Endogenous Liquidity and Defaultable Bonds

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Motivation: Default vs Liquidity

- Default and liquidity are interconnected as evident from recent financial crisis
  - Liquidity: funding liquidity, price impact, transaction costs, etc

- Prevalent in theoretical literature: one-way linkages
  1. Liquidity $\rightarrow$ default or 2. Default $\rightarrow$ liquidity

- Today’s paper: liquidity $\leftrightarrow$ default, two-way feedback
  Endogenous liquidity solved jointly with default decision
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- Important in understanding the liquidity and default premia for corporate bond credit spreads
  - Barclays Capital report (2009) shows high correlation between default and liquidity spreads, both in time-series and cross-section
  - Dick-Nielsen, Feldhütter, and Lando (2012), and Friewald, Jankowitsch, and Subrahmanyam (2012)
  - Yet state-of-art empirical literature additively decomposes spreads into independent liquidity and default premium
Motivation: Corporate Bonds

Source: Huang, Huang '03
Motivation: Corporate Bonds

Source: Edward, Harris, Piwowar ’06 (EHP; BA at median trade size)
Mechanism and Results

**Building blocks** for interaction between fundamental and liquidity:

- How does bond illiquidity arise, and how is it affected by the state of the firm?
  - Over-the-counter market with search friction à la Duffie et al (2005)
- How do corporate decisions interact with secondary market liquidity?
  - Endogenous default and rollover channel à la Leland Toft (1996)

Main results:
- Closed-form solution for bond values and bid-ask spreads, equity values, and default boundary
- Endogenous liquidity allows us to match the cross-sectional pattern of bid-ask spreads and credit spreads
- Liquidity-default spiral and novel default-liquidity decomposition (applied to financial crisis)
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Related Literature

Search in asset markets:
- Duffie, Garleanu, Pedersen ’05 (DGP), ’07
  OTC search market with simplified 'derivative' security

Capital structure models:
- Leland, Toft ’96 (LT)
  Rollover increases exposure of equity holders to fundamental risk
- He, Xiong ’12
  Exogenously given secondary market liquidity affects default decision

Empirical literature:
- Bao, Pan, Wang ’11; Edwards, Harris, Piwowar ’07; Hong, Warga ’00; Hong, Warga, Schultz ’01; Harris, Piwowar ’06; Feldhütter ’11

Feedback models:
- Many many more papers...
Above analysis outside default
Schematic Representation: The Primary Market

Above analysis outside default
Schematic Representation: The Secondary Market

Above analysis outside default
Model: The Firm

Preferences: Everyone risk-neutral with common discount rate $r$

Cash flows:
- Cash-flow rate $\delta_t$, $d\delta_t = \mu \delta_t dt + \sigma \delta_t dZ^Q_t$
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Debt structure:
- Debt in place with aggregate face value \( p \) and coupon \( c \)
- Stationary principal & staggered maturity (as in LT):
  - Uniform maturity structure \( \Rightarrow \) Mass \( 1/T \) matures every instant
  - Maturing bonds reissued with identical contract terms \( (c, p, T) \)
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Rollover:
- Primary market with transaction costs $\kappa$, debt reissued at $D_H$

$$Net\text{CashFlow}_t = \delta_t - (1 - \pi) c + \frac{1}{T} \left[ (1 - \kappa) D_H (\delta_t, T) - p \right]$$

Endogenous default:
- Equity defaults at $\delta_b$ when absorbing further losses unprofitable
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\]

Endogenous default:

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Idiosyncratic liquidity shock to bond investors:

- Asset holding restriction \( \{0, 1\} \) as in DGP '05
- Uninsurable i.i.d. liquidity shock results in two types of agents:
  - **H type**: subject to liq shock with intensity \( \xi \) before default, \( \xi_b > \xi \) after default
  - **L type**: currently in liquidity shock state, holding cost \( \chi \) pre-default \( (\chi_b r - \mu) \) post-default) until asset sold.

- Type dependent bankruptcy value \( D_i (\delta_b, \tau) = \alpha_i \frac{\delta_b}{r - \mu}, i \in \{H, L\} \)
  where \( \alpha_i \) determined by frictions in post-bankruptcy market
- Simplifying assumption: No recovery from liq shock, \( L \) types exit market after sale. [Assumption for expositional purposes only]
Model: Investors, Liquidity Shocks & Search

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Trade & search friction:

- \( L \) sellers, \( H \) buyers, all meet OTC dealers with intensity \( \lambda \)
- Competitive interdealer market, no inventory, transaction price \( M \)
- Agents have bargaining power \( \beta \) vis-a-vis a dealer
Model: Secondary Market - A Seller’s Market

Seller’s market Assumption:
Mass sellers $\mu_L$ smaller than mass buyers $\mu_H$, i.e., $\mu_L < \mu_H$
Model: Secondary Market - Bid and Ask

**Nash-bargaining:**
- Let $\Pi$ be generic surplus. Then Nash-bargaining splits it
  $\beta \Pi \rightarrow Investor$ \hspace{1cm} $(1 - \beta) \Pi \rightarrow Dealer$
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Pre-default market:
- $L$-dealer (seller) surplus $\Pi_L$, $H$-dealer (buyer) surplus $\Pi_H$
- Bertrand competition in interdealer market erodes $H$-dealer surplus
  - Why? Any positive surplus would be outbid as there is more potential buyers than sellers
- Ask price $A$ ($H$ is buying at), bid price $B$ ($L$ is selling at)
  - Buy side: $A = D_H - \beta \Pi_H = M = D_H$ and $\Pi_H = 0$
  - Sell side: $B = D_L + \beta \Pi_L$ and $\Pi \equiv \Pi_L = D_H - D_L > 0$
- Key tractability from $D_{L0} = 0$ (by previous assumption) and $D_{H0} = 0$ (as no surplus to buyers in a seller’s market)
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Endogenous liquidity:
- Bid-ask spread $A - B = (1 - \beta) (D_H - D_L)$ proportional to valuation wedge
As surplus from buying is zero, $H$-investors indifferent between buying on primary market, secondary market and being on sideline.
Model: Valuation Equations

**Debt:** Boundary conditions $D_{i}(\delta, 0) = p$, $D_{i}(\delta_b, \tau) = \alpha_{i} \frac{\delta_b}{r-\mu}$

$$rD_{H}(\delta, \tau) = \underbrace{A^{\delta} D_{H}(\delta, \tau)}_{CF \ dynamics} - \underbrace{\frac{\partial D_{H}}{\partial \tau}(\delta, \tau)}_{Maturity} \underbrace{(\delta, \tau) + c + \zeta \left[D_{L}(\delta, \tau) - D_{H}(\delta, \tau)\right]}_{Liquidity \ shock}$$

$$rD_{L}(\delta, \tau) = \underbrace{A^{\delta} D_{L}(\delta, \tau)}_{CF \ dynamics} - \underbrace{\frac{\partial D_{L}}{\partial \tau}(\delta, \tau)}_{Maturity} \underbrace{(\delta, \tau) + c - \chi + \lambda \left[B(\delta, \tau) - D_{L}(\delta, \tau)\right]}_{Secondary \ market}$$
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\[
\begin{align*}
    rD_H (\delta, \tau) &= A^\delta D_H (\delta, \tau) - \alpha H \frac{D_H}{\partial \tau} (\delta, \tau) + c + \frac{\delta_b}{r-\mu} \left[ D_L (\delta, \tau) - D_H (\delta, \tau) \right] \\
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**Observational equivalence:**

- As \( \lambda [B - D_L] = \lambda \beta [D_H - D_L] \), pricing equivalent to world with exogenous \( H \leftrightarrow L \) switching intensities \( \zeta \) and \( \lambda \beta \).
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- As $\lambda [B - D_L] = \lambda \beta [D_H - D_L]$, pricing equivalent to world with exogenous $H \leftrightarrow L$ switching intensities $\xi$ and $\lambda \beta$

**Equity:** Boundary condition $E (\delta_b) = 0$, optimality condition $E' (\delta_b) = 0$

\[
rE = \delta - (1 - \pi) c + A^\delta E (\delta) + \frac{1}{T} [(1 - \kappa) D_H (\delta, T) - p]
\]
Analytic Solutions and Comparative Statics

Closed form solutions:

- Closed form solutions for debt $D_{H/L}$ (mix of two LT solutions), equity $E$ and optimal default boundary $\delta_b$
- Consequently, closed form solutions for absolute and proportional bid-ask spread, $A - B = (1 - \beta) \Pi_L$ and $\Delta = \frac{A - B}{\frac{1}{2}A + \frac{1}{2}B}$, respectively
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Analytic comparative statics:
1. If wedge at default, $\Pi = (\alpha_H - \alpha_L) \frac{\delta_b}{\mu - r}$, greater than wedge at $(\delta, \tau) \to (\infty, \infty)$, $\Pi = \frac{\lambda}{r + \xi + \lambda \beta}$, then $\partial_\delta (A - B) < 0$.
2. If additionally $\partial_\delta D_H > 0$ (condition provided), then also $\partial_\delta \Delta < 0$.
3. If $\alpha_H > \alpha_L$, then $\partial_\tau (A - B) > 0$.

Interpretation:
1. + 2. Controlling for time-to-maturity, both abs and prop bid-ask spreads decreasing in $\delta$ (pro-cyclical liquidity)
3. Controlling for dist-to-default, abs bid-ask spread is increasing in $\tau$. 
Liquidity and Default: Full Feedback Loop

Counterfactual: Fixed illiquidity / transaction cost

- Fixed transaction cost $k$ (bid-ask spread of $\frac{k}{1-k/2}$) with immediate sale after shock (as in Amihud Mendelson '86, He Xiong '12)
- Suppose investors believe they are in this counterfactual world
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Thought Experiment:

- When $\delta$ shrinks, to investors, transaction costs 'unexpectedly' rise
- Higher transaction costs mean investors value the bond less
- Rollover $\frac{1}{T} \left[ (1 - \kappa) D_H (\delta, T) - p \right]$ more costly for each $\delta$
- Costlier rollover implies earlier default $\delta_b$
Liquidity and Default: Full Feedback Loop

Equilibrium feedback loop:

- Cash-flow $\delta$ declines
- Liquidity decreases
- Debt values decline
- Equity holders default earlier
- Debt rollover more expensive

- Fixed point $\delta_b$ outcome of this spiral
# Calibration: Baseline Parameters

<table>
<thead>
<tr>
<th>Firm Characteristics</th>
<th>Secondary Market</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Param.</strong></td>
<td><strong>Interpretation</strong></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Volatility</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Drift</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Tax shield</td>
</tr>
<tr>
<td>$p$</td>
<td>Principal</td>
</tr>
<tr>
<td>$c$</td>
<td>Coupon</td>
</tr>
<tr>
<td>$T$</td>
<td>Bond maturity</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Issuance costs</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>Initial Cash Flow</td>
</tr>
</tbody>
</table>

- Pre-default holding costs: $\chi = \chi_p p + \chi_c c$. Pick $\chi_p/c$ targeting bid-ask spread of A/BBB-rated bonds.
- Recovery $\alpha$’s derived form Moody’s Default and Recovery Database
- BA-spread at default $\approx 200\text{ bps}$ in line with EHP data for defaulted bonds for median trade size
- Map $\delta$ into quasi leverage via $QL(\delta) = \frac{p}{p + E(\delta)}$
Solid: Adjust $c$ so issued at par (newly issued bonds); Dashed: Constant $c$ (stale bonds)
Calibration: Liquidity

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Model-Based Decomposition: Methodology

- Longstaff et al '05: CDS back out default component $\hat{y}_{DEF}$. How much of default component is caused by liquidity?

- Structural model allows finer decomposition of *credit spread*:

$$\hat{y} = \hat{y}_{\text{pureDEF}} + \hat{y}_{\text{LIQ} \rightarrow \text{DEF}} + \hat{y}_{\text{pureLIQ}} + \hat{y}_{\text{DEF} \rightarrow \text{LIQ}}$$

- **Pure default** $\hat{y}_{\text{pureDEF}}$: fully liquid secondary bond market (LT 96), default at $\delta_{LT}^*$

- **Liquidity-driven Default** $\hat{y}_{\text{LIQ} \rightarrow \text{DEF}}$: additional default due to earlier default at $\delta_b^* > \delta_{LT}^*$ (but full liquidity in trading)

- **Pure Liquidity** $\hat{y}_{\text{pureLIQ}}$: riskless bond spread with illiquid secondary bond market (DGP 05)

- **Default-driven Liquidity** $\hat{y}_{\text{DEF} \rightarrow \text{LIQ}}$: additional illiquidity part due to default

- **Goal**: Separate *causes* from *consequences*
Decomposition: Application to Financial Crisis

- Set $\delta_0$’s to target the credit spreads. Crisis: $-50\%$ shock to $\delta_0$’s.

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<tr>
<td></td>
<td>Normal</td>
<td>Crisis</td>
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<td><strong>Panel A: Data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Spread bps</td>
<td>97</td>
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Based on Friewald et al (2012)
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**Panel A: Data** based on Friewald et al (2012)

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**Panel B: Model**

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<tr>
<td>Yield Spread bps</td>
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<td>318</td>
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<tr>
<td></td>
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<tr>
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<td><strong>Panel C: Decomposition</strong></td>
<td></td>
<td></td>
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<tr>
<td>Pure Default</td>
<td>32</td>
<td>168</td>
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<tr>
<td>Liquidity-driven Def</td>
<td>8</td>
<td>20</td>
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</tr>
<tr>
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Microfoundation of Bankruptcy Wedge

Bankruptcy payout delay:

- Bankruptcy recovery $\alpha < 1$ of unlevered firm value $\frac{\delta_b}{r-\mu}$
- Recovery payout at exponential ($\theta$) time due to legal delay
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Post-default market:
- Search market characterized by $(\theta, \xi_b, \lambda_b, \chi_b, \beta_b, \delta_b)$
- Ask price $A^b = D^b_H$, bid price $B^b = D^b_L + (1 - \beta) \Pi^b_L$
- *Seller’s market* assumption: Competitive interdealer price $M^b$ erodes all surplus of buyers
Microfoundation of Bankruptcy Wedge

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Effective bankruptcy recovery for $H$ and $L$ investors:
- Closed form $D^b_H = \alpha_H \frac{\delta_b}{r-\mu} > D^b_L = \alpha_L \frac{\delta_b}{r-\mu}$
  $\Rightarrow$ Pre-default liquidity, via $\delta_b$, affects post-default liquidity
- Interpretation of default as firm-wide liquidity event that is endogenously triggered
Calibration of Bankruptcy Wedge

Other data sources:

- EHP: Bid-ask spread right before default $\approx 200$ bps
- Chen 2011: Bankruptcy recovery from trading price at default $\alpha_L = 50\%$

Moody’s default and recovery database:

- Avg time to emergence: 501 days $\Rightarrow \theta = 0.73$
- Annual buy-and-hold return: 36\% (relative to S&P benchmark) $\Rightarrow$ Eventual recovery $\alpha = \alpha_L \times 1.52 = 0.75$
- Benchmark perfect liquidity model has $\alpha_{LT} = \frac{\theta}{r+\theta} \alpha$

Parameters that give $\alpha$’s:

- Data only allows identification of two deeper structural parameters
- Assuming $(\beta_b, \lambda_b)$ unchanged, $(\chi_b, \xi_b) = (0.1855, 16.5)$
Optimal Maturity: Rollover Risk vs Liquidity

**Negative: Short-term debt leads to earlier default**
- Higher rollover frequency increases equity’s exposure to $\delta$

\[
Rollover\ gain/loss_t = \frac{1}{T} \times \left[ (1 - \kappa) D_H(\delta_t, T) - p \right]
\]

- Higher exposure to $\delta$ leads to higher default boundary $\delta_B$

**Positive: Short-term debt provides liquidity**
- Short maturity improves bargaining outcome between seller & dealer
- Issuing to $H$ types more frequently improves allocative efficiency as it 'recycles' $L$ types to $H$ types quicker

⇒ Finite maturity $T^* < \infty$ optimal if moderate initial leverage; $T^*$ lower the less liquid secondary market (i.e. the lower $\lambda$)
Optimal Maturity: Rollover Risk vs Liquidity

Negative: Short-term debt leads to earlier default

- Higher rollover frequency increases equity’s exposure to $\delta$

$$\text{Rollover gain/loss}_t = \frac{1}{T} \times \left[ (1 - \kappa) D_H (\delta_t, T) - p \right]$$

- Higher exposure to $\delta$ leads to higher default boundary $\delta_B$

$\Rightarrow$ LT, He and Xiong ’12: Infinite maturity debt *always* optimal ex-ante
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- Short maturity improves bargaining outcome between seller & dealer
- Issuing to $H$ types more frequently improves allocative efficiency as it ‘recycles’ $L$ types to $H$ types quicker
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$\Rightarrow$ Finite maturity $T^* < \infty$ optimal if moderate initial leverage; $T^*$ lower the less liquid secondary market (i.e. the lower $\lambda$)
{Recall: Orders \textit{batched}, i.e. intermediation intensity $\lambda$ constant across $\tau$}

\textbf{Steady-state Distribution:}

- Let $p_H(\tau) / p_L(\tau)$ be the proportion of H/L types of maturity $\tau$ that hold the bond. Then

$$p_H(T) = 1 \quad \& \quad \frac{\partial p_H(\tau)}{\partial \tau} = \lambda p_L(\tau) - \xi p_H(\tau)$$

$$\Rightarrow p_H(\tau) = \frac{\lambda + \xi e^{(\tau-T)(\lambda+\xi)}}{\lambda + \xi}$$
Trading Volume

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\textbf{Empirical implication:}

- Trade volume decreasing in $\tau$: $V(\tau) = \frac{\lambda \xi}{T(\lambda + \xi)} \left[ 1 - \xi e^{(\tau-T)(\lambda+\xi)} \right]$

- Results driven by
  - Placement tech. ('recycling' maturing L types to H types)
  - Constant intermediation intensity $\lambda$ across maturities
A Full Equilibrium Search Market

Recovery:
- Introduce recovery shock $\xi_{LH}$ that hits agents of type $L$.
- Effective recovery rate for pricing $\xi_{LH}^{\text{effective}} \equiv (\lambda \beta + \xi_{LH})$.

Accounting:
- Total mass $\mu$ of agents: $\mu = \mu_{H0} + \mu_{H1} + \mu_{L0} + \mu_{L1}$.
- Type-only distributions, $\mu_{H}$ and $\mu_{L}$, are independent of trading.
- Let $\lim_{t \to \infty} \mu_{H}(t) = \mu_{ss}$.

From ODE $\dot{\mu}_{H} = \xi_{LH} \mu_{L} - \xi_{HL} \mu_{H} = \xi_{LH} \mu - (\xi_{HL} + \xi_{LH}) \mu_{H}$, we know $\mu_{H}$ monotone and $\mu_{ss} = \xi_{LH} \mu_{x}$.

Micro-foundations of Seller's Market assumption:
- $\mu_{H0} = \mu - \mu_{H1} - \mu_{L0} - \mu_{L1} = \mu - (1 - \mu_{L1}) - \mu_{L1} = \mu_{H} - 1 + \mu_{L1}$.

Note that $\xi_{bHL} > \xi_{HL}$ implies $\mu_{ss}$, $b_{H} < \mu_{ss}$.
- Then $\mu_{H0}(t) > \mu_{L1}(t) \iff \min\{\mu_{H}(0), \mu_{ss}, b_{H}\} > 1$. 
A Full Equilibrium Search Market

Recovery:

- Introduce recovery shock $\xi_{LH}$ that hits agents of type $L$
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- Let $\lim_{t \to \infty} \mu_H(t) = \mu_{H}^{ss}$. Then from ODE

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- Note that $\xi_{HL}^b > \xi_{HL}$ implies $\mu_{H}^{ss,b} < \mu_{H}^{ss}$
- Then $\mu_{H0}(t) > \mu_{L1}(t) \iff \min \left\{ \mu_H(0), \mu_{H}^{ss,b} \right\} > 1$
Future Work: Liquidity over the Business Cycle

Aggregate shocks:
- Model so far only has aggregate shocks in $\delta$
- Introduce (Poisson) aggregate shocks to parameters to capture macro-risks

Changes to model:
- Sacrifice deterministic maturity, use random maturity to handle shifts in aggregate state while maintaining tractability / closed-forms:
  - **Good** period with normal cash-flows and well intermediated OTC markets
  - **Bad / Crisis** period with shock to intermediation intensity (financial crisis), riskier cash-flows, and higher price of risk (Chen 2010)
Conclusion

**Fully solved non-stationary dynamic search model:**
- Closed form solution for debt, equity, default boundary

**Liquidity-default spiral:**
- Lower liquidity in secondary market lowers the distance to default, which further lowers liquidity in secondary market,...

**Yield-spread decomposition:**
- Focus on *causes* instead of *consequences*

**What about adverse selection?**
- Definitely reasonable but challenging. Probably generates similar empirical illiquidity pattern (Crotty, Back ’13)
- For understanding the role of liquidity in credit spreads, search framework (simple, easy to be integrated) delivers first-order effects

**On-going empirical work:**
- Incorporating macroeconomy & aggregate liquidity states, to better understand *liquidity/default interaction*