Debt Financing in Asset Markets*

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Short-term debt such as overnight repos and commercial paper was heavily used by financial institutions to fund their investment positions during the asset market boom preceding the financial crisis of 2007-2008, and later played a major role in leading to the distress of these institutions during the crisis. What explains the popularity of short-term debt in financing asset market investments? Geanakoplos (2010) presents a dynamic model of the joint equilibrium of asset markets and credit markets, in which optimistic buyers of a risky asset use the asset as collateral to raise debt financing from less optimistic creditors. The asset's collateral value depends on the marginal creditor's asset valuation, and determines the buyers' purchasing capacity to bid up the asset price. Geanakoplos posits that short-term debt allows the asset buyers to maximize riskless leverage.[1] However, short-term debt exposes the asset buyers to rollover risk (e.g., Acharya, Gale, and Yorulmaker (2011), and He and Xiong (2011, 2012)), which is reflected in the fact that the asset buyers in Geanakoplos' model may not be able to obtain refinancing after a negative fundamental shock. As a result, they are forced to transfer their asset holdings to the creditors, causing a leverage cycle to emerge. The presence of such rollover risk prompts a fundamental question regarding the optimality of short-term debt financing.

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[1] The existing literature also offers several other advantages of short-term debt. First, short-term debt can act as a disciplinary device by giving the creditors the option to pull out if they discover that firm managers are pursuing value-destroying projects (e.g., Calomiris and Kahn (1991)). Second, the short maturity also makes short-term debt less information sensitive and thus less exposed to adverse-selection problems (e.g., Gorton and Pennacchi (1990)).

Moreover, one might argue that as creditors are less optimistic and undervalue risky debt issued by optimistic asset buyers, it is not desirable for the buyers to use risky financing. Following Geanakoplos' framework, Simsek (2011) examines a setting in which asset buyers have to hold their positions until the asset matures. He shows that optimistic asset buyers may use risky debt financing despite their debt being undervalued by the creditors. The contrast between Simsek's and Geanakoplos' analysis suggests that the asset's tradability in the interim periods may affect the debt financing of asset buyers.

This paper examines the optimal use of debt financing by extending the dynamic framework of Geanakoplos along two dimensions. First, we allow the asset's fundamental value to follow a generic binomial tree, which takes multiple values after multiple periods, as opposed to the two values in Geanakoplos' model. Second, we allow two groups of agents to have time-varying beliefs about the asset's fundamental, and, in particular, to have more dispersed beliefs after a negative fundamental shock. The more dispersed beliefs make debt refinancing more expensive to the optimistic asset buyers and thus expose them to greater rollover risk. The rollover risk motivates long-term debt financing. To uncover the role played by the asset's tradability, we also contrast two settings: the main setting whereby agents can freely trade the asset on any date, and a benchmark setting whereby agents cannot trade after the initial date. The non-tradability is due to the presence of informational and trading frictions that prevent agents from promptly responding to fundamental fluctuations, and makes the benchmark setting essentially static and analogous to that of Simsek.

Our analysis delivers several results. First, to our surprise, the maximum riskless short-term leverage is optimal despite the presence of rollover risk. While long-term debt financing can dominate short-term debt financing when the rollover risk is sufficiently high (i.e., belief dispersion becomes sufficiently high after a negative shock), the improved investment opportunity after interim negative fundamental shocks makes saving cash for that state more desirable than acquiring the position. Thus, long-term debt financing is dominated either by short-term debt financing or by saving cash. This result explains the pervasive use of short-term debt in practice. It also verifies the statement put forward by Geanakoplos, albeit due to optimists' cash saving incentives rather than his argument that short-term debt allows optimists to maximize riskless leverage.

Second, we highlight the role played by the asset's tradability. By making it possible to buy the liquidated collateral on an interim date, the asset's tradability motivates some optimists to save cash, which, in turn, boosts the creditors' valuation of the collateral on the initial date, as well as the optimists' purchasing capacity to bid up the asset price.

Finally, despite the market incompleteness due to the borrowing constraints faced by the optimists, we derive a risk-neutral representation of the equilibrium prices of the asset and debt contracts collateralized by the asset. This is because the payoff of a collateralized debt contract is monotonic with respect to the asset's value and thus shares the same marginal investor as the asset. This representation facilitates our analysis and could prove useful in analyzing issues related to collateralized debt financing in more complex settings.

The paper is organized as follows. Section 1 describes the model. We derive the static benchmark setting in Section 2, and analyze the dynamic setting in Section 3. We provide an online appendix to cover more details and all the technical derivations. The online Appendix also extends the two-period model described in the paper to N periods.

1 The Model

Consider a model with three dates $t = 0, 1, 2$ and a long-term risky asset. The asset's final payoff on date 2 is determined by a publicly observable binomial tree (Figure 1). The asset's final payoff $\widetilde{\theta}$ at the end of the path uu is 1, at the end of ud and du is θ, and at the end of dd is θ^2, where $\theta \in (0, 1)$. The probability of the tree going up in each period is unobservable. Two groups of risk-neutral agents differ in their beliefs about these probabilities. We collect an agent's beliefs on the three intermediate states, one on date 0 and two on date 1 (u and d), by $\{\pi_0^i, \pi_u^i, \pi_d^i\}$ where

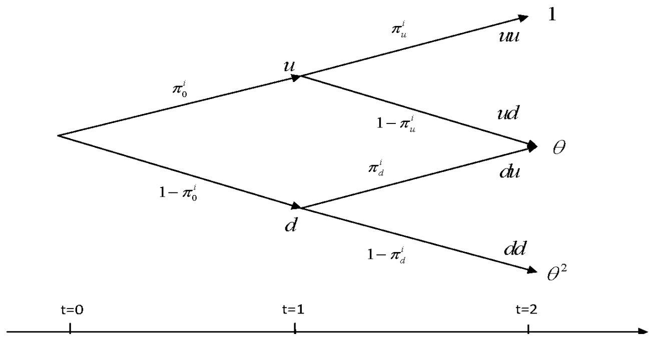

Figure 1: Timeline.

$i \in \{h, l\}$ indicates the agent's type. We assume that the h-type agents are always more optimistic than the l-type agents (the superscript "h" and "l" stand for high and low): $\pi_n^h \geq \pi_n^l$ for any $n \in \{0, u, d\}$. We emphasize that each agent's belief changes over time, driven by either his learning process or sentimental fluctuation. As a result, the belief dispersion between the optimists and pessimists varies over time. Our analysis highlights the case where the optimists face rollover risk in the intermediate state d when the belief dispersion between the optimists and pessimists diverges (i.e., $\pi_d^h - \pi_d^l \geq \pi_0^h - \pi_0^l \geq 0$).[2]

The pessimists (l-type agents) who have infinite amount of cash (deep pockets) are initially endowed with all of the asset, normalized to be one unit. On date 0, each optimist (an h-type agent) is endowed with c dollars of cash. We assume that both the risk-free interest rate and the agents' discount rate are zero and short sales of the asset are not allowed. We focus on non-contingent debt contracts, which specify constant debt payments at maturity unless the borrowers default.

2 The Static Setting

We first consider a benchmark setting in which agents cannot trade the asset on date 1 and are restricted from using debt contracts that mature on date 1. This setting is effectively static with

[2] We simplify the continuum of constant beliefs considered by Geanakoplos (2009) to two types. By focusing on homogeneous optimists, we can isolate an optimist's incentive to save cash from a pure belief effect.

three possible final states on date 2. Like Geanakoplos (2010), we allow the optimists to use their asset holdings as collateral to obtain debt financing.

Suppose that an optimist uses a debt contract, collateralized by one unit of the asset with a promised payment F due on date 2. The debt's eventual payment is $\widetilde{\theta} \wedge F \equiv \min\left(F, \widetilde{\theta}\right)$. A pessimistic creditor will grant a credit of $D_0 = \mathbb{E}_0^l\left[\widetilde{\theta} \wedge F\right]$, where $\mathbb{E}_0^l$ is the expectation operator under the pessimist's belief. To establish the asset position, the optimist has to use $p_0 - \mathbb{E}_0^l\left[\widetilde{\theta} \wedge F\right]$ of his own cash. This amount is the so-called haircut. With a cash endowment of c, the optimist can purchase $c/\left(p_0 - \mathbb{E}_0^l\left[\widetilde{\theta} \wedge F\right]\right)$ units of the asset. Each unit gives an expected payoff of $\mathbb{E}_0^h\left[\widetilde{\theta} - \widetilde{\theta} \wedge F\right]$. Maximizing the expected profit leads to the optimist's optimal choice of leverage F^*.

In the joint equilibrium of the asset and credit markets, each optimist maximizes his expected profit. As an optimist could also choose to acquire the debt issued by other optimists instead of the asset, another equilibrium condition requires that the marginal value of investing one dollar in the asset is as good as investing in debt issued by other optimists. Furthermore, if the optimists are the marginal investors of the asset, the market clearing condition requires that the asset price is determined by the optimists' aggregate purchasing power (the sum of their cash endowment c and the borrowed credit $\mathbb{E}_0^l\left[\widetilde{\theta} \wedge F^*\right]$). In the online Appendix, we describe these conditions in detail and derive the equilibrium. Here, we highlight the key characteristics of the equilibrium.

The equilibrium crucially depends on whether the belief dispersion between the optimists and pessimists is concentrated in the highest state uu (i.e., the belief dispersion about uu being higher than about the three upper states $\{uu, ud, du\}$):

$$\frac{\pi_0^h \pi_u^h}{\pi_0^l \pi_u^l} \geq \frac{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h}{\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l}. \tag{1}$$

This condition is consistent with the monotonic belief ordering imposed by Simsek (2011). To facilitate our interpretation, suppose $\pi_u^h = \pi_u^l$ (i.e., there is no belief divergence at state u after the interim good news). Then, (1) can be rewritten as

$$\frac{\pi_d^h}{\pi_d^l} \leq \frac{\left(1-\pi_0^l\right)\pi_0^h}{\left(1-\pi_0^h\right)\pi_0^l}.$$

As the optimists with short-term debt financing have to roll over their debt after the interim bad news in state d of date 1 and as the belief divergence in this state (π_d^h/π_d^l) captures their refinancing cost, (1) requires the rollover risk faced by the optimists be modest relative to their initial speculative incentives on date 0. As we are mainly interested in scenarios with significant rollover risk, our later analysis will focus on the case whereby (1) fails.

Under the monotone belief dispersion specified in (1), the optimists will use risky debt with a promised payment θ to finance their asset acquisition when their cash endowment is sufficiently low (Proposition 2 of the online Appendix). Despite their debt being undervalued by the creditors, the optimists believe that the asset price is sufficiently attractive to offset the high financing cost. This result is consistent with that of Simsek (2011), and builds on the three-point distribution of the asset's fundamental on the final date, as opposed to the two-point distribution in Geanakoplos (2010). On the other hand, if (1) is not satisfied, the optimists use only riskless debt with a promised payment θ^2 to finance their asset acquisition (Proposition 3 of the online Appendix). This is the case that we focus on.

3 The Dynamic Setting

We now allow agents to trade the risky asset on the interim date—date 1—after the two possible states, u and d, are revealed to the public. As a result, each optimist can also make endogenous investment and leverage decisions in these two interim states. Moreover, each optimist has the choice to use either short-term debt maturing on date 1 or long-term debt maturing on date 2 to finance his asset position on date 0.

3.1 Risk-Neutral Representation of Prices

Despite the financing constraints faced by the optimists, we can still establish a risk-neutral representation of the prices of the asset and debt contracts collateralized by the asset. It is realistic to allow the collateralized debt contracts to be tradable on dates 0 and 1. The payoff of a collateralized debt contract is monotonic with respect to the asset's value, and thus must share the same marginal investor, whose identity can be either optimist or pessimist and can be different across states. We can represent the price P_s of the asset or any debt contract collateralized by the asset in any intermediate state $s \in \{u, d, 0\}$ by a risk-neutral form:

$$P_u = \phi_u P_{uu} + (1 - \phi_u) P_{ud}, \quad P_d = \phi_d P_{ud} + (1 - \phi_d) P_{dd}, \;\; P_0 = \phi_0 P_u + (1 - \phi_0) P_d,$$

where ϕ_s is the marginal investor's effective belief in state s.

These effective beliefs are directly linked with the optimists' marginal value of cash. Consider the intermediate state d. The asset price p_d must be between the optimists' and pessimists' asset valuations: $p_d \in \left[\mathbb{E}_d^l\left(\widetilde{\theta}\right), \mathbb{E}_d^h\left(\widetilde{\theta}\right)\right]$ and $\phi_d \in \left[\pi_d^l, \pi_d^h\right]$. One dollar of cash allows an optimist to acquire the asset at a discount to his valuation. Each dollar, levered up by using a debt contract collateralized by one unit of the asset and with a promise of θ^2 (the maximum riskless promise), allows him to acquire a position of $1/\left(p_d - \theta^2\right)$ units of the asset and thus gives a marginal value of $v_d = \pi_d^h\left(\theta - \theta^2\right)/\left(p_d - \theta^2\right) = \pi_d^h/\phi_d \in \left[1, \pi_d^h/\pi_d^l\right]$. Similarly, his marginal value of cash in state u is $v_u = \pi_u^h(1 - \theta)/(p_u - \theta) = \pi_u^h/\phi_u$. On date 0, the optimists can either use the cash, levered up by collateralized debt, to acquire the asset or save it for date 1. Thus, his marginal value of cash at date 0 is the maximum of the two choices:

$$v_0 = \max\left(u_0, \pi_0^h v_u + \left(1 - \pi_0^h\right) v_d\right), \text{ where } u_0 = \frac{\pi_0^h v_u\left(p_u - p_d\right)}{p_0 - p_d} = \frac{\pi_0^h v_u}{\phi_0}.$$

3.2 An Optimist's Problem

An individual optimist faces three alternatives on date 0. He can establish an asset position by using short-term debt financing; he can use long-term debt financing; or he can simply save cash for establishing a position later on date 1.

Suppose that he uses a debt contract $\widetilde{F}$ (either long-term or short-term) collateralized by each unit of asset to finance a position. By selling the debt to the marginal investor in the market he obtains a credit of $D_0\left(\widetilde{F}\right) = \phi_0 D_u\left(\widetilde{F}\right) + (1-\phi_0)\, D_d\left(\widetilde{F}\right)$, where we denote D_s the debt value in state s. Thus, each dollar of cash allows him to establish a position of $x = 1/\left(p_0 - D_0\right)$ units of the asset. This position gives an expected profit of

$$V_0\left(\widetilde{F}\right) = \frac{\left(\pi_0^h v_u p_u + \left(1-\pi_0^h\right) v_d p_d\right) - \left(\pi_0^h v_u D_u\left(\widetilde{F}\right) + \left(1-\pi_0^h\right) v_d D_d\left(\widetilde{F}\right)\right)}{p_0 - D_0\left(\widetilde{F}\right)}. \tag{2}$$

This equation reflects two effects. One is a leverage effect: a more aggressive debt contract allows the optimist to establish a greater position, as shown by the denominator of (2). The other is a debt-cost effect: a more aggressive debt contract also implies a higher expected debt payment in the future, as shown by the term $\pi_0^h v_u D_u + \left(1-\pi_0^h\right) v_d D_d$ in the numerator of (2).

Debt Maturity Choice In light of (2), when the optimist compares a pair of long-term and short-term debt contracts with the same initial credit, he prefers the one with a lower expected debt cost. For these two contracts to give the same initial credit, we must have

$$\phi_0 D_u^S + (1-\phi_0)\, D_d^S = \phi_0 D_u^L + (1-\phi_0)\, D_d^L,$$

where, with potential abuse of notation, D_s^S (D_s^L) denotes the value of the long-term (short-term) debt in state $s \in \{u, d\}$. We define the optimist's cash-value-adjusted probability as $\widehat{\pi}_0^h \equiv \pi_0^h v_u / \left(\pi_0^h v_u + \left(1-\pi_0^h\right) v_d\right)$. The short-term contract gives the lower expected debt cost to the optimist, and thus preferable to the long-term debt, if and only if $\widehat{\pi}_0^h \geq \phi_0$ (Proposition 7 of the online Appendix).

This result establishes that the optimist's debt maturity choice depends on his cash-value-adjusted belief relative to the creditor's belief. The basic intuition works as follows: refinancing the short-term debt allows the borrower to trade a higher payment in state d for a lower payment in state u. Whether this trade is preferable depends on whether the borrower's belief about the probability of the up state, after adjusting for his marginal value of cash in the future states, is higher than the creditor's belief.

We can further show in the online Appendix that $\widehat{\pi}_0^h \geq \phi_0$ holds when (1) holds. Thus, when rollover risk is relatively high, long-term debt could dominate short-term debt in financing the optimists' asset positions. This result contrasts the standard intuition that optimists always prefer short-term debt financing if they have to borrow from pessimistic creditors.

Cash Saving The optimist can also choose to save cash for the next date. Interestingly, saving cash dominates over using long-term debt to finance an asset position if $\widehat{\pi}_0^h < \phi_0$ (Proposition 8 of the online Appendix). This is because the asset appears to be overvalued to the optimist after adjusting for his marginal value of cash on the following date. This result shows that when long-term debt dominates short-term debt for financing the optimist's asset position, saving cash dominates for establishing the position. Thus, the optimist will never strictly prefer long-term debt in the equilibrium. He could be indifferent between using long-term and short-term debt in financing his position only if $\widehat{\pi}_0^h = \phi_0$.

Optimal Debt Financing In summary, if $\widehat{\pi}_0^h > \phi_0$, the optimist will establish an asset position by using short-term debt with the maximum riskless promise to pay p_d on date 1; if $\widehat{\pi}_0^h < \phi_0$, he will save cash; if $\widehat{\pi}_0^h = \phi_0$, he is indifferent between saving cash and establishing the position (using either long-term or short-term debt).

3.3 The Equilibrium

The pessimists are initially endowed with all of the asset. Due to their risk neutrality and deep pockets, their asset valuations put lower bounds on the prices of the asset and any collateralized debt contract. On date 0, the pessimists are the marginal investor if and only if $\phi_0 = \pi_0^l$. They are indifferent between selling and retaining the asset in this case, and prefer selling if $\phi_0 > \pi_0^l$.

The joint equilibrium of the asset and credit markets can be characterized by the prices of the asset on date 0 and in states u and d of date 1: $\{p_0, p_u, p_d\}$; the fraction of optimists who establish asset positions on date 0: $\alpha \in [0, 1]$; and the fraction of pessimists who sell their asset endowments on date 0: $\lambda \in [0, 1]$. In this equilibrium, each optimist and each pessimist solve his optimal decisions based on our earlier discussion, and the markets for the asset and the debt used by optimists must clear. In particular, if the optimists are the marginal investor in the markets, their aggregate purchasing power needs to be sufficient to support the asset price. We present the specific equilibrium conditions for each intermediate state $s \in \{0, u, d\}$ in the online Appendix, which also characterizes various cases that arise in the equilibrium.

Now we provide a numerical illustration to contrast the dynamic setting with the static benchmark setting. We set $\theta = 0.4$, $\pi_0^h = 0.6$, $\pi_0^l = 0.55$, $\pi_u^h = \pi_u^l = 0.5$, $\pi_d^h = 0.75$, and $\pi_d^l = 0.4$. The specified belief structure does not satisfy the monotone belief dispersion given in (1), and thus captures the rollover risk discussed after (1) in Section 2.

Figure 2 illustrates the effects of varying optimists' cash endowment c from 0.42 to 0.07. Panel A depicts the date-0 asset price p_0. The two horizontal lines at the levels of 0.5 and 0.556 represent the pessimists' and the optimists' expectation of the asset's fundamental value, i.e., $\mathbb{E}_0^l(\widetilde{\theta})$ and $\mathbb{E}_0^h(\widetilde{\theta})$. The equilibrium price has to fall between these two benchmark levels. The dotted-and-dashed line is the asset price under the static setting discussed in Section 2. When c is above 0.4, the equilibrium price is equal to the optimists' valuation. Once c falls below 0.4, the asset price starts to fall with c and is determined by the optimists' purchasing power rather than their valuation.

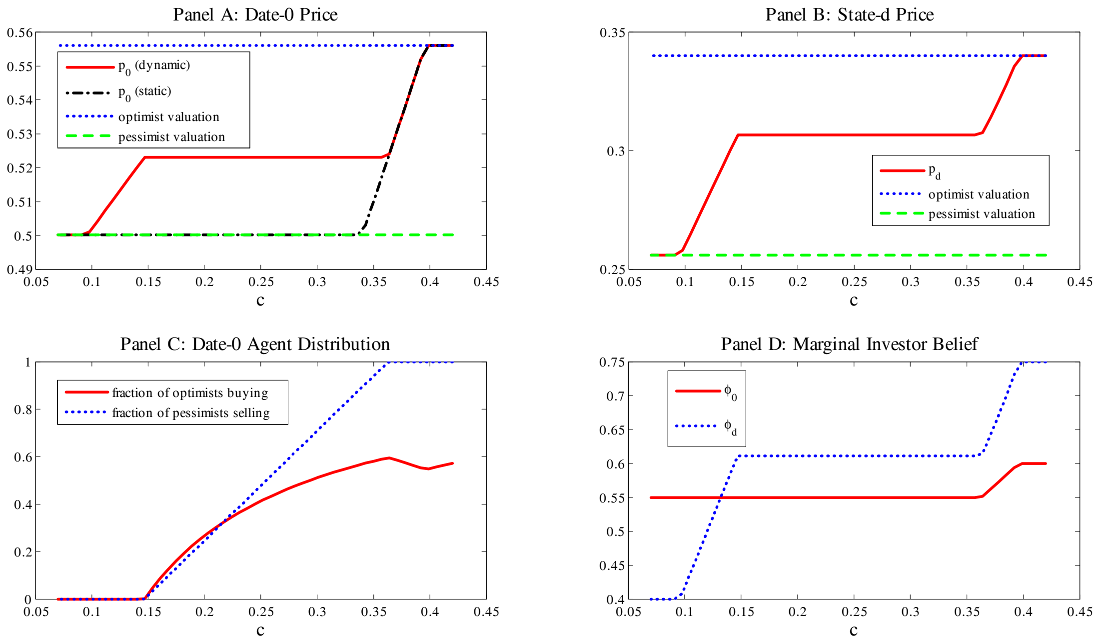

Figure 2: The effects of optimists' cash endowments based on the following parameters: $\theta = 0.4$, $\pi_0^h = 0.6$, $\pi_0^l = 0.55$, $\pi_u^h = \pi_u^l = 0.5$, $\pi_d^h = 0.75$, and $\pi_d^l = 0.4$.

As c falls below 0.336, the marginal investor shifts to the pessimists and the asset price is equal to the pessimists' valuation.

The solid line in Panel A gives the asset price in the dynamic setting discussed in this section. Like the dotted-and-dashed line, when c is above 0.4, the equilibrium price is equal to the optimists' valuation; and the asset price starts to fall with c once c falls below 0.4. Interestingly, as c drops below 0.357 but above 0.14, the price levels off at the level 0.523. In this range, the asset price is substantially higher than that in the static setting. Panel C reveals the key source of this difference—as c falls, a greater fraction of the optimists $(1 - \alpha)$ choose to save cash on date 0 and more pessimists hold their asset endowments (λ). As further shown by Panels B and D, in this range the optimists' cash saving sustains the asset price p_d and the implied belief of the marginal investor ϕ_d in state d of date 1 at constant levels, and, in particular, ensures the optimists as the marginal investor in this state (i.e., $\phi_d > \pi_d^l$). Panel D also shows that the marginal investor on date 0 is the pessimists ($\phi_0 = \pi_0^l = 0.55$). Despite this, the date-0 asset price p_0 is higher than that in the static setting because the pessimists anticipate that p_d will be determined by the optimists.

As c gets lower than 0.14 but higher than 0.09, all optimists save cash on date 0 ($\alpha = 0$) and all pessimists hold their asset endowments ($\lambda = 0$). In this case, the optimists remain the marginal investor of the asset in state d. As the optimists have saved all of their cash endowments to state d, the asset price in this state now falls with c, which in turn causes the asset price on date 0 to fall with c as well. Nevertheless, the date-0 price p_0 is higher than that in the static setting because the pessimists, the marginal investor on date 0, anticipate selling their asset holdings to optimists in state d. Only when c falls below 0.09 do the pessimists become the marginal investor of the asset both on date 0 and in state d of date 1. As a result, the prices in the dynamic and static settings coincide.

Taken together, Figure 2 demonstrates a significant impact of the asset's tradability when the optimists' cash endowment c is in the intermediate range between 0.09 and 0.357. In this range, the tradability induces at least some optimists to preserve cash from date 0 to state d, where the marginal value of cash is highest. These optimists' cash saving supports the asset price in state d, which in turn motivates the pessimists to assign a higher collateral value to the asset on date 0.

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