A Theory of Debt Maturity:
The Long and Short of Debt Overhang

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This draft: January 2011
(First draft: January 2010)

Abstract

Maturing risky short-term debt can impose a stronger debt overhang effect than long-term does, distorting the firm’s investment decisions. We derive the optimal maturity structure based on the trade-off between long-term overhang in good times and (stronger) short-term overhang in bad times. The theory has implications on empirical studies of debt maturity structure, understanding the excessive defaults and underinvestment during recessions, market-based pricing of credit lines, and firm’s cash holdings.

Key words: Wealth transfer, short-term debt crisis, underinvestment, endogenous default.

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1 Booth School of Business, University of Chicago, and NBER; and Booth School of Business, University of Chicago. We thank seminar participants at MIT Sloan, OSU Fisher, Chicago Booth, Columbia, Yale, Harvard, NBER Corporate Finance meeting, AFA 2011 in Denver, Nittai Bergman, Hui Chen, Gustavo Manso, Gregor Matvos, Victoria Ivashina, Henri Pages, Raghu Rajan, Berk Sensoy, Jeremy Stein, Rene Stulz, and especially Stewart Myers and Charles Kahn for insightful comments.
1. Introduction

Myers (1977) shows that risky debt that matures in the future leads to underinvestment today. The insight is that part of the cash flows generated by investment goes to debt holders at maturity, and unfortunately the equity holders who make the investment decision will not internalize this benefit. The truncation of cash flows (and implied sharing of them) can distort investment incentives. Myers (1977) therefore suggests the solution of short-term debt to the debt overhang problem, because if all debt matures before the investment opportunity, the firm can make the investment decision as if an all-equity firm.

The short-term debt in Myers (1977) is better viewed as debt that has matured (safely) yesterday before the investment decision, rather than the one that is going to mature soon. We stress the importance of timing here, as this paper studies the situation where the firm is making a series of investment decisions (say, expansion, maintenance, or even default) given both short-term and long-term debt maturing in the future. This framework fits well with the empirical literature in which short-term debts are often classified as those with maturity within three years (e.g., Johnson (2003)).

Our model features two new elements relative to Myers (1977). First, the firm’s debt maturity structure choice (debt maturity at t=1 or t=2) is made at t=-1 before the information of asset-in-place gets realized at t=0. Second, the short-term debt is going to mature but still before the firm’s investment decision at t=0. Specifically, at t=0 the firm is considering taking a long-term investment opportunity which generates cash flows at t=2. However, equity holders not only face the long-term debt that matures at t=2, but also the short-term debt maturing at t=1 which requires refinancing. As a result, both long-term and short-term debts may impose overhang effect on the investment at t=0.

In general, from Myers (1977) we know that any riskless debt (regardless short-term or long-term) imposes no overhang. When there is interim bad news on firm’s asset-in-place, the short-term debt might turn risky, which gives arise to wealth transfer to short-term debt holders and in turn debt overhang. In this paper, we show that the potentially risky short-term debt might impose stronger overhang effect than the (risky) long-term debt does.

The economic mechanism underlying the stronger short-term overhang is simple. The core of debt overhang is wealth transfer from equity holders to debt holders. Because short-term debt is going to be paid earlier than the long-term debt, there is less uncertainty resolved over the shorter
time, and the wealth transfer to potentially risky short-term debt holders (through new investment) might be greater. Although the general theme is the same, in this paper we identify two specific channels that short-term debt might impose stronger overhang.

The first is the increasing leverage effect given our running timing assumption that the firm issued debt before certain significant news arrives regarding the value of asset-in-place. Specifically, when the interim bad news hits, the short-term debt that features a higher refinancing (rollover) face value will have a greater debt value (higher leverage) compared to the long-term debt with fixed face value. This in turn causes a greater overhang due to short-term debt in bad times.

In examples studied in Section 2, we assume that the firm issues either exclusively short-term or long-term debt. When the asset-in-place value is high, the short-term debt can easily get refinanced, and consequently it imposes no overhang. In contrast, the long-term debt is risky due to greater uncertainty in the long-run, leading to a positive overhang.

However, when the asset-in-place value deteriorates, things might change. Compared to long-term debt, the value of short-term debt drops less due to bad news, leading to a higher leverage and greater overhang. In our (extreme) example, given bad news the firm always has trouble in refinancing its short-term debt at interim date \( t=1 \) if the firm does not invest at \( t=0 \). In fact, the short-term debt imposes 100% leverage on the firm, and the entire investment benefit at this state goes to the short-term debt holders (i.e., the greatest overhang). Long-term debt, on the other hand, will have a leverage below 100% and therefore less overhang, because the firm might still survive at \( t=2 \) when subsequent good shocks hit and equity holders can recoup some investment benefit. Short-term debt which sometimes becomes risky after issue leads to a higher fluctuation of debt overhang than long-term debt.

In Section 2.3 we also show that risky short-term debt may impose greater overhang even on a date when the market value is identical for both long-term and short-term debt, i.e., even if we control for leverage. This implies that even if there is no significant news about asset-in-place that arrives between the debt maturity decision and the investment decision, it is still possible that the firm would prefer to issue long-term debt to reduce overhang. We show that this occurs when the firm’s asset-in-place exhibits higher volatility for low asset value levels, which implies that the asset value distribution is negatively skewed. Intuitively, the firm issuing short-term debt is similar to an owner of a down-and-out option (an option that expires worthless if the value of the
underlying asset reaches a boundary, even if it recovers in the money before maturity). If assets display higher volatility following interim bad news, then equity holders with short-term debt will be knocked out and lose more of the investment benefits that come in the future. Therefore, our theory predicts that all else equal, firms with higher degree of countercyclical volatilities will use more long-term debt if they want to maximize their incentive to invest.

Our finding implies that short-term debt is not a free lunch in coping with debt overhang. Although riskless short-term debt imposes no overhang, potentially risky short-term debt that truncates the cash flows received by date 0 equity holders might do more evil than long-term debt in dwarfing the firm’s investment incentives. Therefore, the firm faces a trade-off in choosing the optimal debt maturity structure before the value of the asset-in-place is known. The formal model in Section 3 endogenizes the interior debt maturity structure (chosen at $t=-1$) based on the trade-off that we illustrate in Section 2.

In our model, the interpretation of “investment” can be spending capital to either establish new projects, or keep old projects alive. For instance, “maintenance” is a form of investment that requires capital expenditure to keep the existing project operating efficiently, and “foregoing maintenance” represents underinvestment. The extreme version of “foregoing maintenance” is just “default,” where the firm essentially gives up the old project. In fact, the endogenous default, which usually occurs given a sufficiently low asset-in-place value, is a symptom of short-term debt overhang (see Leland (1994) and related literature on endogenous default).² This is because the new financiers understand that to prevent the firm from defaulting, the fresh capital that they put in pays the maturing short-term debt first, resulting in a full subsidy to short-term debt holders. Although it is true that bailing out the firm also benefits long-term debt holders who have claim on the firm’s future cash flows, the subsidy to long-term debt holders will not be as large, since new financiers can recoup some benefit if the firm recovers tomorrow.

Debt overhang has been an active research topic since Myers (1977).³ Our paper emphasizes the role of debt maturity on debt overhang. We show that the short-term overhang is in a zero-one nature. To be precise, when the firm’s asset-in-place value is high and short-term debt is riskless, short-term debt imposes no overhang effect. However, once the firm has experienced a series of

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² The interpretation of endogenous default given debt burden as “underinvestment” due to debt-overhang, is mentioned in, for example, Lambrecht and Myers (2008) and He (2010).
³ To name a few recent studies on debt overhang, see Philippou and Schnabl (2009) and Diamond and Rajan (2010)
negative shocks and short-term debt becomes risky, the short-term debt overhang quickly becomes overwhelming.\footnote{Moyen (2007) studies the role of maturity on debt overhang in a different aspect. In Moyen (2007), firms with short-term debt can freely adjust their leverage each period in response to fundamental shocks for tax benefit reasons, while firms with long-term debt cannot. Therefore, firms with short-term debt adjust leverage upward in good time, leading to a greater overhang. Note that our model switches off the “flexibility” effect of short-term debt. And, it is unclear what will happen if firms with long-term debt can also adjust leverage in Moyen (2007).}

In many financial crises, short-term debt is often implicated as contributing factor to defaults. However, it is often attributed to a “run” by short-term debt (e.g., Diamond and Dybvig (1983)). Meanwhile, researchers acknowledge the debt-overhang effect in motivating government bail outs of financial firms, but mostly attribute the overhang effect to existing long-term debt. By showing that the distortion due to overhang is not a feature of long-term debt exclusively, this paper suggests that the short-term overhang can be another important contributing factor to financial crises. In fact, the results in Veronesi and Zingales (2009) suggest that the wealth transfer in the 2008 United States government bailout of financial firms was concentrated on the short-term debt. Moreover, the prediction that short-term overhang leads to underinvestment is consistent with the empirical findings in Duchin, Ozbus, and Sensoy (2009) and Almeida et. al (2009) who study firms’ underinvestment during the 2007/2008 crisis.

Our theoretical results have important implications for empirical testing of the Myers (1977) debt-overhang theory, especially the prediction that growth firms (presumably with more investment opportunities) should use more short-term debt. The existing empirical evidence on this prediction has been mixed.\footnote{For instance, Barclay and Smith (1995) and Guedes and Opler (1996) document a negative relation between maturity and growth opportunities, while Stohs and Mauer (1996) and Johnson (2003) find a positive relation once controlling for firm leverage. See Section 4.2.3 for more details.} In light of our theory, these mixed results are not surprising. Since this literature often classifies short-term debts as those are going to mature in three years (e.g., Johnson (2003)), our timing assumption that investment is made before short-term debt matures fits quite well. Then, early default for growth firms might be more costly which pushes optimal maturity structure toward long-term. Furthermore, our theory emphasizes the importance of correlation between investment opportunities and asset-in-place values: long-term debt is preferable for those maturing firms where maintenance (investment when asset-in-place value is low) is particularly valuable.

The optimal maturity structure in our paper is based on the trade-off due to long-term and short-term overhang. It is different from existing theories of optimal maturity structure which
focus on the disciplinary role to short-term debt in curbing the managers’ asset substitution or other misbehavior, e.g., Calomiris and Kahn (1991), Diamond and Rajan (2001), Flannery (1994) and Leland (1998), or private information of borrowers about their future credit ratings, e.g., Flannery (1986) and Diamond (1991).  

The rest of paper is organized as follows. We give a series of examples in Section 2 to illustrate the key intuition of this paper. Section 3 gives the formal model, and Section 4 provides discussion and extensions. In Section 5 we conclude.

2. Examples

2.1 Example Structure

The following examples deliver the main idea of this paper. We are standing at t=0, where the firm has some asset-in-place, and is considering making a new investment which requires new outside financing (as the firm has no cash holdings; see discussion in Section 4.5). Figure 1 depicts two states G (Good) and B (Bad) at t=0 with different asset-in-place values.

At t=0, equity holders face a debt maturity structure which was determined at t=-1, a decision that will be formally studied in Section 3. The premise here is that we will focus on the case that there is some significant amount of information revealed regarding the asset-in-place after the maturity decision. Although Section 2.3 also investigates the possibility of maturity choice at t=0, we believe the information setting of maturity choice at t=-1 is more relevant empirically.

We assume risk neutral agents and a zero interest rate. Short-term debt will be maturing at t=1, while long-term debt will be maturing at t=2. Importantly, since there are no interim cash flows, short-term debt needs refinancing at t=1. To study debt overhang, we assume existing debt to be senior to any new financings. We rule out renegotiation, and bankruptcy (if occurs) cost is zero.

To clearly see the trade-off, let us consider the case where the firm is allowed to take either exclusively short-term or long-term debt. The full analysis in Section 3 will consider the optimal mix of long-term debt and short-term debt.

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6 Benmelech (2006) argues that entrenchment can lead the self-interested manager to take long-term debt, and provide supporting evidence for this theory. Brunnermeier and Yogo (2009) stress the option value of using short-term financing so that the firm can readjust the debt maturity before the firm has experienced sufficiently negative shocks. In our model, the short-term debt matures after the sufficiently negative shock arrives. And, Brunnermeier and Oehmke (2009) study the maturity rat race between creditors; there, short-term creditors impose negative externalities on long-term creditors, leading to excessive short-term debt in equilibrium. He and Xiong (2010) study the impact of bond market illiquidity on credit risk; the trade-off in that paper is based on the stronger overhang effect in rolling over short-term debt and its greater liquidity in the secondary market.
Figure 1: A binomial example with two date 0 asset-in-place realization $X$. Left (left) panel is state G (B) with high (low) asset-in-place $X=21$ (4). Following the previous node, each branch occurs with a probability of 0.5. Long-term debt face value $F_2=14.75$, and short-term debt face value $F_1=11$.

The firm’s asset-in-place follows a standard binomial tree depicted in Figure 1. With probability 0.5 the state $u$ ($d$) occurs at $t=1$, and with probability 0.5 further uncertainty resolves. In state $dd$ the asset-in-place has a value of 1. In state $ud$ or $du$, the asset-in-place’s value is $X$, and in $uu$ state the value is $X^2$. The left (right) panel stands for state G (B), which features a higher (lower) asset-in-place $X$.

Suppose that the new investment of $I < 1$ at $t=0$ leads to a constant payoff of 1 at $t=2$, which implies a positive NPV of $1 - I$. When equity holders invest at $t=0$, debt overhang implies that part of the investment payoffs goes to the debt holders (i.e., wealth transfer). Equivalently, this wealth transfer is reflected in the value increment of debt (either short-term or long-term) due to the new investment. Since there is no other dead-weight loss in this example, equity holders will invest if and only if the NPV of this investment, i.e., $1 - I$, exceeds the wealth transfer to the debt holders.

2.2 Example 1: Increasing Leverage Given Bad Interim News

Throughout this example, the short-term debt has a face value of $F_1=11$, and the long-term debt has a face value of $F_2=14.75$. These face values allow the firm to raise the same amount of
debt financing independent of debt maturity at $t=-1$, if we assume that Case 1 (good state) and Case 2 (bad state) occurs equally likely standing at $t=-1$. Also note that since $F_2$ is above the asset-in-place at state $dd$ (which is 1) for both cases, long-term debt is always risky.

2.2.1 Analysis

Any debt, if it is riskless, imposes no overhang. Therefore, the existence of short-term overhang depends on the realized date 0 value of asset-in-place. In the first (second) case, the asset-in-place has a relatively high (low) value and the short-term debt is riskless (risky). Throughout this paper we say that the debt is risky if the firm’s asset-in-place is insufficient to pay debt holders in full, given no investment opportunities. We adopt this convention because we are interested in the firm’s investment incentives once it faces investment opportunities.

State G: High asset-in-place and riskless short-term debt (Figure 1 left panel)

Short-term debt only. The short-term debt with face value $F_1=11$ can be refinanced even at the state $d$, as the asset-in-place has a market value of $0.5*(21+1)=11$. Hence the short-term debt is riskless and has a market value of 11 at $t=0$. Because the value of riskless short-term debt cannot be improved further, the wealth transfer to debt holders is zero by taking the investment.

Long-term debt only. Long-term debt with face value $F_2=14.75$ is risky, which leads to debt overhang. Without investment, the long-term debt value is $0.75*14.75+0.25=11.3125$. But with investment, the long-term debt value increases to $0.75*14.75+0.25*2=11.5625$. As a result, the wealth transfer to long-term debt involved in the new investment is 0.25, which implies that equity holders will pass on investment opportunities with NPV less than 0.25. This example illustrates the long-term overhang effect discussed in Myers (1977).

State B: Low asset-in-place and risky short-term debt (Figure 1 right panel)

Short-term debt only. Now in this case the asset-in-place is low. Without investment, the short-term debt with face value 11 cannot be refinanced in either state $d$ or $u$---even at state $u$ the asset-in-place only has a value of $0.5*16+0.5*4=10<F_1=11$. Therefore the firm will default, and the date-0 market value of short-term debt is just the firm’s asset-in-place value 6.25.

How much does the short-term debt gain if the firm invests? Since the firm is totally underwater in this case (100% leverage as shown in Table 1), one would expect short-term debt to gain all of the initial increment of value from new investment. This is indeed true here. If the firm

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7 This property of equal debt financing holds if we pre-fix the future date 0 investment strategy (either invest or not) at $t=-1$. In other words, in this example we ignore the feedback from endogenous investment decisions to debt valuation. In Section 3 we fully consider this feedback effect.
invests at $t=0$, at state $d$ the firm defaults (and short-term debt holders receive $0.5 \times 5 + 0.5 \times 2 = 3.5$), while at $u$ the asset-in-place value 11 is just sufficient to refinance the short-term debt. As a result, the short-term debt value at $t=0$ increases from 6.25 to 7.25. In other words, in this extreme example with sufficiently low asset-in-place value at $t=0$, the short-term debt will truncate the entire subsequent cash flows due to investment, leading to a wealth transfer of 1 from equity holders to short-term debt holders.

**Long-term debt only.** The analysis for long-term debt is similar to the Case 1, and it is easy to check that the wealth transfer to long-term debt holders due to investment is 0.75.

In sum, the wealth transfer to short-term debt (1) is greater than that to long-term debt (0.75). Although long-term debt benefits only at the bottom states $\{du, ud, dd\}$ with a total probability of 0.75, short-term debt benefits on all the $t=2$ states. More importantly, this example also illustrates two mechanisms that short-term debt holders receive wealth transfer due to new

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**Table 1: Increasing Leverage Effect of Short-term Debt Overhang**

<table>
<thead>
<tr>
<th></th>
<th>Case G: High Asset-in-Place, Riskless ST Debt</th>
<th>Case B: Low Asset-in-Place, Risky ST Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ST Debt only</td>
<td>LT Debt only</td>
</tr>
<tr>
<td>Asset-in-place Value at $t=0$</td>
<td>121</td>
<td>121</td>
</tr>
<tr>
<td>Debt Face Value</td>
<td>11</td>
<td>14.75</td>
</tr>
<tr>
<td>Debt Value at $t=0$ (w/o investment)</td>
<td>11</td>
<td>11.3125</td>
</tr>
<tr>
<td>Leverage at $t=0$ (w/o investment)</td>
<td>9%</td>
<td>9.3%</td>
</tr>
<tr>
<td>Debt Value at $t=0$ with investment</td>
<td>11</td>
<td>11.5625</td>
</tr>
<tr>
<td>Wealth Transfer to Debt holders</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>Investment Policy</td>
<td>NPV $\geq$ 0</td>
<td>NPV $\geq$ 0.25</td>
</tr>
</tbody>
</table>
investment when short-term debt turns risky. The first mechanism is default, which is at work in state $d$ at $t=1$ in Case 2. There, since pledging out all future asset payoffs is insufficient to pay down the short-term debt, the short-term debt holders take over the defaulting firm to claim all date-2 cash flows from then on. The second mechanism is refinancing, i.e., the firm raises new financings (say, new equity) to pay the date 1 short-term debt in full, where these new financiers anticipate to recover these payments from asset payoffs at date 2. In this way, short-term debt holders essentially set a new refinancing face value of 17 at $t=2$, which is higher than the long-term face value 14.75. Thus, compared to long-term debt, refinancing at in interim $t=1$ state $u$ allows the short-term debt holders to obtain the additional cash flows at state $uu$.\footnote{In this example we push to the extreme that given date 0 bad news the short-term debt which matures at $t=1$ becomes risky even at state $u$. This property is not necessary for short-term debt to create overhang, as wealth transfer will be at work as long as short-term debt becomes risky at state $d$.}

### 2.2.2 Increasing leverage effect in log-normal setting

Recall that in this example, fixing the $t=0$ investment policy (with or without investment) at $t=-1$, the $t=-1$ debt value is the same across both maturity structures.\footnote{For example, without investment, the date -1 debt value is $(11+6.25)/2=(11.3125+5.9375)/2=8.625$ as both cases are equally likely.} However, when the news about the asset-in-place arrives at $t=0$, short-term debt tends to be a “hard” claim which does not share gain/loss with equity holders. That is, through adjusting refinancing rate, the value of short-term debt is less sensitive to the new information than that of long-term debt (similar to Myers and Majluf (1984)). In our example, after good (bad) news at $t=0$, the short-term debt value rises (drops) by 2.375, while the long-term debt value rises (drops) by 2.6875. These different sensitivities to news translate to different contingent amounts of leverage for the different maturities; particularly, contingent on bad news, short-term debt with higher value exhibits a higher leverage and in turn stronger overhang at $t=0$. As a result, short-term debt which is raised before the information may hurt the firm’s ex post investment incentives more than the long-term debt does.

This “increasing leverage” effect, which only relies on the fact that short-term debt is less sensitive to information about firm value than is long-term, is quite general. For further illustration, consider another example based on the standard Black-Scholes framework. Suppose that the firm’s asset-in-place value $V_2$ at $t=2$ is

$$V_0 \exp \left( \bar{x}_1 - 0.5\sigma^2 + \bar{x}_2 - 0.5\sigma^2 \right),$$

where $\bar{x}_1$ and $\bar{x}_2$ are the asset-in-place values at dates 1 and 2, respectively, and $\sigma$ is the volatility.
where $\tilde{X}_1$ and $\tilde{X}_2$ follow i.i.d. normal distributions with standard deviation of $\sigma$. This implies that the firm value at $t=0$ is $V_0$, and at $t=1$ is $V_0 \exp \left( \frac{1}{2} \tilde{X}_1 - 0.5 \sigma^2 \right)$.

We are interested in the date-0 investment incentives, given short-term debt with face value $F_1$ or long-term debt with face value $F_2$. Since debt value is firm value minus equity value (which is a call option $E(V_0)$ on the firm value), the debt overhang (OH), as wealth transfer to debt holders, can be measured as the marginal improvement of debt value when the firm considers investment to improve $t=0$ firm value $V_0$:

$$OH(V_0, F_i, t) = 1 - \frac{\partial E(V_0, F_i, t)}{\partial V_0} = 1 - \Delta(V_0, F_i, t).$$

where $\Delta$ is the delta-hedge for the call option in the Black-Scholes formula. Economically, imagine the firm’s incremental investment projects to be that, each dollar of expansion on $V_0$ only requires $1 - b \in (0,1)$ dollar of investment, i.e., the project’s NPV is $b>0$. Now because equity holders understand that each unit of expansion only benefits them by $\Delta$, they invest if their investment outlay exceeds the investment benefit, i.e., $1 - b \geq \Delta$. In other words, equity holders pass investment projects with NPV $b \leq 1 - \Delta$. Hence, the above wealth transfer measure $1-\Delta$ directly gives the NPV threshold of the investment policy taken by equity holders. Finally, as any investment requires some positive outlay, an overhang of 1 would imply that the firm would never invest. We have the following Proposition.
Proposition 1. Fix face values $F_1$ and $F_2$. There exists a threshold $\hat{V}_0$ so that, for $V_0 > \hat{V}_0$ we have $OH(V_0, F_1, 1) < OH(V_0, F_2, 2)$, while for $V_0 \leq \hat{V}_0$ we have $OH(V_0, F_1, 1) \geq OH(V_0, F_2, 2)$.

The intuition for Proposition 1 is simple. Debt overhang increases with the bankruptcy probability. Short-term debt, through refinancing at $t=1$, features a higher $t=2$ face value given bad interim news. Relative to using long-term debt with fixed face value, a sufficiently unfavorable interim news $V_0 \leq \hat{V}_0$ implies a higher bankruptcy probability (therefore a greater overhang) by using short-term debt. To the extreme, consider the ultra-short debt with time-to-maturity $t$ converging to 0. Once $V_0 < F$, $\Delta(V_0, F, t)$ goes to 0, implying an overhang of $1-\Delta=1$.

This corresponds to the situation with the greatest debt overhang, as equity holders get zero and debt holders who have immediate claims on the insolvent firm receives everything. Intuitively, the ultra-short debt does not share any risk with equity holders; however, once it is risky, it does not share any return with equity holders either!

We can also link the potential stronger short-term overhang to information insensitiveness of short-term debt. When information arrives between date -1 and date 0 short-term debt features a less volatile debt value at $t=0$. In particular, when bad news hits, the higher leverage leads to a greater short-term debt overhang, and equity holders are more likely to pass up investment projects with positive NPV.\(^{10}\)

In Figure 2, we choose $(F_1, F_2)$ to give the same value for short-term and long-term debts when $V_{-1} = 1$. A deviation of $V_0$ from $V_{-1} = 1$ can be viewed as new information arrives about the firm’s asset-in-place at $t=0$. The left panel in Figure 3 illustrates that when bad news hits ($V_0$ drops below 0.7), short-term debt imposes stronger overhang than long-term debt. This is the increasing leverage effect: As illustrated at the right panel, short-term debt value $D_1$ is less sensitive to the bad news (i.e., flatter than the long-term debt $D_2$), and $D_1 > D_2$ when $V_0$ is low.

We have established one dark side of short-term debt: risky short-term debt imposes stronger overhang when some bad news arrives after raising the debt and the firm becomes more levered. We will further examine this information structure in a fuller model in Section 3. The next

\(^{10}\) One tends to think that a lower information sensitivity of short-term debt indicates a higher equity delta, therefore a lower overhang. This logic misses the important timing structure. The investment occurs at date 0 (therefore only date 0 delta is relevant), while the information sensitivity is regarding the information between date -1 and date 0. In fact, given bad news, the increasing leverage effect leads to a lower date 0 equity delta for short-term debt raised at date -1.
section examines another form of short-term debt overhang where there is no significant information arriving after debt issuance.

2.3 Example 2: Overhang differences by maturity for given leverage

2.3.1 Analysis

This section modifies the information structure and asks a further question: what if the firm chooses the maturity structure right after the bad news hits at t=0, as opposed to at t=-1 before news arrives? Put differently, once we force the short-term and long-term debt to have the same value (therefore eliminating the increasing leverage effect studied in Section 2.2), can short-term debt still impose stronger overhang?

There are three reasons why this question is interesting. First, it clarifies that a driving force of stronger short-term overhang can be independent of leverage. Second, even the formal model in Section 3 later studies a different information structure where maturity choice is made before any news shock arrives (at t=-1), this effect will be present. It is because if the firm wants to replace short-term debt with long-term debt (maintaining the same total debt value) even given bad news
at \( t=0 \), then the firm definitely would like to do so at \( t=-1 \). Last but not the least, the answer to this question offers certain guidance for firms how to rebalance their maturity structure in bad times (when all debt issued might need to be risky).

The following example given in Table 2 provides a positive answer. In words, it is possible that given that short-term debt and long-term debt have the same market value at \( t=0 \), the short-term debt imposes stronger overhang. Consider the Case 2 with asset-in-place depicted in Figure 1, but with \( F_1=3.5 \) or \( F_2=11/3<4 \). One can easily verify that both debts have a value of 3. However, short-term overhang is 0.5, which is greater than long-term overhang 0.25.

Unlike the first effect identified in Section 2.2, this stronger effect is present only for some probability distributions of firm value.\(^1\) The driving force of this effect is as follows. The firm with interim bad news at \( t=1 \) (i.e., at state \( d \)) cannot survive without new investment under short-term debt, and all investment benefits following on this path (\( du \) and \( dd \)) go to the short-term debt. However, under long-term debt, the firm has some chance to come back at \( t=2 \) (the state \( du \)), which allows equity holders to recoup the investment benefit at state \( du \).

As suggested by this example, the reason that the (risky) short-term debt could impose stronger overhang, even controlling for the debt value, is because there is less uncertainty to be resolved over a shorter time horizon. Through refinancing or default, short-term debt holders may capture the investment benefit in the subsequent states---even for the states (i.e., \( du \) state) after sufficiently good follow-up news that long-term debt holders are not able to capture.

Of course, the opposite logic might work at state \( u \)--i.e., things might go worse after the good interim news---and this leads to stronger overhang for long-term debt. In our example with discrete states, this effect is absent as the negative shock following date 1 good news is not that severe (i.e., at \( ud \) state the asset value is 4>\( F_2 \)). However, this observation does suggest that, in general, short-term debts impose stronger overhang when the firm may improve dramatically after interim bad news, while there will be insignificant shocks after interim good news. Based on this idea, in the next subsection we give an example that this effect holds in a modified Black-Scholes case.

\(^{11}\) We sincerely thank Stewart Myers for pushing us on this direction.
2.3.2 An Example with State-Dependent Volatility

To illustrate how the distribution of value affects the severity of overhang for the two maturities, consider the following modification of Black-Scholes framework as a continuation of Section 2.2.2. The firm’s asset-in-place value at t=2 is

\[ V_0 \exp \left( \bar{X}_1 - 0.5\tilde{\sigma}_1^2 + \bar{X}_2 - 0.5\tilde{\sigma}_2^2 \right) \]

where \( \bar{X}_1 \) and \( \bar{X}_2 \) follow normal distribution with variances \( \sigma_1^2 \) and \( \tilde{\sigma}_2^2 \), respectively. Recall that the value on date t=1 is \( V_0 \exp \left( \bar{X}_1 - 0.5\sigma_1^2 \right) \). We allow the volatility \( \tilde{\sigma}_2^2 \), to be dependent on the t=1 realization \( \bar{X}_1 \). Particularly, we set

\[ \tilde{\sigma}_2 = \begin{cases} \sigma_L & \text{when } \bar{X}_1 > Q \\ \sigma_H & \text{when } \bar{X}_1 \leq Q \end{cases} \]

where \( \sigma_L \leq \sigma_H \). This corresponds to the situation where volatility is higher in low fundamental states (or, negatively skewed distribution), which is a common feature in many macro models with financial frictions. In fact, this pattern can be generated by the existence of volatility that is not scaled with the asset value (say, randomness in the fixed cost.) A fixed absolute volatility becomes a larger percentage of volatility when asset values are decreased.

The following proposition shows that the existence of conditional volatility is the driver of stronger short-term debt overhang for a given leverage. For the ease of analysis, we focus on the case where the second period variance is close to zero.

**Proposition 2.** Fix \( Q \) and \( \sigma_1^2 \). Suppose that long-term debt face value \( F_2 = V_0 \exp \left( Q - 0.5\sigma_1^2 \right) \), and adjust the short-term debt face value \( F_1 \) to ensure the same market value for both debts. Then for small \( \varepsilon > 0 \),

1. Without conditional volatility, i.e., when \( \sigma_L = \sigma_H = \varepsilon > 0 \), long-term debt imposes stronger overhang than short-term debt.

2. With conditional volatility, i.e., when \( \sigma_L = 0 \) and \( \sigma_H = \varepsilon > 0 \), short-term debt imposes stronger overhang than long-term debt.

See the proof in Appendix A.1. To provide intuition for this result, imagine that the first period shocks \( \bar{X}_1 = Q \pm \eta \) where \( \eta > 0 \) is sufficiently small, and consider long-term debt overhang.

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Recall that $\tilde{X}_1 = Q \pm \eta$ implies that the $t=1$ firm asset value $V_1 = V_0 \exp(Q \pm \eta - 0.5\sigma_i^2)$. Given the long-term debt face value $F_2 = V_0 \exp(Q - 0.5\sigma_i^2)$, if there are no second period shocks, then we do not have overhang for the realization of $\tilde{X}_1 = Q + \eta$ while there is overhang for $\tilde{X}_1 = Q - \eta$. Now the force of conditional volatility kicks in. With conditional volatility, we have asymmetric effects on these two realizations: The realization $\tilde{X}_1 = Q + \eta$ follows no risk, and there is still no overhang; while the point $\tilde{X}_1 = Q - \eta$ follows some risk, and the overhang is reduced (as equity holders receive some value for good realizations of $\tilde{X}_2$). This effect leads to a lower long-term debt overhang. In contrast, without conditional volatility, a higher risk for both realizations generate a greater long-term debt overhang overall.

Figure 3 plots a numerical example when we increase the volatility $\sigma_H$ after bad interim news from $\sigma_H = 0.01 = \sigma_L$ to $\sigma_H = 0.12$. The left panel plots the debt face values where we adjust the long-term debt face value $F_2$ to ensure the same market value for both debts at $t=0$. The right panel plots the resulting short-term and long-term overhang. Since we fix $\sigma_i^2$ and $F_1$, the short-term debt overhang is a constant. However, when we increase the volatility $\sigma_H$ in low fundamental state $\tilde{X}_1 < Q$, the long-term debt overhang drops below the short-term debt overhang. The reason is just as explained before: given the greater volatility after bad news, it is more likely
for the firm’s value to jump back above the long-term debt face value $F_2$, and the probability mass that $V_2$ below $F_2$ (therefore overhang) becomes smaller.

### 2.3.3 Discussion

Analogous to the “increasing leverage” effect studied in Section 2.2, here the risky short-term debt allows the debt holders to claim future incremental investment payoffs in states more than the long-term debt holders do. The overhang of short-term debt depends primarily on how much firm value is below debt face value and less on volatility (because there is little time for more resolution of uncertainty). In contrast, the level of volatility is relatively important for long-term debt, and this difference drives the role played by the countercyclical volatility. After good news, long-term overhang decreases substantially because the lower volatility going forward means that the value is likely to remain high for the long term. After bad news, long-term overhang increases by a smaller amount, because the higher volatility going forward implies a greater chance of a sufficient recovery to avoid default in the longer period after short-term debt would have matured.

This mechanism can also be intuitively illustrated by the an analogy with a down-and-out option that expires worthless if the value of the underlying asset reaches a boundary, even if it recovers in the money before maturity. When the asset displays a higher volatility following interim bad news (i.e., a negatively skewed distribution), the equity of a firm issuing short-term debt becomes more similar to the owner of a down-and-out option, and the firm loses everything going forward after an interim bad news. This explains why short-term debt is particularly harmful to investment incentives when there is negative skewness.

The above findings have important implications for firms who are deciding maturities along with investment at $t=0$, or considering readjusting its maturity structure at $t=0$, conditional on the firm’s value of asset-in-place. Clearly, when the asset-in-place is relatively high, the firm would be better off by replacing risky long-term debts with riskless short-term debts, as any riskless debt imposes no overhang. But once the firm’s asset-in-place deteriorates, risky short-term debt can impose stronger overhang than the long-term debt does, if the firm’s assets displays higher volatility following further bad shocks. At this scenario, the firm would be better off by replacing risky short-term debt with long-term debt with the same market value. Although, keep in mind
that, if this debt replacement is not allowed by its contract, a firm will not be able to unilaterally make such a change.\footnote{The maturity adjustment studied here implicitly involves renegotiation with debt holders, which is ruled out in our analysis from the beginning (to focus on debt overhang). To see this, “holding debt values constant” would mean that the firm pays the debt holders their market value, not face value, for giving up the debt claim. Alternatively, the debt must be non-standard in which some covenants give the firm an option to make this adjustment.}

3. The Formal Model

We have illustrated the state-contingent effect of short-term overhang, i.e., that short-term debt imposes no overhang when the firm’s asset-in-place value is high while stronger overhang when the value is low. This suggests a trade-off when the firm makes the debt maturity decision at $t=-1$, and now we consider a formal model which incorporates the feedback effect from investment policy to $t=-1$ debt value. The main objective of this section is to show that our key insight remains under the general setting that 1) the firm can raise a mixture between short-term and long-term debt, and 2) the date -1 debt value takes into account the endogenous date 0 investment decision (which will depends on the date -1 maturity structure). Readers can skip this section without hindering the reading of Section 4.

At $t=-1$ the firm needs to raise debt financing $D_{\text{target}}$ for initial investment. To focus on debt maturity choice, we fix the debt level $D_{\text{target}}$ in the main analysis. Section 4.4 endogenizes $D_{\text{target}}$ by considering an entrepreneur who requires outside capital to finance this project.

We assume that the credit market is competitive so that all debt is fairly priced. In this model shocks are all independent. All information is public. No cash flows from the asset occur before date 2, and we ignore cash holding.\footnote{We study the implication of cash holdings in this model in Section 4.5.}

After the firm chooses maturity structure at $t=-1$, at $t=0$ it faces the situation described in Section 2. As shown in Figure 4, there are two possible states of nature at $t=0$, $S=G$ (good) or $S=B$ (bad), and the firm’s asset-in-place is higher (lower) in good (bad) state. Specifically, asset-in-place generates cash flows $2X^S$ in state $Suu$, $X^B$ in state $Sdu$ or $Sud$, and 0 in state $Sdd$.

The firm faces investment opportunity $I^S$ at $t=0$, which yields a constant payoff $Y^S$ at $t=2$, $S \in \{G,B\}$. This implies that the investment has a positive NPV of $\gamma^S = Y^S - I^S$. Throughout we will assume that the NPV $\gamma^S$ is relatively moderate so that the necessary condition for investment at $t=0$ state $S$ is successful refinancing at state $Su$. 

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3.1 Short-term and Long-term Debt Contracts

We only consider standard debt contracts. The short-term (long-term) debt, with $t=-1$ market value $D_1 (D_2)$ has a face value $F_1 (F_2)$. We can summarize the firm’s debt policy as $(F_1, F_2)$ so that $D_1 + D_2 = D_{\text{target}}$. We impose the following assumption throughout the paper to focus on the debt overhang problem.

**Assumption.** Neither short-term debt nor long-term can be renegotiated (perhaps because debt holders are diverse). The long-term debt has a covenant stating that any new financing will be junior to the long-term debt.

The firm may default if it cannot repay its maturing debt. We impose no bankruptcy cost in the main model. Perhaps more importantly, as we are only interested in the wealth transfer to total debt, our analysis does not depend on the seniority rule between the long-term and short-term debts in bankruptcy.
3.2 Solving the Model

3.2.1 Good State G

Consider the state $Gd$ at $t=1$ first. Without new investment, the maximum new financing that the firm can obtain, i.e., refinancing capacity, is $\frac{1}{2} \max (X^G - F_2, 0)$. The new financing can be raised from existing equity holders/Managers or outside equity holders.\(^\text{14}\) Since it is long-term debt holders that get their payment $F_2$ first (they are senior to any new financings), new financiers get repaid only when $X^G$ realizes with prob. 1/2. This reflects the standard debt overhang in Myers (1977).

The firm needs to use this refinancing capacity to pay the maturing short-term debt $F_1$. We focus on the case that $X^G > 2F_1 + F_2$, so that short-term debt can get refinanced at $t=1$ even without investment. In other words, the short-term debt is riskless, and therefore imposes no overhang. Note that this condition implies that in state $Gu$ (with asset-in-place value $1.5X^G$) not only the short-term debt is riskless, but also the long-term debt is riskless (as $X^G > F_2$).

To determine the firm’s investment incentives at $t=0$, we calculate the value increment of long-term debt due to new investment. Without investment, the long-term debt date-0 value is $0.75 \cdot F_2$. With investment, the long-term debt value becomes $0.75 \cdot F_2 + 0.25 \cdot \min (Y^G, F_2)$. Therefore, the firm will invest if and only if

$$0.25 \cdot \min (Y^G, F_2) < \gamma^G$$

We assume throughout that $0.25 \cdot Y^G > \gamma^G$; otherwise the firm invests always regardless of $F_2$. Therefore, the firm will invest if and only if $F_2 < 4\gamma^G$, which is the standard debt overhang of Myers (1977).

\(^\text{14}\) This differs from Diamond (1991), where it is the manager’s non-pleageable control rent combined with the need for outside funding drives the inefficiency. In Diamond (1991), if the manager had deep pockets to refinance the firm, then he would internalize the loss of non-pleageable control rent and eliminate the inefficiency. In this paper’s model without control rent, there is no distinction between existing equity holders/managers or outside equity holders.
3.2.2 Bad State B

Without investment. When bad news about the firm’s asset-in-place hits, $X^B$ may be sufficiently low that the firm cannot successfully refinance the short-term debt at state $B_d$. Formally, we assume that

$$X^B < 2F_1 + F_2,$$

and the firm defaults on its short-term debt at $t=1$. Therefore, without investment, the total debt value (the sum of short-term and long-term) at state $B_d$ at $t=1$ is $0.5 \cdot X^B$, which occurs with probability 0.5.

Now we study state $Bu$. Because we are mainly interested in preserving the first best investment policy, and since investment NPV is moderate, we focus on the situation that $1.5X^B > F_1 + F_2$. This implies that at state $Bu$ both debts are riskless, and as a result equity holders will recover all investment benefit there. Therefore, without investment the total debt value at state $Bu$ is $F_1 + F_2$.

In sum, without investment the total debt value at $t=0$ is

Figure 5: $(F_1,F_2)$ space with different investment policy and $t=1$ debt values $D^{-1}$ in the model.
With investment. Now we calculate the total debt value given investment. The next lemma shows that in order for the firm to invest at \( t=0 \), it must be the case that the firm with investment can successfully refinance the short-term date at state \( Bd \).

**Lemma 1.** Assume that \( \gamma^B < 0.5 \cdot Y^B \). Then the necessary condition for the firm to invest at \( t=0 \) is successful refinancing short-term debt at state \( Bd \).

See the proof in Appendix. Given this lemma, we only need to discuss two cases depending on whether the long-term debt is risky or not.

**Case 1.** When \( Y^B \geq F_2 \) so long-term debt is paid in full in state \( Bdd \). This implies that both short-term debt and long-term debt become riskless, and the total debt value at \( t=0 \) is \( F_1 + F_2 \). Therefore the value increment due to new investment is \( 0.5 \cdot (F_1 + F_2) - 0.25 \cdot X^B \). Comparing to positive NPV \( \gamma^B \), the necessary condition for investment is

\[
F_1 + F_2 \leq 2\gamma^B + 0.5 \cdot X^B.
\]

**Case 2.** When \( Y^B < F_2 \) so the long-term debt only recovers \( Y^B \) at the state \( Bdd \). The total debt value given investment at \( t=0 \) becomes \( F_1 + 0.75 \cdot F_2 + 0.25 \cdot Y^B \). Therefore, relative to (1), the debt value increment due to new investment is

\[
\frac{1}{2} \left( F_1 + F_2 \right) + \frac{Y^B}{4} - \frac{X^B}{4},
\]

and the necessary condition for investment is

\[
\frac{1}{2} \left( F_1 + F_2 \right) + \frac{Y^B}{4} - \frac{X^B}{4} \leq \gamma^B \iff F_1 + \frac{F_2}{2} \leq 2\gamma^B - \frac{Y^B}{2} + \frac{X^B}{2}.
\]

In this case the new investment bails out short-term debt, but cannot make long-term debt holders to be paid in full.

### 3.3 Model Solution

The firm is choosing debt maturity \( (F_1, F_2) \) at \( t=-1 \) to minimize overhang at \( t=0 \). In the following analysis, we are mainly interested in characterizing the optimal debt maturity structure that achieves the maximum \( t=-1 \) target debt value \( D^{\text{target}} \) while preserving the first-best investment policies at both states at \( t=0 \). We illustrate the firm’s investment decisions on the space of debt
structure \((F_1, F_2)\) in Figure 5. As the firm tries to meet the target debt value \(D_{\text{target}}\) at \(t=-1\), we also calculate the total debt value \(D^{-1}\) at \(t=-1\) for all regions.

We have shown that raising \(F_2\) and reducing \(F_1\) at \(t=-1\) is a good idea from the standpoint of state B. However, the analysis in state G implies that the firm cannot raise \(F_2\) too much: there, the riskless short-term debt is harmless, and it is only \(F_2\) that imposes overhang effect. This trade-off determines the optimal interior debt maturity that we are after.

In Figure 5, only region R1 achieves the first best investment in both states. There are two sub-regions, depending on whether \(F_2 > Y^B\), i.e., whether long-term debt given investment is riskless or not. This is also the kink point shown in Figure 5. According to results after Lemma 1 in Section 3.2.2, when \(F_2 \leq Y^B\), both long-term and short-term debt are risk free, with \(t=-1\) value \(D^{-1} = F_1 + F_2\), and the boundary line is \(F_1 + F_2 = 2\gamma^B + 0.5 \cdot X^B\). When \(F_2 > Y^B\), long-term debt only defaults at the state \(Bdd\), and \(D^{-1} = F_1 + \frac{3F_2 + Y^B}{4}\), with the boundary line \(F_1 + \frac{F_2}{2} = 2\gamma^B - \frac{Y^B}{2} + \frac{X^B}{2}\).\(^{15}\)

From Figure 5, we see that the optimal debt maturity structure that achieves the first-best investment policies and the maximum \(t=-1\) target debt value \(D_{\text{target}}\) is the intersection point between \(F_2 = 4\gamma^G\) and \(F_1 + \frac{F_2}{2} = 2\gamma^B - \frac{Y^B}{2} + \frac{X^B}{2}\), i.e., (the asterisk in Figure 5)

\[
F_1^* = \frac{X^B}{2} + 2\gamma^B - \frac{Y^B}{2} - 2\gamma^G, \quad F_2^* = 4\gamma^G.
\]

With this debt maturity structure, the maximum \(t=-1\) debt value that preserves the optimal investment policies is

\[
D_{\text{target},*} = \frac{X^B}{2} + 2\gamma^B + \gamma^G - \frac{Y^B}{4}.
\]

\(^{15}\) Note that in Figure 5 we have implicitly assumed that \(4\gamma^G > Y^B\). This condition also requires that \(Y^G > Y^B\) (more precisely, the investment payoff at state G is greater than that in state B) given our earlier assumption \(4\gamma^G > Y^G\) in Section 3.2.1. This assumption allows for the case that the state-G first-best long-term face \(F_2 = 4\gamma^G\) exceeds the state-B lower bound cash flows \(Y^B\). As a result, even with investment, in state B the long-term debt with \(F_2 = 4\gamma^G\) can still be risky at \(t=2\) given the worst cash flow realization, while short-term debt which gets paid in \(t=1\) becomes riskless. There are other ways, e.g., random investment opportunities (symmetric to both states), to achieve this goal, but we deem that the unnecessary modeling complicity outweighs the potential benefit.
We briefly discuss the situation where the firm’s date -1 target debt level $D_{\text{target}^-} > D_{\text{target},*}$, so that the firm is forced to choose some point in the non-first-best region. Here the trade-off between different states kicks in. If $\gamma^B > \gamma^G$, then the region R2 is chosen where relatively more long-term debt is used to maximize the investment incentives at state B. On the other hand, if $\gamma^B < \gamma^G$, then R3 is chosen where the firm takes relatively more short-term debt to maximize the investment incentives in state G. As our main objective of this paper is to show the trade-off between short-term and long-term debt, the detailed characterization is less interesting.

4. Extensions and Discussions

4.1 Endogenous Costly Default

Costly endogenous default, the extreme version of underinvestment, is just one symptom of short-term debt overhang. Endogenous default has been analyzed in Bulow and Shoven (1978), Black and Cox (1976), and Leland (1994).\(^{16}\) For instance, in Leland and Toft (1996) and He and Xiong (2009b), equity holders, facing a low value of firm assets, might not choose to keep absorbing the financial losses in rolling over maturing debt. As a result, equity holders default, leaving debt holders to bear the bankruptcy cost. Essentially, it is because equity holders do not want to subsidize debt holders, especially the maturing ones.

To see the equivalence, let us examine the endogenous default decision at $t=1$ state $d$ in this model. Ignore the investment decision at $t=0$, and assume a zero liquidation value of the firm. We can interpret it as that bankruptcy involves a substantial dead-weight loss, which essentially implies that default represents severe underinvestment. As shown in Section 3.2, the new financiers can at most recover $0.5 \cdot (X - F_2)$ from the $t=2$ cash flows (suppose that the asset-in-place $X$ at state $du$ is above $F_2$), and therefore are not willing to refinance the maturing short-term debt if and only if

$$0.5 \cdot X < F_1 + 0.5 \cdot F_2.$$

When this condition holds, the firm inefficiently defaults, leaving both long-term and short-term debt worthless. This inefficient default is neither because the firm cannot get fairly priced outside-financing (due to informational problems or financial market disruption), nor because the firm has some non-pledgeable part of future cash flows (a la Diamond (1991)) that new financiers cannot

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\(^{16}\) The interpretation of endogenous default given debt burden as “underinvestment” due to debt-overhang, is mentioned in, for example, Lambrecht and Myers (2008) and He (2009).
internalize. Rather, it is because in order to bail out the firm, the firm/new financiers need to repay its short-term debt fully, and also subsidize the long-term debt holders with $0.5 \cdot F_2$. This reflects debt overhang, as the inefficiency is rooted in the fact there cannot be renegotiations between existing debt holders who demand payment (either immediate as short-term debt, or future as long-term debt), and the firm who makes the default decision.

The interesting point regarding endogenous default is that, when the overhang effect is about failing to attract new financings to avoid firm’s inefficient early default, it is the maturing short-term debt which demands immediate repayment that plays a more significant role. This point is clearly reflected in the bankruptcy threshold $F_1 + 0.5 \cdot F_2$, which puts a greater weight on short-term debt. The intuition is that the short-term debt gets paid sooner than long-term debt, and there are more uncertainties resolved (especially good shocks) in the future to reduce the long-term overhang. More specifically, in bailing out the firm from default, new financiers pay the maturing short-term debt in full, subsidizing them one-to-one. However, by keeping the firm alive the wealth transfer from new financiers to long-term debt holders is typically less than one-to-one (in our example it is the probability $\frac{1}{2}$ of the $du$ realization at $t=2$), because it is possible for the firm to escape the default region tomorrow.

4.2 Discussions on Empirical Predictions

4.2.1 Implications on the crisis in 2007/2008

In many financial crises, short-term debt is often implicated as contributing factor to defaults. However, the literature often emphasizes the “run” by short-term debt (Diamond and Dybvig (1983), Diamond and Rajan (2001), Goldstein and Pauzner (2005), and He and Xiong (2009)). Meanwhile, researchers acknowledge the debt-overhang effect in motivating government bailouts of financial firms, but mostly attribute the overhang effect to existing long-term debt. In contrast, we show that overhang is not a feature of long-term debt exclusively. In fact, the results in Veronesi and Zingales (2009) suggest that the wealth transfer in the recent government bailout of financial firms concentrates on the short-term debt. Moreover, the prediction that short-term debt overhang leads to underinvestment is consistent with the empirical findings in Duchin,

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17 Veronesi and Zingales (2009) evaluate US Treasury secretary Paulson’s plan announced after the Lehman bankruptcy in October 2008. Based on the banks’ CDS prices for debts with different maturities, that paper constructs the Bank Run index, and finds that the index drops dramatically after the announcement of Paulson’s plan. Because the Bank Run index is negatively related to the CDS price of short-term debt, this empirical finding offers supporting evidence to our theory.
Ozbus, and Sensoy (2009) and Almeida et. al (2009) who study firms’ underinvestment during the 2007/2008 crisis.\footnote{Almeida et. al (2009) design a quasi-natural experiment, in which they examine two groups of firms with similar amount of total long-term debt, but with different current portion of what was originally long-term debt when it was issued. In this sense, they are comparing two otherwise identical firms but with different maturity structure. They find that firms with larger current portion of long-term debt cut back investment more than those with smaller current portion, and attribute this result to the disruption of credit market during the 2007/2008 crisis. However, in our model even though the ability to raise new funds from financial markets is perfect, the larger current portion of long-term creates stronger short-term overhang effect, which can also leads firms to cut back their investment. Therefore, their empirical design is not perfect in separating the story of disruption of credit market from that of short-term debt overhang (which is driven by lower firm fundamental).} Therefore, the short-term overhang can be another important contributing factor to financial crises (see also Diamond-Rajan (2009)).

4.2.2 Risky short-term debt

We have shown that short-term debt is not a free solution to debt overhang. Once the firm has experienced a series of negative shocks and short-term debt becomes risky, the short-term debt overhang quickly becomes overwhelming. State differently, short-term debt is helpful only if it is riskless. Because renegotiation (to resolve the overhang effect) is more costly for public debt, this explains that usually firms avoid risky short-term public debt (commercial papers are only issued by safe firms rated P1/P2 or A1/A2), and that risky short-term debt is often renegotiable private (bank) debt.

The above argument suggests that firms’ ex ante optimal maturity choice should ameliorate the short-term debt overhang to a great extent. Although it is true in a static view, sometimes firms with long-term debt might get into unintended/unforeseen situation where they face effectively risky short-term public debt. For instance, in Almeida et. al (2009), the current portion of long-term debt during the 2007/2008 crisis becomes effectively risky short-term debt. Therefore, our analysis has first-order relevance in studying corporations’ behaviors when long-term debt eventually becomes short-term.

4.2.3 Empirical predictions on maturity choice

There are several empirical papers testing the prediction that growth firms (presumably with more investment opportunities) should use more short-term debt. This literature often classifies short-term debts as those with maturity within three years, which fits our timing assumption that investment is made before short-term debt matures quite well. The empirical evidence from this literature has been mixed. For instance, Barclay and Smith (1995) and Guedes and Opler (1996) find a negative relation between maturity and growth opportunities, a result consistent with Myers (1977). However, these studies do not control for firm leverage. Since leverage usually is
positively correlated with maturity, their finding could be due to that growth firms tend to have lower leverage. In contrast, Johnson (2003) argues that maturity and leverage are jointly endogenously determined. Using the standard two-stage instrumental variables regression technique, Johnson finds a positive relation between maturity and growth opportunities. In light of our theory, these mixed results may not be surprising, as early default for growth firms might be more costly which pushes optimal maturity structure toward long-term.

Our theory, by emphasizing the correlation between investment opportunities and asset-in-place values, offers a new perspective in firms’ optimal debt maturity choice. Ignoring costly default, young growth firms---in that they have better investment opportunities when their asset-in-place value is high---should indeed use less long-term debt. This is because relative to short-term debt, long-term debt imposes stronger overhang exactly when the asset-in-place value is high. Furthermore, long-term debt is preferable only for those maturing firms where maintenance (investment when asset-in-place value is low) is particularly valuable.

Last but not the least, our analysis in Section 2.3 implies that countercyclical (percentage) volatilities can lead to a greater short-term debt overhang, even controlling for leverage. Therefore, all else equal, firms/industries with greater countercyclical (percentage) volatilities should use less short-term debt.

4.3 Credit Lines (Revolvers) and Market Based Pricing

In practice, many firms have standing credit lines (also called revolving lines of credit or revolvers) issued by banks. The mechanism of credit line works as an insurance contract: The firm typically pays a fee up front to secure the line, and will later draw down the line if the precommitted rate is below the market one. During the 2007/2008 crisis, the credit line drawdowns accounted for the major part of loans extended by commercial banks (Ivashina and Scharfstein (2009)).

Existing theory about credit lines emphasizes on helping firms overcome liquidity shocks or alleviate risk shifting incentives (e.g., Boot and Thakor (1994)). Our model suggests that credit lines also alleviate short-term overhang by fixing the refinancing cost in bad times. We provide two discussions regarding the debt overhang, credit line, and its recent market-based pricing.

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19 In fact, Stohs and Mauer (1996) control for firm leverage directly (i.e., unfitted value) and also find a significantly positive relationship between maturity and growth opportunities.

20 This argument is more subtle than the one that growth firms require more investment, which is usually invoked in this literature.
4.3.1 Can the seniority of credit line resolve debt overhang?

In practice, the credit lines/revolvers from banks have the distinct feature that draw-downs are often senior to any existing debt. One tends to think that this seniority of credit line should resolve the debt overhang problem completely, because it directly attacks the heart of debt overhang: Banks who issued revolvers do not have to worry that the first dollar out of the new investment goes to existing senior debt holders.

There is a subtle but important difference between the incentives of new financiers (banks in the credit line case) and the incentives of the firm who decides whether to draw down the line. In fact, because banks are already obligated to provide financing if the firm decides to do so, the seniority of drawdowns plays no role at all in the firm’s investment incentives. The only thing that matters in this scenario is the pricing of drawdowns. Given the relatively expensive drawdowns which get to be repaid first later on, the firm may decide not to draw the line if almost all the future investment benefit goes to the bank. For illustration, consider the following extreme example with zero risk-free rate. Because revolvers must be priced to break even when they are issued, usually banks set a future drawdown rate \( r > 0 \) higher than the risk-free rate. Clearly, the firm will decide not to draw the line for the positive NPV projects that yields constant returns below \( r \).

4.3.2 Market based pricing

A recent innovation to the contract of credit-lines/revolvers is market-based pricing; that is, the interest rate of new-drawdowns is partially tied to the firm’s current strength. This is a form of performance-based pricing which is common in the bank debt (e.g., Asquith, Beatty, and Weber (2005)). More specifically, when the firm draws on the remaining line, the drawn spread is positively tied to its credit default swap (CDS) spread.

The repricing of the cost of borrowing against the line leaves the firm’s cost of borrowing higher when the firm’s prospects are bad, much like short-term debt. To the extreme case where the drawdown rate is fully market based, this essentially takes away the insurance that the bank

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21 Of course, the ex post seniority helps the bank set a low drawdown rate ex ante, which alleviates overhang.
22 Typically these revolvers with market-based-pricing specify floor and cap which are the minimum and maximum of the drawn spreads. If the benchmark CDS spread is lower than the floor, the floor applies for the drawn margin. Conversely, if the CDS spread at the time of the draw is higher than the cap, the cap is applied. Finally, if the CDS spread at the time of the draw falls in between the floor and cap, certain formula will apply. Source: Thomson Reuters LPC, Markit on Reuters 3000 Xtra / Credit Views.
23 For a rigorous study of debt with performance-based-repricing and its implications on default, see Manso, Strulovici, Tshistyi (2010).
offered to the firm. As a result, there is no difference between new financiers and the bank, and we are back to the model we have analyzed before.

However, in our model, debt overhang is not all about insurance. The heart of overhang lies in the firm’s investment “incentives,” which can be positively related to market-based pricing.

This point suggests that a properly designed market-based pricing on credit lines is better than credit lines without market-based-pricing. The firm knows that its investment may improve its long-term debt CDS, and this will in turn reduces the firm’s future financing cost. As a result, market-based pricing provides the firm extra incentives to invest at t=0. Essentially, a properly designed market-based-pricing should combine two components. The first is the insurance that protects the firm from those states with deteriorating asset-in-place, and the second is the performance-based sensitivity that entices the firm’s investment. Therefore, the core idea here is similar to optimal contracting with moral hazard, which is to reward/punish the firm for its actions (here, maintenance) but not for fundamental states beyond its actions.

Interestingly, this further suggests that although long-term debt CDS has its own advantages (e.g., more liquid and accurate pricing) over short-term debt CDS to be the market-base in designing the market-based-pricing scheme, the ideal market-base-measure would be state-dependent. In bad times, stronger short-term overhang implies that short-term debt CDS might contain more action-based information than long-term debt CDS does, simply because the wealth transfer due to investment is greater for short-term debt when the firm is close to default.

4.4 Endogenizing Leverage $D_{\text{target}}$ at $t=-1$

Now we endogenize the target date-0 debt value. A tradeoff between saving taxes versus increasing bankruptcy costs will yield a positive value of $D_{\text{target}}$, for standard reasons. There is not much special about this approach in our framework. An alternative is to account for the control role of debt, and of short-term debt in particular. This section describes such an approach. This added structure will prove to be useful in discussion of cash reserve in Section 4.4.

Suppose that at $t=-1$ the entrepreneur owns the patent of the project. To start this project, the entrepreneur with no personal wealth needs 1 dollar of initial investment. He can raise this initial investment through debt or (outside) equity, and becomes the manager of the firm afterwards. We assume that equity holders are soft claims that are subject to renegotiation.

We will use a simple model motivated by Jensen (1986), Hart and Moore (1994), Diamond and Rajan (1999), and Diamond (2004, 2006). It introduces managerial “equity overhang,” where
managers take a fraction $\lambda$ of all free cash flow in excess of debt payments. We follow Diamond (2004, 2006) and assume that default on debt allows the legal system to prevent the manager from consuming any cash they divert instead of paying out or investing (giving the manager nothing if the legal sanction is imposed).$^{24}$ Legal sanctions remove any of the benefit of diversion which occurs that period, but not the benefit of diversion in previous periods. Contracts are written such that this legal sanction is imposed if a debt contract is not paid on its due date. The threat of this legal sanction ensures that debt is paid when cash is available, and because debt cannot be renegotiated, the default automatically imposes the sanction. However equity contracts (which are soft) do not have automatic sanctions so that equity holders have no right to impose the legal sanction for default. We simply assume that the manager can take a fraction $\lambda$ of remaining cash flows, which can be motivated by that the manager is able to directly divert all free cash and retain a fraction $\lambda$ of it (while destroying a fraction $1-\lambda$) in a way that cannot be verified or recovered without legal sanctions. As a result, owners of outside equity allow the manager to take a fraction $\lambda$ of current free cash flow each period, if the manager so desires, given its effect on his current or future payoffs.$^{25}$

To recap, at $t=-1$ the firm raises equity and debt to carry out the initial investment.$^{26}$ At $t=0$ the firm requires new investment as analyzed in Section 3, and at $t=1$ the refinancing decision is controlled by the existing shareholders. Because issuing any new equity only benefits the entrepreneur and dilutes their own value, the new financing is in the form of (junior) debt. And at $t=2$ the manager can get (at least) $\lambda$ fraction of free cash flows after the debt payment.

Denote by $M_{-1}$ the value of entrepreneur/manager, $E_{-1}$ the value of (outside) equity, and $D_{-1}$ the value of total long-term and short-term debt, all evaluated at $t=-1$. The agency problem at $t=2$ implies that $M_{-1} \geq \frac{\lambda}{1-\lambda} E_{-1}$, i.e., the manager has to have sufficient inside stake for him to behave. The initial investment requires that $E_{-1} + D_{-1} \geq 1$; if this inequality holds strictly then the entrepreneur/manager can consume the difference at date -1. Therefore the entrepreneur’s date -1

$^{24}$ This model is a much simplified of that in Diamond (2006). We simplify by assuming that no debt can be renegotiated and that the legal sanctions of debt default completely eliminate proceeds from managerial diversion on the date when default occurs.

$^{25}$ So essentially we are modeling outside equity in this setup. See Myers (2000) for another way of modeling.

$^{26}$ We assume that the manager can commit to invest funds properly on the instant they are received, but cannot commit not to divert cash flows obtained in excess of immediate investment needs or cash flows obtained from the returns to investments.
value is \( M_{-1} + (E_{-1} + D_{-1} - 1) \). Finally, denote by \( v(D_{-1}) \) the firm value as a function of \( D_{-1} \) (which is an decreasing function) where the firm value is determined by the optimal maturity structure studied in Section 3, and the accounting identity implies that \( M_{-1} + E_{-1} + D_{-1} = v(D_{-1}) \). Therefore, the manager who chooses the \( t=-1 \) financial structure solves the following problem:

\[
\max M_{-1} + (E_{-1} + D_{-1} - 1)
\]

\[
\text{s.t. } E_{-1} + D_{-1} \geq 1, M_{-1} \geq \frac{\lambda}{1-\lambda} E_{-1}, E_{-1} + M_{-1} + D_{-1} = v(D_{-1})
\]

**Proposition 3.** Assume that \( (1-\lambda)v(0) < 1 \) and \( v(1) > 1 \). Then the optimal date-0 debt value \( D_{\text{target}} \in (0,1) \) is the smallest solution to the equation \( D_{-1} + (1-\lambda)(v(D_{-1}) - D_{-1}) = 1 \).

The first restriction \( (1-\lambda)v(0) < 1 \) implies that using outside equity only cannot raise enough capital to cover initial investment; and the second condition \( v(1) > 1 \) implies that it is feasible to raise the entire investment capital by debt. Then the optimal date 0 debt will be an interior solution. Finally, the equation in Proposition 1 simply says that the debt holders and outside equity holders (who have a \( 1-\lambda \) fraction of total equity value \( v(D_{-1}) - D_{-1} \)) contribute the entire initial investment 1 (as the manager has zero initial wealth.)

### 4.5 Cash Reserve?

One potential solution to the debt overhang is that the firm maintains cash reserves. We will first investigate the role of cash reserve by ignoring the agency issue of managerial diverting that we introduced in Section 4.4, then discuss the interesting interaction between the managerial diverting and debt overhang.

#### 4.5.1 State-contingent maturity and callable long-term debt

It is clear that raising cash reserves that are not subject to agency problems (diverting, dividend payout, etc.), while holding total debt issuance fixed, could alleviate debt-overhang—simply because we can interpreted cash as negative debt. The more interesting question is, can the firm reduce overhang by issuing more debt at \( t=-1 \), say \( D_{-1} + C \) where \( C > 0 \), and keeping \( C \) inside the firm as cash reserve? The answer is yes.

In our model, if the firm can issue debt with state-contingent maturity, then the optimal contract will be short-term debt in state G (so there is no overhang) and long-term debt in state B (so there is less overhang). This result indicates that the callable feature of long-term debt can
help on this dimension, because the firm who issued callable long-term debt at \( t=-1 \) can choose to call these debt at \( t=1 \) if state \( G \) realizes.\(^{27}\) Of course, in order to motivate the firm to call back the debt in full, the call price should be at a discount, i.e., below the long-term debt value given investment. Otherwise, the same extent of wealth transfer suggests that the firm will decide to do nothing. Also, a pre-determined call price cannot deal with random investment opportunities.

Interestingly, the state-contingent callable feature can also be generated by a cash reserve. It is because cash allows the firm to have a state-contingent repayment policy. Specifically, in bad states, the firm can use the cash to pay part of short-term debt at \( t=1 \), while in good states the firm will save these cash and use them to pay part of long-term debt at \( t=2 \). Therefore, this state-contingent repayment policy help the firm transform some long-term (short-term) debt to short-term (long-term) debt in state \( G \) (\( B \)), which is value improving in this model.

### 4.5.2 Debt overhang on manager’s diverting decision

Of course, the very reason to have debt in the first place, as discussed in Section 4.3, is because the manager can divert some of the firm’s free cash flows. This militates against having the firm pile up extra cash, because for any dollar that sitting in the firm at \( t=1 \) in excess of short-term debt payment, the manager can divert it to obtain \( \lambda \) at \( t=1 \) (and the date 2 default is too late to recover it).\(^{28}\) Interestingly, different from the standard argument (e.g., DeMarzo and Fishman (2007)) that the manager’s inside equity stake \( \lambda \) will prevent him from diverting the cash at the interim date \( t=1 \) (because he can get \( \lambda \) fraction of free cash flows at \( t=2 \)), in our setting the manager will strictly prefer to divert at \( t=1 \), if the outside equity holders cannot promise to the manager more than \( \lambda \) fraction of \( t=2 \) free cash flows. The reason is just debt overhang: the manager understands that the cash left in the firm goes to the debt holders first at \( t=2 \), and therefore he has a strict incentive to divert at today rather than wait to share the cash tomorrow.

For illustration, suppose that there is one dollar of free cash flow in the firm at \( t=1 \), and in our model at \( t=2 \) the firm is solvent with probability \( \frac{1}{2} \). Then diverting today gives the manager a utility of \( \lambda \), while the expected value by waiting to share the free cash flows with outside equity holders at \( t=2 \) is only \( \lambda/2 \). Here, the manager will divert earlier along the equilibrium path because the debt coming due tomorrow hangs over his own “inside-equity.” The bottom line is,

\(^{27}\) Bodie and Taggart (1978) suggest that callable long-term debt can alleviate overhang. However, Bodie and Taggart (1978) still deem it as a puzzle why firms do not simply roll over their short-term debt, which suggests that the authors do not realize that roll over short-term debt can impose stronger overhang in some states.

\(^{28}\) Here, for the sake of argument, we only consider the manager’s diverting incentives at \( t=1 \).
holding extra cash does not overcome the short-term debt overhang of injecting new cash, because the manager’s payoff is very much like that of newly-injected equity.

5. Conclusion

Debt overhang influences the investment and default decisions of those whose claims are like equity. Long-term debt causes overhang, because it prevents equity from receiving any payoff from investment when the ex-post payoff is low enough that there is a default. Short-term debt with the possibility of default can impose even greater overhang, simply because there is less uncertainty resolved over the shorter time until it matures, and as a result most part of the initial increase in value (due to investment or bailout to avoid default) will not result in any payoff to equity. Short-term debt shares less risk (which includes both losses and gains) with equity than long-term debt does. As a result, short-term debt then imposes either no overhang (if riskless it leaves all marginal returns to equity) or large overhang (if likely to default, it leaves very little marginal returns to equity). This implies that a given issue of risky short-term debt has an overhang effect that fluctuates more than that of long-term debt as assets in place fluctuate in value after debt is issued. In addition, even the prospect of future fluctuations in value can lead short-term debt to impose stronger overhang than long-term debt. Similar to a down-and-out option, short-term debt eliminates greater investment incentives when the volatility of firm asset values is higher after interim bad news. The timing of debt maturity can have a major impact on investments, especially on investments that can help avoid default.

The problems caused by large impending debt maturity go beyond the risk of runs and limited access to liquidity. The timing of repayments, access to lines of credit, and the pricing of credit lines all combine to either amplify or reduce the risks of potential default.

Appendix

A.1 Proof for Proposition 1

Without loss of generality we set $\sigma^2 = 1$, and we can easily calculate the overhang difference as

$$\text{OH}(V_0, F_1, 1) - \text{OH}(V_0, F_2, 2) = N\left(\ln F_1 - \ln V_0 + 0.5\right) - N\left(\frac{\ln F_2 - \ln V_0 + 1}{\sqrt{2}}\right).$$

When $V_0 \to 0$ so that

$$\ln V_0 \to -\infty, \ln F_1 - \ln V_0 + 0.5 > \frac{\ln F_2 - \ln V_0 + 1}{\sqrt{2}},$$

and therefore $\text{OH}_1 > \text{OH}_2$. Similarly we have
\( OH_1 < OH_2 \) when \( V_0 \to \infty \). And, since there is a unique solution \( \hat{V}_0 = \frac{\sqrt{2} \ln F_1 - \ln F_2 + \sqrt{2}/2 - 1}{\sqrt{2} - 1} \)

so that \( OH(V_0, F_1, 1) = OH(V_0, F_2, 2) \), our claim follows. Q.E.D.

A.2 Proof for Proposition 2

Consider \( V_2 = V_0 \exp \left( \tilde{X}_1 - 0.5 + \tilde{X}_2 - 0.5 \tilde{\sigma}_2^2 \right) \), where \( Q = 0 \). Without conditional volatility, \( \tilde{\sigma}_2 = \sigma = \varepsilon \), while with conditional volatility, we have

\[
\tilde{\sigma}_2 = \begin{cases} 
0 & \text{when } \tilde{X}_1 > 0 \\
\sigma = \varepsilon & \text{when } \tilde{X}_1 \leq 0
\end{cases}
\]

where \( \varepsilon \) is sufficiently small. For either case, when \( \sigma = 0 \), the second period adds no risk, and as a result long-term debt is identical to short-term debt. As a result, both debts have the same value and overhang. We will set \( F_2 = \exp(-0.5) \), so that potential second period noise occurs exactly when the long-term debt is at the money. As we have seen in the discussion, this gives the best chance to have smaller long-term overhang.

Our road map is as follows. We consider a perturbation from \( \sigma = 0 \) to \( \sigma = \varepsilon > 0 \), and comparing the change of overhang effects on each debt. Of course we need to control for debt value, and we will adjust \( F_1 \) (which is much easier) to ensure the same debt value. Then we can check the following object (where we denote debt overhang by \( OH_i = 1 - \Delta_i \)):

\[
\frac{dOH_1}{d\sigma} = \frac{dOH_2}{d\sigma} = \frac{dOH_1}{dF_1} \frac{dF_1}{d\sigma} - \frac{dOH_2}{dF_1} \frac{dF_1}{d\sigma} = \frac{dOH_1}{dD_1/d\sigma} - \frac{dOH_2}{dD_1/d\sigma}
\]

(2)

We aim to show that in the first case (2) is negative, and in the second case (2) is positive. This implies our result, because when \( \sigma = 0 \) both debts have the same overhang.

Without conditional volatility, short-term debt and long-term debt differ only by the fact that short-term debt has an asset variance of \( \sigma_1^2 = 1 \), while the long-term debt has an asset variance of \( \sigma_1^2 + \sigma_2^2 = 1 + \varepsilon^2 \). By writing the asset variance as \( \Sigma \), we have

\[
D = N \left( \frac{\ln F_1 - 0.5 \Sigma}{\sqrt{\Sigma}} \right) + FN \left( -\frac{\ln F + 0.5 \Sigma}{\sqrt{\Sigma}} \right), \quad OH = N \left( \frac{\ln F + 0.5 \Sigma}{\sqrt{\Sigma}} \right).
\]

Therefore, evaluating at the point of \( \Sigma = 1 \),

\[
\frac{dOH_1}{dF_1} = n(0) \frac{1}{F_1}, \quad \frac{dD_1}{dF_1} = n(0), \quad \frac{dD_2}{\Sigma} = -F_2 n(0), \quad \frac{dOH_2}{\Sigma} = n(0)
\]
Then by the fact that \( F_1 = F_2 = \exp(-0.5) \), and \( n(0) < N(0) \), plugging in (2) we obtain our first result
\[
\frac{dOH_1}{d\sigma} < \frac{dOH_2}{d\sigma}.
\]

Now we consider the second case with conditional volatility. We will show that raising \( \sigma \) from 0 has no first order effect on long-term debt value \( D_2 \), i.e.,
\[
\frac{dD_2}{d\sigma}\bigg|_{\sigma=0} = 0 \quad \text{while} \quad \frac{dOH_2}{d\sigma}\bigg|_{\sigma=0} < 0.
\]
Therefore, since \( dD_1/dF_1 > 0 \), we obtain our result. To show these results, we have (note that \( Q = \ln F_2 + 0.5 \))
\[
D_2(\sigma) = F_2 \int_{-\infty}^\infty n(x) dx + \int_{-\infty}^0 \left\{ \int_{-\infty}^{-\frac{x^2}{\sigma^2}} \exp\left(x - \frac{1 + \sigma^2}{2} + y\right) n\left(\frac{y}{\sigma}\right) \frac{1}{\sigma} dy + F_2 \int_{-\infty}^{\frac{x^2}{\sigma^2}} n\left(\frac{y}{\sigma}\right) \frac{1}{\sigma} dy \right\} n(x) dx
\]
\[
= F_2 \int_{0}^\infty n(x) dx + F_2 \int_{0}^0 \left\{ \exp(x) \int_{-\infty}^{-\frac{x^2}{\sigma^2}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left( -\left(\frac{y-x^2}{2\sigma^2} \right) \right) dy + \int_{-\infty}^{\frac{x^2}{\sigma^2}} n\left(\frac{y}{\sigma}\right) \frac{1}{\sigma} dy \right\} n(x) dx
\]
\[
\text{let } x = \frac{y-x^2}{\sigma} \rightarrow \frac{\sigma}{\sigma^2} \int_{-\infty}^\infty \exp(s) \int_{-\infty}^{\frac{\sigma}{\sigma^2}} n(t) dt + \int_{\frac{\sigma}{\sigma^2}}^\infty n(t) dt \right\} n(x) dx
\]
Therefore
\[
\frac{dD_2(\sigma)}{d\sigma} = F_2 \int_{-\infty}^\infty \left\{ \exp(x) n\left( -\frac{x}{\sigma} + \frac{\sigma}{2}\right) \left( \frac{x}{\sigma^2} + \frac{1}{2} \right) - n\left( -\frac{x}{\sigma} + \frac{\sigma}{2}\right) \left( \frac{x}{\sigma^2} - \frac{1}{2} \right) \right\} n(x) dx
\]
\[
= F_2 \int_{-\infty}^\infty \left\{ n\left( -\frac{x}{\sigma} + \frac{\sigma}{2}\right) \left( \frac{x}{\sigma^2} - \frac{1}{2} \right) - n\left( -\frac{x}{\sigma} + \frac{\sigma}{2}\right) \left( \frac{x}{\sigma^2} + \frac{1}{2} \right) \right\} n(x) dx
\]
\[
= -F_2 \int_{-\infty}^0 n\left( -\frac{x}{\sigma} + \frac{\sigma}{2}\right) n(x) dx
\]
\[
= -\sigma F_2 \int_{-\infty}^0 n\left( -u + \frac{\sigma}{2}\right) n(\sigma u) du
\]
Which is zero when \( \sigma = 0 \). However, since
\[
OH_2(\sigma) = 1 - \int_{0}^\infty n(x) dx - \int_{-\infty}^0 \int_{-\infty}^{x^2/\sigma^2} n\left(\frac{y}{\sigma}\right) \frac{1}{\sigma} dy n(x) dx = \int_{-\infty}^0 n(x) dx - \int_{-\infty}^0 \int_{-\infty}^{x^2/\sigma^2} n(t) dt n(x) dx
\]
And the first order effect on overhang by raising \( \sigma \) is
\[
\frac{d\text{OH}_2(\sigma)}{d\sigma} = \int_{-\infty}^{0} n\left(\frac{-x + \sigma \sigma}{2}\right) n(x) dx = \int_{-\infty}^{0} n\left(\frac{-x}{\sigma} + \frac{\sigma}{2}\right) n(x) \frac{1}{\sigma} dx
\]

\[
= \int_{-\infty}^{\sigma} n(-t + \sigma) \tan\left(\sigma t - \frac{\sigma^2}{2}\right) dt \quad \text{let } \sigma = 0 \quad \frac{1}{\sqrt{2\pi}} < 0.
\]

Therefore we proved our second result. QED.

A.3 Proof of Lemma 1

Suppose not. At Bd the total debt value is the firm value \(0.5 \cdot X^B + Y^B\), and at Bu the debt holders get paid in full. Relative to the situation without investment, this implies that standing at \(t=0\) the debt holders gain by \(0.5 \cdot Y^B\). Given that \(\gamma^B < 0.5 \cdot Y^B\), equity holders cannot break even. QED.

A.4 Proof of Proposition 3

The manager’s optimization problem is equivalent to \(\max v(D) \text{ s.t. } E + D \geq 1, \text{ and } M \geq \frac{\lambda}{1-\lambda} E\), where we use D, E, M to indicate their \(t=-1\) value. Because \(v(D)\) is decreasing in \(D\), the first constraint is binding; otherwise lowering \(D\) improves. Then the second constraint is binding as well, because otherwise a higher \(M\) raises \(E\) which in turn reduces \(D\). As a result, we have that

\[
\frac{1}{1-\lambda} E + D = v(D) \quad \Rightarrow D + (1-\lambda)(v(D) - D) = 1.
\]

Let \(Q(D) = D + (1-\lambda)(v(D) - D)\). Under our assumptions, \(Q(1) > 1\) and \(Q(0) < 1\). Also, as we have seen in Figure 5, if there is any discontinuity on \(v(D)\), it only jumps downward where certain efficient investment starting to be cut. Hence, \(Q(D)\) only jumps downward, and there always exists a solution \(D \in (0,1)\) to the above equation (take the smallest one if multiple solutions exist). Q.E.D.
References


