The Small Worlds of Corporate Governance

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Social networks characterize many types of human interaction, including research communities. The community behind this book surely became a small world, starting from rather distant relationships among us to the realization of joint papers and articles. At the start of the project, the idea was to create an international data set to study if robust patterns of corporate governance could be discovered. Focusing on two types of network nodes and their ties, owners and boards of director, the design was to capture two critical dimensions of governance: the property right to control and the allocation of decision rights to exercise that control.

There was a second motivation too, namely to create an ‘open research community’ in which the price of membership was simply the contribution of a data set and the ultimate goal was to post data and results at a public site. The pace of social science research seems slow compared to that of the natural sciences. There are many reasons for this difference: natural science communities are large (but so are social science communities if they would permit this realization), they are better funded and have a more developed division of labor among themselves to achieve large projects, natural science is more sharp-edged and permits greater clarity regarding advancement in results.

There is though another possibility regarding simply the openness of natural science. In the United States, the largest social science community is economics, which through the offices of the National Bureau of Economic Research (NBER) has an unusual capacity to launch new research initiatives with internal funding, to organize summer
camps for young scholars, and conferences and events. It has a major influence on the making of economic and social policy in the United States. It has hard to run the counterfactual of what would be the state of economic research and policy if it should not have existed, but the loss would surely be substantial.

Because of the capability of economics to mobilize resources, economic research can move quickly, even if sometimes from faulty premises. It has a high power to adapt through the force of large numbers of well-financed researchers. This power is surely seen in the work on corporate governance, which began entirely on the wrong footing in the early 1990s but by the end of the 1990s was making a volte face in pointing to the importance of institutions and especially law –even if the causal statements here were too strong as latter research has shown. Clearly, one of the most important factors in the success of this research program (which was in fact the most highly cited research program in economics in the 1990s) was the creation and dissemination of data on the rule of law and legal institutions. Other social sciences had long studied institutions, and yet they ended up either incorporating the economic research into their domain, or defining themselves by opposition to it.

The trajectory of self-correction etched by economic research on governance is, however, far from complete –more simply said, some of the results and claims are, we believe, false. The wide variance among rich nations in governance despite high convergence in their national outcomes (e.g. wealth) is poorly explained by theories stating, for example, a given type of corporate law is superior. Even within economics, it was realized that contradictions existed, such as well-functioning capital markets despite
concentrated ownership without substantial minority shareholder protection. The belief, almost ideology, that business groups are bad poorly specifies the counterfactual alternative as a state of the world of perfect competition. The many contradictions have lead to vague appeals to social norms and culture.

Our aim is to complement the impressive accomplishments in the economics of governance by examining the social networks and small worlds that support and influence governance. This project of small worlds and corporate governance sought to harness the power of an open research community in order to develop explicit measures and research on the social foundations to corporate governance. The objective from the start had been to create social network data and to disseminate these data broadly. The design of this community was oriented towards the rapid sharing and accumulation of learning, the creation of joint research projects (including the chapters to this book), and the sharing and public posting of data. In this regard, it has been quite successful.

Lacking an NBER for sociological research at this international scale, the project still garnished support for four conferences and for the salaries to support a centralized research capability. The central research infrastructure was partly supported by a grant through the European Union’s funded project called Measuring and Modeling Complex Networks across Domains. We would like to thank the director of this program, Felix Reed-Tsochas at the Said Business School at Oxford University, for his encouragement and support. The members of this community financed their research and data collection from local national funding, which they gratefully acknowledge.
The first meeting of the open community to study “The Small Worlds of Corporate Governance” was held on the campus of INSEAD, in Fontainebleau, on January 10 to 12, 2004. Subsequent meetings were held at IESE, Business School of the University of Navarra, in Barcelona, in May 19 to 21, 2005, at University of Bologna, April 27 to 29, 2006, and finally at Columbia University, New York, on February 20, 2009. These meetings were financed by generous support the Wendel International Centre for Family Enterprise of INSEAD, Center for Globalization and Strategy at IESE Business School, ALMA Graduate School at the University of Bologna and the Sanford C. Bernstein & Co. Center for Leadership and Ethics at Columbia University.

We have many people to thank for their contributions. Stephanie Paille at INSEAD was a superb administrator of the project through 2007. Mariano Belinky and Jordi Colomer gave tremendous support to the project, both intellectually and laboriously – sloshing through 20 plus national data sets proved often to be an overwhelming task. From writing original code to the creation of new name matching algorithms, they were the bedrock of this project. Pietro Urso was a major intellectual companion to this project and helped us generously in defining and coding of algorithms and in defining major concepts. Yotam Stanger provided us with editorial assistance. Moreover, our editor at MIT Press, John Covell, ran a superb review process and excellent reviewers who helped us to improve greatly the manuscript.

The data to this project are posted at the web site of the Sanford C. Bernstein & Co. Center for Leadership and Ethics. These data include network measures and economic and social indicators at the national level, plus network measures for individual firms in each of
the data bases. You are welcome to contribute your data on national ownership and directors or correct errors. This is an open community that believes our individual ambitions are best served by the fostering of collective projects and the sharing of social science research data.
Chapter 8

Generating Rules and the Social Science of Governance

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The chapters in this book have compared country networks in order to inform an institutional analysis of corporate governance. This chapter approaches the analysis of governance from the perspective of the simulation methodologies of complex adaptive systems. Up to ten years ago, the dominant approach to the study of networks in management and social sciences had focused on static empirical graphs that related particular decisions, such as adoption of managerial practices, to ties among firms or other organizational entities. At times, particular network attributes were proposed as especially meaningful, such as centrality, closure, structural holes, or blocks. Although a few papers analyzed the evolution and replication of structure (e.g. Ahuja, 2000; Walker, Kogut, & Shan, 1997), most research correlated behavioral outcomes to static network properties. We have relied upon, and extended this conventional approach in the previous chapters by ‘comparing comparative statics’, namely by comparing the governance networks in a country at two distinct times.

A next step is to understand how networks grow and evolve over time from a bottom up perspective. The evolution of networks involves the emergence of new macrostructures that arise from the cumulative micro-behaviors of individual entities, or agents. Since macro-structure and micro-behaviors interact, the analysis of network structures confronts difficult theoretical and methodological hurdles. Simulation has been very useful as a tool by which to address these hurdles.

We have already utilized simulations as an important tool in our analysis of small worlds, since we generated the small world statistics of Robins-Alexanders by rewiring the empirical bipartite networks to generate random networks. Once we attained the
randomized networks, we calculated the average path length and average clustering values to normalize the empirical measures. Rewiring static networks was also the original innovation proposed and utilized by the Watts-Strogatz (1998) paper. The degree of rewiring was a tunable parameter that graphically generated the deep insight into small worlds as characterized by low path lengths but high clustering.

Dynamic evolutionary models rely upon ‘generative rules’ to drive micro-structural changes that accumulate into broad macro-structural patterns (Kogut, 2000). This approach goes beyond static rewiring of a network holding the number of nodes and links constant towards the growth of new nodes and links. To illustrate the idea of generative rules, consider the figure at the end of this paper. There are three graphs (topologies) that began from the same starting point of 27 nodes and one link. For each case, a different algorithm is used to generate a link between nodes. For random, links were assigned randomly to any two nodes. For preferential attachment, the rich became richer as described in the first chapter; the nodes with higher degrees were favored to receive links. The most interesting from a perspective of sociology and economics is transitivity, where triangular closure is favored; by closure, we mean the tendency for an open triangle to be closed: if node firms A and B are partners and B and C are partners, it is likely that A and C will become partners. These three algorithms are abstract statements of generating rules, but they correspond to social and business behaviors that we empirically observe.

All the graphs share the same sub-structure: they begin with 27 nodes ordered. The generating rule for random creates a benchmark comparison: there is no evident clustering or pattern. Preferential attachment creates the familiar outcome of a few degrees having
high degrees (many links) and most nodes fewer. If the network was larger, this rule would generate a power law distribution of degrees. Finally, transitivity creates, of course, a pattern of closed triangles among the players. The differences in these structures are examples of how different generating rules lead to different network topologies.

The point of interest in this exercise is to show the relationship between generative rules (i.e. micro actions) and macro-structures. Imagine a social science that has evolved sufficiently to understand how the rules of day-to-day interactions lead to particular structures. The so-called ecological fallacy of inferring individual motivations from group statistics litters the comparative literature. The rule of law is associated with more developed finance markets is an example. How much more comforting it would be if we could simulate, if not observe, the effect of law on individual interactions and from that, aggregate these actions into structures. Then we might truly be able to say per our chapter 1 discussion that Thai governance operates as family groups who obey particular genealogical rules, thus creating business groups whose partitions map onto lineages.

In this chapter, we offer three illustrations of this approach. The first establishes not only that governance networks are not random, but that we can also pinpoint what set of nodes to the bipartite graph drives the social process. By this, we mean, whether the social network is driven by owner or firm choice, by board or director choice. Having established the meaning of a random graph, the second illustration answers the question, how far is a given country from an ideal type called the Anglo-Saxon governance system. The solution we offer relies upon rewiring country networks holding constant the number of nodes and degree. The second question asks how much of a quota is required to achieve
a possible, but not yet attained world of corporate governance marked by greater gender equality. This analysis requires a complex agent-based system approach, relying upon generative rules that change the nodal and link parameters of the empirical networks. Thus these two cases illustrate the static and dynamic simulation models that generate important statistical results (e.g. robustness) or analysis of the emergent properties of a social network.

**Rewiring: Are the Networks Random?**

The most primary assumption behind the thesis of this book is the claim that the governance networks are not random. After all, it would be a devastating result to the presumption that they do matter if in fact the assignment of links between the nodes (e.g. the directors between boards) is simply random. A tempting approach is to rely on visualization tools, such as the ubiquitous Pajek program, to generate pictures of the graphs. (Figure 1A and Figure 1B in Appendix 1 provide these images for Brazil and Mexico.) While such visualizations, when the graphs are small, can provide useful insights, it is hard to detect if a graph is random or if two graphs are isomorphic or different. In this sense, we are looking for a parallel to a statistical test.

We can do better than just test if a graph is random. One of the critical challenges in network analysis is simply to know whether a graph property (e.g. density, structural hole, path length) can be used to compare two graphs that differ dramatically in size. If both graphs are very large, we might be able to assume that asymptotically the network statistics converge to a common even if unknown distribution. The problem of comparison
exists though for graphs (networks) that they are small and medium size. What can we do then?

We cannot rely upon the standard significance tests because we don’t know the distribution of the network statistics. However, the mathematics of Erdös and Rényi (see for example their 1959 and 1960 papers (Erdös & Rényi, 1959)) provides a useful benchmark in the identification of a random graph whose link assignment is Poisson distributed. (However, we offer the caution that these results are asymptotic and thus may be poor approximations for small networks.) Consequently, we know the expectation of some important statistics for a given number of nodes and links. For example, the expected path length or clustering coefficient (which is closely related to the Burt’s idea of a structural hole) is asymptotically ln(n)/ln(k) and k/n, respectively (Watts & Strogatz, 1998). The brute force approach described in chapter 1 (and appendix 1) provides an alternative way to estimate the random small world statistics. We can estimate the statistics for an arbitrary degree random graph through simulations that take the average of a statistic upon sufficient iterations that re-assigns links between nodes.

Once we know the expected statistic for a random graph of the same nodal and link size, we can then compare it to the empirical estimation from the actual data. By this comparison, we can say that the empirical statistic is close to the random graph statistic, and hence not meaningful. Or if the empirical statistic is substantially far away, it can be argued that we are sufficiently confident that the process of assignment is not random – thus implying that a social mechanism is responsible for governing the link assignment between nodes. Finally, we can also make the comparison of one empirical graph
normalized by the random statistic to another graph which is normalized by its corresponding random graph. From this comparison, we can compare graph statistics across different networks of different sizes just like we can compare Z-score and other statistics across different samples of varying size.

It was this insight that Duncan Watts and Stephen Strogatz exploited brilliantly in their work on small worlds (Watts & Strogatz, 1998; Watts, 1999a). Watts and Strogatz defined a small world as a case in which people are on average close to one another and yet they tend to be located in neighborhoods where everyone knows everyone. In his monograph, Watts (1999a) wanted to know what will happen when the connections are randomly rewired if you start with many kinds of graph topologies which have high clustering and high path lengths relative to the random graph. For many kinds of topologies, he performed this simulation of rewiring and observed the changes in the network statistics. He found that the path lengths quickly converge towards the lengths found for a random graph, but the clustering values remained high substantially longer. (This approach is described in detail in chapter 1 and appendix 1). This use of simulations provided us with a methodology that guided the approach used in this book:

1. Starting with the empirical country networks for owners and for boards, we use the rewiring simulations to generate the mean and variance to the statistics of interest drawn from the simulated random graph, holding constant the same number of nodes and links as found in the empirical graphs. The random statistics serve as an ‘expectation’ drawn from a normal distribution.
2. The observed statistics from the empirical graphs (that is, our country networks) can then be compared to the random and can be statistically tested against the random expectation.

3. Apart from statistical testing, the random statistics can also be used to normalize the empirical observations. Once normalized, these statistics are ‘scale free’ and do not depend on the size of the network from which they are measured. It is thus possible to compare statistics observed in different size networks. In practical terms, this normalization permits, say, the average path length statistic drawn from the small Israeli network to be compared to the average path length statistic drawn from the much larger US network.

Using this methodological philosophy, let’s return to our task at hand to determine if the ownership and director networks are non-random. To do this, we follow a method designed by Gary Robins and Malcolm Alexander (2004) and start with a bi-partite graph explained above. We have two bi-partite graphs: the equity investments (links) from owners to firms (nodes) and the membership (links) of directors to two or more boards (nodes). An appealing way to visualize a bi-partite graph is the joining of a top and bottom half (see figure 1 in chapter 1). We would like to know if the link assignment between nodes is simply random or whether they are generated by social processes and rules.

This identification of social processes is a formidable task, requiring not only an arsenal of sophisticated statistical tools (e.g. Tom Snijder's Siena program) but also a parsimonious theory of the motivations and interactions of agents (directors, firms,
An alternative is to decompose the governance networks according to the essential structural building blocks of any network, independent of broader theoretical considerations. A simple way to achieve this decomposition is to describe the local structure of the bi-partite network through a ‘census’ of ‘configurations’. (See the methodological appendix for the full details.) For example, we might want to ask how many directors belong to two boards, or three boards) and how many of these directors are linked to other boards by directors who sit on their boards. We can also take the other perspective of boards and do the same census, namely, how many boards are linked to two or three directors and how many are boards are linked to each other via their directors who sit on two or more boards.

Robins and Alexander provide this census which builds upon the census given in Wasserman and Faust for simple unipartite graphs (Wasserman & Faust, 1994: 512ff., 564ff.). The census takes into account whether one anchors the configuration in the top or bottom set of nodes (using top and bottom literally in reference to figure 1A in chapter 1). The Robins-Alexander bipartite configuration census is given here in Figure 2 in Appendix 1. The most important configuration for us is what Robins and Alexander call SA3 and SP3, which we reproduce in Figure 2 to this chapter. This configuration is the bipartite equivalent to the clustering coefficient for unipartite graphs, where the clustering coefficient is constructed from the number of closed triangles in a neighborhood. It should be clear that the projection of either of these figures results in a triangle. (The projection eliminates the nodes of the top or bottom half of the bipartite graph and connects directly the nodes in the other half by the links that were intermediated in the bipartite graph –i.e.

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1 Snijder’s Siena program is downloadable at http://stat.gamma.rug.nl/siena.html.
all nodes in the projected graph have a link indicating that they are one-step away from each other, while they are two-steps away from each other in the bipartite graph.)

Our test for randomness starts with this census. For every country governance network (be it director or owner), we count the name of configurations of each type in the empirically observed graph. We then re-wire the empirical graph over many simulations to generate random graphs from which we calculate the mean and variance for every configuration. Then, following Robins and Alexander, we can accept or reject the null hypothesis that the empirical census count is statistically the same as the random by a means test under a normal distribution.

The results of this method are summarized in figures 2.A and 2.B. Since the graphs are bi-partite, the census is conducted for ‘each side’ of the graph. The red represents the t-statistic for the census count for the owner or board side; the blue gives the t-statistic for the firm or director side. There are two observations to be made. First, it is clear that these networks are not random, since the blue colored configurations are almost always significantly different than the random expectations. Second, the social network for directors is generated by the director side; for owners, by the owner side. There are two exceptions in the owner graph, where the UK shows firms being more important and France showing slightly more importance attached to firms as well –but the difference is insignificant.

These results are important to the theorizing about corporate governance. Essentially, it indicates that friendships (i.e. social capital) among directors as opposed to ties among boards matter. This observation is consistent with the study by Palmer et al.
(1983) that show the ties between firms are not frequently constituted when a director that is responsible for the board to board link departs. It is personal ties, not the business relationship, that drives the social process behind board links. In a parallel way, owners drive the process by which firms are linked rather than the opposite. This result also makes sense, since owners generally are principals to firms; they decide to own or now and their ties among each other will influence with whom they co-invest. It is possible that firms may invite owners to invest, but this seems rare, with only the UK indicating that this process may apply.

**Rewiring: How Far Away Is Korea from an Anglo-Saxon World?**

A tenet in the comparative governance literature is the dominance of the Anglo-Saxon model, which is associated with a handful of countries sharing common law practices and equity-dominanted financial systems. Our approach has defined Anglo-Saxon companies in terms of their small world properties; we have proposed that the ideal type of such an economy should be marked by atomistic exchanges. The chapter 2 by Martin Conyon and Andrew Shipilov found considerable variance for the Anglo-Saxon countries regarding their small world properties. Leaving aside the empirical reality, the theoretical definition of an Anglo-Saxon network is the random graph: a pure market economy absent all social influences trades solely on the basis of price. Surely, this definition can be complicated by noting that any economy runs on trust and thus on society. However, it is useful to define the limiting case as in a mathematical proof and to say that as an economy moves toward perfect information and perfect competition, the appropriate graph is
random. The Anglo-Saxon paradigm is the competitive market and thus its governance should result in a random graph, since everyone has access to the same information and competition prevails.

A more complicated line of reasoning, although also more interesting, is to predict the structure of a governance network conditioned on knowledge of the country under inspection. What does it mean for a country to be labeled Korea or the United States in respect to governance? Statistical analysis might treat this as a fixed effect, but this hardly answers the question. Furthermore, fixed effects handle poorly the tight coupling of institutions and behavior that constitute a country. In a comparative analysis of many countries, this high-order dimensionality of interactions leads to discrete and highly differentiated institutional configurations, as Ragin (1987) early on suggested and as we discussed in chapter one. In other words, you don’t get the United States by adding a bit more of salt to Korea. The linear model, or log-linear model, is unlikely to get you there.

An ideal answer to the question of what does it mean to be Korea would entail in our view an understanding of the micro-rules that guide the choice whether to own or to sell a firm or whether to invite a director to a board and whether the director should accept. Multiple studies on directors show that the choice of a director is strongly conditioned on the social network, such as homophily that means ‘birds of a feather flock together’: men choose men, graduates of Seoul National choose other graduates of the same university, etc. There are also legal rules, e.g. the prohibition of banks to own industrial firms in the United States. It is a daunting, but nevertheless feasible, task should the data exist to estimate the rules in each country that incorporate these institutional and sociological features.
Let’s imagine for example that we estimate the coefficient to triangular closure (i.e. friends are also friends) in two countries called North and South and find that they differ by a factor of 2; everything else is the same. Then by tuning this coefficient, it would be possible to grow the North from the South. In the language of Epstein, if you didn’t grow it, you didn’t explain it. The astute observation is that if you grow, it may be insufficient to explain it; the converse is not clearly true since there may be, and often are, multiple models to grow a given topological structure. Still, the rule is useful as a deduction by elimination in order to disprove a causal claim.\(^2\) Such simplicity in differences among countries are not to be found, because we do not have yet precise theories about the social rules that generate governance networks. Thus, it will indeed be very difficult as a means to arrive at clean results without theoretical guidance.

There is, however, a simpler way to gain insight into the distance among countries through choosing a common ideal type for which there is considerable theoretical and empirical work. The Anglo-Saxon model qua ideal type is especially useful, for we assert that a random graph is the appropriate representation for the perfect market economy: social ties should not matter. By the previous analysis of the Robins-Alexander census, we should have a strong prior that the graphs will not be random; however, we did not offer a measure by which to compare their distances. We will focus on one network statistic, clustering, as it captures well local differences, where local means the formation of neighborhoods and the rules by which neighbors are chosen. (Neighborhood has a technical graph definition, as defined in appendix 1.) This concept has been critical to

\(^2\) For an example, see Kogut, Urso, and Walker (2007), who rule out preferential attachment since a power law did not characterize the degree distribution,
Burt’s notion of a structural hole, for in fact a clustering coefficient is equal to 1 - SH, where SH is the structural hole value. Clustering is also Coleman’s definition of social capital, whereby a more connected neighborhood has the social properties of high trust as well as monitoring.

Having chosen now the ideal type of a market economy as a random graph and defined clustering as the appropriate network statistic, we are prepared to analyze the difference of a country from the Anglo-Saxon polar case. The measure we propose relies upon this proposition: the nodal expected clustering coefficient for a random graph is the density of the graph. We begin by noting two definitions. A random graph can be denoted as $G(N,p)$, where $N$ is the number of nodes and $p$ is the probability of a link between any two nodes. By definition, density is equal to $p$. Next, we wish to show that the probability of closing a triangle is the same for the probability of a link being formed between two nodes in general. We assert that, as would be the case in a randomized experimental design, $\text{cc} = P \ X_{ij} = 1 \ \exists \ k, X_{ik} = X_{kj} = 1) = P \ X_{ij} = 1 = \text{density}$, since there is no local correlation for a random graph.

We exploit this property to measure how much a network differs from a typical random network as the difference between the empirical average clustering coefficient and the density for any given country. We call this difference ‘distance to random’. Its value lies in a (-1,1) interval. Obviously, this value will be 0 if the governance network is random, and will take on a value up to the maximum value of 1, otherwise; theoretically, the maximum value of 1 can never be reached, since we can’t have $\text{cc}=1$ and $\text{density}=0$. Positive values increase monotonically in the importance of social and institutional influences on local link
formation. We can interpret this measure as the decrease in the clustering coefficient that the network would experience through a rewiring process. Negative values are also possible, and we would obtain them on graphs with a smaller clustering coefficient than the expected on the randomized graph, imagine for instance a graph with cc=0 such as a two dimensional lattice, its clustering coefficient would increase instead of decreasing on a rewiring process. Since we are only interested on the distance to random, we suggest to take the absolute value that would lie on the (0,1) interval.

Figure 4.A shows the results of this exercise for the director network and figure 4.B shows the results for the ownership network and we ignore the firm and board networks given the previous section’s results indicating they are essentially random; for ease of exposition, we focus only on the second panel (~ year 2000). The distance to random figure for the directors indicates that the countries are quite far from the Anglo-Saxon (random) ideal, but it is notable that the US, UK, Canada, and Australia are all to the right-hand side. Japan’s director network is remarkably more random than the rest, reflecting the low prestige of the Japanese board and the tendency to load them with many inside directors who do not sit on other boards. (See chapter 3.) Poland is the furthest away from a random graph.

Turning to the owner networks, the Anglo-Saxon countries are again to the far right of the owners’ distance to random figure (we do not have data for Australia). Given the clustering of the Anglo-Saxon countries towards the random value, there is thus face validity to the construct. Still, it is notable that Japan is again the closest to random, but this time Poland is a close second.
There are two observations to be made. First, the Anglo-Saxon countries are relatively consistently close to the random. Second, countries look far more distant from their randomized graphs in respect to directors. Boards indeed look clubby.

There are important caveats. These networks are not large, and including smaller firms and supply chains, would potentially capture the importance of vertical business groups in such countries as Korea and Japan. Moreover, path lengths are also revealing, though we should recall that as a graph is re-wired, path lengths ‘flatten’ out and approach the random value much more quickly than clustering does. In other words, clustering values will tend to be farther away from random values and thus more telling.

How far away is Korea from the ideal Anglo-Saxon governance network? About .86 for the director network and .26 for the owner network. Alternatively, these values respectively are .25 and .03 in relation to the US. As Davis, Woo, and Baker (2003) found, the US after all is a country of fragmented owners but interlocked boards. Would a description of Korea in 2000 as a ‘boys’ club’ with looser crossholdings among the big firms ring true? No doubt, a larger network to pick up the smaller firms would help but it is not a bad description of big Korean business.

Social Rules and Agent-based Models: Generating Possible and more Equitable Worlds

The previous section generated statistical measures by static rewiring of the empirical graphs, thereby permitting comparisons among graphs of different sizes. Arguably, this approach to social networks fails to separate out the structural properties of

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3 See Watts, (D. J. Watts, 1999b) 1999
static relationships from the *generating rules* that governed the evolution of the structure in the first place (Kogut, 2000). These generating rules are the summary motivations for the decisions taken by a class of agents, such as firms. Examples of generating rules would be to seek relationships with powerful partners or to rely on trusted partners. Despite that almost all the research on networks implicitly relied upon an assumption of such rules, few of them sought to address dynamically the implications for network evolution.

Management studies are not alone in this neglect. However, there has been in recent years substantial progress made in allied fields of computer science, artificial intelligence, and social physics, and this progress is also quite visible in the social sciences, if not most in sociology. In fact, once one starts to look for them, they are quite easy to find.

Consider for example the long-standing interest in the ‘rich get richer’ phenomena, labeled as a Matthew Effect by Robert Merton (Merton, 1968). Models by Simon (Simon, 1955) and Price (Price, 1976) showed that a stochastic process in which new nodes (such as those in networks among firms, or among science papers and their authors) favor the construction of links to nodes with high degree. In their model of internet connectivity, Barabasi and Albert (Barabási & Albert, 1999) proposed a mechanism of ‘preferential attachment’ which is a special case of the Simon model. These models derive a network property (degree distribution) by working from theories of micro-behavior.

In this sense, the standard approach to predict micro-behaviors from network structure is reversed. Structure is the product of micro-decisions. Of course, we have chosen rather simple examples and empirically, many rules will operate in noisy environments. Thus, it has been statistically challenging to isolate the effects of rules on
structure. Considerable progress has been made in the last years, especially in relation to
the ERGM models proposed by Pip Pattison and her colleagues (Robins, et al., 2007; Stanley
Wasserman & Pattison, 1996) and the dynamic methods of Tom Snijders (2001). These
models are often extremely sensitive to initial values and the parameter space. This
sensitivity is to be expected and supports the theoretical claim that small differences can
make big changes. But from a statistical estimation perspective, the estimates are often not
robust and require sophisticated understanding of the boundary conditions to which they apply.

Agent-based modeling and “bottom up” models of networks

Simulations offer the service of being able to isolate the macro effects from simple
rules that can be easily manipulated to test for robustness and emergent properties.
Hedström (2005), Flache and Macy (2004), this alternative research strategy seeks to
identify the rules that govern the interaction among agents and that are capable of
dynamically generating the observed structure. Because governance networks dynamics
are driven by many social and economic forces, it is appealing to see what can be learned
by isolating a few rules and evolving the consequent macro-structures. Just as above we
applied static measures to compare empirical networks, the simulated networks can also
be compared to empirical networks by the same means.

There is another virtue to simulations that has considerable potential in the
application to the formation of public policies. Simulations permit explorations of possible
worlds –those worlds that may exist but do not currently. For such an exploration to be
useful, a possible world should not vary in too many properties. We utilize this idea below to explore a possible world of greater gender diversity among boards of directors. We begin with an empirical network of Belgium and choose two rules by which to increase the number of women on boards. The objective is to show that by only small changes in rules, large macro-social change is possible.

Corporate boards and gender

Boards of directors vary in their importance across countries in their powers, but are nevertheless important not only insofar as their governance responsibilities, but also as public testimonies of the status of their directors. The chapters to this book have found that director networks are small worlds. In addition, several recent studies have analyzed cross-sectional panels and found that boards are small worlds (Battiston & Catanzaro, 2004; Conyon & Muldoon, 2006; Davis, et al., 2003; Kogut & Walker, 2001; Newman, 2001; Newman, Strogatz, & Watts, 2001; Watts & Strogatz, 1998; Watts, 2004). However interesting, these topological findings do not tell us about the underlying behavioral processes.

A good behavioral candidate is the concept of homophily. Homophily is a social rule that people prefer associates who are similar to themselves. This rule generates a macro-structure of segregation. If one type of person, say males, dominates, the macro-pattern is likely to show in-groups of male clubs; the women will be scattered throughout the network as long as they lack critical mass to self-organize also into clubs.
There is an important empirical literature that analyzes the relation of networks to the career prospects of women in management. These studies indicate strong gender stratification by job and industry, but find mixed evidence for tokenism and homophily (Petersen, Saporta, & Seidel, 2000; Reskin & Bielby, 2005). More recent studies find homophily operative at the level of job referrals (the supply side) but absent or weak at the level of job offers (the demand side) (Fernandez & Fernandez-Mateo, 2006).

However, the academic research on top management and boards has consistently found tokenism and dominant male homophily. In one of the first statistical studies on social networks, gender and managers, Ibarra (1992) found that work relations among men were more homophilous. Women’s personal networks were also homophilous, but women relied heavily upon male networks for instrumental goals. The heterophilous patterns of women managers reflected the strategy of a minority group below a critical threshold, forced to rely disproportionately upon the male managerial majority. This strategy comes at a cost. Burt (1998) found that women rely upon ‘borrowed social capital’ of their usually male boss, which would be an example of an instrumental strategy. This strategy resulted in lower chances for promotion and lower pay. In conjunction with the Ibarra (1992) findings, these results suggest that instrumental networks may in fact be limited for advancement. This implication is surely consistent with the lower percentage of female top managers.

The study by Cohen et al. (1998) poses explicitly the question of how can women gain entry into top managerial positions in the absence of homophilous social networks and at sub-critical proportions. They found that women are less likely to be promoted to
positions where they are not already present. They conclude that “it may be that what is thought of as a glass ceiling is actually a glass door, which can only be opened by women if other women have opened it previously. If so, patterns of sex segregation in managerial ranks are unlikely to change drastically through processes endogenous to employing organizations”.

The previous chapter seven provides an important insight into the likely gender dynamics behind board choice for three Scandinavian countries. The authors Edling et al. find that diverse boards have a higher degree of connectivity and they also tend to export this diversity; the latter result suggests that diversity grows by a process of contagion through local neighborhoods. Their study thus points to the possibility that after a critical threshold is reach, women directorships diffuse through the network. In comparison, the studies by Ibarra and Cohen et al. cited indicate that for the US top management, women are below this critical point.

Given this implication of sub-criticality, it is not surprising, therefore, that there is discussion in many countries to mandate minimum quotas for women on boards. Norway has been the most bold in this regard, requiring as of January 2008, all boards of public corporations to have attained 40% women or to be fined or closed. Clearly, Norway is not relying on endogenous processes to attain more gender egalitarianism. Nevertheless, the process by which women will be selected is not mandated by law and thus will be characterized by social conventions.

Explanations of such social phenomena can be greatly enhanced with the use of multi-agent simulations (Epstein, 2006; Epstein & Axtell, 1996; Hedström, 2005; Miller &
The role of simulation in such analysis provides a test bed for behavioral rules at the firm level, which allow for mechanism-based explanations based on ‘as-if’ propositions (Hedström, 2005). In all, such ‘as if’ investigations into existing social networks are rare.

An appealing property of a simulation is to investigate possible worlds in order to address important social questions. In the context of this study, an obvious question is: what should be the recommended quota that would be sufficient to endogenize homophilous patterns of female director recruitment? It might seem that an imposition of a relatively high quota (40%) is required. A more subtle approach is to set the quota just high enough to achieve criticality and to provide the structural conditions that would endogenize female director recruitment. Criticality should yield a structure that has many highly central women brokering providing direct intermediation among women and male directors, such that an egalitarian structure self-replicates.

We study through computer simulation the implications of externally mandated quotas in the context of Belgian board membership for the year 2000 using data collected by Malika Hamadi. This analysis follows a methodology developed in Kogut, Belinky, and Colomer (2010). Interlocking boards of directors have been extensively studied (Davis & Yoo, 2003; Kogut & Walker, 2001; Mizruchi, 1996).

We analyze the director network as a bipartite graph of directors and boards. We work with the directors’ one-mode projection of this graph, meaning that two directors are connected if they sit on the same board. In the following analysis, we will work also with
the gender-specific sub-graphs of the general director network. We construct this sub-graph by using the director-firm ties where all directors belong to a specific gender.

The effects of a quota on eliminating tokenism will be dependent upon the initial topology of the network and also upon particular parameters influencing the rate of director replacement and choice of directors. We start with the existing empirical network. In our simulations, boards are forced to invite female directors if their ratio is below the exogenously established threshold. Once they are above this threshold, the gender of the director joining the board will follow the global gender distribution, i.e. if the network has 75% male directors, then the probability of the joining director being male is .75.

The parameters to the simulation tune a re-wiring process by which directors can join new boards and a decay process by which directors can leave a board or retire altogether. The rewiring rules by which a director is invited to join a board are either by random selection from the whole network or by random selection from the neighboring boards. Obviously, the second rule of selecting a woman from a nearby board will have important structural implications. Moreover, a third option exists when new entrants join the network by joining a board.

The decay phase summarizes two processes that correspond to the career histories of directors, as they either retire from professional life, thus ceasing their participation in all boards, or reach the end of their tenure as members of a specific board, creating an “empty seat” in a given board. To maintain simplicity, we keep the board sizes constant. For the renewal process, at each iteration we allow for a small number of new directors to join
the network to compensate for the departing directors. We also allow for a small increment in population size.

We explore a few simple strategies the network of boards can engage in to increase the ratio to a mandated quota. As a test case we use the Belgian network of boards for the year 2000. This network has 141 firms and 1167 directors. Out of these 1167 directors, 954 are male, 81 are female and 132 are unknown (on account of them being company seats). The ratio for the known directors is about 8.46%. We randomly assign gender to the unknown 132 directors following the same existing ratio. In the end we are left with 1076 male and 91 female directors, for a ratio of 8.49%

Figure 5.A provides a histogram of the number of boards by the ration of women to men directors. As can be seen, many boards in Belgian had no women directors in 2000. Figure 5.B provides the histogram of board sizes. Most boards are in the environ of ten, with a few very large boards. Such large boards create high clustering in the one-mode projection (i.e. the direct to director network), thus illustrating why it is important to correct the director network statistics for the distribution of board sizes. (See the discussion in chapter 1 and Appendix 1.) The empirical small world statistics for Belgium are 2.94 for average path length and 0.94 for the clustering coefficient. Standardizing by the random values for a bipartite network permits the calculation of the Kogut-Walker SW statistic (standardized clustering over the standardized path length), giving a value of 3.12. The director network of Belgium is a small world.

The simulation has the objective to see what happens to the betweenness centrality of female directors as their numbers per board increases due to an imposed quota. We
impose and compare two quotas of 10% and 40%. The simulation design question is what rules should be imposed to achieve the quota. The first is how to permit entry of new directors and retirement of old. The more theoretically interesting rules are the social rules that used to choose female directors. A standard, and helpful, baseline is random selection. A women director is chosen randomly from the existing network. A second rule is, to stay faithful to the notion of local clustering, to select a female director in the neighborhood of a board, where again neighborhood means those boards that have a common director with a given board.

In table 1, we summarize these rules of entry, exit, random selection, and trust your neighbor and provide the parameter values we have chosen. These parameter values are estimated from the US network in the study by Kogut et al. (2010). By an analysis of the 2006 and 2007 panels, we estimated the transition rates for departure and renewal of directors. Using the empirical rates as guidance, we fixed the rate at which directors leave a seat at 11% and the complete retirement from the network rate at 1% per iteration (or year). The renewal rate is calculated at each iteration, based on the number of directors that left the network, in order to keep the size of the network constant.

Once a board has reached the target quota (either 10% or 40%), we still follow the selection method but we don’t force anymore the new director to be a woman, and the gender is expected to be in proportion to the gender ratio of the network. The entire simulation ends when all the boards have reached the quota.
Centrality Results

An appealing way to measure the impact of a quota on women representation is their average centrality. Betweenness centrality has the nice property of measuring the importance of a director in relation to the number of geodesic that she or he intermediate. A maximal centrality score is achieved when a node sits in the middle of a star. This node is equivalent to occupying a structural hole, since all communication must pass by it. A minimal score is achieved for a fully decentralized star, when all nodes are directly connected.

To render this simulation more tangible, we list in Table 2 the names of the top ten male and female directors by their centrality. These names are quite famous in Belgium and Europe. Etienne Davignon, for example, was a top commissioner in the European Union for industrial policy. Christine Morin-Postel was the head of Société Générale de Belgique, the most powerful conglomerate in Belgium at that time, until 2001. As can be seen from the column called centrality, the male directors are far more central on average.

Comparing now the results for random and for trust your neighbor for each of the quotas, we observe that a 10% quota does remarkably well in creating very central female directors. The values given in Table 2 do not characterize the whole network, though they surely indicate an improvement in the centrality of female directors. Figure 6 provides this description that shows the ratio of male and female centrality over iterations. Since it is easier to satisfy a 10% than a 50% quota, the simulation ends earlier for the lower quota, since all boards have satisfied this lower bound. Not surprisingly, the centrality values are somewhat below those of the higher quota, but asymptotically, the two quotas achieve
similar levels of centrality for male and female directors. While this parity should not mask that numerically women directors are far fewer than male directors, it is an indication that low quotas can achieve structural equality.

These results of this agent-based modeling indicate that mild quotas can have surprisingly important implications for increasing the balance of power between gender. Since a mild quota is more politically feasible than a large one, the result is a nice illustration of the utility of simulations to explore possible worlds and to highlight feasible policies by which to attain them. Of course, these results depend upon the social rules that indeed prevail in a country, and it is unlikely that the rule of ‘trust your neighbor’ is the only generative rule. But it might also well be that other rules are complementary, and in all, the possibility for social change is more accessible than otherwise suggested by the raw statistics.

Conclusions

This chapter investigated the use of simulations for the analysis of social networks and in particular governance networks. Two of the simulations were used to generate measures based on the re-wiring of static networks, one applying the Robins-Alexander bipartite census to test for what ‘half’ of the bipartite network drove the social process in link formation, the second applying our proposal for measuring distance to the Anglo-Saxon network.
The third simulation relied on agent-based models to explore the accessibility to possible worlds that many would find desirable. The goal of simulation of an evolving network is to unravel the macro-structure to recover the underlying rules that generate the topology. It is in this exercise that we can return to an earlier era of studies in which culture was proposed to explain the differences among firms and countries. If a culture promotes that transitivity and clustering prevail (i.e. friends become friends of friends), then we will see a structure of many closed triangles. This type of triadic closure would represent well a keiretsu in which the firms have a high density of ties with each other, either if measured by equity or by product flows. Taken to an extreme, this type of generating rule would not generate a power law distribution in degree and thus the network would not be marked by the self-organizing pattern found in many physical or biological networks.

Diversity has the important contribution of providing enough variance in a society, and in a social network, to balance between centripetal forces of homophily and centrifugal forces for in-breeding. It is this important role that the previous chapter on women in Scandinavian boards has proposed. Of course, there is a social justice argument for diversity. However, it should also not be neglected that diversity can be anticipated to have positive effects on the performance of society. This claim is worthy of further exploration.
### Table 8.1. Generating Rules and Parameter Values

<table>
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<tr>
<th>Strategy</th>
<th>Quota</th>
<th>Random selection</th>
<th>Neighbor Selection</th>
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<tr>
<td>Random low quota</td>
<td>0.1</td>
<td>0.95</td>
<td>0.05</td>
</tr>
<tr>
<td>Random high quota</td>
<td>0.3</td>
<td>0.95</td>
<td>0.05</td>
</tr>
<tr>
<td>Neighbor low quota</td>
<td>0.1</td>
<td>0.05</td>
<td>0.95</td>
</tr>
<tr>
<td>Neighbor high quota</td>
<td>0.3</td>
<td>0.05</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Table 8.2. Male versus Female Centrality: Simulations

<table>
<thead>
<tr>
<th>Male</th>
<th>Initial</th>
<th>Random Low Quota</th>
<th>Random High Quota</th>
<th>Neighbor Low Quota</th>
<th>Neighbor High Quota</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1081</td>
<td>0.0163</td>
<td>0.0127</td>
<td>0.0139</td>
<td>0.004</td>
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<tr>
<td>2</td>
<td>0.1003</td>
<td>0.0136</td>
<td>0.0046</td>
<td>0.0133</td>
<td>0.0061</td>
<td>PHILIPPE VLERICK</td>
</tr>
<tr>
<td>3</td>
<td>0.0588</td>
<td>0.0046</td>
<td>0.0072</td>
<td>0.008</td>
<td>0.0054</td>
<td>RONALD EVERAERT</td>
</tr>
<tr>
<td>4</td>
<td>0.0493</td>
<td>0.0118</td>
<td>0.0013</td>
<td>0.0061</td>
<td>0.007</td>
<td>YVES BOEL</td>
</tr>
<tr>
<td>5</td>
<td>0.0475</td>
<td>0.0097</td>
<td>0.0053</td>
<td>0.0081</td>
<td>0.0046</td>
<td>ANDRE LEYSSEN</td>
</tr>
<tr>
<td>6</td>
<td>0.042</td>
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<td>0.0075</td>
<td>0.0059</td>
<td>PHILIPPE SAVERYS</td>
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<td>7</td>
<td>0.0323</td>
<td>0.0096</td>
<td>0.0099</td>
<td>0.0012</td>
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<td>8</td>
<td>0.0322</td>
<td>0.0153</td>
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<td>0.0196</td>
<td>0.0045</td>
<td>CHRISTIAN VARIN</td>
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<td>0.0069</td>
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<td>10</td>
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<td>0.01</td>
<td>0.0147</td>
<td>0.0108</td>
<td>PAUL DE MEESTER</td>
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<td>0.012</td>
<td>0.0075</td>
<td>0.0099</td>
<td>0.0055</td>
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<table>
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<tr>
<th>Female</th>
<th>Initial</th>
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<th>Random High Quota</th>
<th>Neighbor Low Quota</th>
<th>Neighbor High Quota</th>
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<td>0.0235</td>
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<td>0.0127</td>
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<td>0.0124</td>
<td>0.0035</td>
<td>0.0098</td>
<td>0.0023</td>
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<td>0.0009</td>
<td>0.0028</td>
<td>0.0018</td>
<td>0.0026</td>
<td>0.0001</td>
<td>SOLANGE SCHWENNICKE</td>
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<tr>
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<td>0.0074</td>
<td>0.003</td>
<td>0.0001</td>
<td>0</td>
<td>GIUSEPPE SANTINO</td>
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<tr>
<td>130</td>
<td>0.0005</td>
<td>0.003</td>
<td>0.0087</td>
<td>0.0035</td>
<td>0.0027</td>
<td>DIANA DU MONCEAU DE BERGENDAL</td>
</tr>
<tr>
<td>134</td>
<td>0.0004</td>
<td>0.0045</td>
<td>0.0035</td>
<td>0.0004</td>
<td>0.0051</td>
<td>JOHANNE IWEINS D'ECKHOUTTE</td>
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<td>147</td>
<td>0.0002</td>
<td>0.0051</td>
<td>0.0106</td>
<td>0.0026</td>
<td>0.0033</td>
<td>MONIQUE PAQUOT-NEVEN</td>
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<td>162</td>
<td>0</td>
<td>0.0036</td>
<td>0.0041</td>
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<td>0.0057</td>
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<td>Average</td>
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<td>0.0065</td>
<td>0.0047</td>
<td>0.0046</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Generating structure from rules, Results from three simulations.

Random

Preferential

Transitivity
The squares are firms/boards and the circles our owners/directors. As you can see in the vertical charts legend, the blue bars correspond to the configurations SP2 or SP3 and red is SA2 or SA3. In brief, blue means owners/directors and red firms/boards.
Figure 3

A. Owners (blue) and Firms (red) for Panel 2 (~2000): the SP3 and SA3 Configurations
Figure 3

B. Directors (blue) and Boards (red) for Panel 2 (~2000): the SP3 and SA3 Configurations
A. Distance to Random for the Director Graph for Panel 2 (~2000)
B. Distance to Random for the Owner Graph for Panel 2 (~2000)
Figure 5
Descriptive Data for the Belgium Boards in 2000

A. Gender Ratio Distribution

B. Board Size Distribution
Figure 5

Female and Male Average Betweenness Ratio for Belgium 2000