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Debt and Creative Destruction: Why Could Subsidizing Corporate Debt Be Optimal?

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The existing theoretical literature provides little justification for a corporate debt subsidy. We illustrate the welfare benefit of this subsidy and study how the social costs and benefits change with the duration of industry distress. In our model, two firms engage in socially wasteful competition for survival in a declining industry. Firms differ on two dimensions: exogenous productivity and endogenously chosen amount of debt financing, resulting in a two-dimensional war of attrition. Debt financing increases incentives to exit, which, although costly for the firm, is socially beneficial. These benefits decline as industry distress shortens. Our normative model sheds light on why the debt tax subsidy still persists around the world. Analogously, the model can also rationalize a seemingly ad hoc feature of the U.S. tax system, which subsidizes the conflict of interest between debt and equity regarding firm liquidation.

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1. Introduction

Corporations can deduct interest payments from their profits. This subsidy to financing with debt rather than equity is massive, comprising 9.7% of firm value in the United States (Graham 2000) and can have important consequences for firms' capital structure choices (Heider and Ljungqvist 2015, Panier et al. 2013). Despite its importance in the real world, the existing theoretical literature provides little justification for such a subsidy. Most theories suggest that, if debt financing has externalities, then they are negative (e.g., Lorenzoni 2008), implying corporate debt should be taxed, not subsidized. This belief has been the driving force behind policy proposals to eliminate the debt subsidy over the last two decades. For example, the President's Advisory Panel on Federal Tax Reform (2005) recommended that the tax system should "provide a more level treatment of debt and equity financing for large businesses" (p. 102) and justified it in the following way:[1]

> The tax bias against corporate equity encourages firms to rely on debt more than they would if the tax system imposed no such bias. The use of higher debt levels by corporations, known as "leveraging," may increase the risk of bankruptcy and financial distress during temporary industry or economy-wide downturns.

The backbone of these models is that *temporary* shocks cause bankruptcy of efficient firms. However, as early as Schumpeter (1934), economists have recognized that liquidating inefficient firms can also be important when economies are transitioning from *permanent* demand and technological innovation shocks, which render some industries obsolete and at overcapacity. These declining industries are an important part of the economy, since industries become large by supplanting industries, which were large in the past. In the United States, for example, 56% of manufacturing output in 2010 was produced in declining industries.[2]

Weak firms in declining industries frequently do not exit quickly and may even continue to invest, as is eloquently described by Jensen (1993, p. 847):

> In industry after industry with excess capacity, managers fail to recognize that they themselves must downsize; instead they leave the exit to others while they continue to invest. When all managers behave this way, exit is significantly delayed at substantial cost of real resources to society.

[1] President's Advisory Panel on Federal Tax Reform (2005, p. 100). Several tax reform proposals during the Clinton administration had proposed the elimination of unequal treatment of debt and equity (Congressional Budget Office 1997). Recently, the Chairmen of the Senate Finance and House Ways and Means Committee had requested a study of tax treatment of debt and equity (McKinnon 2011).

[2] Declining industries are those whose real output has declined from 2000 to 2010. Data source: Bureau Labor Statistics data on three-digit NAICS industries.

Restructuring of industries can be a lengthy and painful process.[3] Debt financing, and resulting bankruptcy, in particular, have been shown to play an important role in reducing capacity in declining industries (Wurgler 2000, Maksimovic and Phillips 1998).

Consider the recent developments in the brick-and-mortar book retail industry. Amazon's entry had decreased demand for purchasing books in physical stores. As early as 2005, bookselling had become "a game of stealing market share from competitors" (Strauss 2005) with two major players, Borders and Barnes and Noble (B&N). Nevertheless, neither Borders nor B&N were reducing the number of stores they operated. In fact, Borders was investing in stores for most of the 2000s (Newman 2011). It was Borders' *bankruptcy* on February 16, 2011 that finally significantly reduced the number of stores.[4] B&N declined to purchase the liquidated Borders stores, since 70% were within five miles of their own stores. Instead, these stores were converted to sell other products (Mowbray 2011). Moreover, B&N is expected to cut 10% of its stores in the near future (Austen 2011). The bankruptcy of Borders terminated a costly war for market share, increasing the value of the whole industry. Our model aims to study the effect of subsidized debt financing on firms' exit in declining industries by capturing the forces in this example.

Our model embeds the classic conflict of interest between equity and debt (Jensen and Meckling 1976, Leland 1994) into a war of attrition model (e.g., Fudenberg and Tirole 1986, Bulow and Klemperer 1999). In this conflict, equity holders ex post fail to internalize the bankruptcy costs that accrue to debt holders and therefore file for bankruptcy earlier than would be optimal from the perspective of the firm as a whole. Similar to standard models, this conflict lowers firm value. In our model, however, the conflict also alleviates the socially wasteful war of attrition. Therefore, in sharp contrast to much of the previous literature, we emphasize that the conflict of interest between debt and equity within a firm can be socially desirable and illustrate a benefit of subsidizing debt.

The histories of how the debt tax subsidy was initially implemented differ across a wide variety of countries. We do not attempt to provide a unified explanation for why the subsidy was enacted. The primary goal of our paper is to provide a normative model highlighting the benefits of the subsidy. If grossly inefficient subsidies and taxes should erode over time, then a normative model may shed some light on why the debt tax subsidy still persists around the world, decades, and in some cases almost a century, after it was enacted. Interestingly, our model captures some of the richness in how the tax subsidy is implemented in the U.S. tax system. The Internal Revenue Service (IRS) encourages a conflict of interest by requiring that debt holders and equity holders are distinct entities, and have conflicting liquidation preferences, if debt is to be subsidized. These actions are consistent with those of a social planer in our model.

We start with a baseline model in which we demonstrate the benefits of subsidizing debt. Two firms engage in an asymmetric information war of attrition as in Bulow and Klemperer (1999). The industry is at overcapacity and supports only one firm: the presence of a firm exerts a nonpecuniary externality on the other firm. Before the war of attrition, each firm chooses its capital structure: equity holders borrow from a competitive debt market. During the war of attrition, equity holders decide when to default, leading to liquidation. The social planner can subsidize firms' debt repayment.

The distinguishing feature of our model is the two-dimensional type that determines firms' effective strength in the war of attrition. The first dimension is exogenously given productivity (real strength), and the second is endogenous amount of debt (financial strength). Because debt payments are tax subsidized, raising debt increases firm value. Increasing debt also distorts firm's exit time, lowering firm value. A firm's choice of debt equalizes these two effects on the margin.

We solve for equilibrium exit times and debt schedule in closed form. We first prove that, in equilibrium, more productive firms are stronger in the war of attrition even after accounting for their endogenous debt choice. Therefore, in equilibrium, from a given firm's point of view, its opponent's type becomes effectively one dimensional. This tractability of the model not only allows us to solve for the unique symmetric equilibrium of this game, but also has efficiency implications. One concern with subsidizing debt is that, in order to exploit the subsidies, productive firms might "over leverage" and exit sooner than less productive firms. Monotonicity in effective strength implies that there are no inefficiencies in the sorting of firms' exits: if a firm is exiting, then any less productive firms have exited already.

Although the socially optimal allocation requires that the relatively weaker firm exits immediately,

[3] This phenomenon was illustrated in the cereals, whiskey, and tin plate industries in the late nineteenth century; gasoline refining, baby food, leather tanning (Harrigan and Porter 1983), and steel industries (DeAngelo and DeAngelo 1991); defense industries after the cold war (Jensen 1993), as well as department stores with the entry of big box retailers, mainframe computers in 1970s and 1980s, the tire industry with the switch to radial tires (Jensen 1993), and (more recently) brick-and-mortar retailing with the advent of Internet retail.

[4] Borders started decreasing capacity in its international operations before bankruptcy. It also closed some of its U.S. stores before bankruptcy, but these closures were caused by its problems of servicing debt payments.

firms engage in a socially wasteful war of attrition. A debt subsidy induces all firms to increase their debt levels in equilibrium. Shorter exit times improve welfare by shortening wasteful wars of attrition and do not induce inefficient sorting of firms' exits. Therefore, a planner increases welfare through subsidizing debt. In fact, in the baseline model, in which the industry never recovers, a higher subsidy always increases welfare.

Our baseline model is designed to highlight the welfare benefits of debt, and not the costs, which have been well understood in the literature heretofore. Section 4 then shows that the trade-off between the benefits and costs of subsidizing debt hinges on the *duration* of industry distress. Relative to the baseline model, the industry recovers in the future. If the industry recovers quickly then debt may induce more exit (bankruptcy) than is socially optimal. This modification introduces the standard cost of subsidizing debt into the model, which the planner then has to trade off with the benefits outlined above. As the duration of industry distress increases, the benefit of subsidizing debt rises and the cost falls. Interestingly, using our setting we show that in an economy comprising several different industries with different durations of shocks, this trade-off always favors a strictly positive subsidy to debt. Moreover, for an economy with heterogenous industries, the optimal subsidy is not unbounded: raising it beyond a certain point can be welfare destroying. In this sense, our theory is robust to the economy with heterogeneous industries at different growth stages.

We consider an extension of coupon paying debt, which allows us to examine several possible policies of subsidizing debt. The policy implemented in practice provides subsidies only to profitable firms. We show this policy is preferred to an alternative policy that subsidizes debt payments to loss-making firms as well; the intuition is that not subsidizing loss-making firms raises the cost of fighting and incentivizes earlier exit. Therefore, the planner directs the subsidy at profitable firms, even though the goal of the policy is to entice low productivity firms to take on debt.

This result is particularly interesting, as alternative models have a difficult time generating it. Consider models in which bankruptcy is welfare destroying. The implementation of the subsidy as a corporate tax deduction seems at odds with this view: firms receive subsidies when they are in little danger of bankruptcy, but lose subsidies when they need them most to avoid the socially inefficient bankruptcy. Within our framework, on the other hand, this policy emerges naturally: subsidizing debt of loss-making firms lowers their costs of fighting, prolonging a war of attrition relative to the policy that does not do that.

Our model aims to show that, in contrast to most leading theories on incomplete financial markets, subsidizing corporate debt financing might provide welfare benefits. We take wars of attrition as given: examining the optimal way to terminate them is beyond the scope of this paper. We do consider several private and public means of reducing wars of attrition. Coase theorem logic suggests that a third party could purchase both firms and internalize the within-firm externality; this mechanism resembles mergers or large common investors. Section 5.3 shows that private information about productivity, which drives the war of attrition, is so extensive that a potential buyer incurs a loss any time the buyer attempts to internalize externalities. Therefore, there is room for the government to intervene with a debt subsidy.

One could also conceive of government policies, which could be finely tuned to target only industries at overcapacity, and a debt subsidy targeted at overcapacity industries is one of them. Our model suggests that it is more efficient to subsidize only profitable firms in overcapacity industries. In reality, subsidies are targeted at profitable firms even in profitable industries, which do not generate social benefits within our model. One possible reason is that narrowly targeted industrial policies can be subject to regulatory capture (Stigler 1971), which is less of a concern when facing broad-based interventions such as a debt subsidy. Analyzing alternative regulatory concerns, e.g., capture, although interesting, is well beyond the scope of this paper.

Last but not least, our model does not speak to the optimality of another large interest tax subsidy: the mortgage tax deduction. It is unclear that there is a mapping from inefficiently slow exit of firms in declining industries to residential home ownership.

Relationship to Past Literature. The purpose of our theory is to illustrate the welfare benefits of subsidizing debt financing. Debt financing can result in positive externalities through creative destruction.[5] In contrast, most leading theories on incomplete financial markets focus on the negative externality of debt when firms suffer temporary shocks.[6] Our approach is complementary to this literature. Instead of focusing on temporary shocks to productivity, we focus on permanent shocks that leave some industries obsolete and show that the duration of shocks is critical to the trade-off between costs and benefits of debt subsidies.

[5] For early work on creative destruction, see Schumpeter (1934); for recent work, see Jovanovic and Tse (2006).

[6] One particular mechanism wherein one firm's borrowing imposes negative externalities on others is the fire-sale channel (Shleifer and Vishny 1992). Given temporary shocks in productivity, efficient firms will be forced to cut back investment or even go bankrupt when collateral values decline.

Our modelling of capital structure is in the spirit of Leland (1994). In that model, optimal leverage balances the dead-weight bankruptcy cost due to the equity-debt conflict of interest with the benefit of the debt tax subsidy. Our model provides a normative benchmark for the existence of the subsidy. In our model, bankruptcy is still costly from the perspective of an individual firm, but becomes socially beneficial. The equity-debt conflict of interest leads leveraged equity holders to exit sooner than they would otherwise, which alleviates the socially wasteful war of attrition. However, only a positive tax subsidy can entice firms to borrow and impose the equity-debt conflict of interest on themselves.

Our model, by allowing firms to choose capital structure before entering the war of attrition,[7] introduces endogenous types in a setting similar to Bulow and Klemperer (1999) in order to study the welfare consequences of subsidizing corporate debt financing.[8] Methodologically, introducing an endogenous type is challenging. First, the war of attrition equilibrium derived in Bulow and Klemperer (1999) relies heavily on monotonicity in exit times. Since effective strength in our model is endogenously chosen through subsidized borrowing, it is not clear that more productive firms endogenously choose to be stronger in the war of attrition. We show that monotonicity is indeed preserved without imposing ad hoc properties on the war of attrition equilibrium. Second, the equilibrium borrowing schedule in the financing stage is intertwined with the equilibrium exiting schedule, which have to be solved simultaneously. We show that endogenous monotonicity is the key to solving the system of equilibrium schedules in closed form.

Debt financing has many potential benefits. In models with within-firm frictions, e.g., empire building, unverifiable investment, and asymmetric information, the second-best allocation is achieved through capital structure choices that maximize stakeholders' welfare. Private contracting achieves the constrained optimal allocation, so the social planner cannot increase welfare through intervention.[9] Our innovation is to highlight a between-firm externality so that the social planner can play a role. Our paper is in the same spirit as Almazan et al. (2015), who point out that the monetary and tax policies can affect welfare by influencing individual firms' financing and liquidation policies.

We also complement the literature exploring the interaction between firms' capital structure choices and industrial organization (e.g., Brander and Lewis 1986). Lambrecht (2001) analyzes the strategic exit decisions in a duopoly with perfect information in which firms' debt levels are exogenously given. Endogenous debt levels given a debt tax subsidy are critical for our paper. Empirically, Kovenock and Phillips (1997) document that in concentrated industries firms with more debt close more plants, which is consistent with our view.[10] Our paper is also related to the literature on capital reallocations and its macro consequences, for example, Ramey and Shapiro (1998, 2001) and Eisfeldt and Rampini (2008).

Our paper is complementary to the current literature on welfare consequences of corporate taxation. Although we focus on the welfare consequences of differential tax treatment of debt and equity financing, this literature has mainly explored welfare consequences of taxation of other corporate choices such as payout policy.[11]

2. The Baseline Model with Permanent Distress

2.1. Firms and Market

We work in continuous time without discounting. Consider an industry with two firms, indexed by 1 and 2. Each firm is endowed with a single unit of capital, and has a privately observed productivity parameter θ_i drawn from $\Theta \equiv [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} \geq 0$,

[7] For wars of attrition with asymmetric information, see Kreps and Wilson (1982) and Fudenberg and Tirole (1986). Siegel (2009, 2010) studies contests in which players have multidimensional types in a setting with complete information. Recently, Hopenhayn and Squintani (2011) consider the problem when firms' types stochastically change over time. Acharya et al. (2011) is a recent application of strategic timing games to corporate finance.

[8] Since we introduce a social planner into the war of attrition, the paper is also broadly related to the work on optimal contest structure. See, for example, Moldovanu and Sela (2001) or Che and Gale (2003).

[9] However, if in practice firms face restricted contracting spaces, then private contracting cannot achieve constrained efficiency and as a result there is room for government intervention. Alternatively, if managers are conservative given other agency frictions (say reputation concerns, Zwiebel 1995) and take too little leverage, then subsidy to debt can help as well.

[10] In a similar vein, Zingales (1998) shows empirically that financially weak firms (i.e., firms with high leverage) exit sooner following an industry-wide profitability shock. Chevalier (1995) and Chevalier and Scharfstein (1996) find evidence that supermarkets undergoing leveraged buyouts invest less in future customers by raising prices.

[11] Gordon and Dietz (2006) and Chetty and Saez (2010), for example, evaluate the welfare consequences of dividend tax changes. Their focus is on taxing firms' payouts to investors and the resulting distortions. Therefore, they assume that the only outside financing in their model is equity. In our model, payout policy does not play a role, since we allow equity holders to costlessly inject or remove cash from the firm. See Auerbach (2002) for an extensive review of this literature. The literature on dynamic optimal corporate taxation focuses on optimal capital taxation in general equilibrium models, in which financing does not play a role (e.g., Farhi and Werning 2012). Instead, we hold corporate taxation fixed and explore the planner's problem of subsidizing debt financing.

and a cumulative distribution function $F(\cdot)$. The density $f(\cdot) = F'(\cdot)$ is continuous and strictly positive everywhere; we denote the hazard rate by $h(\theta) \equiv f(\theta)/(1 - F(\theta))$. Firms' productivities θ_i and θ_j are independent and identically distributed. Without loss of generality, we index the more productive firm as firm 1 so that $\theta_1 > \theta_2$. Productivity θ_i is each firm's private information.

The cash flows of a given firm depend on whether its competitor is still present in the industry. Following Bulow and Klemperer (1999), at any point in time t, firm i produces cash flows of $-k$ if its opponent, firm j, has not yet exited the industry, and realizes a lump sum profit of θ_i when firm j exits.[12] We can also interpret the positive constant k as a nonpecuniary externality that the presence of one firm imposes on the other firm. In the book industry example in the introduction, both firms maintain their stores simply to steal business from the other firm. Alternatively, one can consider this k as a wasteful component of investment to maintain market share, for example, through advertising.

At any time, the firm can be liquidated and the unit of capital is then redeployed to an alternative investment that yields a value of l, which we normalize to zero. This liquidation can involve direct disinvestment of capital, but one can also interpret the liquidation as cessation of investment activities that the firm needs in order to stay in business. For simplicity, we do not allow partial liquidation.

In this baseline model, this industry is always at overcapacity: the first-best allocation results in an immediate exit of the less productive firm. In §4, we consider a more realistic model in which industries may recover in the future to support two firms; this allows distress to be temporary rather than permanent. Last, we consider an economy, which comprises multiple industries with different durations of distress. For simplicity, we only focus on firm surplus and do not model the consumer side of the industry. In a model with consumers, once a firm exits, the remaining firm may increase prices and decrease consumer surplus, which should also be a part of the welfare calculation. Our analysis carries through even if there is a wedge between firms' profits and welfare, as long as welfare is increasing in the productivity of the surviving firm (see §3.3.1).

2.2. Financing

Each firm's unit of capital is initially financed entirely by equity, owned by a single party at each firm, the "equity holder." Before engaging in the war of attrition described above, a firm can raise debt financing, a stage that we call "the financing stage." The amount of debt that each firm takes on is unobservable to the competitor. Debt is available from competitive banks who, unlike competitors, observe firms' types. This is consistent with the idea that the bank will perform thorough due diligence once it establishes a financing relationship with the firm. From the theoretical point of view, these informational assumptions allow us to focus on the basic friction in the model, which is the firm's private desire for excess continuation in the war of attrition.

Debt is in the form of a callable bullet loan with face value $b \geq 0$: the loan continuously rolls over and the bank can decide if it wants to collect b at any point in time. Equity has limited liability, and can decide to default and leave the firm to the bank at any point in time. The bank cannot run the firm in the current industry and redeploys the capital to alternative investment.[13] These assumptions imply that as long as both firms are in the industry, the bank will keep rolling the debt over, and the equity holder defaults whenever she decides to exit the industry. In §5.2, we show that the results carry through if financing is in the form of coupon debt, in the manner of Leland (1994).

The tax rate is $\pi \in (0, 1)$. Our focus is the government's tax subsidy for debt financing relative to equity financing for a given level of overall corporate taxation—we do not want the subsidy to affect the overall after-tax cash flows of the firm. We therefore fix *after-tax cash flows:* the fighting cost $-k$, the winning cash flow θ_i, and the debt payment of the firm b are all in after-tax dollars.[14] By doing so, changing the tax rate π only affects the level of debt subsidies, but not the firm's cash flows.

When the war is over, the loser defaults and there is no debt payment to the loser's bank. Suppose the winner takes an after-tax debt payment b; then the winner pays the bank b, but after subsidy the bank

[12] Consider a setting in which firms obtain flow payoffs and the discount rate is r. The profitability index is $\theta_i \in [\underline{\theta}, \bar{\theta}]$. The fixed cost each period is $2k$ and there is no marginal cost of production. There is a market of size Π. With two firms, the profit flow of each firm is $\Pi/2 - 2k + r\theta_i$ and with one firm it is $\Pi - 2k + r\theta_i$. Set $\Pi = 2k$ and $r \to 0$. Then, as two firms compete, the profit flow is $-k + r\theta_i \to -k$, and the present value from winning is $\int_0^\infty e^{-rt} r\theta_i \, dt = \theta_i$.

[13] In other words, we invoke the standard assumption that bankruptcy reduces the actual productivity of the firm: as the bank takes over, it does not, for example, have the human capital required to run the firm efficiently. It is also worth pointing out that cutting back investment rather than redeploying is the key driver of our model. Therefore, our model applies even if the corporate form of financially distressed company survives under Chapter 11, as long as the bankruptcy procedure results in capacity cuts (see, e.g., Borenstein and Rose 2003).

[14] Given the tax rate π, the before-tax fighting cost is $k/(1 - \pi)$, the before-tax winning cash flow is $\theta_i/(1 - \pi)$, and the debt payment is $b/(1 - \pi)$.

receives $b/(1-\pi)$ in total. Effectively, for every dollar received by the bank, the government contributes π dollars and the firm pays $1-\pi$ in after-tax terms.

It is worth pointing out that in the resulting equilibrium the firm earns the subsidy only when it is profitable, which is consistent with the implementation of debt tax shields in practice. In our setting, there are no debt payments while firms fight (and incur losses). Once the war ends, the loser defaults on the debt and no subsidy is paid. Moreover, we later show that endogenously, any winning firm earns positive after-tax profits (Theorem 1), earning a positive debt tax subsidy of $\pi b/(1-\pi)$. In §5.2 we explicitly study the difference between a realistic subsidy, which only subsidizes profitable firms and a subsidy, which also subsidizes debt payments by loss-making firms.

2.3. Equilibrium

We study symmetric pure strategy perfect Bayesian equilibria of the game. Firms first choose debt financing and then engage in a war of attrition in which firms have a two-dimensional type. The first dimension is the exogenously given productivity θ. The second dimension is the amount of debt b, which is endogenously determined.

2.3.1. Equilibrium Debt Schedule and Equilibrium Exit Strategy. Consider firm i with type θ_i. The information that arrives as time passes is whether its opponent, firm j, had exited up to this point. Throughout, we say that firm i follows an exit strategy t (exits at t), if it chooses to exit at t given that firm j has not exited at t^-, with $t^- \equiv \lim_{s\uparrow t} s$ as the left limit of t.[15] Firm i's optimal exit time depends on its productivity θ_i and its debt burden b_i. More importantly, the firm's optimal exit time also depends on the exit strategy of firm j, which we denote by T_j. From the perspective of firm i, who does not observe opponent j's productivity nor its debt choice, the sufficient statistic for firm j's strategy is the distribution of firm j's exit time, which is denoted by $G^j(t) \equiv \Pr(T_j < t)$. We can write firm i's optimal exit strategy as $T(\theta_i, b_i; G^j(\cdot))$.

In any symmetric equilibrium, $G^j(\cdot) = G(\cdot)$. Let $B(\cdot)\colon \Theta \to \mathbb{R}^+$ denote the equilibrium debt schedule, so that the firm with productivity θ_i borrows $B(\theta_i) \geq 0$ in equilibrium. The distribution of (the opponent's) exit time depends on the effective strength of all potential opponents, which in turn depends on the equilibrium debt schedule $B(\cdot)$. From now on, we write $T(\theta_i, B(\theta_i); B(\cdot))$ as firm i's equilibrium exit time strategy. Because of symmetry, $T(\theta, B(\theta); B(\cdot))$ is also the equilibrium exit strategy of its opponent j with type θ, and we have in equilibrium

$$G(t) = \Pr(T(\theta, B(\theta); B(\cdot)) < t). \tag{1}$$

Three points are noteworthy regarding Equation (1). First, in general T might be nonmonotone in firm's productivity θ: more productive firms may exit sooner than some less productive firms. Second, there might be times at which no firms exit, or several types exit at the same time: $G(t)$ may be an integral of disconnected intervals and may involve jumps. Third, Equation (1) embeds a fixed-point relation. The cumulative distribution function of the opponent's exit time $G(t)$ enters $T(\cdot,\cdot;\cdot)$ on the right-hand side through the equilibrium debt schedule, $B(\cdot)$. The equilibrium exit times, $T(\cdot,\cdot;\cdot)$, are a function of the debt schedule, $B(\cdot)$, which, in turn, is a function of equilibrium exit times, $T(\cdot,\cdot;\cdot)$.

2.3.2. Equity Holder's Problem: War of Attrition Stage. We carry out our analysis backward by first studying the war of attrition game. The payoff to equity from a firm with productivity θ, which has chosen debt b and exits at time t, has a payoff of

$$E(\theta, b, t) = \underbrace{(1-G(t))(-kt)}_{\text{Firm exits before opponent}} + \underbrace{\int_0^t ((\theta - b) - kx)\, dG(x)}_{\text{Firm survives after opponent exits}}. \tag{2}$$

The first term captures the event when the firm loses the war of attrition: with probability $1-G(t)$ its opponent exits after t, and the equity holder loses k continuously until t. The second term captures the event when the firm wins the war of attrition. If the opponent exits at time x, the equity holder's value is its payoff from winning, θ, minus the debt that is now due, b, and minus the realized cost of fighting kx:

$$(\theta - b) - kx.$$

The second term in (2) integrates over the opponent's exit time $x \in [0, t]$ according to the distribution $G(\cdot)$. The equity holder chooses exit time to maximize the value of equity during the war of attrition:

$$T(\theta, b; B(\cdot)) = \arg\max_{t\geq 0} E(\theta, b, t). \tag{3}$$

2.3.3. Financing Stage. As is standard (e.g., Leland 1994), equity cannot commit to a particular exit strategy. The bank, therefore, infers a firm's expected default time from its productivity and its choice of debt. Banks' ex post payoff depends on the

[15] The left-limit definition implies that the firm exiting decision only depends on the information set right before t. Intuitively, if both firms set exiting strategy t, then both firms will exit. This treatment plays a role when firm j's strategy is such that, from firm i's perspective, it places a positive probability mass of exiting at t. We show that, in equilibrium, this never occurs so this treatment is innocuous.

before-tax promised debt payment $b/(1-\pi)$, and the probability of repayment. The probability of repayment is $G(T(\theta, b; B(\cdot)))$[16] and the bank's expected payoff, which is also the competitive debt price, is

$$D(\theta, b) = G(T(\theta, b; B(\cdot)))\frac{b}{1-\pi}. \tag{4}$$

There are two implicit assumptions underlying this debt value expression. First, each bank finances only one firm for whom it knows the productivity through due diligence; it does not observe the opponent's productivity nor their debt choice. Second, we have assumed that $\theta > b$ so that the debt is paid in full once the opponent exits. In other words, firms never borrow more than their own productivity, a property that always holds in equilibrium (Lemma 2).

The equity holder chooses the debt of firm θ to maximize her total wealth at the financing stage, which consists of the price obtained for issuing debt b, (4), as well as her equity value, (2). The equilibrium level of debt, $B(\theta)$, solves the following problem:

$$B(\theta) = \arg\max_{b\geq 0}\left\{G(T(\theta, b; B(\cdot)))\frac{b}{1-\pi} + E(\theta, b, T(\theta, b; B(\cdot)))\right\}. \tag{5}$$

Because equity cannot commit to an exit strategy, the debt choice b influences the firm's ex post exit strategy $T(\theta, b; B(\cdot))$, which feeds back into the competitive debt price paid by banks.

The equilibrium in this model consists of the debt schedule $B(\cdot)$ and the exit time strategy $T(\cdot,\cdot; B(\cdot))$ so that competitive banks earn zero profits, i.e., (4) holds; and the equity holder in each firm maximizes her value in each stage, i.e., (3) and (5) hold. Note that for each firm, the conjectured opponent's exit time distribution has to be consistent with the equilibrium debt schedule and the equilibrium exit strategy, i.e., (1) has to hold.

3. Equilibrium with Subsidized Debt

In this section we first show that in equilibrium, more productive firms are always stronger in the war of attrition, even accounting for their endogenous choices of debt. This critical property allows us to solve for the equilibrium exit times, the equilibrium debt schedule, and welfare implications of the model.

3.1. Equilibrium Debt Schedule and Exit Times

3.1.1. Monotone Effective Strength. We define a firm's "effective strength" as the payoff from winning the war of attrition net of debt payments, $\theta - b$. From (2) we see that it is effective strength that determines the payoff from the war of attrition and therefore drives differences in firms' exit times. The next lemma shows that the firm with greater effective strength exits later.

Lemma 1. *The equilibrium exit time $T(\theta, b; B(\cdot))$ is a function of effective strength $\theta - b$ only, so we denote $T(\theta, b)$ by $T(\theta - b)$. $T(\cdot)$ is increasing, and strictly increasing when $G(T(\cdot)) \in (0, 1)$.*

To see the intuition, which is similar to Bulow and Klemperer (1999), assume that $G(t)$ is differentiable with density $g(t) \equiv G'(t)$;[17] then, from (2), the marginal benefit to equity from staying longer is $g(t)(\theta - b)$. Because the marginal benefit is strictly increasing in $\theta - b$, firms with a higher effective strength $\theta - b$ have an incentive to exit later. Of course, once a firm is certain to win the war, $G(T(\theta - b)) = 1$, then increasing effective strength does not affect its exit time, and vice versa for $G(T(\theta - b)) = 0$.

From now on, we denote $T(\theta, b)$ by $T(\theta - b)$. The next lemma is the central result in this section:

Lemma 2. *In equilibrium, a firm's effective strength in the war of attrition, $\theta - B(\theta)$, is strictly increasing in θ.*

The intuition is as follows. Using Lemma 1 such that only the effective strength $\theta - b$ enters the equity value and the exit time, the equity holder maximizes (5):

$$G(T(\theta - b))\frac{b}{1-\pi} + E(\theta - b, T(\theta - b)). \tag{6}$$

Imagine two firms differing in their productivities, $\theta_1 > \theta_2$, but hypothetically having the same effective strength, i.e., $\theta_1 - b_1 = \theta_2 - b_2$. We compare their incentives to marginally increase debt b further. First, equal effective strength implies the same marginal impact on equity value E in (6). The marginal impact of increasing b on debt value $G(T(\theta - b))(b/(1-\pi))$ is

$$G(T(\theta - b))\frac{1}{1-\pi} - G_b(T(\theta - b))\frac{b}{1-\pi}. \tag{7}$$

In (7), $G(T(\theta - b))$ and $G_b(T(\theta - b))$ are the same across both firms, and $G_b(T(\theta - b)) < 0$ as increasing debt leads to earlier exit and less chance of winning. The debt burden "b" in the second term in (7) differs: equal effective strength implies that the more productive firm θ_1 borrows more ($b_1 > b_2$), and thus it has lower incentives for increasing debt. In other words, in equilibrium, the more productive firm wants to decrease its debt burden, raising its effective strength (above the less productive firm).

[16] Suppose that the firm takes the exiting strategy t, but $G(t)$ features a jump so that $G(t) < G(t^+) = \lim_{s\downarrow t} G(s)$, which implies that there is a positive mass of probability that the opponent will exit at t as well. Since we take the convention that the firm i exits according to information at t^-, the chance of survival is $G(t) = \Pr(\hat{T}_j < t)$.

[17] The proof does not rely on this assumption.

3.1.2. Equilibrium Exit Times. Lemma 2 shows that, in equilibrium, from a given firm's point of view, its opponent's type becomes effectively one dimensional. This property greatly simplifies the analysis: we can work directly with the underlying distribution of productivity θ, rather than integrating over two dimensions of opponent's type (θ and b). Given any potential equilibrium debt schedule $B(\cdot)$, we can characterize exit times along the lines of Bulow and Klemperer (1999), with modifications to account potential discontinuities in $B(\cdot)$.

Combining Lemmas 1 and 2, we know that the equilibrium exit time is strictly increasing in firm's productivity. The following proposition derives the equilibrium exit time $\hat{T}(\theta; B(\cdot))$ only as a function of underlying productivity.

Proposition 1. *The equilibrium exit time $T(\theta, B(\theta); B(\cdot))$ is strictly increasing in θ. Given an equilibrium debt schedule $B(\cdot)$, the equilibrium exit time is*

$$\hat{T}(\theta; B(\cdot)) \equiv T(\theta - B(\theta); B(\cdot)) = \int_{\underline{\theta}}^{\theta} h(y)\frac{y - B(y)}{k}\,dy. \quad (8)$$

Weaker firms always exit earlier than stronger firms; i.e., $\hat{T}(\theta; B(\cdot))$ is strictly increasing in θ. And, in equilibrium, the cumulative distribution function for exit time $G(t) = F(\hat{T}^{-1}(t)) = \Pr(\theta < \hat{T}^{-1}(t))$.

To see the intuition behind (8), consider the decision of type θ to fight a bit longer ($d\hat{T}(\theta)$). The cost of fighting is $kd\hat{T}(\theta)$. The benefit is that the opponent may exit over that interval, and the firm reaps the benefit of $\theta - B(\theta)$. The exit times are strictly monotonic in productivity, and the equilibrium is symmetric. Then the conditional probability of an opponent's exit, if she has not exited up to this point, is the hazard rate $h(\theta)d\theta$ from the productivity distribution. To equate marginal cost with marginal benefit so that $kd\hat{T}(\theta) = (\theta - B(\theta))h(\theta)\,d\theta$, the optimal exit time satisfies

$$\frac{d\hat{T}(\theta)}{d\theta} = h(\theta)\frac{\theta - B(\theta)}{k}. \quad (9)$$

To obtain the exit time $\hat{T}(\theta)$, we integrate the marginal times for firms that exit before type θ, and the integration starts from $\underline{\theta}$ who sets $\hat{T}(\underline{\theta}) = 0$.

Proposition 1 shows the effect of the equilibrium debt schedule $B(\cdot)$ on exit times. As we can see in (9), higher debt reduces the payoff from winning the war, thereby inducing faster exit for each individual firm. This direction of distortion is essentially the same as the debt overhang effect in Leland (1994).[18] More interestingly, increasing debt for a given type decreases exit times for all higher types, because exit times are cumulative.

3.1.3. Equilibrium Debt Schedule. We now solve for the equilibrium debt schedule, which is one of the central results of the paper. For exposition purposes we heuristically derive the firm's first order condition (FOC) for choosing debt. A rigorous treatment is provided in the proof of Theorem 1 in Appendix A.

First, we study the direct effect of marginally increasing b by fixing the exit time at the equilibrium level $T(\theta - B(\theta))$. Issuing more debt leads to an increase in debt and a decrease in equity value. Without a tax subsidy, these two forces exactly cancel out, because, for a given exit time, b only affects the allocation of firm value between equity and debt in the spirit of Modigliani and Miller (1958). With a tax subsidy, this direct effect is positive as a higher debt burden increases the subsidy. This is the marginal benefit of debt, MB. Since the total tax subsidy is $(\pi b/(1-\pi))G(t) = (\pi b/(1-\pi))F(\theta)$, the marginal benefit is

$$MB = \frac{\pi}{1-\pi}F(\theta). \quad (10)$$

This direct positive effect is the main force through which the subsidy increases equilibrium debt levels.

Second, we study the indirect effect of increasing debt because of the endogenous change of exit time $T(\theta - b)$. Lemma 1 shows that increasing b shortens a firm's exit time, i.e., $T'(\theta - b) > 0$. Because exit time is chosen to maximize equity value, the envelope theorem suggests a zero first-order change in equity value in (2) from a marginal change in exit time. However, a lower exit time T has a negative first-order impact on debt value $D = (b/(1-\pi))G(T(\theta - b))$, because the probability of winning the war declines. This is the marginal cost of increasing debt:

$$MC = \frac{b}{1-\pi}\frac{dG(T(\theta - b))}{db} < 0. \quad (11)$$

[18] Our result goes through as long as levered firms liquidate/exit earlier than unlevered firms, which is consistent with empirical evidence (Kovenock and Phillips 1997, Zingales 1998). We choose debt overhang in Leland (1994) to be the particular mechanism to achieve this effect. This is the case in the setting above, as well as if we introduce coupon paying debt as in Leland (1994). Debt financing resulting in early liquidation is a feature of a wide class of models. For instance, in Bolton and Scharfstein (1990), debt leads to early liquidation to provide borrowers with incentives to pay back loans; in Rajan (1992) bank loans liquidate the firm early in bad states; and Aghion and Bolton (1992), Hart and Moore (1994), and Dewatripont and Tirole (1994) obtain this early liquidation in incomplete contracting frameworks. Our result is therefore general in this regard. There is another potential debt-induced distortion, risk shifting, in which equity with a close-to-zero value may want to gamble for resurrection. In general, equity holders only internalize the risk on the upside, but not the negative impact of downside risk on debt. If delay leads to significantly more downside risk than exit does, debt may induce delay. In our model, staying in the industry yields a stochastic payoff with upside risk, winning the war, so this "risk-shifting-caused-delay" is absent. We do not deem this theoretical possibility as a major drawback of our theory. From a policy perspective, early liquidation has been emphasized as the most significant *negative* consequence of subsidizing debt. Second, exiting empirical evidence suggests that in practice, the dominating effect of debt is expediting firm exit (even if the opposite force coexists), which is all our theory requires.

Relative to the marginal benefit in (10), the marginal cost in (11) is more complicated: the decrease in exit time potentially depends on the distribution of the opponent's equilibrium exit time $\hat{T}(\cdot; B(\cdot))$, which in turn depends on the underlying productivity distribution and the equilibrium debt schedule $B(\cdot)$.

To calculate the marginal cost in (11) we use the insight from Proposition 1 that the probability of winning the war of attrition only depends on where a firm ranks in the distribution of effective strength. This allows us to compute (11) without relying on the equilibrium exit time $\hat{T}(\cdot; B(\cdot))$. We study how a small increase in debt changes the ranking of firm θ. Increasing b marginally from $B(\theta)$ by $\epsilon > 0$ lowers the firm's effective strength from $\theta - B(\theta)$ to $\theta - B(\theta) - \epsilon$. We can find a type $\theta(\epsilon)$ whose equilibrium effective strength is exactly $\theta - B(\theta) - \epsilon$,[19] i.e.,

$$\theta(\epsilon) - B(\theta(\epsilon)) = \theta - B(\theta) - \epsilon \quad \Rightarrow \quad \theta(\epsilon) = \theta - \frac{\epsilon}{1 - B'(\theta)}.$$

Therefore, increasing debt by ϵ changes the relative ranking by $\epsilon/(1 - B'(\theta))$. This is intuitive. If the slope of equilibrium schedule, i.e., $B'(\theta)$, is close to one, then in equilibrium the effective strength $\theta - B(\theta)$ increases slowly in θ. Consequently, increasing debt above the equilibrium level even by a small amount will cause a large decline in ranking of effective strength in the war of attrition.

To further translate the decrease in ranking into the marginal reduction of winning probability, we need to multiply this impact by the density of opponents $f(\theta)$. As a result, the marginal cost in (11) can be alternatively expressed in a more intuitive way:

$$MC = \frac{B(\theta)}{1 - \pi} \frac{f(\theta)}{1 - B'(\theta)}. \tag{12}$$

Equating the marginal benefit in (10) with the marginal cost in (12), the equilibrium debt schedule must satisfy

$$\pi F(\theta) = \frac{B(\theta) f(\theta)}{1 - B'(\theta)}. \tag{13}$$

Note that the equilibrium exit schedule $T(\cdot; B(\cdot))$ does not enter (13), which determines the equilibrium debt schedule $B(\theta)$. This property relies on endogenous monotonicity of effective strength established in Lemma 2.

In Theorem 1 we give the closed-form solution to the differential Equation (13), which characterizes the equilibrium debt schedule based on the primitives of the model. We further establish the uniqueness of the equilibrium debt schedule and show that there are no profitable global deviations from the debt schedule characterized by the first-order condition in (13).

[19] From the continuity of $\theta - B(\theta)$ (see proof of Lemma 1), we know that such a type exists.

Theorem 1. *There exists a unique symmetric pure strategy perfect Bayesian equilibrium. The equilibrium debt schedule is*

$$B(\theta) = F(\theta)^{-1/\pi} \int_{\underline{\theta}}^{\theta} F(y)^{1/\pi}\, dy, \tag{14}$$

with $B(\underline{\theta}) = 0$ and $B(\theta) < \theta$ for $\theta > \underline{\theta}$. In the war of attrition stage, the equilibrium exit times are given by (8) *in* Proposition 1, *with $B(\cdot)$ given in* (14).

With a positive debt tax subsidy, in equilibrium firms take on debt consistent with Theorem 1, which shortens exit times. Without a subsidy, the equity holder minimizes the conflict of interest by not borrowing:

Corollary 1. *If $\pi = 0$, the equilibrium is all-equity financing $B(\theta) = 0$. The debt schedule $B(\theta; \pi)$ is strictly increasing and exit times $\hat{T}(\theta; B(\cdot; \pi))$ are strictly decreasing in the tax subsidy π for all $\theta > \underline{\theta}$.*

When the debt tax subsidy increases, every firm has an incentive to take on a bit more debt. This direct effect is reinforced in equilibrium: Theorem 1 shows that any increase in debt by a given firm also has an indirect effect on increasing the debt level of all firms with higher productivity; we will come back to this upward indirect effect in §3.3.1. As debt reduces firms' effective strength, the tax subsidy shortens exit times.

3.2. Welfare Implications

In general, a debt tax subsidy could affect welfare in our model through two distinct channels. The first is through sorting conditional on firm exit; i.e., if a firm exits, is it the relatively less productive firm that leaves the industry? Second, even if weaker firms exit first, so that the sorting is efficient, how long is the socially wasteful war of attrition? In this section, we analyze how the debt subsidy affects these two dimensions of welfare.

3.2.1. Efficient Sorting Property with Endogenous Financial Strength. We first discuss whether sorting conditional on exit is efficient. In other words, whether it is the relatively low productivity firms that exit first. In a standard asymmetric information war of attrition without debt financing, sorting is efficient (see Fudenberg and Tirole 1986, Bulow and Klemperer 1999). Proposition 1 shows that sorting is efficient in our model with endogenous debt choice.

That sorting should be efficient is not obvious in our model. When firms enter the war-of-attrition game, their strength has two dimensions: their exogenous productivity θ and their endogenous debt $B(\theta)$. A priori, once debt is subsidized, the equilibrium effective strength in the war of attrition need not be monotone in the underlying productivity. If more productive firms borrow too much, it is possible that they are effectively weaker than less productive firms, which borrowed less aggressively. In fact, a casual argument would suggest that more productive firms should take on more debt to exploit the attractive subsidies. Under this logic, it may be possible to violate efficient sorting.[20] However, Proposition 1 guarantees sorting efficiency in our model: more productive firms never "over borrow" relative to less productive firms, and therefore, in equilibrium, less productive firms always exit earlier. As a result, conditional on firm exit, it is the less productive firm that exits in equilibrium.

3.3. Subsidizing the Equity-Debt Conflict of Interest of Weaker Firms

The previous section implies that welfare only depends on how soon the weak firm exits the industry. We now study the effect of the debt tax subsidy on firms' exit times. Debt induces the well-known conflict of interest between equity holders and debt holders (e.g., Jensen and Meckling 1976, Leland 1994). A debt financed firm defaults earlier than it would otherwise, because ex post equity does not internalize the losses to the bank in the war of attrition stage. But ex ante, at the financing stage, the equity internalizes any cost that it imposes on debt during the war of attrition stage through competitive debt pricing. If there is no tax subsidy, debt financing reduces equity holders' value in the financing stage by distorting exit times in the war of attrition. Without a subsidy, the equity holder minimizes the conflict of interest by not borrowing.

A debt tax subsidy encourages firms to take on debt, and therefore intensifies this conflict of interest between debt and equity. The proponents of abolishing the subsidy argue that this conflict is costly for firms, and therefore subsidizing it is welfare destroying. In our baseline model, on the other hand, this conflict of interest shortens a wasteful war of attrition. Firms do not internalize this benefit, which accrues to the opponent who wins the war. Thus, a debt subsidy is required to encourage the equity-debt conflict of interest, which improves welfare in overcapacity industries. Therefore, in contrast to most of the previous literature, we emphasize that the equity-debt conflict of interest can be socially desirable and needs to be subsidized. We explore the trade-off between this effect and more traditional forces in §4.

[20] More specifically, highly productive firms may borrow a lot and take a chance that their opponent is weak enough. However, they may be unlucky, draw a moderately productive opponent who took on less debt, and lose the war of attrition to that opponent.

3.3.1. Optimality of Positive Debt Tax Subsidy. Denote by $s(\theta, B(\cdot))$ the surplus to a firm with productivity θ, given the equilibrium debt schedule $B(\cdot)$ given in (14) in Theorem 1. Then, we have the following:

Lemma 3. *The firm θ's surplus can be written as*

$$s(\theta, B(\cdot)) = \int_{\underline{\theta}}^{\theta} f(y)[\theta - (y - B(y))]\, dy. \tag{15}$$

The expression in (15) transparently illustrates the effect of debt on equilibrium welfare. A higher debt schedule $B(\cdot)$ decreases exit times, which improves equilibrium welfare in a particularly simple way. At the first-best allocation, the weaker firm $y \in [\underline{\theta}, \theta)$ exits immediately without fighting, and the firm θ's expected surplus is $\int_{\underline{\theta}}^{\theta} f(y)\theta\, dy$. The expression for surplus in (15) is similar, but each weaker opponent y also imposes a dead weight cost on θ through their effective strength $y - B(y)$. The higher the debt schedule, the lower the effective strength of weaker opponents, the higher the welfare.

The total expected social surplus $S(\pi)$ is an integration of individual firm surplus $s(\theta, \pi)$ over all types θ:

$$S(\pi) = \mathbb{E}_{\theta}\left[\int_{\underline{\theta}}^{\theta} f(y) s(y, B(\cdot))\, dy\right]. \tag{16}$$

The following proposition formally shows that subsidizing debt increases welfare. This result is straightforward at this point: Equation (15) shows that higher debt levels lead to higher welfare. Increasing the subsidy π raises the equilibrium debt schedule (Corollary 1), therefore increasing welfare.

Proposition 2. *The expected social surplus $S(\pi)$ is strictly increasing in debt subsidy π, i.e., $S'(\pi) > 0$.*

The impact of the debt subsidy π on total surplus $S(\pi)$ is at work through two distinct forces: the first operates through equilibrium exit times, and the second through the equilibrium debt schedule. Consider the following thought experiment in which only firm θ obtains a higher debt subsidy π, which induces it to borrow a bit more. First, holding the rest of the debt schedule fixed, welfare increases for all firms that are more productive than θ—notice that a weaker firms' borrowing enters in the welfare of more productive firms in (15). This is due to the bottom-up cumulative feature of equilibrium exit times in (8). The second effect is on the equilibrium debt schedule because a higher debt by θ increases the borrowing of all types above θ from Theorem 1. The total welfare effect of a higher subsidy will be the full interaction and amplification of these two forces. Finally, keep in mind that our thought experiment only changed the subsidy for firm θ. When we consider the impact of the subsidy on welfare in Proposition 2, $S'(\pi)$, it is all firms that experience the increase.

Although we do not model the product market, it is useful to discuss whether our welfare results are robust to a modification in which consumer surplus decreases when a firm exits the industry, either because of increased market power of the remaining firm, or smaller availability of differentiated products. Our analysis carries through even if there is a wedge between firms' profits and welfare, as long as welfare is increasing in the productivity of the surviving firm.[21]

Our baseline model is designed to highlight the social benefits of debt only; we have intentionally ignored the associated social costs, which have been better understood in the literature heretofore. Now we move on to §4 to consider one leading cause of such welfare costs in light of the quote from the Report of the President's Advisory Panel on Federal Tax Reform (see the introduction), i.e., industries might recover after experiencing temporary distresses.

4. Temporary Distress and Industry Heterogeneity

In the baseline model above, we analyze an industry that has been hit with a permanent demand or technology shock, from which it can only recover by reducing capacity. In this case, higher subsidies lead to more welfare through faster exits. Suppose instead, that the demand shock is temporary, and the industry will eventually recover to support the full capacity. In this section, we study how changing the duration of industry shocks alters the role of subsidizing corporate debt financing.

We then turn from studying an industry to examining an economy comprised of industries with different durations of distress. We show that even in an economy populated with heterogenous industries, the optimal debt subsidy is still positive.

4.1. Temporary Distress

We study an industry that will eventually recover at time $T_d \geq 0$, at which point the war of attrition exogenously terminates. If both firms persist until T_d, they each obtain their corresponding payoff θ_i's. So an industry with T_d close to zero corresponds to a highly profitable industry; in contrast, the industry in the baseline model is nested as T_d goes to infinity, which demands an exit of one firm before returning to profitability.

4.2. Equilibrium Characterization

We now characterizes the equilibrium with temporary distress based on the results in §3. There is an endogenous threshold type $\hat{\theta}$. Firms below $\hat{\theta}$ have equilibrium debt schedules and exit times that are identical to the baseline permanent distress case. Firms above $\hat{\theta}$ never exit, and they borrow to the extent that their effective strength all equal that of the threshold type $\hat{\theta}$.

Proposition 3. *In the model with temporary distress $T_d \in [0, \infty)$, the equilibrium debt schedule is*

$$B(\theta) = \begin{cases} F(\theta)^{-1/\pi} \int_0^\theta F(y)^{1/\pi}\, dy & \text{for } \theta \leq \hat{\theta}, \\ \theta - \hat{\theta} + B(\hat{\theta}) & \text{for } \theta > \hat{\theta}; \end{cases}$$

and the equilibrium exit times are

$$\hat{T}(\theta) = \begin{cases} \int_{\underline{\theta}}^\theta h(x) \dfrac{x - B(x)}{k}\, dx & \text{for } \theta \leq \hat{\theta}, \\ \infty & \text{for } \theta > \hat{\theta}; \end{cases}$$

where the unique threshold type $\hat{\theta}$ is given by the solution to the equation

$$k(T_d - \hat{T}(\hat{\theta})) = \hat{\theta} - B(\hat{\theta}). \tag{17}$$

And, the threshold $\hat{\theta}$ is decreasing in π and increasing in T_d. If a solution $\hat{\theta}$ to (17) does not exist, $B(\theta)$ and $\hat{T}(\theta)$ are given by the baseline model with permanent distress.

To see the intuition, with temporary distress the equity holder's payoff $E(\theta, b, t)$ given the opponent's exit time distribution $G(t)$ becomes

$$E(\theta, b, t) = \begin{cases} (1 - G(t))(-kt) + \int_0^t ((\theta - b) - kx)\, dG(x) \\ \qquad \text{for } t \leq T_d, \\ (\theta - b) + \int_0^{T_d} (-kx)\, dG(x) \\ \qquad \text{for } t > T_d. \end{cases}$$

In short, if a firm exits before T_d, the equity holder's payoff is exactly the same as in the benchmark case $T_d = \infty$. If the firm decides to stay until T_d, however, the equity holder wins the prize θ for sure. We can follow the same argument as before to show that in the symmetric pure strategy equilibrium, the exit time $\hat{T}(\theta)$ is strictly increasing in θ (before T_d). Thus, there will be a threshold $\hat{\theta}$ such that firms above $\hat{\theta}$ never

[21] Suppose that total welfare, including consumer surplus, during the war of attrition is -2κ, $\kappa > 0$, and welfare after the war of attrition is an arbitrary increasing function of the productivity of the surviving firm $w(\theta) \geq 0$. The first-best allocation in this setting is for one firm to survive. Instead of the total surplus computed in (16), the surplus is $S = \mathbb{E}_\theta[\int_{\underline{\theta}}^\theta f(y)[w(\theta) - (\kappa/k)(y - B(y))]\, dy]$, and it is straightforward to see that welfare is increasing in the debt subsidy π.

exit. For types below $\hat{\theta}$, the game is the same as in the baseline. Because the firm θ's exit strategy only depends on exiting strategies of his weaker opponents, the equilibrium exit time for all types below $\hat{\theta}$ is the same as in the baseline model.

The same logic above implies the same equilibrium debt schedule $B(\cdot)$ for $\theta < \hat{\theta}$. Firms $\theta > \hat{\theta}$ borrow the highest amount of debt such that they never exit, which equalizes their effective strength with that of $\hat{\theta}$. Finally, the equation for the threshold $\hat{\theta}$, Equation (17), is determined from the indifference condition between exiting at $\hat{T}(\hat{\theta})$ and staying forever (beyond T_d) for type $\hat{\theta}$. Intuitively, conditional on staying up to $\hat{T}(\hat{\theta})$, no firms will exit beyond that point, and waiting till T_d has an additional fighting cost of $k(T_d - \hat{T}(\hat{\theta}))$ while the benefit is $\hat{\theta} - B(\hat{\theta})$.

Proposition 3 shows that the debt subsidy in industries with temporary distress affects firm exit on two margins. First, increasing debt reduces exit times of firms who would have exited even without a subsidy. This is the intensive margin of exit, which is also present in the baseline case with permanent distress. When industry distress is temporary, in addition to the intensive margin, subsidizing debt also affects the extensive margin of exit. As the debt tax subsidy increases equilibrium debt burdens, the endogenous threshold productivity $\hat{\theta}$ decreases. This induces firms, which would not have exited otherwise, into bankruptcy.

4.3. Duration of Distress and Welfare

Consider an economy comprised of industries each facing a different duration of distress. For example, there can be several industries with a distress duration T_d close to zero. Such industries are extremely profitable, and firms will exit only for very low realizations of productivity if they are not distorted through government policy. The economy can also comprise some declining industries, in which distress is permanent $T_d = \infty$. We will show that in such an economy, the optimal tax subsidy is still positive.

4.3.1. First Best. Since the industry recovers to support both firms at T_d, it may not always be socially desirable to eliminate the weaker firm. As before, index the stronger firm with θ_1. If both firms survive until T_d, the surplus is $\theta_1 + \theta_2 - 2kT_d$. Alternatively, if the weaker firm exits immediately, then the surplus is θ_1. Hence, the first-best allocation is that the weaker firm exits immediately if θ_2 is below $2kT_d$, which is the total fighting cost in the industry until recovery. In our baseline permanent distress case $T_d = \infty$, so that it is always socially efficient for the weak firm to exit immediately.

4.3.2. Optimality of Positive Debt Subsidy. In the baseline case, Proposition 2 shows that increasing debt subsidies always increases welfare. We now explore how changing the duration of industry distress affects this result. In comparing the allocation in Proposition 3 with the first best, the planner faces a trade-off. Increasing the subsidy π raises the endogenous threshold type $\hat{\theta}$ below which the weaker firm will exit. When $\hat{\theta} < 2kT_d$, increasing the subsidy π always produces welfare benefits. Once the critical point is reached, $\hat{\theta} = 2kT_d$, there are both benefits and costs. Increasing the subsidy improves the intensive margin of exit by accelerating weaker firms' default for $\theta_2 \le 2kT_d$. But it also forces the exit of firms $\theta_2 \in [2kT_d, \hat{\theta}]$ that should not exit under the first-best allocation. The second effect captures the standard argument of why subsidizing debt is socially inefficient: the debt subsidy induces firms to take on too much debt, leading to socially excessive default.

Despite this standard negative force, it is interesting to see that the welfare optimal debt subsidy π is positive for any industry with even temporary distress. Without a debt subsidy, the same argument as in Corollary 1 implies that firms refrain from debt financing. But with only equity financing, according to (17) the threshold type

$$\hat{\theta} = kT_d - k\hat{T}(\hat{\theta}; B = 0) < 2kT_d. \tag{18}$$

As a result, the planner can increase welfare on both the intensive and extensive margin by increasing the debt subsidy π from 0. This is a fairly general result: because a firm's exit benefits the surviving firm, the cutoff below which the planner would want to close the firm is higher than a firms' private choice if there is no subsidy; this is the economic reason behind (18). By slightly increasing the subsidy above zero, the planner can achieve the exit of desirable low productivity firms without exceeding the first-best cutoff. In addition, exit is now quicker. Marginally increasing the subsidy above zero therefore has no marginal cost, but a strictly positive marginal benefit.

On the other hand, when $T_d = 0$ so that the industry can always support two firms regardless of their productivity, any debt subsidy obtains the first-best allocation. Firms borrow the maximal possible amount for the highest possible subsidy. Since there are no negative shocks, firms can always repay their debt and never default.

Suppose that the planner only considers some broad-based interventions such as the debt subsidy, because fine-tuned industry-based tax policies involve prohibitive inefficiencies due to regulatory capture (Stigler 1971). Then, it is always optimal to set a strictly positive debt subsidy. This result is summarized in the following corollary.

Corollary 2. *For any configuration of an economy comprising industries with temporary or permanent distress, a strictly positive debt subsidy $\pi > 0$ is optimal as long as $T_d > 0$. When $T_d = 0$ for all industries, the subsidy does not affect firm exit.*

At this optimal tax subsidy $\pi > 0$, the welfare losses from growing industries in which firms take too much debt and thus default inefficiently early, are always dominated by the welfare benefits from expedited exits in overcapacity industries with either temporary or permanent distress. Furthermore, it is easy to see that our result goes through even if the industry may experience temporary distress in the future, although for the ease of analysis we have assumed that the temporary distress starts at $t = 0$. Then, unless all industries in the economy are in permanent growth phase, it is always optimal to set a strictly positive debt tax subsidy.

Consider the situation in which the planner could target specific industries. They would set a different positive subsidy for every industry, depending on the duration of its shocks. The difference from nontargeted subsidy is that more profitable industries would obtain lower subsidies. Of course, such targeted industry interventions would be subject to extensive lobbying by industries claiming they were distressed. If only nontargeted interventions are allowed, our model suggests that more profitable firms obtain larger subsidies. This result is reinforced in §5.2, when we show that even conditional on an industry subsidy, the subsidy should be awarded only to profitable firms.

Last, the subsidy we consider is not revenue neutral. Funding it may require the planner to raise distortionary taxes to finance it. This need not be the case because the subsidy is implemented through a reduction in corporate taxes, which may be particularly distortionary.

5. Extensions and Discussion

In this section we discuss several important aspects of our model. We first document that, in implementing tax subsidies, the IRS is trying to subsidize a conflict of interest between debt and equity holders. These rules, which seem puzzling from the perspective of existing models, are quite sensible through the lens of our model. We then show that our main result goes through if debt takes the form of perpetual coupon payments, in the manner of Leland (1994), instead of a callable bullet loan in our baseline model, in which payments only occur after a firm wins the war. Moreover, we show that the realistic implementation of debt tax shield, in which only profitable firms are subsidized, is preferred by the planner.

Our analysis takes wars of attrition as given: examining the optimal way to terminate them is beyond the scope of this paper. We do, however, consider some alternative means of reducing wars of attrition. Coase theorem logic suggests that a third party could purchase both firms and internalize the within-firm externality; this mechanism resembles mergers or large common investors. Section 5.3 shows that private information about productivity, which drives the war of attrition, is so extensive that a potential buyer incurs a loss any time the buyer attempts to internalize externalities.

5.1. Debt Tax Subsidy: Implementation

The histories of how the debt tax subsidy was initially implemented across such a wide variety of countries differ, hence a model would be hard pressed to provide a unified explanation for why the subsidy was enacted. If grossly inefficient subsidies and taxes should erode over time, then the normative model presented above may shed some light into why the debt tax subsidy still persists, and has not been removed even if standard theories show that it can lead to destruction of health firms. In that spirit, we discuss how our model captures some of the richness in the IRS tax treatment of securities for tax purposes. Firms can potentially issue a wide range of securities that they could in principle claim to be debt instruments, which therefore deserve tax preferential treatment. In 1994, the IRS set forth factors that they use in determining whether a particular security is considered tax-exempt.[22] From these factors we infer that the IRS is trying to subsidize a conflict of interest between debt and equity holders:

1. First, the IRS considers *"whether there is identity between holders of the instruments and stockholders of the issuer,"* i.e., debt should be held by a different entity than equity. If the same investor held both debt and equity in our model, she would simply maximize the (private) value of the firm, reverting back to the all equity case.[23]

2. Second, the IRS considers whether *"the rights of the holders of the instruments are subordinate to rights of general creditors,"* i.e., whether instrument holders are lower in priority of liquidation than general creditors (e.g., trade credit). Suppose the tax subsidized instrument had the same priority in bankruptcy as equity. Then equity would pari pasu participate in the liquidation of the company, ex post bearing a part of bankruptcy cost, which would partially resolve the

[22] IRS Notice 94-47, 1994-1 C.B. 357.

[23] This factor also increases the cost of equity purchasing the outstanding debt in order to stave off bankruptcy. If equity were to purchase the outstanding debt on the open market, they would not obtain the tax subsidy.

conflict of interest between debt and equity. Consequently, it dampens the incentive of equity holders to liquidate the firm, lengthening the war of attrition.

3. Finally, the IRS requires that the holders of debt should not have the right to participate in the management of the firm, giving equity the decision rights. If debt holders were allowed to participate in the liquidation decision in our model, they would delay liquidation, since they bear the cost ex post. This would allow equity to reap the benefits of the tax subsidy ex ante through debt pricing without shortening the war of attrition.

Even though the IRS probably did not implement these rules with our model in mind, it is intriguing that the IRS subsidizes securities that create a conflict of interest between equity and debt holders, especially in regards to bankruptcy. The literature, starting from Jensen and Meckling (1976), has been emphasizing the dark side of the equity-debt conflict of interest. From that perspective, IRS has willfully implemented rules that destroy welfare and lead to destruction of healthy firms. Our normative model provides a rationalization of these seemingly ad hoc rules.

5.2. Coupon Paying Debt and Subsidizing Only Profitable Firms

In the baseline model, debt is in the form of a callable bullet loan, where the bank continuously rolls over the debt and finally collects the principal payment $b \geq 0$. In this section we consider another polar case, in which firms finance themselves with debt in the form of a consol bond (in the manner of Leland 1994), i.e., the firm pays a constant coupon to the bank continuously until default.

The analysis in this section has two goals. First, we show that our main results are robust to coupon paying debt. Second, the setting with coupon paying debt allows us to explicitly compare two alternative policies of subsidizing debt. We show that the policy of subsidizing profitable firms only, which is implemented in reality, is preferred to a policy that also subsidizes debt payments to loss-making firms. In our baseline model, the difference between these two policies cannot be analyzed. It is because, as discussed at the end of §2.2, in the baseline model debt payments are made only by the winning firm, with a positive profit of $\theta - B(\theta) > 0$ when the war ends.[24]

5.2.1. Setting. We modify the baseline model as follows. The economy has a constant discount rate of $r > 0$. Instead of being modeled as the winning prize, now the privately observed productivity parameter θ_i is the after-tax cash flow that the firm receives every period. Taking into account the fighting cost when competing, firm i produces (after-tax) cash flows of $\theta_i - k$ if its opponent firm j stays in the industry, and produces θ_i if firm j has exited. Recall the hazard rate of the distribution of θ is $h(\theta)$.

Before engaging in a war of attrition, firms can raise debt financing from competitive banks. Debt is in the form of a consol bond, in the manner of Leland (1994), which pays a constant after-tax coupon b to the bank continuously until defaults. When equity chooses to default, the bank takes over the firm and receives zero as the liquidation value.

For ease of analysis, we assume that unlike the baseline model, banks do not observe firms' types. As a result, the first-stage financing game is a signaling game, and in the working paper version of this paper, we show that the key Lemma 2 and a varation of Proposition 1 (with the new equilibrium debt schedule) hold.[25] To highlight the comparison between two subsidy policies, we focus on the pooling equilibrium, where all firms borrow a constant after-tax coupon b, i.e., $B(\cdot) = b$.

5.2.2. Alternative Policies of Subsidizing Debt. Debt subsidies could be implemented in several different ways. The government could simply pay for a part of the interest that the firm owes. Alternatively, as in practice, the debt tax shield is implemented as a corporate tax deduction. In essence, the firm can only use the subsidy if it is profitable.[26] We now compare these two policies.

It can be shown that the relevant range of θ in determining the equilibrium exit schedule is $\theta \in [b, b+k)$, i.e., firms who are loss making during fighting but are profitable once they win the war. This is because only these firms will initially fight, and then decide to exit if their opponents turn out to be sufficiently strong.[27]

First suppose that debt tax subsidies are available even to loss-making firms. Then firms obtain pre-tax cash flows of $(\theta - k - b)/(1 - \pi)$ and a tax subsidy of $(\pi/(1 - \pi))(\theta - k - b)$, resulting in after-tax cash flows of

$$\theta - k - b. \tag{19}$$

This is not the case under the policy under which the tax deduction can only be used by profitable firms. These firms still obtain pre-tax cash flows of $(\theta - k - b)/(1 - \pi)$ but earn a tax subsidy of $(\pi/(1 - \pi)) \max(\theta - k - b, 0)$. For the relevant range

[24] Recall that, in equilibrium, the winning firm is always profitable as firms endogenously choose $b < \theta$—the equilibrium effective strength $\theta - b$ is always positive.

[25] Proof is available upon request.

[26] Losses from the interest debt tax shield can also be carried forward, so in principle the firm can use some of the subsidy once it becomes profitable in the future. The case of General Motors writing down \$39 billion in deferred tax credit is an example of how these subsidies may never come to fruition if the firm is not profitable enough.

[27] Firms with $\theta < b$ default immediately; conversely, firms with $\theta > b + k$ never exit.

of $\theta \in [b, b+k)$, these loss-making firms do not obtain any subsidies, and thus have after-tax cash flows of

$$(\theta - k - b)/(1-\pi). \tag{20}$$

Since $\theta < b+k$, firms suffer greater losses under the second policy. Finally, because winning is profitable ($\theta > b$), firms obtain the same subsidy after winning regardless of the policy, resulting in after-tax cash flows of $(\theta - b)/r$ if they win the war. In sum, for a given debt schedule, the only difference between the two policies is that the loss during fighting is higher if the debt subsidy only accrues to profitable firms.

Let $\mathbb{I}_{T+}$ be the indicator for the policy that only subsidizes profitable firms. We only present the essential results for differentiating the two subsidy policies; the technical proofs as well as proofs for the monotonicity of effective strength follow the arguments in §3.1. Because firms are pooled with a consol bond with coupon b, the type θ's first-order condition in setting its exiting strategy is

$$\frac{dT(\theta; B(\cdot))}{d\theta} = (1 - \mathbb{I}_{T+}\pi)\frac{h(\theta)}{r}\frac{\theta - b}{k+b-\theta} \quad \text{for } \theta \in [b, b+k). \tag{21}$$

The intuition is similar to (9): fighting a bit longer, $dT(\theta; B(\cdot))$, gives a marginal cost of $(k+b-\theta)/(1-\mathbb{I}_{T+}\pi)$, whereas the marginal benefit is the payoff from winning $(\theta - b)/r$, times the hazard rate $h(\theta)\,d\theta$ that the opponent will exit in the next instant.

In the working paper version of this paper, given $B(\cdot) = b$, we show that the equilibrium exiting time $T(\theta; B(\cdot))$ under either tax policy is given by[28]

$$T(\theta; B(\cdot)) = (1-\mathbb{I}_{T+}\pi)\begin{cases} 0 & \text{for } \theta < b, \\ \displaystyle\int_b^\theta h(x)\frac{x-b}{r(k+b-x)}\,dx & \text{for } \theta \in [b, b+k), \\ \infty & \text{for } \theta \geq b+k. \end{cases}$$

Compared to a policy that subsidizes all debt payments, the policy of subsidizing profitable firms decreases exit time by a factor of π, the subsidy rate. By not subsidizing loss-making firms, the social planner raises the cost of fighting, thereby incentivizing firms to exit earlier.

Because in our model earlier exit typically leads to higher welfare, the above result implies that it is socially desirable to have debt tax shield available only to profitable firms.[29] This result is particularly interesting, because most of the literature regarding debt with negative externalities relies on the presumption that bankruptcy is welfare destroying. The particular implementation of the subsidy to debt as corporate tax deduction seems at odds with this view, i.e., firms receive subsidies when they are in little danger of bankruptcy, but lose subsidies when they actually need them most to avoid the socially inefficient bankruptcy. However, this policy emerges naturally in our framework: a debt subsidy targeted at profitable firms is more efficient than a general subsidy to debt because it is more effective at driving out weaker firms.

Therefore, if subsidies cannot be targeted at industries, for example, because of regulatory capture as discussed in §4.3.2, our model would suggest that the planner subsidizes profitable firms only. These biggest beneficiaries of such a subsidy would be profitable firms in profitable industries.

5.3. Coasian Solution

In this subsection we examine a "Coasian" solution to the costly war of attrition. Specifically, we examine whether a third firm (called C) can purchase both firms in the industry, potentially close the less productive firm, and thereby internalize the cost. In that case the government does not need to use tax policy to reduce the war of attrition externality. Because firms' productivity is private information, the buyer faces an adverse selection problem. We show that adverse selection is so severe that it imposes losses on the buyer whenever she tries to internalize this externality. In other words, the Coasian solution fails, leaving room for government intervention.

We first consider static unconditional bidding, and then extend this case to conditional offers and the ability of firm C to make offers over time. We finish this section with a short discussion of mergers and acquisitions.

5.3.1. Static Bidding. Suppose that firm C can bid p to purchase the firms before the war of attrition. Both firms decide simultaneously whether to accept the bid, which is public information and there are no future bids. If a firm is indifferent then it accepts the bid.[30] Upon purchase, C learns the productivity of acquired firms and decides whether and which firms to close. If only one firm accepts the bid, then C inherits that firm's asset and competes with the other firm in the war of attrition.

Proposition 4 first shows that the equilibrium strategy of rejecting bids is monotone in firms' type. In equilibrium, there exists a threshold $\hat{\theta}$, such that

[28] One can verify pooling at $B(\cdot) = b$ with a set of specific off-equilibrium beliefs constitute an equilibrium; the details are available upon request.

[29] In the working paper version, we show this result formally, based on some distributional assumptions of θ (proof is available upon request).

[30] The result is the same if we break the tie in the other way, i.e., reject whenever indifferent.

firms, which are more productive than $\hat{\theta}$, reject the bid, and vice versa. Second, we characterize the unique pure strategy equilibrium of this game. To do so, we have to specify off-equilibrium beliefs. Suppose $\theta_i > \hat{\theta} > \theta_j$. In equilibrium, the strong firm $\theta_i > \hat{\theta}$ rejects the bid and the weak firm $\theta_j < \hat{\theta}$ sells its asset to C. Therefore, it has been revealed that C owns the weak firm, and in equilibrium, C exits immediately. On the off-equilibrium path, C may enter the war of attrition. We impose the off-equilibrium belief that the strong firm θ_i, which rejected the bid, forgets that C's asset is weak, and instead behaves as though the sold firm is drawn from the distribution of firms that should have rejected the bid. Note that this off-equilibrium belief increases incentives of θ_i to sell the firm, which improves the potential for a profitable acquisition by firm C, thereby favoring the Coasian solution. Even under such off-equilibrium beliefs we show that firm C earns strictly negative profits as long as any externalities are internalized, so the private solution always fails.

Proposition 4. *We have the following results*:

1. *Given p, the equilibrium strategy of rejecting bids is monotonically increasing in firm type.*
2. *Under the off-equilibrium belief specified above, given p there exists a unique pure strategy symmetric equilibrium. In this equilibrium, define the cutoff $\hat{\theta}$, which solves $\hat{\theta}F(\hat{\theta}) = p$. All firms below $\hat{\theta}$ accept the bid and all firms above $\hat{\theta}$ reject the bid. If both firms accept, firm C closes the weaker firm. If both firms reject the bid, they compete in a war of attrition with exit times given by Proposition* 1 *but truncated below at $\hat{\theta}$. If firm i rejects, while firm j accepts so the firm C buys firm j, firm C exits immediately* (*and firm i stays forever*).
3. *For any equilibrium in step* 2 *in which firms are sold with positive probabilities, the buyer's expected profits are strictly negative.*

The results in Proposition 4 are intuitive. As firm C raises its bid, it draws in marginally more productive firms, which increases the expected value of acquired firms and the probability that it internalizes the externality. Result 3 shows that this benefit is more than undone, since a price increase also accrues to less productive firms who would have accepted a lower bid as well. Therefore, it is optimal for firm C not to bid. We follow a similar argument in Appendix C to show that an offer, which is conditional on both firms accepting, fails as well.

5.3.2. Time-Varying Bidding Schedule. The previous section considers only static offers. Now suppose that firm C can offer a history dependent pricing schedule $\{p_1(\cdot), p_2(\cdot,\cdot)\}$ and both firms can choose to sell at any time. Each firm can be the first to sell its asset to C at a price of $p_1(t) \geq 0$ at time t or it can reject the bid; the sale is publicly observable. If there is only one firm (say i) that sells, then firm j competes with firm C (with asset θ_i) after t. Firm C can close the firm at any time, and may propose a continuation pricing offer $p_2(s,t) \geq 0$ for $s > t$. If both firms sell to C at the same time, then C may close either of the acquired firms any time. As before, if a firm is indifferent between accepting and rejecting the bid, it accepts the bid. We further require that firm C employs a pricing strategy that earns nonnegative expected profits at any point in time.[31]

This schedule is potentially an improvement over the static offer, because it may induce less productive firms to sell earlier at lower prices, thereby reducing the overall information rent that firm C is paying. However, similar to our earlier results, Proposition 5 shows that the private solution is still not viable. We first establish that, given any possible equilibrium pricing schedule, the equilibrium time of sale is increasing in firm type. We then show that for reasonable classes of equilibria, there does not exist an equilibrium in which firms are purchased with positive probability and C earns nonnegative profits.

Proposition 5. *We have the following results.*

1. *The equilibrium time of accepting the offer is monotonically increasing in firm type.*
2. *In any subsequent war-of-attrition game with common knowledge that one firm is strictly stronger than the other, we restrict attention to the equilibrium outcome in which the weaker firm exits immediately. Then, in the game with time-varying offering prices and without commitment, the C's expected profits are strictly negative in situations in which firms are sold with positive probability.*

5.3.3. Mergers and Acquisitions. A related channel to the Coasian solution are mergers and acquisitions, in which it is firms in the industry that acquire each other. Firms in the industry do not know which firm is stronger, and they face a similar adverse selection problem as the outside buyer in the Coasian solution considered above. In addition, the offer itself can be used as a signal of buyer's productivity and therefore subject to manipulation, making efficient exits impossible (Cramton 1992). Therefore, it is unclear whether the possibility of acquisitions within the industry reduces information frictions relative to the Coasian solution. An explicit analysis of acquisitions necessarily involves repeated bargaining with asymmetric information and is nontrivial to analyze in our framework (see, for example, Fuchs and Skrzypacz 2010, for a model with one-sided asymmetric

[31] This implies that firm C cannot commit to offer a pricing schedule that may become suboptimal (relative to withdrawing the bid) at some point in the future. For an example of selling with commitment, see Board and Skrzypacz (2014).

information). To sum up, private solutions to the war of attrition face potential market breakdown because of asymmetric information, providing a rationale for government intervention with a debt tax subsidy.

6. Conclusion

Allowing corporations to deduct interest payments from corporate taxes is a large government subsidy to debt financing. Such subsidies are common around the world and introduce a large wedge between the cost of equity and debt (Congressional Budget Office 2005). Contrary to practice, the theoretical literature implies that debt and equity should be treated equally, or that debt should be taxed more heavily than equity, if industries experience temporary negative shocks that lead to collateral-based spillovers. We show that, on the other hand, when economies experience shocks of longer duration in which it is socially efficient for low productivity firms to exit, debt financing generates welfare by facilitating efficient exit. Whether the debt subsidy should be abolished, as has been proposed several times, therefore depends on the magnitude of these costs and benefits.

The primary goal of our paper is to provide a normative model highlighting the benefits of the subsidy. Since the histories of how the debt tax subsidy was initially implemented across countries differ, we do not provide an explanation for why the subsidy was enacted. However, if grossly inefficient subsidies and taxes should erode over time, then our normative model may shed some light on why the debt tax subsidy still persists around the world. Analogously, the model can also rationalize a seemingly ad hoc feature of the tax system, which subsidizes debt payments for profitable firms rather than helping firms, which are trying to stave off bankruptcy.

The recent financial crisis has also spurred an extensive debate on subsidizing debt financing of financial intermediaries (e.g., Admati et al. 2013, Kashyap et al. 2010). Our model does not distinguish between financial intermediaries and other firms, so it can, in principle, also be applied to financial intermediaries. Given that financial intermediaries are more vulnerable to fire sales externalities than industrial firms (Diamond and Rajan 2011, Stein 2012), it is more likely that in their case the social cost of subsidizing debt outweighs the social benefit.

Our model does not speak to the optimality of another large interest tax subsidy: the mortgage tax deduction. In our model, the corporate debt tax subsidy is trying to resolve the inefficiently slow exit of firms in declining industries. It is unclear that there is a mapping from our model to residential home ownership. In fact, the current literature suggests that homeowner bankruptcy generates negative spillovers on other homeowners (e.g., Campbell et al. 2011).

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Appendix A. Proofs

For clarity of exposition, we recap some notations: $G(t)$ is the cumulative distribution function of equilibrium exit time in the war of attrition. For firm θ, given its exit strategy t and debt b, the debt value is $D(b, t) = (b/(1-\pi))G(t)$, and its equity value is

$$\begin{aligned} E(\theta, b, t) &= -(1-G(t))kt + \int_0^t [-kx + \theta - b]\, dG(x) \\ &= -k\int_0^t (1-G(x))\, dx + (\theta - b)G(t). \qquad \text{(A1)} \end{aligned}$$

Recall the optimal exit time $T(\theta, b) \in \arg\max_{t\in[0,\infty)} E(\theta, b, t)$, where $E(\theta, b, t)$ is given in (A1). The total payoff of the equity holder who chooses b is

$$\begin{aligned} V(\theta,b) \equiv v(\theta,b,T(\theta,b)) &= \left(\theta + \frac{\pi b}{1-\pi}\right)G(T(\theta,b)) \\ &\quad - k\int_0^{T(\theta,b)} (1-G(x))\, dx. \qquad \text{(A2)} \end{aligned}$$

Focus on the equilibrium exit time range $[0, \bar{T}]$, where the upper bound $\bar{T}$ may take the value of ∞ (in equilibrium it might). We have the preliminary lemma to ensure that $G(t)$ is well behaved in equilibrium.

Lemma 4. *$G(t)$ is continuous and strictly increasing for $t \in [0, \bar{T})$.*

Proof. We first show that $G(t) < G(t')$ for $0 \le t < t' \le T$. Suppose not; then no types exit between t and t'. But types who exit at t' have a strictly profitable deviation to $t + (t'-t)/2$, which saves fighting costs of $k(t'-t)/2$, but does not affect the benefit of winning; a contradiction. This argument implies that in equilibrium, at any point in time, some types are exiting in expectation.

Now suppose that $G(t)$ involves upward jumps, and, without loss of generality, say that $G(t^+) > G(t)$ (recall that we take the convention that G is always left continuous; see Footnote 16). Then types exiting at t profit from waiting a bit longer: for an arbitrarily small increase in exit time ϵ, they obtain a discrete increase in the winning probability but an arbitrarily small increase in the fighting cost:

$$\begin{aligned} &E(\theta, b, t+\epsilon) - E(\theta, b, t) \\ &\quad = -k\int_0^{t+\epsilon} (1-G(x))\, dx + (\theta - b)G(t+\epsilon) \\ &\qquad + k\int_0^t (1-G(x))\, dx + (\theta-b)G(t) \\ &\quad = (\theta - b)(G(t+\epsilon) - G(t)) - k\int_t^{t+\epsilon} (1-G(t))\, dx > 0. \end{aligned}$$

Hence, types exiting at t will deviate to wait longer, a contradiction. Q.E.D.

A.1. Proof of Lemma 1

First, θ and b enter $E(\theta, b, t)$ in (A1) only through the effective strength $\theta - b$. Therefore, if $\arg\max_{t\in[0,\infty)} E(\theta, b, t)$ is a singleton, then our claim that T is a function of $\theta - b$ follows. If $\arg\max_{t\in[0,\infty)} E(\theta, b, t)$ is a nonsingleton set, then equity holders find it optimal to choose the largest exit time, because $D(b, t) = (b/(1-\pi))G(t)$ is strictly increasing in t as implied by 4. As in the main text, we now denote the optimal exit time by $T(\theta - b)$.

To prove monotonicity, consider $\theta_1 - b_1$ and $\theta_2 - b_2$ with $\Delta \equiv \theta_1 - b_1 - (\theta_2 - b_2) > 0$, and denote their optimal exit times by $T_1 = T(\theta_1 - b_1)$ and $T_2 = T(\theta_2 - b_2)$. We show that $T_1 \geq T_2$, and $T_1 > T_2$ if $G(T_i) \in (0, 1)$, i.e., $T_i \in (0, \bar{T})$.

We first show weak monotonicity, $T_1 \geq T_2$. Suppose not, then $T_1 < T_2$. Optimality implies that $E(\theta_1, b_1, T_1) \geq E(\theta_1, b_1, T_2)$ and $E(\theta_2, b_2, T_2) \geq E(\theta_2, b_2, T_1)$. Summing these two equations yields

$$E(\theta_1, b_1, T_1) - E(\theta_2, b_2, T_1) \geq E(\theta_2, b_2, T_2) - E(\theta_1, b_1, T_2).$$

Using (A1), the above inequality implies that $\Delta G(t_1) \geq \Delta G(t_2)$, contradicting Lemma 4.

Now we rule out the case that $T_1 = T_2 = t \in (0, \bar{T})$. Recall that

$$E(\theta_1, b_1, t) = -k\int_0^t (1 - G(x))\,dx + (\theta_1 - b_1)G(t),$$
$$E(\theta_2, b_2, t) = -k\int_0^t (1 - G(x))\,dx + (\theta_2 - b_2)G(t).$$

The optimality of t for θ_2 implies that $(\theta_2 - b_2)[G(t+\epsilon) - G(t)] \leq k\int_t^{t+\epsilon}(1 - G(x))\,dx$ for any ϵ. By taking positive and negative ϵ's, it is easy to show that G is differentiable with $G'(t) \equiv g(t) = k(1 - G(t))/(\theta_2 - b_2) > 0$ for $t < \bar{T}$, and $(\theta_2 - b_2)g(t)\epsilon - k(1 - G(t))\epsilon = o(\epsilon)$. Now consider firm θ_1 with debt b_1; we have

$$\begin{aligned}
&(\theta_1 - b_1)[G(t+\epsilon) - G(t)] - k\int_t^{t+\epsilon}(1 - G(x))\,dx \\
&\quad = (\theta_1 - b_1)g(t)\epsilon - k(1 - G(t))\epsilon + o(\epsilon) \\
&\quad > \Delta g(t)\epsilon + (\theta_2 - b_2)g(t)\epsilon - k(1 - G(t))\epsilon + o(\epsilon) \\
&\quad = \Delta g(t)\epsilon + o(\epsilon) > 0 \quad \text{for some sufficiently small } \epsilon > 0,
\end{aligned}$$

which implies that θ_1 with debt b_1 strictly prefers to wait longer than t. Q.E.D.

A.2. Proof of Lemma 2

We prove the lemma in two steps.

Step 1. First, we show that in equilibrium $\theta - B(\theta)$ is increasing in θ.

Suppose not; then there exists $\theta_1 < \theta_2$, whose corresponding debt choices $B(\theta_1)$ and $B(\theta_2)$ are such that

$$\theta_1 - B(\theta_1) > \theta_2 - B(\theta_2). \tag{A3}$$

Denote $T_1 \equiv T(\theta_1 - B(\theta_1))$ and $T_2 \equiv T(\theta_2 - B(\theta_2))$. Lemma 1 implies that $T_1 \geq T_2$. If both firms choose corner values $\bar{T}$ or 0 in equilibrium, then $T_1 = T_2$. First, consider the case of $\bar{T}$ so both types win the war of attrition for sure; then θ_1 can choose a higher debt $\theta_1 - \theta_2 + B(\theta_2) > B(\theta_1)$ to obtain a larger debt tax subsidy but still win the war of attrition for sure. Second, if both types withdraw immediately and therefore obtain zero, θ_2 can borrow nothing and get a strictly positive payoff. Lemma 1 then implies that $T_1 > T_2$.

The optimality of the debt choice implies that $V(\theta, B(\theta)) \geq V(\theta, x)$ for $x \neq B(\theta)$ with V given in (A2). In particular, consider the deviations such that θ_1 (θ_2) chooses debt to have the effective strength of θ_2 (θ_1) in equilibrium; i.e., θ_2 deviates to choose $b_2' = \theta_2 - \theta_1 + B(\theta_1)$ and θ_1 chooses $b_1' = \theta_1 - \theta_2 + B(\theta_2)$, respectively. Because $\theta_2 > \theta_1$, $b_2' > B(\theta_1) > 0$; and because of (A3), $b_1' > B(\theta_1) > 0$. Therefore, both deviations are feasible. Applying the optimality of equilibrium debt choices to these deviations, we have $V(\theta_1, B(\theta_1)) \geq V(\theta_1, \theta_1 - \theta_2 + B(\theta_2))$, and $V(\theta_2, B(\theta_2)) \geq V(\theta_2, \theta_2 - \theta_1 + B(\theta_1))$. Combining both inequalities gives

$$\begin{aligned}
&V(\theta_1, B(\theta_1)) - V(\theta_2, \theta_2 - \theta_1 + B(\theta_1)) \\
&\quad - (V(\theta_1, \theta_1 - \theta_2 + B(\theta_2)) - V(\theta_2, B(\theta_2))) \geq 0.
\end{aligned} \tag{A4}$$

We now show that this inequality cannot hold. Lemma 1 shows that the exit time T depends on the effective strength $\theta - b$ only:

$$\begin{aligned}
T_1 &= T(\theta_1, B(\theta_1)) = T(\theta_2, \theta_2 - \theta_1 + B(\theta_1)) = T(\theta_1 - B(\theta_1)), \\
T_2 &= T(\theta_2, B(\theta_2)) = T(\theta_1, \theta_1 - \theta_2 + B(\theta_2)) = T(\theta_2 - B(\theta_2)).
\end{aligned}$$

Then (A2) implies that

$$\begin{aligned}
V(\theta_1, B(\theta_1)) - V(\theta_2, \theta_2 - \theta_1 + B(\theta_1)) &= -\frac{\theta_2 - \theta_1}{1 - \pi}G(T_1), \\
V(\theta_1, \theta_1 - \theta_2 + B(\theta_2)) - V(\theta_2, B(\theta_2)) &= -\frac{\theta_2 - \theta_1}{1 - \pi}G(T_2).
\end{aligned}$$

From Lemma 1 we know that $T_1 > T_2$, and hence $G(T_1) > G(T_2)$ (Lemma 4). As a result, the term in (A4) is $((\theta_2 - \theta_1)/(1 - \pi))(G(T_2) - G(T_1)) < 0$, a contradiction.

Step 2. We show that, in equilibrium, $\theta - B(\theta)$ is strictly increasing in θ.

Suppose not; then there exists an interval $[\theta_1, \theta_2]$ with $\theta_2 > \theta_1$ such that $T(\theta - B(\theta)) = t$ for all $\theta \in [\theta_1, \theta_2]$. Then any type θ in this interval can reduce debt by ϵ and exit a bit later than t (exit time is strictly monotone in b from Lemma 1). This leads to a strictly profitable deviation: the firm obtains at least a discrete gain in the winning probability by winning over types in the interval $[\theta_1, \theta_2]$, while only sacrificing debt tax subsidies in the order of ϵ.

A.3. Proof of Proposition 1

Using Lemma 2 we can follow Bulow and Klemperer (1999) to derive equilibrium exit times in the war of attrition. The argument is almost identical to Bulow and Klemperer (1999) by simply replacing the productivity θ by $\theta - B(\theta)$, such that

$$\hat{T}'(\theta; B(\cdot)) = h(\theta)\frac{\theta - B(\theta)}{k} \quad \text{for all } \theta.$$

The only technical difference is that $\theta - B(\theta)$ might contain upward jumps. At these points, the derivative of exit time with respect to θ is not continuous. More specifically, without loss of generality, say that $B(\theta^-) > B(\theta)$. Then at θ, we have

$$\begin{aligned}\lim_{\epsilon\to 0^+}\frac{\hat{T}(\theta+\epsilon;B(\cdot))-\hat{T}(\theta;B(\cdot))}{\epsilon} &= \hat{T}'_+(\theta;B(\cdot))\\ &= h(\theta)\frac{\theta-B(\theta)}{k},\\ \lim_{\epsilon\to 0^-}\frac{(\hat{T}(\theta+\epsilon;B(\cdot))-\hat{T}(\theta;B(\cdot)))}{\epsilon} &= \hat{T}'_-(\theta;B(\cdot))\\ &= h(\theta)\frac{\theta-B(\theta^-)}{k}.\end{aligned}$$

However, the set of points with jumps in $\theta - B(\theta)$ must be of zero Lebesgue measure, because $\theta - B(\theta)$ is a monotone function. Therefore, if we show that $\hat{T}(\theta; B(\cdot))$ is Lipschitz continuous, then $\hat{T}(\theta; B(\cdot))$ is absolute continuous and therefore (8) holds (Royden 1988). To show Lipschitz continuity, choose any $\theta \in [\underline{\theta}, \bar{\theta})$. We know that the (right and left) derivatives $h(x)((x - B(x))/k)$ are bounded for all $x \in [\underline{\theta}, \theta]$ (recall the regularity condition of hazard rate $h(\cdot)$, and boundedness of $B(x) \in [\underline{\theta}, \theta]$) including at the points with jumps in $B(\cdot)$. Therefore, given θ, we choose a sufficient large number $M(\theta)$ such that $T(x; B(\cdot)) - T(y; B(\cdot)) \le M \cdot (x - y)$ for any $x, y \in [\underline{\theta}, \theta]$. As a result, $T(\theta; B(\cdot))$ is Lipschitz continuous on $[\underline{\theta}, \theta]$, and our result follows.

A.4. Proof of Theorem 1

The proof of Theorem 1 comprises several steps. We first establish continuity and differentiability of the equilibrium debt schedule $B(\cdot)$.

Lemma 5. *The debt schedule $B(\theta)$ is continuous on $(\underline{\theta}, \bar{\theta}]$. As a result, without loss of generality, we assume $B(\theta)$ to be continuous on $[\underline{\theta}, \bar{\theta}]$.*

Proof. Suppose that $B(\cdot)$ involves downward jumps at θ (weak monotonicity of $\theta - B(\theta)$ in Lemma 2 rules out upward jump immediately). There are two subcases.

First suppose that $B(\theta^-) > B(\theta)$, i.e., $B(\theta)$ is right continuous only. Then θ can borrow $[B(\theta^-) - B(\theta)]/2$ more, while still maintaining its ranking in the war-of-attrition game. Because of Proposition 1, his optimal exit time t is unchanged. Then from (6), this deviation gives a discrete gain in debt tax subsidies without any other losses.

Now consider the situation where $B(\theta) > B(\theta^+)$; i.e., $B(\theta)$ is left continuous. Then type $\theta+\epsilon$ can deviate to borrow $B(\theta)$, and vice versa. Optimality implies that

$$\begin{aligned}&V(\theta+\epsilon,B(\theta))-V(\theta+\epsilon,B(\theta^+))>V(\theta,B(\theta))-V(\theta+\epsilon,B(\theta^+))\\ &\quad=V(\theta,B(\theta))-V(\theta,B(\theta^+))-[V(\theta+\epsilon,B(\theta^+))-V(\theta,B(\theta^+))].\end{aligned}$$

The first term is positive and at the order of $O(1)$ according to the argument in the first case (θ only loses the debt subsidy because the deviation does not affect its exit time). The second term is at the order of ϵ because exit time is continuous. As a result, $V(\theta+\epsilon, B(\theta)) > V(\theta+\epsilon, B(\theta^+))$, which implies that the deviation is strictly profitable.

The above argument does not apply to $B(\underline{\theta})$. It is because the firm $\underline{\theta}$ in equilibrium exits immediately, and the exact debt amount raised by $\underline{\theta}$ is undetermined. Instead, we will use the continuity at $\underline{\theta}$ to determine $B(\underline{\theta})$ in the next lemma. Q.E.D.

Lemma 6. *The debt schedule $B(\theta)$ is differentiable for $(\underline{\theta}, \bar{\theta}]$ and satisfies*

$$B'(\theta) = 1 - \frac{f(\theta)}{\pi F(\theta)}B(\theta). \tag{A5}$$

Proof. Consider type θ; by revealed preference the equilibrium payoff is higher than taking $B(\theta)+\epsilon$, i.e., (recall the definition of $v(\cdot,\cdot,\cdot)$ in (A2)):

$$\begin{aligned}0 &\ge v(\theta,B(\theta)+\epsilon,T(\theta-B(\theta)-\epsilon))-v(\theta,B(\theta),T(\theta-B(\theta)))\\ &= -\int_{\underline{\theta}}^{T^{-1}(T(\theta-B(\theta)-\epsilon))} f(y)(y-B(y))\,dy\\ &\quad +\theta F(T^{-1}(T(\theta-B(\theta)-\epsilon)))\\ &\quad +\frac{\pi(B(\theta)+\epsilon)}{1-\pi}F(T^{-1}(T(\theta-B(\theta)-\epsilon)))\\ &\quad +\int_{\underline{\theta}}^{T^{-1}(T(\theta-B(\theta)))} f(y)(y-B(y))\,dy-\theta F(T^{-1}(T(\theta-B(\theta))))\\ &\quad -\frac{\pi B(\theta)}{1-\pi}F(T^{-1}(T(\theta-B(\theta)))).\end{aligned}$$

Let $\hat{\theta}$ be the type such that

$$\hat{\theta}-B(\hat{\theta})=\theta-B(\theta)-\epsilon \quad\Leftrightarrow\quad \hat{\theta}-\theta=B(\hat{\theta})-B(\theta)-\epsilon. \tag{A6}$$

Because $\theta - B(\theta)$ is strictly increasing and continuous, such $\hat{\theta} < \theta$ must exist, and $\theta - \hat{\theta} > 0$ converges to zero when ϵ converges to zero. Then we have

$$\begin{aligned}0 &\ge -\int_{\underline{\theta}}^{\hat{\theta}} f(y)(y-B(y))\,dy+\theta F(\hat{\theta})+\frac{\pi(B(\theta)+\epsilon)}{1-\pi}F(\hat{\theta})\\ &\quad +\int_{\underline{\theta}}^{\theta} f(y)(y-B(y))\,dy-\theta F(\theta)-\frac{\pi B(\theta)}{1-\pi}F(\theta)\\ &= \int_{\hat{\theta}}^{\theta} f(y)(y-B(y))\,dy+\theta f(\theta)(\hat{\theta}-\theta)+\frac{\pi(B(\theta)+\epsilon)}{1-\pi}F(\hat{\theta})\\ &\quad -\frac{\pi B(\theta)}{1-\pi}F(\theta)+o(\theta-\hat{\theta})\\ &= f(\theta)(\theta-B(\theta))(\theta-\hat{\theta})+\theta f(\theta)(\hat{\theta}-\theta)\\ &\quad +\frac{\pi B(\theta)}{1-\pi}(F(\hat{\theta})-F(\theta))+\frac{\pi\epsilon}{1-\pi}F(\theta)+o(\theta-\hat{\theta})\\ &= -\frac{B(\theta)}{1-\pi}f(\theta)(\theta-\hat{\theta})+\frac{\pi\epsilon}{1-\pi}F(\theta)+o(\theta-\hat{\theta}).\end{aligned}$$

Note that from (A6) we have

$$\hat{\theta}-\theta=B(\hat{\theta})-B(\theta)-\epsilon \;\Rightarrow\; \epsilon=B(\hat{\theta})-B(\theta)-\hat{\theta}+\theta,$$

which implies that

$$\begin{aligned}0 &\ge -\frac{B(\theta)}{1-\pi}f(\theta)(\theta-\hat{\theta})+\frac{\pi(B(\hat{\theta})-B(\theta)-\hat{\theta}+\theta)}{1-\pi}F(\theta)\\ &\quad +o(\theta-\hat{\theta})\\ &= \left[\frac{\pi}{1-\pi}F(\theta)-\frac{B(\theta)}{1-\pi}f(\theta)\right](\theta-\hat{\theta})\\ &\quad +\frac{\pi(B(\hat{\theta})-B(\theta))}{1-\pi}F(\theta)+o(\theta-\hat{\theta}).\end{aligned} \tag{A7}$$

Therefore, we have

$$B(\theta)-B(\hat{\theta}) \ge \frac{\pi F(\theta)-B(\theta)f(\theta)}{\pi F(\theta)}(\theta-\hat{\theta})+o(\theta-\hat{\theta}) \quad \text{for } \theta-\hat{\theta}>0. \tag{A8}$$

Applying the similar (but opposite) argument to deviation of $B(\theta)-\epsilon$ implies that

$$B(\theta)-B(\hat{\theta}) \leq \frac{\pi F(\theta)-B(\theta)f(\theta)}{\pi F(\theta)}(\theta-\hat{\theta})+o(\theta-\hat{\theta}) \quad \text{for } \theta-\hat{\theta}<0. \tag{A9}$$

Combining both (A8) and (A9), we know that $B(\theta)$ is differentiable and satisfies (A5). Q.E.D.

The general closed-form solution for the ordinary differential equation (A5) is (with C being an arbitrary constant)

$$\begin{aligned}
&\exp\left(-\int_{\underline{\theta}}^{\theta}\frac{f(x)}{\pi F(x)}\,dx\right)\left(\int_{\underline{\theta}}^{\theta}\exp\left(\int_{\underline{\theta}}^{y}\frac{f(x)}{\pi F(x)}\,dx\right)dy+C\right)\\
&=\left(\frac{F(\underline{\theta})}{F(\theta)}\right)^{1/\pi}\left(\int_{\underline{\theta}}^{\theta}\left(\frac{F(y)}{F(\underline{\theta})}\right)^{1/\pi}dy+C\right)\\
&=F(\theta)^{-1/\pi}\left(\int_{\underline{\theta}}^{\theta}F(y)^{1/\pi}\,dy+C\right),
\end{aligned}$$

where the last equality uses integration by parts. Since $F(\underline{\theta})=0$, any solution with $C\neq 0$ explodes at $\underline{\theta}$. Therefore, $C=0$ and

$$B(\theta)=F(\theta)^{-1/\pi}\int_{\underline{\theta}}^{\theta}F(y)^{1/\pi}\,dy,$$

which is unique by construction. It is easy to show that $B(\underline{\theta})=\lim_{\theta\downarrow\underline{\theta}}B(\theta)=0$. Note that as shown in Lemma 5, $B(\underline{\theta})$ itself is not determined, because $\underline{\theta}$ exits immediately so $B(\underline{\theta})$ does not matter. However, we know that types in the neighborhood of $\underline{\theta}$ must borrow almost nothing, otherwise, $\underline{\theta}$ can borrow nothing and beat these types.

Finally, we show the sufficiency of the FOC and confirm that $B(\theta)$ indeed constitutes an equilibrium. Consider type θ with debt $B(\theta)$ in equilibrium. Consider a deviation to debt level b. Without loss of generality, we can focus on deviations such that $\theta-b\in[\underline{\theta}-B(\underline{\theta}),\bar{\theta}-B(\bar{\theta})]$. First, if $\theta-b<\underline{\theta}-B(\underline{\theta})$, then θ is effectively weaker than the lowest productivity type. This is unprofitable because θ exits immediately and realizes a zero payoff. Second, if $\theta-b>\bar{\theta}-B(\bar{\theta})$, then either b is infeasible if $\theta-\bar{\theta}+B(\bar{\theta})<0$. Second, if $\theta-\bar{\theta}+B(\bar{\theta})>0$, then taking debt b below $\theta-\bar{\theta}+B(\bar{\theta})$ provides no gain in winning the war but reduces the debt subsidy.

Now consider a deviation to debt level $b>B(\theta)$. We want to show that the marginal incentive to reduce debt at the deviation is positive, i.e., $V_b(\theta,b)<0$. From continuity of $\theta-B(\theta)$, we can find θ' with $B(\theta')$ who has the same effective strength

$$\theta'-B(\theta')=\theta-b.$$

Because $\theta-B(\theta)$ is increasing in equilibrium (Proposition 2), $\theta'<\theta$, which also implies that $b>B(\theta')$. Also, their exit time is the same as well $T(\theta-b)=T(\theta'-B(\theta'))$. We compare $V_b(\theta,b)$ and $V_b(\theta',B(\theta'))$. Since θ' chooses b' in equilibrium, $V_b(\theta',B(\theta'))=0$. We have

$$\begin{aligned}
V_b(\theta,b)&=\frac{dE(\theta-b,T(\theta-b))}{db}+\frac{G(T(\theta-b))}{1-\pi}\\
&\quad-\frac{b}{1-\pi}g(T(\theta-b))T'(\theta-b),\\
V_b(\theta',B(\theta'))&=\frac{dE(\theta'-B(\theta'),T(\theta'-B(\theta')))}{db}+\frac{G(T(\theta'-B(\theta')))}{1-\pi}\\
&\quad-\frac{B(\theta')}{1-\pi}g(T(\theta'-B(\theta')))T'(\theta'-B(\theta')).
\end{aligned}$$

Because $T(\theta-b)=T(\theta'-B(\theta'))$ the first two terms of $V_b(\theta,b)$ and $V_b(\theta',B(\theta'))$ are identical. We know that $T'(\theta-b)=T'(\theta'-B(\theta'))>0$, and $g(T(\theta-b))=g(T(\theta'-B(\theta')))=f(T^{-1}(T(\theta'-B(\theta'))))/T'(T^{-1}(T(\theta'-B(\theta'))))=f(\theta')k/h(\theta')(\theta'-b')>0$. Because $b>B(\theta')$ the third term for θ' dominates the one for θ. Thus, $V_b(\theta,b)<V_b(\theta',b')=0$. The proof for the negative deviation in debt is analogous. Q.E.D.

A.5. Proof of Corollary 1

Setting $\pi=0$ in (A7), we have $0\geq -B(\theta)f(\theta)(\theta-\hat{\theta})+o(\theta-\hat{\theta})$ for both $\hat{\theta}>\theta$ and $\hat{\theta}<\theta$. This immediately implies that $B(\theta)=0$. Q.E.D.

Direct calculation yields that

$$\begin{aligned}
\frac{\partial B(\theta;\pi)}{\partial\pi}&=\int_{\underline{\theta}}^{\theta}\frac{\partial}{\partial\pi}\left[\left(\frac{F(y)}{F(\theta)}\right)^{1/\pi}\right]dy\\
&=\frac{1}{\pi^2}\int_{\underline{\theta}}^{\theta}\left(\frac{F(y)}{F(\theta)}\right)^{1/\pi}\ln\left(\frac{F(\theta)}{F(y)}\right)dy>0,
\end{aligned}$$

and

$$\frac{\partial T(\theta;B(\cdot;\pi))}{\partial\pi}=\int_{\underline{\theta}}^{\theta}\frac{h(x)}{k}\left[\frac{-\partial B(x;\pi)}{\partial\pi}\right]dx<0. \quad \text{Q.E.D.}$$

A.6. Proof of Lemma 3

Firm θ generates a positive payoff only if its opponent y is weaker, which is $\int_{\underline{\theta}}^{\theta}\theta f(y)\,dy$. The expected fighting costs have two components. First, if the opponents are weaker, $y<\theta$, the firm incurs a total fighting cost of $k\hat{T}(y;B(\cdot))$ as the opponent y exits at $\hat{T}(y;B(\cdot))$. This gives an expected cost of

$$\begin{aligned}
&\int_{\underline{\theta}}^{\theta}k\hat{T}(y,B(\cdot))f(y)\,dy\\
&=-k\int_{\underline{\theta}}^{\theta}\hat{T}(y,B(\cdot))d(1-F(y))\\
&=-k\hat{T}(\theta,B(\cdot))(1-F(\theta))+\int_{\underline{\theta}}^{\theta}f(y)(y-B(y))\,dy,
\end{aligned} \tag{A10}$$

where the second equality uses (8) and integration by parts. Second, with probability $1-F(\theta)$, the opponent is stronger, and the firm θ's fighting cost of $k\hat{T}(\theta,B)$ is a deadweight loss. Adding this cost to (A10) the total expected fighting cost is simply $\int_{\underline{\theta}}^{\theta}f(y)(y-B(y))\,dy$. Summing up costs and benefits we obtain (15).

A.7. Proof of Proposition 2

Direct calculation yields

$$\frac{dS(\pi)}{d\pi}=\mathbb{E}_{\theta}\left[\int_{\underline{\theta}}^{\theta}f(y)\frac{\partial B(y;\pi)}{\partial\pi}\,dy\right]>0. \quad \text{Q.E.D.}$$

Appendix B. Duration of Distress

B.1. Proof of Proposition 3

We have given the main argument of how we derive the equilibrium debt schedule and equilibrium exit times in the main text. The only thing left to show is the existence and uniqueness of threshold $\hat{\theta}$. Because the right-hand side of the equation $kT_d=k\hat{T}(\hat{\theta})+\hat{\theta}-B(\hat{\theta})$ is strictly increasing in $\hat{\theta}$, uniqueness follows. Because the right-hand side is increasing from $\bar{\theta}$ to $k\hat{T}(\bar{\theta})+\bar{\theta}-B(\bar{\theta})$ $\hat{\theta}$ exists if

$$kT_d \in (\bar{\theta}, k\hat{T}(\bar{\theta})+\bar{\theta}-B(\bar{\theta})).$$

Otherwise, $\hat{\theta}=\bar{\theta}$, and the solution is the same as in the benchmark model. Finally, the comparative statics with respect to π and T_d are straightforward.

B.2. Characterization of Threshold

Here we solve analytically for the threshold type $\hat{\theta}$ for the distribution $F(\theta)=\lambda\theta^{\alpha}$. Then $B(\theta;\pi)=(\pi/(\alpha+\pi))\theta$ for $\theta<\hat{\theta}$. Denote the equilibrium exit time without a debt subsidy by

$$T_0(\theta)=\int_0^{\theta}\frac{\lambda\alpha y^{\alpha}}{1-\lambda y^{\alpha}}\,dy.$$

For $\pi>0$, $\hat{\theta}(\pi)-B(\hat{\theta}(\pi))=(\alpha/(\alpha+\pi))\hat{\theta}(\pi)$, and

$$\hat{T}(\hat{\theta}(\pi);B(\theta;\pi))=\frac{\alpha}{\alpha+\pi}T_0(\hat{\theta}(\pi))=\frac{\alpha}{\alpha+\pi}\int_0^{\hat{\theta}(\pi)}\frac{\lambda\alpha y^{\alpha}}{1-\lambda y^{\alpha}}\,dy.$$

$\hat{\theta}(\pi)$ solves the following equation by rearranging (17):

$$kT_0(\hat{\theta}(\pi))+\hat{\theta}(\pi)=\frac{\alpha+\pi}{\alpha}kT_d.$$

One can calculate the increasing function on the left-hand side $kT(y)+y$ as (${}_2\mathbb{F}_1$ is the hypergeometric function)

$$\begin{aligned} kT(y)+y &= \int_0^{y}\frac{\lambda\alpha x^{\alpha}}{1-\lambda x^{\alpha}}\,dx+y \\ &= \alpha y\,{}_2\mathbb{F}_1\left(1,\frac{1}{\alpha},1+\frac{1}{\alpha},\lambda y^{\alpha}\right)+(1-\alpha)y. \end{aligned}$$

Thus, given π we can easily solve for $\hat{\theta}(\pi)$ numerically. Also, the total welfare is

$$\begin{aligned} S(\pi) &= \mathbb{E}_{\theta}[(s(\theta))_{\theta\le\hat{\theta}(\pi)}+(-kT_d+\theta)_{\theta>\hat{\theta}(\pi)}] \\ &= \lambda^2\alpha\frac{\alpha+\pi+\pi\alpha}{(\alpha+\pi)(1+\alpha)}\frac{1}{2\alpha+1}(\hat{\theta}(\pi))^{2\alpha+1} \\ &\quad -(1-\lambda(\hat{\theta}(\pi))^{\alpha})kT_d+\frac{\lambda\alpha}{1+\alpha}[(\bar{\theta})^{\alpha+1}-(\hat{\theta}(\pi))^{\alpha+1}]. \end{aligned}$$

Appendix C. Coasian Solution

C.1. Proof of Proposition 4

For simplicity we assume $\Theta=[0,\bar{\theta}]$, where $\bar{\theta}$ is finite.

Monotonicity in bid rejection. Suppose not; then there exists $\theta_1<\theta_2$, such that θ_1 rejects the offer and θ_2 accepts the offer. Let Θ_Y ($\Theta_N=\Theta\backslash\Theta_Y$) be the set of all types who accept (reject) the offer, and let $\Pr(Y)=\Pr(\theta\in\Theta_Y)$. Denote by $V(\theta;\Theta_1,\Theta_2)$ the payoff to firm θ after sale decisions become public, where firm θ believes that its opponent is from Θ_1, and the opponent believes that she is playing against a firm from Θ_2. The incentive compatibility conditions for both types require that (recall that the rejection of θ_1 implies that θ_1 strictly prefers rejection)

$$\Pr(Y)V(\theta_1;\Theta_Y,\Theta_N)+(1-\Pr(Y))V(\theta_1;\Theta_N,\Theta_N)>p,$$
$$p\ge\Pr(Y)V(\theta_2;\Theta_Y,\Theta_N)+(1-\Pr(Y))V(\theta_2;\Theta_N,\Theta_N).$$

Combining and rearranging, we require that

$$\begin{aligned} &\Pr(Y)[V(\theta_1;\Theta_Y,\Theta_N)-V(\theta_2;\Theta_Y,\Theta_N)] \\ &\quad >(1-\Pr(Y))[V(\theta_2;\Theta_N,\Theta_N)-V(\theta_1;\Theta_N,\Theta_N)]. \end{aligned} \tag{C1}$$

In a war of attrition, θ_2 can follow θ_1 exit strategy and freely dispose $\theta_2-\theta_1>0$. Thus, $V(\theta_1;\Theta_Y,\Theta_N)-V(\theta_2;\Theta_Y,\Theta_N)\le 0$ and $V(\theta_2;\Theta_N,\Theta_N)-V(\theta_1;\Theta_N,\Theta_N)\ge 0$, and (C1) cannot hold, a contradiction.

Existence and uniqueness. Fix p and let $\hat{\theta}$ be the solution to $\hat{\theta}F(\hat{\theta})=p$. We show that the firm $\theta_i=\hat{\theta}$ is indifferent between accepting and rejecting. By accepting, it always obtains p. Suppose firm i rejects the offer. If the other firm, θ_j, rejects, then $\theta_j>\hat{\theta}$, which occurs with probability $1-F(\hat{\theta})$. Firm i enters the war of attrition with this stronger opponent and her expected payoff is zero. If the other firm accepts the offer, with probability $F(\hat{\theta})$, in equilibrium, firm C acquires the assets but does not enter the war, and firm i wins the war and obtains $\hat{\theta}$. Therefore, the indifference condition implies that $\hat{\theta}F(\hat{\theta})=p$. Because $\hat{\theta}F(\hat{\theta})$ is strictly monotone in $\hat{\theta}$, the cutoff $\hat{\theta}$ is unique, and this equilibrium is unique.

Now we verify that the proposed strategies comprise an equilibrium. Given the monotonicity of the bid rejection strategy, we know that it is optimal for types below $\hat{\theta}$ to accept the offer and types above $\hat{\theta}$ to reject it. Now we verify that the war of attrition game is indeed an equilibrium. If both firms reject the offer, the war of attrition is one with the truncated type distribution with lower bound $\hat{\theta}$. If one firm rejects and the other accepts, then it is obvious that the firm that rejected the offer and saw its opponent accepting it, will enter the war (because, in equilibrium, the firm that rejects the offer expects firm C to immediately fold). Finally, after buying a weak firm $\theta\le\hat{\theta}$, firm C will find it optimal not to enter the war of attrition. On this off-equlibrium path, the opponent plays according to standard war of attrition in Proposition 1 with $B(\theta)=0$, with lower bound $\hat{\theta}$ as a common belief. In this game, $\hat{\theta}$ drops out immediately, therefore firm C with $\theta\le\hat{\theta}$ finds it optimal to exit immediately as well.

Negative expected profits. Now we compute the firm C's expected profits for any $p>0$. Since the bidding price $p(\hat{\theta})=\hat{\theta}F(\hat{\theta})$ is monotone in the equilibrium cutoff type $\hat{\theta}$, firm C can effectively choose $\hat{\theta}$. The total equilibrium expected profits as a function of $\hat{\theta}$ are

$$\begin{aligned} 2\Pi(\hat{\theta}) &= \int_0^{\hat{\theta}}\int_0^{\hat{\theta}}(\max(\theta_1,\theta_2)-2p(\hat{\theta}))f(\theta_1)f(\theta_2)\,d\theta_1 d\theta_2 \\ &\quad -2p(\hat{\theta})F(\hat{\theta})(1-F(\hat{\theta})) \\ &= \int_0^{\hat{\theta}}\int_0^{\hat{\theta}}(\max(\theta_1,\theta_2))f(\theta_1)f(\theta_2)d\theta_1 d\theta_2-2\hat{\theta}F^2(\hat{\theta}). \end{aligned}$$

Here, the first term captures the situation where both firms accept the offer. Firm C pays $p(\hat{\theta})$ to each firm and internalizes externalities by closing the less profitable firm. In the situation where only one firm accepts, which occurs with probability $2F(\hat{\theta})(1-F(\hat{\theta}))$, firm C pays $p(\hat{\theta})$ to one firm, and immediately exits. When both firms reject firm C has zero payoffs. Since firm C is effectively choosing $\hat{\theta}$, the first-order impact of $\hat{\theta}$ on profits is

$$\frac{\partial \Pi}{\partial \hat{\theta}} = -\hat{\theta} F(\hat{\theta}) f(\hat{\theta}) - F^2(\hat{\theta}) < 0$$

and strictly negative for $\hat{\theta} > 0$. Further, since $\Pi(\hat{\theta} = 0) = 0$, the strictly negative slope implies that $\Pi(\hat{\theta}) < 0$ for all positive $\hat{\theta}$ (i.e., $p > 0$).

C.2. Proof of Proposition 5

We first show that the firm's equilibrium time of sale is increasing in firms' type. Before either firm i or j sell, both firms face similar payoffs as in (2), with two key differences. First, each firm has the option to sell to firm C at t and receive $p_1(t)$. Second, whichever firm chooses not to sell will compete with firm C, but it can also choose to sell its asset to C for $p_2(s, t)$ at any time $s > t$. Without loss of generality, consider firm i and suppose that its opponent's selling time distribution is G_S. Because we are focusing on symmetric equilibria, G_S is independent of firm's identity. Therefore, the value of the firm with productivity θ and selling time t is

$$V(\theta, t) = (1 - G_S(t))(-kt + p_1(t)) + \int_0^t [-kx + V^C(\theta, x)]\, dG_S(x), \tag{C2}$$

where the first term is firm i's payoff if firm i is the first firm to accept C's offer. The second term is her payoff if the opponent sells first. In this term, the first part is the cost of competing with firm j, and the second term $V^C(\theta, x)$ gives the continuation value of competing with firm C if the opponent sells itself to C at time x.

Suppose that $\theta < \theta'$, but they choose $t > t'$ in equilibrium. Then $V(\theta', t') \geq V(\theta', t)$ and $V(\theta, t) \geq V(\theta, t')$. Moreover, because we break ties such that the firm accepts an offer whenever indifferent,[32] we can show that the first inequality is strict: $V(\theta, t) > V(\theta, t')$. Suppose that $V(\theta, t) = V(\theta, t')$. Because $t > t'$, firm θ must strictly prefer rejecting at t' , i.e., there exits some later selling time $t^* > t'$ such that $V(\theta, t^*) > V(\theta, t') = V(\theta, t)$. This contradicts the fact that $V(\theta, t)$ is the optimal selling strategy.

Combining the inequalities, we obtain

$$V(\theta', t') - V(\theta, t') > V(\theta', t) - V(\theta, t). \tag{C3}$$

Because of free disposal $V^C(\theta, x) \geq V^C(\theta', x)$ for $\theta > \theta'$. Let $\Delta V^C(\theta, x) = V^C(\theta', x) - V^C(\theta, x) \geq 0$. Using (C2), we have $V(\theta', t) - V(\theta, t) = \int_0^t \Delta V^C(\theta, x)\, dG(x)$; therefore, we have

$$\frac{d[V(\theta', t) - V(\theta, t)]}{dt} = \Delta V^C(\theta, t) dG(t) \geq 0.$$

Then $V(\theta', t') - V(\theta, t') \leq V(\theta', t) - V(\theta, t)$ because $t' < t$. This contradicts with (C3), and therefore the time to accept the offer is increasing in firm productivity.

Now we show that firm C cannot earn positive profits. If firms sell to C at different times, it is common knowledge that the first firm C bought is strictly weaker. We focus subgame equilibria in which strictly weaker firms exit immediately. Therefore, firm C exits immediately and loses money in paying the firm who sold first.

Firm C still might be able to earn positive profits if it can induce both firms to sell at the same time. Because of monotonicity, at any point in time, the distribution of types that has not sold yet is the original type distribution but potentially truncated from below at some $\hat{\theta}$. Consider the last time at which both firms might sell. This corresponds to the highest cutoff $\hat{\theta}$. Since $\hat{\theta}$ is bounded above by $\bar{\theta}$, such a time exists. Then we can evaluate the profit of firm C at this highest cutoff. Without loss of generality, firm C must offer a nonzero price—otherwise no firms will sell. This subgame corresponds to the static time-0 game analyzed in Proposition 4, and the same argument as in Proposition 4 implies that firm C will incur a strictly negative profit at $\hat{\theta}$. This violates the requirement that firm C cannot commit to offer prices that incur a negative expected profit. Q.E.D.

[32] The argument is similar if we break ties the other way.

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