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"Membership has its Benefits": Subscription Pricing for Free Delivery Services

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ABSTRACT

Many online retailers now provide consumers the option of subscribing to a membership plan for limited or unlimited free delivery instead of paying delivery charges for each purchase. This paper seeks to understand the benefits, to both firms and consumers, of such free-delivery-subscription (FDS) plans compared to the traditional pay-for-delivery (PFD) option, characterize the firm's optimal FDS policy, and study the effects of product, consumer, and market characteristics on the firm's profits and consumer surplus. We focus on two main factors that make such plans appealing to firms – the "lift" in a PFD customer's demand when she subscribes to the FDS plan, and the possibility of attracting new consumers. We propose a novel utility-based framework to model consumer decisions on which option to use and how much to purchase. Our approach incorporates consumer heterogeneity, both in terms of their utility for the product and their preference for purchasing from the firm. We show that offering the FDS plan increases firm profits even though the optimal subscription fee may not be sufficient to cover the firm's actual shipping cost. Interestingly, the FDS option also benefits consumers. Our analysis of two FDS pricing strategies – a single "Universal" plan or multiple "Tiered" plans with varying subscription fees and free delivery limits – leads to several interesting insights on the drivers and sensitivity of the optimal policy, profits, and market share to model inputs. We also conduct extensive computational experiments to assess the relative profitability of different strategies. The approach and results presented in this paper also extend to subscription pricing in other contexts beyond online retailing.

1. Introduction

In recent years, many retailing and service firms have supplemented their traditional pay-per-use pricing schemes by introducing subscription plans that provide unlimited or limited free (or discounted) service for a fixed annual membership fee. For instance, several online retailers now offer customers the options of either paying delivery charges with each purchase or subscribing to an annual membership plan for free deliveries. Similar choices are available in a variety of other contexts ranging from cell phone service, cloud computing, ride sharing, and content streaming to health clubs and airport lounges. The subscription plans are most appealing to "heavy" users whose annual pay-per-use cost exceeds the subscription fee. But, when firms incur tangible costs of service fulfillment (e.g., to deliver products to customers), these heavy users are least preferred as subscribers since the firm's cost to provide them service will exceed the subscription fee. For instance, in 2015, Amazon's revenues from delivery charges and subscription fees for its Prime membership program covered only 55% of its total shipping costs (Business Insider 2016). Why then do firms offer subscription plans for free service? Two main reasons underlie this strategy: (i) the firm's current pay-per-use customers may purchase more when they subscribe to the membership plan, i.e., the subscription plan can induce a "lift" in the customer's demand; and (ii) if the subscription fee is low enough, new or "external" customers who prefer not to purchase from the firm using the pay-per-use option may switch to the firm if it offers the membership plan. These two sources of additional demand increase the firm's contribution from product sales (when the consumer must first purchase a product to be eligible for free service, as in retailing) as well as its subscription revenues. Hence, the firm may find it profitable to offer a subscription plan for free services despite higher net costs for service fulfillment. Deciding the terms of the subscription policy poses the familiar pricing tradeoff: a lower subscription fee can induce more external consumers to switch to purchasing from the firm, but also reduces the total subscription revenue from all subscribers. Further, when consumers have varying demand, the firm must also decide which consumer types or segments to target for the subscription option, and whether to offer multiple plans with varying subscription fees and free service limits. Thus, subscription pricing for free service, particularly delivering products to customers, poses an interesting, relevant, and challenging problem with multiple tradeoffs. The goals of this paper are to develop an economic model that captures all of the above features for the online retailing context, and determines the profit-maximizing subscription policy, i.e., the subscription fee(s) and free service limit(s). Using this model, we wish to gain insights about the circumstances under which offering the free delivery subscription option can be profitable for the firm, determine whether consumers benefit from this option, guide the firm's subscription pricing strategy, and identify the key drivers of profitability.

Researchers have previously studied subscription pricing mainly in the contexts of information goods, access services, and ancillary services.[1] The online retailing context, with subscription for free deliveries, is more general. For instance, unlike information goods and access services, customer must first purchase a product from the online retailer (which provides a contribution margin to the firm) before taking advantage of the free delivery service. In other words, the product being sold is separate from the service. Moreover, the firm incurs actual and non-negligible cost for delivery to customer (e.g., using a package delivery service), whereas the incremental service cost can be minimal for information goods and some access services. Hence, subscription pricing for free delivery services requires a richer model that incorporates features such as: (i) the "lift" in demand for current pay-for-delivery customers if they subscribe to the free delivery subscription plan, (ii) the possibility of attracting new customers, (iii) consumer heterogeneity in terms of their valuation of the product and preference for purchasing from the firm, (iv) positive costs incurred by the firm to provide the service, and (v) the strategy of simultaneously offering both the pay-for-delivery option and one or more free-delivery-subscription plans. None of the previous models consider all these features together.

To address the subscription pricing problem for free delivery services, we develop a novel economic model that is rich enough to consider a general consumer decision making framework and yields good insights, but is also analytically tractable and captures the core subscription pricing policy-setting tradeoffs facing the firm. Anecdotal and empirical evidence suggests that consumers purchase more when they join a free-delivery subscription plan. For instance, Amazon Prime members spend nearly twice as much on purchases as those who pay for delivery (Levin and Lowitz 2016).[2] There could be several reasons for the lift in demand, including economic reasons as well as possible behavioral (sunk cost) effects of paying the membership fee up front. Instead of treating lift as a given (exogenous) input, our approach focuses on the economic drivers for lift using a utility-based approach. Specifically, we show that, to maximize her surplus, a consumer who chooses to subscribe to a free-delivery subscription (abbreviated as *FDS*) plan purchases more than she would under the pay-for-delivery (abbreviated as *PFD*) option. We consider two dimensions of heterogeneity among consumers: (i) based on their "type," with higher types deriving more utility for a given purchase quantity, and (ii) based on their preference (modeled as a customer-dependent switching or opportunity cost) for purchasing from the firm if it only offered the PFD option. This approach permits us to segment the consumers and capture the increase in demand when the firm introduces an FDS option. We keep the model very general by not assuming any particular functional form for the utility function (as long as the function satisfies some mild and natural assumptions) or any particular distribution for the opportunity cost. Indeed, we do not even impose

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[1] There is also some recent literature on online grocery delivery and shipping cost promotions for individual orders.

[2] In part, this spending differential may reflect a self-selection bias, i.e., consumers who purchase larger quantities are more likely to become members.

conditions to ensure concavity of the firm's profit function. Since our main goals are to understand the drivers and strategies for offering the FDS option, we focus on the FDS policy decisions of setting the subscription fee(s) and free delivery limit(s), for given financial parameters such as product price, product and shipping costs, and PFD delivery charge. The analysis (and computational results) of our versatile economic model sheds light on questions such as the following: Can offering one or more FDS plans increase the firm's profit (compared to offering just the PFD option)? What is the optimal subscription policy, i.e., what subscription fee to charge and whether to limit the purchase quantity for which consumers get the benefits of subscription? What are the drivers of the optimal subscription policy? Under what circumstances would offering multiple "Tiered" subscription plans yield significantly higher profits than a single "Universal" subscription plan with unlimited free deliveries (as some firms do)?

Even with a general model, we are able to identify interesting properties of the firm's optimal subscription pricing policy, and provide comparative statics results on how the optimal profit, subscription fee, and number of customers vary with the product, consumer, and market attributes. We analyze both the Universal plan and the Tiered plan, develop algorithms to find the optimal policies under these plans using as subroutine the procedure to solve the easier problem with a single consumer type. Some of the key managerial insights from our analysis are as follows:

- Offering the FDS option increases the firm's profit regardless of the product, consumer, and market characteristics. Moreover, this result holds even without any external consumers due to the lift in the demand of existing customers when they subscribe to a FDS plan.
- Interestingly, the consumers also benefit, i.e., when the firm offers the FDS option, each consumer's surplus increases or stays the same compared to the surplus under the PFD option.
- The firm's optimal subscription fee can be much lower than actual delivery cost it incurs for each subscriber, implying that anchoring the subscription fee to the shipping cost is not a good strategy.
- Offering unlimited free deliveries with subscription is optimal under many situations (as we often observe in practice), but imposing a free delivery limit can be superior under certain conditions.

In addition to the analytical results for the firm's optimal subscription pricing policy under different strategies, we also provide a comprehensive computational comparison between the Universal plan and Tiered plan. These quantitative comparisons highlight conditions under which one strategy significantly dominates the other. Finally, although this paper focuses on free delivery to customers by internet retailers, the model subsumes features required for problems in other contexts such as information and access services, and so our approach and results also applies to these settings.

The rest of this paper is organized as follows. Section 2 reviews the literature and positions our work relative to existing research. Section 3 motivates and presents the ingredients of our model, examines the consumer decision process, and analyze a base model with a single consumer type. Sections 4 and 5 respectively study the firm's optimal subscription pricing policy under the Universal plan and discusses the optimal Tiered plan. Section 6 presents and interprets our computational results comparing the strategies of offering a Universal plan or a Tiered subscription plan. The proofs for all the Propositions and Lemmas are presented in the online Appendix.

2. Literature Review

Researchers in economics, marketing, and information systems have studied the subscription pricing problem in various service contexts. Depending on the context, the proposed models differ in the way they capture or represent consumer demand under the free service subscription option, heterogeneity among consumers, costs to the firm and consumers, and pricing strategies. We relate our work to prior research on subscription pricing for information goods, access services, ancillary services, and online grocery deliveries, emphasizing the similarities and differences in the model features with our approach.

Sundararajan (2004) considers three options for selling *information goods*: a fixed fee for unlimited usage, a per-usage pricing, and a combination of these two pricing schemes. Unlike the retailing context in which firms incur significant costs for the delivery service, a key feature of information goods is the negligible marginal cost for serving a consumer. Sundararajan shows that, if consumers are heterogeneous in their value for the good and incur positive transaction costs per unit if they select the usage-based pricing option, then the firm's optimal strategy is to offer both a fixed fee (for unlimited usage) and a usage-based pricing plans. Other papers on subscription pricing for information goods do not consider the option of simultaneously offering both the subscription and pay-per use plans. Danaher (2002), Fruchter and Rao (2001), Fishburn et al. (2000), and Balasubramanian et al. (2015) compare the pay-per use vs. subscription pricing option for information goods to address various issues such as the effect of pricing policy on customer attrition, network effects of product adoption, and behavioral effects of pay-per use option on consumer utility. None of the above papers account for the lift in demand when a consumer switches from the pay-per-use option to the subscription option, nor do they consider external consumers. Bhargava and Gangwar (2016) study the problem of designing optimal three-part tariffs for information goods. These tariffs consist of an access fee for consumption upto a specified level plus a unit "overage" price for any consumption beyond the allowance (Lambrecht et al. 2007, Acarza et al. 2012, Iyengar 2004, and Grubb 2009). Bhargava and Gangwar consider consumer heterogeneity along two dimensions, in value and in usage (both of which we also capture in our utility function), but do not consider the possibility of attracting external consumers by offering a subscription plan. The authors numerically compare the pay-per use, unlimited usage subscription, and optimal three-part tariff strategies, and discuss comparative static results with respect to consumer heterogeneity.

Access services, such as wireless services and health clubs, also entail negligible marginal cost for providing the service. For such services, Essgaier et al. (2002) compare a pay-per use pricing option with a two-part pricing scheme that includes both a subscription fee for access and a per-usage price. Assuming that consumers' usage rate under both the options are the same (unlike our model), the authors show that two-part pricing yields higher profits. Other subscription pricing models in the context of access services study the effect of service capacity on the optimal pricing policy (Randhawa and Kumar 2008) and behavioral biases that affect consumer usage (Leider and Sahin 2014). Cachon and Feldman (2011) compare the pay-per-use vs. subscription option for an access service the experiences congestion. Consumers are homogeneous in their value for the service but incur a cost for waiting to receive the service. Since the marginal payment to the firm for using the service under subscription is zero, a consumer's usage rate under subscription increases, increasing the congestion and lowering the willingness to pay for subscription (whereas in the retailing context, higher demand increases the contribution to the firm's profits). The authors show that depending on the waiting costs and service capacity, the subscription option can provide higher profits.

Researchers have also studied subscription pricing models for ancillary services, i.e., discretionary services after purchasing a product, and online grocery delivery. Wang et al. (2015) consider the option of introducing an annual subscription plan for free ancillary services, and study the joint pricing problem for the main product as well as the ancillary subscription plan. They assume that the firm does not incur any cost for providing the ancillary service, and do consider the possibility that more consumers will purchase the ancillary service under the subscription option (as we do, with external consumers); however, unlike our model, the expected usage per consumer does not depend on the pricing option. Belavina et al. (2016) compare the pay-per-order vs. subscription pricing model for online grocery deliveries, and analyze the impact of these revenue models on the firm's profit and environment. The authors show that consumers order more frequently under the subscription option but overall sales are higher under the pay-per-order option (due to higher order sizes). Further, similar to our model, the subscription option can attract more consumers to purchase from the retailer. Using a detailed model for the firm's delivery costs, the authors find that either revenue model can outperform the other depending on the product margin, consumption rate and characteristics of the delivery area.

Finally, researchers have also studied models for shipping cost promotions for individual orders. Gümüş et al. (2013) study the optimal price partitioning strategy – whether to separately charge a retail price and delivery fee, or offer free-shipping but incorporate all or part of the delivery charge in the retail price. Leng and Becerril-Arreola (2009) and Cachon et al. (2016) address contingent free-shipping, i.e., charging a discounted delivery fee if the order quantity exceeds a minimum threshold.

As this review of the literature suggests, our model is more general and combines features found in various previous models, and so is applicable to a variety of contexts.

3. Optimal Free-Delivery Subscription (FDS) Policy: Model Development

Consider an internet retailer that currently offers a *Pay-for-Delivery* or *PFD* option, i.e., customers pay a unit delivery charge for items they purchase. The firm is exploring the possibility of also offering a Subscription-based fulfillment option, which we call the *Free-Delivery-Subscription* (FDS) option. A consumer who subscribes to a membership plan under this option pays an upfront subscription fee, valid for a period of say one year, and gets free delivery for up to a pre-specified number of units during the subscription period. For additional units beyond the free shipping limit, the subscriber pays the same unit delivery charge as the PFD option. A firm may choose to offer multiple FDS plans with different subscription fees and free shipping limits, designed to attract different consumer segments (consumers self-select the plan of their choice from the available menu of plans). Consumers are segmented along two dimensions depending on their demand characteristics or *type* and their inclination to purchase from the firm. Consumer types are differentiated by their utility for the firm's products, with higher types deriving more value or utility. We refer to customers who will purchase from the firm even if it only offers the PFD option (i.e., the firm's current customer base) as *loyal* consumers; other potential customers who prefer not to purchase from the firm using the PFD option are *external* consumers. Each external consumer incurs an opportunity cost to switch to the firm, and the opportunity cost can vary by consumer. Given the product price and cost, the cost for shipping to the customer, the delivery charge payable under the PFD option (as well as other costs for the firm and consumer), and the market and consumer characteristics, the *Subscription pricing problem* entails deciding the subscription fee(s) and free delivery limit(s) for the FDS option so as to maximize the firm's profits.

In the following sections, we develop an appropriate economic model for this problem, characterize consumer behavior, and analyze the case when there is only one consumer type. Section 3.1 introduces our modeling framework and notation, most notably our general utility-based approach to capture the lift in demand for loyal consumers when they subscribe to a FDS plan instead of using the PFD option, and the switching behavior of external consumers. Section 3.2 examines how a consumer's utility function and the costs for the PFD and FDS options govern her choice among these two options and her purchase quantity. This understanding of consumer behavior leads to a characterization of the firm's demand as a function of the subscription fee and shipping limit of a FDS plan. Section 3.3 applies this model to the situation where consumers are heterogeneous only in their current purchasing strategy, but not in their type (i.e., all consumers are of the same type). This analysis serves as the foundation for our later discussions (in Sections 4 and 5) to address subscription pricing with multiple consumer types. All of our discussions (in this and subsequent sections) consider the situation where the firm continues to offer its current PFD option alongside its FDS plans, permitting consumers to choose the option that provides them the highest surplus.

3.1 Model Ingredients

We first introduce the notation for the costs that consumers and the firm incur under PFD and FDS, and then describe our utility-based model to capture consumer behavior and demand.

Let p denote the *retail price* per unit[3] purchased. Under the PFD option, the consumer also pays a positive *delivery charge* of d per unit, and may incur a non-negative transaction cost of g per unit. The transaction cost g can capture any inconvenience that a consumer experiences when paying the delivery charge for each purchase (e.g., Balasubramanian et al. 2015). Let S and L respectively denote the *subscription fee* and the *free delivery limit* of a FDS plan. If a consumer chooses to subscribe to this plan, she pays the fixed fee S per subscription period (say, a year), and the firm waives the delivery charge for up to L units of demand during the subscription period. For purchase quantities above the limit L, the consumer pays the delivery charge of d per unit. If the firm chooses to offer multiple plans, these plans differ in their S and L values, with plans having lower subscription fees also having lower free delivery limits (as required for incentive compatibility; see discussions in Section 5).

The firm incurs a non-negative product cost c and an actual shipping cost e per unit sold (to PFD or FDS consumers). We permit this shipping cost to be different from the delivery charge d that PFD consumers pay. The firm may also incur a non-negative per unit transaction cost τ for units sold under the PFD option (Sundararajan 2004); this cost can represent, for instance, any financial or administrative expenses associated with accounting and billing for the delivery charge.

Note that, for our stylized economic model to address the tradeoff between the firm's additional demand and subscription fees for the FDS option, we make the simplifying assumption that the product and shipping costs are linear in demand (i.e., per unit cost is constant). However, even if these costs are concave functions of demand (reflecting economies of scale), the main insights and qualitative inferences from our model should hold since the concave cost structure essentially makes the FDS option more attractive.

Consistent with the focus of this paper on the effects of introducing the FDS option, we keep the terms of the PFD option fixed by considering the unit price p and delivery charge d as given values; however, we do address comparative statics for various problem parameters. For each FDS plan, the firm must decide the subscription fee S and delivery limit L; we refer to the pair of decisions $\langle S, L \rangle$ as the firm's *FDS pricing policy*. Next, we discuss our treatment of consumer heterogeneity in terms of their valuation of the product and their preference for purchasing from the firm.

Consumer characteristics:

Consumers vary in their utility for the products sold by the firm. To capture these variations, we

[3] In this paper, a *unit* refers to any metric that measures a consumer's purchase quantity of a single product or a representative mix of products sold by the firm.

classify consumers into different "*types*," each with an associated parameter $\theta \in \Theta$, where Θ is the set of all consumer types. Consumers with higher values of θ obtain higher utility from the product, and hence purchase more. Let $u_\theta(y)$ denote the utility that a consumer of type θ derives from purchasing *y* units over the planning horizon. The utility function $u_\theta(y)$ is increasing in both θ and *y*, i.e., larger purchase quantities provide more utility for a given consumer type, and consumers with higher type derive more utility than lower types for a given purchase quantity. For each consumer type θ, we assume that $u_\theta(y)$ is concave and continuously differentiable in the purchasing quantity *y*. Further, we assume that the utility function $u_\theta(y)$ has increasing differences in *y* and θ, i.e., for consumer types θ_1 and θ_2, with $\theta_2 > \theta_1$, and purchase quantities y' and $y" > y'$, $u_{\theta_2}(y") - u_{\theta_2}(y') \geq u_{\theta_1}(y") - u_{\theta_1}(y')$. This property implies that a consumer with higher type obtains higher incremental value from additional units. These assumptions, commonly used in utility-based economic models, hold for several families of utility functions such as the *Quadratic* ($(\theta\rho y - (b-\theta\omega)y^2,\ \ \omega \in [-1,1],\ \ 0 < \theta \leq b,$ for $y \leq \theta\rho / 2(b-\theta\omega)$), *Power* ($\theta y^{1-\alpha} / (1-\alpha)$, $\alpha \in (0,1)$), and *Exponential* ($\theta(1-e^{-\alpha y}) / \alpha,\ \ \alpha > 0$) functions. Our model does not assume any particular functional form for the utility function.

In addition to distinguishing consumers based on their type, we consider a second dimension of consumer heterogeneity based on their preference for purchasing from the firm. Specifically, among consumers of each type θ, we distinguish between: (i) "*loyal*" consumers who prefer to purchase from the firm, using either the PFD option or an FDS plan if they find the terms of this plan to be beneficial (in terms of consumer surplus); and (ii) "*external*" consumers who prefer not to purchase using the firm's PFD option (e.g., compared to this option, they prefer some outside option that can vary by consumer), but may be willing to switch to the firm if it offers an attractive FDS plan. Let β_θ denote the *fraction of external consumers* among all consumers of type θ; the fraction of loyal consumers is $(1-\beta_\theta)$. We normalize the total number of type θ consumers, including both loyal and potential external customers, to one, and use the terms fraction of consumers and number of consumers interchangeably. An external consumer's preference for an outside option (compared to using the firm's PFD option) may arise due to a variety of reasons including her familiarity, proximity, ease of shopping, or loyalty to another retailer. We model this preference as a positive switching or *opportunity cost* δ; by permitting this cost to vary by external consumer, we consider further heterogeneity among external consumers in terms of their inclination to switch to the firm's FDS plan. Let $F_\theta(\delta)$, for $\delta \in (0, \delta_\theta^{max}]$, denote the CDF of the opportunity cost δ among the external consumers of type θ.[4] As we discuss in the next section, by setting the subscription fee *S* sufficiently low, the firm can compensate for the opportunity cost for at least some external consumers, thus incenting them to switch to the firm's FDS plan. Thus, instead of fixing a priori the number or fraction of external consumers who will switch to the firm's FDS plan, our model endogenizes this fraction as a function of the FDS pricing policy. We assume that, if an external consumer subscribes to the firm's FDS option, she switches fully from her outside option to purchasing from the firm.

In the remainder of the paper, when we say that some value (e.g., profit, purchase quantity, surplus) "increases" (or "decreases") we mean that the value increases (decreases) or stays the same. We use "strictly increases" (or "strictly decreases") to identify situations when the value does not stay the same.

3.2 Consumers' Purchasing Decision

Given the PFD option with unit delivery charge *d* and a FDS plan $<S, L>$, each loyal consumer must decide which option to use, and how much to purchase, while external consumers must decide whether to switch to the FDS plan from their current purchasing option. To understand consumers' purchasing behavior, we first determine the optimal PFD and FDS purchase quantities for each consumer type, and compare the surplus for each option to determine which option a loyal consumer would select.

Optimal purchasing quantities under PFD and FDS:

Let $\zeta_\theta^{PFD}(y)$ denote the surplus that a loyal consumer of type θ gets if she chooses to purchase *y* units under PFD. Since the consumer incurs a cost of ($p+d+g$) per unit under the PFD option, $\zeta_\theta^{PFD}(y) = u_\theta(y) - (p+d+g)y$. So, to maximize her surplus, the consumer's optimal **PFD purchase quantity** is the value w_θ that satisfies $du_\theta(y)/dy\big|_{y=w_\theta} = (p+d+g)$.[5]

Under the FDS plan $<S, L>$, for a type θ consumer who subscribes to the plan and wishes to purchase *y* units, the firm will provide free delivery for $min(y, L)$ units, and the consumer needs to pay the delivery charge and incurs the transaction cost for the remaining $(y - min(y, L))$ units. Hence, the subscriber's FDS surplus from purchasing *y* units is $\zeta_\theta^{FDS}(y,S,L) = u_\theta(y) - py - S - (d+g)(y-L)^+$. If the firm offers *unlimited free deliveries*, i.e., when $L = \infty$, this consumer surplus expression reduces to $\zeta_\theta^{FDS}(y,S,\infty) = u_\theta(y) - py - S$. In this case, the consumer's optimal FDS purchase quantity, to maximize her surplus, is the value v_θ that satisfies $du_\theta(y)/dy\big|_{y=v_\theta} = p$. Observe that, since the PFD delivery charge

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[4] As we discuss in Section 3.2, the firm need not consider external consumers with excessively high opportunity cost, i.e., we can assume a finite upper bound δ_θ^{max} without loss of generality.

[5] We assume that $du_\theta(y)/dy\big|_{y=0} > (p+d+g)$; otherwise, the PFD purchase quantity is zero.

d is positive and the utility function is strictly concave, v_θ is finite and greater than w_θ. Moreover, this purchase quantity does not depend on the subscription fee *S* because this fee is a sunk cost once a consumer decides to subscribe. Table 1 provides expressions for the PFD quantity and the FDS quantity (with unlimited free deliveries) for the three illustrative utility functions shown earlier. (We discuss the last two columns of this table later.)

Utility Function family	Utility Function form	PFD qty. w_θ	FDS qty. v_θ (with $L=\infty$)	Lift $\lambda_\theta = v_\theta - w_\theta$ (with $L=\infty$)	Threshold Subscription Fee $\hat{S}_\theta$ (with $L=\infty$)
Quadratic	$\theta\rho y-(b-\theta\omega)y^2$, with $\omega\in[-1,1]$, $0<\theta\le b$	$\dfrac{\theta\rho-(p+d)}{2(b-\omega\theta)}$	$\dfrac{\theta\rho-p}{2(b-\omega\theta)}$	$\dfrac{d}{2(b-\omega\theta)}$	$\dfrac{d(2\rho\theta-2p-d)}{4(b-\omega\theta)}$ $=v_\theta^2-w_\theta^2$
Power	$\dfrac{\theta y^{1-\alpha}}{1-\alpha}$, with $\alpha\in(0,1)$	$\left(\dfrac{\theta}{p+d}\right)^{1/\alpha}$	$\left(\dfrac{\theta}{p}\right)^{1/\alpha}$	$\left(\dfrac{\theta}{p}\right)^{1/\alpha}-\left(\dfrac{\theta}{p+d}\right)^{1/\alpha}$	$\dfrac{\theta}{1-\alpha}(v_\theta^{1-\alpha}-w_\theta^{1-\alpha})$ $-\left(pv_\theta-(p+d)w_\theta\right)$
Exponential	$\dfrac{\theta\left(1-e^{-\alpha y}\right)}{\alpha}$, with $\alpha>0$	$\dfrac{1}{\alpha}ln\left(\dfrac{p+d}{\theta}\right)$	$\dfrac{1}{\alpha}ln\left(\dfrac{p}{\theta}\right)$	$\dfrac{1}{\alpha}ln\left(\dfrac{p}{p+d}\right)$	$(p+d)ln\left((p+d)/\theta\right)$ $-p\ ln\left(p/\theta\right)$ $-\theta^2\left(d/p(p+d)\right)$

Table 1: Example of common utility functions in economic models

When the FDS plan provides only limited free deliveries, with delivery limit $L<v_\theta$, let $v_\theta(L)$ denote a type θ subscriber's optimal *total* purchase quantity, consisting of a portion that qualifies for free delivery plus possibly additional units purchased using the PFD option. We refer to this total quantity (including any units purchased under the PFD option) as the **FDS purchase quantity**. Define $\lambda_\theta(L)\triangleq v_\theta(L)-w_\theta$ as the ***lift*** in a loyal consumer's purchase quantity when she subscribes to the FDS plan <*S*, *L*> instead of purchasing using PFD. Table 1 provides the expression for the lift in demand, when the FDS plan offers unlimited free deliveries, for our three representative utility functions. The next proposition discusses the relationship between $v_\theta(L)$ and *L*, and how the PFD and FDS purchase quantities as well as the lift in demand change as the consumer's costs and type vary.

Proposition 1: Optimal purchase quantities and lift in demand for loyal consumers

(i) The total purchase quantity $v_\theta(L)$ of a consumer of type θ who chooses the FDS option <*S*, *L*> is a piecewise increasing function of *L*, given by $v_\theta(L)=\min\{\max(w_\theta,L),v_\theta\}$.

(ii) The lift $\lambda_\theta(L)\triangleq vx_\theta(L)-w_\theta$ in a loyal consumer's purchase quantity when she chooses the FDS option is positive for $L > w_\theta$ (and zero if $0 \le L \le w_\theta$). Further, the lift is increasing with the delivery charge d and transaction cost g.

(iii) The PFD and FDS purchase quantities, w_θ and $v_\theta(L)$, are both increasing in consumer type θ.

The results in Proposition 1 lie at the core of the firm's benefit from offering the FDS option, and drive the subscription pricing decision. Part (i) shows that the total purchase quantity of a consumer who opts for the FDS plan is an increasing function of the free delivery limit L. Part (ii) establishes the existence of a lift in a loyal consumer's demand when she switches from PFD to FDS, based solely on economic factors, without even considering any behavioral effects that may cause consumers to purchase more after they have incurred the sunk cost (fee) for subscription. This property is consistent with observations from practice that existing customers' purchases increase when they switch to a FDS plan. Part (iii) of the proposition shows that consumers with higher types purchase higher quantities under both the PFD and FDS options. This result stems from the increasing differences property of the utility function, and implies that heterogeneity in consumer types corresponds to differentiating consumers based on their purchase quantities. Interestingly, although consumers with higher types purchase more under both the FDS and PFD options, the lift in purchase quantity can increase, decrease or remain constant as the consumer type θ increases. Figure 1 shows that even within the family of quadratic utility functions, changing the ω parameter can lead to higher, lower, or constant lift as consumer type increases.

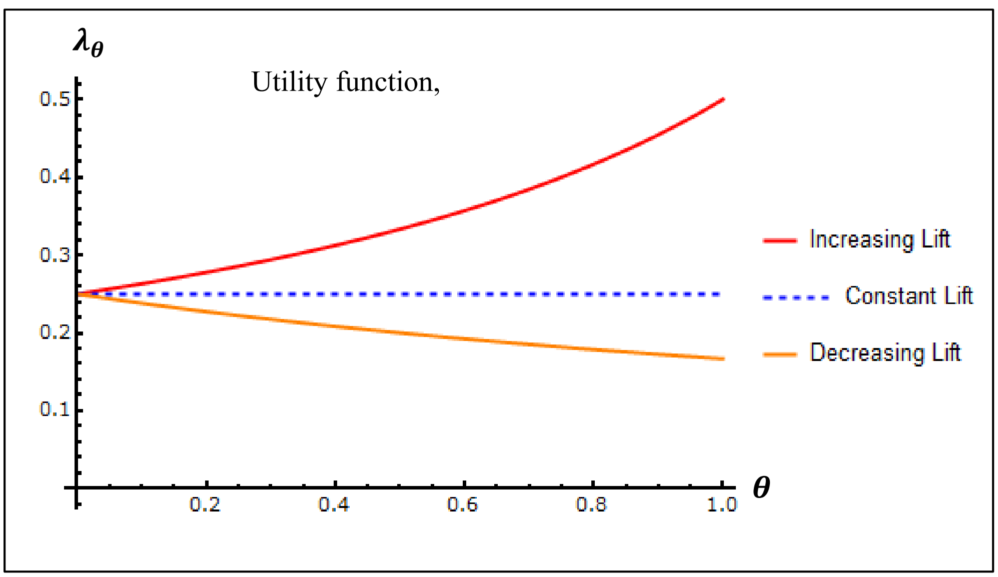

Figure 1: Variation of Lift $\lambda_\theta(L)$ for consumer type θ with Quadratic utility function

Consumer's optimal purchasing strategy:

Given the firm's FDS policy <S, L>, we now examine whether a type θ consumer – loyal or external

– should subscribe to FDS. We first consider *loyal* consumers who face the choice between purchasing using PFD or subscribing to FDS. Under PFD, the consumer obtains a surplus of $\zeta_\theta^{PFD}(w_\theta) = u_\theta(w_\theta) - (p+d+g)w_\theta$, whereas her surplus under the FDS policy <*S*, *L*> is $\zeta_\theta^{FDS}(v_\theta(L), S, L) = u_\theta(v_\theta(L)) - pv_\theta(L) - S - (d+g)(v_\theta(L) - L)^+$. As a convention, we assume that if a consumer is indifferent between the two options, she chooses the FDS option (as we will see later, the firm also prefers this choice). A loyal consumer of type θ will switch from PFD to FDS if and only if $\zeta_\theta^{FDS}(v_\theta(L), S, L) \geq \zeta_\theta^{PFD}(w_\theta)$. Since the FDS surplus increases as the subscription fee *S* decreases, the previous condition implies that the consumer finds the FDS option to be more attractive if and only if:

$$S \leq \hat{S}_\theta(L) \triangleq u_\theta(v_\theta(L)) - u_\theta(w_\theta) + (d+g)\min\left(w_\theta, L\right) - p(v_\theta(L) - w_\theta) . \tag{3.1}$$

The value $\hat{S}_\theta(L)$, which we call the *threshold subscription fee*, is the maximum subscription fee that a loyal type θ consumer is willing to pay in order to select the FDS plan (and increase her purchase quantity) instead of using PFD. Observe that, at this threshold subscription fee, a loyal consumer's surplus is the same for both the PFD and FDS options. We next discuss how $\hat{S}_\theta(L)$ changes with consumer type θ, free shipping limit *L*, delivery charge *d*, and consumer's transaction cost *g*. Define $\hat{S}_\theta$ as the threshold subscription fee for an FDS plan that offers unlimited deliveries (i.e., with $L = \infty$).

Lemma 1: Properties of the threshold subscription fee $\hat{S}_\theta(L)$

For a consumer of type θ,

(i) $\hat{S}_\theta(L)$ equals $(d+g)L$ for $0 \leq L \leq w_\theta$, is strictly concave and increasing from $(d+g)w_\theta$ to $\hat{S}_\theta$ as *L* increases from w_θ to v_θ, and is constant at $\hat{S}_\theta$ for $L \geq v_\theta$.

(ii) $\hat{S}_\theta(L) < (d+g)L$ for all $L > w_\theta$.

(iii) $\hat{S}_\theta(L)$ increases with consumer type θ, and, for a given type, $\hat{S}_\theta(L)$ strictly increases with the delivery charge *d* and consumer transaction cost *g*.

The result in part (i) implies that $\hat{S}_\theta(L)$ is a continuous, concave, and increasing function of *L*, increasing first, for $0 \leq L \leq w_\theta$, at the rate $(d + L)$ (which is the total per unit cost the consumer would have incurred under the PFD option); beyond w_θ, *L* increases at a decreasing rate for $w_\theta < L < v_\theta$, until it attains the constant value $\hat{S}_\theta$ for $L \geq v_\theta$. The threshold subscription fee for consumer type θ increases as *L* increases (part (i)) since a consumer's incremental surplus (excluding the subscription fee) under the FDS option increases with *L*. To interpret part (ii), recall that, for $w_\theta < L < v_\theta$, the consumer's total FDS purchase quantity $v_\theta(L)$ is *L* units. Part (ii) has the important implication that the maximum subscription fee that the loyal consumer is willing to pay in this range of L is strictly less than the cost she would have paid for her purchase quantity under the PFD option. Therefore, if the firm's unit shipping cost is high, i.e., $e \geq d + g$ (e.g., if the firm's shipping charge is the same as the delivery charge, and the consumer's transaction cost g is 0), then the subscription fee is always lower than the firm's total cost for shipping to the customer, i.e., the subscription option is "lossy" for the firm. Interestingly, we show in the next section, that the firm prefers to offer the FDS option even in this situation. Finally, part (iii) shows that a loyal consumer with higher type receives higher incremental surplus (excluding the subscription fee), hence is willing to pay a higher subscription fee to switch from PFD to FDS. Figure 2 shows the threshold subscription fee as a function of L for two consumer types θ_1 and θ_2, with $\theta_2 > \theta_1$. We exploit this monotonic behavior of $\hat{S}_\theta(L)$ with θ when we analyze a model with multiple consumer types in Section 4. As the delivery charge d or consumer's transaction cost g increases, a loyal consumer's surplus under the PFD option decreases, making the FDS option more valuable. Thus, loyal consumers are willing to pay a higher subscription fee.

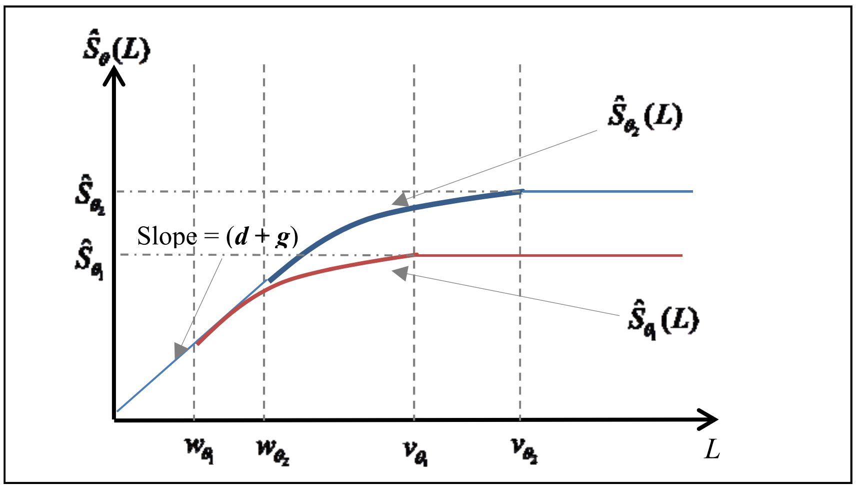

Figure 2: Threshold Subscription Fee $\hat{S}_\theta(L)$ for consumers of types θ_1 and θ_2

Purchasing strategy of external consumers:

External consumers prefer not to purchase from the firm using the PFD option. To model this preference, we posit that each external consumer of type θ has some outside option that provides an additional positive surplus of δ (that can vary by consumer) compared to a loyal consumer's PFD surplus of $\zeta_\theta^{PFD}(w_\theta)$. Thus, δ is a switching or *opportunity cost* for the consumer that must be overcome by the firm to attract the consumer. (Effectively, loyal consumers have an opportunity cost of zero). Recall that $F_\theta(\delta)$, for $\delta \in (0, \delta_\theta^{max}]$, is the CDF of the opportunity cost δ among the external consumers of type θ.

The FDS option provides the means to compensate for an external consumer's opportunity cost by setting a low subscription fee for the FDS plan. Specifically, an external customer with opportunity cost δ will switch to the firm's FDS option if the surplus from this option equals or exceeds $(\zeta_\theta^{PFD}(w_\theta)+\delta)$. Since the FDS surplus for a type θ consumer at the threshold subscription fee $\hat{S}_\theta(L)$ equals $\zeta_\theta^{PFD}(w_\theta)$, the firm needs to reduce its subscription fee to at least δ below the threshold value in order to attract the external consumer with opportunity cost δ. Suppose, for a given shipping limit L, the firm charges a subscription fee of $S \le \hat{S}_\theta(L)$. Then, since the <S, L> FDS plan provides a surplus of $\zeta_\theta^{PFD}(w_\theta)+(\hat{S}_\theta(L)-S)$ to consumer type θ, all external consumers with opportunity cost $\delta \le \hat{S}_\theta(L)-S$ will switch to the firm and subscribe to its FDS option (in addition, all loyal consumers will also purchase using this option). Therefore, for $S \le \hat{S}_\theta(L)$, the fraction $\psi_\theta(S,L)$ of consumers of type θ who purchase using the FDS option <S, L> is:

$$\psi_\theta(S,L)=(1-\beta_\theta)+\beta_\theta F_\theta(\hat{S}_\theta(L)-S)\,. \tag{3.2}$$

The first term corresponds to the loyal consumers, and the second term represents the external consumers who subscribe to the firm's FDS option. Thus, starting with the threshold subscription fee $\hat{S}_\theta(L)$, as the firm reduces its subscription fee, the number of FDS subscribers increases due to more external consumers switching to the firm, and the rate of increase in consumers depends on the distribution of external consumers' opportunity cost δ. If $S > \hat{S}_\theta(L)$, no consumer (loyal or external) will subscribe.

Define $D_\theta(S)=\hat{S}_\theta(L)-S$. This value $D_\theta(S)$ represents a "subsidy," relative to the threshold subscription fee, that the firm provides to loyal consumers in order to attract some external consumers. Since S must be non-negative, the subsidy can never exceed $\hat{S}_\theta(L)$, which in turn is less than or equal to $\hat{S}_\theta$, the threshold subscription fee for the FDS plan with unlimited free deliveries. Hence, only consumers with $\delta \le \hat{S}_\theta$ are potential customers for the firm. We can, therefore, assume without loss of generality, that $\delta_\theta^{\max} \le \hat{S}_\theta$.

Using this characterization of the consumer's purchase decision process and firm's FDS demand function, we next analyze the retailer's optimal subscription pricing policy with a single consumer type and identify different drivers of this policy.

3.3 Optimal FDS Pricing Policy with Single Consumer Type

We begin our analysis of the firm's optimal FDS pricing policy by considering the case when all consumers have the same type θ, but differ in their preference for purchasing from the firm using the PFD option (i.e., in their opportunity cost). Examining this special case permits us to first study the effects of firm preference as the only dimension of consumer heterogeneity on the firm's profit and subscription pricing strategy. Furthermore, the “single-consumer-type” problem also serves as a building block for the analysis with multiple consumer types (Section 4). Equation (3.2) shows that the firm can increase the number of FDS consumers (and hence sales revenue) by increasing the subsidy for the loyal consumers (thereby, losing some subscription revenue). This tradeoff is central to the firm’s optimal FDS pricing policy when all consumers have the same type.

Given the price p, costs c and e, PFD delivery charge d, firm’s transacion cost τ, the firm’s PFD profit is $\Pi_\theta^{PFD} = (1-\beta_\theta)(p-c+(d-e)-\tau)w_\theta$ since only loyal consumers purchase using the PFD option (w_θ units each) and the firm’s profit margin per unit sold under the PFD option is $(p-c+(d-e)-\tau)$. We next specify the firm’s FDS profit function $\Pi_\theta^{FDS}(S,L)$ for a given candidate FDS policy <S, L>, with $S \le \hat{S}_\theta(L)$. (If $S > \hat{S}_\theta(L)$, the firm’s profit is the same as the PFD profit since no consumer subscribes to the FDS option.) Let $\hat{\pi}_\theta(L) = \hat{S}_\theta(L) + (p-c-e)v_\theta(L) + (d-\tau)(v_\theta(L)-L)^+$ denote the firm’s profit *per FDS subscriber* if it sets the subscription fee to be exactly equal to the threshold value, i.e., if $S = \hat{S}_\theta(L)$; we refer to this profit as the *threshold profit per subscriber*. At the subscription fee $S = \hat{S}_\theta(L)$, only loyal consumers subscribe to the FDS plan (since the firm does not provide any subsidy), and so $\Pi_\theta^{FDS}(\hat{S}_\theta(L), L) = (1-\beta_\theta)\hat{\pi}_\theta(L)$. For any subscription fee $S < \hat{S}_\theta(L)$, the firm also attracts all external consumers with opportunity cost $\delta \le \hat{S}_\theta(L) - S$ to FDS. Hence its FDS profit function is:

$$\Pi_\theta^{FDS}(S,L) = ((1-\beta_\theta)+\beta_\theta F_\theta(\hat{S}_\theta(L)-S))(S+(p-c-e)v_\theta(L)+(d-\tau)(v_\theta(L)-K)^+)$$
$$= \psi_\theta(S,L)(\hat{\pi}_\theta(L) - (\hat{S}_\theta(L)-S)).$$

The firm’s PFD transaction cost τ does not affect a consumer’s behavior and only serves to make the FDS plan more attractive than the PFD option for the firm. To focus on whether the FDS plan can be profitable without the savings in transaction cost, we set $\tau = 0$ in all our subsequent analysis. However, our results also hold for any positive transaction cost τ.

Under the FDS option, the firm bears the unit shipping cost e and does not impose a per unit delivery charge on the customer. It would, therefore, be reasonable to assume that $e < p - c$, so that the profit margin per unit sold under the FDS plan, $(p-c-e)$, is positive. However, we initially impose the following less stringent upper bound, $e \le \hat{e}_\theta - c$, where $\hat{e}_\theta = \left(u_\theta(v_\theta) - u_\theta(w_\theta)\right)/\left(v_\theta - w_\theta\right)$ to ensure the viability of the FDS plan. We can show that $\hat{e}_\theta > p$, and so our bound for shipping cost even permits negative (but not too negative) profit margin per FDS unit. The upper-bound $\hat{e}_\theta - c$ reduces to a simple form for many utility functions; for example, $\hat{e}_\theta - c = p + (d/2)$ for the quadratic utility function (Table 1). Proposition 2 discusses properties of the optimal FDS policy with one consumer type.

Proposition 2: Properties of the firm's optimal FDS policy $< S_\theta^*, L_\theta^* >$ with single consumer type θ

(i) Offering the FDS option, with a single subscription plan, strictly increases the firm's profits (even without external consumers), for $e < \hat{e}_\theta - c$. Moreover, under the FDS option, each consumer's surplus is equal to or higher than the surplus from their current purchasing option.

(ii) The firm's optimal free shipping limit L_θ^* is as follows:

$$L_\theta^* = \begin{cases} v_\theta & \text{if } e \le p - c \\ \hat{L}_\theta & \text{if } e > p - c \end{cases}, \text{ where } \hat{L}_\theta \text{ is the value of } L \text{ that satisfies } d\hat{S}_\theta(L)/dL + (p - c - e) = 0 .$$

(iii) The optimal FDS plan offers a subsidy of at most $\min(\beta_\theta \hat{\pi}_\theta(L_\theta^*), \delta_\theta^{\max})$, i.e.,

$$D_\theta^* \triangleq \hat{S}_\theta(L_\theta^*) - S_\theta^* \le \min\left(\beta_\theta \hat{\pi}_\theta(L_\theta^*), \delta_\theta^{\max}\right) .$$

(iv) The FDS plan attracts only loyal consumers (and does not attract any external consumer) if and only if $\beta_\theta \hat{\pi}_\theta(L_\theta^*) \le \left(\beta_\theta + (1-\beta_\theta)/\rho_\theta\right) F_\theta^{-1}(\rho_\theta)$ for all $\rho_\theta \in (0,1]$.

Part (i) establishes the important result that the FDS option *benefits both the firm and consumers*. Since all consumers have the same type, offering just one FDS plan suffices. For the firm, introducing the FDS option increases profits (compared to the PFD profits) even if we ignore external consumers and regardless of the price and cost parameters (except for the upper bound $e \le \hat{e}_\theta - c$), for any utility function $u_\theta(y)$ in the broad class we have considered and any distribution function $F_\theta(\delta)$ of the opportunity cost. This result holds even if $e \ge (d + g)$, in which case the FDS subscription fee is "lossy" compared to the total delivery costs. In our computational tests (Section 6), we show that the subscription fee may actually be only a small fraction of the total shipping costs, depending on the market composition and product features. Intuitively, the main driver of the profitability of FDS for the firm is the lift in a loyal consumer's demand when she switches from PFD to FDS. If the firm sets $S = \hat{S}_\theta(L)$, the loyal consumers are indifferent between PFD and FDS and subscribe to FDS. At this fee, the firm earns a *subscription revenue* of $\hat{S}_\theta(L)$ from each loyal consumer. If $e \le p - c$, i.e., the net contribution for each unit sold under FDS is non-negative, the firm's total contribution from sales also increases due to the lift in demand under FDS, making this option profitable for the firm even if only loyal consumers subscribe. When $e > p - c$, the firm's total contribution decreases with each additional unit sold under subscription. The result (i) implies that if the shipping cost is within reasonable bounds ($e \le \hat{e}_\theta - c$), the total subscription revenue the firm earns from all loyal consumers is higher than the reduction in the total profit contribution (due to the negative margin per unit under FDS). From the consumers' perspective, if all consumers have the same type, every loyal consumer and some or all external consumers (depending on the subsidy) will subscribe to the FDS plan. Since consumers subscribe to this plan only if it provides equal or higher surplus than their current purchasing option, offering the FDS option does not reduce the surplus for any consumer.

Part (ii) shows that, if $e \le p - c$, i.e., the profit margin per FDS unit sold is non-negative, then the optimal free shipping limit L_θ^* is the quantity v_θ that an FDS subscriber would purchase under a plan with unlimited free deliveries. In other words, with a single consumer type, offering an *unlimited* FDS plan is optimal when $e \le p - c$. Intuitively, in this case, for a given subscription fee *S*, since each unit increase in *L* (for $L \in [w_\theta, v_\theta)$) increases both subscription revenue and profit contribution from sales, the firm sets $L_\theta^* = \infty$ (more precisely, $L_\theta^* \ge v_\theta$,) and allows subscribers to purchase their surplus maximizing quantity v_θ. If $e > p - c$, since $(p - c + e + d) > 0$ and $dS_\theta(L)/dL = d + g$ for $L \le w_\theta$, L_θ^* must be greater than or equal to w_θ; so offering a FDS plan with a delivery limit $L_\theta^* \in [w_\theta, v_\theta]$ is optimal.

Part (iii) of the proposition provides a lower-bound on the optimal subscription fee. Offering a subsidy of $\delta_\theta^{\max}$ inserts all external consumers to subscribe to the firm's FDS plan, and so the optimal subsidy never exceeds this value. Moreover, for any subsidy $D_\theta > \beta_\theta \hat{\pi}_\theta(L_\theta^*)$, the firm can increase its profits by reducing the subsidy. Hence, $D_\theta^* \le \min\left(\beta_\theta \hat{\pi}_\theta(L_\theta^*), \delta_\theta^{\max}\right)$. Finally, part (iv) provides a necessary and sufficient condition for the firm's optimal policy to attract any external consumer. If $\delta \sim Uniform(0, \delta_\theta^{\max}]$, this necessary and sufficient condition reduces to the simple condition $\beta_\theta > \delta_\theta^{\max} / (\hat{\pi}_\theta + \delta_\theta^{\max})$.

Thus far, we have permitted the profit margin per FDS unit, $(p - c - e)$, to be negative (as long as $e < \hat{e}_\theta - c$). To keep the exposition simple in our subsequent discussions, **we assume that $e < (p - c)$, i.e., the firm earns a positive profit margin on each unit it sells to a FDS subscriber**. From Proposition 2, we then have $L_\theta^* = \infty$ for the single consumer type case, and so we will drop the argument *L* in the mathematical expressions. All our qualitative results for the single consumer type problem hold even when $e > (p - c)$ and $e \le \hat{e}_\theta - c$ holds.

Since we do not assume any specific distribution for δ (even continuity or differentiability) or a particular functional form for the utility function of a consumer, the FDS profit function is not necessarily concave and even unimodal in *S*. Remarkably, even with this general model, we are able to determine (in Section 4) how the firm's optimal subscription fee, optimal profits, and the total number of FDS consumers vary as the model parameters change.

Next, we restrict the possible distributions of δ to ensure concavity of the profit function and characterize the optimal subscription fee. For this purpose, we assume that the opportunity cost δ of external consumers varies continuously in the interval $(0,\delta_\theta^{\max}]$ and its PDF $f_\theta(\delta)$ is continuously differentiable and satisfies the following *concavity condition*: $f_\theta^{'}(\delta)/f_\theta(\delta)<2/(\hat{\pi}_\theta-\delta)$. Common distributions such as the Uniform and Exponential distributions satisfy this condition. Under thse conditions, Proposition 3 provides a characterization of the optimal subscription fee.

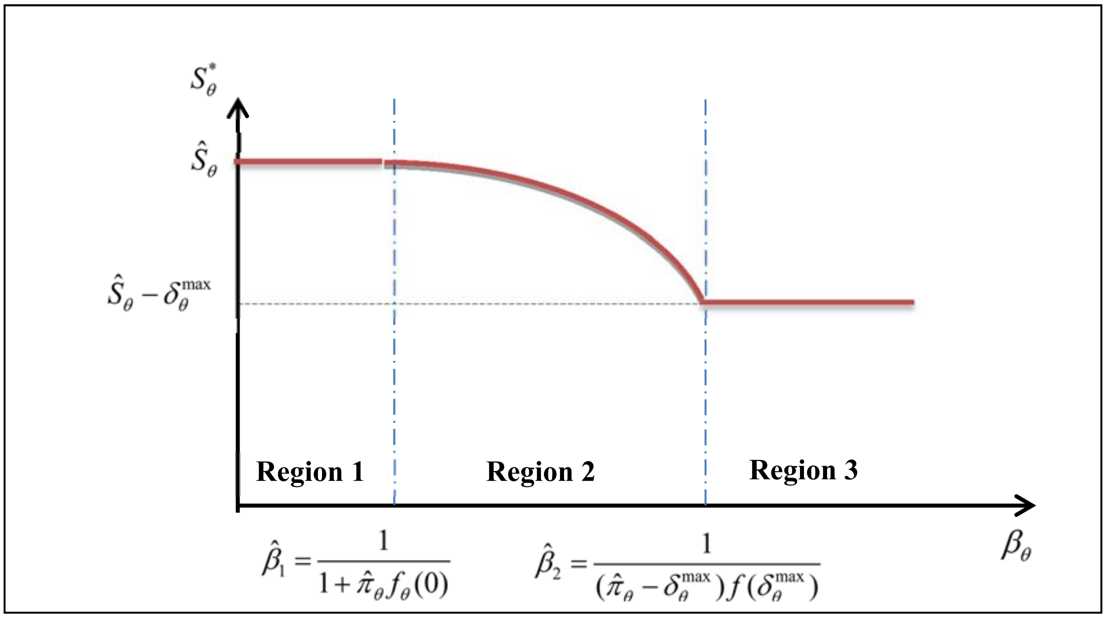

Figure 3: Optimal Subscription Fee as β increases

Proposition 3: Characterization of optimal subscription fees

If $f_\theta(\delta)$ satisfies $f_\theta^{'}(\delta)/f_\theta(\delta)<2/(\hat{\pi}_\theta-\delta)$ for all $\delta\in(0,\delta_\theta^{\max}]$, the optimal subsidy $D_\theta^*\triangleq\hat{S}_\theta-S_\theta^*$ is given by:

$$D_\theta=\begin{cases}0, & \text{if } 0\le\beta_\theta\le\dfrac{1}{1+\hat{\pi}_\theta f_\theta(0)}\\ \tilde{D}_\theta, & \text{if } \dfrac{1}{1+\hat{\pi}_\theta f_\theta(0)}<\beta_\theta\le\dfrac{1}{(\hat{\pi}_\theta-\delta_\theta^{\max})f(\delta_\theta^{\max})}\\ \delta_\theta^{\max}, & \text{if } \beta_\theta>\dfrac{1}{(\hat{\pi}_\theta-\delta_\theta^{\max})f(\delta_\theta^{\max})}\end{cases}, \text{ and}$$

where $\tilde{D}_\theta$ is the unique value of D_θ satisfying $(\hat{\pi}_\theta-D_\theta)f_\theta(D_\theta)-F_\theta(D_\theta)=(1-\beta_\theta)/\beta_\theta$.

Proposition 3 divides the optimal policy into three regions (as shown in Figure 3). In Region 1, the fraction β_θ of external consumers is low, i.e., $\beta_\theta\le 1/(1+\hat{\pi}_\theta f_\theta(0))$; the firm sets the subscription fee to $\hat{S}_\theta$ and does not attract any external consumer to purchase FDS. At the other extreme, if fraction β_θ of external consumers is very high, i.e., $\beta_\theta > 1\big/(\hat{\pi}_\theta - \delta_\theta^{\max}) f(\delta_\theta^{\max})$, the firm lowers its subscription fee significantly to attract all the external consumers to FDS. For any intermediate value β_θ, the firm sets the subscription fee to be between $\hat{S}_\theta$ and $(\hat{S}_\theta - \delta_\theta^{\max})$ and attracts only a fraction of external consumers to FDS. The condition on the distribution function ensures that there always exists a unique solution $\tilde{D}_\theta$ to the equation $(\hat{\pi}_\theta - D_\theta) f_\theta(D_\theta) - F_\theta(D_\theta) = (1-\beta_\theta)/\beta_\theta$ in the intermediate range. Moreover, the optimal subscription fee S_θ^* is a concave, decreasing function of β_θ as shown in Figure 3. An interesting observation here is that both thresholds of β, namely, $\hat{\beta}_1$ at which the firm does not attract any external consumer to FDS and $\hat{\beta}_2$ at which the firm attracts all external consumers to FDS, decrease as the threshold profit per subscriber increases. For Uniformly distributed opportunity cost δ, Table 2 provides the expressions for the optimal profit, subscription fee and the total number of consumers purchasing FDS in each region.

	$\beta_\theta \le \dfrac{\delta_\theta^{\max}}{\hat{\pi}_\theta + \delta_\theta^{\max}}$	$\dfrac{\delta_\theta^{\max}}{\hat{\pi}_\theta + \delta_\theta^{\max}} < \beta_\theta \le \dfrac{\delta_\theta^{\max}}{\hat{\pi}_\theta - \delta_\theta^{\max}}$	$\beta_\theta \ge \dfrac{\delta_\theta^{\max}}{\hat{\pi}_\theta - \delta_\theta^{\max}}$
S_θ^*	$\hat{S}_\theta$	$\hat{S}_\theta - \dfrac{1}{2}\left(\hat{\pi}_\theta - \dfrac{(1-\beta_\theta)}{\beta_\theta}\delta_\theta^{\max}\right)$	$\hat{S}_\theta - \delta_\theta^{\max}$
ψ_θ^*	$(1-\beta_\theta)$	$\dfrac{1}{2}\left((1-\beta_\theta) + \beta_\theta \dfrac{\hat{\pi}_\theta}{\delta_\theta^{\max}}\right)$	1
Π_θ^*	$(1-\beta_\theta)\hat{\pi}_\theta$	$\dfrac{\beta}{4\delta_\theta^{\max}}\left(\hat{\pi}_\theta + \dfrac{(1-\beta_\theta)}{\beta_\theta}\delta_\theta^{\max}\right)^2$	$\hat{\pi}_\theta - \delta_\theta^{\max}$

Table 2: Optimal profit, subscription fee and number of subscribers when $\delta \sim Uniform$

Maximum value of external consumers with limited distribution information:

An interesting question in FDS subscription pricing is how much profit is lost, relative to the optimal FDS policy, if a firm targets only loyal consumers for the FDS option by setting the subscription fee at the threshold value $S = \hat{S}_\theta(L)$, thereby not attracting any external consumers. We next provide an upper bound on the ratio of profits if we target both types of consumers versus only loyal consumers. For this result, we only assume that the firm knows the mean opportunity cost μ among all external consumers, and do not assume any particular distribution of this cost. Proposition 4 contains the results of this distribution-free "worst-case" type result.

Proposition 4: **Optimal FDS profit versus loyal customers-only FDS profit**

If the mean opportunity cost of external consumers is μ, the maximum possible profit ratio with a single consumer type is $\frac{\Pi^{both}}{\Pi^{loyal}} = \frac{1}{(1-\beta_\theta)}\left(1-\frac{\mu}{\hat{\pi}_\theta}\right)$ if $\beta_\theta \hat{\pi}_\theta \geq \delta_\theta^{\max}$, and $\frac{\Pi^{both}}{\Pi^{loyal}} = \left[1+\left(\frac{\beta_\theta}{1-\beta_\theta}\right)\left(1-\frac{\mu}{\delta_\theta^{\max}}\right)\right]$ if $\beta_\theta \hat{\pi}_\theta < \delta_\theta^{\max}$, where Π^{loyal} and Π^{both} respectively denote the profits considering only loyal consumers and both loyal and external consumers.

The simple and intuitive expressions for the maximum possible profit ratio highlights the key drivers affecting the firm's trade-off between attracting external consumers and subsidizing loyal consumers. As the mean opportunity cost μ increases, the relative benefit from attracting external consumers decreases since the firm has to offer a higher subsidy to the loyal consumers. Similarly, as the threshold profit per subscriber increases, the firm can benefit more by attracting external consumers. Interestingly, the result shows that the mean opportunity cost and the threshold profit per subscriber can balance each other so that the firm's profit does not change.

The analysis in this section of optimal FDS subscription pricing with a single consumer-type analysis provides interesting insights into the trade-off between attracting higher number of external consumers and the profitability from loyal consumers. Introducing an additional dimension of heterogeneity, namely, multiple consumer types introduces the additional tradeoff of which consumer type to target for the FDS plan. In the next two sections, we extend the analysis of this section to address the more complex problem when there are multiple consumer types.

4. Single FDS Plan with Multiple Consumer Types

For the single consumer type problem we studied in Section 3.3, all loyal consumers and possibly some or all external consumers subscribe to the optimal FDS policy (which outperforms the PFD option). So, the subscription pricing problem reduces to deciding what fraction of external consumers to attract by appropriately setting the subscription fee and free delivery limit. We now study the FDS subscription pricing problem when consumers differ not only in their opportunity cost but are also heterogeneous in their value for the product, i.e., they have different types. Correspondingly, the subscription pricing problem becomes more complex: the firm must now decide which consumer segments to target for the FDS option and what fraction of external consumers in these segments to attract to FDS (these two decisions are interrelated). The consumer types that the firm chooses to attract to the subscription plan depends on the lift in demand for each customer type, the market composition, i.e., number of consumers of each type in the market, the fraction of loyal and external consumers within each type, and the distribution of opportunity cost δ for external consumers of each type. As Figure 1 shows, a consumer with lower type can have a larger lift than a consumer with higher type, depending on the utility function. Thus, the firm may find it beneficial to substantially reduce its subscription fee in order to attract consumers of lower types, particularly if a large fraction of these consumers are external. Therefore, identifying the best subscription policy requires making tradeoffs, across multiple consumer types, between reducing overall subscription revenues and increasing contribution from product sales due to lift and additional customers. We extend our previous general model to incorporate multiple consumer types, and analyze this model to generate insights about the structure and sensitivity of the optimal policy.

In practice, online retailers often offer only one FDS plan (in addition to the PFD option); moreover, this plan often provides *unlimited free deliveries*, i.e., $L = \infty$. We refer to such a plan as a "Universal" FDS plan, abbreviated as the *Universal plan*. This section analyzes the subscription fee setting problem for this Universal plan; Section 5 considers the more general family of Tiered FDS subscription plans. The firm continues to also offer the existing PFD option. Each consumer then self-selects whether to switch to the firm's subscription plan or continue with her current purchasing option.

One of the key properties we exploit in of our analysis of the model for the Universal plan with multiple consumer types (as well as our later model) is that, if a customer of a particular type subscribes to a given Universal FDS policy, then consumers of all higher types will also subscribe. So, the subscription pricing problem reduces to deciding the lowest customer type to target for the FDS policy and what fraction of external consumers of this and higher types to attract. Based on this property, we develop a method that uses a single-consumer-type solution procedure to find the optimal Universal plan. This method relies on a decomposition of the space of possible subscription fees into a number of intervals such that the subscribing consumer types does not change within each interval, applying the single-consumer-type procedure to a modified single-consumer-type problem, and selecting the best interval. Besides leading to this overall solution procedure, the decomposition also permits us to use insights from Section 3.1 for the single-consumer-type problem to characterize the optimal solution and conduct comparative statics with respect to the model parameters.

We first introduce some additional notation for the discussions in this section. Let K be the number of different consumer types, and let θ_k be the parameter associated with the k^{th} consumer type. We index the consumer types from 1 to K in increasing order of θ_k, i.e., $\theta_1 < \theta_2 < ... < \theta_K$. Henceforth, we use consumer type k to refer to a consumer with type parameter θ_k. Further, we replace the previous subscript θ for customer type (e.g., for purchase quantities, threshold subscription fee, etc.) with the subscript k when referring to consumer type k. Let γ_k be the fraction of consumers (loyal and potential external consumers) of type k, with the total market size normalized to 1, i.e., $\sum_{k=1}^{K} \gamma_k = 1$. Consumers of each type are further classified into loyal and external consumers. Let β_k be the fraction of external consumers among all consumers of type *k* (consequently, $(1-\beta_k)$ is the fraction of loyal consumers among consumers of type *k*). Moreover, the external consumers of type *k* also vary in their opportunity cost δ , where $\delta \in (0, \delta_k^{\max}]$ with CDF $F_k(\delta)$. Since the Universal plan provides unlimited free deliveries, we use only the subscription fee *S* as the argument for various functions (e.g., profit), omitting the delivery limit *L* as an argument.

Universal Plan Profit Functions:

At a subscription fee *S*, a consumer of type *k* subscribes to FDS only if $S \le \hat{S}_k$; otherwise type *k* loyal consumers choose PFD. Let $IK(S) = \{k \mid \hat{S}_k \ge S\}$ be the set of consumer-types that purchase FDS, and $PK(S) = \{k \mid \hat{S}_k < S\}$ the types that purchase PFD when the Universal subscription fee is *S*. Since the threshold subscription fees for different consumer types satisfy the ordering $\hat{S}_1 < \hat{S}_2 < ... < \hat{S}_K$ (from Lemma 1), if a loyal consumer of type *k* does *not* subscribe to FDS, then no consumer of type $k' < k$ will subscribe to FDS. We refer to this property as the *nested FDS subscribers* property. Hence, the set *IK*(*S*), if not empty, is a contiguous set of consumer types starting from type *K*. Let $r^{PFD} = ((p-c)+(d-e))$ and $r^{FDS} = (p-c-e)$ respectively denote the firm's profit margin from each unit it sells under the PFD option and the Universal plan. Then, the firm's profit function under the Universal plan for any *S* is given by:

$$\Pi^U(S) = \sum_{k \in IK(S)} ((1-\beta_k) + \beta_k F_k(\hat{S}_k - S))(S + r^{FDS} v_k) + \sum_{k' \in PK(S)} (1-\beta_{k'}) r^{PFD} w_{k'} \quad (4.1)$$

The first term of the profit function is the profit from all (loyal and external) FDS subscribers and the second term is the profit from all loyal PFD consumers.

Proposition 5: Benefit of offering the Universal plan

Offering the Universal plan increases the firm's profits (even without external consumers) compared to offering only the PFD option. Moreover, with such a plan, every consumer's surplus increases compared to their surplus from their current purchasing option.

As for the single customer type case, the proposition shows that both the firm and the consumers benefit when the firm offers the Universal plan. This plan's subscription fee attracts at least the loyal consumers of type *K* (highest consumer type) to subscribe to the FDS plan. The lift in their demand raises the firm's profit over the PFD profits.

To understand the tradeoffs in subscription pricing for the Universal plan, let us examine how reducing its subscription fee incents additional consumers to switch to the FDS plan. Starting with the threshold subscription fee $S = \hat{S}_K$ for the highest consumer type (type *K*), as we decrease *S*, it first attracts external consumers of type *K*, then attracts loyal consumers of type (*K* – 1) when $S = \hat{S}_{K-1} < \hat{S}_K$

and so on. Correspondingly, the first term in the profit function (4.1) increases and the second term decreases. Interestingly, when a loyal consumer of type k' subscribes to FDS (at $S = \hat{S}_{k'}$) some of the external consumers of with higher type $k > k'$ may not have not switched to FDS since the subsidy of $D_k(\hat{S}_{k'}) = \hat{S}_k - \hat{S}_{k'}$ may not be adequate to overcome the opportunity cost for these consumers. This situation occurs if $\delta_k^{\max} > \hat{S}_k - \hat{S}_{k'}$.

Lower consumer types may have high demand lift (e.g., see Figure 1) and/or have high potential number of external consumers. Hence, the firm's profits can increase even when we lower the subscription fee substantially to attract these consumers. Thus, the Universal plan requires making tradeoffs across customer types to decide which (nested) segment to target for the FDS plan, and then deciding what fraction of external consumers to attract (among the consumer types who will subscribe to the plan). The Universal plan profit function is non-concave and discontinuous in subscription fee *S*. The profit function is discontinuous at $S = \hat{S}_k$ for each $k = 1,...,K$ where loyal consumers of type *k* switch from PFD to FDS (Proposition 2).

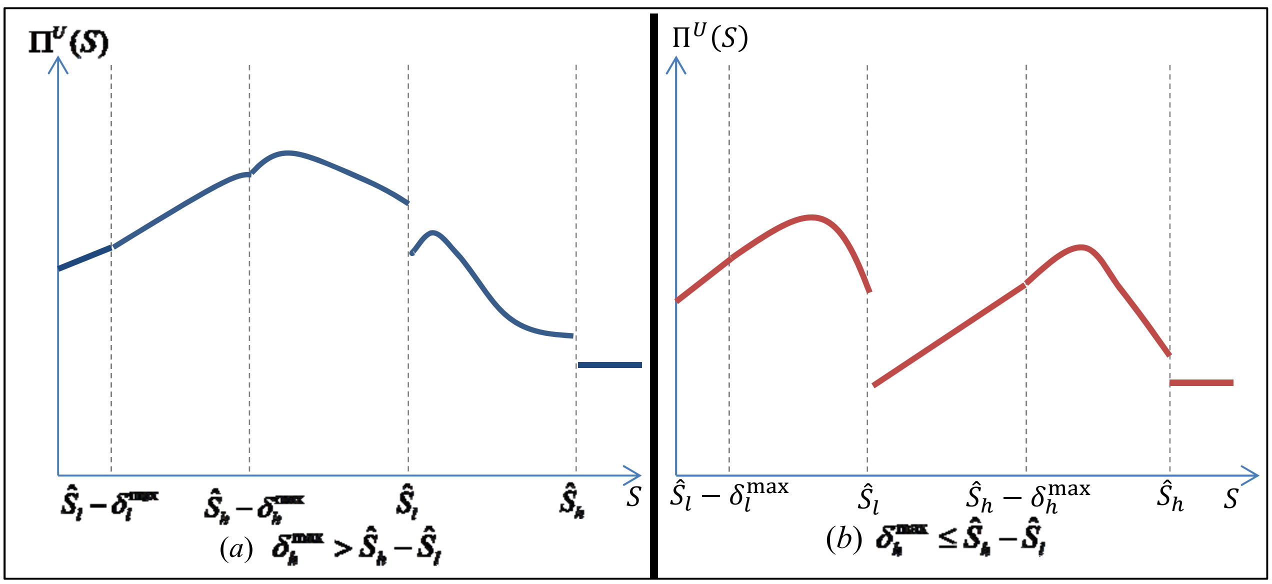

Figure 4: Examples of Universal plan profit function for two consumer types: *High* (*h*) and *Low* (*l*).

Figure 4 illustrates this complex behavior of the Universal plan profit as a function of the subscription fee, for a problem with just two consumer types, High (*h*) and Low (*l*). The consumer types that purchase FDS at any subscription fee *S* depend on the threshold subscription fees $\hat{S}_k$ for $k = 1,...,K$ and the maximum opportunity cost $\delta_k^{\max}$ for external consumers of each type *k*. Figure 4(a) shows case when $\delta_k^{\max} > \hat{S}_k - \hat{S}_{k'}$ for consumer types k, k' with $k > k'$, and Figure 4(b) illustrates the behavior when

$\delta_k^{\max} \le \hat{S}_k - \hat{S}_{k-1}$ for each consumer type $k = 2,...,K$. We refer to this latter situation as the *Low opportunity costs* case. Under this special case, if any consumer of type *k* switches to FDS then *all* consumers of type $k+1,\ldots,K$ must have already switched to FDS. As we discuss later, this property makes the subscription pricing problem somewhat easier to solve.

The non-concavity and discontinuity of the profit function makes it difficult to obtain a closed-form expression for the optimal Universal subscription fee. We develop a method to find the optimal solution by solving a sequence of *O*(*K*) "modified" single-consumer-type sub-problems. The validity of the approach rests on the fact that the optimal Universal subscription fee is the optimal solution to one of these subproblems. Next, we briefly discuss this solution procedure.

For the Universal plan problem with K consumer types, the optimal subscription fee S^* lies in the range $[\min_{k\in K}(\hat{S}_k - \delta_k^{\max}), \hat{S}_K]$. To find the optimal value, we divide this range into at most (2*K* – 1) *intervals* by arranging the values of $\hat{S}_k$ and $\hat{S}_k - \delta_k^{\max}$, for each *k*, in increasing order. Let *H* denote this set of intervals, with each interval $h \in H$ defined by $(\tilde{S}_{h-1}, \tilde{S}_h]$. For example, in Figures 4(a) and 4(b), we get three intervals (with two consumer types). For the scenario in Figure 4(a), the set *H* is $\{(\hat{S}_l - \delta_l^{\max}, \hat{S}_h - \delta_h^{\max}], (\hat{S}_h - \delta_h^{\max}, \hat{S}_l], (\hat{S}_l, \hat{S}_h]\}$, and for Figure 4(b), the set *H* is $\{(\hat{S}_l - \delta_l^{\max}, \hat{S}_l], (\hat{S}_l, \hat{S}_h - \delta_h^{\max}], (\hat{S}_h - \delta_h^{\max}, \hat{S}_h]\}$. Let S_h^* be the optimal subscription fee within an interval $h \in H$. Since the optimal Universal subscription fee S^* lies in one of the $|H|$ intervals, S^* must be the best (i.e., one with maximum profit) among the optimal solutions S_h^* for $h \in H$.

Our scheme to partition the range of possible Universal fee values into $|H|$ intervals is motivated by the fact that, within each interval, the set of consumer types who subscribe to the FDS plan for subscription fees in that interval does not change. Specifically, in the interval $(\tilde{S}_{h-1}, \tilde{S}_h]$, all consumer types *k* with $\hat{S}_k \ge \tilde{S}_h$ subscribe to FDS (although not all external consumers of this type may subscribe), and none of the consumer types *k*' with $\hat{S}_{k'} < \tilde{S}_{h-1}$ subscribe to FDS (all the loyal consumers of these types purchase using PFD). Hence, the problem of finding the optimal fee within interval *h* reduces to a generalized version of the single-consumer-type problem. The generalization is needed to account for the fact that decreasing the fee within interval *h* attracts additional consumers with different types whose FDS purchase quantities are different. Figure 5 outlines the steps of the overall procedure to find the optimal value of the Universal fee.

Universal plan algorithm

- Arrange all consumer types in increasing order from $1,\ldots,K$.
- Let *H* be the set of intervals created by arranging $\hat{S}_k$ and $\hat{S}_k - \delta_k^{\max}$ in increasing order for each *k*. Define each interval $h \in H$ by $(\hat{S}_{h-1}, \hat{S}_h]$
- For each interval $h \in H$, define the following for $S \in (\hat{S}_{h-1}, \hat{S}_h]$:

  Number of FDS subscribers at *S*, $A_h(S) = \sum_{k \in IK(S)} \gamma_k [(1-\beta_k) + \beta_k F_k(\hat{S}_k - S)]$

  Total purchase volume by subscribers at *S*, $B_h(S) = \sum_{k \in IK(S)} \gamma_k [(1-\beta_k) + \beta_k F_k(\hat{S}_k - S)] v_k$

  Total purchase volume of all PFD consumers in interval *h*, $C_h = \sum_{k \in PK(S)} \gamma_k (1-\beta_k) w_k$
- Define the profit function for each interval *h* as $\Pi_h(S) = A_h(S) + B_h(S).r^{FDS} + C_h.r^{PPD}$ for $S \in (\hat{S}_{h-1}, \hat{S}_h]$
- Obtain the optimal solution S_h^* within each interval *h* by solving the corresponding single-consumer-type subproblem
- The optimal Universal plan subscription fee $S^* = \arg\max_{S_h^*} \{\Pi_h(S_h^*) : h \in H\}$

Figure 5: Solution procedure for Universal plan pricing problem

Thus, using solution method for the single-consumer-type subscription pricing subproblem, we can solve the Universal plan subscription pricing problem by applying this method *O*(*K*) times. We remark that our strategy of decomposing the overall problem into the simpler single-type subproblems not only facilitates solving the problem but also permits us to identify characteristics of the optimal solution while retaining our general modeling framework. In particular, since analyzing a single-consumer-type solution is easy, we can readily extend the properties of this solution to assess the behavior of the optimal Universal subscription fee, profit, and the total number of FDS subscribers as the problem parameters change (see later discussion of comparative statics). The next proposition formally establishes the validity of the Universal plan solution method, and describes a simpler method for the Low opportunity costs case.

Proposition 6: Optimal Universal subscription fee

(i) The Universal FDS plan algorithm finds the optimal Universal FDS subscription fee.

(ii) Let $S1_k^*(\alpha_k, v_k)$ be the optimal solution to the single-consumer-type problem considering only type *k* consumers, with α_k loyal and v_k external consumers. With low opportunity costs, i.e., if $\delta_k^{\max} \le \hat{S}_k - \hat{S}_{k-1}$ for $k = 2,\ldots,K$, the optimal Universal FDS subscription fee S^* is the best (profit maximizing) value among $S1_k^*(\tilde{\alpha}_k, v_k)$, for *k* = 1, 2, …*K,* where $\tilde{\alpha}_k = \gamma_k(1-\beta_k) + \sum_{k'>k} \gamma_{k'}$ and $v_k = \gamma_k \beta_k$.

The simplified method for the Low opportunity costs special case, described in part (ii), exploits two properties of this special case. First, we need to consider only K intervals, one for each customer type k and defined as $[\hat{S}_k - \delta_k^{\max}, \hat{S}_k]$, in the partition of the range for the Universal subscription fee. Hence, we only need to solve K subproblems. Second, for subscription fees within the k^{th} range, *all* consumers (loyal as well as all external consumers) of higher types $k' > k$ will definitely subscribe to the FDS plan. So, we can effectively treat all these consumers as "loyal" consumers when deciding the best fee in the k^{th} range. The parameter $\tilde{\alpha}_k = \gamma_k(1-\beta_k) + \sum_{k'>k} \gamma_{k'}$ specifies the adjustment needed in the fraction of "loyal" consumers (to account for all higher consumer types) before we apply the method used to solve the single-consumer-type problem discussed in Section 3.3.

To better understand the optimal pricing strategy, we develop a visual representation (and closed form expressions) for the optimal solution in various scenarios by considering an example with only two consumer type – *High* (h) and *Low* (l) – where distributions $F_h(.) = F_l(.) \sim Uniform(0, \delta^{\max}]$, fraction of external consumers $\beta_h = \beta_l = \beta$, and low opportunity costs $\delta^{\max} \le \hat{S}_h - \hat{S}_l$. Thus, for a fixed $\delta^{\max}$, by varying β and γ, the fraction of low-type consumers, we can generate all possible problem settings or scenarios over which to study how the optimal policy changes. We refer to this special case as the specialized two consumer-type problem. Lemma 2 provides sufficient conditions for the firm to attract only high-type consumers to FDS or to attract both high and low-type consumers in this special case.

Lemma 2: Let $\overline{\beta} = (\hat{S}_h - \hat{S}_l)/\hat{\pi}_h$, $\chi = (\hat{\pi}_l - \pi_l^{PFD})/\hat{\pi}_h$ and $\overline{\gamma}(\beta) = (\overline{\beta} - \beta)/\left((\overline{\beta} + \chi) - (1+\chi)\beta\right)$.

Then, for the specialized two consumer-type problem,

(i) if $\beta \le \overline{\beta}$ and $\gamma \le \overline{\gamma}(\beta)$, the firm only attracts the high-type consumers to FDS; and,

(ii) if $\gamma > \overline{\gamma}(0) \triangleq \overline{\beta}/(\overline{\beta} + \chi)$, the firm attracts both low and high-type consumers to FDS for all values of β.

The sufficient condition in part (i) implies that if the fractions of low-type consumers and external consumers are lower than the given thresholds, the firm only attracts high-type consumers to the FDS plan. Part (ii) provides a sufficient condition, in terms of a lower bound on γ, for the firm's optimal Universal plan to attract low type consumers (regardless of the value of β).

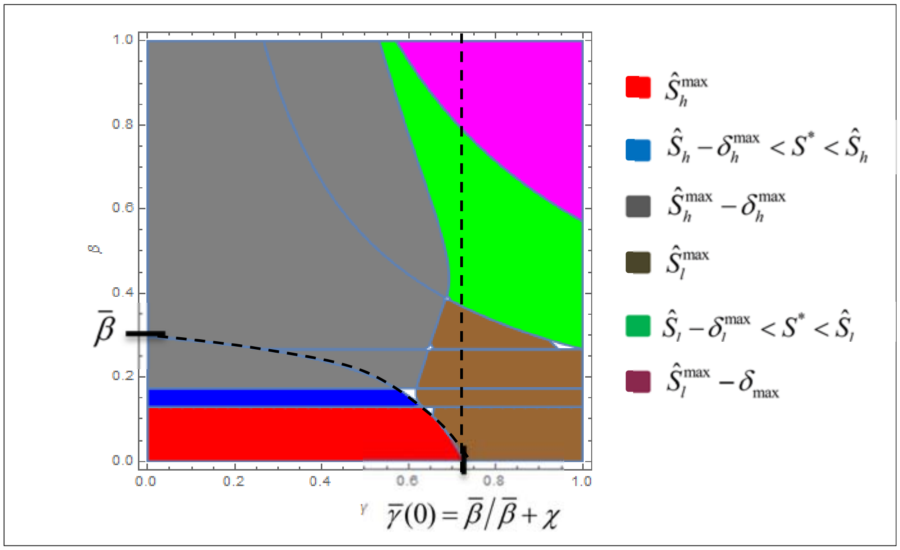

Figure 6: Ranges of optimal Universal subscription fee for specialized example

Figure 6 shows the optimal Universal subscription fee for the specialized two consumer-type problem as the fraction γ of low-type consumers and fraction β of external consumers (for both high- and low-type consumers) change simultaneously. The online appendix contains the expressions for the optimal subscription fee for this special case. For the figure, we assume a quadratic utility function, and low opportunity costs. Each shaded area represents a particular composition of FDS consumers, e.g., Loyal high-type, all external high-type, loyal low-type, and some external low-type, in the optimal solution. Each such composition corresponds to the interval of subscription fee values shown for each region in the figure. As expected, when both γ and β are small, the firm only attracts high-type consumers to FDS; on the other hand, when both γ and β are high, the firm attracts both high- and low-type consumers to FDS. The figure also shows the two sufficiency conditions from Lemma 2. Note that the threshold of γ above which the firm starts attracting low-type consumers to FDS changes non-linearly as β increases. Hence, for a particular value of γ (e.g., $\gamma = 0.65$), the firm's optimal policy may alternate between attracting both high and low-type consumers and only the high-type consumers as β increases.

Comparative Statics:

We next focus on the properties of the firm's Universal plan profits and subscription fee as the market composition, consumer characteristics, and product features change. For these comparative statics, we consider the general model in which the proportion β_k of external consumers as well as the distribution $F_k(.)$ of opportunity cost can vary by consumer types. We assume that the perturbations in the value(s) of parameter(s) are small enough so that the firm's FDS targeting strategy does not change, i.e., the consumer types that purchase FDS remain the same. That is, if the optimal Universal subscription fee lies in an interval $h \in H$, i.e., $S^* \in (\hat{S}_{h-1}, \hat{S}_h]$, then the new optimal subscription fee S' (in response to change in any parameter) also lies in *h*, i.e., $S' \in (\hat{S}_{h-1}, \hat{S}_h]$. We also assume that, for any consumer type *k*, the fraction of consumers γ_k, fraction of external consumers β_k, and distribution of opportunity costs $F_k(\delta)$ do not depend on the other model parameters (e.g., costs). Table 3 summarizes the comparative statics for different parameters of the general model. Propositions 7 to 10 provide detailed results on how the optimal Universal plan profit, subscription fee, number of consumers purchasing FDS, and the additional surplus for each FDS subscriber change as different parameters change and also consider some special cases.

Increase in parameter	**Optimal profit Π^***	**Optimal subscription fee S^***	**Number of FDS subscribers**	**Additional surplus for each subscriber**
Product cost *c*	↓	↑	↓	↓
Shipping cost *e*	↓	↑	↓	↓
Delivery charge *d*	–	–	↑	↑
Consumer type θ_k	↑	–	↑	↑
Fraction of external consumers of type *k* β_k	↓	↓	–	–

"↑": Value increases, "↓": Value decreases, and "–": Value can increase or decrease

Table 3: Summary of Comparative statics for Universal plan

Proposition 7: Comparative statics for product and consumer characteristics

(i) With higher product cost *c* or higher shipping cost *e*, the subscription fee increases, the number of FDS subscribers decreases, and the firm's profit strictly decreases. Further, consumer welfare also decreases.

(ii) At higher delivery charge *d*, the number of FDS subscribers under the Universal plan increases. The profit from FDS subscribers increases, but the profit from PFD consumers, if any, decreases. Total profit can increase or decrease with *d*.

(iii) At higher values of θ_k for any *k*, the firm's profit strictly increases and the number of FDS subscribers increases, but the optimal subscription fee can increase or decrease.

The product cost *c* and the shipping cost *e* (part(i)) only affect the firm's net profit margins (both PFD and FDS) per unit and not consumer behavior. So, as expected, the firm's profit decreases and the subscription fee increases for products with higher product and shipping costs. The delivery charge *d* (part(ii)), however, affects consumer choice (and the net PFD profit margin per unit). Recall from Lemma 1 that the threshold subscription fee $\hat{S}_k$ for each $k \in K$ increases as *d* increases. Hence, with a higher delivery charge *d*, the firm can charge a higher subscription fee while maintaining the same number of FDS consumers. If no consumer purchases PFD at the firm's current optimal solution (with the original delivery charge), then the firm can charge a higher subscription fee while keeping the same number of consumers, and the profit increases. If there are some PFD consumers at the current subscription fee, the firm's profit can decrease with *d* since the purchase volume of PFD consumers decrease. Interestingly, the firm *may* set a lower subscription fee for higher *d*. Since the loyal consumers are willing to pay a higher subscription fee to switch to FDS, the firm may find it beneficial to offer them a higher subsidy to attract additional external consumers (depending on the distributions $F_k(\delta)$ for each *k* of external consumers). Part (iii) of the proposition considers the case when the value for the product changes for some consumer type. Since a consumer's purchase quantity (under both PFD and FDS options) increases as her type increases, she is willing to pay a higher subscription fee to switch to FDS, increasing the firm's profits. Note that the firm may charge a lower subscription fee even though consumers of type *k* are willing to pay a higher subscription fee to switch to FDS. For instance, suppose only some consumers of type *k* purchase FDS at the current subscription fee. As θ_k increases, more type *k* consumers purchase FDS at the same subscription fee. The firm can increase profits by increasing its subscription fee to attract the same number of consumers as in the original solution. The additional revenue from type *k* consumers (at the current optimal subscription fee) can also prompt the firm to reduce the subscription fee in order to attract more external consumers. Thus, as the type of a consumer increases, the optimal policy may reduce the subscription fee.

Proposition 7 considers independent changes in the delivery charge and shipping cost. In practice, firms may set the PFD delivery charge equal to their shipping cost i.e., the firm neither subsidizes nor profits from shipping. Hence, when *e* changes, the firm also changes *d*. Proposition 8 addresses the comparative statics for this simultaneous change.

Proposition 8: Comparative statics when shipping cost equals delivery charge

If the firm's delivery charge is same the shipping cost, i.e., $d = e$, at higher values of shipping cost, the subscription fee increases, the number of FDS subscribers decreases, and the firm's profit decreases.

At a higher shipping cost, the firm's profit margins as well as consumer behavior change. As *e* increases, the firm can increase its revenue by charging a higher subscription fee without losing any FDS consumer, but the firm's total delivery cost also increases. The result shows that the decrease in the contribution from sales (due to higher delivery costs) is higher than the increase in subscription revenue as shipping cost (and delivery charge) increases, thus lowering the overall profit. The firm charges a higher subscription fee to offset the higher delivery costs and attracts fewer FDS subscribers.

We next discuss the comparative statics results as the market composition, i.e., fraction β_k of external consumers among type *k* consumers, fraction γ_k of type *k* consumers, and distribution $F_k(\delta)$, change. For the comparative static results with respect to γ_k, we assume that as the fraction γ_k increases (for $k < K$), the fraction $\gamma_{\tilde{k}}$ for one or more higher consumer types $\tilde{k} = k+1,\ldots,K$ decreases, so that

$\sum_{\tilde{k}=k+1}^{K}\gamma_{\tilde{k}}+\gamma_k$ remains the same.

Proposition 9: Comparative statics with respect to market composition

(i) For higher fraction β_k of external consumers, the subscription fee decreases, and the fraction of external consumers purchasing FDS increases, and the firm's profit decreases. The total number of FDS subscribers can either increase or decrease.

(ii) Suppose $\delta_{\tilde{k}}^{\max} \le \hat{S}_{\tilde{k}} - \hat{S}_k$ for all $\tilde{k} > k$. At higher fraction γ_k of consumer type *k* (for *k* < *K*), the subscription fee decreases, the number of FDS subscribers decreases, and the firm's profit strictly decreases.

(iii) For consumer type *k*, if $G_k(\delta)$ is first order stochastically larger than $F_k(\delta)$, then the firm's optimal profit under $G_k(\delta)$ is smaller than the firm's profit under $F_k(\delta)$.

Below we consider two special cases of $G_k(\delta)$ being first order stochastically larger than $F_k(\delta)$:

(a) If $G_k(\delta)$ dominates $F_k(\delta)$ in the sense of monotone probability ratio, i.e., $G_k(\delta)/F_k(\delta)$ is non-decreasing in $\delta \in (0, \delta_k^{\max}]$, the firm's optimal subscription fee decreases.

(b) If $G_k(\delta)$ is obtained by shifting $F_k(\delta)$ to right, the number of FDS subscribers under $G_k(\delta)$ is no larger than the number under $F_k(\delta)$.

The fraction of consumers of each type that are loyal (or external) to the firm depends on several factors such as the popularity of the firm's PFD option, nature of competition, loyalty of consumers towards the firm, and the product category. For example, an established firm with a niche product may have high fraction of consumers who are loyal while a new entrant in the market may have a high fraction of external consumers. Part (i) of the proposition shows that a firm with a high fraction of external consumers offers a lower subscription fee and earns a lower optimal profit. By setting a low subscription fee, the firm attracts a higher fraction of the external consumers but the total number of FDS subscribers can be lower (or higher). Part (ii) of the proposition considers the case where some consumers of a higher type shift to a lower type *k*. Specifically, the proportion γ_k of the consumers of type $k < K$ increases and the fraction $\gamma_{\tilde{k}}$, for $\tilde{k} = k+1, \ldots, K$, decreases. (Note that the decrease in fraction of higher consumer types $\tilde{k}$ can vary by consumer types.) The result implies that, in general, even if the total number of higher type consumer decreases, the firm's profit may not decrease. The profit may increase because the number of consumers purchasing FDS at the current subscription fee may increase as the proportion of consumer type changes[6]. With $\delta_{\tilde{k}}^{\max} \le \hat{S}_{\tilde{k}} - \hat{S}_k$ for all $\tilde{k} > k$, however, we can specify the direction in

[6] We can demonstrate this using a simple example when there are only two types of consumers, *high* and *low*. Let

which the firm's profits and the subscription fee change. With low opportunity costs, the increase in the number of consumers of type *k* purchasing FDS (at the current subscription fee) is less than the decrease in the number of higher consumer types- lowering the overall profits. In response, the firm decreases its subscription fee to attract a higher fraction of consumers of type *k*. Part (iii) of the proposition considers the case when consumers' opportunity costs increase, i.e., the distribution $F_k(\delta)$ of external consumers of type *k* changes to $G_k(\delta)$ that is first order stochastically larger than $F_k(\delta)$, i.e., $G_k(\delta) \le F_k(\delta)$ for $\delta \in (0, \delta_k^{\max}]$. At the current subscription fee, the number of FDS subscribers decreases under $G_k(\delta)$, thus reducing profits. Unfortunately, the first order stochastically larger order does not guarantee, by itself, a monotonic shift in the subscription fee or the number of FDS consumers. We consider two, more restrictive, cases than the first order stochastically larger condition. First, suppose the distribution $G_k(\delta)$ dominates $F_k(\delta)$ in the sense of monotone probability ratio (MPR). Maskin and Riley (2000) show that the MPR order implies that $G_k(\delta)$ is first order stochastically larger than $F_k(\delta)$. With the distributions $G_k(\delta)$ and $F_k(\delta)$ satisfying the MPR order, the firm charges a lower subscription fee under $G_k(\delta)$. Even with the MPR order, the number of FDS subscribers purchasing under $G_k(\delta)$ can be higher or lower than that under $F_k(\delta)$. Second, suppose the distribution $F_k(\delta)$ shifts to the right so that $g_k(\delta + \epsilon) = f_k(\delta)$ for some $\epsilon \ge 0, \delta \in (0, \delta_k^{\max}]$ and $g_k(\epsilon') = 0$ for $0 \le \epsilon' < \epsilon$. Note that the right-shifted distribution $G_k(\delta)$ is first order stochastically larger than $F_k(\delta)$. Shifting the distribution to the right by ϵ is equivalent to reducing the threshold subscription fee by ϵ for the original distribution. With a lower threshold subscription fee, the original fee provides less subsidy and attracts fewer external consumers. So, the firm may raise or lower its subscription fee relative to the original subscription fee.

We next examine how changes in the parameters affect the *additional* surplus that consumers obtain when the firm introduces the FDS option. The additional surplus a loyal consumer receives is the difference between her FDS and PFD surplus. The additional surplus an external consumer receives is the net gain in surplus under FDS over her opportunity cost δ to switch to the firm. The next proposition provides comparative static results for the additional surplus of each consumer with changes to the problem parameters.

Proposition 10: Comparative statics for additional surplus of each consumer

(i) For higher product cost *c* or shipping cost *e*, the additional surplus for each consumer decreases.

(ii) For higher delivery charge *d*, the additional surplus for each consumer increases.

$\gamma_l = 0.2$ and $\gamma_h = 0.7$. Let 90% of the low-type consumers and 50% of the high-type consumers purchase FDS at the current solution. Thus, 58% of the market purchases FDS at the current solution. Suppose γ_l increases to 0.3 (consequently $\gamma_h = 0.7$). At the same subscription fee, 62% of the market purchases FDS now.

(iii) For higher values of θ_k, the additional surplus of each consumer increases.

The additional surplus a consumer receives by switching to FDS depends on how her willingness to pay for the FDS option changes and how the firm adjusts its subscription fee as any parameter changes. The results in Proposition 10 intuitively follow from the results in Propositions 7, 8 and 9. We only focus on some key observations from the results. For a higher delivery charge *d*, even though we cannot say whether the optimal subscription fee increases or decreases, the overall consumer surplus increases. Recall from Proposition 7 that if no consumers purchase using PFD at the current solution, the firm's profit increases as *d* increases. In this case, the consumer's additional surplus also increases. Similarly, for higher values of θ_k, the firm may charge a higher or lower subscription fee but the additional surplus for each consumer as well as the firm's profit increase.

In summary, this section builds upon the foundations established in Section 3 to characterize and gain many insights on the subscription pricing problem for the Universal plan under very general conditions. The next section considers the case where the firm offers a menu of subscription plans. Recall that the Universal plan, common in practice, provides unlimited free deliveries. Although limiting the quantity that is eligible for free delivery can be beneficial for the firm under certain circumstances, our later numerical studies confirm that permitting unlimited free deliveries does not significantly reduce profits, i.e., offering limited free deliveries provides little benefit and under very few scenarios. We analyze limited free deliveries in the next section where we discuss the strategy of offering multiple FDS plans tailored for different consumer segments (i.e., subsets of consumer types).

5. Tiered Subscription Plans

A key trade-off in Universal plan subscription pricing is whether, and how much, to subsidize higher type consumers to attract lower type consumers. The firm can exploit the heterogeneity between consumer types by offering a menu of subscription plans – with different free delivery limits and subscription fees – customized for each consumer type. Given such a menu of subscription plans, each consumer self-selects the plan that maximizes her surplus. Such price discrimination strategies are quite prevalent in practice and have been widely discussed in past research. Depending on the context and type of heterogeneity among consumers, researchers have studied different forms of price discrimination such as intertemporal price discrimination, damaged goods, coupons (or rebates), versioning of information goods.

In our model, each consumer type's FDS purchase quantity and hence surplus depends on the firm's free delivery limit. Thus, a natural way to price discriminate between consumer types is to offer a hierarchy of plans with decreasing subscription fees and free delivery limits. We refer such a menu of subscription plans as a Tiered FDS plan, abbreviated as *Tiered plan*. This section studies the properties and characterizes the firm's optimal Tiered plan.

For expositional simplicity, we only consider two types of consumers – *High* (h) and *Low* (l). The firm's Tiered plan has two subscription plans – *Hi-plan* $< S_{hi}, L_{hi} >$ and *Lo-plan* $< S_{lo}, L_{lo} >$, with $S_{hi} > S_{lo}$ and $L_{hi} > L_{lo}$. Given the two plans, each consumer (of either type) selects the plan that provides her the highest surplus. One possible, but suboptimal, approach to designing a Tiered plan is to set the subscription fee and shipping fee for each tier as the optimal values of the single-consumer-type subscription pricing problem for the high and low consumer types. We refer to such a Tiered plan as the *Independent plan*. Let $< S1_h^*, v_h >$ be the optimal single-consumer-type solution with only high-type consumers, and $< S1_l^*, v_l >$ the optimal single-consumer-type solution with only low-type consumers. Then, the firm's Independent Hi-plan is $< S1_h^*, v_h >$ and the Independent Lo-plan is $< S1_l^*, v_l >$. If the firm can identify each consumer's type at the time of purchase and choose a single subscription plan to offer, the firm should offer each consumer the Independent plan corresponding to her type. Thus, given complete information and the ability of the firm to customize its subscription plan according to each consumer, the Independent plan provides the highest profit the firm can earn under any Tiered pricing policy. For most practical scenarios, however, a consumer's type is her private information and the firm cannot customize its subscription plan for each consumer. So, the Independent plan may not be optimal since the high-types may find the Lo-plan more attractive and/or low-types may find the Hi-plan more appealing. To adequately account for the demand under each tier and hence the overall profits under the Tiered plan, the firm must develop *incentive compatible* plans – subscription plans under which each consumer purchases the plan designed for her. The next result provides the incentive compatibility constraints required for the optimal Tiered plans.

Observation: Incentive Compatibility constraints: Any incentive compatible Tiered plan $< S_{hi}, L_{hi} >$ and $< S_{lo}, L_{lo} >$, with $S_{hi} > S_{lo}$ and $L_{hi} > L_{lo}$, must satisfy the following constraints:

IC for high-type consumer: $S_{hi} \le S_{lo} + \hat{S}_h(L_{hi}) - \hat{S}_h(L_{lo})$, and

IC for low-type consumer: $S_{hi} \ge S_{lo} + \hat{S}_l(L_{hi}) - \hat{S}_l(L_{lo})$.

The IC constraint for high-type (low-type) ensures that high-type (low-type) consumers prefer the Hi-plan (Lo-plan). We illustrate the intuition underlying the incentive compatibility constraints using the examples in Figure 7. All loyal high-type consumers and external high-type consumers with $\delta \le \hat{S}_{hi}(L_{hi}) - S_{hi}$ prefer subscribing to the Hi-plan $< S_{hi}, L_{hi} >$ over their current purchasing option. However, if $\hat{S}_h(L_{lo}) - S_{lo} \ge \hat{S}_h(L_{hi}) - S_{hi}$, then the loyal high-type consumers prefer purchasing the Lo-plan over the Hi-plan since a loyal high-type consumer's surplus at $\hat{S}_h(L)$ is same as her PFD surplus, for any $L \ge 0$. Similarly, all external high-type consumers who wish to switch to the firm's FDS plan will also find the Lo-plan more attractive. For any Tiered plan $< S_{hi}, L_{hi} >$ and $< S_{lo}, L_{lo} >$ that does not satisfy the IC constraint for high-type consumers (Figure 7(a)), all loyal high-type consumers and external high-type consumers with $\delta \le \hat{S}_h(L_{lo}) - S_{lo}$ purchase the Lo-plan. In this case, if the firm reduces the Hi-plan subscription fee S_{hi} so that $S_{hi} = S_{lo} + \hat{S}_h(L_{hi}) - \hat{S}_h(L_{lo})$ in order to attract all current high-type FDS consumers to the Hi-plan, the firm's profit increases due to the lift in demand of the high-type consumers and a higher subscription revenue. In the revised plan, high-type consumers purchase the Hi-plan. Next, suppose the Tiered plan $< S_{hi}, L_{hi} >$ and $< S_{lo}, L_{lo} >$ does not satisfy the IC constraints for low-type consumers (Figure 7(b)). All loyal low-type and external low-type consumers with $\delta \le \hat{S}_l(L_{lo}) - S_{lo}$ obtain a non-negative net surplus from the Lo-plan but prefer the Hi-plan. Even if the firm reduces the Lo-plan fee S_{lo} to $S_{lo} = S_{hi} - (\hat{S}_l(L_{hi}) - \hat{S}_l(L_{lo}))$ and all low-type consumers purchase the Lo-plan, the firm's profit remains the same. Thus, when offering a Tiered plan, the plan can be designed so that low-type consumers purchase the Lo-plan and high-type consumers purchase the Hi-plan without losing any profits.

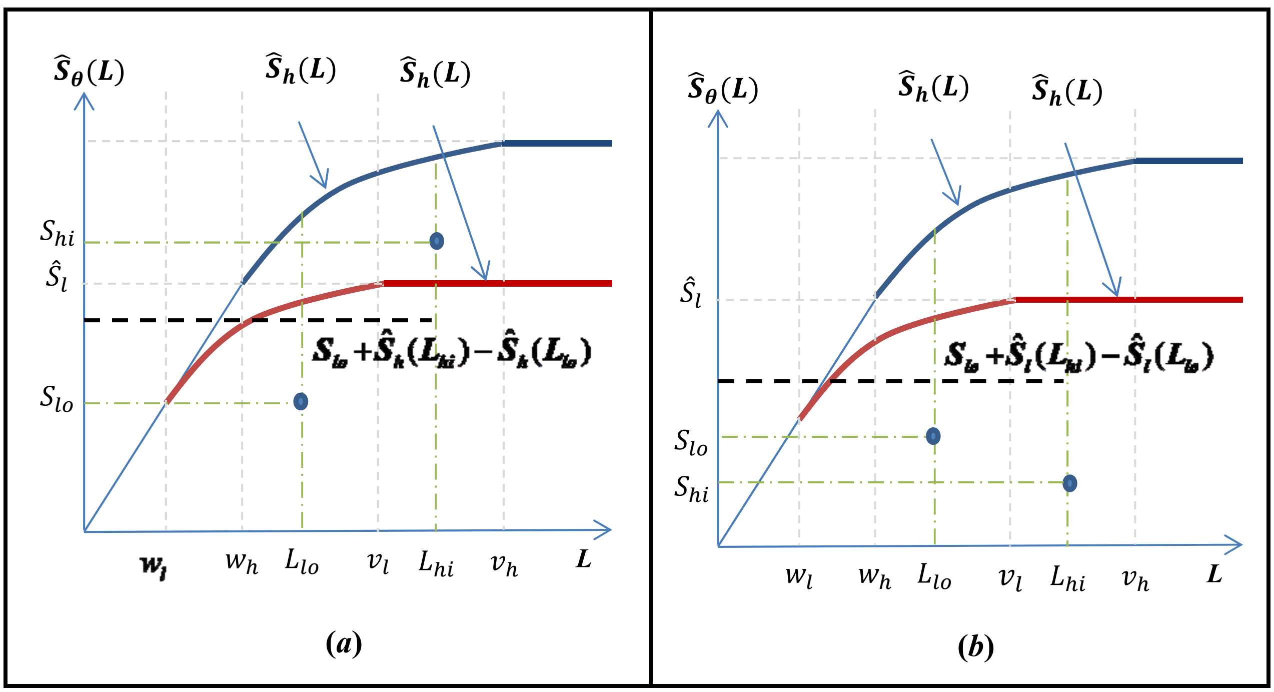

Figure 7: Examples of non-incentive compatible Tiered plans – (*a*) Tiered plan violates High-type IC constraints, (*b*) Tiered plan violates Low-type IC

Given the incentive compatibility constraints, the problem of finding the optimal strategy with multiple plans can be expressed as the following Multiple Plan Optimization model [MPO]:

$$max_{S_{hi}, S_{lo}, L_{hi}, L_{lo}} \Pi_h(S_{hi}, L_{hi}) + \Pi_l(S_{lo}, L_{lo})$$

High-type IC: $S_{hi} \le S_{lo} + \hat{S}_h\left(L_{hi}\right) - \hat{S}_h\left(L_{lo}\right)$

Low-type IC: $S_{hi} \ge S_{lo} + \hat{S}_l\left(L_{hi}\right) - \hat{S}_l\left(L_{lo}\right)$

High-type IR: $S_{hi} \le \hat{S}_h\left(L_{hi}\right)$

Low-type IR: $S_{lo} \le \hat{S}_l\left(L_{lo}\right)$

If [MPO] has an optimal "separating" solution, i.e., with $S^*_{hi} > S^*_{lo}$ and $L^*_{hi} > L^*_{lo}$, then this solution corresponds to a Tiered plan. If it has an optimal "pooled" solution, with $S^*_{hi} = S^*_{lo}$ or $L^*_{hi} = L^*_{lo}$, we refer to this solution as a *Pooled* plan. Such a Pooled plan is essentially equivalent to a single plan with $S = \min(S^*_{hi}, S^*_{lo})$ and $L = \max(L^*_{hi}, L^*_{lo})$. But, unlike the Universal plan of Section 4, the Pooled plan may only permit limited deliveries.

The next proposition discusses conditions under which [MPO] has an optimal Tiered solution, and relates this solution to the Independent plan. For this proposition, we assume that the profit functions for each single-consumer-type problem is unimodal in the subscription fee *S*.

Proposition 11: Optimal Tiered subscription plans

(i) The Independent plan $< S1^*_h, v_h >$ and $< S1^*_l, v_l >$ is an optimal Tiered plan if and only if $S1^*_l < S1^*_h \le S1^*_l + \hat{S}_h(v_h) - \hat{S}_h(v_l)$.

(ii) [MPO] has a strictly separating solution, i.e., a Tiered plan solution if and only if $S1^*_h > S1^*_l$; otherwise, the firm's optimal strategy is to offer a single Pooled plan.

(iii) If $S1^*_h > S1^*_l + \hat{S}_h(v_h) - \hat{S}_h(v_l)$, i.e., a Tiered plan is optimal for [MPO] but the Independent plan is not incentive compatible, then $L^*_{lo} \in [w_l, v_l]$, $S1^*_l \le S^*_{lo}$, $L^*_{hi} = v_h$ and $S^*_{hi} = \min[S^*_{lo} + \hat{S}_h - \hat{S}_h(L^*_{lo}), S1^*_h] \le S1^*_h$.

Proposition 11 provides interesting insights about the firm's optimal Tiered pricing policy under very general conditions. Part (i) provides necessary and sufficient condition for the Independent plan to be optimal for problem [MPO]. These conditions simply ensure that the Independent plan satisfies [MPO]'s incentive compatibility requirements. The property holds because the Independent plan $< S1^*_h, v_h >$ and $< S1^*_l, v_l >$ is an optimal solution to the relaxed version of [MPO], obtained by dropping the incentive compatible constraints. So, if this solution satisfies the omitted IC constraints, it is solves the original [MPO] problem; moreover, it is a Tiered plan.[7] This possibility shows that, even with asymmetric

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[7] If the Independent plan does not satisfy the IC constraints, both consumer types choose either $< S1^*_h, v_h >$ or $< S1^*_l, v_l >$, when offered the Independent plans. This situation is equivalent (in terms of profits) to the firm

information, the firm can achieve the same profits as it gets with perfect information about each consumer. The condition for the existence of an optimal Tiered plan solution to [MPO] holds if the consumer types are different enough, as confirmed in our later computational experiments. Part (ii) of the proposition provides a simple condition based on the Independent plan (single-consumer-type) solution to determine whether [MPO] has a separating solution (Tiered plan). If the condition is not satisfied, we do not need to consider the strategy of offering multiple FDS plans. In a situation where the Independent plan is not incentive compatible and [MPO] has a Tiered optimal solution, Part (iii) provides guidance on how to modify the Independent plan to obtain the incentive-compatible Tiered solution. For the high-type consumers, the firm sets $L_{hi}^{*} = v_h$, i.e., offers unlimited free delivery, and charges an equal or lower subscription fee than the Independent Hi-plan. Conversely, the subscription fee for the Tiered Lo-plan is greater than or equal to that of the Independent Lo-plan and the shipping limit lies between the PFD and unconstrained FDS quantities for low-type consumers. This result is similar to the standard result in price discrimination literature where, to satisfy incentive compatibility, the firm may provide an additional surplus to high-type consumers while it reduces the surplus for the low-type consumers (compared to the Independent plans). These observations show that the fee and shipping limit adjustments needed to obtain the optimal Tiered plan entail resolving tradeoffs across customer types.

This section has addressed an online retailer's option to offer multiple (tiered) FDS plans as a price discrimination tactic across consumer types in order to further increase profits (compared to offering only one FDS plan). For simplicity, we focused on the two-consumer-type case. For the Tiered plan, we require incentive compatibility constraints to ensure that the subscription fees and shipping limits for each tier are such that the High-type (Low-type) consumers select the Hi- (Lo-) plan. Building upon the earlier discussions, we are able to identify conditions when offering a Tiered plan is optimal, and how this plan relates to single consumer type (Independent) solutions.

6. Computational Results

The previous three sections focused on obtaining analytical properties of the optimal subscription policy for the Universal plan and Tiered plans. In this section, we focus on quantitative comparisons of the different subscription pricing strategies as the market composition, consumer characteristics, and product features change based on a comprehensive set of computational runs. Specifically, we compare the optimal Universal plan, Tiered plan and Independent plan in terms of their performance (optimal profit) and differences in policy (optimal subscription fee) under different parameter settings. For our computational study, we only consider the low opportunity costs case. The results in Section 5 show that,

offering only the better of these two FDS options, which may yield lower profits than the optimal value of [MPO].

for this case, the Tiered plan yields at least as much profit as the Universal plan. Even if the Tiered plan has strictly higher profit, the firm may still prefer to offer the Universal plan because it is easier to administer and consumers may also prefer a simple pricing policy. Our computational results show the conditions under which the Tiered plan significantly outperforms the Universal plan (and when the Universal plan performs nearly as good as the Tiered plan). We also show conditions under which the Tiered plan achieves profits comparable to that of the Independent plan (which ignores incentive compatibility), thus providing an assessment about the value of obtaining additional information about consumer types. We compare the subscription fees under different strategies to understand the relative separation between the Hi and Lo-plans and the effect of the tradeoffs the firm faces under the Universal plan on the optimal subscription fee.

Computational setup:

We consider only two consumer type – *High* (*h*) and *Low* (*l*) and the Quadratic utility function, $u_\theta(y) = \theta\rho y - ((b-\theta\omega)y^2/2)$, with $b = 2$, $\rho = 7$, and $\omega = 0$. We set the proportion of external consumers among each consumer type to be the same, i.e., $\beta_l = \beta_h = \beta$, and assume that the opportunity costs δ for both high and low-type consumers have same Uniform distribution, i.e., $\delta \sim Uniform(0, \delta^{\max}]$. Starting with a *base value* for each parameter, we change the value of one parameter at a time across a set of values to gauge this parameter's effect on the firm's profits and subscription fee. Table 4 lists the base value and the other trial (perturbed) values for each parameter. In this table, $r = p - c$ is the per unit profit contribution.

Parameter	**Set of values**	**Base value**
Difference in consumer types	Fix $\theta_h = 1$, and vary $\theta_l \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$	$\theta_l = 0.5$
Fraction of Low-type consumers, γ_l	$\gamma_l \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$	$\gamma_l = 0.7$
Fraction of External consumers, β	$\beta \in \{0.1, 0.3, 0.5, 0.70.9\}$	$\beta = 0.7$
Shipping cost, e^* (= d)	$e \in \{0.1r, 0.3r, 0.5r, 0.7r, 0.9r\}$	$e = d = 0.5r$
Maximum opportunity cost, $\delta^{\max}$	$\delta^{\max} \in \{0.05, 0.1, 0.3, 0.5, 0.7\}$	$\delta^{\max} = 0.3$

* We assume delivery charge *d* is same as the shipping cost *e*

Table 4: Different parameter values for the computational tests

Varying γ_l corresponds to changing the market composition from a *high-type dominant* $(\gamma \le 0.3)$ to a *low-type dominant* $(\gamma \ge 0.7)$ market. Similarly, changing β alters market composition from a *loyal-dominated* $(\beta \le 0.3)$ to *external-dominated* $(\beta \ge 0.7)$ market. For the base case, we consider $\gamma = 0.7$ and $\beta = 0.7$ to represent an internet retailer whose customers are mainly low-type and external. To understand the effects of changing the consumer characteristics, we increase or decrease the difference between consumer types by keeping θ_h fixed at 1 and varying θ_l. For our computational results, we assume that the firm's delivery charge d is same as the shipping cost e, which in turn is lower than the product margin, i.e., $e<(p-c)$. We consider scenarios from very low product cost to very high product cost, simulating different product categories. Next, we compare the firm's optimal profits and subscription fee as the market composition, consumer characteristics and product shipping costs changes.

Comparison of Optimal Profits and Subscription Fees under different strategies:

We first compare the firm's profits and the subscription fee with changes in parameters. For all the comparisons, we normalize base scenario's (with all parameters at their base value) optimal Universal plan profit to 100 and Universal subscription fee to 100 (and accordingly scale profits and subscription fees for all other scenarios). In the following charts, if the Universal plan attracts only high-type consumers to FDS, we highlight the corresponding profit and subscription fee with a circular marker.

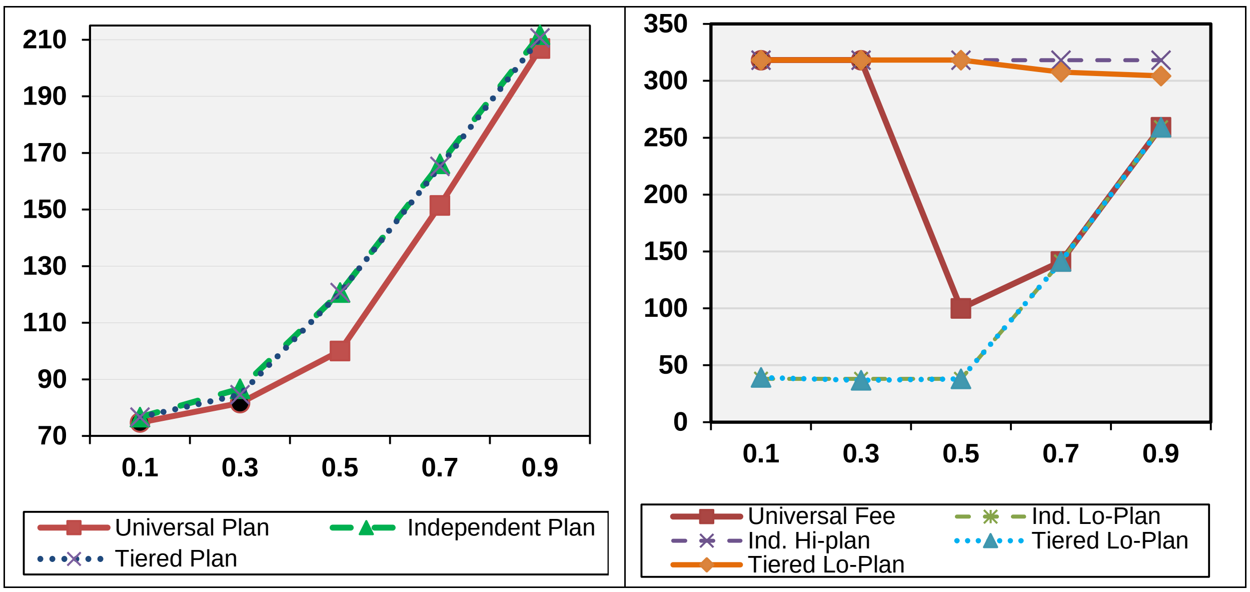

Figure 8: Profit and Subscription Fee comparison as θ_l / θ_h varies

(i) Ratio of consumer type θ_l / θ_h increases

Figure 8 shows the firm's optimal profits and subscription fees under the Universal plan, Tiered plan, and Independent plan as the difference between the consumer types decreases. Since we have fixed θ_h, only θ_l increases along the x-axis. As expected, the profit under each strategy increases as θ_l increases. Interestingly, the Universal plan's profit is similar to the Tiered plan profit when the two consumer types are either too distinct or too similar. With very distinct consumers, the Universal plan attracts only the high-type consumers to FDS. The Tiered plan attracts the low-type consumers to subscribe, but achieves only slightly higher profits (2-4%) since these consumers purchase a very low quantity. As the consumer types become less distinct, the firm faces stronger trade-offs under the Universal plan and lowers the subscription fee to attract low-type consumers. At $\theta_l = 0.5\theta_h$, the Tiered plan charges three times higher subscription fee to the high-types and attracts a more low-type consumers than the Universal plan, earning nearly 20% higher profit than the Universal plan. As consumers become homogenous, the relative benefit of the Tiered plan decreases. Interestingly, the Tiered plan performs nearly as good as the Independent plan (only <1% difference in profits) for all scenarios including those with highly homogenous consumers, when the Tiered Hi-plan's subscription fee decreases compared to Independent Hi-plan subscription fee because of incentive compatibility.

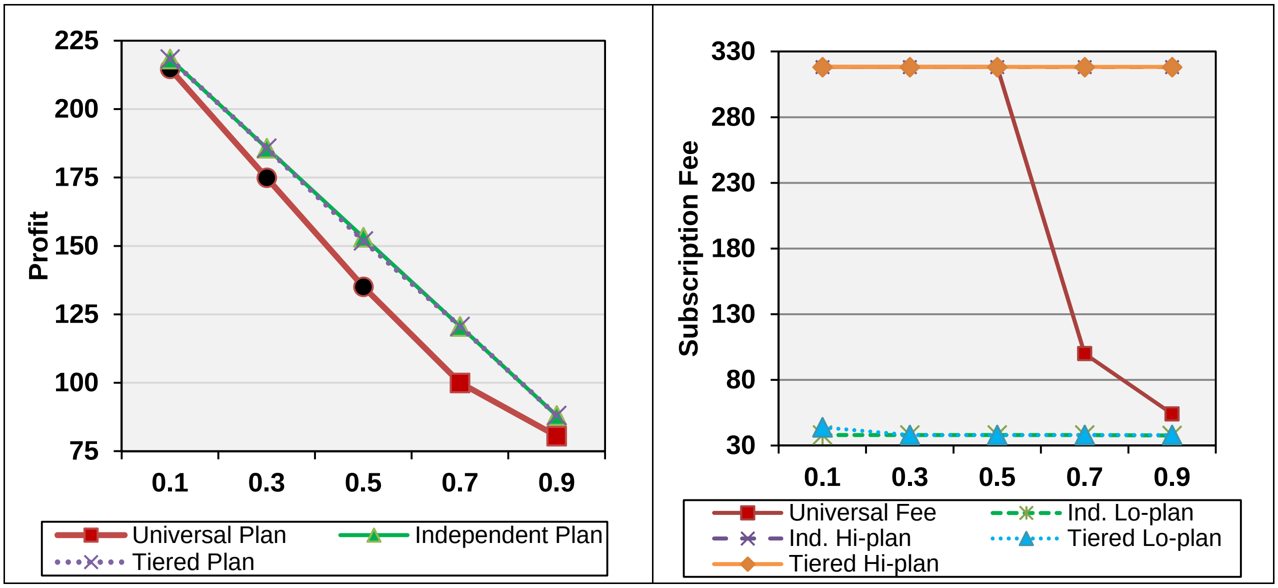

Figure 9: Profit and Subscription Fee comparison as γ varies

(ii) Proportion of low-type consumers increases

As the market changes from high-type dominant to low-type dominant, the firm's profit decreases under each strategy, as expected (Figure 9). The Tiered plan achieves similar profits as the Independent plan and is quite robust to the change in market composition. The Universal plan, however, achieves similar profits as the Tiered plan when the market is either dominated heavily by high types or low types. In an even market, the Tiered plan performs much better than the Universal plan. Specifically, even if 50% of consumers are low-type, the Universal plan only attracts high-type consumers to purchase FDS, losing significant revenue from the low-type consumers. For $\gamma > 0.5$, the firm starts attracting the low-type consumers under the Universal plan. For $\gamma \in [0.5, 0.7]$, the Tiered plan earns 16% more profit than the Universal plan on average.

(iii) Proportion of external consumers increases

As the fraction β of external consumers increases (among both high- and low-type consumers), the firm's optimal profit and optimal subscription decreases under each strategy (refer to Figure 10). In a loyal-dominated market (i.e., $\beta \le 0.3$), the Universal plan attract only high-type consumers to subscribe, whereas the Tiered plan benefits mainly from the lift of loyal low-type subscribers, which yields 3% higher profits. Even when $\beta = 0.1$, i.e., 63% of the market consists of loyal low-type consumers, the Universal plan attracts only the loyal high-type consumers to FDS. In fact, the Universal plan subscription fee is 2% higher than the Tiered Hi-plan subscription fee because the firm lowers the Hi-plan subscription fee to satisfy incentive compatibility. The relative benefit of the Tiered plan increases as the market becomes external-dominated (i.e., $\beta \ge 0.7$). By charging separate subscription fees to high- and low-types, the Tiered plan benefits both from the lift in the loyal low-type consumers' demand and by attracting a higher fraction of external low-type consumers. The Universal plan charges a low-subscription fee to attract some external low-type consumers, losing subscription revenue from high-type consumers. In practice, a new entrant in the market will typically have a higher β, and this result shows that the Tiered plan is much better strategy for such a player.

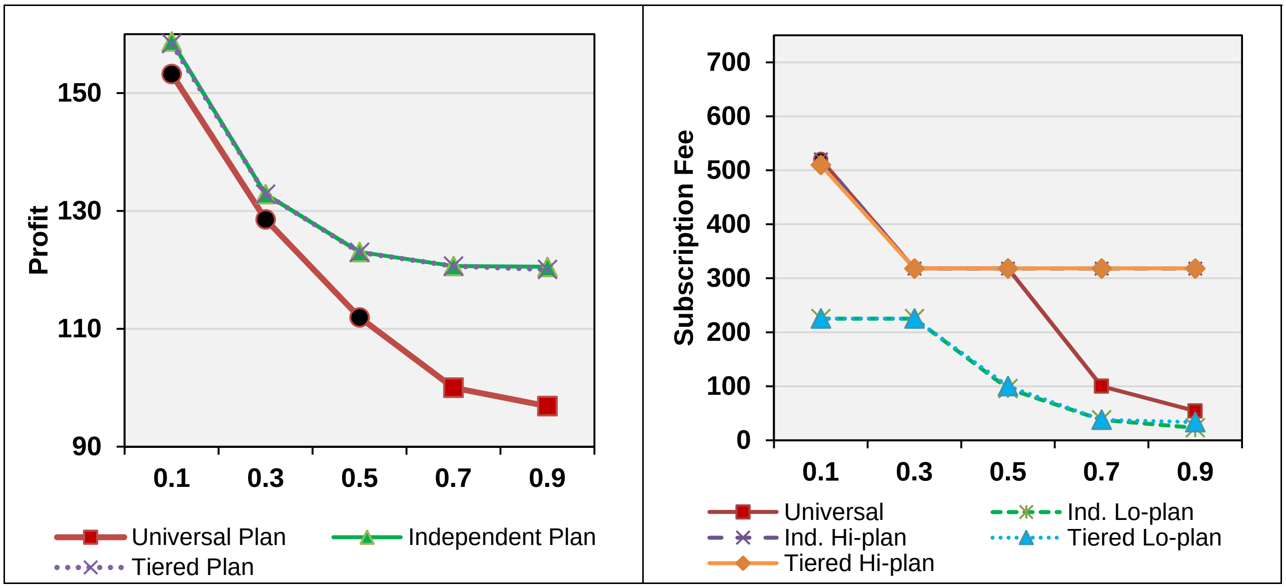

Figure 10: Profit and Subscription Fee comparison as β varies

(iv) Spread of δ changes

Recall that $\delta \sim Uniform(0, \delta^{\max}]$ for both low- and high-type external consumers. As $\delta^{\max}$ increases, the distribution becomes stochastically larger. As expected, the firm's profit decreases as $\delta^{\max}$ increases (Figure 11). Interestingly, the gap between the Tiered and Universal plan profits decreases as $\delta^{\max}$ increases. Moreover, the Universal subscription fee as well as the Lo-plan subscription fee can change non-monotonically as $\delta^{\max}$ increases. When $\delta^{\max}$ is small, the firm sets the subscription fee so that it can attract all the consumers under the Universal subscription fee, and all the low-type consumers under the Tiered Lo-plan. So the firm's subscription fee decreases as $\delta^{\max}$ increases up to a point where the firm does not find it attractive to sell FDS to all consumers. As $\delta^{\max}$ increases, the firm does not attract all consumers. When the firm attracts only a fraction of external consumers, the subsidy the firm offers decreases as $\delta^{\max}$ increases and the subscription fee increases. When $\delta^{\max}$ is very high, the separation between the firm's Lo-plan and Hi-plan is very small and the Universal plan yields nearly the same profits as the Tiered plan.

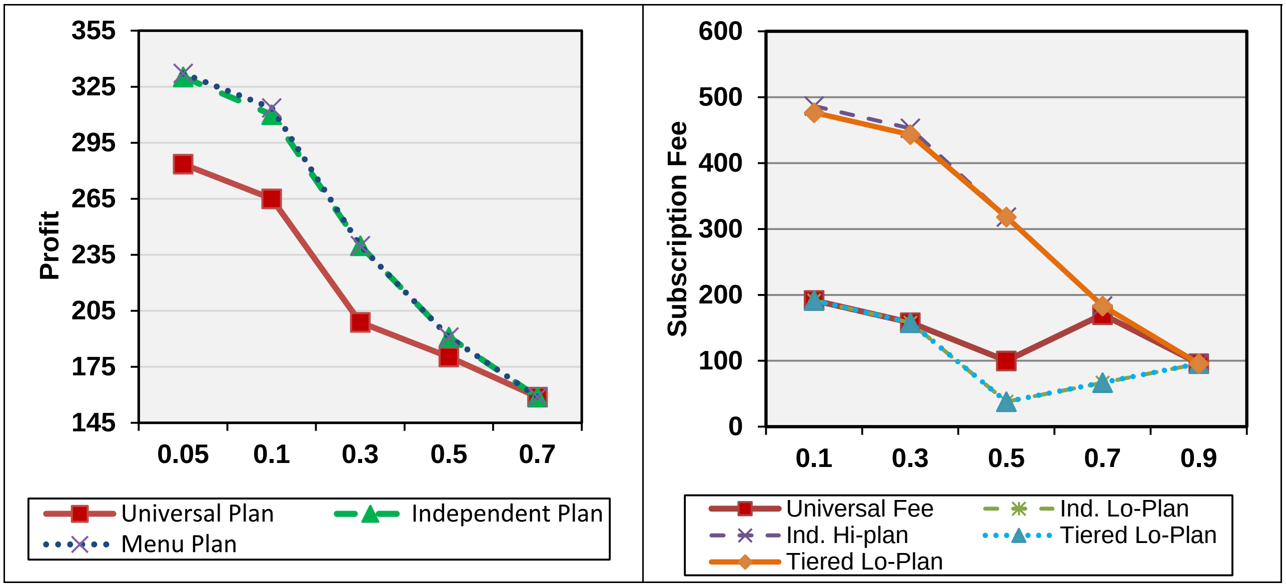

Figure 11: Profit and Subscription Fee comparison as $\delta^{\max}$ varies

(v) Shipping Cost increases

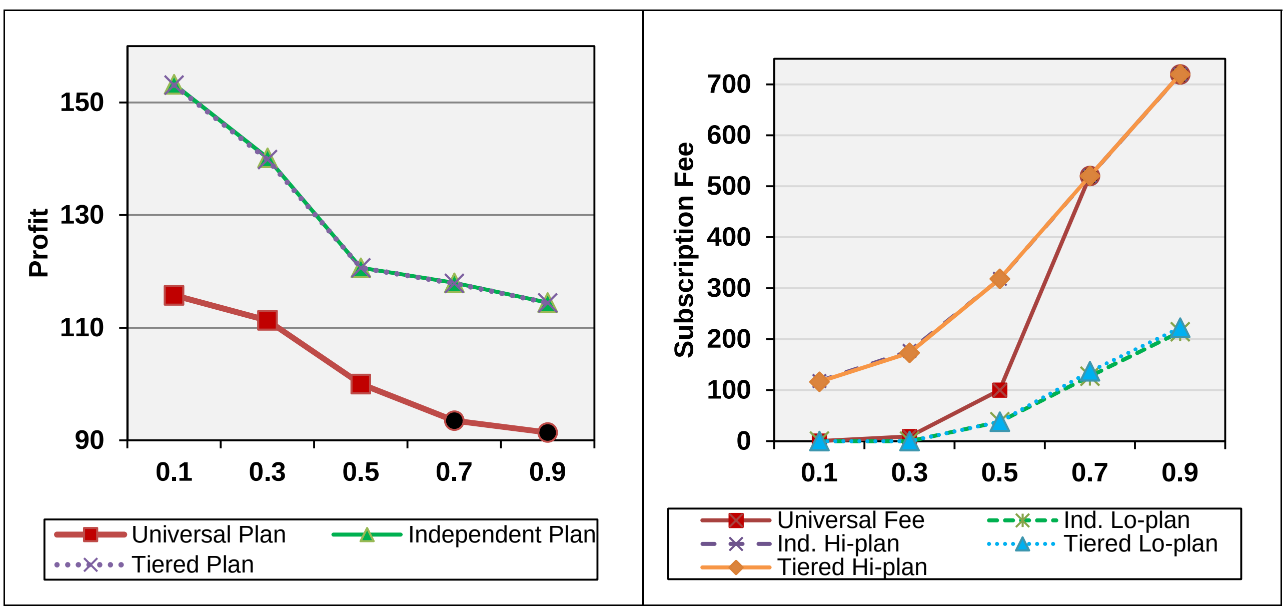

Figure 12: Profit and Subscription Fee comparison as $e/(p-c)$ varies

Figure 12 compares the optimal profit and subscription fee as the shipping cost (as a fraction of product margin) increases. This comparison might shed light on the differences in pricing strategy across firms selling different product types. Since the firm's delivery charge is same as the shipping cost, the profits decreases under each strategy as shipping cost increases. The Tiered plan again performs as well as the Independent plan, and earns 25% higher profits than the Universal plan on average across products with different shipping costs. For low shipping cost products, the firm charges a low-subscription fee and both types of consumers purchase FDS under the Universal plan. As shipping cost increases, the delivery costs increases and the firm increases its subscription fee. For high shipping cost products, the firm attracts only the high-type consumers to purchase FDS.

Shipping Cost Recovery from Subscription Fee:

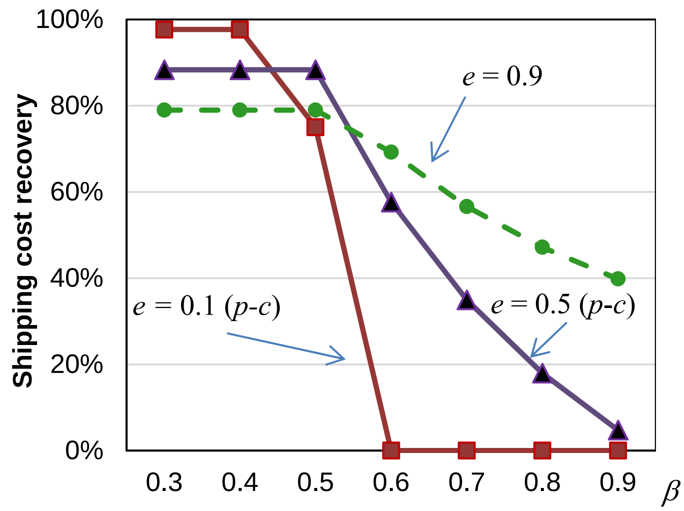

Figure 13: Shipping Cost Recovery

We observe in our analytical results that the firm can benefit by offering an FDS option even if the subscription fee is "lossy" with respect to the total delivery costs. Figure 13 shows a comparison of the fraction of the delivery costs that the optimal subscription fee recovers as β changes, for different values of the shipping cost/delivery charge (e.g., for different product categories). For simplicity, we show this result for only a single-consumer-type problem but similar insights also hold for the Universal plan with multiple consumer types. First, we note that under many circumstances, the subscription fee may recover a very low fraction of the shipping costs. Thus, the subscription fee acts as a lever to both attract new consumer demand and take advantage of the lift in loyal consumers' demand; the additional profit margin due to increased sales more than compensates for the loss in shipping costs. Second, for products with low shipping costs, when the firm attracts only loyal consumers, it recovers nearly 100% of the shipping cost. But, the shipping cost recovery decreases very rapidly when the firm starts attracting external consumers. Intuitively, for products with low shipping costs, the lift is small (since the delivery charge is also small), so the firm sets the threshold subscription fee near the total delivery cost for each FDS consumer. For products with higher shipping costs (and delivery charge) the firm sets a lower threshold subscription fee since the lift is high. But, as the firm starts attracting external consumers, for low shipping cost products, the firm decreases subscription fee significantly to attract a large number of external consumers (since the total delivery cost of the firm does not increase by a lot).

7. Conclusion

Subscription plans for free delivery services have become common among internet retailers. This paper has developed a new utility model to capture the two main drivers underlying such plans – the lift in a loyal consumer's demand (based solely on economic reasons) and the possibility of attracting new consumers when a retailer introduces a FDS plan to supplement their pay-for-delivery option. Our model is both rich in its features and general in its structure. We capture two dimensions of consumer heterogeneity, due to differences in valuation and in the opportunity cost to purchase from the firm, impose only mild conditions on the utility functions, and permit arbitrary distributions of the opportunity cost. Despite the profit function not being concave, we are able to characterize the optimal FDS pricing policy both for a Universal plan with unlimited free deliveries (as is common in practice) and a Tiered strategy that offers multiple plans tailored to different consumer segments. Building upon the method to solve a simpler base model with a single consumer type, we developed an algorithm to find the optimal Universal plan with multiple consumer types. We also generated several managerial insights based on our comparative statics analysis exploring the sensitivity of the optimal policy, profits, and market share to product, consumer, and market characteristics. Our analytical results and extensive computational tests validate several observations from practice. For instance, in the optimal FDS plan, the cost to ship to FDS subscribers can far exceed the subscription revenues from these consumers. And, the Universal plan with unlimited free deliveries often generates profits that are close to those of the optimal Tiered plan.

Although motivated by the online retailing context, our model and results also apply to other contexts such as information goods and access services whose features are covered by our model. Our approach and results also point to several directions for future work. For instance, we might augment the model to include behavioral reasons for lift in demand. Characterizing the behavioral lift, validating it using experiments, and quantifying the effect of the behavioral lift on the firm's optimal subscription policy offers interesting avenues for further investigation. The model lends itself quite readily to extensions that incorporate "partly" external consumers who currently purchase a portion of their requirements using the firm's PFD option, but may be willing to switch all of their demand to the firm under a FDS plan that provides appropriate subsidy. Other possible extensions include simultaneously optimizing the FDS subscription fee and delivery limits and the PFD delivery charge, and understanding the effect of competition when other retailers also introduce FDS plans.

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