Dynamic Compensation Contracts with Private Savings

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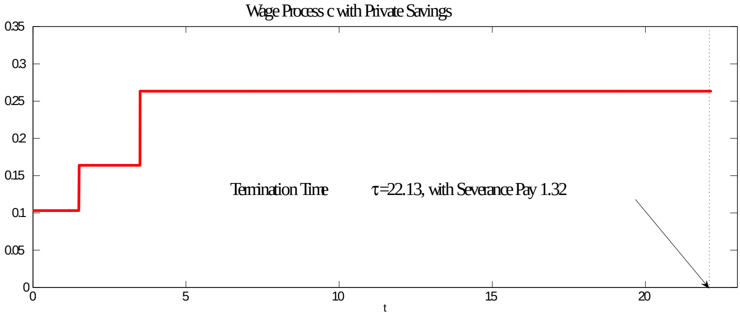
Introduction (1)

- Recent dynamic agency literature: Risk-neutral agent;
  - DeMarzo and Fishman (2007), DeMarzo and Sannikov (2007), Biais et al. (2007).
- Several important and realistic aspects are missing;
  - The agent is risk-averse;
  - Risk-aversion triggers the motive for private (unobservable, hidden) savings.
    - The agent saves to self-insure income risk, which unwinds the incentive scheme.
- This paper: a risk-averse agent who can privately save.

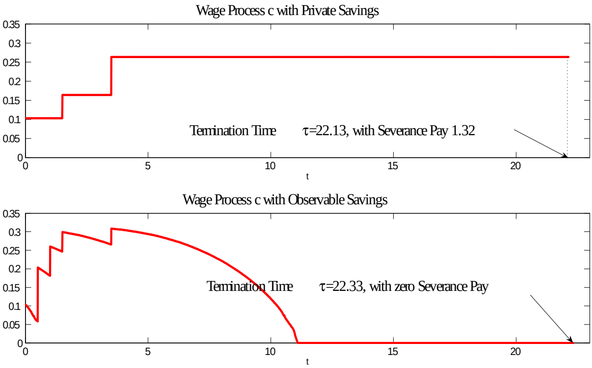
Introduction (2)

- In the optimal contract derived in this paper,
  - Downward rigid wages;
  - Severance pay if the agent is fired due to poor performance.



Introduction (3)

The possibility of private saving makes a dramatic difference.



For many jobs, the contract with private savings appears more empirically plausible.

Brief Literature Review (1)

- Dynamic contracting with risk-neutral agent;
  - DeMarzo and Fishman (2007), DeMarzo and Sannikov (2007), Biais et al. (2007), Tchistyi (2006), Sannikov (2007), He (2007a), Piskorski and Tchistyi (2007), etc.
- Dynamic contracting with risk-averse agent, but ruling out private saving: Rogerson (1985), Sannikov (2006); etc.
  - Rogerson (1985) shows that expected marginal utility is increasing over time—front-loaded compensation.
  - Under the optimal contract, the agent would want to save.
- Kocherlakota (2004): unemployment insurance contract with private saving. Implementing interior effort.
  - My paper studies a more general setup with endogenous termination.
  - Mitchell and Zhang (2007).

Brief Literature Review (2)

- CARA preferences with monetary effort cost, no wealth effect. Holmstrom and Milgrom (1987).
  - Contracting with private saving is tractable: Williams (2006), He (2007b).
- Harris and Holmstrom (1982): downward rigid wages.
  - Risk-averse worker; may quit once his productivity rises.
  - No moral hazard. Extension: Berk et al. (2007).
- Atkeson and Cole (2005): downward rigid wages.
  - Dynamic CSV model. Different agency issue.
  - No private saving, no terminations.

Technology

- The firm generates cashflow YdN_t at t; Y is constant;
  - $\{N_t\}$ is a Poisson process with intensity $\{a\}$;
- Agent's unobservable effort controls $a \in \{0, p, \overline{p}\}$, $\overline{p} > p > 0$;
  - $a_t = 0$ shirking; $a_t = p$ working; $a_t = \overline{p} = p + \epsilon$, myopic action.
  - $a_t = p$ is the first-best and the second-best.
- Termination: at τ (to be determined endogenously), liquidate the firm for L and fire the agent.
  - $L\left(\{a\}_0^\tau\right)$. Taking the myopic action $a = \overline{p}$ hurts L.
  - Investors' liquidation value $L(a)$ is noncontractible (e.g., realization of L is observed only by investors.)
  - Other general interpretation of myopic actions: multi-task setting; loss realized after the agent's tenure; ...
- The optimal contract implements $a_t = p$.

Agent

- The agent has no initial wealth, his outside option is zero.
- Agent's utility $U(c_t, a_t) = u(c_t) - \frac{b}{p}(a_t - p)$, where $b > 0$.
  - When shirking, he enjoys private benefit bdt.
- Focus on moderate wealth effect:

$$\inf_{c \geq 0} u'(c) = \underline{\gamma} > 0.$$

  - This bounds the monetary-equivalent effort cost: $\frac{b}{u'(c)} \leq \frac{b}{\underline{\gamma}}$.

Contract and Agent's Problem

- Contract Π specifies wages $\{c_t \geq 0 : 0 \leq t < \tau\}$, a termination time τ, and a lump-sum transfer $F_\tau \geq 0$ at termination.
  - All elements are $\{N_t\}$-history dependent.
- The agent has a private savings account with market interest rate r.
- Agent's consumption: $\{\widehat{c}\}_0^\tau$, and $\widehat{c}_\tau$ after termination (unemployed).
- With private savings $S_t \geq 0$, the agent solves

$$\max_{\{a\}_0^\tau, \{\widehat{c} \geq 0\}_0^\tau, \widehat{c}_\tau} \quad \mathbb{E}^a \left\{ \int_0^\tau e^{-rt} \left[u\left(\widehat{c}_t\right) - \frac{b}{p}\left(a_t - p\right) \right] dt + e^{-r\tau} \frac{u\left(\widehat{c}_\tau\right)}{r} \right\}$$

$$s.t. \quad dS_t = rS_t dt + c_t dt - \widehat{c}_t dt,\ S_0 = 0,\ S_t \geq 0,\ t \in [0, \tau]$$

$$\widehat{c}_\tau = r\left(F_\tau + S_\tau\right).$$

  - Borrowing is ruled out. Consistent with private-saving.

Investors' Problem

- W.l.o.g, consider Π s.t. $\widehat{c} = c$ (no savings NS), and $a = p$ (IC)

- The optimal contract maximizes the investors' expected payoff, subject to NS and IC.

$$\max_{\Pi \text{ is IC, NS}} \mathbb{E}^{a=p}\left[\int_0^\tau e^{-rt}\left(YdN_t - c_t\right)dt + e^{-r\tau}L - e^{-r\tau}F_\tau\right]$$

  - The agent enjoys a positive rent, and his participation constraint never binds.

Continuation Payoff and State Variables

- Agent's continuation payoff

$$W_t = \mathbb{E}_t \left[\int_t^{\tau} e^{-r(s-t)} u(c_s)\, ds + e^{-r(\tau-t)} \frac{u(rF_\tau)}{r} \right];$$

  - The agent's expected discounted payoff under equilibrium strategies (working + no saving);
  - W_t has to be the optimal value among all strategies;
    - i.e., agent's deviations cannot improve his value.

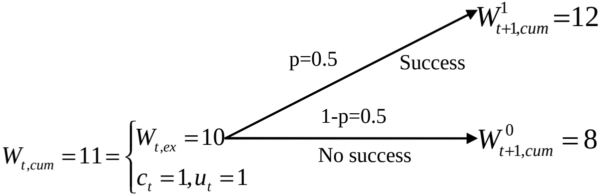
- Two state variables in optimal contracting, which summarize the histroy:
  1. W_t. Spear and Srivastava (1987). Effort incentives.
  2. The agent's marginal utility $M_t = u'(c_t)$. Saving incentives.

- Using "W_t is the agent's optimal value" to derive necessary conditions for the evolution of W_t and M_t.

- Discrete-time example. Use wage level c_t as the state variable.

Binding Incentive-Compatibility Constraint

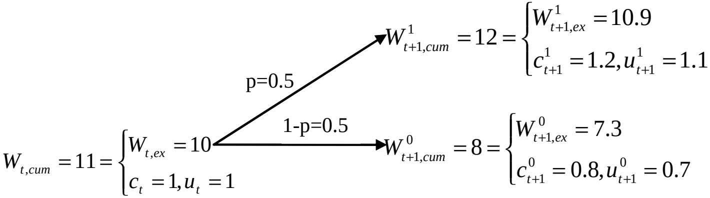
1. **Fixing consumption. Single effort-deviation$\Rightarrow$ binding IC constraint.**
   - Discrete-time example: $r = 0$, $p = 0.5$, and $b = 2$. $u(1) = 1$.



   - $W_{t,nc} = 10 = 0.5 \times W^1_{t+1,c} + 0.5 \times W^0_{t+1,c}$. Promise-keeping condition.
   - The difference $W^1_{t+1,c} - W^0_{t+1,c}$ provides incentives.
   - To implement interior effort p, $W^1_{t+1,c} - W^0_{t+1,c} = 4 = \frac{b}{p}$.
     - The agent solves $\max_{a \in \{0, p, \overline{p}\}} -\frac{b}{p}(a - p) + aW^1_{t+1,c} + (1-a) W^0_{t+1,c}$.
   - The agent is indifferent between working and shirking, given the consumption plan.

Joint Deviation

2. **Joint deviation of shirking + saving $\Rightarrow$ wage never falls without success.**
   - Consider c_{t+1}. The following contract is infeasible.



   - Wage cut after poor performance $c^0_{t+1} = 0.8$ triggers a joint-deviation.
   - The agent's time-t pre-consumption deviation value is
     - By shirking only, he gets $11 = 1 + 2 + 8$ still;
     - In the meantime by saving 0.1 for $t+1$, the agent gets $11 + 2u(0.9) - u(1) - u(0.8) > 11 = W_{t,c}$.
   - The possibility of *shirking and saving* implies $c^0_{t+1} \geq c_t$, or $M^0_{t+1} \leq M_t$.

Evolution of Continuation Payoff W_t

- Martingale Representation Theorem: there exists $\{\beta_t\}$ s.t.

$$dW_t + u(c_t)\,dt = rW_t dt + \beta_t (dN_t - pdt).$$

- Drift part $\mathbb{E}_t [dW_t + u(c_t)\,dt] = rW_t dt$. Promise-keeping.
- Volatility part: β_t controls the agent's incentives.
  - β_t captures the responsiveness of future pay to the agent's performance dN_t.
  - $\beta_t = \frac{b}{p}$ as in discrete-time example.
  - Binding IC constraint, and the agent is indifferent between p and 0.
- Utility $U(M_t)$. $dW_t = rW_t dt - U(M_t) + \frac{b}{p}(dN_t - pdt)$.

Evolution of Marginal Utility M_t

- No private saving $\Rightarrow$ in expectation, M_t nonincreasing.
  - Under equilibrium measure $a = p$,

$$\mathbb{E}_t^{a=p}\left[M_{t+dt}\right] = M_{t+}^0\left(1 - pdt\right) + M_{t+}^1 pdt \leq M_t.$$

  - The agent controls the measure. No savings under $a = 0$:

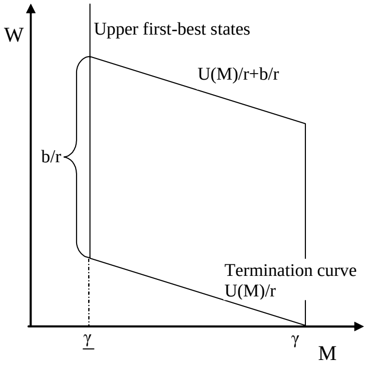
$$\mathbb{E}_t^{a=0}\left[M_{t+dt}\right] = M_{t+}^0 \leq M_t.$$

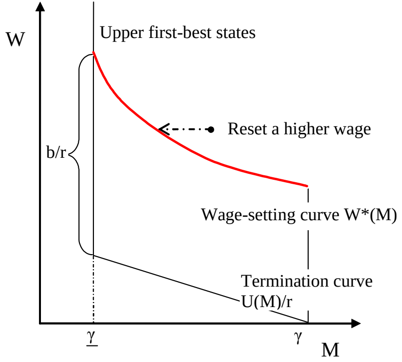
    - One needs the indifference between shirking and working here.

- In the optimal contract, $M_{t+}^0 = M_t$, $M_{t+}^1 \leq M_t$. Downward rigid wages.
  - Without success, $dW_t = rW_t dt - U\left(M\right) dt - bdt$, M constant.
  - With success, $W_{t+}^1 = W_t + \frac{b}{p}$; investors may adjust $M_t^1 \leq M_t$.

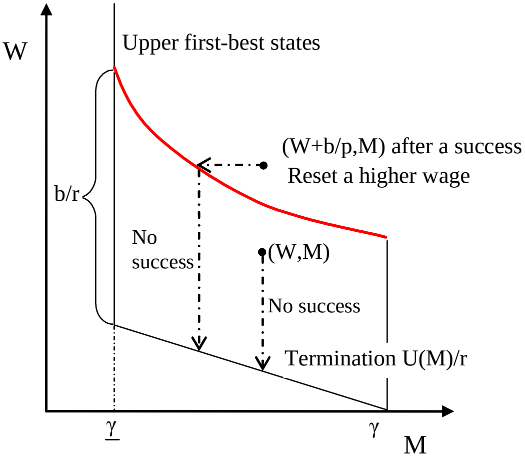
Optimal Contracting: State Space (1)

- Given M, $\frac{U(M)}{r}$ is the utility from the perpetual consumption $c(M)$.
- When $W = \frac{U(M)}{r}$, the agent has zero future rents, termination.
- Assume $u(c) = 1 - e^{-\gamma c}$, and risk-neutral when $M = \underline{\gamma}$.



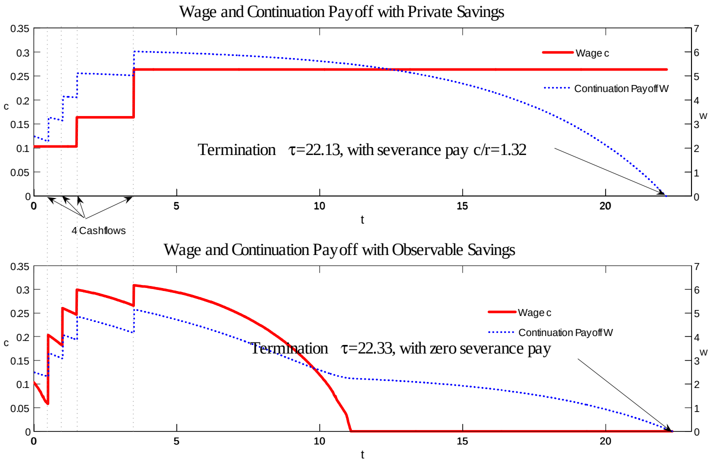


Optimal Contracting: State Space (2)



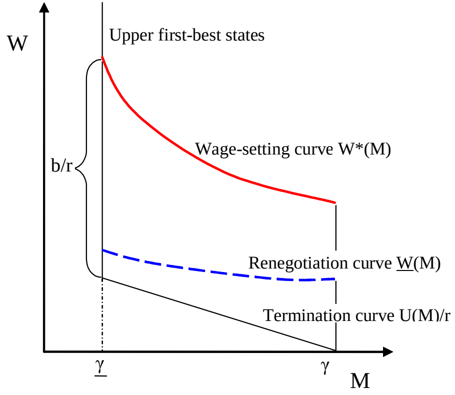
Optimal Contract

- Agent is protected by a promised life-time wage.
  - In dismissal, the agent walks away with $\frac{c(M)}{r}$.
- Agent works for future pay raises—they are lost in termination.



Extensions (1)

- Renegotiation-proof (RP) contracts.
- Investors' value function J, RP requires that $J_W \leq 0$. Otherwise both parties would agree to renegotiate to $W' > W$.
- Optimal contract features a renegotiation boundary $W^{RP}(M)$.



Extensions (2)

- Adverse selection: What if the agent has privately observed initial wealth?
- In this case investors should offer a menu of contracts.
- In the paper I give a sufficient condition for each contract to take the form derived above.
- Conjecture: for screening purposes, raising the wage even without success might be optimal.
  - Reward the agent even though performance is poor.

Summary

- Optimal contracting with risk-averse agent and private saving.
- In the optimal contract,
  - Wages are rigid, insensitive to the agent's performance;
  - Termination serves as a punishment, though the agent will walk away with severance pay.
- Reasonable contracting frictions can make "inefficient" compensation schemes optimal.