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Coalition Stability in Assembly Models

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In this paper, we study dynamic supplier alliances in a decentralized assembly system. We examine a supply chain in which n suppliers sell complementary components to a downstream assembler, who faces a price-sensitive deterministic demand. We analyze alliance/coalition formation between suppliers, using a two-stage approach. In Stage 1, suppliers form coalitions that each agree to sell a kit of components to the assembler. In Stage 2, coalitions make wholesale price decisions, whereas the assembler buys the components (kits) from the coalitions and sets the selling price of the product. Stage 2 is modeled as a competitive game, in which the primary competition is vertical (i.e., supplier coalitions compete against the downstream assembler), and the secondary competition is horizontal, in that coalitions compete against each other. Here, we consider three modes of competition—Supplier Stackelberg, Vertical Nash, and Assembler Stackelberg models—that correspond to different power structures in the market. In Stage 1, we analyze the stability of coalition structures. We assume that suppliers are farsighted, that is, each coalition considers the possibility that once it acts, another coalition may react, and a third coalition might in turn react, and so on. Using this framework, we predict the structure of possible supplier alliances as a function of the power structure in the market, the number of suppliers, and the structure of the demand.

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1. Introduction

Consider a decentralized assembly system where n suppliers of complementary components sell to a downstream assembler, who faces market demand. The assembler buys components from the suppliers and assembles the final product to meet demand. Examples of such supply chains are numerous and span industries from automobiles and various electronic and high-tech sectors, to retail (Starbucks, for example, can be thought of as an assembler selling a final product that comprises of milk, sugar, coffee, etc.) and service settings (tour operators can be thought of as assemblers who put together vacation packages that bundle several complementary services). We assume that the downstream assembler, who faces deterministic price-sensitive market demand, buys components from the suppliers, assembles them, and sets the selling price of the final product. In this setting, we are interested in examining coalition/alliance formation between the component-suppliers.

Decentralized assembly systems have been the topic of several papers in the operations management literature. For a detailed discussion of various issues in decentralized assembly supply chains, see Bernstein and DeCroix (2004a, b) and Wang (2006). However, much of the existing literature (with very few exceptions) focuses on the contracting and coordination issues between suppliers and the assembler, resulting equilibrium inventory levels, etc. Our interest is different, and is to study the alliances that suppliers may possibly form in this supply chain. Evidence from real-world assembly systems, in which coalitions of complementary component-suppliers selling kits (bundles) are formed, is not uncommon. Examples include auto-parts suppliers (see, for example, Stallkamp 2001), travel and tourism, various service sectors, the health-care industry, etc. For more examples of supplier coalitions in assembly systems, see Nagarajan and Bassok (2008).

In our analysis, we look at the previously described supply chain and propose a two-stage game. In the first stage, suppliers form coalitions and determine the coalition structure (a partition of the set $\{1, 2, \ldots, n\}$). Coalition members are component-suppliers that tacitly agree to set the joint wholesale price for the kit consisting of the components supplied by all coalition members. We impose the condition that individual coalition partners sign a binding agreement to this effect. This assumption keeps the model tractable and is not unusual in the literature. Moreover, this assumption is consistent with some of the examples we have illustrated and resonates with the philosophy of suppliers selling kits to a downstream assembler. In the second stage, the different coalitions and the assembler compete with each other by setting wholesale and retail prices, respectively. In analyzing the second stage, we use three models of competition. In the first model, suppliers act as Stackelberg leaders and set the wholesale prices first, and, after observing these prices, the assembler determines the order quantity (i.e., sets the retail price). In the second model, all parties determine their prices simultaneously through a Nash equilibrium. In the third model, the assembler is the Stackelberg leader and determines his markup first, and the suppliers set their prices after observing his decision. In every case, the equilibrium prices determine the individual profits of the n suppliers and the assembler. Thus, the profit of an individual player depends on the coalition structure in the market, the coalition that he is a member of (if the firm is a supplier), and the prices set by coalitions. Using this framework and working backwards, we conjecture the characteristics of the stable outcomes in the first stage. That is, we predict the stable supplier coalition structures (the alliances in the market) as a function of the competition and the number of players. We want to lay special emphasis on the word "stability." The market can, over time, see changes in coalition structures (through defections and regrouping) before some sort of stability is attained. It is our interest to include these dynamics in our analysis. Perhaps more importantly, the existing theoretical literature on coalitions in assembly models assumes that players are myopic, and thus completely ignores this issue (for exceptions, see Nagarajan and Bassok 2008 and Granot and Yin 2008). Thus, we believe that incorporating a notion of stability that captures some of these dynamics is of theoretical importance and practical value. Towards this end, we use two different concepts—the farsighted stability concept resulting in the largest consistent set (LCS), proposed by Chwe (1994); and the equilibrium process of coalition formation (EPCF), proposed by Konishi and Ray (2003). We primarily use the LCS concept, and then apply the EPCF to check the robustness of our results and to provide refinements of the LCS.

In general, the reasons for forming supplier alliances in supply chains are manifold. Tactical operational reasons include, for example, potential cost savings through shared resources and economies of scale, risk pooling, improved capacity utilization, benefits that arise due to collaboration in design, etc. Strategic reasons to form alliances include the ability to attract better contractual terms due to increased bargaining power. In this paper, we examine a very simple assembly model, in which many of the previously mentioned reasons for coalition formation among suppliers are absent. We consider a deterministic price-sensitive downstream demand with linear costs. Moreover, suppliers sell complementary components. Thus, we remove tactical operational advantages arising from risk pooling and potential cost savings due to economies of scale. We look at a very simple contract between the players, so as to isolate the effect of vertical and horizontal competition. We assume that each supplier (or a coalition of suppliers) charges a per-unit wholesale price to the assembler.[1] Despite the lack of obvious tactical reasons, due to vertical competition with the assembler, suppliers have an incentive to form alliances. However, horizontal competition between the supplier alliances leads to some interesting dynamics. For example, a set of suppliers may benefit from creating a new alliance structure by defecting from a large alliance. Thus, it is not immediately clear what a stable alliance structure may look like. We are interested in answering the following questions: (1) If component-suppliers are allowed to freely form coalitions, what would be the resulting stable coalitional outcomes? and (2) How does the specific model of competition and the number of suppliers affect the resulting stable coalitional outcomes?

Our analysis provides some answers to the above questions. After an initial analysis by the LCS and subsequent refinements, our main results are as follows. When suppliers are "powerful" (Stackelberg leaders—small n), they form the grand coalition. When the assembler becomes relatively more powerful (either because n increases and the suppliers compete with each other more intensely, or when we move toward a model where all players move simultaneously, whereby the suppliers lose some of their first-movers' advantage), we also find defections from the grand coalition to two coalitions being stable. When all players compete simultaneously (i.e., when there is no Stackelberg leader), the grand coalition is no longer stable, and coalition structures with two roughly equal-sized coalitions become uniquely stable. At the other extreme, when the suppliers are relatively at their weakest (i.e., when the assembler moves first and when n is large), the grand coalition of all suppliers again shows up as the unique stable outcome. We note that these results are contingent on the assumption that agents are farsighted; myopic concepts of stability will lead to results that are very different. If we include membership and friction cost in our analysis, we find that many of our results continue to hold for reasonable costs, but the friction cost can remove the grand coalition from the set of stable outcomes. We also analyze a few nonlinear demand models. Our analysis suggests that the results from the costless linear demand model continue to hold for nonlinear models with large n as long as the elasticity of demand is not constant. When demand is isoelastic, for the parameters to remain meaningful, we only analyze relatively small values of n. In this case, for every model of competition, we show the stability of the grand coalition and a structure with a coalition containing all but one supplier. This difference in results between models with isoelastic demand and demand with nonconstant elasticity should not be surprising because previous channel literature (e.g., Choi 1991) shows that isoelastic demand may reverse many results obtained for linear demand functions. In this paper, we try to provide some intuition on some of these differences.

We believe that our results are potentially interesting to practitioners. The structure of the assembly systems has several important operational and strategic implications. From the assembler's perspective, the ability to predict the number of suppliers he deals with, the structure of coalitions they form, and resulting kits they sell can impact the profit, and perhaps even have implications on the design and variety of the final product. For a component-supplier, the ability to predict stable outcomes provides insights on whom to align with and the long-term stability of his decisions. Also of interest to managers is the dependence of these results on the elasticity of the demand and its curvature. We note that in much of our analysis, we ignore the costs involved in forming an alliance and manufacturing a kit. Clearly, design issues and associated costs may work toward reversing the trend toward consolidation. We partially analyze the effects of operational costs and derive some insights on the structure of alliances when such costs are included in §6.1, and find that many of our results continue to hold when these costs are not too prohibitive.

Our findings may provide some motivation for the impetus that component-suppliers face toward forming strategic alliances in the automobile industry. Indeed, in the last several years, the number of individual component-suppliers for the big-three car manufacturers has dramatically decreased. Although the increased supplier consolidation is driven by the assemblers who want to make the final assembly process more streamlined and flexible, the final number of the alliances also depends on the suppliers' decisions regarding alliance membership. Our analysis may shed partial insights on the suppliers' willingness to participate. Our findings may also explain the situation in the fiber-optics systems, where assemblers routinely buy kits from a few large coalitions. In general, our results will have greater predictive power in industries where alliance formation and coordination of alliance members are relatively inexpensive and less complicated.

From a theoretical perspective, our model of competition in Stage 2 presents some interesting dynamics among supplier alliances in assembly systems. The stability analysis in Stage 1 presents some interesting challenges both in proof techniques and in computational complexity.

The structure of this paper is as follows. In the next section, we provide a brief literature review. In §3, we introduce the model with linear demand. In particular, we describe Stage 2 for the three models of competition. Section 4 briefly introduces, as stability criteria, farsighted notions of coalition stability (the LCS and the EPCF), which are subsequently used in our analysis. We also describe the methodology behind the computations of the stable outcomes in our setting, which may be of theoretical interest and perhaps useful to future researchers. In §5, we characterize stable outcomes. Section 6 discusses some extensions of our analysis. Here, we first analyze the impact of including operating costs in coalition formation, and then briefly discuss nonlinear demand models, with an emphasis on isoelastic demand. We conclude by summarizing the main insights from our results and possible future work in §7. Due to space restrictions, all proofs and several technical discussions related to the analysis in §6 are given in the online appendix. An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

2. Literature Review

In this section, we provide a brief literature review of decentralized assembly systems and stability analysis for coalitional games.

Predictions about coalitions in assembly systems have received very little attention in the economics literature. Tirole (2003) provides references to related models in which the analysis suggests that the coalition of all suppliers yields the maximum surplus. In these studies, analysis of arbitrary coalition structures and associated stability issues are ignored. The operations literature on decentralized assembly models traditionally focuses on the equilibrium capacity (Wang and Gerchak 2003), inventory, capacity and price values (Bernstein and Decroix 2004a, b), or on contracting issues in decentralized assembly systems (Wang and Gerchak 2003, Gurnani and Gerchak 2007). Wang (2006) and Jiang and Wang (2005) study contracting issues in an assembly system with price-sensitive stochastic demand. These papers use noncooperative models of competition and do not analyze supplier coalitions.

Nagarajan and Bassok (2008) use a bargaining framework to analyze profit allocations in an assembly system in which farsighted suppliers form coalitions, and do not consider price as a decision variable. Granot and Yin (2008) analyze push-and-pull contracting systems between an assembler and his suppliers and investigate their various aspects (alliance formation, profit allocation, and members' preferences over the two systems). Price is again not a decision variable, and the stability concept they use is different from ours. Yin (2006) looks at an assembly system in which price and inventory are decision variables, but uses a myopic notion of stability. In contrast to them, we keep the Stage 2 model simple, depend on price, and allow for various modes of competition. In Stage 1, we use a rich model of farsighted stability that allows us to predict stable outcomes as a function of the competition. Moreover, we validate our results using two completely different models of stability.

3. The Assembly Model

In this section, we describe Stage 2 of our model. Consider n suppliers, each of whom sells a complementary component to a downstream assembler facing market demand. We assume that this demand is a simple linear function, given by $D(p) = a - bp$, and that there is no downstream cost of assembling the components. It should be clear from the proofs that introducing a linear assembling cost offers no further insight on the coalition stability, and thus we scale this cost to zero. In general, this reasoning does not apply if there are economies of scale in costs associated with assembling the components.

Each supplier; i, has a marginal cost of production c_i and sells to the assembler at a per-unit wholesale price of w_i, $i = 1, \ldots, n$. To avoid the possibility of negative demand, we assume that $D(C) = a - bC \geqslant 0$, where $C = \sum_1^n c_i$. If we let $W = \sum_{i=1}^n w_i$, the assembler generates a profit

$$\Pi_A(p, w_1, \ldots, w_n) = (p - W)(a - bp), \tag{1}$$

while the profit of a supplier, i, can then be written as $\Pi_i(p, w_i) = (w_i - c_i)(a - bp)$.

We consider three possible models of competition between the assembler and the n suppliers. First, we consider the *Supplier Stackelberg* game (SS), where the suppliers are Stackelberg leaders. In this model, each supplier simultaneously determines a wholesale price w_i, and the assembler then sets the retail price of the product. This case represents the market with several powerful suppliers selling to relatively smaller assemblers. The market is controlled by suppliers, who play the role of Stackelberg leaders and take the assemblers' reaction function into consideration when making their wholesale price decisions. With only a few strong suppliers in the market, suppliers may learn competitors' prices when making their decision. Next, we consider the *Vertical Nash* (VN) game, where suppliers and the assembler move simultaneously. VN games are usually used to model situations where all players are small or medium sized. In this setting, no player benefits from the first-mover advantage. In the VN game, a supplier may not be able to determine competing suppliers' wholesale prices (due to a smaller size and a larger number of players in the market), but can observe the final retail price on which he conditions his decision. The assembler cannot observe suppliers' reaction functions, so she conditions her decision on the suppliers' wholesale prices. Finally, we consider the *Assembler Stackelberg* (AS) game, where the assembler determines her margin, and each supplier then determines his wholesale price. This case assumes that the assembler has larger influence on the market than the suppliers and endows her with the first-mover advantage. The assembler uses the suppliers' reaction functions when determining the retail price, whereas the suppliers condition their decision on the final retail price. These three models of competition have been used in the literature to model market power (see, for example, Choi 1991).

3.1. Supplier Stackelberg (SS) Model

We start with the case wherein the suppliers act as Stackelberg leaders. The assembler's first-order conditions given $(w_1, \ldots, w_n)$ can be derived from (1):

$$\frac{\partial \Pi_A}{\partial p} = a - 2bp + bW = 0, \tag{2}$$

with second derivative $-2b < 0$, satisfying the second-order condition for maximum. It is then immediate that the assembler sets a price $p^{\mathrm{SS}}(w_1, \ldots, w_n) = (a + bW)/2b$. Using the reaction function $p^{\mathrm{SS}}(w_1, \ldots, w_n)$, the suppliers' equilibrium wholesale prices can be derived from the first-order conditions of their profit-maximization problems, $\partial \Pi_i(p(W), w_1, \ldots, w_{i-1}, w_{i+1}, \ldots, w_n)/\partial w_i = \frac{1}{2}(a - bW - bw_i + bc_i) = 0$, and each supplier at equilibrium sets a wholesale price w_i^{SS} that satisfies

$$w_i^{\mathrm{SS}}(w_1, \ldots, w_{i-1}, w_{i+1}, \ldots, w_n) = \frac{a + bc_i - b\sum_{j \neq i} w_j}{2b}. \tag{3}$$

Therefore, the equilibrium prices are $p^{\mathrm{SS}} = (a(2n+1) + bC)/2b(n+1)$, $w_i^{\mathrm{SS}} = c_i + (a - bC)/b(n+1)$.

3.2. Vertical Nash (VN) Model

Next, we consider the case wherein all players act simultaneously. We assume that each supplier takes as a given the final retail price and the assembler's margin, which we denote by m. Thus, $p = W + m$. The first-order conditions for a supplier, i, who is maximizing profit function $\pi_i(w_i, p, m) = (w_i - c_i)(a - bp)$, are given by $\partial \Pi_i / \partial w_i = a - bp - b(w_i - c_i) = 0$. Each supplier at equilibrium sets a wholesale price w_i^{VN} that satisfies

$$w_i^{\mathrm{VN}}(p, m) = \frac{a - bp}{b} + c_i, \tag{4}$$

hence,

$$W^{\mathrm{VN}} = \frac{n(a - bp)}{b} + C. \tag{5}$$

The assembler conditions his margin on the total wholesale price, W. As a result, his first-order conditions correspond to (2), and the equilibrium prices are $p^{\mathrm{VN}} = (a(n+1) + bC)/b(n+2)$, $w_i^{\mathrm{VN}} = c_i + (a - bC)/b(n+2)$.

3.3. Assembler Stackelberg (AS) Model

Finally, we assume that the assembler first determines his margin, m, and the suppliers then determine their prices. Thus, the reaction function for the suppliers is given by (4), and expression (5) holds for W. The assembler is maximizing his profit function, given by $\Pi_A(p, W(p)) = [p(n+1) - na/b - C](a - bp)$, and the equilibrium prices are $p^{\mathrm{AS}} = (a(2n+1) + bC)/2b(n+1)$, $w_i^{\mathrm{AS}} = c_i + (a - bC)/2b(n+1)$.

3.4. Comparisons

The three models endow different market power on the players. In many models of vertical competition, Stackelberg leaders are naturally endowed with a first-mover advantage. We find that this is not altogether true in assembly systems. To see this, first note that $w_i^{\mathrm{AS}} \leqslant w_i^{\mathrm{VN}} \leqslant w_i^{\mathrm{SS}}$. This implies that when the assembler moves first, the wholesale prices are the lowest, whereas the suppliers charge the highest wholesale price when they are the Stackelberg leaders. One can also note that $p^{\mathrm{VN}} \leqslant p^{\mathrm{AS}} = p^{\mathrm{SS}}$. That is,

Table 1. The suppliers' and the assembler's profits with linear demand.

Profit	SS	VN	AS
Supplier i	$\frac{(a-bC)^2}{2b(n+1)^2}$	$\frac{(a-bC)^2}{b(n+2)^2}$	$\frac{(a-bC)^2}{4b(n+1)^2}$
Assembler	$\frac{(a-bC)^2}{4b(n+1)^2}$	$\frac{(a-bC)^2}{b(n+2)^2}$	$\frac{(a-bC)^2}{4b(n+1)}$

higher retail prices are set when there is a leader in the market as compared to the VN system. This is not surprising because in the VN model, the competition among the players is the most intense and, consequently, the final price that consumers see is at its lowest value. Although prices (both wholesale and retail) follow a somewhat predictable pattern and endow the Stackelberg leader with some advantage, the relationship between the individual profits that players make across the three models is not so clear (see Table 1). Note that, for the simplest case (i.e., when $n = 1$), as intuition would suggest, Stackelberg leaders enjoy a favorable outcome. However, when $n > 1$, the VN model endows more profit on every supplier than either Stackelberg model. This is somewhat counterintuitive because VN makes players compete and removes any first-mover advantage that a Stackelberg game endows upon the leader. However, as the suppliers charge higher prices in the SS model, the assembler reduces the quantity ordered, and consequently the suppliers' profit decreases. In addition, although in all three models the equilibrium retail prices exceed the first-best price, $(a + bC)/2$, the additional degree of competition in the VN model drives the retail price lower, which increases the supply chain profits. Finally, we note that in any model of competition, the profit of all players decreases as n increases.

3.5. Coalition Analysis

We now let suppliers freely form coalitions among themselves. We assume that each coalition sells a kit of products (one from each member) and sets a wholesale price for that kit. Thus, if there are l supplier coalitions, the assembler faces a vector $(w_1, \ldots, w_l)$ and consequently sets the retail price $p^*(w_1, \ldots, w_l)$ of the product.

Let us first introduce some notation. We denote by $N = \{1, 2, \ldots, n\}$ the set of all players (suppliers). A subset $S \subseteq N$ is called a *coalition*, and the set N of all players is referred to as the *grand coalition*. Any partition of N, $\mathscr{Z} = \{Z_1, \ldots, Z_l\}$, $\bigcup_{i=1}^{l} Z_i = N$, $Z_j \cap Z_k = \varnothing$, $j \neq k$, corresponds to a *coalition structure*. We identify certain coalition structures that play an important role in the analysis. Assume that the set of all players is partitioned so that k players form a coalition, C, while each of the remaining players acts independently. We denote this coalition structure by $\mathscr{Z}_k^n$, and refer to it as a *basic coalition structure*. Thus, $\mathscr{Z}_1^n$ represents the outcome in which all players act independently. Basic coalition structures describe market settings when there is one large alliance and several individual players. These are commonplace in markets where substitutable products are sold—examples include dairy cooperatives, raw material suppliers, etc. We let $|Z_k|$ denote the size of the coalition Z_k. An arbitrary coalition structure $\mathscr{Z} = \{Z_a, Z_b, \ldots, Z_l\}$ with $|Z_a| = a, |Z_b| = b, \ldots, |Z_l| = l$, where $|Z_i| = 1$ for at most $l - 2$ coalitions, will be denoted by $\mathscr{Z}_{a,b,\ldots,l}$. Thus, $\mathscr{Z}_{k,n-k}^n$ denotes a coalition structure with one k-member and one $n - k$-member coalition. We use $\mathbf{Z}$ to denote the set of all possible coalition structures.

Let $\Pi_i^{\mathscr{Z}}$ denote the profit obtained by player i in coalition structure $\mathscr{Z}$. Thus, Π_i^N denotes the profit of any player when the grand coalition is formed. We denote the cost incurred by coalition Z_k by $c^{Z_k} = \sum_{j \in Z_k} c_j$, equilibrium price charged by coalition Z_k by w^{Z_k}, and the corresponding equilibrium price charged by the assembler by $p^{\mathscr{Z}}$. Clearly, some of these notations are incomplete because the prices set and profit realized by a player depend on both the coalition structure and his individual membership. However, these notations will be made clear depending on the context.

Before analyzing equilibrium prices and profits of coalitions, we need to resolve the issue of surplus allocation among coalition members. Let $\mathscr{Z} = \{Z_1, \ldots, Z_l\}$ be an arbitrary coalition structure. It is easy to see that at equilibrium the assembler purchases equal quantity, say Q, from each supplier coalition and sets p so that the demand equals Q units. It follows from (3) and (4) that for every coalition Z_k,

$$b(w^{Z_k} - c^{Z_k}) = \begin{cases} a - bW & \text{for the SS model,} \\ a - bp & \text{for the VN and AS models.} \end{cases}$$

Consequently, the equilibrium wholesale prices satisfy

$$w^{Z_r} - c^{Z_r} = w^{Z_s} - c^{Z_s}, \quad r, s \in \{1, \ldots, l\}. \tag{6}$$

This also implies that the net profit obtained by each coalition is the same, independent of its membership. As a result, we assume that coalition members divide the profit equally.[2]

Every coalition at equilibrium charges the same markup. Consider an arbitrary coalition, $Z_k \in \mathscr{Z}$, and a supplier, $i \in Z_k$. Table 2 describes equilibrium prices and profits. We can observe that each supplier's profit depends on the number of alliances in the coalition structure and the number of suppliers in the coalition that he belongs to. The following proposition summarizes some key results.

Proposition 1. *Let $\mathscr{Z} = \{Z_1, \ldots, Z_l\}$ be a coalition structure with at least two nonempty coalitions.*

(1) *If $i \in Z_k$, $j \in Z_m$, and $|Z_k| > |Z_m|$, then $\Pi_i^{\mathscr{Z}} < \Pi_j^{\mathscr{Z}}$.*

(2) *Suppose that $i \in Z_k$ leaves the coalition, thereby changing the coalition structure to $\mathscr{Z}' = \{Z_1, \ldots, Z_k \backslash \{i\}, \{i\}, \ldots, Z_l\}$. Then, $\Pi_i^{\mathscr{Z}'} \geqslant \Pi_i^{\mathscr{Z}}$.*

(3) *In the SS model and the AS model, $\sum_1^n \Pi_i^{\mathscr{Z}} < \sum_1^n \Pi_i^N$.*

(4) *In the VN model, $\sum_1^n \Pi_i^{\mathscr{Z}} < \sum_1^n \Pi_i^{\mathscr{Z}_{k,n-k}^n}$ for any $0 < k < n$ and $\mathscr{Z}$ with $l \neq 2$.*

Table 2. Equilibrium prices and profits in coalition structure $\mathscr{Z} = \{Z_1, \ldots, Z_l\}$ with linear demand.

Equil.	SS	VN	AS
$w^{Z_k} - c^{Z_k}$	$\frac{a-bC}{(l+1)b}$	$\frac{a-bC}{(l+2)b}$	$\frac{a-bC}{2(l+1)b}$
$Q^{\mathscr{Z}}$	$\frac{a-bC}{2(l+1)}$	$\frac{a-bC}{l+2}$	$\frac{a-bC}{2(l+1)}$
$\Pi_i^{\mathscr{Z}}$	$\frac{(a-bC)^2}{2b(l+1)^2\lvert Z_k\rvert}$	$\frac{(a-bC)^2}{b(l+2)^2\lvert Z_k\rvert}$	$\frac{(a-bC)^2}{4b(l+1)^2\lvert Z_k\rvert}$

This proposition merits some discussion. Item 3 states that the grand coalition maximizes the total profit generated by all suppliers in the AS and the SS models, whereas item 4 states that in the VN model, the same holds for any outcome in which there are exactly two coalitions. This brings about the following discussion. In general, coalitional games in which the grand coalition is Pareto efficient (as in the Stackelberg models) have some nice properties. In our setting, this aids somewhat in the characterization of stable outcomes. The VN model does not possess this property, and thus adds another layer of complexity to the analysis.

Item 1 implies that whenever there are two different-sized supplier coalitions, a member of a larger coalition realizes smaller profit than a member of a smaller one. This follows directly from the fact that each coalition makes equal profit irrespective of its size and, as per the discussion before, coalition members share the coalition profit equally among themselves. Thus, even though forming coalitions may be advantageous, this pushes players toward memberships in smaller-sized alliances.

Item 2 is an important property that all three models of competition exhibit—from any status quo outcome, an immediate defection by a member who decides to compete by himself benefits the defector. Thus, if one were to use a myopic concept of stability, all coalitional outcomes (with the possible exception of the outcome where no coalitions are formed, $\mathscr{Z}_1^n$) are unstable. Now the outcome $\mathscr{Z}_1^n$ is dominated by the grand coalition for all players, so a myopic concept may in fact predict that stability will never be reached. As will be seen, a dynamic concept of stability (which assumes that players are farsighted) will not necessarily lead to this conclusion.

To summarize, Pareto efficiency sets the stage for the trade-offs that a player or a coalition faces when deciding to defect from existing coalitions and/or when contemplating forging new alliances. On one hand, Pareto efficiency implies a large total pie, whereas on the other hand, defecting from such a state may lead to an outcome in which a coalition may enjoy a larger share of a smaller pie. Thus, in the dynamic process of defecting and forming alliances, suppliers continuously face the trade-off between obtaining a smaller fraction of a larger pie versus a larger fraction of a smaller pie. Further, if they are farsighted, they fully anticipate future ramifications of their immediate moves.

Finally, items 1 and 3 above imply the following useful observation, which gives us a handle on when merging coalitions benefit. This observation is also useful in the proofs of stability.

Corollary 1. *In the AS and the SS models, the profit for the merging coalitions increases only when there is a merger from a coalition structure with two coalitions to form the grand coalition.*

This result somewhat decreases the complexity of the search for stable outcomes in the Stackelberg models. Item 4, on the other hand, complicates the analysis of coalition stability, and we use some novel ideas and techniques to compute the stable outcomes in the VN model.

4. Stability Concepts

In this section, we analyze the stability of supplier alliances. Before we describe the exact methodology, we will briefly try to motivate our framework.

Game-theoretical concepts of stability are usually static. In noncooperative strategic form games, the often-used concept is the Nash equilibrium, which only considers deviations by individual players. In our setting, we assume that all coalitions can communicate among themselves and can join or leave alliances at their will. Thus, we may expect that they will consider both unilateral and multilateral deviations from a given coalition structure. The *strong Nash equilibirium* (SNE) in strategic form games (Aumann 1959) admits this extension. The *coalition structure core* (Aumann and Dreze 1974) is the cooperative analogue of the SNE. However, these solution concepts, along with the majority of solution concepts commonly used in the analysis of coalition structures stability, including the *core* (Gillies 1959) and *coalition-proof Nash equilibrium* (Bernheim et al. 1987), share the same problem that afflicts all static concepts. This can be illustrated as follows: Consider the SS model with linear demand that we analyzed in the previous section. Assume that the status quo position is the grand coalition of all suppliers. We know that it is beneficial for a player to defect from the grand coalition. The existing static concepts will immediately conclude that the grand coalition is *not* stable. There are potentially two fundamental problems with this logic. First, does this mean that the outcome in which a player competes with a coalition of $n-1$ players is stable? If not, why should we conclude that the move from the grand coalition will ever happen? Second, the static analysis does not check if a further defection from the outcome with the coalition of $n-1$ players will occur. Indeed, we know that a second defection by a player may be anticipated. In fact, it may possibly happen that an initial defection triggers a sequence of further defections that eventually leads to an outcome in which the defecting parties accrue a lower payoff than the status quo. If this were the case, farsighted players may not choose to defect in the first place, and thus an outcome that we thought was possibly not stable, may actually prove to be stable. A static concept, by definition, does not handle such trade-offs.

A solution concept that allows players to consider multiple possible further deviations is *the largest consistent set*, introduced by Chwe (1994). It is defined below, and is used as a stability criterion in our analysis of stable alliance structures.

4.1. Largest Consistent Set (LCS)

Let us denote by $\prec_i$ the players' strong preference relations, described as follows: for two coalition structures, $\mathscr{Z}_1$ and $\mathscr{Z}_2$, $\mathscr{Z}_1 \prec_i \mathscr{Z}_2 \Leftrightarrow \Pi_i^{\mathscr{Z}_1} < \Pi_i^{\mathscr{Z}_2}$, where $\Pi_i^{\mathscr{Z}}$ is a retailer i's profit in the coalition structure $\mathscr{Z}$. If $\mathscr{Z}_1 \prec_i \mathscr{Z}_2$ for all $i \in S$, we write $\mathscr{Z}_1 \prec_S \mathscr{Z}_2$. Denote by $\rightharpoonup_S$ the following relation: $\mathscr{Z}_1 \rightharpoonup_S \mathscr{Z}_2$ if the coalition structure $\mathscr{Z}_2$ is obtained when S deviates from the coalition structure $\mathscr{Z}_1$. For a given coalition S, let $F_S(\mathscr{Z})$ denote the set of coalition structures achievable by a one-step coalitional move by S from $\mathscr{Z}$. We say that $\mathscr{Z}_1$ is *directly dominated* by $\mathscr{Z}_2$, denoted by $\mathscr{Z}_1 < \mathscr{Z}_2$, if there exists an S such that $\mathscr{Z}_2 \in F_S(\mathscr{Z}_1)$ and $\mathscr{Z}_1 \prec_S \mathscr{Z}_2$. We say that $\mathscr{Z}_1$ is *indirectly dominated* by $\mathscr{Z}_m$, denoted by $\mathscr{Z}_1 \ll \mathscr{Z}_m$, if there exist $\mathscr{Z}_1, \mathscr{Z}_2, \mathscr{Z}_3, \ldots, \mathscr{Z}_m$ and $S_1, S_2, S_3, \ldots, S_{m-1}$ such that $\mathscr{Z}_{i+1} \in F_{S_i}(\mathscr{Z}_i)$ and $\mathscr{Z}_i \prec_{S_i} \mathscr{Z}_m$ for $i = 1, 2, 3, \ldots, m-1$.

A set Y is called *consistent* if $\mathscr{Z} \in Y$ if and only if for all $\mathscr{V}$ and S, such that $\mathscr{V} \in F_S(\mathscr{Z})$, there is a $\mathscr{B} \in Y$, where $\mathscr{V} = \mathscr{B}$ or $\mathscr{V} \ll \mathscr{B}$, such that $\mathscr{Z} \not\prec_S \mathscr{B}$. Chwe (1994) proves the existence, uniqueness, and nonemptiness of the largest consistent set. Because every coalition considers the possibility that, once it reacts, another coalition may react, and then yet another, and so on, the LCS incorporates *farsighted* coalition stability. The LCS describes all possible stable outcomes and has the merit of "ruling out with confidence." That is, if $\mathscr{Z} \notin$ the LCS, $\mathscr{Z}$ cannot be stable. For a more detailed analysis of farsighted coalition stability, see Chwe (1994). Some applications of analysis of stability using Chwe's LCS criterion include Masuda et al. (2000), Granot and Sošić (2005), Nagarajan and Bassok (2008), Sošić (2006), and Nagarajan and Sošić (2007).

4.2. Equilibrium Process of Coalition Formation (EPCF)

Although Chwe (1994) establishes the existence of an LCS, a criticism of this solution concept is that it may be too inclusive. Konishi and Ray (2003) propose an alternate dynamic approach to stability of coalition structures, which they call the *equilibrium process of coalition formation*, and establish a possible link between the LCS and the limit states of absorbing deterministic EPCFs. In this section, we define and describe the EPCF. For some essential differences between the LCS and the EPCF, see Nagarajan and Sošić (2007).

For each player, recall that $\Pi_i^{\mathscr{Z}}$ denotes player i's profit in the coalition structure $\mathscr{Z}$, and let δ_i denote his discount factor. Then, i's profit from a sequence of coalition structures $\{\mathscr{Z}_t\}$ may be written as $\sum_{t=0}^{\infty} \delta_i^t \Pi_i^{\mathscr{Z}_t}$. Clearly, $\delta \to 0$ will correspond to myopic solution concepts, and being concerned with farsightedness, we are more interested in values of δ close to one.

A process of coalition formation (PCF) is a transition probability p: $\mathbf{Z} \times \mathbf{Z} \to [0, 1]$, so that $\sum_{\mathscr{V} \in \mathbf{Z}} p(\mathscr{Z}, \mathscr{V}) = 1$ $\forall \mathscr{Z} \in \mathbf{Z}$. A PCF p induces a value function v_i for each player i, which represents the infinite-horizon payoff to a player starting from any coalition structure $\mathscr{Z}$ under the Markov process p, and is the unique solution to the equation $v_i(\mathscr{Z}, p) = \Pi_i^{\mathscr{Z}} + \delta_i \sum_{\mathscr{V} \in \mathbf{Z}} p(\mathscr{Z}, \mathscr{V}) v_i(\mathscr{V}, p)$. We say that S has a strictly profitable move from $\mathscr{Z}$ under p if there is $\mathscr{V} \in F_S(\mathscr{Z})$, $\mathscr{V} \neq \mathscr{Z}$, such that $v_i(\mathscr{V}, p) > v_i(\mathscr{Z}, p)$ $\forall i \in S$. A move $\mathscr{V}$ is called *efficient* for S if there is no other move $\mathscr{W}$ for S such that $v_i(\mathscr{W}, p) > v_i(\mathscr{V}, p)$ $\forall i \in S$. Now, we can define EPCF—a PCF is an equilibrium PCF if: (1) whenever $p(\mathscr{Z}, \mathscr{V}) > 0$ for some $\mathscr{V} \neq \mathscr{Z}$, then there is S such that $\mathscr{V}$ is a profitable and efficient move for S from $\mathscr{Z}$, and (2) if there is a strictly profitable move from $\mathscr{Z}$, then $p(\mathscr{Z}, \mathscr{Z}) = 0$ and there is a strictly profitable and efficient move $\mathscr{V}$ with $p(\mathscr{Z}, \mathscr{V}) > 0$. Thus, a deviation from one coalition structure to another occurs only if all members of the deviating coalition agree to move and they cannot find a strictly better alternative. In addition, the deviation from a coalition structure must occur if there is a strictly profitable move. Note that this definition does not require that every strictly profitable move has a positive probability. Konishi and Ray (2003) show that there is an equilibrium process of coalition formation.

If $p(\mathscr{Z}, \mathscr{V}) \in \{0, 1\}$ $\forall \mathscr{Z}, \mathscr{V}$, a PCF, is called *deterministic*. A coalition structure $\mathscr{Z}$ is said to be *absorbing* if $p(\mathscr{Z}, \mathscr{Z}) = 1$, whereas a PCF is absorbing if, for every coalition structure $\mathscr{V}$, there is some absorbing coalition structure $\mathscr{Z}$ such that $p^{(k)}(\mathscr{Z}, \mathscr{V}) > 0$, where $p^{(k)}$ denotes the k-step transition probability. Konishi and Ray (2003) show that when the δs are large enough, the set of all absorbing states under all deterministic EPCFs is a subset of the LCS. In addition, under the LCS, different coalitions may posses different conjectures about what will happen at a given state. If all players have common beliefs (as they must in an EPCF), this possibility cannot arise. This is one reason why the set of all absorbing states (under all deterministic absorbing EPCFs) is typically a strict subset of the LCS.

4.3. Methodology

We now discuss techniques that we use in this paper to characterize the LCS and the absorbing states of the EPCF. First, note that a coalitional outcome, $\mathscr{Z}$, is in the LCS only if it is a member of some consistent set Y. From the definition of a consistent set, we need to ensure that every defection from $\mathscr{Z}$ is deterred, which implies that we need to find elements of Y that can act as $\mathscr{B}$ in the definition of the LCS. This immediately provides a sufficient condition for an outcome to be stable—$\mathscr{Z}$ is stable if, for all $\mathscr{V}$ and S such that $\mathscr{V} \in F_S(\mathscr{Z})$, we have $\mathscr{V} \ll \mathscr{Z}$. To characterize stability for certain small values of n, we use this definition successfully. Two outcomes, the grand coalition (N) and the independent structure ($\mathscr{Z}_1^n$), require special attention; this is particularly true in the context of the Stackelberg models, where the grand coalition is efficient. If the players are symmetric and items 1–3 in Proposition 1 hold, the grand coalition can be shown to be uniquely stable if some players in any outcome with unequal coalitions make less than in $\mathscr{Z}_1^n$. We use this technique in characterizing the grand coalition as the unique stable outcome for small n in the SS and AS models (see Proposition 2).

Although the above definition is useful, checking the above condition using brute force can be done only for small values of n. In general, checking the indirect dominance can involve analyzing an exponential number of possibilities, and there is no known efficient oracle that can check for indirect dominance. Indeed, computing the LCS is, in general, NP-hard. There are two potential recipes that get around this issue; both use some properties of the assembly model, and include those in Proposition 1. The first technique uses the idea that the LCS can be thought of as a fixed point of a suitably defined map $f: 2^{\mathbf{Z}} \to 2^{\mathbf{Z}}$. It then represents coalition structures as equivalence classes of an equivalence relation. Using relatively well-known topological ideas on evaluating fixed points associated with maps on equivalence classes (see Tarski 1955), many elements of the LCS can be characterized in polynomial time for large values of n for several problems (the assembly model being one of them). This exercise is computational and does not often lead to insights on the structure of the outcomes. The second technique uses the fixed-point characterization to derive the LCS as the intersection of a finite number of inductively defined sets. This representation lends itself more suitably to analysis of the structure of the stable coalitions. In particular, it not only helps predict stable outcomes, but also helps eliminate unstable coalition structures. In conjunction with this approach, we use a property of the LCS referred to as *external stability*. External stability implies that, given an outcome $\mathscr{X}$ not in the LCS, we can show that there exists some $\mathscr{Z}$ in the LCS such that $\mathscr{Z}$ indirectly dominates $\mathscr{X}$. We prove and use variants of this property in our analysis. These techniques are used in our proofs, especially when n is large. For example, in Theorem 1, they help rule out certain outcomes from being stable (items 2 and 3), and help verify the stability of other outcomes (items 1 and 4). We note that despite enormous simplifications achieved by these techniques, a complete characterization of the LCS was not achieved in Theorem 1 for the SS model. In the VN model, the news is better. Even though the grand coalition is not Pareto dominant, the fact that there are approximately $n/2$ coalition structures with two coalitions that can potentially be used to eventually deter initial defections enables us to exactly characterize the LCS (Theorem 3). Thus, the techniques described above are more fruitful in this case.

As mentioned, the LCS is liberal in picking candidates for stability, and is difficult to completely characterize even for simple games; thus, we use the EPCF to validate and refine our results. We focus on deterministic PCFs and are interested in computing their associated set of absorbing states. For sufficiently high foresight, the deterministic PCF refines the LCS and thus narrows down the findings of the LCS. Moreover, sometimes the EPCF may be easier to calculate than the LCS. When this is the case, one may be able to use the EPCF to indirectly find outcomes in the LCS! An illustration of this is item 1 in Theorem 2, where a complete characterization of EPCF with high foresight produces outcomes in the LCS that were not calculated in Theorem 1. This weakness of the LCS is well recognized. This sentiment and the use of the EPCF to refine the LCS to get more meaningful results can be seen in Xue (1998) and Konishi and Ray (2003). Dutta et al. (2003) use a dynamic programming formulation for capturing farsighted stability in network games, akin to the EPCF, and use this to refine outcomes potentially suggested by consistent sets.

In general, complete characterizations of the EPCF is hard. Recall that we are interested in calculating associated absorbing states of transition probability functions that satisfy $p(\mathscr{Z}, \mathscr{V}) \in \{0, 1\}\ \forall \mathscr{Z}, \mathscr{V}$, along with the conditions required for an EPCF. There are two difficulties here. First, given a deterministic PCF, we need to evaluate dynamic programs with large state spaces to evaluate if it is an equilibrium process. Second, there is an exponential number of possible PCFs. For small values of n, we can enumerate the PCFs and check their absorbing states. For large values of n, we once again note that the absorbing states of deterministic PCFs can be written as fixed points of suitably defined set-valued maps. Characterizing these fixed points (i.e., sets) leads to efficient approximation algorithms for computing the absorbing states, but often does not provide characterizations. In this paper, we use the route of indirect objections—they consider sequences with finite moves, from a given state, that lead to a potential absorbing state. Because the payoffs for any given state are easily computed in the assembly model, and because myopic defections by lone players are beneficial, one can derive bounds on payoffs that inhibit moves from any given state (see the proofs of Theorems 2 and 4). This idea aids in the characterizations.

Finally, we employ *external stability*, whenever required, to resolve ambiguities about stable outcomes. External stability has been a consistent requirement for a stable outcome in coalitional games. This sentiment was initiated by von Neumann and Morgenstern (1953) and is echoed by Greenberg (1990). We use the idea that the smallest subset of the LCS that possesses external stability and withstands refinement by the EPCF has the highest likelihood of being stable. The idea of using the smallest subset of stable candidates with external domination as a refinement has some merits. Dynamic concepts of stability that incorporate the idea of caution (Mauleon and Vannetelbosch 2004, Kamat and Page 2001) end up predicting the smallest set of outcomes that are obtained by refinements using external stability. External stability is used to refine outcomes in games with a dynamic formulation such as the EPCF (Deroian 2003, Bhattacharya and Ziad 2003). Mathematically, when solutions are characterized by multiple fixed points (as in our case), the smallest set of externally stable outcomes guarantees the least sensitivity to the actual trajectory of the defections (Roth 1977). Finally, if we cast dynamic coalition games as a directed graph, the smallest set of externally stable outcomes is akin to subgame perfection in noncooperative models (van den Nouweland 1993, Chvatal and Lovasz 1972). We characterize such outcomes in our setting and discuss the implication of our findings.

5. Stable Outcomes

The analysis of supplier coalitions in §3 showed the following trade-off: by forming large coalitions, on one hand total profits may increase, but on the otherhand, players myopically benefit by defections. For the previously mentioned reasons, this leads itself nicely to a dynamic analysis when players are farsighted. A myopic concept would in our setting often predict that the set of stable outcomes is empty; this is an undesirable result because it sheds little light on what the behavior of the suppliers would be. As we show, a dynamic analysis provides richer and more satisfactory predictions.

In what follows, we first try to calculate the LCS. For the SS model, our characterization is incomplete (Theorem 1), while we are able to characterize the entire LCS for the VN model (Theorem 3). We then use the EPCF with high degrees of foresight and completely characterize the absorbing states for both the SS and VN models.[3] An interesting consequence of the characterization is that the EPCF in the SS model (Theorem 2, item 1) indirectly identifies some elements of the LCS. Finally, whenever the refinement by the EPCF leaves us with multiple stable outcomes, we use the idea of the smallest set of outcomes that possesses external dominance.

Before we proceed, we explain what we refer to as *equal-sized coalition structures* through an example. Consider a set N, $|N| = n$, and a coalition structure $\mathscr{Z}$ that consists of three sets of players, $\mathscr{Z} = \{Z_1, Z_2, Z_3\}$. We will consider $\mathscr{Z}$ to be an equal-sized coalition structure if the following holds:

(1) If $n = 3k$, $\mathscr{Z}$ contains three coalitions of the same size, $|Z_1| = |Z_2| = |Z_3| = k$;

(2) If $n = 3k+1$, $\mathscr{Z}$ has two coalitions of the same size, $|Z_1| = |Z_2| = k$, while $|Z_3| = k+1$;

(3) If $n = 3k+2$, $\mathscr{Z}$ has two coalitions of the same size, $|Z_1| = |Z_2| = k+1$, while $|Z_3| = k$.

Let $\lfloor x \rfloor = \max\{z\colon z \leqslant x, z \text{ integer}\}$. Then, if $\mathscr{Z}$ contains l coalitions, the coalition sizes are either $\lfloor n/l \rfloor$ or $\lfloor n/l \rfloor + 1$, and the difference in the coalitions' sizes is at most one.

5.1. Supplier Stackelberg Model

In this section, we characterize stable outcomes for a small number of players, and partially characterize them for large n. We start with some cases for which the grand coalition is the only stable outcome.

Proposition 2. *In the SS model with linear demand, LCS $= \{N\}$ when $n < 6$.*

With a small number of suppliers, the only stable outcome is the one in which all suppliers join the alliance. Note that a lone supplier always benefits from a defection; hence, the grand coalition is never myopically stable, whereas it is always farsighted stable. As the number of suppliers increases, the alliance of all players ceases to be the only stable outcome—structures with smaller supplier alliances become stable as well (e.g., $\mathscr{Z}^n_{n-1}$). As n becomes large, evaluating all stable outcomes (or even testing the membership of outcomes in the LCS) becomes intractable. However, using the techniques discussed in §4.3, we were able to generate partial results for an arbitrary number of suppliers. Let $\lceil x \rceil = \min\{z\colon z \geqslant x, z \text{ integer}\}$. The following result summarizes our findings.

Theorem 1. *In the SS model with linear demand, the following statements hold*:

(1) *The grand coalition is always in the LCS.*

(2) *The coalition structure where all players act independently is never in the LCS.*

(3) *Let* $\hat{k} = n - \lceil (\sqrt{4n+5} - 1)/2 \rceil + 1$. *Then, no basic coalition structure* Z^n_k, $k \leqslant \hat{k}$, *is in the LCS.*

(4) *For large n*, $\mathscr{Z}^{2k}_{k,k}$ *or* $\mathscr{Z}^{2j+1}_{j,j,1}$ *are in the LCS.*

The first finding is stability of the grand coalition, N. Recall that suppliers truly enjoy Stackelberg leadership only when $n = 1$. Thus, by forming the grand coalition, they imitate the situation when $n = 1$. This by itself does not explain the grand coalition's stability. In fact, we know that a lone player is better off by defecting from N and competing with the remaining $(n-1)$-member coalition. Thus, almost all myopic concepts will rule out N as being stable. However, the fact that players contemplating a move take into account future defections ensures that N is actually a member of the LCS. Item 2 above states that $\mathscr{Z}^n_1$, in which no coalition is formed, is unstable. Note that the move from $\mathscr{Z}^n_1$ to N is myopically beneficial for all players. When players are farsighted, this by itself simply does not preclude stability. The intuition behind this result is that the realization that $\mathscr{Z}^n_1$ completely fritters away any benefits of being a Stackelberg leader is sufficient to make it unstable when players are farsighted; similar reasoning can be applied to basic coalition structures with many loose suppliers. Note that, in general, when n is large,[4] suppliers prefer to form few large coalitions. This suggests, for example, that in the automobile industry, in which assemblers often encourage suppliers to form alliances, suppliers by themselves have an incentive to do so. Outcomes such as $\mathscr{Z}^{2k}_{k,k}$ offer a compromise between forming the grand coalition and the fact that coalitions may benefit in the short term by defecting from N.

The LCS still predicts somewhat liberally—stable outcomes are not unique. Moreover, we cannot provide its complete characterization. We next use the EPCF to address this issue. For a general n, denote the set of absorbing states of an EPCF by $EP(n)$.[5] As mentioned earlier, for large enough δ, $EP(n) \subseteq LCS$. In our setting, arguments discussed in §4.3 lead to the following result.

THEOREM 2. *Consider the SS model with linear demand. Let* $\underline{k} = \lceil 4/9n \rceil$, $\bar{k} = \lfloor 5/9n \rfloor$, *and* $\hat{l} = \lceil 2\sqrt{n} - 1 \rceil$.

(1) *There exists* Δ^1 *such that for* $1 > \delta > \Delta^1$,

$$EP(n) = \{N\} \cup \{\mathcal{Z}^n_{k,\,n-k},\, k \leqslant \underline{k}\} \cup \{\mathcal{Z}^{2j+1}_{j,\,j,\,1} \text{ if } n = 2j+1 \text{ odd}\}.$$

(2) *For* $1 > \delta > 0.5$,

(i) $\mathcal{Z}^n_{k,\,n-k} \notin EP(n)$ *for* $\underline{k} < k < \bar{k}$;

(ii) $\mathcal{Z} = \{Z_1, \ldots, Z_l\} \notin EP(n)$ *whenever* $l \geqslant \hat{l}$.

The above result has a few implications. Item 1 completely characterizes the EPCF for players with high foresight. In this process, we also recover members ($\mathcal{Z}^n_{k,\,n-k}$ with $k \leqslant \underline{k}$) of the LCS that we could not directly characterize in Theorem 1. The EPCF rules out some members of the LCS—one such outcome is $\mathcal{Z}^{2k}_{k,\,k}$. However, note that the grand coalition is not ruled out as a stable outcome. One advantage of the EPCF is that we can speculate what the stable outcomes would be with even moderate foresight. Item 2 indicates that as soon as δ is greater than 0.5, certain outcomes ($\mathcal{Z}^{2k}_{k,\,k}$, $\mathcal{Z}^n_1$, etc.) will never end up as being stable. In addition, item 2 (ii) provides an upper bound on the number of coalitions in stable outcomes: with $n = 20$, we will observe at most 7 coalitions; with $n = 90$, there are at most 17 coalitions in a stable outcome.

In summary, based on the above two theorems, one can predict that the grand coalition, certain outcomes with only two unequal coalitions, and $\mathcal{Z}^{2j+1}_{j,\,j,\,1}$ are possible stable outcomes. In an ensuing section, we seek the smallest set of externally stable outcomes among these.

5.2. Vertical Nash Model

The VN model is used when both parties are closer to being equally powerful. Recall that in this model of competition, we have shown that the grand coalition is not an efficient outcome. For the suppliers, outcomes with two coalition structures are efficient. As a result, the process of determining stable outcomes for large n may become much more computationally complex. However, the fact that we now have several efficient candidates to deter initial defections enables us to provide a complete characterization of the LCS for the VN game.

THEOREM 3. *In the VN model with linear demand, the following statements hold*:

(1) $LCS = \{\mathcal{Z}^n_1\}$ *when* $n = 2$ *or* $n = 3$. $\mathcal{Z}^n_1 \notin LCS$ *for any other n.*

(2) $LCS = \{N\}$ *when* $n = 5$ *or* $n = 7$. $N \notin LCS$ *for any other n.*

(3) *Equal-sized coalition structures with two or three coalitions are the only members of the LCS when* $n = 4$, $n = 6$, *and* $n \geqslant 8$.

Recall that the VN model dominates the SS and AS models for the suppliers only when $n \geqslant 2$. This explains the reluctance that suppliers have in forming the grand coalition. In fact, as item 2 states, N is only stable when $n = 5$ and $n = 7$. This is due to the fact that we cannot split the set of suppliers into two or three coalitions of exactly the same size, and the profits with a small number of players are very sensitive to the coalition size. Item 3 shows the persistence of equal-sized coalitions as being stable. Note that, if one looks at large values of n in the SS model, inherent supplier competition is so high that the VN model is not very far away from capturing the trade-offs for individual suppliers. This may partially explain item 3. Finally, $\mathcal{Z}^n_1$ is again unstable when we have at least four players, for similar reasons as in the SS case. In general, the trend in the VN model is for suppliers to consolidate into a few large coalitions.

We once again analyze the absorbing states of the EPCF, which leads to our next result. The proof is quite similar to that of the SS model, and thus we omit it.

THEOREM 4. *Consider the VN model with linear demand.*

(1) $EP(n) = \{\mathcal{Z}^n_1\}$ *when* $n \in \{2, 3\}$; $EP(n) = \{N\}$ *when* $n \in \{5, 7\}$.

(2) *When* $1 > \delta > 2/3$, *equal-sized coalition structures with more than three coalitions are never in* $EP(n)$.

(3) *When* $n = 4$, $n = 6$, *or* $n \geqslant 8$, *there is* Δ^* *such that for* $1 > \delta > \Delta^*$, *equal-sized coalition structures with two or three coalitions are the only members of* $EP(n)$.

The main finding is that equal-sized coalition structures with two and three coalitions are predicted as the only absorbing states when players are sufficiently farsighted. Note that each player in an equal-sized coalition structure with at least four coalitions makes less than any player in an equal-sized coalition structure with two coalitions. Thus, we cannot find an EPCF with an absorbing state in an equal-sized coalition structure with more than three coalitions when players exhibit some foresight ($\delta > 2/3$).

Based on the previous two theorems, we can conclude that coalitions structures with two or three equal-sized coalitions are the likely stable outcomes in the VN model, with the exception of small values of n. In §5.4, we look at possible refinements of these outcomes via external stability.

5.3. Assembler Stackelberg Model

In the AS model, the expressions for the coalition profits are very similar to those in the SS model. In fact, the profit obtained by a coalition, Z_k, is half of what Z_k obtains in the SS model. This fits in with the idea that, when the assembler moves first, suppliers are worse off than in the SS game. This phenomenon is not unique to our assembly model—similarity of profit expressions in situations where either the upstream or downstream player(s) is the Stackelberg leader can be seen in games when products are substitutes (Choi 1991) or a combination of substitutes and complements (Shugan and Jeuland 1988) under linear demand assumption. The underlying reason for this seems to be that the Stackelberg leader in a linear demand model enjoys a first-mover advantage due to his ability to manipulate the derived elasticity of the demand function that he sees as a response to his move. When the downstream price setter (the assembler in our model, the retailer in other channel settings) moves first, the benefit of being the leader is marginally (if any) different from the case where the upstream player moves first because she already directly controls the original demand curve. In our assembly model, the first move made by the assembler does not lead to an increase in her power relative to the SS model, but inflicts relative losses on both the channel and the supplier. However, the difference with the SS model is not of an order of magnitude, and thus our analysis from before carries through. This may also explain results in the literature in which the first-mover advantage may simply not exist in vertical competition with constant demand elasticity. In §6.2, we discuss this and related issues of coalition stability and its relationship to the shape of the demand.

As a result, when we analyze the stable outcomes for small n in the AS model, we obtain the same results as in the SS case. After extending our analysis to an arbitrary number of players, we can show that the results are similar to Theorems 1 and 2.

5.4. Stable Outcomes and Competition Models

In this section, we use external stability to further refine the set of stable outcomes, and we try to draw a theme of how coalitions evolve as a function of the mode of competition.

Let us first reflect on how the stable outcomes evolved. We started with the SS model with small values of n, where N was uniquely stable. As the value of n increased, the LCS and EPCF started picking outcomes that were different from the grand coalition (although N was never ruled out). In particular, the outcomes that we were left with after Theorem 2 are: N, $\mathcal{L}^n_{k,\,n-k}$ with $k \leqslant \underline{k}$, and $\mathcal{L}^{2j+1}_{j,\,j,\,1}$ for $n = 2j+1$. Now, the smallest subset of this set that is sufficient for external stability is $\{N\}$. One can make an even stronger statement in this case—from the observation in the proof of Theorem 1, we can notice that $\{N\}$ is in fact the smallest such subset of the LCS, although we do not know the LCS fully. As a result, we may conclude confidently that the grand coalition alone is the most likely stable outcome in the SS model—the SS model gives the suppliers first-mover advantage, which they lose by forming several coalitions. Even though a sequence of myopic defections can be beneficial and leads to some indecision (the reason why both LCS and EPCF were somewhat liberal), the grand coalition eventually trumps.

Next, when we move to the VN model, we consider the set of outcomes with two or three equal-sized coalitions. If we put this set to the test of external stability, we get the outcome with two equal-sized coalitions. Recall that even though external stability refines the outcomes $\mathcal{L}^n_{k,\,n-k}$, $k \leqslant \underline{k}$, and $\mathcal{L}^{2j+1}_{j,\,j,\,1}$ as not being likely in the SS model, the EPCF picks them as candidates for stability. This may indicate that there is some likelihood that they may crop up as being stable in certain SS settings, especially when n increases. In fact, our calculations indicate that these outcomes are absorbing states even with moderate foresight (say $\delta > 0.5$). When n is large in the SS model, the suppliers fritter away their first-mover advantage and the game may tend toward the VN game, in which players are more or less equally powerful. In such a case, we can observe the following pattern: we start from the grand coalition when the suppliers are powerful; the outcomes with two splits emerge when their power diminishes, leading to the point in which they consolidate to two equal-sized coalitions in the VN game. Recall that in Proposition 1, we show that the split into two coalitions maximizes the total profit generated in the VN model. That is, this is the Pareto-optimal outcome for the supplier coalitions. Thus, on one hand, suppliers have some incentive to reach an outcome with two coalitions. However, on the other hand, this may not simply pan out, because players may (and do) have an incentive to defect. When coalitions are unequal, players from the larger coalition have a greater incentive to defect, and such defections are harder to deter even in a farsighted sense. This pushes the refinements to only possibly include coalitional structures with equal outcomes, and this leads precisely to the result in Theorem 4. These results are reminiscent of those obtained in Nagarajan and Bassok (2008). For example, they show that suppliers who are equally powerful as the assembler may organize themselves into two equal-sized coalitions. When the assembler is weaker, the suppliers organize themselves into a grand coalition and two coalitions with unequal sizes. Their model is very different then ours and we are loathe to draw further comparisons.

In the AS model, the results revert back to coalitions structures predicted in the SS model. Note that the AS model assumes that we have a powerful assembler; hence, it is more likely that we will see larger values of n. To increase their "power," the suppliers are likely to form one large coalition. It may also happen that suppliers who join the market later (e.g., a new part is required for the product and a new supplier is added to the pool) prefer not to join other suppliers, which corresponds to basic structures with a few lone suppliers. We note that the profits and the resulting stable coalition structures seem to be sensitive to the presence of a Stackelberg leader, irrespective of who the leader is. This is reminiscent of the results in Choi (1991).

We revisit this in §6.2, where we discuss the shape of the demand.

We also note that the behavior of the suppliers when they are at their strongest (SS model, small n) and when they are at their weakest (AS model, large n) is identical (i.e., they prefer the grand coalition). This result, we believe, is somewhat interesting and is seen in union negotiations in the pattern-bargaining and the wage-bargaining literature (Murillo 2001).

6. Extensions

In this section, we discuss two important extensions to the assembly model. First, in all our above analysis, we have assumed that there are no exogenous costs involved in operating alliances. We have also ignored friction costs when players defect from a status quo outcome and form a new coalition. We include these two costs and analyze some of our results. Second, we extend our analysis to certain nonlinear models of demand and discuss the stable outcomes thereof.

6.1. Models with Friction and Costs

To the best of our knowledge, the extant theoretical literature in coalition formation and stability usually ignores the exogenous costs involved in forming, maintaining, and defecting from coalitions. We try to include them in our analysis. In practice, these costs are complex, and our attempts may be simplistic and perhaps abstract. We hope that the ideas will be thought of as first steps and be used and further developed by future researchers to incorporate costs in dynamic coalition games.

We explicitly consider two kinds of costs. First, we assume that there is a cost to being a part of a coalition, which we call the *membership cost* and denote by f. This cost may include, for example, a variety of administrative costs entailed in managing an alliance. Next, we assume that every time a coalition is formed, a certain cost is incurred by the members of the defecting coalition. We call it the *friction cost* and denote it by g. Due to the symmetry in our setting and our assumptions, f and g only depend on the size of the coalition on which they are evaluated and are thus simple cardinality functions—they can be thought of as $f, g\colon \mathbb{N} \to \mathbb{R}$, where $\mathbb{N}$ is the set of natural numbers. More precisely, if $\mathscr{X} \rightharpoonup_S \mathscr{Y}$, each member of S incurs the cost $g(S)/\|S\|$. For any $\mathscr{Z} = \{Z_1, \ldots, Z_l\} \in P(N)$, each member of Z_i incurs $f(Z_i)/\|Z_i\|$. If we assume that g is convex (f concave), these functions induce a set-valued supermodular (submodular) function on the subsets of $\{1, 2, \ldots, n\}$. This may nicely lend itself to the general idea that membership costs are submodular and defection costs may be supermodular.

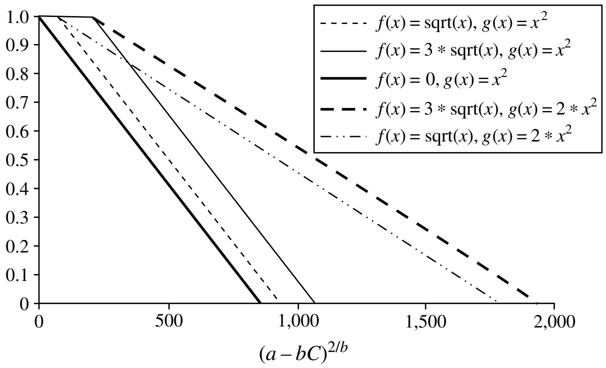
The existence of these costs is well documented by practitioners (Doz 1992) and in the organization theory literature (Doz 1996, Ishii 2000). For a review of this literature, we refer the reader to White (2005). The literature consistently agrees that membership cost increases with the size of the coalition and is likely to be concave. Friction costs seem to be rarely considered in game-theoretic models, and in the setting of alliance formation are even less understood. We were unable to find in the literature a consistent set of properties that this cost usually exhibits. Moreover, the property of cost incurred by an individual player ($g(x)/x$ and $f(x)/x$, $x \in \mathbb{N}$) can be fairly arbitrary. We provide in the online appendix a more detailed analysis of our model, and summarize our main results below.

The magnitude of the membership cost, f, decreases the profit that players may obtain by forming a coalition. In fact, if this cost is significant, certain coalitions may never be feasible. Thus, membership cost can be thought of as mostly affecting the size of coalition profit and feasibility of certain outcomes. To deal with the cost of defections (friction), g, we use a novel approach that abstracts away from the details of friction. Intuitively, the introduction of friction translates to curbing the length of the chains that one may need to consider in calculations. Loosely speaking, friction is related to the notion of bounded rationality (Rubinstein 1997). To show that a certain defection is deterred in the context of the LCS implies that one may need to restrict oneself to a bounded number of possible future defections. In the case of the EPCF, intuition suggests that friction costs deflate the value of δ. We defer details of the modeling of these costs to the online appendix.

Our main results are as follows. With zero friction costs and positive membership cost ($g = 0, f > 0$), we find (see Theorem B1 in the online appendix) that for reasonable values of membership costs (e.g., $f(x) = \alpha x$ or $f(x) = \alpha\sqrt{x}$ for some $\alpha > 0$), most results of Theorem 1 (e.g., stability of N, $\mathscr{Z}^{2k}_{k,k}$, and $\mathscr{Z}^{2j+1}_{j,j,1}$) and Theorem 3 (e.g., stability of equal-sized coalition structures with two and three coalitions) continue to hold. However, some additional structures with smaller coalitions may become stable as well (e.g., equal-sized coalition structures with four coalitions in the VN game). When we subject these outcomes to refinements using the EPCF[6] and external dominance, the SS game predicts the grand coalition and the VN game uniquely predicts the outcome with two equal outcomes (for sufficiently large n). This matches with our predictions from §5.4.

In the analysis with zero membership costs and positive friction costs[7] ($g > 0, f = 0$), we initially find that the LCS rules out the grand coalition in the Stackelberg models, but leaves some other predictions from Theorem 1 (items 2 and 4) intact. In addition, some new basic coalition structures emerge as stable. Interestingly, when further refinements[8] are used, only basic coalition structures remain stable. The intuition behind this result is as follows. Although it makes sense myopically for players to defect from a large coalition, fully farsighted players realize the ill effects of their actions and mutually enforce the grand

Figure 1. Lower bounds for values of δ for which the grand coalition is stable as a function of ρ.



coalition (or few large coalitions). In other words, stability of N is contingent on players contemplating a fairly long and sophisticated sequence of moves. Now, however, they contemplate only a few steps of action, so defections, up to a degree, seem to be a good idea. This results in basic coalitions being stable. We also note that analyzing the VN game seems very challenging. We can show that N and basic coalition structures are not stable. Numerical experiments indicate that outcomes with two equal or close-to-equal coalitions are unstable as well. The only stable outcomes seem to be structures with a few lone players and two larger coalitions. This coincides with the intuition described above for the SS model.

Obtaining even simple theoretical results regarding the effect of both f and g on $EP(n)$ seems to be a daunting task. We show some simple cases in the online appendix, which also illustrates the ideas used in the above refinements. We show the calculations for a model with $n = 3$, for which N was the only stable outcome in the model without costs (see Figure 1). This continues to be the case when the costs are not too large and the suppliers exhibit reasonable levels of farsightedness. We show in the online appendix how the stability of N depends on $\rho := (a - bC)^2/b$ and the level of players' farsightedness: we find that N may cease to be stable for smaller values of ρ when the membership cost f is high. However, when ρ is sufficiently large, N is stable even for high values of f and a low level of farsightedness. On the other hand, the "rate" at which N becomes stable depends more on the friction cost g—as g increases, a higher level of farsightedness is required to achieve stability of N for the same value of ρ. Note that the remaining two coalition structures ($\mathcal{Z}_1^3$ and $\mathcal{Z}_2^3$) can be stable only when ρ is large and the suppliers are not too farsighted (that is, δ is rather low).

6.2. Nonlinear Demand

Although the linear demand model is often used because of its simplicity and its ability to lead to tractable results, there are many instances in which demand exhibits nonlinear characteristics. In this section, we discuss nonlinear demand models, and focus on three types of demand: $D_1(p) = (a - bp)^\gamma$, $\gamma > 0$,[9] $D_2(p) = ae^{-bp}$, and $D_3(p) = ap^{-b}$. Note that $D_1(p)$ and $D_2(p)$ are models of demand with nonconstant elasticity, whereas $D_3(p)$ corresponds to the model with isoelastic demand. We defer much of the analysis to the online appendix, and here provide a few key insights. For the models with nonisoelastic demand, our results are similar to those obtained when $D(p) = a - bp$. When we consider the isoelastic model, $D_3(p)$, we get different results.[10]

The first difference between $D_3(p)$ and the elastic demand models is that, irrespective of the competition type (SS, VN, or AS), the grand coalition is Pareto efficient and stable for the isoelastic demand (in contrast to the VN model under D_1 and D_2, where N was not Pareto nor stable). When one undertakes a complete analysis (that is, first uses the LCS, then refines using the EPCF, and then looks at the smallest externally dominating set of outcomes), we get that N and $\mathcal{Z}_{n-1}^n$ are the only stable candidates across all modes of competition. Thus, in summary, as compared to the elastic demand models, two key differences emerge: (1) in Stackelberg models, a basic coalition structure also becomes stable; (2) in the VN model, the grand coalition also becomes stable. Recall that this assumes $b \geqslant n$ and $n < 6$, so that b is not too large. In what follows, we briefly explain the cause of these differences. We leave some of the technical details for the online appendix.

Farsighted stability depends on a few important factors: first, the difference in profits realized by different coalition structures (as a function of the number of coalitions), and second, given a mode of competition, the relative advantage to a player of being a leader (or a follower). We measure the first factor by the ratio of the profits obtained by the suppliers in a coalition structure with l coalitions and $l + 1$ coalitions. This is formalized using the function $RI(l)$ (defined in the online appendix), which we call the *relative incentive to defect*. One measure of the second factor above seems to be elasticity of the demand. A first mover who sets a wholesale price (or margin) has an advantage by virtue of the elasticity of the derived demand (i.e., the reaction from the second mover) that the first mover manipulates to his advantage. A necessary condition for this to happen is that the price elasticity of the demand is not constant and is a function of the price (otherwise, manipulating w, and hence p, will not change it). When demand is nonisoelastic, this manifests itself as an advantage for the suppliers in the SS model. In the AS model, the assembler (who sets the final price) does not have a derived demand per se because she already controls the direct demand through her pricing decision. Thus, being a leader does not give her a strong relative advantage in the AS model. In the isoelastic demand case, the Stackelberg models yield the first mover and the system lower profits than the VN game. Thus, the fact that the first mover has no control of the elasticity not only negates his first-mover advantage, but makes him vulnerable. This logic can be formalized to some extent using the simple curvature of the demand, $C(p)$ (Munkres 1966). When $C(p) > 1$, the demand is logconvex (as in the isoelastic case). This usually implies (i) little or no advantage to the first mover, and (ii) a decreasing $RI(l)$. A decreasing incentive to defect can be used to explain why $\mathcal{L}_{n-1}^{n}$ is not deterred. We also note that $C(p) \leqslant 1$ for $D_1(p)$ and $D_2(p)$; hence, $RI(l)$ is increasing in l in both these models. This explains, to some degree, the similarity in stability results for these models when demand elasticity is a function of price.

Finally, if we temporarily suspend realism and allow b to grow large in the isoelastic model, we note that $C(p) \to 1$. In this case, one may argue that the simple curvature of the isoelastic model starts to look like that of a demand model with price-sensitive elasticity, which implies that the grand coalition emerges as uniquely stable for large n. This indeed occurs for the SS model with isoelastic demand.

7. Discussion and Future Research

We believe that our analysis of component-supplier coalitions that sell to a downstream assembler provides some insights that may be valuable to practitioners as well as academicians. We show how supplier alliances evolve in decentralized assembly systems, and analyze their dependence on the number of players in the supply chain, the mode of competition, and the type of demand.

Understanding the coalitions between component-suppliers could be of value to an assembler. Component-suppliers sell kits of products, and availability of these kits can have ramifications on the design of the final assembled product and the features that the product exhibits. This is particularly true in service settings, where the characteristics of the final product are affected by the service bundles that are available. For a supplier, understanding the nature of the alliances in the market and their stability can be crucial. This is especially true when a supplier enters a market and wants to decide on his course of action regarding his membership with a particular alliance, or when suppliers in an existing coalition are contemplating a defection.

We believe that we make some theoretical contributions in this paper. From a modeling perspective, we show how competition affects supplier coalitions in a price-sensitive demand setting. In this process, we make some observations about Stackelberg leadership issues in assembly models. In computing and characterizing the LCS and the EPCF, we use some novel ideas that we believe can be used in other settings. Because analysis of dynamic alliance formation is relatively new, we use two concepts to predict the market structure, and thus check the robustness of our results. We also briefly discuss how one may formally incorporate costs involved in dynamic coalition games, at least in the context of assembly systems.

We realize that we have ignored several important aspects of coalition formation. Many of these increase the benefits of alliances. These would strengthen our results, in that more consolidations would occur. We have established a basic framework to examine dynamic alliance formation with price competition in an assembly system. To do so, we have used a simple model in Stage 2. Adding other features to this model, we believe, will make it more intractable. We hope future researchers will enrich and build on this model and analysis.

8. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

Endnotes

1. If we were to introduce a two-parameter contract (e.g., a two-part tariff), we would be endowing the player who offers the contract with too much power, and coalition dynamics would become moot. Moreover, because our intent is to look at vertical and horizontal competitions and not, say, study coordination, we desist from examining other contracts.
2. Note that our model assumes that all components are equally important, that is, it does not look at a potential number of suppliers of individual components. One possible extension of this model would assign lower importance, and lower profit share, to suppliers of commodity-type components.
3. We note that the results for the AS model bear close resemblance to those obtained for the SS model.
4. "Large" clearly depends on the parameters. Our analysis suggests that $n > 12$ is sufficient in most cases.
5. We abuse notation by making this independent of δ, but this will be clear from the context.
6. We only check whether the outcomes for which we showed the LCS membership can be obtained as absorbing states of some EPCFs to verify their robustness. Thus, it is a restrictive analysis.
7. We make specific assumptions on the friction costs, as can be seen in the online appendix.
8. The EPCF is done only with consideration to PCFs using starting points obtained from the LCS. This turns out to be sufficient due to bounds on the length of the chains.
9. $D_1(p)$ with $\gamma = 1$ is exactly the demand that we have analyzed thus far in the paper—the analysis in this case is simplified due to tractable expressions we obtain for the equilibrium profit of the supplier coalitions.
10. Note that in this case we need $b > n$ for the Stackelberg models and $b > n + 1$ for the VN model. This implies that, for realistic demand scenarios (Tellis 1988 shows that b in $D_3(p)$ generally ranges between 1 and 3), we need to restrict ourselves to small values of n; we choose $n < 6$.

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