On Fragmented Markets

Ahmad Peivandi[*], Rakesh V. Vohra[†]

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Abstract

Centralized markets reduce the costs of search for buyers and sellers. Their 'thickness' increases the chance of order execution at competitive prices. In spite of the incentives to consolidate, some markets, securities markets being the most notable, have fragmented into multiple trading venues. We argue in this paper that fragmentation is an unavoidable feature of any centralized exchange except in certain special circumstances.[1]

1 Introduction

A centralized exchange reduces the costs of clearing, settlement and search compared to a market consisting of multiple trading venues. Were these costs to decline because of technological innovation, a centralized exchange should still dominate a fragmented market because traders would prefer the venue that offers the highest probability of order execution and the most competitive prices. Each additional trader on an exchange reduces the execution risk for other potential traders, attracting more traders. This positive feedback should encourage trade to be concentrated in a single exchange.

In spite of the incentives to consolidate, some markets, securities markets being the most notable, have spawned multiple trading venues.[2] Futhermore, they are not

[*]Risk and Insurance Management Department, Robinson College of Business, Georgia State University

[†]Department of Economics and Department of Electrical and Systems Engineering, University of Pennsylvania

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[2]For fragmentation in labor markets see Roth and Xing (1994).

restricted to assets that are non-standardized or have low trading volume. Traditional exchanges now face a host of competitors such as ECNSs (electronic communication networks), ATSs (alternative trading systems) and the trading desks of broker-dealer firms.[3] These venues use a variety of pricing rules, may not broadcast the bids they receive, and, in some cases allow traders to restrict who they will transact with. Madhavan (2000) calls this the *network externality puzzle* and writes: "Despite strong arguments for consolidation, many markets are fragmented and remain so for long periods of time."

A variety of explanations, summarized below (not entirely mutually exclusive), have been offered for why centralized markets fragment.

1. Regulation: Fragmentation enhances efficiency because competition between exchanges forces them to narrow their bid-ask spreads (e.g., Pagano (1989); Biais, Martimort, and Rochet (2000)). Fragmentation in securities markets can be traced to regulation in the 80s and 90s designed to limit the abuse of market power by operators of centralized exchanges.[4] Fragmentation can also enhance efficiency (total welfare) by limiting the market power of participants (Malamud and Rostek (2014)).[5]

2. Heterogenous Preferences: Alternative trading venues arise to cater to traders who differ in their preferences for order size, anonymity and likelihood of execution (Harris (1993), Ambrus and Argenziano (2009) and Petrella (2009)).

3. Congestion: As a market becomes thicker, the time to select, evaluate, and process offers lengthens, during which time prices may change. This encourages participants to transact earlier, fragmenting the market in time (see Roth and Xing (1994)).[6]

4. Informational: Traders seek out alternative venues so as to conceal private information (see Madhavan (1995)), other venues spring up to attract uninformed

[3]According to the Securities Industry and Financial Markets Association and the Bank of International Settlements, daily volume reaches, on average, \$5.4T in the global foreign exchange market, \$2.3B in the U.S. interest-rate derivative market, and \$0.8T in the U.S. bond market.

[4]Regulation National Market System in the US and the Market in Financial Instruments Directive (MiFID) in Europe.

[5]Malmund and Rostek (2014) provide examples of markets where a market fracture can increase the total welfare of market participants, however, the payoff of some agents may be lower post fracture. We consider the incentives of a group of agents to fragment the market.

[6]Congestion can cause fragmented markets to persist as agents tradeoff thickness in one venue for less competition in another (Ellison and Fudenberg (2003), Ellison, Fudenberg and Mobius (2004)).

traders from the incumbent exchange (Easley (1996)) or competing venues affect the incentives to acquire information (Glode and Opp (2016)).

We don't consider the reasons listed above to be fundamental, because they can all be eliminated, *in principle*, by a suitable (but possibly impractical) mechanism. In the first case, the operator could be mandated to implement the constrained efficient mechanism. In the remaining cases, a mechanism that allowed agents to use a richer message space to communicate preferences could be employed.

This paper argues that fragmentation is an unavoidable feature of *any* centralized exchange. Within the model in which we make this point, the reasons for fragmentation enumerated above do not apply. Our setting is the standard model of bilateral trade (Myerson and Satterthwaite (1983)) where each seller has one unit of a homogeneous good and each buyer is interested in purchasing at most one unit of the same good. The private type of each buyer is their marginal value for the good and the private type of each seller is the opportunity cost of their endowment. Thus, agents are all interested in the same order size. Holding prices equal, they are indifferent about who they trade with. There is no common values component in the private information of agents making them equally informed (or uninformed). Trade takes place in a single time period, so the timing of trades is irrelevant.

We model trade within the incumbent exchange as being conducted via an individually rational, weakly budget balanced and incentive compatible mechanism. The first condition is uncontroversial. The second prevents the operator of the incumbent exchange from subsidizing trades. The third recognizes that trading agents will act strategically. Even if the mechanism in the incumbent exchange was not incentive compatible, by the revelation principle, there would be a corresponding incentive compatible direct mechanism that would replicate the outcome of the mechanism in the incubment exchange. We are interested in when the incumbent mechanism is stable, in the sense that no subset of agents has an incentive to deviate and trade among themselves using a different mechanism, called the blocking mechanism. Our message is that budget balance and incentive compatibility conspire to make this impossible. We interpret this to mean it is the violation of the price taking assumption that makes an exchange vulnerable to fragmentation.

Formalizing the idea of blocking raises two conceptual difficulties. First, the decision to participate in the blocking mechanism reveals something about one's type which should be incorporated into the beliefs of potential counterparties. Second, payoffs in the incumbent mechanism will depend on the equilibrium played in that mechanism, which can be affected by the presence of a blocking mechanism. For this reason, we consider two related notions of blocking that depend on the equilibrium being "played" in the incumbent mechanism. The first, from Peivandi (2013), assumes that agents in the incumbent mechanism play a dominant strategy equilibrium. The second, introduced in this paper, assumes that the agents play a Bayesian equilibrium of the incumbent mechanism. We distinguish between them by calling the first D-blocking and the second B-blocking. They differ from prior notions of blocking used in the theory of cooperative games by allowing agents to condition their beliefs about counterparties based on which mechanism they are participating in. Roughly speaking, an incumbent mechanism is *blocked* by a coalition of agents and a blocking mechanism if the blocking mechanism gives to each member of the blocking coalition, for a critical subset of their types, at least as much surplus as they would obtain if they remained in the incumbent mechanism. Furthermore, no agent with a type outside of the critical subset of types will participate in the blocking mechanism. Two features of this notion of blocking differentiate it from similar notions in the literature (see Section 3). First, each agent commits to participating in either the incumbent mechanism or the alternative, but not both, *before* knowing the outcome of each. Second, each agent recognizes that a decision by a counterparty to defect from the incumbent mechanism (or not) reveals some information about the counterparty's type, which should be used. We argue that the conditions under which a mechanism is immune to blocking are very restrictive. From this, we conclude that centralized markets are vulnerable to fragmentation.

We offer two sets of results. In the first, we restrict attention to deterministic mechanisms that are ex-post individually rational (EIR) and ex-post (weakly) budget balanced (EBB), features enjoyed by many observed trading rules. We do not specify a particular mechanism but consider all mechanisms that are robust to the beliefs of agents. Like Hagerty and Rogerson (1987) we model this by requiring the mechanism to be dominant strategy incentive compatible (DSIC). This is often touted as a desirable feature for mechanisms. Dominant strategy incentive compatibility does not exclude the possibility that the mechanism can depend on the designer's beliefs. For example, the designer could select a single price at which all trade must take place *a priori,* which depends on the designer's beliefs about the distribution of types of the agents. We show that for any EIR, EBB, and DSIC mechanism, there is a distribution over types, for which this mechanism can be "D-blocked" by another EIR, EBB, and DSIC mechanism. In particular, the posted price that maximizes ex-ante efficiency is not immune to blocking.

If the distribution of buyer and seller types satisfies the monotone hazard rate condition, there is only one EIR, EBB, and DSIC mechanism immune to D-blocking. It is a posted price mechanism: a price p is fixed a priori, and a pair of buyer and seller who wish to transact do so at price p.[7] However, any fixed price p will *not* suffice. It must lie between the optimal monopsony price set by the highest type buyer and the optimal monopoly price set by the lowest type seller. Therefore, the only EIR, EBB and DSIC mechanisms immune to D-blocking must be *sensitive* to the underlying distribution of types.

If one desires a mechanism to be independent of the beliefs of the designer as well, then, no mechanism (within the class considered) is immune to D-blocking. We show that every mechanism (in the class) can be D-blocked by a simple mechanism called a positive spread posted price mechanism. In such a mechanism, two prices $p_1 \leq p_2$ are posted. If buyer and seller agree to trade, the seller is paid p_1 and the buyer pays p_2. The spread of a posted price mechanism is $p_2 - p_1$, and this is what the designer pockets. A posted price mechanism is one where $p_1 = p_2 = p$. Thus, every EIR, EBB, and DSIC mechanism can be D-blocked by a mechanism that gives the operator of the blocking mechanism positive expected profit. In other words, the tendency to of fragmentation cannot be hindered unless the incumbent mechanism is a money pump for its participants. This is the first reason for our assertion that fragmentation is an unavoidable feature of a centralized exchange.

Our second result focuses on the double bid auction. With one buyer and seller, the price is set between the bid and ask (provided they cross). It is observed in practice, satisfies EIR and EBB, but is not DSIC. It is not even Bayesian incentive compatible (BIC). The analysis of blocking, in this case, is more subtle. The Bayesian equilibrium played in the incumbent mechanism depends on the putative blocking mechanism that agents can participate in. Its presence changes the beliefs of agents about their counterparties in the incumbent mechanism. We show that there is a Bayesian equilibrium of the double auction that cannot be B-blocked by any positive spread posted price mechanism. We believe this result of interest for two reasons. First, it shows that the rules of the mechanism alone do not determine its stability but the equilibrium played. Hence, two markets using the same pricing and allocation rule need not share the same vulnerability to blocking. Second, it shows that a non-DSIC mechanism is, in one sense,

[7]One caninterpret this as a reason for why posted price mechanisms are widely used in practice, see Einav et al. (2013).

more robust than a DSIC mechanism. However, it also shows that agents must settle on exactly the right equilibrium to prevent fragmentation.

In the case when types are uniformly distributed in $[0, 1]$ it is well known that there is a constrained efficient equilibrium of the double bid auction. However, the equilibrium that prevents blocking is not constrained efficient. This runs counter to the intuition that a mechanism that produces efficient outcomes is robust to blocking.

In the next section of this paper we introduce notation and give a precise definition of D-blocking. Subsequently we contrast D-blocking with prior notions of the core of games with incomplete information. The subsequent section states and proves the main results concerning D-blocking. In section 5 we introduce B-blocking and its application to the double bid auction. Section 6 concludes.

2 D-blocking

Let $N = \{1, 2, 3, ..., n\}$ be the set of agents. The value of agent i for a unit of the good is $v_i \in V_i$ where $V_i \subset \mathbb{R}^+$ is bounded. Each v_i is the private information of agent $i \in N$ and is independently distributed. Each agent i has an endowment $\omega_i \in \{0, 1\}$ of the good, which is common knowledge. If $\omega_i = 1$, then agent i is a seller, and if $\omega_i = 0$, agent i is a buyer. Agent preferences are quasilinear; that is, agent i's payoff from receiving a quantity q of the good (interpret as probability) for a payment of t is $qv_i - t$.

A direct mechanism is defined by an allocation rule and a payment rule. The allocation rule maps profiles of reports of the private information of agents to an allocation of the good. If Q is the allocation rule, denote the component of Q that corresponds to agent i's allocation by q_i. Thus, $q_i : \prod_{i \in N} V_i \to \mathbb{R}^+$. As agent i has an endowment of ω_i, we require that an allocation rule be feasible in the sense that for all $i \in N$ and all profiles $\mathbf{v} \in \prod_{i \in N} V_i$ that

1. $1 \geq q_i(\mathbf{v}) + \omega_\mathbf{i} \geq \mathbf{0}$, and,

2. $\sum_{i \in N} q_i(v) = 0$.

The payment rule maps each profile $\mathbf{v} \in \prod_{i \in N} V_i$ to a *per-unit* price each agent must pay. If P is the payment rule, the component of P that corresponds to agent i's per-unit payment is denoted p_i. Thus, $p_i : \prod_{i \in N} V_i \to \mathbb{R}^+$.

We now define dominant strategy incentive compatibility. Let $\mathbf{v} = (v_i, v_{-i})$ and $\hat{\mathbf{v}} = (\hat{v}_i, v_{-i})$ be two profiles of valuations in $\prod_{i \in N} V_i$. Observe that $\hat{\mathbf{v}}$ differs from $\mathbf{v}$ in that agent i only changes the report of his marginal value. Agents can misreport their marginal value or opportunity cost but not their role as buyer or seller. Note that we only need to impose incentive compatibility on deviations from profiles that result in feasible outcomes. The mechanism (Q, P) is DSIC if for all $\mathbf{v}$ and $\hat{\mathbf{v}}$:

$$q_i(\mathbf{v})(v_i - p_i(\mathbf{v})) \geq q_i(\hat{\mathbf{v}})(v_i - p_i(\hat{\mathbf{v}})).$$

Mechanism (Q, P) is ex-post individually rational if for all profiles $\mathbf{v} \in \prod_{i \in N} V_i$ and all $i \in N$

$$q_i(\mathbf{v})(v_i - p_i(\mathbf{v})) \geq 0.$$

Mechanism (Q, P) is (weakly) ex-post budget balanced if for all $\mathbf{v} \in \prod_{i \in N} V_i$

$$\sum_{i \in N} p_i(\mathbf{v}) q_i(\mathbf{v}) \geq 0.$$

In mechanism (Q, P), the utility that agent $i \in N$ under profile $\mathbf{v}$ enjoys is

$$u_i(\mathbf{v}, Q, P) = q_i(\mathbf{v})(v_i - p_i(\mathbf{v})).$$

The expected utility that agent $i \in N$ enjoys when her type is v_i is

$$E_{v_{-i}}[u_i(\{v_i, v_{-i}\}, Q, P)].$$

Now suppose an alternative feasible, DSIC, EIR mechanism $(\hat{Q}, \hat{P})$:

$$\hat{p}_i : \prod_{i \in A} V_i \to \mathbb{R}^+$$
$$\hat{q}_i : \prod_{i \in A} V_i \to \mathbb{R}^+ \ \forall i \in A$$

We will give a definition of what it means for $(\hat{Q}, \hat{P})$ to D-block the incumbent mechanism (Q, P) by a subset $A \subseteq N$ of the agents. Imagine that before participating in the mechanism (Q, P), each agent in A (and only A) is invited to participate in $(\hat{Q}, \hat{P})$. If at least one of the agents in A declines the invitation, all agents are required to participate in (Q, P); in this case we say the D-block is unsuccessful. If every agent in A accepts the invitation, this becomes common knowledge among them, and they enjoy the outcome delivered by $(\hat{Q}, \hat{P})$. The agents now face a Bayesian game in which they must first decide which of the two mechanisms to participate in and subsequently what to report in their chosen mechanism. As each mechanism is DSIC, we assume truthful reporting. We say the set A D-blocks (Q, P) if there is Bayesian equilibrium of the game, where with positive probability all agents in A choose $(\hat{Q}, \hat{P})$. Formally, we need for each $i \in A$, a positive measure subset $V_i' \subseteq V_i$ and an equilibrium where each $i \in A$ chooses $(\hat{Q}, \hat{P})$ if their type is in V_i' and (Q, P) otherwise. Call V_i' the **critical** set of types for agent $i$and for each $i \in A$ let T_i be the event that each agent $j \in A \setminus \{i\}$ has a type in V_j'. The set A D-blocks (Q, P) with respect to $\Pi_{i \in A} V_i'$ if the five conditions listed below hold.

1. If $v_i \in V_i'$, then,

$$E_{-i}[u_i(\{v_i, v_{-i}\}, Q, P)|T_i] \leq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i] \ \forall i \in A \tag{1}$$

2. If $v_i \notin V_i'$ then,

$$E_{-i}[u_i(\{v_i, v_{-i}\}, Q, P)|T_i] \geq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i] \ \forall i \in A. \tag{2}$$

3. For all $\bar{v} \in \prod_{i \in A} V_i'$

$$\sum_{i \in A} \hat{q}_i(\bar{\mathbf{v}}) = 0. \tag{3}$$

4.

$$\forall \bar{\mathbf{v}} \in \prod_{i \in A} V_i' \quad \sum_{i \in A} \hat{q}_i(\bar{\mathbf{v}}) \hat{p}_i(\bar{\mathbf{v}}) \geq 0 \tag{4}$$

5.

$$E[\sum_{i \in A} \hat{q}_i(\bar{\mathbf{v}}) \hat{p}_i(\bar{\mathbf{v}}) | \bar{\mathbf{v}} \in \prod_{i \in A} V_i'] > 0. \tag{5}$$

Condition (1) states that if each $i \in A$ has a type in V_i', then every agent in A choosing to participate in $(\hat{Q}, \hat{P})$ is a best response to the other agents in A doing so. Condition (2) states that if $i \in A$ is the only member with a type not in V_i', then choosing to participate in(Q, P) is a best response for agent i. Condition (3) ensures that the sum of the net trades is zero. Condition (4) states that the mechanism is weakly ex-post budget balanced. Condition (5) requires that, on some profile, the D-blocking mechanism generates a positive surplus. There is a technical and a substantive reason for this condition. The strict inequality means that there is a strict incentive for someone to offer the D-blocking mechanism. In the prior notions of blocking, it is required that the analogue of inequality (1) holds strict for some agent i; however, we eliminate the possibility of an exactly budget balanced mechanism being D-blocked by itself by requiring a strict budget balanced condition on the blocking mechanism.

We have assumed that if any agent in A declines the invitation, all agents must participate in the incumbent mechanism. This makes D-blocking harder. To see why, suppose one buyer and one seller only. If all agents who accept the invitation must trade in the alternative mechanism, there would be two equilibria: one where both agents always choose the incumbent and one where both always choose the alternative. We also assumed that once the agents participate in the blocking mechanism, and this becomes common knowledge, they cannot change their minds and return to the incumbent mechanism. As the incumbent mechanism is DSIC, allowing agents to return to the incumbent mechanism after observing the participants in the blocking mechanism does not alter subsequent results.

3 Prior Notions of Blocking

Immunity to D-blocking (or B-blocking) can be interpreted as a notion of the core of a cooperative game of incomplete information. Forges, Minelli, and Vohra (2002) provides a brief survey of various notions of the core for cooperative games of incomplete information. These notions differ on two dimensions. First, is the decision to block made at the ex-ante or interim stage? Second, are incentive constraints relevant? In our case, the decision to block is made at the interim stage and incentive constraints are certainly relevant. For this reason, we don't discuss either the ex-ante core or the coarse core.[8] The corresponding incentive versions of these core concepts and their drawbacks are summarized in Dutta and Vohra (2005). In response to these drawbacks, Dutta and Vohra (2005) propose the credible core and this is the notion most relevant to this paper. We first contrast the credible core with the notion of D-blocking.

In the credible core, the incumbent mechanism can be any incentive compatible mechanism, not just a DSIC. In this paper, a subset A of agents form a potential block

[8]The notion of durable decision rules due to Holmstrom and Myerson (1983) is concerned with blocking by the grand coalition only.

when each of their types lies in their respective critical sets. As stated in this paper, conditional on an agent in A having a type in the critical set, it is an equilibrium to choose the alternative mechanism. In Dutta and Vohra (2005) the alternative mechanism is only incentive compatible assuming types lie in their respective critical set. It implies that the alternative mechanism bars participation by types not in the critical set. This is accomplished by choosing 'no-trade' in the event than an agent in A reports a type outside their critical set. This is the second major difference with the notion of D-blocking. In our case the alternative mechanism does not rely on such restrictions because it is DSIC.

The differences between B-blocking and the credible core are even starker. To begin with, the incumbent mechanism in our case is not required to be incentive compatible. Second, in one of the versions of B-blocking, conditional on types being in the critical set, we allow the equilibrium in the incumbent mechanism to condition on this fact. This is not permitted in the definition of the credible core.

Dutta and Vohra (2005) is not the last word on the subject. We briefly summarize subsequent contributions highlighting differences. Myerson (2007), using the virtual utility construct, proposes a blocking notion that, in addition to the credibility requirements, considers random coalition formation and random allocations for each coalition. Serrano and Vohra (2007) use coalitional voting in an incomplete information environment to incorporate endogenous information transmission among members of a coalition. Finally, Liu et al. (2014) study the implications of common knowledge of stability of a two-sided match when one side of the market has incomplete information about the other side. The literature on competing mechanisms (common agency) has also focused on how agents will choose between two alternative mechanisms. However, the setting is one-sided in that agents are all buyers (or all workers), and they are choosing between alternative mechanisms in which to purchase something. The need for budget balance, for example, is absent. See Peters (2014) for a survey.

4 Vulnerability to D-blocking

To provide some intuition, we restrict attention to one buyer and seller. In this case, the set A of agents that could possibly D-block an incumbent mechanism will be the set of all agents. We focus on positive (or zero) spread posted price mechanisms for the reason stated below.

Theorem 1. *Every deterministic, dominant strategy incentive compatible, (weakly) ex-post budget balanced, and ex-post individually rational mechanism that generates positive expected profits can be implemented as a positive spread posted price mechanism.*

The proof follows Hagerty and Rogerson (1987) and is omitted. We give two sufficient conditions on the distribution of types such that the only EIR, EBB and DSIC mechanism immune to D-blocking is a posted price mechanism. One of these will follow from a characterization of mechanisms immune to D-blocking. As the characterization is hard to interpret we do not emphasize it.

4.1 Bilateral Trade

Consider a positive spread posted price mechanism. It is easy to see that such a mechanism can always be D-blocked by a positive spread posted price mechanism with a smaller spread. Thus, the only mechanisms (within the class considered) that might be immune to D-blocking are posted price mechanisms. But, what should the posted price be? Let agent 1 be the seller with an opportunity cost of $c \in [0,1]$ and $\omega_1 = 1$ and agent 2 the buyer with a value of $v \in [0,1]$ and $\omega_2 = 0$. Assume c and v are private information distributed independently with atomless density functions $g(c)$ and $f(v)$ respectively. Denote the corresponding distribution functions by G and F. Endowments are common knowledge.

Theorem 2. *If* $x(1-F(x))$ *and* $(1-x)G(x)$ *are concave and arg* $\max_{x\in[0,1]} x(1-F(x)) \geq \arg\max_{x\in[0,1]}(1-x)G(x)$*, then, any posted price mechanism with a price*

$$p \in [\arg\max_{x\in[0,1]}(1-x)G(x), \arg\max_{x\in[0,1]} x(1-F(x))]$$

is immune to D-blocking. The left-hand endpoint of this interval is the optimal posted price set by a buyer whose value is 1; the optimal monopsony price of the highest type buyer. The right-hand endpoint is the optimal monopoly price set by the lowest type seller.

Proof. Let $p \in [\arg\max_{x\in[0,1]}(1-x)G(x), \arg\max_{x\in[0,1]} x(1-F(x))]$ be the posted price of the incumbent mechanism. Consider a positive spread posted price mechanism (p', p'') with $p' < p''$ as a possible D-blocking mechanism. As there are only two agents (one buyer and one seller), the D-blocking coalition will consist of just these two agents.

It remains to identify a critical set of types. We can do this by "reverse" engineering. There are three cases:

1. **Case 1:** $p' < p'' < p$: A buyer with type $v \geq p''$ strictly prefers the D-blocking mechanism conditioned on a seller being present. Thus, the critical set of types of the buyer will be $[1, p'']$. Now, we find the critical set of types for the seller that would make them prefer the D-blocking mechanism. A seller with type $c < p'$ will join the D-blocking mechanism only if:

   $$(p - c)Pr(v \geq p | v \geq p'') \leq (p' - c) \;\Rightarrow\; \frac{1 - F(p)}{1 - F(p'')} \leq \frac{p' - c}{p - c}.$$

   The right-hand side is maximized at $c = 0$; therefore, the posted price p *cannot* be D-blocked by the positive spread posted price mechanism (p', p'') if the following holds:

   $$\frac{1 - F(p)}{1 - F(p'')} > \frac{p'}{p}.$$

   This is equivalent to $p(1 - F(p)) > p'(1 - F(p''))$. Therefore, if for all $p' < p$,

   $$p(1 - F(p)) > p'(1 - F(p')), \tag{6}$$

   the posted price mechanism cannot be blocked with prices lower than p. This is clearly true given the choice of p.

2. **Case 2:** $p < p' < p''$: In this case, the seller with opportunity cost $c < p'$ joins the D-blocking mechanism conditional on a buyer being present. A buyer with type $v > p''$ joins the D-blocking mechanism if

   $$(v - p)Pr(c \leq p | c \leq p') \leq v - p''.$$

   As in Case 1 this does not happen if:

   $$\forall\, p' > p\; (1 - G(p)) > (1 - p')G(p'). \tag{7}$$

3. **Case 3:** $p' < p < p''$: In this case no agent will join the D-blocking mechanism.

□

Our second sufficient condition is based on the hazard rates of the distribution of types and is based on a characterization of the EIR, EBB, and DSIC mechanisms immune to D-blocking. Recall that the hazard rate of the buyer is defined as $v - \frac{1-F(v)}{f(v)}$ while the hazard rate of the seller is defined as $c + \frac{G(c)}{g(c)}$.

Theorem 3. *Assume that the hazard rate of both buyer and seller are increasing. Suppose there exists $p \in [0, 1]$ such that $p - \frac{1-F(p)}{f(p)}$ and $1 - p - \frac{G(p)}{g(p)}$ are both non-positive. Then, a posted price mechanism with price p is immune to D-blocking by a positive spread posted price mechanism.*

Proof. Let $\mathcal{M}$ be any EBB and DSIC mechanism for the case of bilateral trade. Denote by $u_b(v, c)$ and $u_s(v, c)$ the buyer's and seller's payoff, respectively under $\mathcal{M}$. We first identify conditions under which $\mathcal{M}$ is immune to D-blocking by a positive spread posted price mechanism.

Lemma 4. *$\mathcal{M}$ is immune to D-blocking by a positive spread posted price mechanism if and only if for all $0 \le y < x \le 1$ the following holds:*

$$E[u_b(x, c)|c \le y] + E[u_s(v, y)|v \ge x] \ge x - y. \tag{8}$$

If for some $0 \le y < x \le 1$ inequality (8) is violated, we construct a posted price blocking mechanism. Let $V_b = [x, 1]$ and $V_s = [0, y]$ be the critical set of types for buyer and the seller respectively. As inequality (8) is violated, there exists $0 \le p_1 < p_2 \le 1$ such that the following holds:

$$E[u_b(x, c)|c \le y] = x - p_2, \tag{9}$$

$$E[u_s(v, y)|v \ge x] = p_1 - y. \tag{10}$$

For a candidate D-blocking mechanism we choose the positive spread posted price mechanism with prices (p_1, p_2). This mechanism is clearly dominant strategy incentive compatible and budget balanced. We now verify that all types in the critical set weakly prefer the D-blocking mechanism to the mechanism $\mathcal{M}$.

Let $a(v, c)$ be the probability of trade in $\mathcal{M}$ when the the profile of types is (v, c). Recall, from Myerson and Satterthwaite (1983) that $u_b(\alpha, \beta) = \int_0^\alpha a(t, \beta)dt$ and $u_s(\alpha, \beta) =$

$\int_\beta^1 a(\alpha,t)dt$. Therefore, for all $1 \geq v' \geq x$ and $y \geq c' \geq 0$ the following holds:

$$
\begin{aligned}
E[u_b(v',c)|c \leq y] &= E[u_b(x,c)|c \leq y] + \int_x^{v'} E[a(v,c)|c \leq y]dv \\
&\leq E[u_b(x,c)|c \leq y] + (v'-x) = v' - p_2, \qquad (11)\\
E[u_s(v,c')|v \geq x] &= E[u_s(v,y)|v \geq x] + \int_{c'}^{y} E[a(v,c)|v \geq x]dc \\
&\leq E[u_s(v,y)|v \geq x] + (y-c') = p_1 - c'. \qquad (12)
\end{aligned}
$$

Equations (11) and (12) ensure that all types in the critical set weakly prefer the D-blocking mechanism to $\mathcal{M}$. It is straightforward to check that when an agent's type is outside the critical set, this agent does not prefer the blocking mechanism to $\mathcal{M}$.

To prove the reverse we show that if there is a positive spread posted price D-blocking mechanism, inequality (8) is violated for some $0 \leq y < x \leq 1$. Let $0 \leq p_1 < p_2 \leq 1$ be the prices in the D-blocking mechanism and V_b and V_s be the associated critical set of types. As the sets V_b and V_s have positive measure, there exists $x \geq p_2$ and $y \leq p_1$ such that $x \in V_b$ and $y \in V_s$. For all such x, y the following must hold:

$$E[u_b(x,c)|c \in V_s] \leq E[(x-p_2)I_{\{c \leq p_1\}}|c \in V_s]. \qquad (13)$$

The left-hand side of (13) is the expected payoff to the buyer when she participates in $\mathcal{M}$ knowing that the seller has a type in the critical set V_s. The right-hand side is the expected payoff to the buyer when she chooses to participate in the D-blocking mechanism conditional on the seller's type being in the critical set and the seller participating in the D-blocking mechanism. A similar observation yields:

$$E[u_s(v,y)|v \in V_b] \leq E[(p_1-y)I_{\{v \geq p_2\}}|v \in V_b]. \qquad (14)$$

Thus, rewriting inequality (13) yields:

$$
\begin{aligned}
&\frac{\int_{c \in V_s} u_b(x,c)g(c)dc}{Pr(c \in V_s)} \leq \frac{(x-p_2)Pr(V_s \cap [0,p_1])}{Pr(c \in V_s)} \\
&\Longleftrightarrow \frac{\int_{c \in V_s} u_b(x,c)g(c)dc}{Pr(V_s \cap [0,p_1])} \leq x - p_2 \\
&\Longleftrightarrow \frac{\int_{c \in V_s \cap [0,p_1]} u_b(x,c)g(c)dc}{Pr(V_s \cap [0,p_1])} \leq x - p_2 \\
&\Longleftrightarrow E[u_b(x,c)|c \in V_s \cap [0,p_1]] \leq x - p_2 \qquad (15)
\end{aligned}
$$

Similarly, the following inequality holds:

$$E[u_s(v,y)|v \in V_b \cap [p_2,1]] \leq p_1 - y. \tag{16}$$

Inequalities (15) and (16) allow us to assume $V_b \subseteq [p_2,1]$ and $V_s \subseteq [0,p_1]$. Let $x^* = \inf V_b$ and $y^* = \sup V_s$. As the distribution of types is atomless, we may assume $x^* \in V_b$ and $y^* \in V_s$. The following inequalities hold:

$$E[u_b(x^*,c)|c \in V_s] \leq x^* - p_2, \tag{17}$$

$$E[u_s(v,y^*)|v \in V_b] \leq p_1 - y^*. \tag{18}$$

Note that the payoff to a seller with type $c \in V_s \cap [p_1,1]$ is zero in the D-blocking mechanism. Therefore, if a seller has type in $c \in V_s \cap [p_1,1]$, it must receive a payoff of zero in $\mathcal{M}$, i.e., almost surely $\forall v \in V_b$ $u_s(v,c) = 0$.This is similar to a buyer whose type is in $V_b \cap [p_2,1]$. If $a(x^*,y)$ is constant for all $y \leq y^*$, then $u_b(x^*,c) = u_b(x^*,c')$ for any two $c,c' \in V_s$. It follows from (17) that $u_b(x^*,c) = x^* - p_2$ for all $c \leq y^*$. Hence

$$E[u_b(x^*,c)|c \leq y^*] \leq x^* - p_2, \tag{19}$$

Similarly, if $a(x,y^*)$ is constant for all $x \geq x^*$ we deduce that

$$E[u_s(v,y^*)|v \geq x^*] \leq p_1 - y^*. \tag{20}$$

Thus, if $a(x,y)$ is constant in the relevant ranges, the proof is complete. Suppose, for a contradiction, this is not true. Consider the case $x > x^*$ (a similar argument applies when $y < y^*$). For all $x > x^*$ the following holds:

$$\begin{aligned} E[u_b(x,c)|c \in V_c] &= E[u_b(x^*,c)|c \in V_c] + \int_{x^*}^{x} E[a(s,c)|c \in V_c]ds \\ &\leq (x^* - p_2) + (x - x^*) = x - p_2 \end{aligned} \tag{21}$$

If inequality (21) holds with equality for any $\bar{x} > x^*$, it must be the case that for all $x > x^*$ and almost all $c \in V_c$, $a(x,c) = 1$. To see why, note that equality for $x = \bar{x}$ implies that $a(x,c) = 1$ for all $x^* < x \leq \bar{x}$. However, $a(\cdot,c)$ is monotone in its first component by dominant strategy incentive compatibility. Therefore, $a(x,c) = 1$ for all

$x > x^*$. This means that $a(x,c)$ is constant and (19) applies.

Suppose then that inequality (21) is strict for all $x > x^*$. Therefore, $E[u_b(x,c)|c \in V_c] < x - p_2$ for all $x > x^*$. Hence, $x \in V_b$ for all $x > x^*$. A similar argument shows that $y \in V_s$ for all $y < y^*$. This proves the lemma.

Consider a posted price mechanism that selects a price according to density $h(p)$. Lemma 8 implies that this mechanism is D-blocked by a positive spread posted price mechanism if for all $1 \geq x > y \geq 0$ the following holds:

$$\begin{aligned} &\int_y^x (x-p)h(p)dp + \frac{\int_0^y \int_0^p (x-p)g(c)h(p)dcdp}{G(y)} \\ &+ \int_y^x (p-y)h(p)dp + \frac{\int_x^1 \int_p^1 (p-y)h(p)f(v)dvdp}{1-F(x)} \\ &\geq x - y \end{aligned} \tag{22}$$

The right-hand side of inequality (22) can be rewritten as follows:

$$(x-y)(H(x)-H(y)) + \int_x^1 (p-y)\frac{1-F(p)}{1-F(x)}h(p)dp + \int_0^y (x-p)\frac{G(p)}{G(y)}h(p)dp$$

Using integration by parts inequality (22) can be written as follows:

$$\begin{aligned} &\int_0^y \int_x^1 H(v)(v - \frac{1-F(v)}{f(v)} - y)f(v)g(c)dvdc + \int_0^y \int_x^1 H(c)(c + \frac{G(c)}{g(c)} - x)f(v)g(c)dvdc \\ &\geq (x-y)G(y)(1-F(x)) \end{aligned} \tag{23}$$

Inequality (23) provides a necessary and sufficient condition for immunity of a trade mechanism to D-blocking by a positive spread posted price mechanism.

To prove this, consider a randomized posted price mechanism that randomizes only over prices for which both hazard rates are negative. Note that if $H(v) = 1$ for all $v \geq x$ and $H(v) = 0$ for all $v \leq y$, then inequality (23) holds with equality. Such a randomized posted price mechanism sets $H(v) < 1$ in the first part of the integral only if $v - \frac{1-F(v)}{f(v)} \leq 0$ and it sets $H(v) > 0$ in the second integral only if $\frac{G(c)}{g(c)} - 1 \geq 0$. Note that

$$v - \frac{1-F(v)}{f(v)} \leq 0 \Rightarrow v - \frac{1-F(v)}{f(v)} - y \leq 0 \text{ and } \frac{G(c)}{g(c)} - 1 \geq 0 \Rightarrow \frac{G(c)}{g(c)} - x \geq 0.$$

Therefore, inequality (23) holds for this mechanism. This proves the theorem. □

4.2 The General Case

We now allow for more than one buyer and seller.

Theorem 5. *Fix a EIR, EBB and DSIC mechanism that is robust to the beliefs of the designer. For this mechanism there is an atomless distribution over types under which the mechanism can be D-blocked by a group of agents.*

Proof. Suppose the mechanism cannot be D-blocked under any atomless distribution over types. We show that such a mechanism must be ex-post efficient. The theorem follows from the fact that such a mechanism does not exist. Let $I \subset N$ be the set of sellers and $J \subset N$ be the set of buyers. Consider a profile of valuations $x = (x_I, x_J) \in \prod_{i \in N} V_i$. Let $I' \subseteq I$ and $J' \subseteq J$ be the subset of the sellers and buyer that should trade in an efficient allocation. Note that $|I'| = |J'|$. Let W_i be the event that the types of the sellers in $I' \setminus \{i\}$ are below the $x_{I' \setminus i}$ and the type of buyers in $J' \setminus i$ are below $x_{J' \setminus i}$s for all agents in $I' \cup J'$. Formally,

$$W_i = \{v \in \prod_{i \in N} V_i | \forall k \in I' \setminus \{i\} \ v_k \leq x_k \text{ and } \forall k \in J' \setminus \{i\} \ v_k \geq x_k\}.$$

If the following inequality is violated one can construct a D-blocking mechanism as in the the special case: _

$$\sum_{i \in I' \cup J'} E[u_i(x_i, v_{-i}) | W_i] \geq \sum_{k \in J'} x_k - \sum_{k \in I'} x_k. \tag{24}$$

Inequality (24) must hold for all possible atomless distributions. Consider a sequence of the atomless distributions that converge to the distribution that puts probability one on the event that the type profile is x. Therefore, the following must hold:

$$\sum_{i \in I' \cup J'} E[u_i(x) | W_i] \geq \sum_{k \in J'} x_k - \sum_{k \in I'} x_k. \tag{25}$$

Inequality (25) implies that the mechanism must be efficient. □

If we have the same number of buyers and sellers and choose the posted price p so that $1 - F(p) = G(p)$, as the number of agents increases we should converge to the

Walrasian outcome. As the Walrasian outcome is in the core, this appears to contradict Theorem 5. It does not. As the number of agents increases, the expected profit of the blocking mechanism will decrease but still be positive. It is only in the continuum limit that the expected profit of a blocking mechanism falls to zero. We interpret this to mean that D-blocking can only take place if the price-taking assumption is violated.

5 B-blocking

In this section, we modify the notion of D-blocking to account for the possibility that the incumbent mechanism is not DSIC. In this case, we assume the agents play a Bayesian equilibrium of the incumbent mechanism. In a Bayesian equilibrium, the distribution of types matters. The presence of an alternative mechanism changes the distribution of types participating in the incumbent mechanism. Therefore, if we allow agents to return to the incumbent mechanism after observing the decision of other agents in the putative blocking coalition, the equilibrium played in the incumbent mechanism may change. We provide two notions of Bayesian Blocking (B-blocking). In the first, we allow agents to revert to the incumbent mechanism after observing the decision of other agents in the blocking coalition, and in the second, we do not.

5.1 B-Blocking with Possibility of Return

Denote the incumbent mechanism, not necessarily direct, by (M, P, Q) where $M = \prod_{i \in N} M_i$ is the message space. Denote the putative blocking mechanism by $(\hat{P}, \hat{Q})$. The payoff of agent i with type v_i when all agents send message profile m is denoted $u_i(v_i, \{m\}, Q, P)$. We use the notation $(m_i(v_i))_{i \in N}$ for a Bayesian Equilibrium of the incumbent mechanism. As in the case of D-blocking, before participating in the incumbent mechanism (Q, P), each agent in the putative blocking coalition A (and only A) is invited to participate in $(\hat{Q}, \hat{P})$. If at least one of the agents in A declines the invitation, all agents are required to participate in (Q, P); in this case the block fails. If every agent in A accepts the invitation, this selection is revealed to all agents. At this point, each agent in A is given the option of returning to (Q, P). If any one of them chooses to return, all agents in A must return. If all agents in A elect not to return, they enjoy the outcome delivered by $(\hat{Q}, \hat{P})$. Hence, when agents in the Aevaluate their payoffs from joining the blocking mechanism and the incumbent mechanism, they update their belief and assume that the type of agents in A are in their respective critical sets. Therefore, any Bayesian equilibrium of the incumbent mechanism should be consistent with this updated belief and hence must satisfy,

$$\forall i \in N \text{ and } v_i \in V_i \;\; m_i(v_i) = argmax_{m_i \in M_i} E[u_i(v_i, \{m_i, m_{-i}(v_{-i})\}, Q, P)|T_i].$$

The agents in A and a mechanism $(\hat{P}, \hat{Q})$ B-block the incumbent mechanism if there exists a non-zero measure subset of types $(V_i')_{i \in A}$ (the critical set) such that the following inequalities hold for all consistent Bayesian equilibria of the incumbent mechanism, $m_i(v_i)$,

1. If $v_i \in V_i'$, then,

$$E_{-i}[u_i(v_i, \{m_i(v_i), v_{-i})\}, Q, P)|T_i] \leq E_{-i}[u_i(\{v_i, v_{A\setminus\{i\}}\}, \hat{Q}, \hat{P})|T_i] \; \forall i \in A \quad (26)$$

$$T_i = \{(v_k)_{k \in A} | v_k \in V_k', \; \forall k \in A \setminus \{i\}\} \quad (27)$$

2. If $v_i \notin V_i'$ then,

$$E_{-i}[u_i(v_i, \{m_i(v_i), v_{-i}\}, Q, P)|T_i] \geq E_{-i}[u_i(\{v_i, v_{A\setminus\{i\}}\}, \hat{Q}, \hat{P})|T_i] \; \forall i \in A. \quad (28)$$

3. For all $\bar{v} \in \prod_{i \in A} V_i'$,

$$\sum_{i \in A} \hat{q}_i(\bar{v}) = 0. \quad (29)$$

4.

$$E[\sum_{i \in A} \hat{q}_i(\bar{v}) \hat{p}_i(\bar{v}) | \bar{v} \in \prod_{i \in A} V_i'] > 0. \quad (30)$$

We can interpret these conditions in terms of the following "exit" game: agents in A simultaneously decide to join $(\hat{P}, \hat{Q})$ or not. Equations (26) and (28) imply that the "exit" game has a Bayesian equilibrium where agents in A choose the exit option if their types are in their respective critical set of types. Equation (29) is the market clearing condition. Finally, (30) requires that that the blocking mechanism generate positive expected surplus (conditional on types being in the critical set). The important difference between D-blocking and B-blocking is that in B-blocking agents may report messages different from those reported when there was no alternative mechanism.

5.2 Double Auctions

Double Auctions are widely used in practice. Here is how we define them: assume there are n buyers and m sellers. Each buyer $i$reports bid, b_i, and each seller i reports an ask, c_i. Bids and asks are positive real numbers. Index the agents so that $b_1 \geq b_2 \geq b_3 \geq ... \geq b_n$ and $c_1 \leq c_2 \leq c_3 \leq ... \leq c_m$. Let k be the largest index where $b_k > c_k$ and k' be the smallest index where $b_{k'} < c_{k'}$. All buyers and sellers with indices no less than k and a subset of buyers and sellers with indices in $\{k+1, ..., k'-1\}$ trade at price $p(b_k, c_k, ..., b_{k'-1}, c_{k'-1})$. It is required that if a buyer (seller) with bid (ask) b (c) trade, then $v \geq p$ ($c \leq p$). Thus, the price is allowed to depend on the profile of reported bids. If the number of trading buyers (seller) is more than the number of trading sellers (buyers), then sellers (buyers) must be rationed.

The definition of B-blocking requires the mechanism to be blocked for all consistent Bayesian equilibria of the incumbent mechanism. The following result rationalizes this definition:

Theorem 6. *For any double auction that is ex-post budget balanced, there exists a Bayesian Nash Equilibrium of the double auction that can be blocked; i.e., equations (26), (28), (29) and (30) are satisfied.*

Proof. All double auctions have a no trade equilibria, where for all types, trade occurs with zero probability. If the no-trade equilibrium obtains, then all agents for all of their types prefer to participate in the blocking mechanism.

□

Because of Theorem (6), in the definition of B-blocking we require the alternative mechanism to block the incumbent mechanism for all consistent Bayesian equilibria of the incumbent mechanism. This issue does not arise in the DSIC case, because dominant strategy equilibria of the incumbent mechanism are not eliminated by the existence of a blocking mechanism.

We show despite Theorem (5), the double auction is immune to B-blocking by positive-spread posted price mechanisms.

Theorem 7. *Assume there is one buyer and one seller, then all double bid auctions are immune against B-blocking by positive-spread posted price mechanisms.*

Proof. Consider a potential positive-spread posted-price mechanism with prices p and p' such that $p < p'$. Suppose, for a contradiction, that it B-blocks the double auction.

We show the set of buyer types who visit the blocking mechanism is $[x, 1]$ and the set of seller types is $[0, y]$, for some x and y such that $p' < x$ and $y < p$. Set V'_b and V'_s to be the type of agents. Let $x = \inf\{v | v \in V'_b\}$ and $y = \sup\{s | s \in V'_s\}$. If $V'_b \neq [x, 1]$, there exists $x' > x$ such that $x' \notin V'_b$. In that case, the following inequalities must hold:

$$E_s[u_i(x, \{m_b(x), m_s(s)\}, Q, P) | s \in V'_s] \leq x - p', \tag{31}$$

$$E_s[u_i(x', \{m_b(x'), m_s(s)\}, Q, P) | s \in V'_s] > x' - p. \tag{32}$$

Inequalities (31) and (32) imply:

$$E_s[u_i(x', \{m_b(x'), m_s(s)\}, Q, P) | s \in V'_s] - E_s[u_i(x, \{m_b(x), m_s(s)\}, Q, P) | s \in V'_s] > x' - x. \tag{33}$$

Set $a(v)$ to be the probability that buyer with type v trades when the seller's type is in V'_s. The right-hand side of inequality (33) is equal to $\int_x^{x'} a(v)dv$. Note that since $a(v) \leq 1$, $\int_x^{x'} a(v)dv \leq x' - x$ which contradicts with (33). The contradiction proves $V'_b = [x, 1]$, similar proof shows $V'_s = [0, y]$.

Consider a Bayesian equilibrium where the buyer and the seller both report the same bid p'', such that $p < p'' < p'$. Note that this equilibrium is consistent, however, equation (26) is violated.

□

Assume one buyer and one seller with types selected from the uniform distribution over $[0, 1]$. Consider the double bid auction that selects the mid-point between the bid and the ask. It is well known that there is an equilibrium of this double bid auction that is constrained efficient. This equilibrium is not the one described in the proof of theorem above. Hence, efficiency is not a shield against fragmentation. If anything, it suggests that the threat of an alternative mechanism induces inefficiency in the incumbent mechanism.

5.3 B-Blocking without possibility of return

Now, we describe a notion of B-blocking when agents don't have the option of returning to the incumbent mechanism. In this case, fix a Bayesian equilibrium of the incumbent mechanism. If a block fails, the agents in the putative blocking coalition A are assumed to continue playing the original Bayesian equilibrium of the incumbent mechanism. The definition of B-blocking changes to the following:

The agents in A and a mechanism $(\hat{P}, \hat{Q})$ B-block the incumbent mechanism with equilibrium $(m_i(.))_{i \in N}$ if there exists a non-zero measure subset of types $(V_i')_{i \in A}$ (the critical subset) such that the following inequalities hold,

1. If $v_i \in V_i'$, then,

$$E_{-i}[u_i(v_i, \{m_i(v_i), v_{-i})\}, Q, P)|T_i] \leq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i] \; \forall i \in A \quad (34)$$

$$T_i = \{(v_k)_{k \in A} | v_k \in V_k', \; \forall k \in A \setminus \{i\}\} \quad (35)$$

2. If $v_i \notin V_i'$ then,

$$E_{-i}[u_i(v_i, \{m_i(v_i), v_{-i}\}, Q, P)|T_i] \geq E_{-i}[u_i(\{v_i, v_{A \setminus \{i\}}\}, \hat{Q}, \hat{P})|T_i]. \quad (36)$$

3. For all $\bar{v} \in \prod_{i \in A} V_i'$,

$$\sum_{i \in A} \hat{q}_i(\bar{v}) = 0. \quad (37)$$

4.

$$E[\sum_{i \in A} \hat{q}_i(\bar{v}) \hat{p}_i(\bar{v}) | \bar{v} \in \prod_{i \in A} V_i'] > 0. \quad (38)$$

A mechanism is immune to B-blocking if it has an equilibrium for which no blocking occurs. In the buyer's bid double auction with m buyers and m sellers, buyers and sellers submit bids and asks, and the mechanism designer sorts the bids and asks in an increasing order, the price is the $m+1$'th number in the order, sellers with an opportunity cost strictly less than the price and buyers with a valuation greater or equal to the price trade.

Theorem 8. *Assume there is one buyer and one seller and that types are drawn independently from an atomless distribution with continuous probability density function. Then, the buyer's bid double auction is immune to B-blocking by a positive-spread posted price mechanisms.*

Proof. Satterthwaite and Williams (1989) prove that the buyer's bid double auction has an equilibrium in which the seller submits his opportunity cost and the buyer with valuation v submits bid $b(v)$, and $b(v)$ is increasing and continuous in terms of v. We show that this equilibrium cannot be blocked by any positive-spread posted price mechanism. Assume, on the contrary, that a block exists. A similar argument as in the proof of theorem (7) shows that the critical sets should be $V_b' = [x, 1]$ and $V_s' = [0, y]$, for the buyer and the seller respectively, for some $1 \geq x > y \geq 0$. We show the following inequality holds, preventing a block from forming.

$$E_s[u_b(x, \{b(x), s\}, Q, P)|s \in [0, y]] + E_v[u_s(y, \{b(v), y\}, Q, P)|v \in [x, 1]] \geq x - y. \quad (39)$$

We consider two cases:

1. $b(x) \geq y$**:** note that $u_b(x, \{b(x), s\}, Q, P))$ is decreasing in s and $u_s(y, \{b(v), y\}, Q, P)$ is increasing in v, therefore,

   $$E_s[u_b(x, \{b(x), s\}, Q, P)|s \in [0, y]] + E_v[u_s(y, \{b(v), y\}, Q, P)|v \in [x, 1]] \geq$$
   $$u_b(x, \{b(x), y\}, Q, P) + u_b(x, \{b(x), y\}, Q, P).$$

   Buyer with type x trades with seller with type y with price $b(x)$, therefore,

   $$u_b(x, \{b(x), y\}, Q, P) + u_b(x, \{b(x), y\}, Q, P) = x - y.$$

2. $b(x) < y$: note that the buyer with valuation x does not trade with seller whose opportunity cost exceeds y. Therefore,

   $$E_s[u_b(x, \{b(x), s\}, Q, P)|s \in [0, y]] = \frac{E_s[u_b(x, \{b(x), s\}, Q, P)]}{Pr(s \in [0, y])}. \quad (40)$$

   Note that a bid equal to $b(x)$ maximizes the surplus of a buyer with valuation x. In particular, the buyer weakly prefers to bid $b(x)$ instead of y; therefore,

   $$E_s[u_b(x, \{b(x), s\}, Q, P)] \geq (x - y)Pr(s \in [0, y]). \quad (41)$$

   Equations (40) and (41) imply:

   $$E_s[u_b(x, \{b(x), s\}, Q, P)|s \in [0, y]] \geq x - y. \quad (42)$$

□

6 Conclusion

We have shown that only when a posted price is tailored to the distribution of types and those distributions are well-behaved it is immune to D-blocks by a positive spread posted price mechanism. When the type distributions are unknown, there is no mechanism that is robust against D-blocks by positive spread posted price mechanisms. When we consider B-blocking, we show that double auctions are immune against the B-blocks by a positive spread posted price mechanism, however, the immunity is present only if agents play the "right" Bayesian Equilibrium in the incumbent mechanism. Our analysis shows that when the price taking assumption is violated, the conditions under which an incumbent mechanism is immune to blocking are both restrictive and fragile. For this reason we argue that centralized markets are vulnerable to fragmentation.

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