) \), such that strategy \( \sigma^{d,k}(\theta) \) is an equilibrium of the direct sourcing game for this \( \delta \). Similarly, define \( \pi^{m,k}(\delta) = \max_{\nu} \pi^{m,k}(\nu) \), such that strategy \( \sigma^{m,k}(\nu) \) is the equilibrium of the mediated sourcing game for this \( \delta \). For any given type of relationship \( k \), \( \pi^{d,k}(\delta) \) and \( \pi^{m,k}(\delta) \) are the highest sourcing profits that are achievable as equilibria, considering all different possible allocations of business.

4.1. Ability to Sustain Relationships. The next theorem compares the ability of direct and mediated sourcing in sustaining a given type of relationship.

**Theorem 1.** ∀δ, and for each type of relationship, \( k \in \{s_1, s_2, d\} \), sourcing through an intermediary earns higher sourcing profits than both buyers sourcing directly with the same relationship:

\[
\forall \delta, k \quad \pi^{m,k}(\delta) \geq \pi^{d,k}(\delta) + \pi^{d,k}(\delta),
\]

with strict inequality for some \( \delta \).

**Proof.** A formal proof is provided in the Appendix (Page 29).

**Sketch of the Proof:** For any given type of relationship \( k \), we can write the best sourcing profit for direct sourcing as

\[
\pi^{d,k}(\delta) + \pi^{d,k}(\delta) = \max_{\theta_1, \theta_2} \mathcal{E}^0 \left( \Pi^k(\theta_1, \theta_2) \right),
\]

s.t. \( \theta_1, \theta_2 \in \mathcal{D}^k(\theta_1, \theta_2) \),

where the set \( \mathcal{D}^k \) denotes the feasible set defined by the equilibrium conditions for direct sourcing strategy \( k \) and, as before, the optimization is over a sequence of functions \( \theta_t \equiv \{\theta^t_t(X^t_i)\}, t \geq 0 \).

Interestingly, the mediated sourcing profit, \( \pi^{m,k} \), can be written with exactly the same objective function, but with a different feasible set, \( \mathcal{M}^k \):

\[
\pi^{m,k}(\delta) = \max_{\nu_1, \nu_2} \mathcal{E}^0 \left( \Pi^k(\nu_1, \nu_2) \right),
\]

s.t. \( \nu_1, \nu_2 \in \mathcal{M}^k(\nu_1, \nu_2) \).

This suggests that the difference between mediated and direct sourcing can be understood by examining the set of equilibrium conditions, \( \mathcal{D}^k(\theta_1, \theta_2) \) and \( \mathcal{M}^k(\nu_1, \nu_2) \). It is most instructive to

---

7 The inequality is strict for \( \delta < \delta^k \); \( \delta^k \) is the smallest \( \delta \) such that strategy \( k \) with responsive allocation is an equilibrium.

8 \( \Pi^k(x, y) = \{\Pi^{k,t}(x', y'), t \geq 0\}, \Pi^{k,t}(x', y') \equiv (x' + y')u_b(a_b^k) + x'x^t_i + y'x^t_j + (2 - (x' + y'))u_b(a_{b_2}^k), a_{i}^k \) denotes \( a_{i}^k (\sigma^k) \) or \( a_{b_2}^k (\sigma^k) \) or \( a_{i}^k (\sigma^k) \) depending on the context.
compare the conditions that come from the incentives of suppliers, for example, consider supplier 1’s incentives.\(^9\)

<table>
<thead>
<tr>
<th>Direct, (\mathcal{D}(\theta_1, \theta_2))</th>
<th>Mediated, (\mathcal{M}(\nu_1, \nu_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D.1) (\theta^{t+1}_1 - d\theta^t_1 \geq \gamma^t_1)</td>
<td>(M) (\nu^{t+1}_1 + \nu^{t+1}_2 - d(\nu^t_1 + \nu^t_2) \geq \gamma^t_1 + \gamma^t_2)</td>
</tr>
<tr>
<td>(D.2) (\theta^{t+1}_2 - d\theta^t_2 \geq \gamma^t_2)</td>
<td></td>
</tr>
</tbody>
</table>

where \(\gamma^t_i \equiv \theta^{t+1}(1 - F_i(0))\) \(u_s(a^{\mathcal{N}})\), \(d \equiv \frac{1 - \delta}{\delta} G_s(u_s(a^{\mathcal{C}}))\), \(1 - F_i(x) \equiv \{1 - F_i(x), t \geq 0\}\).

Sourcing directly, buyers 1 and 2 must each individually ensure that their stream of orders, \(\theta_1\) or \(\theta_2\), is such that the supplier has an incentive to continue the relationship. Conditions (D.1) and (D.2) reflect this. In mediated sourcing, on the other hand, the intermediary must only ensure that the combined stream of orders on behalf of buyer 1 and 2, \(\nu_1 + \nu_2\), is such that the supplier has an incentive to continue the relationship. Condition (M) reflects this. Essentially, the condition for maintaining a mediated relationship is the sum of the conditions for maintaining equivalent direct relationships. Thus, the equilibrium conditions for direct relationships are a subset of the conditions for mediated relationships, and mediated sourcing always (weakly) outperforms direct sourcing for a given relationship. Also, note by looking at the RHS of the above equations that the combined stream of orders, while potentially larger than any single buyer’s order stream, must also cross a higher threshold. Put differently, the intermediary does indeed have more scale than any individual buyer, but this scale cuts both ways, providing more incentives to stay in the relationship, but also proportionally more gains from cheating or deviating from the relationship. Thus, the advantage of mediated sourcing that drives the above result is not simply arising from the greater scale of an intermediary.

To better understand the above effect, consider the following three cases:

**Case I:** The discount factor is high enough that neither of the constraints, (D.1) or (D.2), are binding. Now, direct buyers can choose the responsive allocation stream and achieve the highest profits. In such a setup, we also show that the constraint (M) will not be binding, and mediated sourcing will earn the same profits. Thus, direct and mediated sourcing perform equally well.

**Case II:** Next, consider the case where the discount factor is a bit lower, and one of the two constraints, (D.1) or (D.2), becomes binding while the other has some slack. This happens when the buyers are heterogeneous in their “long-run preferences” over suppliers, i.e. \(\gamma^t_1 \neq \gamma^t_2\) or equivalently \(\theta^{t+1}_1 \neq \theta^{t+1}_2\), i.e. the discounted probability that buyer 1 prefers supplier 1 is not equal to the discounted probability that buyer 2 prefers supplier 1. For example, when buyer 1 in the long-run prefers supplier 1 more than buyer 2, \(\exists \delta\), where constraint (D.1) is not binding and (D.2) is binding.

\(^9\)Supplier 2’s incentives, if applicable (i.e. if \(k = d \text{ or } s_2\)), follow along the same lines.
Now while one order stream, $\theta_2$, is constrained in a specific fashion, the other, $\theta_1$, is not constrained and can be set to the responsive order stream– the unconstrained optimal. In the case of mediated sourcing, the only constraint, constraint M, is the sum of the constraints D.1 and D.2, and as a result, it is not binding and the order streams on behalf of buyer 1 and the order stream on behalf of buyer 2 both can be set to their responsive or unconstrained maximization values. Essentially, if one buyer long-run prefers a supplier more than the other buyer, mediated sourcing makes it possible to use this buyer’s bias to compensate for the other buyer’s weaker interest. In direct sourcing, the buyer that prefers the particular supplier would find it in its interest to provide more business than strictly necessary, whereas the other buyer would be forced to provide more business than it wants to just to sustain the relationship. Pooling the order streams eliminates this inefficient situation, and the level of business accruing to the supplier on behalf of both buyers can be adjusted to the minimum level sufficient for sustaining the relationship, achieving a responsive allocation. In this fashion, an intermediary can exploit the differences between buyers in their long-run preferences over suppliers to outperform direct sourcing.

Case III: Finally, consider a case where the discount factor is such that both constraints, (D.1) and (D.2), are binding. This arises for low enough $\delta$ or when the buyers are symmetric. Now, constraint M will also be binding, but the order streams in mediated sourcing will still earn higher profits by being more responsive. Say in direct sourcing, the constrained optimal order streams are $\theta_1^*$ and $\theta_2^*$. Now construct order streams, $\nu_1$ and $\nu_2$ as follows: when $X_{1t}^i \geq X_{1t}^i$,\(^{10}\) set $\nu_1^t (X_{1t}^1, X_{1t}^2) = \min \{1, \theta_1^t (X_{1t}^1) + \theta_2^t (X_{1t}^2)\}$ and $\nu_2^t (X_{1t}^1, X_{1t}^2) = \theta_1^t (X_{1t}^1) + \theta_2^t (X_{1t}^2) - \min \{1, \theta_1^t (X_{1t}^1) + \theta_2^t (X_{1t}^2)\}$. Hence, by construction, $\forall X_{1t}^1, X_{1t}^2 \nu_1^t + \nu_2^t = \theta_1^t + \theta_2^t$. This order stream is constructed such that from the supplier’s point of view, the orders coming from the two separate buyers or from the intermediary are identical. However, the intermediary can better adapt the composition of the orders to the current realization of the relative cost advantages. In particular, the intermediary ensures that whatever quantity of orders must be sent to the supplier, its composition is such that to the maximum possible degree, it is composed of orders on behalf of the buyer who has a cost advantage of sourcing from this supplier in this sourcing period. Again, the intermediary uses one buyer’s stronger preference for a supplier, $X_{1t}^i \geq X_{1t}^i$, to compensate for the other buyer’s weaker preference. However, this time the difference in preference arises out of the random draws of the relative cost advantage, $X_{1t}^i \neq X_{1t}^i$, or what we call myopic preferences. Thus, an intermediary can exploit the myopic bias of one buyer for a supplier to ensure that the allocation of business is such that the composition of the business allocated to the suppliers is the most advantageous. On the

\(^{10}\)For $i \in \{1, 2\}$, $\bar{i} = 3 - i$ (the other buyer)
other hand, direct buyers do not have the flexibility to change the composition of the orders going to a supplier, and thus, they often end up with a suboptimal composition of orders.

To summarize, mediated sourcing performs better than direct sourcing by adjusting the level of sourcing business allocated to a supplier when the buyers have heterogeneous long-run preferences over suppliers, or by responsively adjusting the composition of sourcing business allocated to a supplier when the buyer’s have different myopic preferences over suppliers. Essentially, with heterogeneous long-run preferences over suppliers, one buyer wants to allocate more business than necessary to ensure cooperative behavior, whereas the other may want to allocate less business than necessary. An intermediary that pools the order streams from both buyers can use one buyer’s above-requirement allocation to compensate for the other buyer’s below-requirement business. Similarly, with different myopic preferences, the supplier can be provided the same incentives for cooperative behavior as in direct sourcing but the composition of that business can be adjusted responsively.

**Corollary. Relationship between Buyers’ Preferences over Suppliers**

1. **Perfectly Correlated Preferences:** If \( \forall t, X_1^t = \alpha X_2^t \),
   
   (a) If \( \alpha = 1 \), \( \forall \delta, k \) mediated sourcing has no advantage over direct sourcing:
   
   \[ \pi^{mk} (\delta) = \pi^{d_1 k} (\delta) + \pi^{d_2 k} (\delta). \]

   (b) If \( \alpha \neq 1 \), \( \forall \delta, k \) mediated sourcing is better at maintaining a given relationship than the direct buyers: \( \pi^{mk} (\delta) \geq \pi^{d_1 k} (\delta) + \pi^{d_2 k} (\delta) \), with strict inequality for some \( \delta \).

2. **Identically Distributed Preferences:** If \( X_1^t, X_2^t \sim F^t (x) \), \( \forall \delta, k \) mediated sourcing is better at maintaining a given relationship than the direct buyers: \( \pi^{mk} (\delta) \geq \pi^{d_1 k} (\delta) + \pi^{d_2 k} (\delta) \), with strict inequality for some \( \delta \).

3. **Deterministic Preferences:** If \( X_1^t = x_i \), when \( t = 2T \), and \( X_1^t = -x_i \), when \( t = 2T + 1 \), where \( T \in \{0, 1, 2, \ldots\} \),
   
   (a) If \( x_1 = x_2 \), \( \forall \delta, k \) mediated sourcing has no advantage over direct sourcing:
   
   \[ \pi^{mk} (\delta) = \pi^{d_1 k} (\delta) + \pi^{d_2 k} (\delta). \]

   (b) If \( x_1 \neq x_2 \), \( \forall \delta, k \) mediated sourcing is better at maintaining a given relationship than the direct buyers: \( \pi^{mk} (\delta) \geq \pi^{d_1 k} (\delta) + \pi^{d_2 k} (\delta) \), with strict inequality for some \( \delta \).

If \( \alpha = 1 \), the realizations of each buyer’s preferences over suppliers will always be identical and there are no benefits from changing the level or composition of orders to a supplier. However, even if the draws are perfectly correlated, with \( \alpha \neq 1 \), the two draws will be different and the intermediary can exploit the difference. Further, if buyer preferences are identically distributed, or if on average both buyers prefer the same supplier, there are no long-run differences between
buyer preferences, \( \forall t, \gamma_1^t = \gamma_2^t \), but in each period there is still a chance that the realizations of each buyer’s preferences over suppliers will be different, \( \Pr \{ X_1^t \neq X_2^t \} > 0 \), and the intermediary can exploit myopic differences as described above. Finally, if there is no risk involved, that is the shocks are deterministic, but there is still a difference in the buyers’ preferences over suppliers in every period \( x_1 \neq x_2 \), the intermediary can continue to exploit the resultant differences in myopic and long-run preferences as described above. The above corollary starkly demonstrates that the effects highlighted above accrue from differences in buyer preferences over suppliers. These could arise from myopic differences in preferences over suppliers and/or from systematic or long-run heterogeneity in preferences over suppliers— but as long as there is a possibility that the realized preferences of buyers over suppliers are different at some point in time, mediated sourcing can better maintain relationships. This illustrates that our argument extends beyond the pooling of randomness in preferences to the pooling of random, systematic and temporal differences in preferences.

4.2. The Preferred Supply Chain Structure. In the above section, we illustrated how intermediaries are better at maintaining any given relationship. However, the choice of the preferred supply chain structure depends on the achievable sourcing profits that take into account both the ability to maintain a given relationship and the choice of which relationship to maintain. In this section, we consider both of these effects and identify the preferred supply chain structures.

For any \( \delta \), the best achievable sourcing profit in direct sourcing, \( \pi^d(\delta) \), is \( \max_k \pi_{d1k}(\delta) + \max_k \pi_{d2k}(\delta) \); in mediated sourcing it is \( \pi^m(\delta) = \max_k \pi_{mk}(\delta) \), where \( k \in \{ T, s_1, s_2, d \} \).

**Theorem 2.** Mediated sourcing outperforms direct sourcing, i.e. \( \forall \delta \pi^m(\delta) \geq \pi^d(\delta) \), with strict inequality for some \( \delta \), if the same strategy \( k \) is the solution to both \( \max_k \pi_{d1k}(\delta) \) and \( \max_k \pi_{d2k}(\delta) \). This condition always holds when the buyers are ex-ante symmetric in their preferences over suppliers i.e. \( \forall t, F_1^t = F_2^t \).

**Proof.** The formal proof is provided in the Appendix (Page 29). \( \square \)

Figure 4.1 illustrates the comparison between direct and mediated sourcing as described in Theorem 2. For the highest values of the discount factor, (region (v)) in both direct and mediated sourcing, the firms can achieve first-best profit, since the responsive allocation stream satisfies the dual relationship equilibrium conditions. For lower values, (region (iv)) one of the buyers’ responsive allocation streams is no longer sufficient for sustaining the dual relationship. In direct sourcing, this buyer must now shift to a less responsive allocation stream, but the intermediary can use the slack in the other buyer’s responsive allocation to still satisfy the supplier (long-run differences). Although, for even lower discount factors (region (iii)), both buyer’s responsive allocation streams may now
be insufficient for the supplier(s), mediated sourcing can still exploit the changing preferences that lead to myopic differences to earn higher sourcing profits. For even lower values of the discount factor, the same effects repeat for single relationships (region (ii), (ij)).

The above result highlights that if the same relationship structure is used by the two direct buyers, the intermediary will be able to better maintain that relationship. However, it is possible that the two direct buyers may prefer to maintain relationships with different sets of cooperative suppliers. In such cases, the intermediary will have to choose one of the two sets of cooperative suppliers or relationship structures, whereas the direct buyers can each choose their preferred relationship structure. Thus, direct sourcing may perform better, as direct buyers have more selectivity in choosing their relationships; in particular, they are not obliged to each have the same set of relationships, as is the case when an intermediary acts on their behalf. For example, in direct sourcing with a single relationship, each buyer must choose supplier 1 or 2 as the cooperative supplier. This can be the same supplier for both buyers or a different supplier for each buyer. If this is the same supplier for both, the above theorem applies and mediated sourcing outperforms direct sourcing. If the preferred supplier, is different for the two buyers, in direct sourcing both buyers can choose their desired partner. But, the intermediary, being constrained to choosing one supplier for both buyers, might find itself in a disadvantaged position. Thus, independent decisions on the type of relationship (k) of the two buyers in direct sourcing effectively gives the buyers more selectivity in

![Figure 4.1. Mediated Sourcing Outperforms Direct Sourcing](image-url)
choosing the preferred supplier, whereas the intermediary being limited to choosing one type of relationship for both buyers has lower selectivity. The next theorem formalizes this.

**Theorem 3.** If all of the following conditions hold for all \( t \geq 0 \), then there exists \( \hat{\delta} \in (0, 1) \) such that buyer \( i \in \{1, 2\} \) prefers a single relationship with supplier 1 and buyer \( \bar{i} \) with supplier 2. Consequently, direct sourcing outperforms mediated sourcing, \( \pi^d(\hat{\delta}) > \pi^m(\hat{\delta}) \).

\[
F^i_t(-\eta_b) = 0, F^i_t(\eta_b) = 1; (2) \mu^i_t > 0, \mu^i_\bar{t} < 0; (3) \delta^{t+1}(\eta_b + \mu_i) > \delta^{t+1}(\eta_b - \mu_{i+}) > \frac{\eta_b G_b}{\epsilon_a}.
\]

Where \( E[X^i_t] = \mu^i_t \) and \( E[X^i_t|X^i_t \geq 0] = \mu^i_{t+} \), \( E[X^i_t|X^i_t < 0] = \mu^i_{t-} \); \( \eta_b = \{\eta_b, t \geq 0\}, \mu_i = \{\mu^i_t, t \geq 0\} \).

**Proof.** A more general statement of the theorem and its proof are provided in the Appendix (Page 32).

The conditions in the theorem above ensure that the two direct buyers wish to enter into relationships with different suppliers. Condition (1) implies that at time \( t \), the expected discounted profit of buyer \( i \) from cooperation with supplier 1 amounts to \( \delta^t(\eta_b + \mu_i) \) and with supplier 2 to \( \delta^t(\eta_b) \).

Condition (2) ensures that for all \( t \), buyer \( i \) prefers to source from supplier 1, \( \delta^t(\eta_b + \mu_i) > \delta^t(\eta_b) \), and buyer \( \bar{i} \) from supplier 2, \( \delta^t(\eta_b) > \delta^t(\eta_b + \mu_{i+}) \). Further, condition (3) ensures that there exist \( \hat{\delta} \) for which these sourcing strategies are the most profitable equilibrium strategies.

Note that the above effect arises only as the mediated single relationship is constrained to be either cooperative or non-cooperative, but the intermediary can’t choose to source part of the order cooperatively and the remaining part non-cooperatively from the supplier it has a relationship with. If the intermediary could have such "partial cooperation" with one supplier, corresponding to different behavior when sourcing for the two client buyers, this disadvantage of intermediation would not arise and an intermediary would always outperform direct sourcing, as illustrated in Theorems 1 and 2. Taken together, our analyses demonstrate that mediated sourcing is better at maintaining relationships, while direct sourcing is better at letting buyers choose which supplier to get into a relationship with. In particular, there are three key phenomena that differentiate direct and mediate sourcing; the ability to use long-run and myopic differences that favor mediated sourcing, and the better selectivity of direct sourcing.

These three phenomena are distinct from the transactional and informational advantages of intermediation. There are no information asymmetries or information aggregation effects in our setup. Further, the intermediary is not using the aggregated scale of buyer transactions to defray fixed transaction costs. The key drivers of our effects are incomplete contracting and the difference in buyer preferences over suppliers at any given point in time.
We conjecture that these effects may provide an explanation for the phenomenal recent growth in mediated sourcing. With an increasingly volatile business environment, there is increasing uncertainty in buyer preferences over suppliers, which leads to more changes and higher differences in buyer preferences. We also believe that as firms are outsourcing increasingly critical inputs and more complex parts of their businesses, sourcing is characterized more and more by incompleteness of contracts, which increases the value of maintaining relationships which, as per our analysis, is a key advantage of intermediaries. Finally, our effects are agnostic to the scale of the sourcing company, and thus might also explain the adoption of mediated sourcing by companies larger than predicted by existing theory.

Notice that our key effects are all driven by changing preferences of buyers over suppliers. Thus, one may conjecture that mediation is most useful in industries with a wide and diverse buyer/supplier base, like the apparel industry. On the other hand, industries such as aerospace or semiconductors, with a concentrated supplier base perhaps derive fewer benefits.

5. Extensions

In our model of mediated sourcing, we assume that buyers transfer all their profit-relevant actions to the intermediary, and thus have no control over buyer-side surplus or sourcing profits. Arguably, this assumption unfairly favors the mediated sourcing model—by assuming a perfect transfer of the actions from the buyers to the intermediary, we assume that the addition of the intermediary to the supply chain does not create any new incentive conflicts, or that the incentives of the buyer and the intermediary are perfectly aligned. However, it is possible for the buyer and intermediary to work at cross-purposes and this incentive conflict would destroy some of the value created by mediation.

To address this concern, we developed and analyzed an alternate model of mediated sourcing that explicitly models the buyer-intermediary transaction as another generic extensive form game.\footnote{The detailed models, their analyses, and the formal results discussed in all extensions described in Section 5 are available from the authors upon request.} In this model, in addition to allowing inefficiency in the supplier-intermediary transaction, we also allow for an additional inefficiency in the buyer-intermediary transaction. Specifically, we assume that the Nash behavior in the buyer-intermediary transaction decreases the sourcing surplus as compared to the model presented in the paper. Only when the buyer and intermediary behave cooperatively in their interaction is there no additional loss in efficiency.

Our analysis indicates that all the effects mentioned in this paper that drive the advantages of mediation continue to hold with this extension. However, when buyer-intermediary incentives are
misaligned, as expected, there is an increased potential for opportunism in mediated sourcing compared to direct that lowers some of the gains from mediation; however, surprisingly, we find that there is also a “policing effect” that actually increases the gains from mediation. The increased potential for opportunism serves as a bigger deterrent against opportunism for some players. Specifically, intermediary’s opportunism can be punished by actions from both the buyers and suppliers.

In our model, we allowed for buyer preferences over suppliers to change over time, but the suppliers are indifferent between buyers. Suppliers may also have preferred buyers and these preferences may change over time. We also developed an extension to the model presented in the paper that allows for both buyer and supplier preferences to change over time. Our analysis indicates that even in this setting the effects described in the paper continue to operate, and mediated sourcing continues to outperform direct sourcing in establishing relationships. Note that while we have labeled one party as the supplier and the other as buyer, our model is agnostic to actual product flows. Thus, the results presented here are all equally valid if the roles are reversed.

Finally, note that we develop all the results in our paper for a cooperation outcome $a^C$. In the game $\Gamma$ there are various actions that the firms could take that correspond to different levels of cooperation. For example there might exist another cooperative outcome $a^c$ that is associated with a smaller gain from deviation by the buyer or the supplier, and it might be possible to sustain $a^c$ when outcome $a^C$ cannot be sustained. The results presented continue to hold true in such a case, our analysis is agnostic to the specific action that the players choose to cooperate on.

References


NOTATION FOR THE ENGAGEMENT GAME $\Gamma$. Let $\Xi$ be the collection of initial nodes of the subgames of game $\Gamma$, with $\xi^0$ being the initial node. The subgame of $\Gamma$ with initial node $\xi \in \Xi$ is denoted by $\Gamma_\xi$, hence $\Gamma_{\xi^0} = \Gamma$. It is partially ordered by precedence relation, where $\xi < \xi'$, if $\xi'$ is a node in $\Gamma_\xi$. A set of terminal nodes is denoted by $Y$, with typical element $y$. An action for player $s$ ($b$) specifies a move for player $s$ ($b$) at each information set owned by that player. At the end of the period, the players observe terminal node $y$ reached. A unique terminal node is reached under a path of play implied by $a$. Given a node $\xi \in \Xi$, $u_{s(b)}(a|\xi)$ is player $s$’s ($b$’s) payoff from $\Gamma_\xi$, given the moves in $\Gamma_\xi$ implied by $a$. The terminal node reached by $a$ conditional on $\xi$ is denoted by $y(a|\xi)$.

APPENDIX B. FORMALIZATION AND PROOFS FOR SECTION 3.2 (DIRECT SOURCING)

B.1. Notation for The Direct Sourcing Stage Game at time $t$, $G^{it}$. The collection of the initial nodes of the subgames of game $G^{it}$ is $\Xi^{G^{it}} \equiv \{\xi^{0_t} \} \cup \{\Xi^{\Gamma_{i1}} \times \Xi^{\Gamma_{i2}} \}$ where $\xi^{0_t}$ is the sourcing fraction selection node, and $\Xi^{\Gamma_{ij}}$ is the set of initial nodes of the subgames of game $\Gamma$ played between buyer $i$ and supplier $j$, i.e. we will add superscript $ij$ to all nodes, so the initial node of game $\Gamma$, $\xi^0$ will become $\xi^{0ij}$ and so the initial node of the transaction step will be $\xi^{01} \xi^{02}$. The set of terminal nodes is $Y^{G^{it}} = Y^{\Gamma_{i1}} \times Y^{\Gamma_{i2}}$.

B.2. Notation for The Repeated Direct Sourcing Game $G^{i\infty}$. The set of period $t$, $t \geq 0$, ex-ante histories is given by $\mathcal{H}^t = (\mathcal{X}_t \times \mathcal{A})^t$, identifying the state $(X^t_i)$ and action profile $(A)$ in each previous period; $\mathcal{X}_t^i$ is the support of $F^t_i$, $\mathcal{A} \equiv \{\theta_i\} \times A \times A$. The set of period $t$, $t \geq 0$, ex-post histories is given by $\tilde{\mathcal{H}}^t = (\mathcal{X}_t \times \mathcal{A})^t \times \mathcal{A}^{t+1}$, identifying the state and action profile in each previous period and identifying the current state. Let $\mathcal{H} = \cup_{t=0}^\infty \mathcal{H}^t$, $\tilde{\mathcal{H}} = \cup_{t=0}^\infty \tilde{\mathcal{H}}^t$, we set $\mathcal{H}^0 = \{\emptyset\}$, hence $\tilde{\mathcal{H}}^0 = \mathcal{X}_i^0$. The pure strategy for player $n$ is a mapping $\sigma_n : \tilde{\mathcal{H}} \rightarrow \mathcal{A}_n$, associating an action with each ex-post history.

B.3. Additional Lemmas.

Lemma 3. Direct Sourcing: The Stage Game Equilibrium

$\alpha^*_{d_i}$ is a subgame-perfect Nash equilibrium of $G^{it}$. 


Proof. By definition, an action profile $\alpha^*_d$ is a subgame-perfect equilibrium if for every node $\xi \in \Xi^{G^{lt}}$, the profile $\alpha^*_d|\xi$ is a Nash equilibrium of subgames $G^{lt}_\xi$. Stage game $G^{lt}$ starts with a choice of supply fractions– initial node $\xi^{l1}$, i.e. $G^{lt} = G^{lt}_{\xi^{l1}}$, and is followed by subgames of game $\Gamma$. We know that $a^N$ is a subgame-perfect equilibrium of $\Gamma$. Hence, due to additive separability of players’ utilities, for every node $\xi \in \{\Xi^{lt1} \times \Xi^{lt2}\}$, $\alpha^*_d|\xi = (a^N, a^N)|\xi$, and so $\alpha^*_d|\xi^{l1} \xi^{l2}$ is a subgame-perfect equilibrium of the transaction step subgame. Hence, we only need to show that $\alpha^*_d|\xi^{l1}$ is a Nash equilibrium of $G^{lt}$, i.e. $\theta^*_i(\mathcal{X}^i_1) \cdot (u_b(a^N) + X^i_1) + (1 - \theta^*_i(\mathcal{X}^i_1)) \cdot u_b(a^N) \geq \theta^*_i(\mathcal{X}^i_1) \cdot (u_b(a^N) + X^i_1) + (1 - \theta^*_i(\mathcal{X}^i_1)) \cdot u_b(a^N)$. The prescribed choice, $\theta^*_i = \tilde{\theta}^*_i \equiv I (X^i_1 \geq 0)$, satisfies this inequality.\footnote{For concise representation, at times, we suppress the argument of $\theta^*_i(\mathcal{X}^i_1)$ and use $\theta^*_i$.} So, $\alpha^*_d$ is a subgame-perfect equilibrium of $G^{lt}$.

B.4. Proof of Lemma 1 (Section 3.2, Page 10). Part 1. $\alpha^*_d$, with $\theta^*_i = \tilde{\theta}^*_i$, is a subgame-perfect equilibrium of $G^{lt}$ (Lemma 3) and hence is a subgame-perfect equilibrium of $G^{\infty}$. No other $\theta^*_i$ can be maintained as, in any period, the buyer could deviate to $\tilde{\theta}^*_i$ and improve his profit.

Part 2. In order to establish this, we need to show that for all histories $(\tilde{h}^t, \xi)$, $t \geq 0$, $(1 - \delta) u^*_n (\alpha_k|\xi) + \delta \mathcal{E}^{t+1} (u_n ((\sigma^k_{(\tilde{h}^t, y_n, (\alpha_k|\xi)}))) \geq (1 - \delta) u^*_n (\alpha_n, \alpha_k|\xi) + \delta \mathcal{E}^{t+1} (u_n ((\sigma^k_{(\tilde{h}^t, y_n, (\alpha_n, \alpha_k|\xi)})))).$ Further, from Lemma 3 we know $\forall \xi: u^*_n (\alpha^*_d|\xi) \geq u^*_n (\alpha_n, \alpha^*_n|\xi)$.

We start with histories $\tilde{h}^t$ that include a deviation. Following such a history, the strategy $\sigma^k$ is prescribing the stage game equilibrium to be played forever after, hence $\mathcal{E}^{t+1} (u_n ((\sigma^k_{(\tilde{h}^t, y_n, (\alpha_k|\xi)})))) = \mathcal{E}^{t+1} (u_n ((\sigma^k_{(\tilde{h}^t, y_n, (\alpha_n, \alpha_k|\xi)}))))$. Further, from Lemma 3 we know $\forall \xi: u^*_n (\alpha^*_d|\xi) \geq u^*_n (\alpha_n, \alpha^*_n|\xi)$.

Taken together, these ensure that condition (L1) is satisfied. Further, for histories $\tilde{h}^t$ that do not include a deviation, the strategy is prescribing $\alpha_k$ to be played if no deviations are observed, and the stage game equilibrium following any deviation. Next, we divide all initial nodes, $\xi \in \Xi^{G^{lt}}$, into two classes: the ones on and off the equilibrium path.

For all $\xi$ that are off the equilibrium path, $\alpha^k|\xi = \alpha^*_d|\xi$ and, hence, $\mathcal{E}^{t+1} (u_n ((\sigma^k_{(\tilde{h}^t, y_n, (\alpha_k|\xi)})))) = \mathcal{E}^{t+1} (u_n ((\sigma^k_{(\tilde{h}^t, y_n, (\alpha_n, \alpha_k|\xi)}))))$, as the strategy prescribes the stage game equilibrium to be played forever after. Then we only need to show that $u^*_n (\alpha^*_d|\xi) \geq u^*_n (\alpha_n, \alpha^*_n|\xi)$, which is established in Lemma 3. Taken together, these ensure that condition (L1) is satisfied.

Next, consider all $\xi$ that belong to the equilibrium path. For the non-cooperating supplier, denoted by $s_N$, (in strategy $d_1s_1$ it is supplier 2, in $d_2s_2$ it is 1) $\mathcal{E}^{t+1} (u_{s_N} ((\sigma^k_{(\tilde{h}^t, y_n, (\alpha_k|\xi)})))) = \mathcal{E}^{t+1} (u_{s_N} ((\sigma^k_{(\tilde{h}^t, y_n, (\alpha_n, \alpha_k|\xi)}))))$, even if she deviates, this would not influence the continuation of cooperation among cooperating players. Thus we only need to ensure $(1 - \delta) u_{s_N} (\alpha^k|\xi) \geq$
Lemma 4. Mediated Sourcing: The Stage Game Equilibrium

\(\alpha_m^*\) is a subgame-perfect Nash equilibrium of \(G^{lt}\).
Proof. From the additive separability of the utilities of the intermediary and the suppliers with respect to actions \( a_{i1} \) and \( a_{i2} \) in the merged games, and that \( a^N \) is a subgame-perfect equilibrium of \( \Gamma \), it follows that \( \alpha^N \) is a subgame-perfect equilibrium of \( G^{tt}_{\xi_0} \). Thus, we only need to show optimality in the supplier selection choice, or \( \sum_{i=1}^{t} \left( \nu^t_i \left( u_b(a^N) + X^t_i \right) + (1 - \nu^t_i) u_b(a^N) \right) \geq \sum_{i=1}^{t} \left( \nu^t_i \left( u_b(a^N) + X^t_i \right) + (1 - \nu^t_i) u_b(a^N) \right). \) The prescribed choice satisfies this inequality. \( \square \)

C.4. Proof of Lemma 2 (Section 3.3, Page 14). The proof follows along the same lines as the proof of Lemma 1, noting that suppliers have \( \vartheta_1 = \nu_1 + \nu_2 \) and \( \vartheta_2 = 2 - (\nu_1 + \nu_2) \) orders on hand on which they can deviate. In \( md \), the intermediary is sourcing \( \vartheta_1 + \vartheta_2 = 2 \) orders cooperatively, in \( m_{s1} - \vartheta_1 \), in \( m_{s2} - \vartheta_2 \), the maximal possible deviations in transaction steps follow.

APPENDIX D. PROOFS FOR SECTION 4 (THE BENEFITS OF INTERMEDIATION)

D.1. Proof of Theorem 1 (Section 4, Page 16). 1. For given strategy \( k \), as \( a^t_{ij} (\sigma^k) = a^t_{i2} (\sigma^k) = a^t_{ij} (\sigma^k) = a^t_{ij}, j \in \{1, 2\} \), the sourcing profits for direct and mediated structures are given by (\( X_t = \{X^t_t, t \geq 0\} \)):

\[
\pi^{d_1k}(\delta) + \pi^{d_2k}(\delta) = \max_{\theta_1, \theta_2} \left( \theta_1 + \theta_2 \right) u_b \left( a^k_1 \right) + \theta_1 X^t_1 + \theta_2 X^t_2 + (2 - (\theta_1 + \theta_2)) u_b \left( a^k_2 \right),
\]

\[
\pi^{mk}(\delta) = \max_{\nu_1, \nu_2} \left( \nu_1 + \nu_2 \right) u_b \left( a^k_1 \right) + \nu_1 X^t_1 + \nu_2 X^t_2 + (2 - (\nu_1 + \nu_2)) u_b \left( a^k_2 \right).
\]

2. Further, we need to ensure that all cooperative players have sufficient incentives to maintain this strategy. In order for supplier \( s_j \) to cooperate in respective games with buyer \( i \) or the intermediary, as per Lemmas 1, 2, the following constraints should be satisfied for all \( t \) (\( \gamma = \frac{u_s(a^N)}{u_s(a^N)}, d = \frac{1 - \delta}{\sigma} \Rightarrow G^t_{\xi} \)):

\[
s_1 : \begin{align*}
(D.1) \quad & \quad \delta^{t+1}(\theta_1) - d\bar{\theta}_1^t \geq \gamma_1^t; \quad \text{(M)} \quad \delta^{t+1}(\nu_1 + \nu_2) - d\bar{\nu}_1^t \geq \gamma_2^t;
\end{align*}
\]

\[
s_2 : \begin{align*}
(D.1) \quad & \quad \delta^{t+1}(1 - \theta_1) - d(1 - \theta_1^t) \geq \gamma - \gamma_1^t; \quad \text{(M)} \quad 2 - \delta^{t+1}(\nu_1 + \nu_2) - d\left(2 - \nu_1^t + \nu_2^t\right) \geq 2\gamma - \gamma_1^t - \gamma_2^t.
\end{align*}
\]

Where \( \bar{\theta}_1^t \equiv \max X^t_i \theta_1^t, \bar{\theta}_2^t \equiv \min X^t_i \theta_1^t, \bar{\nu}_1^t + \bar{\nu}_2^t \equiv \max X^t_1, X^t_2 (\nu_1^t + \nu_2^t), \overline{\nu}_1^t + \overline{\nu}_2^t \equiv \min X^t_1, X^t_2 (\nu_1^t + \nu_2^t).

3. Next, we show that mediated dual relationship performs better than direct dual relationship, using the following strategy. Denote by \( \theta_1^*, \theta_2^* \) solution to respective direct buyers’ problems. Set \( \nu_1^*, \nu_2^* \) as follows: for \( i \) such that \( X^t_i \geq X^t_1 \), set \( \nu_i^t = \min \{1, \theta_1^t + \theta_2^t\} \) and \( \nu_i^t = \theta_1^t + \theta_2^t - \min \{1, \theta_1^t + \theta_2^t\} \). Doing so, in every period \( t \), the mediated system gains over direct:

\[
\int_{x_1^t}^{x_2^t} \int_{x_2^t}^{x_3^t} \min \{1 - \theta_1^t, \theta_2^t\} (x_1 - x_2) f^t(x_1, x_2) dx_1 dx_2 + \int_{x_1^t}^{x_2^t} \int_{x_1^t}^{x_2^t} \min \{1 - \theta_2^t, \theta_1^t\} (x_2 - x_1) f^t(x_1, x_2) dx_1 dx_2.
\]
By looking at the boundaries of the integral one can see that this expression is always non-negative.

4. Next, we need to insure that in following the above described strategy all the incentive constraints are satisfied. In direct sourcing, buyer $i$ will choose $\theta^t_i$, so as to satisfy respective direct sourcing constraints. The supplier-side constraints in mediated sourcing, having $\nu^*_1 + \nu^*_2 = \theta^t_1 + \theta^t_2$, are just the sum of the constraints that direct buyers have to satisfy. Hence, the supplier-side constraints, as defined in part 2, are satisfied. Further, the buyer-side constraints in direct sourcing can be expressed as $\pi^{d,k}_i (\delta|t + 1) - \pi^{d,T}_i (\delta|t + 1) \geq \frac{1-\delta}{\theta} G^{kt}_{b_i}$, where by $\pi^{d,k}_i (\delta|t + 1)$ we denote the expected normalized sourcing profit starting from period $t + 1$ onwards, and $G^{kt}_{b_i}$ is the highest gain buyer $i$ can gain in period $t$ from deviation given that strategy $k$ is played. Having $\nu^*_1 + \nu^*_2 = \theta^t_1 + \theta^t_2$, the constraint of the intermediary can be written as $\pi^{m,k}_i (\delta|t + 1) - \pi^{m,T}_i (\delta|t + 1) \geq \frac{1-\delta}{\theta} (G^{kt}_{b_1} + G^{kt}_{b_2})$.

In part 3 we established that $\pi^{m,k}_i (\delta|t + 1) \geq \pi^{d,k}_i (\delta|t + 1) + \pi^{d,T}_i (\delta|t + 1)$, further in transactional sourcing intermediary and the buyers make the same sourcing profit, $\pi^{m,T}_i (\delta|t + 1) = \pi^{d,T}_i (\delta|t + 1) + \pi^{d_2T}_i (\delta|t + 1)$ (transactional sourcing has no cooperation, hence none of the constraints specified in part 2 apply and both direct and mediated sourcing solve the same unconstrained optimization problem). Hence, the constraint of the intermediary is also satisfied.

5. We can show that the intermediary might be able to further improve profits. From expressions in part 4, the supplier-side constraints are just the sum of the constraints that direct buyers have to satisfy for this supplier. Hence, if for given $\delta$, $\theta^t_1$ is chosen so that the constraint of supplier 1 is binding, but $\theta^t_2$ satisfies the respective constraint with slack, then the suppliers-side constraint in mediated sourcing is satisfied with slack. Hence, the intermediary can further improve the sourcing profit by choosing $\nu^*_1, \nu^*_2$ so as to remove the undesired slack and improve the profit. To further illustrate the proof of the Theorem we present a specific example in the next section.

D.2. Example. Consider the following example with stationary distributions: $F^t_i = F_i$, for all $t \geq 0$; further, set $F_1 \sim U[-50, 50]$, $F_2 \sim U[-70, 30]$. Finally, $u_s (a^N) = 0$. Hence, $\gamma = \gamma^*_1 = \gamma^*_2 = 0$. 

<table>
<thead>
<tr>
<th>Allocation $\theta_i$</th>
<th>Responsive Allocation, $\tilde{\theta}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>$s_2$</td>
</tr>
<tr>
<td>(D.1) $E(\theta_1) \geq d\bar{\theta}_1$</td>
<td>$1 - E(\theta_1) \geq d (1 - \theta_1)$</td>
</tr>
<tr>
<td>(D.2) $E(\theta_2) \geq d\bar{\theta}_2$</td>
<td>$1 - E(\theta_2) \geq d (1 - \theta_2)$</td>
</tr>
<tr>
<td>(M) $E(\nu_1 + \nu_2) \geq d(\bar{\nu}_1 + \bar{\nu}_2)$</td>
<td>$2 - E(\nu_1 + \nu_2) \geq d (2 - \nu_1 + \nu_2)$</td>
</tr>
</tbody>
</table>

Table 1. Example: Supplier-side constraints that ensure that a dual relationship strategy is equilibrium for the specified allocation.
\( \theta^t_i = \theta_i \) and \( \sigma^{t+1}(\theta_i) = E(\theta_i) \). For allocations, \( \theta_i \) and \( \nu_i \), the constraints required to sustain a dual relationship are in columns 2 & 3 of Table 1 (derived from part 2 of the proof of Theorem 1). In the last two columns of the table the responsive allocation values, \( \theta_i = \nu_i = \tilde{\theta}_i = I(X_i \geq 0) \), are used.

For \( d \leq 0.3 \) (\( \delta \) corresponding to region (v)\(^{14} \) in Figure D.1(a)), the responsive allocation itself ensures that all of the equilibrium constraints are satisfied, so \( \theta^*_i = \nu^*_i = \tilde{\theta}_i \) (shown in Figure D.1(b)). Next, for \( d \in (0.3, 0.4) \) (region (iv)\(^{14} \) of Figure D.1(a)), direct buyer 2 can not satisfy constraint of supplier 1 with responsive allocation, while buyer 1 still can do so. In fact the constraint is satisfied with a slack. However, the intermediary can easily satisfy constraints for both suppliers with responsive allocation, \( \nu^*_i = \tilde{\theta}_i \), for \( d \in (0.3, 0.4) \). Effectively in this region intermediary is using the slack that buyer 1 has to subsidize buyer 2’s weaker interest. We depict functions \( \theta^*_i, \nu^*_i \) in Figure D.1(c), the dotted lines apply to direct sourcing, the solid to mediated and the dash-dot lines to both. This corresponds to section 5 of the proof and case II of the sketch of the proof of Theorem 1. Lastly, for \( d > 0.4 \) (region (iii)\(^{14} \) of Figure D.1(a)) the intermediary must also depart from responsive allocation to satisfy the constraints. In Figure D.1(d) we depict how \( \theta^*_1, \nu^*_2 \) would look like for \( d = 1 \) if the buyers establish a dual relationship. To satisfy the constraints, direct buyers must in every period source half of their orders from each supplier, \( \theta^*_i = 1/2, \tilde{\theta}_i = \theta_i = 1/2 \) (top of Figure D.1(d)). In this case, the intermediary can source the same amount per period from each supplier (one unit from each), but if \( X_1 > X_2 \) (buyer 1 has higher preference for supplier 1) he will source buyer 1’s order from supplier 1, and buyer 2’s from supplier 2, \( \nu^*_1 = 1, \nu^*_2 = 0 \), and vice-versa if \( X_2 \geq X_1 \) (Figure D.1(d) bottom). This achieves a more efficient allocation of orders, earning higher profits. This corresponds to section 3 of the proof and case III of the sketch of the proof of Theorem 1.

\(^{14}\)Labeling of the regions corresponds to labeling in Figure 4.1.
D.3. Proof of Corollary to Theorem 1 (Section 4, Page 19). All statements of the Corollary follow from the gain of mediation established in Part 3 of the proof of Theorem 1.

D.4. Proof of Theorem 3 (Section 4, Page 22). In our setup we assume that whenever the intermediary enters into a relationship with the supplier it needs to always source cooperatively from this supplier (independent of the buyer it is sourcing for). This Theorem allows us to demonstrate how restrictive this assumption can be. We will do so by constructing a specific set of conditions which lead to worse performance of mediated sourcing. Below we formulate more general version of Theorem 3. Theorem 3 follows if \( F^t_i (-\eta_b) = 0, F^t_i (\eta_b) = 1 \).

**Theorem 4.** If all of the following conditions hold for all \( t \geq 0 \), there exist \( \delta \in (0, 1) \) such that buyer \( i \in \{1, 2\} \) prefers single relationship with supplier 1 and buyer \( \tilde{i} \) with supplier 2, \( \pi^d (\delta) = \pi^{d,s_1}(\delta) + \pi^{d,s_2}(\delta) \), and direct sourcing outperforms mediated sourcing, \( \pi^d (\delta) > \pi^m (\delta) \).

\[
E[X_t' | \eta_b \geq X_t' \geq -\eta_b] + \eta_b \cdot (1 - F^t_i(-\eta_b) - F^t_i(\eta_b)) > 0, E[X_t' | \eta_b \geq X_t' \geq -\eta_b] + \eta_b \cdot (1 - F^t_i(-\eta_b) - F^t_i(\eta_b)) < 0; \\
\delta^{t+1}((1 - F^t_i(0)) (1 - \gamma) + F^t_i(0) - F^t_i(-\eta_b)) - \delta F^t_i((0) (1 - \gamma) - F^t_i(0) + F^t_i(\eta_b)) \geq 1 - \gamma, \text{ where } \gamma \equiv \frac{u_a(u^n)}{u_a(x^c)}; \\
\delta^{t+1}(E[X_t | 0 \geq X_t \geq -\eta_b] + \eta_b \cdot (1 - F^t_i(-\eta_b))), \delta^{t+1}(\eta_b \cdot F^t_i(-\eta_b)) - E[X_t | \eta_b \geq X_t \geq 0]) > \frac{u_{a,u}}{u_{a,x}}.
\]

1. Suppose that in single relationship sourcing with responsive allocation of orders between co-operative and non-cooperative suppliers, we are going to refer to these profits as \( \pi^{d,s_1}(t) \), buyers sourcing directly prefer to establish a relationship with different suppliers, i.e. for some \( i \in \{1, 2\} \) \( \pi^{d,s_1}(t) > \pi^{d,s_2}(t) \) and \( \pi^{d,i}(t) > \pi^{d,s_1}(t) \). Writing out these conditions in terms of the parameters of the model, we get the first set of constraints stated in the theorem.

2. We need to ensure that at least for some \( \delta \) buyers will use single relationship with responsive allocation, \( \pi^d_i (\delta|t) = \pi^{d,s_1}(t) \) and \( \pi^d_i (\delta|t) = \pi^{d,s_2}(t) \). Having supplier-side constraints for maintaining dual relationship both in direct and mediated sourcing, one can derive that for delta lower than \( \delta^d_1 = \frac{G_1}{\eta_b + G_1} \), no dual relationship strategies can be maintained both in direct and mediated sourcing. Similarly, we can find the lowest delta where the preferred single relationship responsive strategy for each of the buyers can be sustained, \( \delta^{d,s_1}_1 \) and \( \delta^{d,s_2}_1 \). The second set of conditions requires that single responsive sourcing with the preferred supplier can be maintained for a wider range of \( \delta \) than any of the dual relationship strategies, \( \delta^{d,s_1}_1, \delta^{d,s_2}_1 \leq \delta^d_2 \). The last set of constraints ensures that at \( \delta^d_2 \), buyer-side constraints are satisfied with a slack, establishing existence of \( \delta \).

3. Lastly, at \( \delta \) direct sourcing perform better than mediated, as the best profit mediated sourcing can achieve: \( \pi^m (\delta|t) = \max \{ \pi^{d,s_1}(t), \pi^{d,s_2}(t) \} < \pi^d (\delta|t) = \pi^{d,s_1}(t) + \pi^{d,s_2}(t) \).
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