

# A Theory of Debt Maturity: The Long and Short of Debt Overhang

Douglas W. Diamond and Zhiguo He<sup>1</sup>

April 2010

## Abstract

Maturing risky short-term debt can impose a stronger debt overhang effect than long-term does, distorting the firm's investment decisions. We derive the optimal maturity structure based on the trade-off between long-term overhang in good times and (stronger) short-term overhang in bad times. The theory has implications on empirical studies of debt maturity structure, understanding the excessive defaults and underinvestment during recessions, market-based pricing of credit lines, and firm's cash holdings.

**Key words:** Wealth transfer, short-term debt crisis, underinvestment, endogenous default.

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<sup>1</sup> Booth School of Business, University of Chicago, and NBER; and Booth School of Business, University of Chicago. We thank seminar participants at MIT Sloan, OSU Fisher, Chicago Booth, Nittai Bergman, Hui Chen, Gustavo Manso, Gregor Matvos, Victoria Ivashina, Henri Pages, Raghu Rajan, Berk Sensoy, and Rene Stulz for insightful comments.

# 1. Introduction

Myers (1977) shows that risky debt that matures in the future leads to underinvestment today. The insight is that part of the cash flows generated by investment goes to debt holders at maturity; unfortunately the equity holders who make the investment decision will not internalize this benefit. The truncation of cash flows (and implied sharing of them) can distort investment incentives. Myers (1977) therefore suggests the solution of short-term debt to the debt overhang problem, because if all debt matures before the investment opportunity, the firm can make the investment decision as if an all-equity firm.

The short-term debt in Myers (1977) is better viewed *as debt that has matured (safely) yesterday, rather than the one that is going to mature soon*. We stress the importance of *timing* here, as this paper studies the situation where the firm is making investment decisions given both short-term and long-term debt *maturing in the future*. To our best knowledge, this empirically relevant situation has not been explored in the literature.

Our study focuses on the maturity effect of debt overhang on the firm's long-term investment decisions. The new element relative to Myers (1977) is the short-term debt that is going to mature but before the firm's investment decision. Specifically, in our model at date 0 the firm is considering to take a long-term investment opportunity, i.e., its cash flows will realize at date 2. However, the firm has not only the long-term debt which matures at date 2, but also the short-term debt maturing at date 1 which requires refinancing. Since new investment involves wealth transfers to both debts, both long-term and short-term debts may impose overhang effect on the investment at date 0. Interestingly, we find that the relative strength of overhang effects caused by the long-term and short-term debt is state-contingent, where the state is the firm's asset-in-place (or can be equivalently interpreted as the firm's fundamental).

We start our analysis in Section 2 by studying one example. At date 0, the firm is considering a positive NPV project, facing the debt maturity that has been determined before date 0. Suppose that the firm takes either exclusively short-term or long-term debt. We consider two cases where the only difference is the value of the firm's asset-in-place.

In the first case, the asset-in-place has a relatively high value. As a result, the short-term debt can be easily refinanced, and therefore is riskless even without investment. It immediately implies that short-term debt holders, who will be paid in full even without investment, cannot extract any additional value from new investment. In contrast, if the firm has used long-term debt

instead, then the long-term debt is risky due to higher uncertainty in the long-run. Because new investment benefits long-term debt holders, the presence of risky long-term debt reduces the firm's investment incentives at date 0. This is exactly the standard overhang effect in Myers (1977).

The second case is more interesting, where we set a lower asset-in-place value. There, without investment, the firm has trouble in refinancing the short-term debt, and as a result the short-term debt becomes risky. Of course, if the firm instead has used long-term debt, the long-term debt will be risky as well. However, in sharp contrast to the first case with riskless short-term debt, now the risky short-term debt imposes stronger overhang than the risky long-term debt. In other words, the investment at date 0 benefits the short-term debt much more than the long-term debt. This finding implies that short-term debt is not a free lunch in coping with debt overhang: even though riskless short-term debt (when the firm's asset-in-place value is high) impose no overhang, risky short-term debt (when the firm's asset-in-place deteriorates) does more evil than the long-term debt in dwarfing the firm's investment incentives. Therefore, the firm faces a trade-off in choosing the optimal debt maturity structure before the state containing about asset-in-place realizes.

The economic mechanism underlying the stronger short-term overhang is simple. The core of debt overhang is about wealth transfer from equity holders to debt holders. Because short-term debt is going to be paid earlier than the long-term debt, there is less uncertainty resolved over the shorter time, and the wealth transfer to potentially risky short-term debt holders (through new investment) will be greater. Imagining the extreme case where the firm with short-term debt is forced to default at date 1 even with investment. Then, short-term debt holders take the firm over and claim all the investment benefits from that point on. In contrast, if the firm has used long-term debt instead, then there is no default at date 1, and the firm can recoup some investment benefit at date 2 if firm's asset-in-place improves.

Section 3 presents a formal model which endogenizes the debt maturity structure based on the trade-off illustrated above. There, at date -1 the firm can use a mix of short-term and long-term debt, and at date 0 the firm faces two states of the nature (G as Good, or B as Bad) with different asset-in-place value and investment opportunities. We are mainly interested in the debt maturity structure that achieves the highest date -1 debt value while preserving the first-best investment policies in both states at date 0. Not surprisingly, the desired optimal maturity

structure is an interior solution in balancing the long-term debt overhang in state G and (stronger) short-term debt overhang in state B.

In our model, the interpretation of “investment” can be spending capital to either establish new projects, or keep old projects alive. For instance, “maintenance” is a form of investment that requires capital expenditure to keep the existing project operating efficiently, and “foregoing maintenance” represents underinvestment. The extreme version of “foregoing maintenance” is just “default,” where the firm essentially gives up the old project. In fact, the endogenous default, which usually occurs given a sufficiently low asset-in-place value, is a symptom of short-term debt overhang (see Leland (1994) and related literature on endogenous default).<sup>2</sup>

To see this point clearly, imagine a firm who has difficulty in refinancing short-term debt at date 1 by raising fresh capital against existing assets. The new financiers understand that to prevent the firm from defaulting, the fresh capital that they put in pays the maturing short-term debt first, resulting in a full subsidy to short-term debt holders. It is true that bailing out the firm also benefits the long-term debt holders who have claim on the firm’s future cash flows. However, the subsidy to long-term debt holders will not be as large, because new financiers can recoup some benefit if the firm recovers tomorrow. The differential intensity of wealth transfer (to debt holders) across different maturity just reflects the stronger short-term debt overhang when the short-term debt becomes risky. Consequently, this wealth transfer distorts the equity holder’s default decision, leaving debt holders to absorb the potential bankruptcy cost.

Debt overhang has been an active research topic since Myers (1977).<sup>3</sup> This paper emphasizes the role of debt maturity on debt overhang. We show that the short-term overhang effect is in a zero-one nature. To be precise, when the firm’s asset-in-place value is high and short-term debt is riskless, short-term debt imposes no overhang effect. However, once the firm has experienced a series of negative shocks and short-term debt becomes riskless, the short-term debt overhang quickly becomes overwhelming. This in turn could lead to sizable underinvestment or even default.

In many financial crises, short-term debt is often implicated as contributing factor to defaults. However, often times it is attributed to “run” caused by short-term debt (Diamond and Dybvig (1983), Diamond and Rajan (2001), Goldstein and Puzner (2005), and He and Xiong (2009a)).

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<sup>2</sup> The interpretation of endogenous default given debt burden as “underinvestment” due to debt-overhang, is mentioned in, for example, Lambrecht and Myers (2008) and He (2009).

<sup>3</sup> To name a few recent studies on debt overhang, see Hennessy (2004) and Philippon and Schnabl (2009).

Meanwhile, researchers acknowledge the debt-overhang effect in bailing out financial firms, but mostly attribute the overhang effect to existing long-term debt. By showing that the distortion due to overhang is not a feature of long-term debt exclusively, this paper suggests that the short-term overhang can be another important contributing factor to the current crisis. In fact, the results in Veronesi and Zingales (2009) suggest that the wealth transfer in the recent government bailout of financial firms concentrates on the short-term debt.<sup>4</sup>

Moreover, the prediction that short-term debt overhang leads to underinvestment is consistent with the empirical findings in Duchin, Ozbus, and Sensoy (2009) and Almeida et al (2009) who study firms' underinvestment during the 2007/2008 crisis. Almeida et al (2009) design a quasi-natural experiment, in which they examine two groups of firms with similar amount of total long-term debt, but with different current portion of what was originally long-term debt when it was issued. In this sense, they are comparing two otherwise identical firms but with different maturity structure. They find that firms with larger current portion of long-term debt cut back investment more than those with smaller current portion, and attribute this result to the disruption of credit market during the 2007/2008 crisis. However, in our model even though the ability to raise new funds from financial markets is perfect, the larger current portion of long-term creates stronger short-term overhang effect, which can also leads firms to cut back their investment/maintenance. Therefore, their empirical design is not perfect in separating the story of disruption of credit market from that of short-term debt overhang (which is driven by lower firm fundamental).<sup>5</sup>

Our theoretical results have important implications for empirical testing of the Myers (1977) debt-overhang theory, especially the prediction that growth firms (presumably with more investment opportunities) should have more short-term debt in their debt maturity structure. The existing empirical evidence on this prediction has been mixed. For instance, Barclay and Smith (1995) and Guedes and Opler (1996) find a negative relation between maturity and growth opportunities, a result consistent with Myers (1977). However, these studies do not control for

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<sup>4</sup>Veronesi and Zingales (2009) evaluate US Treasury secretary Paulson's plan announced after the Lehman bankruptcy in October 2008. Based on the banks' CDS prices for debts with different maturities, that paper constructs the Bank Run index, and finds that the index drops dramatically after the announcement of Paulson's plan. Because the Bank Run index is negatively related to the CDS price of short-term debt, this empirical finding offers supporting evidence to our theory.

<sup>5</sup> As supporting evidence to the credit supply story, Almeida et al (2009) show that their results do not hold in non-crisis times. But since non-crisis times might coincide with high fundamental episodes, this evidence lacks the power to tell the credit supply channel apart from the short-term overhang channel.

firm leverage. Since leverage usually is positively correlated with maturity, their finding could be due to that growth firms tend to have less leverage. In contrast, Johnson (2003) acknowledges that maturity and leverage are jointly endogenously determined in reality. Using the standard two-stage instrumental variables regression technique, Johnson finds a *positive* relation between maturity and growth opportunities.<sup>6</sup> In light of our theory, these mixed results are not surprising, because 1) early default for growth firms might be more costly which pushes optimal maturity structure toward long-term, and 2) the presence of short-term and long-term debt has state-contingent overhang effect on investment.

The optimal maturity structure in our paper is based on the trade-off due to long-term and short-term overhang. It is different from existing theories of optimal maturity structure which focus on the disciplinary role to short-term debt in curbing the managers' asset substitution or other misbehavior, e.g., Calomiris and Kahn (1991), Diamond and Rajan (2001), Flannery (1994) and Leland (1998), or private information of borrowers about their future credit ratings, e.g., Flannery (1986) and Diamond (1991).<sup>7</sup> Benmelech (2006) argues that entrenchment can lead the self-interested manager to take long-term debt, and provide supporting evidence for this theory. He and Xiong (2009b) study the impact of bond market illiquidity on credit risk. Although they also point out the overhang effect on the firm's endogenous default is stronger when rolling over short-term debt, the trade-off is different as in their model the benefit of short-term debt comes from its greater liquidity in the secondary market.

The rest of paper is organized as follows. We give an example in Section 2 which illustrates the key intuition of this paper. Section 3 gives the formal model, and Section 4 provides discussion and extensions. In Section 5 we conclude. All proofs are placed in Appendix.

## **2. An Example and Brief Discussion**

### **2.1 An Example**

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<sup>6</sup> In fact, Stohs and Mauer (1996) control the firm leverage directly (i.e., unfitted value) and also find a significantly positive relationship.

<sup>7</sup> There are other studies on firm's debt maturity structure. Hart and Moore (1994, 1998) study the optimal debt maturity that persuades the entrepreneur to pay out cash flows; their analysis is based on renegotiation, which is ruled out in this paper for debt overhang to exist in the first place. Brunnermeier and Yogo (2009) stress the option value of using short-term financing so that the firm can readjust the debt maturity *before* the firm has experienced sufficiently negative shocks. In our model, the short-term debt matures *after* the sufficiently negative shock arrives. And, Brunnermeier and Oehmke (2009) study the maturity rat race between creditors; there, short-term creditors impose negative externalities on long-term creditors, leading to excessive short-term debt in equilibrium.

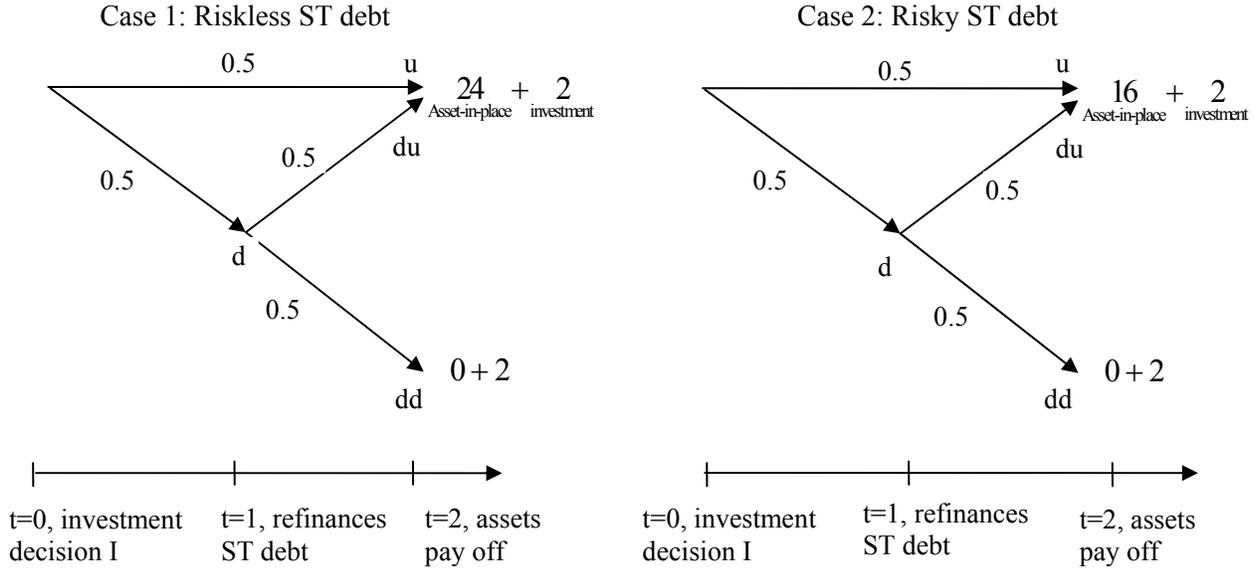


Figure 1: An example with two cases. Case 1 (2) has a high (low) asset-in-place value so the short-term debt will be riskless (risky).

The following example illustrates the main intuition of this paper. We begin our analysis at  $t=0$ , where the firm has some asset-in-place, and is considering making a new investment which requires new outside financing (as the firm has no cash holdings). The agents are risk neutral and interest rate is zero.

At  $t=0$  the equity holders face a debt maturity structure which was determined at  $t=-1$  (this ex ante decision will be studied in Section 3). Short-term debt will be maturing at  $t=1$ , while long-term debt will be maturing at  $t=2$ . Since there are no interim cash flows, short-term debt needs refinancing at  $t=1$ . To study debt overhang, the existing debt must be senior to any new financings. We rule out renegotiation, and bankruptcy (if occurs) cost is zero.

To clearly see the trade-off, let us consider the case where the firm is allowed to take either exclusively short-term or long-term debt. The full analysis in Section 3 will consider the optimal mix of long-term debt and short-term debt. Throughout the example, the short-term debt which matures at  $t=1$  has a face value of  $F_1=10$ , and the long-term debt which matures at  $t=2$  has a face value of  $F_2=12$ .

The firm's asset-in-place follows the tree depicted in Figure 1. With probability 0.5 the state  $u$  occurs at  $t=1$ , and there is no future uncertainty. With probability 0.5 the state  $d$  occurs at  $t=1$ ;

following that, at  $t=2$  the state might be  $du$  or  $dd$  with equal probabilities. For simplicity, asset-in-place has the same positive value in both state  $u$  and  $du$ , which occurs with a total probability of 0.75. The left (right) panel has a higher (lower) value of final realization of asset-in-place. In the state  $dd$ , which occurs with probability 0.25 standing at  $t=0$ , the asset-in-place has zero value. This setup implies that long-term debt is always risky, while short-term debt can be either risky or riskless.

Suppose that the new investment has a cost of  $I < 2$  at  $t=0$  leads to a constant payoff of 2 at  $t=2$ , which implies a positive NPV of  $2-I$ . When equity holders invests at  $t=0$ , debt overhang implies that part of the investment payoffs goes to the debt holders, and it is this wealth transfer that dwarfs the equity holders' investment incentives. Equivalently, this wealth transfer is reflected in the value increment of debt (either short-term or long-term) due to the new investment. Since there is no other dead-weight loss in this example, equity holders will invest if and only if the NPV of this investment, i.e.,  $2-I$ , is greater than the wealth transfer to the debt holders.

The extent of short-term overhang critically depends on whether the short-term debt is risky or not, which in turn depends on the profitability of the asset-in-place. In the first (second) case, the asset-in-place has a relatively high (low) value and the short-term debt is riskless (risky). Throughout this paper we say the debt is risky if the firm's asset-in-place is insufficient to pay debt holders in full, *given no investment opportunities*. We adopt this convention because we are interested in the firm's investment incentives once it faces investment opportunities.

### **Case 1: Riskless short-term debt in the left panel of Figure 1**

**Short-term debt only.** The short-term debt with face value 10 can be easily refinanced even at the state  $d$ , as the asset-in-place there has a market value of 12. Therefore the short-term debt is riskless and has a market value of 10. Because the value of riskless short-term debt cannot be improved further, the wealth transfer from equity holders to debt holders is zero if equity holders take the investment.

**Long-term debt only.** Long-term debt is risky in this example, which leads to debt overhang. Without investment, the long-term debt value is  $0.75 \cdot 12 + 0.25 \cdot 0 = 9$ . But with investment, the long-term debt value increases to  $0.75 \cdot 12 + 0.25 \cdot 2 = 9.5$ . As a result, the wealth

transfer to long-term debt involved in the new investment is 0.5, which implies that equity holders will pass on investment opportunities with NPV less than 0.5.

This example illustrates the overhang effect discussed in Myers (1977), and shows that shortening maturity can help resolve debt overhang. Note that here riskless short-term debt features a higher leverage than long-term debt, which implies that long-term overhang effect stems from maturity rather than leverage.

### **Case 2: Risky short-term debt in the right panel of Figure 1**

**Short-term debt only.** Without investment, the short-term debt with face value 10 cannot be refinanced in state  $d$ , as the asset-in-place only has a value of 8. Therefore the firm will default, and the short-term debt holders who take the firm over receive 8 there. On the other hand, the short-term debt gets refinanced at state  $u$ . As a result, the date-0 market value of short-term debt is  $0.5*8+0.5*10=9$ .

How much does the short-term debt gain if the firm invests? Here, if the firm invests at  $t=0$ , at state  $d$  the total firm value becomes  $0.5*(16+2)+0.5*2=10$ , so that the short-term debt just gets refinanced successfully at state  $d$ . Therefore, the investment makes the short-term debt riskless, and its  $t=0$  market value increases from 9 to 10. As a result, the wealth transfer from equity holders to short-term debt holders is 1, and equity holders will pass investment projects with NPV below 1.

**Long-term debt only.** The analysis for long-term debt is identical to the Case 1. The equity holders will pass on investment opportunities with NPV less than 0.5.

We summarize the results in Table 1, with additional information of firm leverage and debt spread for both cases. The stronger short-term debt overhang in Case 2 is particularly interesting in the following two aspects. First, in this example both the long-term and short-term debts have the same leverage. In other words, here we have controlled for leverage, and all the action is from maturity structure. Second, in this example, the short-term debt is less risky as it has a lower credit spread than the long-term debt; yet it imposes stronger overhang than long-term debt. When fixing maturity, Myers (1977) shows the riskier the debt, the stronger the overhang effect. Our result indicates that if the variation of debt riskiness comes from maturity, the opposite might hold.

Table 1: An Example of Short-term Debt Overhang

	Case 1: High Asset-in-Place, Riskless ST Debt		Case 2: Low Asset-in-Place, Risky ST Debt	
	ST Debt only	LT Debt only	ST Debt only	LT Debt only
	Panel A: Leverage and Credit Spread w/o Investment			
Asset-in-place Value at t=0	18	18	12	12
Debt Face Value	10	12	10	12
Debt Value at t=0 (w/o investment)	10	9	9	9
Leverage at t=0 (w/o investment)	55.6%	50%	75%	75%
Credit Spread <sup>8</sup> at t=0	0	15.5%	11.1%	15.5%
Panel B: Debt Overhang and Wealth Transfer				
Debt Value at t=0 with investment	10	9.5	10	9.5
Wealth Transfer to Debt holders	0	0.5	1	0.5
Investment Policy	NPV $\geq$ 0	NPV $\geq$ 0.5	NPV $\geq$ 1	NPV $\geq$ 0.5

## 2.2 Brief Discussion

From the analysis in Myers (1977), it is clear that riskless debt cannot impose overhang. This is the mechanism that underlies the Case 1 where short-term debt is riskless. However, from Case 2, we observe that once the short-term debt becomes risky, any new investment benefits more to the short-term debt holders than to the long-term debt holders. As a result, firms with short-term debt will pass more positive NPV projects than firms with long-term debt.

### 2.2.1 Risky short-term debt imposes stronger overhang

In Case 2, the new investment raises payoffs by 2 on all three states  $u$ ,  $du$ , and  $dd$  at  $t=2$ . Long-term debt benefits only at the bottom state  $dd$  which occurs with probability  $1/4$ , and this

<sup>8</sup> The credit spread is defined as the yield  $y$  such that  $D_n = F_n / (1+y)^n$ , where  $n=1$  (2) stands for short-term (long-term) debt. Note that risk free rate here is zero.

explains the wealth transfer  $0.5=2*\text{Prob}(dd)=2*0.25$  to long-term debt holders. Interestingly, because short-term debt is to be paid at  $t=1$ , the short-term debt holders at state  $d$  collect the investment benefit at all the subsequent states  $du$  and  $dd$ , and therefore a greater wealth transfer  $2*[\text{Prob}(dd)+\text{Prob}(du)]=2*1/2=1$ .

Apparently, the key difference is at state  $du$ . Long-term debt holders, who only have claim over date 2 cash flows below  $F_2$ , cannot claim the marginal cash flows on  $du$  with cash flows of 18. In contrast, short-term debt holders do receive marginal cash flows at state  $du$ . Then how can the short-term debt holders become the marginal claimants for the cash flows on the state  $du$ ?

There are potentially two mechanisms, and both are relevant in practice. The first mechanism is through refinancing, i.e., the new financiers (say, equity) will pay the date 1 short-term debt in full as they will recover these payments from asset payoffs at date 2, especially the cash flows at state  $du$ . Here, new financiers work as intermediary to transform cash flows across different states and timings. The second mechanism is through default, which can be viewed as an extreme of the refinancing channel. There, pledging out entire future asset payoffs is not enough to pay down the short-term debt, and as a result the short-term debt holders take over the defaulting firm to claim the entire date-2 asset payoffs from then on, including the cash flows in state  $du$ . In our particular example, the firm with investment has just enough expected date-2 asset payoffs to successfully refinance the maturing short-term debt, and it is immaterial to the value of existing debt for the firm to default or not at the state  $d$  (recall the bankruptcy is costless).

The fundamental reason that the (risky) short-term debt imposes stronger overhang is because there is less uncertainty to be resolved over a shorter time horizon, and debt value decreases with uncertainty. In this example, given investment the short-term debt becomes riskless at  $t=1$ , while the long-term debt who suffers further uncertainties still remains risky. Following the above example where the firm defaults at state  $d$  even after new investment; there, short-term debt holder becomes the owner of the firm to claim all the cash flows subsequent to  $d$  (i.e.,  $du$  and  $dd$  in Figure 1), while long-term debt holders who face greater uncertainty only benefit on state  $dd$ . In sum, if debt is risky so that wealth transfer due to new investment is possible, the subsidy to short-term debt holders who get paid sooner will be greater than that to long-term debt holders whose claim will mature in the future.

### 2.2.2 State contingent short-term debt overhang

The only difference between Case 1 and Case 2 in our example is the value of asset-in-place, which determines whether the short-term debt will be risky or not. Case 1 gives the scenario where the asset-in-place is relatively high. There, the firm—if it can readjust its debt maturity structure at  $t=0$ —will be willing to substitute the long-term debt by the riskless short-term debt. In fact, the maximum riskless short-term debt that the firm can accommodate without triggering any overhang is  $0.5 \times 24 = 12$ , which is the expected asset-in-place value at state  $d$ . In other words, when the firm's asset-in-place has a high value, the firm would be better off with the riskless short-term debt which imposes no overhang.

But once the firm's asset-in-place deteriorates so that the short-term debt becomes risky, it creates much stronger overhang effect than the long-term debt does. At that scenario, if the firm had known this situation when it picked its debt maturity, or could readjust it today at  $t=0$ , the firm would be better off with the long-term debt than the risky short-term debt. In fact, in Case 2, the short-term debt has the same market value of 9 as the long-term debt, and the firm with short-term debt would like to replace the short-term debt with long-term debt to reduce the overhang effect, while leaving the debt holders indifferent. Of course, this change would require renegotiation unless the debt gave the firm an option to make this change.

This state-contingent maturity effect of debt-overhang, i.e., that short-term debt imposes no overhang when the firm's asset-in-place value is high while stronger overhang when the value is low, suggests a trade-off when the firm makes the debt maturity decision at  $t=-1$ . The next section presents our main model based on this trade-off.

## 3. The Model

Consider the following model which is the extension of the example studied in Section 2. At  $t=-1$  the firm needs to raise financing  $D^{\text{target}}$  in debt for initial investment. To focus on the implication of debt-overhang on the firm's debt maturity choice, we fix the debt level  $D^{\text{target}}$  in the main analysis. Section 4.3 endogenizes  $D^{\text{target}}$  by considering an entrepreneur who requires outside capital to finance this project.

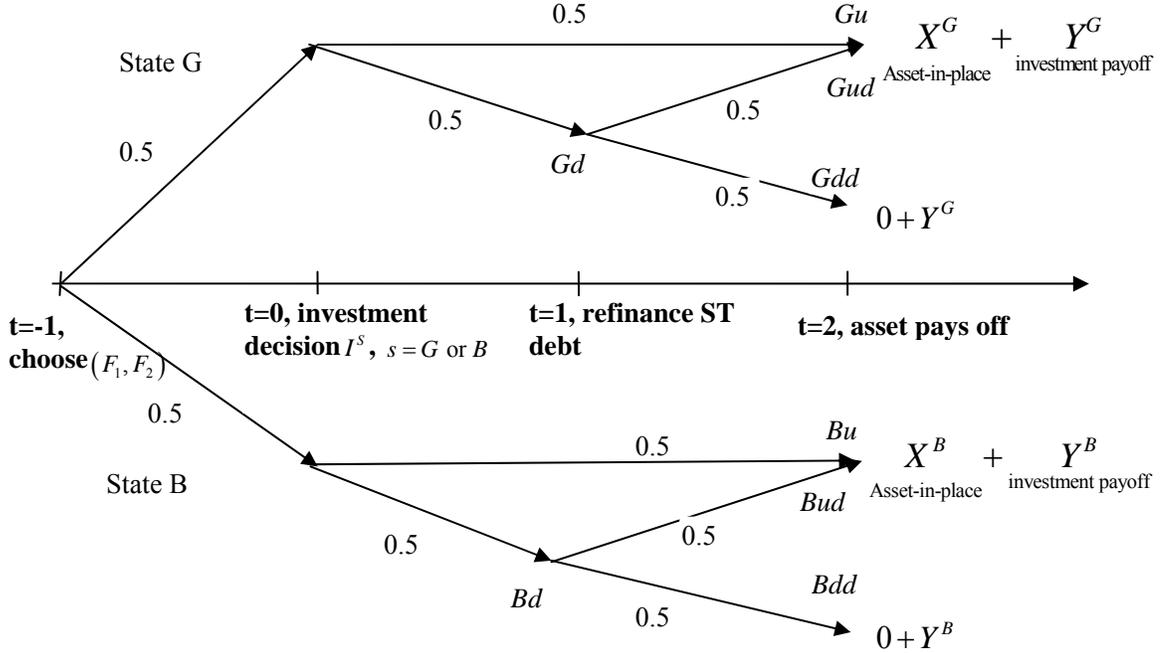


Figure 2: Timeline.

The risk-free rate is zero, and agents are all risk neutral. In this model shocks are all independent. All information is public. No cash flows from the asset occur before date 2, and we ignore cash holding.<sup>9</sup>

At  $t=0$  the firm faces the situation described in the example in Section 2. There are two possible states of nature,  $S=G$  (good) or  $S=B$  (bad), which realizes with equal probabilities. The firm decides to make investment  $I^s$ , which yields a constant payoff  $Y^s$  at  $t=2$ ,  $S \in \{G, B\}$ . This implies that the investment has a positive NPV of  $\gamma^s = Y^s - I^s$ . The potentially different investment profitability in different states gives us more flexibility for this model.

### 3.1 Short-term and Long-term Debt Contracts

We only consider standard debt contracts. The short-term debt, with  $t=-1$  market value  $D_1 > 0$  has a face value  $F_1 \geq 0$ . In other words, the firm needs to repay/refinance  $F_1$  at  $t=1$ . The long-term debt, with a date 0 market value  $D_2 \geq 0$ , has a face value  $F_2 \geq 0$  that the firm needs to repay at  $t=2$ . We can summarize the firm's debt policy as  $(F_1, F_2)$  so that  $D_1 + D_2 = D^{\text{target}}$ . The firm faces a competitive credit market so that all debt is priced to provide lenders with an

<sup>9</sup> We study the implication of cash holdings in this model in Section 4.4.

expected return of zero. We impose the following assumption throughout the paper to focus on the debt overhang problem.

**Assumption 1.** *Neither short-term debt nor long-term can be renegotiated (perhaps because debt holders are diverse). The long-term debt has a covenant stating that any new financing will be junior to the long-term debt.*

The firm might default if it cannot refinance the maturing short-term debt. We impose no bankruptcy cost in the main model. Perhaps more importantly, as we are only interested in the wealth transfer to total debt, our analysis does not depend on the seniority rule between the long-term and short-term debts in bankruptcy.

## 3.2 Solving the Model

### 3.2.1 Good State G

Consider the state G first, which is the standard debt overhang setting studied in Myers (1977). At  $t=1$  state  $Gd$ , without new investment payoff, the maximum new financing that the firm can raise, i.e., refinancing capacity, is

$$\frac{1}{2} \max(X^G - F_2, 0)$$

The new financing can be raised from existing equity holders/managers or outside equity holders.<sup>10</sup> Note that new financiers get repaid only when  $X^G$  (with probability 1/2) realizes, and it is long-term debt holders that get their payment  $F_2$  first (they are senior to any new financings). This reflects the standard debt overhang in Myers (1977).

The firm needs to use this refinancing capacity to pay the maturing short-term debt  $F_1$ . We focus on the case that

$$X^G > 2F_1 + F_2, \tag{2}$$

so that short-term debt can get refinanced at  $t=1$ , even without investment. In other words, the short-term debt is riskless. A similar argument in Case 1 in Section 2 implies that short-term debt impose no overhang on new investment at  $t=0$ .

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<sup>10</sup> This differs from Diamond (1991), where it is the manager's non-pledgeable control rent combined with the need for outside funding drives the inefficiency. In Diamond (1991), if the manager had deep pockets to refinance the firm, then he would internalize the loss of non-pledgeable control rent and eliminate the inefficiency. In this model without control rent, there is no distinction between existing equity holders/managers or outside equity holders.

To determine the firm's investment incentives at  $t=0$ , we need to calculate the value increment of long-term debt due to new investment. Without investment, the long-term debt date-0 value is  $0.75 \times F_2$ . With investment, the long-term debt value becomes

$$\frac{3}{4} \times F_2 + \frac{1}{4} \times \min(Y^G, F_2)$$

Therefore, the firm will invest if and only if

$$\frac{1}{4} \min(Y^G, F_2) < \gamma^G$$

To make the problem interesting, we assume throughout that  $\frac{1}{4} Y^G > \gamma^G$ ; otherwise the firm invests always independent of  $F_2$ . Therefore, the firm will invest if and only if

$$F_2 < 4\gamma^G.$$

This is the standard debt overhang of Myers (1977): The higher the promised long-term face value  $F_2$ , the less likely that the firm will make the positive NPV projects at  $t=0$ .

### 3.2.2 Bad State B

Now consider the state B. Here,  $X^B$  may be sufficiently low that the firm cannot successfully refinance the short-term debt at state *Bd*. Formally, we assume that

$$X^B < 2F_1 + F_2,$$

and without investment the firm will default on its short-term debt at  $t=1$ . As illustrated in the example Case 2 in Section 2, the risky short-term debt may lead to stronger overhang effect.

To determine the firm's investment incentives, we calculate the debt value change due to new investment. It is clear that without investment, the total debt value, i.e., the sum of short-term debt and long-term debt, is

$$0.5 \times (F_1 + F_2) + 0.5 \times \frac{X^B}{2}$$

Here, we focus on the case that  $X^B > F_1 + F_2$ , so that at state  $u$  (with probability 0.5) both debts are riskless (and the total debt value there is  $F_1 + F_2$ ). At state  $Gd$  (with probability 0.5), the firm defaults, and total debt value is just the firm value  $\frac{X^B}{2}$ .<sup>11</sup>

The calculation for the debt value given investment is a bit involved. The next lemma shows that in order for the firm to invest at  $t=0$ , it must be the case that the firm with investment can successfully refinance the short-term debt at state  $Bd$ .

**Lemma 1.** The necessary condition for the firm to invest at  $t=0$  is successful refinancing short-term debt at state  $Bd$ .

**Proof.** Suppose not. Then at  $Bd$  the total debt value is  $\frac{X^B}{2} + Y^B$ , i.e., the firm value at  $Bd$ . Since at  $Bu$  the debt holders get paid in full, this implies that standing at  $t=0$  the debt holders gain by  $0.5 \times Y^B$ . Because we assume that the NPV  $\gamma^B < \frac{1}{4} Y^B$ , equity holders cannot break even. QED.

Now depending on whether the long-term debt is risky or not given investment, we have two cases to consider.

**Case 1.** When  $Y^B \geq F_2$  so long-term debt is paid in full in state  $Bdd$ . This implies that not only short-term debt becomes riskless, but also the long-term debt becomes riskless. Therefore the total debt value at  $t=0$  becomes  $F_1 + F_2$ , and the value increment due to new investment is

$\frac{1}{2}(F_1 + F_2) - \frac{1}{4} X^B$ . Comparing to positive NPV  $\gamma^B$ , the necessary condition for investment is

$$F_1 + F_2 \leq 2\gamma^B + \frac{X^B}{2}.$$

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<sup>11</sup> The seniority rule between short-term and long-term does not matter as we are only interested in the total debt value.

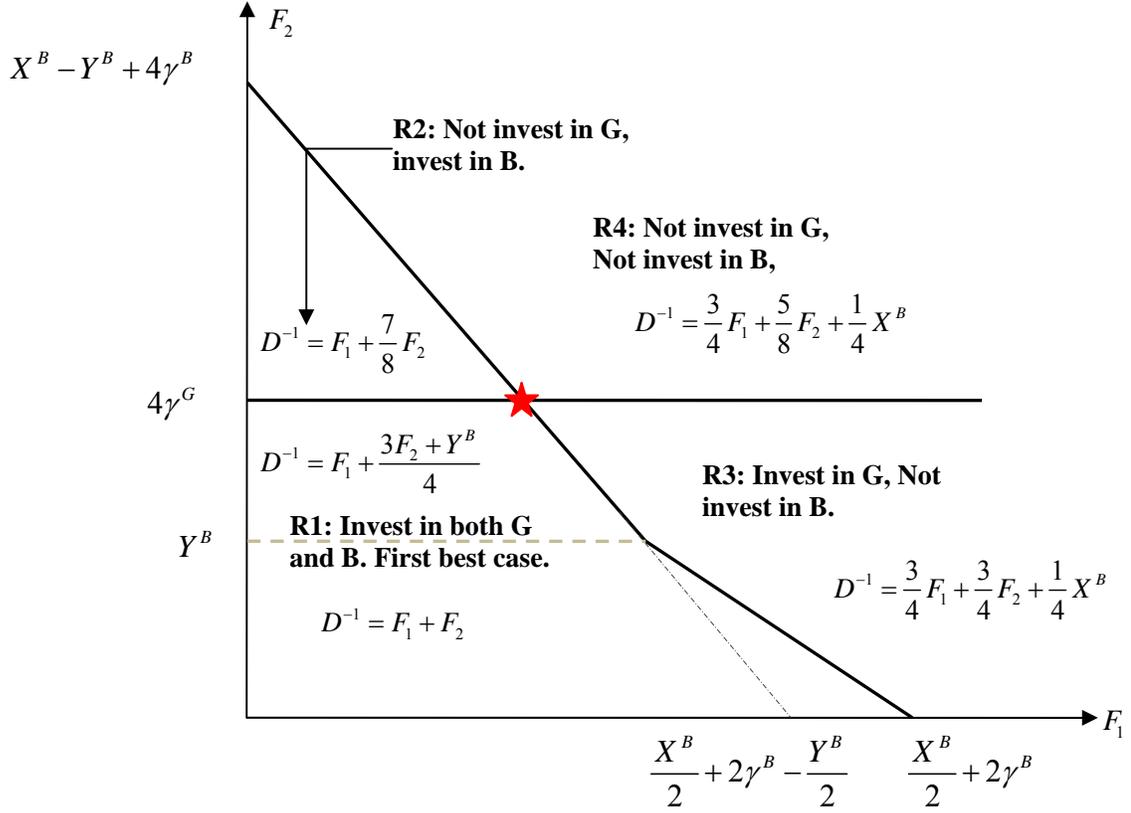


Figure 3:  $(F_1, F_2)$  space with different investment policy and  $t=1$  debt values  $D^{-1}$  in the model.

Let us pause to discuss the economics of this analysis. In this case, because the new investment makes both long-term debt and short-term debt riskless, the wealth transfer to both debts is the same. This is drastically different from the risky short-term debt example illustrated in Section 2, where new investment bails out the short-term debt fully (so the short-term debt becomes riskless due to investment) while the long-term debt, because of future uncertainty resolved in  $t=2$ , remains risky. The next case capture this idea, and this is scenario where short-term debt may create (strictly) stronger overhang than long-term debt does.

**Case 2.** When  $Y^B < F_2$  so the long-term debt only recovers  $Y^B$  at the state  $Bdd$ . The total debt value given investment at  $t=0$  becomes  $F_1 + \frac{3}{4}F_2 + \frac{1}{4}Y^B$ . Therefore, the debt value increment due

to new investment is  $\frac{1}{2}\left(F_1 + \frac{F_2}{2}\right) + \frac{Y^B}{4} - \frac{X^B}{4}$ , and the necessary condition for investment is

$$\frac{1}{2}\left(F_1 + \frac{F_2}{2}\right) + \frac{Y^B}{4} - \frac{X^B}{4} \leq \gamma^B \Leftrightarrow F_1 + \frac{F_2}{2} \leq 2\gamma^B - \frac{Y^B}{2} + \frac{X^B}{2}.$$

As just mentioned, this is the case that captures our key mechanism that risky short-term debt imposes stronger overhang than long-term debt when asset-in-place is relatively low. Note that because new investment cannot make long-term debt holders to be paid in full, the total debt value increment is  $\frac{1}{2}\left(F_1 + \frac{F_2}{2}\right) + \frac{Y^B}{4} - \frac{X^B}{4}$ . We observe that the short-term face value  $F_1$  and the long-term face value  $F_2$  receive different weights in this condition; to be specific, the short-term debt has a greater weight, which is 1, than the long-term debt has, which is  $\frac{1}{2}$ . This difference is the underlying reason why short-term debt holders are able to extract more value increment from new investment than long-term debt holders. Fundamentally, the weight difference reflects the fact that conditional on state  $Bd$ , the short-term debt gets paid in full there, while for long-term debt with probability 0.5 the state  $Bdd$  occurs and the firm default on long-term debt.

### 3.3 Model Solution

The firm is choosing debt maturity  $(F_1, F_2)$  at  $t=-1$  to minimize the overall overhang effect in state G and B at  $t=0$ . In the following analysis, we are mainly interested in characterizing the optimal debt maturity structure that achieves the maximum  $t=-1$  target debt value  $D^{\text{target}}$  while preserving the first-best investment policies at both states at  $t=0$ .

We just showed that raising  $F_2$  and reducing  $F_1$  at  $t=-1$  is a good idea from the stand point of state B, because it can reduce the ex post wealth transfer from equity holders to debt holders due to new investment there. However, the firm cannot raise  $F_2$  too much. As our analysis for the state G indicates, in state G the riskless short-term debt is harmless, and it is only  $F_2$  that imposes overhang effect. This trade-off determines the optimal interior debt maturity that we are after.

We illustrate the firm's investment decisions on the space of debt structure  $(F_1, F_2)$  in Figure 3. As the firm tries to meet the target debt value  $D^{\text{target}}$  at  $t=-1$ , we also calculate the total debt value  $D^{-1}$  at  $t=-1$  for all regions.

In Figure 3, only region R1 achieves the first best, which features first-best investment in both states. There are two sub-regions, depending on whether  $F_2 > Y^B$ , i.e., whether long-term debt given investment is riskless or not. This is also the kink point shown in Figure 3. When  $F_2 \leq Y^B$ , both long-term and short-term debt are risk free, with  $t=-1$  value  $D^{-1} = F_1 + F_2$ , and the boundary line is  $F_1 + F_2 = 2\gamma^B + \frac{X^B}{2}$ . When  $F_2 > Y^B$ , long-term debt only defaults at the state  $Bdd$ , and  $F_1 + \frac{3F_2 + Y^B}{4}$ , and the boundary line is  $F_1 + \frac{F_2}{2} = 2\gamma^B - \frac{Y^B}{2} + \frac{X^B}{2}$ .

Note that in Figure 3 we have assumed that

$$4\gamma^G > Y^B.$$

This condition also requires that  $Y^G > Y^B$  (or, the investment payoff at state G is greater than that in state B) given  $\frac{1}{4}Y^G > \gamma^G$ . This assumption is rather technical to ensure the interesting case of risky long-term debt given investment at state B. To see this, geometrically this assumption implies that there are two sub-regions in the first-best region R1 in Figure 3. Essentially, it allows for the case that the state-G first-best long-term face  $F_2 = 4\gamma^G$  exceeds the state-B lower bound cash flows  $Y^B$ . As a result, even with investment, in state B the long-term debt with  $F_2 = 4\gamma^G$  can still be risky at  $t=2$  given the worst cash flow realization, while short-term debt which gets paid in  $t=1$  becomes riskless. This is the only reason we impose this assumption. There are other ways, e.g., random investment opportunities (symmetric to both states), to achieve this goal, but we deem that the unnecessary modeling complicity outweighs the potential benefit.

From Figure 3, we see that the optimal debt maturity structure that achieves the first-best investment policies and the maximum  $t=-1$  target debt value  $D^{\text{target}}$  is the intersection point

between  $F_2 = 4\gamma^G$  and  $F_1 + \frac{F_2}{2} = 2\gamma^B - \frac{Y^B}{2} + \frac{X^B}{2}$ , i.e., (the asterisk in Figure 3)

$$F_1^* = \frac{X^B}{2} + 2\gamma^B - \frac{Y^B}{2} - 2\gamma^G, F_2^* = 4\gamma^G.$$

With this debt maturity structure, the maximum  $t=-1$  debt value that preserves the optimal investment policies is

$$D^{\text{target},*} = \frac{X^B}{2} + 2\gamma^B + \gamma^G - \frac{Y^B}{4}.$$

We briefly discuss the situation where the firm's date -1 target debt level  $D^{\text{target}} > D^{\text{target},*}$ , so that the firm is forced to choose some point in the non-first-best region. Here the trade-off between different states kicks in. If  $\gamma^B > \gamma^G$ , then the region R2 is chosen where relatively more long-term debt is used to maximize the investment incentives at state B. On the other hand, if  $\gamma^B < \gamma^G$ , then R3 is chosen where the firm takes relatively more short-term debt to maximize the investment incentives in state G. As our main objective of this paper is to show the trade-off between short-term and long-term debt, we leave more detailed study for future research.

## 4. Extensions and Discussions

### 4.1 Endogenous Costly Default

The extreme version of underinvestment is costly endogenous default, which is just one symptom of short-term debt overhang. The endogenous default has been analyzed, e.g., in Bulow and Shoven (1978) and Leland (1994).<sup>12</sup> For instance, in Leland and Toft (1996) and He and Xiong (2010), equity holders, facing a low firm fundamental value, might find suboptimal to keep absorbing the financial losses in rolling over maturing debt. As a result, equity holders default, leaving debt holders to bear the bankruptcy cost. Essentially, it is because equity holders do not want to subsidize debt holders, especially the maturing ones.

To see the equivalence, let us examine the endogenous default decision at  $t=1$  state  $d$  in this model. Ignore the investment decision at  $t=0$ , and assume a zero liquidation value of the firm (i.e., bankruptcy involves a substantial dead-weight loss). As shown in Section 3.2, the new financiers are not willing to refinance the maturing short-term debt if and only if

$$\frac{X}{2} < F_1 + \frac{F_2}{2},$$

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<sup>12</sup> The interpretation of endogenous default given debt burden as “underinvestment” due to debt-overhang, is mentioned in, for example, Lambrecht and Myers (2008) and He (2009).

where  $X$  is the asset-in-place value at state  $du$ . When this condition holds, the firm inefficiently defaults, leaving both long-term and short-term debt worthless. This inefficient default is neither because the firm cannot get fairly priced outside-financing (due to informational problems or financial market disruption), nor because the firm has some non-pledgeable part of future cash flows (a la Diamond (1991)) that new financiers cannot internalize. Rather, it is because in order to bail out the firm, the firm/new financiers need to repay its short-term debt fully, and also subsidize the long-term debt holders with  $\frac{F_2}{2}$ . This reflects debt overhang, as the fundamental problem is rooted in the fact there cannot be renegotiations between existing debt holders who demand payment (either immediate as short-term debt, or future as long-term debt), and the firm who makes the investment/default decision.

The interesting point regarding endogenous default is that, when the overhang effect is about failing to attract new financings to avoid firm's inefficient early default, it is the maturing short-term debt which demands immediate repayment that plays a more significant role. This point is clearly reflected in the bankruptcy threshold  $F_1 + \frac{F_2}{2}$ , which puts a greater weight on short-term debt. The intuition is similar to the one in Section 2 where risky short-term debt requires more wealth transfer because it gets paid sooner than long-term debt. In words, in bailing out the firm from default, new financiers subsidize the maturing short-term debt holders one-to-one. However, by keeping the firm alive the wealth transfer from new financiers to the long-term debt holders is typically less than one-to-one (in this model it is  $\frac{1}{2}$  which is probability of realization of  $ud$  at  $t=2$ ; for a similar discussion regarding the relative weight, see the end of Section 3.2.2) Therefore, the burden on the maturing short-term debt will distort the default decision more than the long-term debt.

## 4.2 Credit Lines (Revolvers) and Market Based Pricing

In practice, many firms have standing credit lines (also called revolving lines of credit or revolvers) issued by banks. The mechanism of credit line works as an insurance contract: The firm typically pays a fee up front to secure the line, and will later draw down the line if the precommitted rate is below the one offered in the market. During the current 2007/2008 crisis, the credit line draw-downs accounted for the major part of loans extended by commercial banks (Ivashina and Scharfstein (2009)).

Existing theory about credit lines emphasizes on helping firms overcome liquidity shocks or alleviate risk shifting incentives (e.g., Boot and Thakor (1994)). Our model suggests that credit lines also alleviate short-term overhang by fixing the refinancing cost in bad times. In other words, part of new financiers (here the bank who have issued credit lines to the firm) contractually committed to subsidize the maturing short-term debt holders, making the firm less likely to default in state B. If endogenous default is the only inefficiency involved, then the insurance contract should help.

#### **4.2.1 Can the seniority of credit line resolve debt overhang?**

In practice, the credit lines/revolvers from banks have the distinct feature that draw-downs generally are senior to any existing debt. One tends to think that this seniority of credit line should resolve the debt overhang problem completely, because it directly attacks the heart of debt overhang: Banks who issued revolvers do not have to worry that the first dollar out of the new investment goes to existing senior debt holders.

There is a subtle but important difference between the incentives of new financiers (banks in the credit line case) and the incentives of the firm who decides whether to draw down the line. In fact, because banks are already obligated to provide financing if the firm decides to do so, the seniority of drawdowns plays no role at all in the firm's investment incentives. The only thing matters in this scenario is the pricing of drawdowns. Given the relatively expensive drawdowns which get to be repaid first later on, the firm may decide not to draw the line if almost all the future investment benefit goes to the bank.<sup>13</sup> Imagine the following extreme example with zero risk-free rate. Because revolvers needs to break even when they are issued, usually banks set a future drawdown rate  $r > 0$  higher than the risk-free rate. Clearly, the firm will decide not to draw the line for the positive NPV projects that yields constant returns below  $r$ .

#### **4.2.2 Market based pricing**

A recent innovation to the contract of credit-lines/revolvers is *market-based pricing*; that is, the interest rate of new-drawdowns is partially tied to the firm's current strength. This is a form of performance-based pricing which is common in the bank debt (e.g., Asquith, Beatty, and

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<sup>13</sup> Of course, the ex post seniority helps the bank set a low drawdown rate ex ante, which alleviates overhang.

Weber (2005)). More specifically, when the firm draws on the remaining line, the drawn spread is positively tied to its credit default swap (CDS) spread.<sup>14</sup>

The repricing of the cost of borrowing against the line leaves the firm's cost of borrowing higher when the firm's prospects are bad, much like short-term debt. To the extreme case where the drawdown rate is fully market based, then this essentially takes away the insurance that the bank offered to the firm. As a result, there is no difference between new financiers and the bank, and we are back to the model we have analyzed before.

However, in our model, debt overhang is not all about insurance. There is a key difference between endogenous default decision and the investment decision. Insurance, by forcing a subsidy from bank to maturing debt holders, always alleviates the distortion of default decision. However, for investment, the heart of overhang lies in the firm's "incentives," which can be positively related to market-based pricing.

This point suggests that a properly designed market-based pricing on credit lines is better than credit lines without market-based-pricing. The firm knows that its investment may improve its long-term debt CDS, and this will in turn reduce the firm's future financing cost. As a result, market-based pricing provides the firm extra incentives to invest at  $t=0$ . Essentially, a properly designed market-based-pricing should combine two components. The first is the insurance that protects the firm from those states with deteriorating asset-in-place, and the second is the performance-based sensitivity that entices the firm's investment. Therefore, the core idea here is similar to optimal contracting with moral hazard, which is to reward/punish the firm for its actions (here, maintenance) but not for fundamental states beyond its actions.

Interestingly, this further suggests that although long-term debt CDS has its own advantages (e.g., more liquid and accurate pricing) over short-term debt CDS to be the market-base in designing the market-based-pricing scheme, the ideal market-base-measure would be state-dependent. In bad times, stronger short-term overhang implies that short-term debt CDS might

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<sup>14</sup> Typically these revolvers with market-based-pricing specify floor and cap which are the minimum and maximum of the drawn spreads. If the benchmark CDS spread is lower than the floor, the floor applies for the drawn margin. Conversely, if the CDS spread at the time of the draw is higher than the cap, the cap is applied. Finally, if the CDS spread at the time of the draw falls in between the floor and cap, certain formula will apply. Source: Thomson Reuters LPC, Markit on Reuters 3000 Xtra / Credit Views.

contain more action-based information than long-term debt CDS does, simply because the wealth transfer due to investment is greater for short-term debt when the firm is close to default.

### 4.3 Endogenizing Leverage $D^{\text{target}}$ at $t=-1$

Now we endogenize the target date-0 debt value. A tradeoff between saving taxes versus increasing bankruptcy costs will yield a positive value of  $D^{\text{target}}$ , for standard reasons. There is not much special about this approach in our framework. An alternative is to account for the control role of debt, and of short-term debt in particular. This section describes such an approach. This added structure will prove to be useful in extending our results below.

Suppose that at  $t=-1$  the entrepreneur owns the patent of the project. To start this project, the entrepreneur with no personal wealth needs 1 dollar of initial investment. He can raise this initial investment through debt or (outside) equity, and becomes the manager of the firm afterwards. We assume that equity holders are soft claims that are subject to renegotiation.

We will use a very simple model motivated by Jensen (1986), Hart and Moore (1994), Diamond and Rajan (1999), and Diamond (2004, 2006). It introduces managerial “equity overhang,” where managers take a fraction  $\lambda$  of all free cash flow in excess of debt payments. We follow Diamond (2004, 2006) and assume that default on debt allows the legal system to prevent the manager from consuming any cash they divert instead of paying out or investing (giving the manager nothing if the legal sanction is imposed).<sup>15</sup> Legal sanctions remove any of the benefit of diversion which occurs that period, but not the benefit of diversion in previous periods. Contracts are written such that this legal sanction is imposed if a debt contract is not paid on its due date. The threat of this legal sanction ensures that debt is paid when cash is available, and because debt cannot be renegotiated, the default automatically imposes the sanction. However equity contracts (which are soft) do not have automatic sanctions so that equity holders have no right to impose the legal sanction for default. We simply assume that the manager can take a fraction  $\lambda$  of remaining cash flows, which can be motivated by  $\lambda$  being the relative bargaining power of the manager in dealing with equity holders. Equivalently, we can assume that the manager is able to directly divert all free cash and retain a fraction  $\lambda$  of it (while

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<sup>15</sup> This model is a much simplified of that in Diamond (2006). We simplify by assuming that no debt can be renegotiated and that the legal sanctions of debt default completely eliminate proceeds from managerial diversion on the date when default occurs.

destroying a fraction  $1-\lambda$ ) in a way that cannot be verified or recovered without legal sanctions. As a result, owners of outside equity allow the manager to take a fraction  $\lambda$  of current free cash flow each period, if the manager so desires, given its effect on his current or future payoffs.<sup>16</sup>

To recap, at  $t=-1$  the firm raises equity and debt to carry out the initial investment.<sup>17</sup> At  $t=0$  the firm requires new investment as analyzed in Section 3, and at  $t=1$  the refinancing decision is controlled by the existing shareholders. Because issuing any new equity only benefits the entrepreneur and dilutes their own value, the new financing is in the form of (junior) debt. And at  $t=2$  the manager can get (at least)  $\lambda$  fraction of free cash flows after the debt payment.

Denote by  $M_{-1}$  the value of entrepreneur/manager,  $E_{-1}$  the value of (outside) equity, and  $D_{-1}$  the value of total long-term and short-term debt, all evaluated at  $t=-1$ . The agency problem at  $t=2$  implies that  $M_{-1} \geq \frac{\lambda}{1-\lambda} E_{-1}$ , i.e., the manager has to have sufficient inside stake for him to behave. The initial investment requires that  $E_{-1} + D_{-1} \geq 1$ ; if this inequality holds strictly then the entrepreneur/manager can consume the difference at date -1. Therefore the entrepreneur's date -1 value is  $M_{-1} + (E_{-1} + D_{-1} - 1)$ . Finally, denote by  $v(D_{-1})$  the firm value as a function of  $D_{-1}$ , where the firm value is determined by the optimal maturity structure determined in Section 3. Because the only agency issue that hurts the firm value is debt overhang,  $v(D_{-1})$  is decreasing in  $D_{-1}$ . Finally, the accounting identity implies that  $M_{-1} + E_{-1} + D_{-1} = v(D_{-1})$ . Therefore, the manager who chooses the  $t=-1$  financial structure solves the following problem:

$$\begin{aligned} & \max M_{-1} + (E_{-1} + D_{-1} - 1) \\ & \text{s.t. } E_{-1} + D_{-1} \geq 1, M_{-1} \geq \frac{\lambda}{1-\lambda} E_{-1}, E_{-1} + M_{-1} + D_{-1} = v(D_{-1}) \end{aligned}$$

**Proposition 1.** *Assume that  $(1-\lambda)v(0) < 1$  and  $v(1) > 1$ . Then the optimal date-0 debt value*

*$D^{\text{target}} \in (0,1)$  is the smallest solution to the equation  $D_{-1} + (1-\lambda)(v(D_{-1}) - D_{-1}) = 1$ .*

<sup>16</sup> So essentially we are modeling outside equity in this setup. See Myers (2000) for another way of modeling.

<sup>17</sup> We assume that the manager can commit to invest funds properly on the instant they are received, but cannot commit not to divert cash flows obtained in excess of immediate investment needs or cash flows obtained from the returns to investments.

The first restriction  $(1-\lambda)v(0) < 1$  implies that using outside equity only cannot raise enough capital to cover initial investment; and the second condition  $v(1) > 1$  implies that it is feasible to raise the entire investment capital by debt. Then the optimal date-0 debt will be an interior solution. Finally, the equation in Proposition 1 simply says that the debt holders and outside equity holders (who have a  $1-\lambda$  fraction of total equity value  $v(D_{-1}) - D_{-1}$ ) contribute the entire initial investment 1 (as the manager has zero initial wealth.)

#### 4.4 Cash Reserve?

One potential solution to the debt overhang is that the firm maintains cash reserves. We will first investigate the role of cash reserve by ignoring the agency issue of managerial diverting that we introduced in Section 4.3, then discuss the interesting interaction between the managerial diverting and debt overhang.

##### 4.4.1 State-contingent maturity and callable long-term debt

It is clear that raising cash reserves that are not subject to agency problems (diverting, dividend payout, etc.), *while holding total debt issuance fixed*, could alleviate debt-overhang— simply because we can interpret cash as negative debt. The more interesting question is, can the firm reduce overhang by issuing more debt at  $t=-1$ , say  $D_{-1} + C$  where  $C > 0$ , and keeping  $C$  inside the firm as cash reserve? The answer is yes.

In our model, if the firm can issue debt with state-contingent maturity, then the optimal contract will be short-term debt in state G (so there is no overhang) and long-term debt in state B (so there is less overhang). This result indicates that the callable feature of long-term debt can help on this dimension, because the firm who issued callable long-term debt at  $t=-1$  can choose to call these debt at  $t=1$  if state G realizes.<sup>18</sup> Of course, in order to motivate the firm to call back the debt in full, the call price should be at a discount, i.e., below the long-term debt value *given investment*. Otherwise, the same extent of wealth transfer suggests that the firm will decide to do nothing. Also, a pre-determined call price cannot deal with random investment opportunities.

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<sup>18</sup> Bodie and Taggart (1978) suggest that callable long-term debt can alleviate overhang. However, Bodie and Taggart (1978) still ask the question why firms do not simply roll over their short-term debt, which suggests that the authors do not realize that roll over short-term debt can impose stronger overhang in some states.

Interestingly, the state-contingent callable feature can also be generated by the cash reserve. It is because cash allows the firm to have a state-contingent repayment policy. To be specific, in state B, the firm can use the cash to pay part of short-term debt at  $t=1$ , while in state G the firm will save these cash and use them to pay part of long-term debt at  $t=2$ . Because cash is raised by a mixture of short-term and long-term debt at  $t=-1$ , this state-contingent repayment policy help the firm transform some long-term (short-term) debt to short-term (long-term) debt in state G (B), which is value improving in this model.

#### 4.4.2 Debt overhang on manager's diverting decision

Of course, the very reason to have debt in the first place, as discussed in Section 4.3, is because the manager can divert some of the firm's free cash flows. This militates against having the firm pile up extra cash, because for any dollar that sitting in the firm at  $t=1$  in excess of short-term debt payment, the manager can divert it to obtain  $\lambda$  at  $t=1$  (and the date 2 default is too late to recover it).<sup>19</sup> Interestingly, different from the standard argument (e.g., DeMarzo and Fishman (2007)) that the manager's inside equity stake  $\lambda$  will prevent him from diverting the cash at the interim date  $t=1$  (because he can get  $\lambda$  fraction of free cash flows at  $t=2$ ), in our setting the manager will strictly prefer to divert at  $t=1$ , *if the outside equity holders cannot promise to the manager more than  $\lambda$  fraction of  $t=2$  free cash flows*. The reason is just *debt overhang*: the manager understands that the cash left in the firm goes to the debt holders first at  $t=2$ , and therefore he has a strict incentive to divert at today rather than wait to share the cash tomorrow. For illustration, suppose that there is one dollar of free cash flow in the firm at  $t=1$ , and in our model at  $t=2$  the firm is solvent with probability  $\frac{1}{2}$ . Then diverting today gives the manager a utility of  $\lambda$ , while the expected value by waiting to share the free cash flows with outside equity holders at  $t=2$  is only  $\lambda/2$ . Here, the manager will divert earlier along the equilibrium path because the debt coming due tomorrow hangs over his own "inside-equity." As a result, holding extra cash does not overcome the short-term debt overhang of injecting new cash because the manager's payoff is very much like that of newly-injected equity.

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<sup>19</sup> Here, for the sake of argument, we only consider the manager's diverting incentives at  $t=1$ .

## 5. Conclusion

Debt overhang influences the investment and default decisions of those whose claims are like equity. Long term debt causes overhang, because it prevents equity from receiving any payoff from investment when the ex-post payoff is low enough that there is a default. Short-term debt with the possibility of default can impose even greater overhang, simply because there is less uncertainty resolved over the shorter time until it matures, and as a result most of the first part of any initial increase in value (due to investment or bailout to avoid default) will not result in any payoff to equity. Short-term debt then imposes either no overhang (if riskless) or large overhang (if likely to default). The timing of debt maturity can have a major impact on investments, especially on investments that can help avoid default.

The problems caused by large impending debt maturity go beyond the risk of runs and limited access to liquidity. The timing of repayments, access to lines of credit, and the pricing of credit lines all combine to either amplify or reduce the risks of potential default.

## Appendix

### A.2 Proof of Proposition 1

The manager's  $t=-1$  optimization problem is equivalent to  $\max v(D)$  s.t.  $E + D \geq 1$ , and

$M \geq \frac{\lambda}{1-\lambda} E$ , where we use  $D$ ,  $E$ ,  $M$  to indicate the value at  $t=-1$ . Because  $v(D)$  is decreasing in

$D$ , the first constraint is binding; otherwise lowering  $D$  improves. Then the second constraint is binding as well, because otherwise a higher  $M$  raises  $E$  which in turn reduces  $D$ . As a result, we have that

$$\left. \begin{array}{l} \frac{1}{1-\lambda} E + D = v(D) \\ E + D = 1 \end{array} \right\} \Rightarrow D + (1-\lambda)(v(D) - D) = 1.$$

Let  $Q(D) = D + (1-\lambda)(v(D) - D)$ . Under our assumptions,  $Q(1) > 1$  and  $Q(0) < 1$ . More importantly, note that as we have seen in Figure 3, if there is any discontinuity on  $v(D)$ , it only jumps downward where certain efficient investment starting to be cut. As a result,  $Q(D)$  only

jumps downward, and there always exists a solution  $D \in (0,1)$  to the above equation (if multiple solutions exist then the optimal one is the smallest  $D$  . Q.E.D.

## Reference

- Almeida, H., M. Campello, B. Laranjeira, and S. Weisbenner, 2009, Corporate Debt Maturity and the Real Effects of the 2007 Credit Crisis, NBER working paper 14990.
- Asquith, P., A. Beatty, and J. Weber, 2005, Performance Pricing in Bank Debt Contracts, *Journal of Accounting and Economics* 40, 101-128.
- Barclay, M., and C. Smith, Jr., 1995, The Maturity Structure of Corporate Debt, *Journal of Finance* 50, 609-632.
- Benmelech, E., 2006, Managerial Entrenchment and Debt Maturity: Theory and Evidence, Working paper, Harvard University.
- Bodie, Z. and R. Taggart, 1978, Future Investment Opportunities and the Value of the Call Provisions on a Bond. *Journal of Finance* 33, 1187-1200.
- Boot, A., and A. Thakor, 1994, Moral Hazard and Secured Lending in an Infinitely Repeated Credit Market Game, *International Economic Review* 35, 899-920.
- Brunnermeier, M., and M. Oehmke, 2009, The Maturity Rat Race, Working paper, Princeton University and Columbia University.
- Brunnermeier, M. and M. Yogo, 2009, A Note on Liquidity Risk Management, *American Economic Review: Papers and Proceedings* 99, 578--583.
- Bulew, J., and J Shoven, 1978, The Bankruptcy Decision, *Bell Journal of Economics* 9, 437-56.
- Calomiris, C., and C. Kahn, 1991, The Role of Demandable Debt in Structuring Optimal Banking Arrangements, *American Economic Review* 81, 497-513.
- DeMarzo, P., and M. Fishman, 2007, Optimal Long-term Financial Contracting, *Review of Financial Studies* 20, 2079-2128.
- Diamond, D., 1991, Debt Maturity Structure and Liquidity Risk, *Quarterly Journal of Economics* 106, 709-737.
- Diamond, D., 2004, Committing to Commit: Short-term Debt When Enforcement is Costly, *Journal of Finance* 59, 1447-1479.

- Diamond, D., 2006, Delegated Monitoring and Legal Protection, Working paper, University of Chicago.
- Diamond, D., and P. Dybvig, 1983, Bank Runs, Deposit Insurance, and Liquidity, *Journal of Political Economy* 91, 401-419.
- Diamond, D., and R. Rajan, 2000, A Theory of Bank Capital, *Journal of Finance* 55, 2431-2465.
- Diamond, D., and R. Rajan, 2001, Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking, *Journal of Political Economy* 109, 287-327.
- Flannery, Mark, 1986, Asymmetric Information and Risky Debt Maturity Choice, *Journal of Finance* 41, 19-37.
- Duchin, R, O. Ozbus and B. Sensoy, 2009, Costly External Finance, Corporate Investment, and Subprime Mortgage Crisis, forthcoming in *Journal of Financial Economics*.
- Flannery, Mark, 1994, Debt Maturity and the Deadweight Cost of Leverage: Optimally Financing Banking Firms, *American Economic Review* 84, 320-331.
- Goldstein, I., and A. Pauzner, 2005, Demand-Deposit Contracts and the Probability of Bank Runs, *Journal of Finance* 60, 1293-1327.
- Guedes, J., and T. Opler, 1996, The Determinants of the Maturity of Corporate Debt Issues, *Journal of Finance* 51, 1809-1834.
- Hart, O., and J. Moore, 1994, A Theory of Debt Based on the Inalienability of Human Capital, *Quarterly Journal of Economics* 109, 841-879.
- Hart, O., and J. Moore, 1998, Default and Renegotiation: A Dynamic Model of Debt, *Quarterly Journal of Economics* 113, 1-41.
- He, Z., 2009, A Model of Dynamic Compensation and Capital Structure, Working paper, University of Chicago.
- He, Z., and W. Xiong, 2009a, Dynamic Debt Runs, Working paper, University of Chicago and Princeton University.
- He, Z., and W. Xiong, 2009b, Rollover Risk and Credit Risk, Working paper, University of Chicago and Princeton University.
- Hennessy, C., 2004, Tobin's Q, Debt Overhang, and Investment, *Journal of Finance* 59, 1717-1742.
- Ivashina, V. and D. Scharfstein, 2009, Bank Lending during Financial Crisis of 2008, Working paper, Harvard Business School.

- Jensen, M., 1986, Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers, *American Economic Review* 76, 323-29.
- Johnson, S., 2003, Debt Maturity and the Effects of Growth Opportunities and Liquidity Risk on Leverage, *Review of Financial Studies* 16, 209-236.
- Lambrecht, Bart M. and Stewart C. Myers, 2008, Debt and managerial rents in a real-options model of the firm, *Journal of Financial Economics* 89, 209-231.
- Leland, H., 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* 49, 1213-1252.
- Leland, H., 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance* 53, 1213-1243.
- Myers, S., 1977, The Determinants of Corporate Borrowing, *Journal of Financial Economics* 5, 147-175.
- Myers, S., 2000, Outside Equity, *Journal of Finance* 55, 1005-1037.
- Philippon T., and P. Schnabl, 2009, Efficient Recapitalization, Working paper, NYU.
- Stohs, M. and D. Mauer, 1996, The Determinants of Corporate Debt Maturity Structure, *Journal of Business* 69, 279-312.
- Veronesi, P. and L. Zingales, 2009, Paulson's Gift, Working paper, University of Chicago.