Endogenous Liquidity and Defaultable Bonds

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Motivation: Default vs Liquidity

- **Default** and **liquidity** are interconnected as evident from recent financial crisis
  - Liquidity: funding liquidity, price impact, transaction costs, etc

- Prevalent in theoretical literature: one-way linkages

  1. Liquidity $\rightarrow$ default *or* 2. Default $\rightarrow$ liquidity

- Today's paper: liquidity$\rightleftharpoons$default, two-way feedback
  Endogenous liquidity solved **jointly** with default decision

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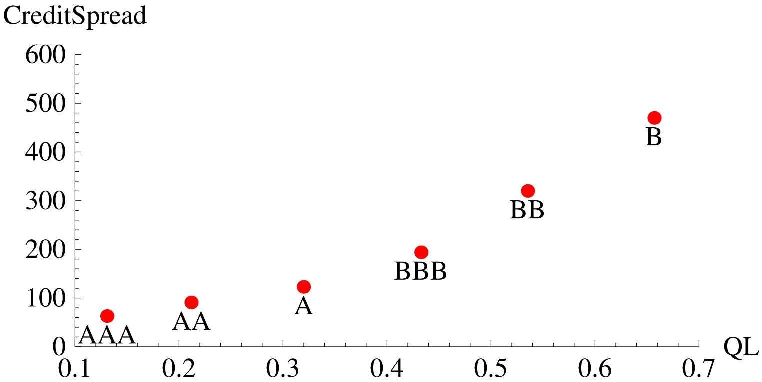
- Today's paper: liquidity⇌default, two-way feedback
  Endogenous liquidity solved **jointly** with default decision
- Important in understanding the liquidity and default premia for corporate bond credit spreads
  - Barclays Capital report (2009) shows high correlation between default and liquidity spreads, both in time-series and cross-section
  - Dick-Nielsen, Feldhütter, and Lando (2012), and Friewald, Jankowitsch, and Subrahmanyam (2012)
  - Yet state-of-art empirical literature additively decomposes spreads into independent liquidity and default premium

Motivation: Corporate Bonds



Source: Huang, Huang '03

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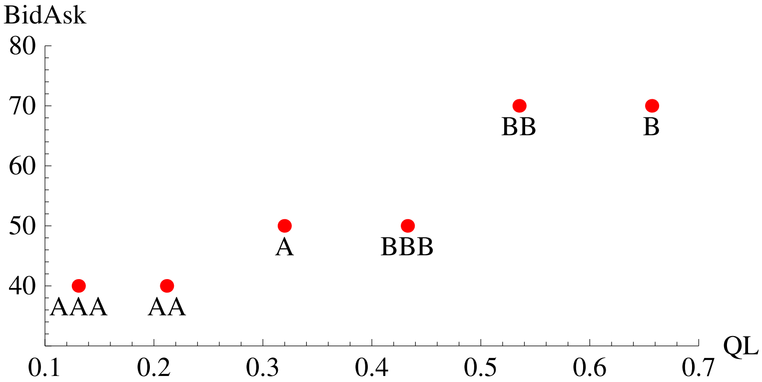

Source: Edward, Harris, Piwowar '06 (EHP; BA at median trade size)

Mechanism and Results

Building blocks for interaction between fundamental and liquidity:

- How does bond illiquidity arise, and how is it affected by the state of the firm?
  - Over-the-counter market with search friction à la Duffie et al (2005)
- How do corporate decisions interact with secondary market liquidity?
  - Endogenous default and rollover channel à la Leland Toft (1996)

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Main results:

- Closed-form solution for bond values and bid-ask spreads, equity values, and default boundary
- Endogenous liquidity allows us to match the cross-sectional pattern of bid-ask spreads and credit spreads
- Liquidity-default spiral and novel default-liquidity decomposition (applied to financial crisis)

Related Literature

Search in asset markets:

- Duffie, Garleanu, Pedersen '05 (DGP), '07
  OTC search market with simplified 'derivative' security

Capital structure models:

- Leland, Toft '96 (LT)
  Rollover increases exposure of equity holders to fundamental risk
- He, Xiong '12
  Exogenously given secondary market liquidity affects default decision

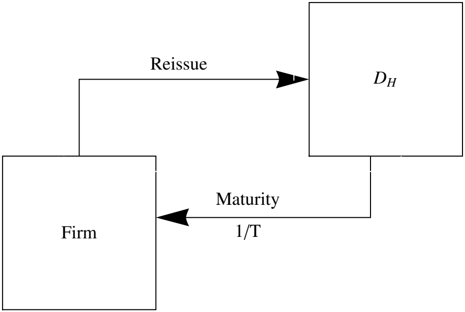
Empirical literature:

- Bao, Pan, Wang '11; Edwards, Harris, Piwowar '07; Hong, Warga '00; Hong, Warga, Schultz '01; Harris, Piwowar '06; Feldhütter '11

Feedback models:

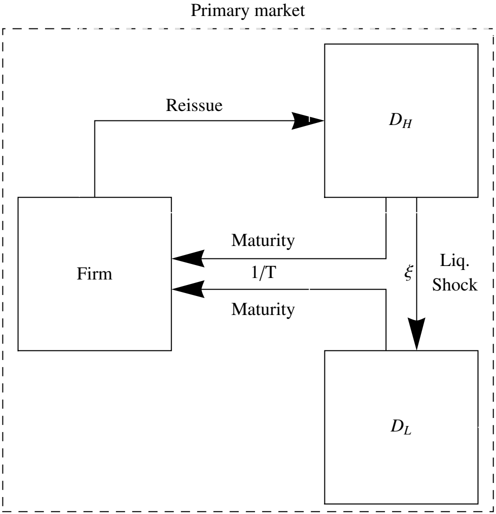
- Many many more papers...

Schematic Representation: Leland Toft 1996



Above analysis outside default

Schematic Representation: The Primary Market



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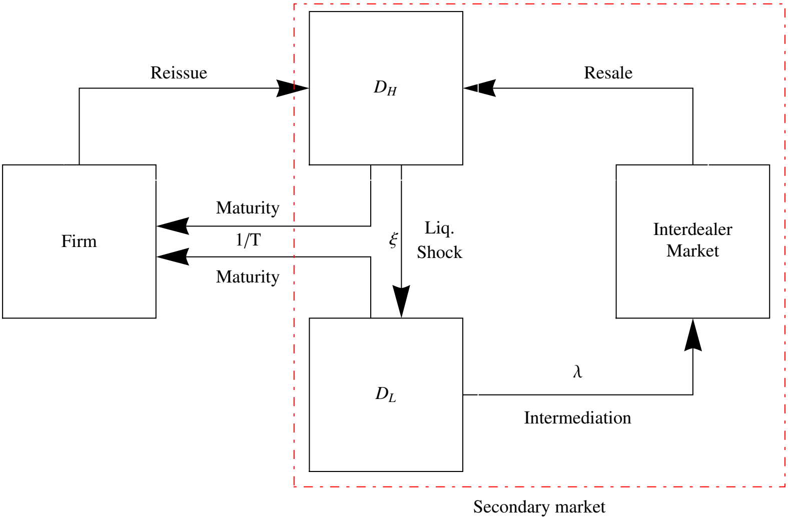
Schematic Representation: The Secondary Market



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Model: The Firm

Preferences: Everyone risk-neutral with common discount rate r

Cash flows:

- Cash-flow rate δ_t, $d\delta_t = \mu\delta_t dt + \sigma\delta_t dZ_t^{\mathbb{Q}}$

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Debt structure:

- Debt in place with aggregate face value p and coupon c
- Stationary principal & staggered maturity (as in LT):
  - Uniform maturity structure $\Rightarrow$ Mass $1/T$ matures every instant
  - Maturing bonds reissued with identical contract terms (c, p, T)

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Rollover:

- Primary market with transaction costs κ, debt reissued at D_H

$$NetCashFlow_t = \underbrace{\delta_t}_{CF} - \underbrace{(1-\pi)\,c}_{Coupon} + \underbrace{\overbrace{1/T}^{Mass\ maturing} \quad \overbrace{\left[(1-\kappa)\,D_H\,(\delta_t, T) - p\right]}^{Repricing}}_{Rollover\ gain/\ loss}$$

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Endogenous default:

- Equity defaults at δ_b when absorbing further losses unprofitable

Model: Investors, Liquidity Shocks & Search

Idiosyncratic liquidity shock to bond investors:

- Asset holding restriction $\{0, 1\}$ as in DGP '05
- Uninsurable i.i.d. liquidity shock results in two types of agents:
  - **H type**: subject to liq shock with intensity ξ before default, $\xi_b > \xi$ after default
  - **L type**: currently in liquidity shock state, holding cost χ pre-default ($\chi_b \frac{\delta_b}{r-\mu}$ post-default) until asset sold.
- Type dependent bankruptcy value $D_i(\delta_b, \tau) = \alpha_i \frac{\delta_b}{r-\mu}, i \in \{H, L\}$ where α_i determined by frictions in post-bankruptcy market
- Simplifying assumption: No recovery from liq shock, L types exit market after sale. [Assumption for expositional purposes only]

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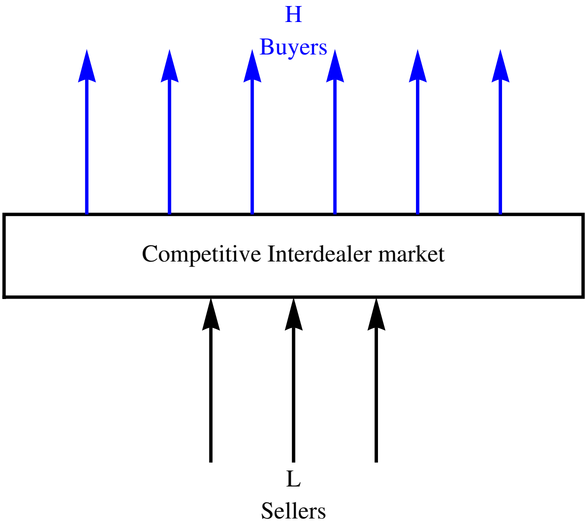
Trade & search friction:

- L sellers, H buyers, all meet OTC dealers with intensity λ
- Competitive interdealer market, no inventory, transaction price M
- Agents have bargaining power β vis-a-vis a dealer

Model: Secondary Market - A *Seller's Market*



Seller's market Assumption:
Mass sellers μ_{L1} smaller than mass buyers μ_{H0}, i.e., $\mu_{L1} < \mu_{H0}$

Model: Secondary Market - Bid and Ask

Nash-bargaining:

- Let Π be generic surplus. Then Nash-bargaining splits it
  $\beta\Pi \rightarrow$ *Investor* $\quad (1-\beta)\,\Pi \rightarrow$ *Dealer*

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Pre-default market:

- L-dealer (seller) surplus Π_L, H-dealer (buyer) surplus Π_H
- Bertrand competition in interdealer market erodes H-dealer surplus
  - Why? Any positive surplus would be outbid as there is more potential buyers than sellers
- Ask price A (H is buying at), bid price B (L is selling at)
  - Buy side: $A = D_H - \beta\Pi_H = M = D_H$ and $\Pi_H = 0$
  - Sell side: $B = D_L + \beta\Pi_L$ and $\Pi \equiv \Pi_L = D_H - D_L > 0$
- Key tractability from $D_{L0} = 0$ (by previous assumption) and $D_{H0} = 0$ (as no surplus to buyers in a *seller's market*)

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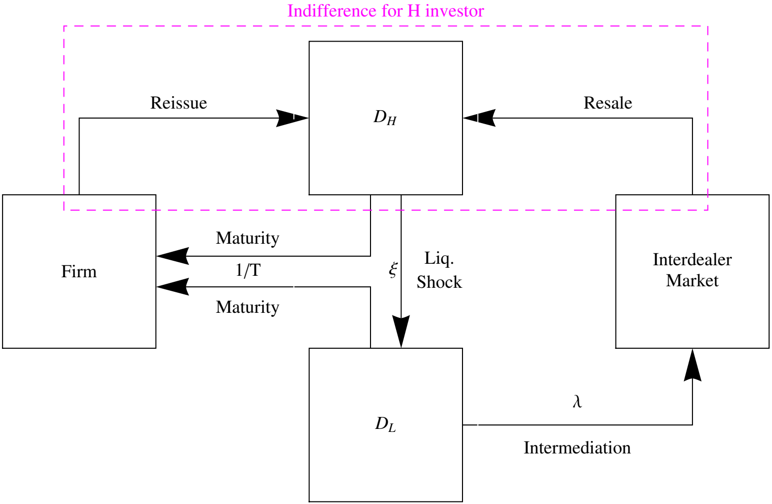
Endogenous liquidity:

- Bid-ask spread $A - B = (1-\beta)\,(D_H - D_L)$ proportional to valuation wedge

Schematic Representation: Indifference of H investors



As surplus from buying is zero, H-investors indifferent between buying on primary market, secondary market and being on sideline

Model: Valuation Equations

Debt: Boundary conditions $D_i(\delta, 0) = p$, $D_i(\delta_b, \tau) = \alpha_i \frac{\delta_b}{r-\mu}$

$$rD_H(\delta,\tau) = \underbrace{\mathcal{A}^{\delta} D_H(\delta,\tau)}_{\text{CF dynamics}} - \underbrace{\frac{\partial D_H}{\partial \tau}(\delta,\tau)}_{\text{Maturity}} + c + \underbrace{\xi\left[D_L(\delta,\tau) - D_H(\delta,\tau)\right]}_{\text{Liqidity shock}}$$

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Observational equivalence:

- As $\lambda[B - D_L] = \lambda\beta[D_H - D_L]$, pricing equivalent to world with exogenous $H \leftrightarrow L$ switching intensities ξ and $\lambda\beta$

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Equity: Boundary condition $E(\delta_b) = 0$, optimality condition $E'(\delta_b) = 0$

$$rE = \delta - (1-\pi)c + \mathcal{A}^\delta E(\delta) + \frac{1}{T}\left[(1-\kappa) D_H(\delta, T) - p\right]$$

Analytic Solutions and Comparative Statics

Closed form solutions:

- Closed form solutions for debt $D_{H/L}$ (mix of two LT solutions), equity E and optimal default boundary δ_b
- Consequently, closed form solutions for absolute and proportional bid-ask spread, $A - B = (1 - \beta)\,\Pi_L$ and $\Delta = \frac{A-B}{\frac{1}{2}A+\frac{1}{2}B}$, respectively

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Analytic comparative statics:

1. If wedge at default, $\Pi = (\alpha_H - \alpha_L)\,\frac{\delta_b}{\mu - r}$, greater than wedge at $(\delta, \tau) \to (\infty, \infty)$, $\Pi = \frac{\chi}{r+\xi+\lambda\beta}$, then $\partial_\delta\,(A - B) < 0$.
2. If additionally $\partial_\delta D_H > 0$ (condition provided), then also $\partial_\delta \Delta < 0$.
3. If $\alpha_H > \alpha_L$, then $\partial_\tau\,(A - B) > 0$.

Interpretation:

1.+ 2. Controlling for time-to-maturity, both abs and prop bid-ask spreads decreasing in δ (**pro-cyclical liquidity**)

3. Controlling for dist-to-default, abs bid-ask spread is increasing in τ.

Liquidity and Default: Full Feedback Loop

Counterfactual: Fixed illiquidity / transaction cost

- Fixed transaction cost k (bid-ask spread of $\frac{k}{1-k/2}$) with immediate sale after shock (as in Amihud Mendelson '86, He Xiong '12)
- Suppose investors believe they are in this counterfactual world

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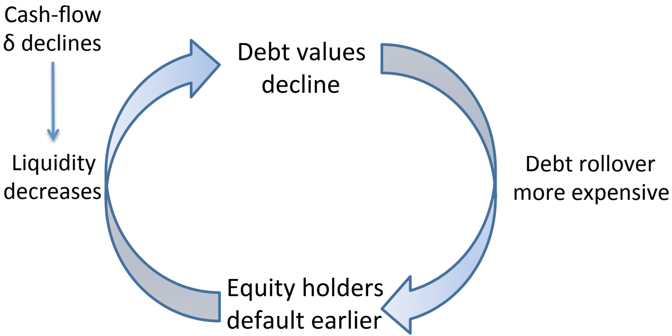
Thought Experiment:

- When δ shrinks, to investors, transaction costs 'unexpectedly' rise
- Higher transaction costs mean investors value the bond less
- Rollover $\frac{1}{T}\left[(1-\kappa)\, D_H\left(\delta, T\right) - p\right]$ more costly for each δ
- Costlier rollover implies earlier default δ_b

Liquidity and Default: Full Feedback Loop

Equilibrium feedback loop:



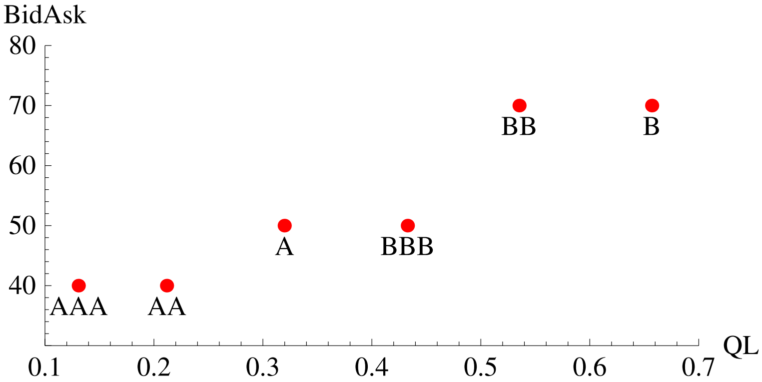
- Fixed point δ_b outcome of this spiral

Calibration: Baseline Parameters

Firm Characteristics			Secondary Market		
Param.	Interpretation	Value	Param.	Interpretation	Value
σ	Volatility	25%	r	Interest rate rate	2%
μ	Drift	0%	χ_0	Holding cost prop to p	0.3%
π	Tax shield	27%	χ_1	Holding cost prop to c	0.24
p	Principal	1	ξ	Intensity of liquidity shock	1
c	Coupon	3.9%	λ	Intensity to meet dealers	26
T	Bond maturity	10	β	Bargaining power investors	5%
κ	Issuance costs	1%	α_H	Recovery value H type	51%
δ_0	Initial Cash Flow	0.055	α_L	Recovery value L type	50%

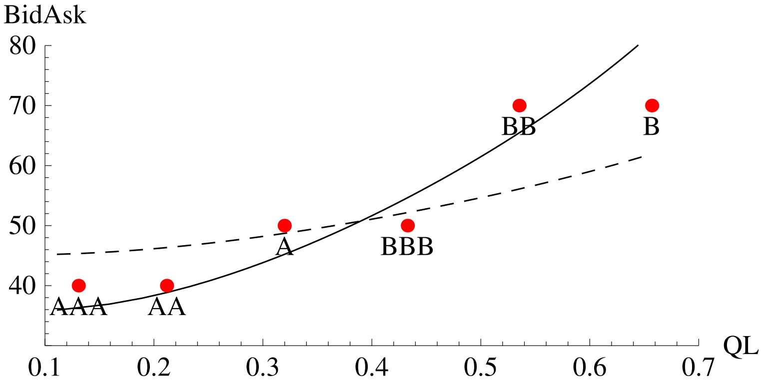
- Pre-default holding costs: $\chi = \chi_p p + \chi_c c$. Pick $\chi_{p/c}$ targeting bid-ask spread of A/BBB-rated bonds.
- Recovery α's derived form Moody's Default and Recovery Database
- BA-spread at default $\approx 200bps$ in line with EHP data for defaulted bonds for median trade size
- Map δ into quasi leverage via $QL(\delta) = \frac{p}{p+E(\delta)}$

Calibration: Liquidity



Solid: Adjust c so issued at par (newly issued bonds); Dashed: Constant c (stale bonds)

Calibration: Liquidity



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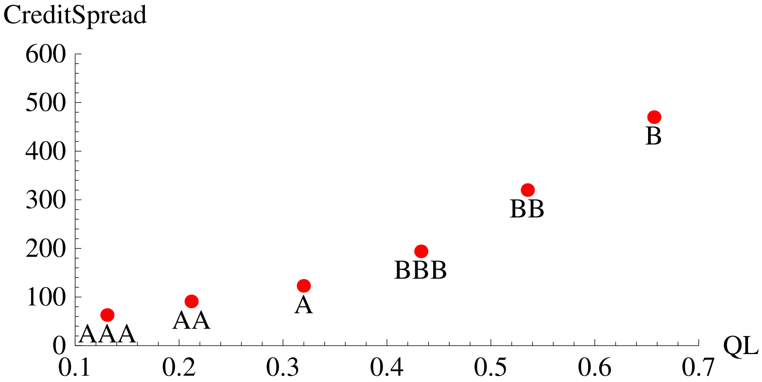
Calibration: Credit Spread



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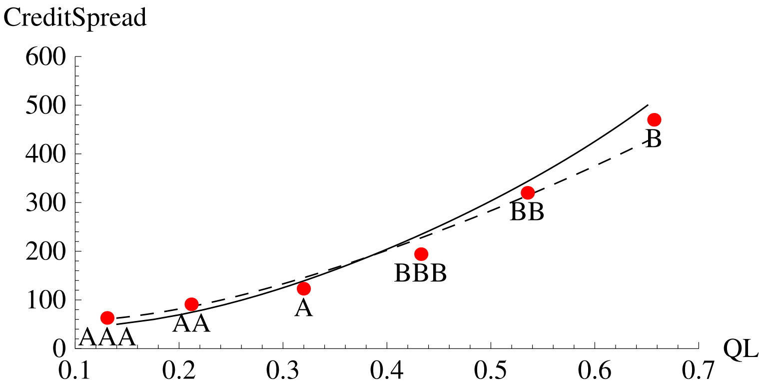

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Model-Based Decomposition: Methodology

- Longstaff et al '05: CDS back out default component $\hat{y}_{DEF}$. How much of default component is caused by liquidity?

- Structural model allows finer decomposition of *credit spread* :

$$\hat{y} = \overbrace{\hat{y}_{pureDEF} + \hat{y}_{LIQ\rightarrow DEF}}^{\text{Default Component } \hat{y}_{DEF}} + \overbrace{\hat{y}_{pureLIQ} + \hat{y}_{DEF\rightarrow LIQ}}^{\text{Liquidity Component } \hat{y}_{LIQ}}$$

  - **Pure default** $\hat{y}_{pureDEF}$: fully liquid secondary bond market (LT 96), default at δ^*_{LT}
  - **Liquidity-driven Default** $\hat{y}_{LIQ\rightarrow DEF}$: additional default due to earlier default at $\delta^*_b > \delta^*_{LT}$ (but full liquidity in trading)
  - **Pure Liquidity** $\hat{y}_{pureLIQ}$: riskless bond spread with illiquid secondary bond market (DGP 05)
  - **Default-driven Liquidity** $\hat{y}_{DEF\rightarrow LIQ}$: additional illiquidity part due to default

- **Goal**: Separate *causes* from *consequences*

Decomposition: Application to Financial Crisis

- Set δ_0's to target the credit spreads. Crisis: -50% shock to δ_0's.

	Investment Grade			Speculative Grade		
	Normal	Crisis	Change	Normal	Crisis	Change
Panel A: Data based on Friewald et al (2012)						
Yield Spread bps	97	321	224 $(3.3\times)$	348	1082	734 $(3.1\times)$
Illiquidity	1	1.5	.5 $(1.5\times)$	1	2	1 $(2\times)$

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Panel B: Model						
Yield Spread bps	100	318	218 (3.2×)	350	1130	780 (3.2×)
BA spread bps	42	51	9 (1.2×)	67	127	60 (1.9×)

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Panel C: Decomposition						
Pure Default	32 (32%)	168 (53%)	136 (62%)	172 (49%)	660 (58%)	488 (63%)
Liquidity-driven Def	8 (8%)	20 (6%)	11 (5%)	28 (8%)	65 (6%)	37 (5%)
Pure Liquidity	41 (41%)	41 (13%)	0 (0%)	59 (17%)	59 (5%)	0 (0%)
Default-driven Liq	19 (19%)	89 (28%)	71 (32%)	92 (26%)	346 (31%)	254 (33%)
Total	100 (100%)	318 (100%)	218 (100%)	350 (100%)	1130 (100%)	780 (100%)

Microfoundation of Bankruptcy Wedge

Bankruptcy payout delay:

- Bankruptcy recovery $\alpha < 1$ of unlevered firm value $\frac{\delta_b}{r-\mu}$
- Recovery payout at exponential (θ) time due to legal delay

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Post-default market:

- Search market characterized by $(\theta, \xi_b, \lambda_b, \chi_b, \beta_b, \delta_b)$
- Ask price $A^b = D_H^b$, bid price $B^b = D_L^b + (1-\beta)\,\Pi_L^b$
- *Seller's market* assumption: Competitive interdealer price M^b erodes all surplus of buyers

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Effective bankruptcy recovery for H and L investors:

- Closed form $D^b_H = \alpha_H \frac{\delta_b}{r-\mu} > D^b_L = \alpha_L \frac{\delta_b}{r-\mu}$
  $\Rightarrow$ Pre-default liquidity, via δ_b, affects post-default liquidity
- Interpretation of default as *firm-wide liquidity event* that is endogenously triggered

Calibration of Bankruptcy Wedge

Other data sources:

- EHP: Bid-ask spread right before default $\approx 200bps$
- Chen 2011: Bankruptcy recovery from trading price at default $\alpha_L = 50\%$

Moody's default and recovery database:

- Avg time to emergence: $501days \Rightarrow \theta = 0.73$
- Annual buy-and-hold return: 36% (relative to S&P benchmark) $\Rightarrow$ Eventual recovery $\alpha = \alpha_L 1.52 = 0.75$
- Benchmark perfect liquidity model has $\alpha_{LT} = \frac{\theta}{r+\theta}\alpha$

Parameters that give α's:

- Data only allows identification of two deeper structural parameters
- Assuming (β_b, λ_b) unchanged, $(\chi_b, \xi_b) = (0.1855, 16.5)$

Optimal Maturity: Rollover Risk vs Liquidity

Negative: Short-term debt leads to earlier default

- Higher rollover frequency increases equity's exposure to δ

$$\textit{Rollover gain/loss}_t = \underbrace{1/T}_{\textit{Rollover frequency}} \times \underbrace{\left[(1-\kappa)\, D_H(\delta_t, T) - p\right]}_{\textit{Repricing}}$$

- Higher exposure to δ leads to higher default boundary δ_B

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$\Rightarrow$ Finite maturity $T^* < \infty$ optimal if moderate initial leverage;
T^* lower the less liquid secondary market (i.e. the lower λ)

Trading Volume

{Recall: Orders *batched*, i.e. intermediation intensity λ constant across τ}

Steady-state Distribution:

- Let $p_H(\tau)\,/\,p_L(\tau)$ be the proportion of H/L types of maturity τ that hold the bond. Then

$$p_H(T) = 1 \quad \& \quad \frac{\partial p_H(\tau)}{\partial \tau} = \lambda p_L(\tau) - \xi p_H(\tau)$$

$$\Rightarrow p_H(\tau) \quad = \quad \frac{\lambda + \xi e^{(\tau - T)(\lambda + \xi)}}{\lambda + \xi}$$

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Empirical implication:

- Trade volume decreasing in τ: $V(\tau) = \frac{\lambda\xi}{T(\lambda+\xi)}\left[1 - \xi e^{(\tau-T)(\lambda+\xi)}\right]$
- Results driven by
  - Placement tech. ('recycling' maturing L types to H types)
  - Constant intermediation intensity λ across maturities

A Full Equilibrium Search Market

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- Total mass μ of agents: $\mu = \mu_{H0} + \mu_{H1} + \mu_{L0} + \mu_{L1}$
- Type-only distributions, μ_H and μ_L, are independent of trading
- Let $\lim_{t\to\infty} \mu_H(t) = \mu_H^{ss}$. Then from ODE

$$\dot{\mu}_H = \xi_{LH}\mu_L - \xi_{HL}\mu_H = \xi_{LH}\mu - (\xi_{HL} + \xi_{LH})\,\mu_H$$

we know μ_H monotone and $\mu_H^{ss} = \frac{\xi_{LH}\mu}{\xi_{HL}+\xi_{LH}}$

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Micro-foundations of Seller's Market assumption:

- $\mu_{H0} = \mu - \mu_{H1} - \mu_{L0} - \mu_{L1} = \mu - (1 - \mu_{L1}) - \mu_L = \mu_H - 1 + \mu_{L1}$
- Note that $\xi_{HL}^b > \xi_{HL}$ implies $\mu_H^{ss,b} < \mu_H^{ss}$
- Then $\mu_{H0}(t) > \mu_{L1}(t) \iff \min\left\{\mu_H(0), \mu_H^{ss,b}\right\} > 1$

Future Work: Liquidity over the Business Cycle

Aggregate shocks:

- Model so far only has aggregate shocks in δ
- Introduce (Poisson) aggregate shocks to parameters to capture macro-risks

Changes to model:

- Sacrifice deterministic maturity, use random maturity to handle shifts in aggregate state while *maintaining* tractability / closed-forms:
  - **Good** period with normal cash-flows and well intermediated OTC markets
  - **Bad / Crisis** period with shock to intermediation intensity (financial crisis), riskier cash-flows, and higher price of risk (Chen 2010)

Conclusion

Fully solved non-stationary dynamic search model:

- Closed form solution for debt, equity, default boundary

Liquidity-default spiral:

- Lower liquidity in secondary market lowers the distance to default, which further lowers liquidity in secondary market,...

Yield-spread decomposition:

- Focus on *causes* instead of *consequences*

What about adverse selection?

- Definitely reasonable but challenging. Probably generates similar empirical illiquidity pattern (Crotty, Back '13)
- For understanding the role of liquidity in credit spreads, search framework (simple, easy to be integrated) delivers first-order effects

On-going empirical work:

- Incorporating macroeconomy & aggregate liquidity states, to better understand *liquidity/default interaction*