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Rollover Risk and Credit Risk

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ABSTRACT

Our model shows that deterioration in debt market liquidity leads to an increase in not only liquidity premium of corporate bonds but also credit risk. The latter effect originates from firms' debt rollover. When liquidity deterioration causes a firm to suffer losses in rolling over its maturing debt, equity holders bear the losses while maturing debt holders are paid in full. This conflict leads the firm to default at a higher fundamental threshold. Our model demonstrates an intricate interaction between liquidity premium and default premium and highlights the role of short-term debt in exacerbating rollover risk.

THE YIELD SPREAD OF a firm's bond relative to the risk-free interest rate directly determines the firm's debt financing cost, and is often referred to as its credit spread. It is widely recognized that the credit spread reflects not only a default premium determined by the firm's credit risk but also a liquidity premium due to illiquidity of the secondary debt market (e.g., Longstaff, Mithal, and Neis (2005), and Chen, Lesmond, and Wei (2007)). However, academics and policy makers tend to treat both the default premium and the liquidity premium as independent, and thus ignore interactions between them. The financial crisis of 2007 to 2008 demonstrates the importance of such an interaction—deterioration in debt market liquidity caused severe financing difficulties for many financial firms, which in turn exacerbated their credit risk.

In this paper, we develop a theoretical model to analyze the interaction between debt market liquidity and credit risk through so-called rollover risk: when debt market liquidity deteriorates, firms face rollover losses from issuing new bonds to replace maturing bonds. To avoid default, equity holders need to bear the rollover losses, while maturing debt holders are paid in full. This

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intrinsic conflict of interest between debt and equity holders implies that equity holders may choose to default earlier. This conflict of interest is similar in spirit to the classic debt overhang problem described by Myers (1977) and has been highlighted by Flannery (2005) and Duffie (2009) as a crucial obstacle to recapitalizing banks and financial institutions in the aftermath of various financial crises, including the recent one.

We build on the structural credit risk model of Leland (1994) and Leland and Toft (1996). Ideal for our research question, this framework adopts the endogenous-default notion of Black and Cox (1976) and endogenously determines a firm's credit risk through the joint valuation of its debt and equity. When a bond matures, the firm issues a new bond with the same face value and maturity to replace it at the market price, which can be higher or lower than the principal of the maturing bond. This rollover gain/loss is absorbed by the firm's equity holders. As a result, the equity price is determined by the firm's current fundamental (i.e., the firm's value when it is unlevered) and expected future rollover gains/losses. When the equity value drops to zero, the firm defaults endogenously and bond holders can only recover their debt by liquidating the firm's assets at a discount.

We extend this framework by including an illiquid debt market. Bond holders are subject to Poisson liquidity shocks. Upon the arrival of a liquidity shock, a bond holder has to sell his holdings at a proportional cost. The trading cost multiplied by bond holders' liquidity shock intensity determines the liquidity premium in the firm's credit spread. Throughout the paper, we take bond market liquidity as exogenously given and focus on the effect of bond market liquidity deterioration (due to either an increase in the trading cost or an increase in investors' liquidity shock intensity) on the firm's credit risk.

A key result of our model is that even in the absence of any constraint on the firm's ability to raise more equity, deterioration in debt market liquidity can cause the firm to default at a higher fundamental threshold due to the surge in the firm's rollover losses. Equity holders are willing to absorb rollover losses and bail out maturing bond holders to the extent that the equity value is positive, that is, the option value of keeping the firm alive justifies the cost of absorbing rollover losses. Deterioration in debt market liquidity makes it more costly for equity holders to keep the firm alive. As a result, not only does the liquidity premium of the firm's bonds rise, but also their default probability and default premium.

Debt maturity plays an important role in determining the firm's rollover risk. While shorter maturity for an individual bond reduces its risk, shorter maturity for all bonds issued by a firm exacerbates its rollover risk by forcing its equity holders to quickly absorb losses incurred by its debt financing. Leland and Toft (1996) numerically illustrate that shorter debt maturity can lead a firm to default at a higher fundamental boundary. We formally analyze this effect and further show that deterioration in market liquidity can amplify this effect.

Our calibration shows that deterioration in market liquidity can have a significant effect on credit risk of firms with different credit ratings and debt maturities. If an unexpected shock causes the liquidity premium to increase by 100 basis points, the default premium of a firm with a speculative grade B rating and 1-year debt maturity (a financial firm) would rise by 70 basis points, which contributes to 41% of the total credit spread increase. As a result of the same liquidity shock, the increase in default premium contributes to a 22.4% increase in the credit spread of a BB rated firm with 6-year debt maturity (a nonfinancial firm), 18.8%for a firm with an investment grade A rating and 1-year debt maturity, and 11.3% for an A rated firm with 6-year debt maturity.

Our model has implications for a broad set of issues related to firms' credit risk. First, our model highlights debt market liquidity as a new economic factor for predicting firm default. This implication can help improve the empirical performance of structural credit risk models (e.g., Merton (1973), Leland (1994), Longstaff and Schwartz (1995), and Leland and Toft (1996)), which focus on the so-called distance to default (a volatility-adjusted measure of firm leverage) as the key variable driving default. Debt market liquidity can also act as a common factor in explaining firms' default correlation, a phenomenon that commonly used variables such as distance to default and trailing stock returns of firms and the market cannot fully explain (e.g., Duffie et al. (2009)).

Second, the intrinsic interaction between liquidity premia and default premia derived from our model challenges the common practice of decomposing firms' credit spreads into independent liquidity-premium and default-premium components and then assessing their quantitative contributions (e.g., Longstaff et al. (2005), Beber, Brandt, and Kavajecz (2009), and Schwarz (2009)). This interaction also implies that in testing the effect of liquidity on firms' credit spreads, commonly used control variables for default risk such as the credit default swap spread may absorb the intended liquidity effects and thus cause underestimation.

Third, by deriving the effect of short-term debt on firms' rollover risk, our model highlights the role of the so-called maturity risk, whereby firms with shorter average debt maturity or more short-term debt face greater default risk. As pointed out by many observers (e.g., Brunnermeier (2009) and Krishnamurthy (2010)), the heavy use of short-term debt financing such as commercial paper and overnight repos is a key factor in the collapse of Bear Stearns and Lehman Brothers.

Finally, our model shows that liquidity risk and default risk can compound each other and make a bond's betas (i.e., price exposures) with respect to fundamental shocks and liquidity shocks highly variable. In the same way that gamma (i.e., variability of delta) reduces the effectiveness of discrete delta hedging of options, the high variability implies a large residual risk in bond investors' portfolios even after an initially perfect hedge of the portfolios' fundamental and liquidity risk.

Our paper complements several recent studies on rollover risk. Acharya, Gale, and Yorulmazer (2011) study a setting in which asset owners have no capital and need to use the purchased risky asset as collateral to secure short-term debt funding. They show that the high rollover frequency associated with short-term debt can lead to diminishing debt capacity. In contrast to their model, our model demonstrates severe consequences of short-term debt even in the absence of any constraint on equity issuance. This feature also differentiates our model from Morris and Shin (2004, 2010) and He and Xiong (2010), who focus on rollover risk originated from coordination problems between debt holders of firms that are restricted from raising more equity. Furthermore, by highlighting the effects of market liquidity within a standard credit-risk framework, our model is convenient for empirical calibrations.

The paper is organized as follows. Section I presents the model setting. In Section II, we derive the debt and equity valuations and the firm's endogenous default boundary in closed form. Section III analyzes the effects of market liquidity on the firm's credit spread. Section IV examines the firm's optimal leverage. We discuss the implications of our model for various issues related to firms' credit risk in Section V and conclude in Section VI. The Appendix provides technical proofs.

I. The Model

We build on the structural credit risk model of Leland and Toft (1996) by adding an illiquid secondary bond market. This setting is generic and applies to both financial and nonfinancial firms, although the effects illustrated by our model are stronger for financial firms due to their higher leverage and shorter debt maturities.

A. Firm Assets

Consider a firm. Suppose that in the absence of leverage, the firm's asset value $\{V_t : 0 \le t < \infty\}$ follows a geometric Brownian motion in the risk-neutral probability measure

$$\frac{dV_t}{V_t} = (r - \delta)\,dt + \sigma\,dZ_t, \tag{1}$$

where r is the constant risk-free rate,[1] δ is the firm's constant cash payout rate, σ is the constant asset volatility, and $\{Z_t : 0 \le t < \infty\}$ is a standard Brownian motion, representing random shocks to the firm's fundamental. Throughout the paper, we refer to V_t as the firm's fundamental.[2]

When the firm goes bankrupt, we assume that creditors can recover only a fraction α of the firm's asset value from liquidation. The bankruptcy cost $1 - \alpha$ can be interpreted in different ways, such as loss from selling the firm's real

[1] In this paper, we treat the risk-free rate as constant and exogenous. This assumption simplifies the potential flight-to-liquidity effect during liquidity crises.

[2] As in Leland (1994), we treat the unlevered firm value process $\{V_t : 0 \le t < \infty\}$ as the exogenously given state variable to focus on the effects of market liquidity and debt maturity. In our context, this approach is equivalent to directly modeling the firm's exogenous cash flow process $\{\phi V_t : 0 \le t < \infty\}$ as the state variable (i.e., the so-called EBIT model advocated by Goldstein, Ju, and Leland (2001)). For instance, Hackbarth, Miao, and Morellec (2006) use this EBIT model framework to analyze the effects of macroeconomic conditions on firms' credit risk.

assets to second-best users, loss of customers because of anticipation of the bankruptcy, asset fire-sale losses, legal fees, etc. An important detail to keep in mind is that the liquidation loss represents a deadweight loss to equity holders ex ante, but ex post is borne by debt holders.

B. Stationary Debt Structure

The firm maintains a stationary debt structure. At each moment in time, the firm has a continuum of bonds outstanding with an aggregate principal of P and an aggregate annual coupon payment of C. Each bond has maturity m, and expirations of the bonds are uniformly spread out over time. This implies that during a time interval $(t, t+dt)$, a fraction $\frac{1}{m}dt$ of the bonds matures and needs to be rolled over.

We measure the firm's bonds by m units. Each unit thus has a principal value of

$$p = \frac{P}{m} \tag{2}$$

and an annual coupon payment of

$$c = \frac{C}{m}. \tag{3}$$

These bonds differ only in the time-to-maturity $\tau \in [0, m]$. Denote by $d(V_t, \tau)$ the value of one unit of a bond as a function of the firm's fundamental V_t and time-to-maturity τ.

Following the Leland framework, we assume that the firm commits to a stationary debt structure denoted by (C, P, m). In other words, when a bond matures, the firm will replace it by issuing a new bond with identical maturity, principal value, and coupon rate. In most of our analysis, we take the firm's leverage (i.e., C and P) and debt maturity (i.e., m) as given; we discuss the firm's initial optimal leverage and maturity choices in Section IV.

C. Debt Rollover and Endogenous Bankruptcy

When the firm issues new bonds to replace maturing bonds, the market price of the new bonds can be higher or lower than the required principal payments of the maturing bonds. Equity holders are the residual claimants of the rollover gains/losses. For simplicity, we assume that any gain will be immediately paid out to equity holders and any loss will be paid off by issuing more equity at the market price. Thus, over a short time interval $(t, t+dt)$, the net cash flow to equity holders (omitting dt) is

$$NC_t = \delta V_t - (1-\pi)C + d(V_t, m) - p. \tag{4}$$

The first term is the firm's cash payout. The second term is the after-tax coupon payment, where π denotes the marginal tax benefit rate of debt. The third and fourth terms capture the firm's rollover gain/loss by issuing new bonds to replace maturing bonds. In this transaction, there are dt units of bonds maturing. The maturing bonds require a principal payment of pdt. The market value of the newly issued bonds is $d(V_t, m)dt$. When the bond price $d(V_t, m)$ drops, equity holders have to absorb the rollover loss $[d(V_t, m) - p]dt$ to prevent bankruptcy.

When the firm issues additional equity to pay off the rollover loss, the equity issuance dilutes the value of existing shares. As a result, the rollover loss feeds back into the equity value. This is a key feature of the model—the equity value is jointly determined by the firm's fundamental and expected future rollover gains/losses.[3] Equity holders are willing to buy more shares and bail out the maturing debt holders as long as the equity value is still positive (i.e., the option value of keeping the firm alive justifies the expected rollover losses). The firm defaults when its equity value drops to zero, which occurs when the firm fundamental drops to an endogenously determined threshold V_B. At this point, the bond holders are entitled to the firm's liquidation value αV_B, which in most cases is below the face value of debt P.

To focus on the liquidity effect originating from the debt market, we ignore any additional frictions in the equity market such as transaction costs and asymmetric information. It is important to note that while we allow the firm to freely issue more equity, the equity value can be severely affected by the firm's debt rollover losses. This feedback effect allows the model to capture difficulties faced by many firms in raising equity during a financial market meltdown even in the absence of any friction in the equity market.

We adopt the stationary debt structure of the Leland framework, that is, newly issued bonds have identical maturity, principal value, coupon rate, and seniority as maturing bonds. When facing rollover losses, it is tempting for the firm to reduce rollover losses by increasing the seniority of its newly issued bonds, which dilutes existing debt holders. Leland (1994) illustrates a dilution effect of this nature by allowing equity holders to issue more pari passu bonds. Since doing so necessarily hurts existing bond holders, it is usually restricted by bond covenants (e.g., Smith and Warner (1979)).[4] However, in

[3] A simple example works as follows. Suppose a firm has one billion shares of equity outstanding, and each share is initially valued at \$10. The firm has \$10 billion of debt maturing now, and because of an unexpected shock to the bond market liquidity, the firm's new bonds with the same face value can only be sold for \$9 billion. To cover the shortfall, the firm needs to issue more equity. As the proceeds from the share offering accrue to the maturing debt holders, the new shares dilute the existing shares and thus reduce the market value of each share. If the firm only needs to roll over its debt once, then it is easy to compute that the firm needs to issue $1/9$ billion shares and each share is valued at \$9. The \$1 price drop reflects the rollover loss borne by each share. If the firm needs to rollover more debt in the future and the debt market liquidity problem persists, the share price should be even lower due to the anticipation of future rollover losses. We derive such an effect in the model.

[4] Brunnermeier and Oehmke (2010) show that if a firm's bond covenants do not restrict the maturity of its new debt issuance, a maturity rat race could emerge as each debt holder would demand the shortest maturity to protect himself against others' demands to have shorter maturities. As shorter maturity leads to implicit higher priority, this result illustrates a severe consequence of not imposing priority rules on future bond issuance in bond covenants.

practice covenants are imperfect and cannot fully shield bond holders from future dilution. Thus, when purchasing newly issued bonds, investors anticipate future dilution and hence pay a lower price. Though theoretically interesting and challenging, this alternative setting is unlikely to change our key result: if debt market liquidity deteriorates, investors will undervalue the firm's newly issued bonds (despite their greater seniority), which in turn will lead equity holders to suffer rollover losses and default earlier.[5] Pre-committing equity holders to absorb ex post rollover losses can resolve the firm's rollover risk. However, this resolution violates equity holders' limited liability. Furthermore, enforcing ex post payments from dispersed equity holders is also costly.

Under the stationary debt structure, the firm's default boundary V_B is constant, which we derive in the next section. As in any trade-off theory, bankruptcy involves a dead-weight loss. Endogenous bankruptcy is a reflection of the conflict of interest between debt and equity holders: when the bond prices are low, equity holders are not willing to bear the rollover losses necessary to avoid the deadweight loss of bankruptcy. This situation resembles the so-called debt overhang problem described by Myers (1977), as equity holders voluntarily discontinue the firm by refusing to subsidize maturing debt holders.

D. Secondary Bond Markets

We adopt a bond market structure similar to that in Amihud and Mendelson (1986). Each bond investor is exposed to an idiosyncratic liquidity shock, which arrives according to a Poisson occurrence with intensity ξ. Upon the arrival of the liquidity shock, the bond investor has to exit by selling his bond holding in the secondary market at a fractional cost of k. In other words, the investor only recovers a fraction $1 - k$ of the bond's market value.[6] We shall broadly

[5] Diamond (1993) presents a two-period model in which it is optimal (even ex ante) to make refinancing debt (issued at intermediate date 1) senior to existing long-term debt (which matures at date 2). In that model, better-than-average firms want to issue more information-sensitive short-term debt at date 0. Because making refinancing debt more senior allows more date-0 short-term debt to be refinanced, it increases date-0 short-term debt capacity. Although the information-driven preference of short-term debt is absent in our model, this insight does suggest that making refinancing debt senior to existing debt can reduce the firm's rollover losses. However, the two-period setting considered by Diamond misses an important issue associated with recurring refinancing of real-life firms. To facilitate our discussion, take the infinite horizon setting of our model. Suppose that newly issued debt is always senior to existing debt, that is, the priority rule in bankruptcy now becomes inversely related to the time-to-maturity of existing bonds. This implies that newly issued bonds, while senior to existing bonds, must be junior to bonds issued in the future. Therefore, although equity holders can reduce rollover losses at the default boundary (because debt issued right before default is most senior during the bankruptcy), they may incur greater rollover losses when further away from the default boundary (because bonds issued at this time are likely to be junior in a more distant bankruptcy). The overall effect is unclear and worth exploring in future research.

[6] As documented by a series of empirical papers (e.g., Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), Mahanti et al. (2008), and Bao, Pan, and Wang (2011)), the secondary markets for corporate bonds are highly illiquid. The illiquidity is reflected

attribute this cost to either the market impact of the trade (e.g., Kyle (1985)), or the bid-ask spreads charged by bond dealers (e.g., Glosten and Milgrom (1985)).

While our model focuses on analyzing the effect of external market liquidity, it is also useful to note the importance of firms' internal liquidity. By keeping more cash and acquiring more credit lines, a firm can alleviate its exposure to market liquidity.[7] By allowing the firm to raise equity as needed, our model shuts off the internal-liquidity channel and instead focuses on the effect of external market liquidity. It is reasonable to conjecture that the availability of internal liquidity can reduce the effect of market liquidity on firms' credit spreads. However, internal liquidity holdings cannot fully shield firms from deterioration in market liquidity as long as internal liquidity is limited.[8] Indeed, as documented by Almeida et al. (2009) and Hu (2011), during the recent credit crisis nonfinancial firms that happened to have a greater fraction of long-term debt maturing in the near future had more pronounced investment declines and greater credit spread increases than otherwise similar firms. This evidence demonstrates the firms' reliance on market liquidity despite their internal liquidity holdings. We leave a more comprehensive analysis of the interaction between internal and external liquidity for future research.

II. Valuation and Default Boundary

A. Debt Value

We first derive bond valuation by taking the firm's default boundary V_B as given. Recall that $d(V_t, \tau; V_B)$ is the value of one unit of a bond with a time-to-maturity of $\tau < m$, an annual coupon payment of c, and a principal value of p. We have the following standard partial differential equation for the bond value:

$$rd(V_t, \tau) = c - \xi k d(V_t, \tau) - \frac{\partial d(V_t, \tau)}{\partial \tau} + (r - \delta) V_t \frac{\partial d(V_t, \tau)}{\partial V} + \frac{1}{2}\sigma^2 V_t^2 \frac{\partial^2 d(V_t, \tau)}{\partial V^2}. \tag{5}$$

by a large bid-ask spread that bond investors have to pay in trading with dealers, as well as a potential price impact of the trade. Edwards et al. (2007) show that the average effective bid-ask spread on corporate bonds ranges from eight basis points for large trades to 150 basis points for small trades. Bao et al. (2011) estimate that in a relatively liquid sample, the average effective trading cost, which incorporates bid-ask spread, price impact, and other factors, ranges from 74 to 221 basis points depending on the trade size. There is also large variation across different bonds with the same trade size.

[7] Bolton, Chen, and Wang (2011) recently model firms' cash holdings as an important aspect of their internal risk management. Campello et al. (2010) provide empirical evidence that during the recent credit crisis, nonfinancial firms used credit lines to substitute cash holdings to finance their investment decisions.

[8] In particular, when the firm draws down its credit lines, issuing new ones may be difficult, especially during crises. Acharya, Almeida, and Campello (2010) provide evidence that aggregate risk limits availability of credit lines and Murfin (2010) shows that a shock to a bank's capital tends to cause the bank to tighten its lending.

The left-hand side rd is the required (dollar) return from holding the bond. There are four terms on the right-hand side, capturing expected returns from holding the bond. The first term is the coupon payment. The second term is the loss caused by the occurrence of a liquidity shock. The liquidity shock hits with probability ξdt. Upon its arrival, the bond holder suffers a transaction cost of $kd(V_t, \tau)$ by selling the bond holding. The last three terms capture the expected value change due to a change in time-to-maturity τ (the third term) and a fluctuation in the value of the firm's assets V_t (the fourth and fifth terms). By moving the second term to the left-hand side, the transaction cost essentially increases the discount rate (i.e., the required return) for the bond to $r + \xi k$, the sum of the risk-free rate r and a liquidity premium ξk.

We have two boundary conditions to pin down the bond price based on equation (5). At the default boundary V_B, bond holders share the firm's liquidation value proportionally. Thus, each unit of bond gets

$$d(V_B, \tau; V_B) = \frac{\alpha V_B}{m}, \quad \text{for all } \tau \in [0, m]. \tag{6}$$

When $\tau = 0$, the bond matures and its holder gets the principal value p if the firm survives:

$$d(V_t, 0; V_B) = p, \text{ for all } V_t > V_B. \tag{7}$$

Equation (5) and boundary conditions (6) and (7) determine the bond's value:

$$d(V_t, \tau; V_B) = \frac{c}{r + \xi k} + e^{-(r+\xi k)\tau}\left[p - \frac{c}{r + \xi k}\right](1 - F(\tau)) + \left[\frac{\alpha V_B}{m} - \frac{c}{r + \xi k}\right]G(\tau), \tag{8}$$

where

$$F(\tau) = N(h_1(\tau)) + \left(\frac{V_t}{V_B}\right)^{-2a} N(h_2(\tau)),$$

$$G(\tau) = \left(\frac{V_t}{V_B}\right)^{-a+\widehat{z}} N(q_1(\tau)) + \left(\frac{V_t}{V_B}\right)^{-a-\widehat{z}} N(q_2(\tau)),$$

$$h_1(\tau) = \frac{(-v_t - a\sigma^2\tau)}{\sigma\sqrt{\tau}}, \; h_2(\tau) = \frac{(-v_t + a\sigma^2\tau)}{\sigma\sqrt{\tau}},$$

$$q_1(\tau) = \frac{(-v_t - \widehat{z}\sigma^2\tau)}{\sigma\sqrt{\tau}}, \; q_2(\tau) = \frac{(-v_t + \widehat{z}\sigma^2\tau)}{\sigma\sqrt{\tau}},$$

$$v_t \equiv \ln\left(\frac{V_t}{V_B}\right), \; a \equiv \frac{r - \delta - \sigma^2/2}{\sigma^2}, \; \widehat{z} \equiv \frac{[a^2\sigma^4 + 2(r + \xi k)\sigma^2]^{1/2}}{\sigma^2}, \tag{9}$$

and $N(x) \equiv \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$ is the cumulative standard normal distribution. This debt valuation formula is similar to the one derived in Leland and Toft (1996) except that market illiquidity makes $r + \xi k$ the effective discount rate for the bond payoff.

The bond yield is typically computed as the equivalent return on a bond conditional on its being held to maturity without default or trading. Given the bond price derived in equation (8), the bond yield y is determined by solving

$$d(V_t, m) = \frac{c}{y}(1 - e^{-ym}) + pe^{-ym}, \tag{10}$$

where the right-hand side is the price of a bond with a constant coupon payment c over time and a principal payment p at the bond maturity, conditional on no default or trading before maturity. The spread between y and the risk-free rate r is often called the credit spread of the bond. Since the bond price in equation (8) includes both trading cost and bankruptcy cost effects, the credit spread contains a liquidity premium and a default premium. The focus of our analysis is to uncover the interaction between the liquidity premium and the default premium.

B. Equity Value and Endogenous Default Boundary

Leland (1994) and Leland and Toft (1996) indirectly derive equity value as the difference between total firm value and debt value. Total firm value is the unlevered firm's value V_t, plus the total tax benefit, minus the bankruptcy cost. This approach does not apply to our model because part of the firm's value is consumed by future trading costs. Thus, we directly compute equity value $E(V_t)$ through the following differential equation:

$$rE = (r - \delta)V_t E_V + \frac{1}{2}\sigma^2 V_t^2 E_{VV} + \delta V_t - (1 - \pi)C + d(V_t, m) - p. \tag{11}$$

The left-hand side is the required equity return. This term should be equal to the expected return from holding the equity, which is the sum of the terms on the right-hand side.

- The first two terms $(r - \delta)V_t E_V + \frac{1}{2}\sigma^2 V_t^2 E_{VV}$ capture the expected change in equity value caused by a fluctuation in the firm's asset value V_t.
- The third term δV_t is cash flow generated by the firm per unit of time.
- The fourth term $(1 - \pi)C$ is the after-tax coupon payment per unit of time.
- The fifth and sixth terms $d(V_t, m) - p$ capture equity holders' rollover gain/loss from paying off maturing bonds by issuing new bonds at the market price.

Limited liability of equity holders provides the following boundary condition at V_B: $E(V_B) = 0$. Solving the differential equation in (11) is challenging because it contains the complicated bond valuation function $d(V_t, m)$ given in (8). We manage to solve it using the Laplace transformation technique detailed in the Appendix. Based on the equity value, we then derive equity holders' endogenous bankruptcy boundary V_B based on the smooth-pasting condition $E'(V_B) = 0$.[9]

[9] Chen and Kou (2009) provide a rigorous proof of the optimality of the smooth-pasting condition in an endogenous-default model under a set of general conditions, which include finite debt maturity and a jump-and-diffusion process for the firm's unlevered asset value.

The results on the firm's equity value and endogenous bankruptcy boundary are summarized in the next proposition.

PROPOSITION 1: *The equity value $E(V_t)$ is given in equation (A7) of Appendix A. The endogenous bankruptcy boundary V_B is given by*

$$V_B = \frac{\dfrac{(1-\pi)C + (1-e^{-(r+\xi k)m})\left(p - \dfrac{c}{r+\xi k}\right)}{\eta} + \left\{\left(p - \dfrac{c}{r+\xi k}\right)[b(-a)+b(a)] + \dfrac{c}{r+\xi k}[B(-\widehat{z})+B(\widehat{z})]\right\}}{\dfrac{\delta}{\eta-1} + \dfrac{\alpha}{m}[B(-\widehat{z})+B(\widehat{z})]}, \tag{12}$$

where $a \equiv \frac{r-\delta-\sigma^2/2}{\sigma^2}$, $z \equiv \frac{(a^2\sigma^4+2r\sigma^2)^{1/2}}{\sigma^2}$, $\eta \equiv z - a > 1$, $\widehat{z} \equiv \frac{[a^2\sigma^4+2(r+\xi k)\sigma^2]^{1/2}}{\sigma^2}$,

$$b(x) = \frac{1}{z+x} e^{-(r+\xi k)m}[N(x\sigma\sqrt{m}) - e^{rm}N(-z\sigma\sqrt{m})],$$

$$B(x) = \frac{1}{z+x}[N(x\sigma\sqrt{m}) - e^{\frac{1}{2}[z^2-x^2]\sigma^2 m}N(-z\sigma\sqrt{m})].$$

III. Market Liquidity and Endogenous Default

Many factors can cause bond market liquidity to change over time. Increased uncertainty about a firm's fundamental can cause the cost of trading its bonds (i.e., k) to go up; less secured financing due to redemption risk faced by open-end mutual funds and margin risk faced by leveraged institutions (i.e., deterioration in funding liquidity a la Brunnermeier and Pedersen (2009)) can also cause bond investors' liquidity shock intensity (i.e., ξ) to rise. Through the increase of one or both of these variables, the liquidity premium ξk will increase. In this section we analyze the effect of such a shock to bond market liquidity on firms' credit spreads.

Figure 1 illustrates two key channels for a shock to ξ or k to affect a firm's credit spread. Besides the direct liquidity premium channel mentioned above, there is an indirect rollover risk channel. The increased liquidity premium suppresses the market price of the firm's newly issued bonds and increases equity holders' rollover losses. As a result, equity holders become more reluctant to keep the firm alive even though the falling bond price is caused by deterioration in market liquidity rather than the firm's fundamental. In other words, the default threshold V_B rises, which in turn leads to a greater default premium in the credit spread. This indirect rollover risk channel is the main focus of our analysis.

As ξ and k affect the bond price in equation (8) symmetrically through the liquidity premium, we use an increase in ξ to illustrate the effect. Specifically, we hold constant the firm's debt structure (i.e., leverage and bond maturity). This choice is realistic as bond covenants and other operational restrictions prevent real-life firms from swiftly modifying their debt structures in response

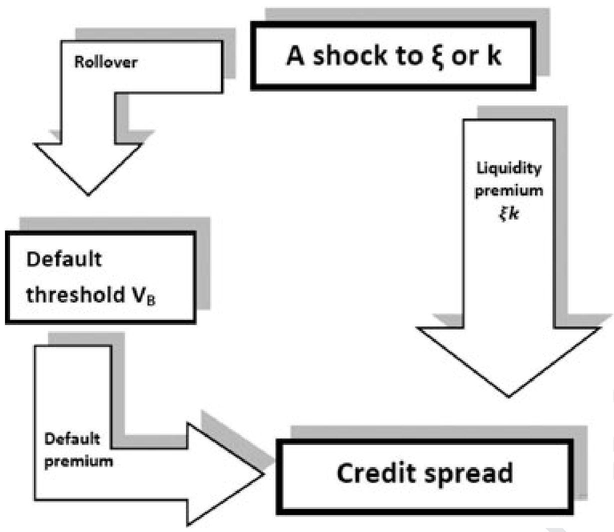

Figure 1. The key channels of liquidity effects on credit spreads.

to sudden market fluctuations. For simplicity, we also treat the increase in ξ as permanent in the analysis.[10]

A. Model Parameters

To facilitate our analysis, we use the set of baseline parameters given in Table I. We choose these parameters to be broadly consistent with those used in the literature to calibrate standard structural credit risk models. We set the risk-free rate r to 8%, which is also used by Huang and Huang (2003). We use a debt tax benefit rate of $\pi = 27\%$ based on the following estimate. While tax rate of bond income is 35%, many institutions holding corporate bonds enjoy a tax exemption. We use an effective bond income tax rate of 25%. The formula given by Miller (1977) thus implies a debt tax benefit of $1 - \frac{(1-35\%)(1-15\%)}{1-25\%} = 26.5\%$, where 35% is the marginal corporate tax rate and 15% is the marginal capital gains tax rate.[11]

We first focus on calibrating our model to firms with a speculative-grade BB rating. In Section III.D below, we also calibrate the model to firms with an

[10] In an earlier version of this paper (NBER working paper #15653), we extend our model to incorporate a temporary liquidity shock. Specifically, an increase in ξ mean-reverts back to its normal level according to a Poisson occurrence. This extension becomes more technically involved and requires numerical analysis. The numerical results nevertheless show that as long as debt maturity is comparable to the expected length of the liquidity shock, treating the increase in ξ as permanent or temporary only leads to a modest difference in its impact on the firm's credit spread.

[11] The formula works as follows. One dollar after-tax to debt holders costs a firm $1/(1−25%) = $1.33. On the other hand, if $1.33 is booked as firm profit and paid out to equity holders, the after-tax income is only $1.33×(1 − 35%) × (1 − 15%) = $0.735, which implies a tax benefit of 26.5% to debt holders.

Table I
Baseline Parameters

General Environment	
Interest rate	$r = 8.0\%$
Debt tax benefit rate	$\pi = 27\%$
Firm Characteristics	
Volatility	$\sigma = 23\%$
Bankruptcy recovery rate	$\alpha = 60\%$
Payout rate	$\delta = 2\%$
Bond Market Illiquidity	
Transaction cost	$k = 1.0\%$
Liquidity shock intensity	$\xi = 1$
Debt Structure	
Maturity	$m = 1$
Current fundamental	$V_0 = 100$
Annual coupon payment	$C = 6.39$
Aggregate principal	$P = 61.68$

investment-grade A rating. According to Zhang, Zhou, and Zhu (2009), BB-rated firms have an average fundamental volatility of 23% and A-rated firms have an average of 21%. We therefore choose $\sigma = 23\%$ as the baseline value in Table I, and use $\sigma = 21\%$ in our later calibration of firms with an A rating. Chen (2010) finds that across nine different aggregate states, bonds have default recovery rates of around 60%. We set $\alpha = 60\%$. Huang and Zhou (2008) find that in a sample of firms the average payout rate is 2.14%, and, more specifically, the average for BB-rated firms is 2.15% and for A-rated firms is 2.02%. Given the small variation across different ratings, we use $\delta = 2\%$ throughout the paper.

Edwards et al. (2007) and Bao et al. (2011) find that the cost of trading corporate bonds decreases with bond rating and trade size. Consistent with their estimates, we choose $k = 1.0\%$ for BB-rated bonds and $k = 0.5\%$ for A-rated bonds. Furthermore, we set bond investors' liquidity shock intensity ξ to one, which is broadly consistent with the average turnover rate of corporate bonds in the sample analyzed by Bao et al. (2011).

As a firm's rollover risk is determined by its overall debt maturity rather than the maturity of a particular bond, we calibrate debt maturity in the model to firms' overall debt maturities. Guedes and Opler (1996) find that firms with different credit ratings have very similar debt maturities. According to Custodio, Ferreira, and Laureano (2010), the medium time-to-maturity of nonfinancial firms is 3 years, which implies an initial debt maturity of 6 years if debt expirations are uniformly distributed. Financial firms tend to have shorter debt maturities as they rely heavily on repo transactions with maturities from 1 day to 3 months and commercial paper with maturities of less than 9 months.

To highlight the rollover risk of financial firms, we choose $m = 1$ as the baseline value in Table I. We also report more modest but nevertheless significant effects of rollover risk in Section III.D for nonfinancial firms by varying m from one to three, six and ten.

Without loss of generality, we normalize the firm's current fundamental $V_0 = 100$ and choose its leverage to match its 1-year credit spread to the average spread of BB-rated bonds. Rossi (2009) summarizes the yield spread for different maturities and credit ratings in the TRACE data. He finds that the average spread for BB-rated bonds is 331 basis points when maturity is either 0–2 years or 3–10 years. For A-rated bonds, the average spread is 107 basis points if maturity is 0–2 years and 90 basis points if maturity is 3–10 years. Based on these numbers, we choose $C = 6.39$ and $P = 61.68$ so that the firm issues 1-year bonds at par and these bonds have a credit spread of 330 basis points. In our calibration in Section III.D, we set the target bond yield at 100 basis points for A-rated bonds.

B. Liquidity Premium and Default Premium

Figure 2 demonstrates the effects of an increase in ξ on the firm's rollover loss, endogenous default boundary, and credit spread by fixing other parameters as given in Table I. Panel A depicts equity holders' aggregate rollover loss per unit of time $d(V_t, m; V_B) - p$ against ξ. The line shows that the magnitude of rollover loss increases with ξ. That is, as bond holders' liquidity shock intensity increases, the increased liquidity premium makes it more costly for equity holders to roll over the firm's maturing bonds. Panel B shows that the firm's default boundary V_B consequently increases with ξ. In other words, when bond market liquidity deteriorates, equity holders will choose to default at a higher fundamental threshold. We formally prove these results in Proposition 2.

PROPOSITION 2: *All else equal, an increase in bond holders' liquidity shock intensity ξ decreases the firm's bond price and increases equity holders' default boundary V_B.*

Panel C of Figure 2 depicts the credit spread of the firm's newly issued bonds against ξ, and shows that it increases with ξ. More specifically, as ξ increases from one to two, the credit spread increases from 330 basis points to 499.6. Panel D further decomposes the bond spread into two components. One is the liquidity premium ξk, which, as shown by the dotted line, increases linearly with ξ. The residual credit spread after deducting the liquidity premium captures the part of the credit spread that is related to the firm's default risk. We call this component the default premium. Interestingly, the solid line shows that the default premium also increases with ξ. This result is in line with our earlier discussion: by raising the firm's default boundary, deterioration in bond market liquidity also increases the default component of the firm's credit spread. Specifically, as ξ increases from one to two, the liquidity premium rises by 100 basis points while the default premium increases by 69.6 basis points (which contributes to 41% of the total credit spread increase).

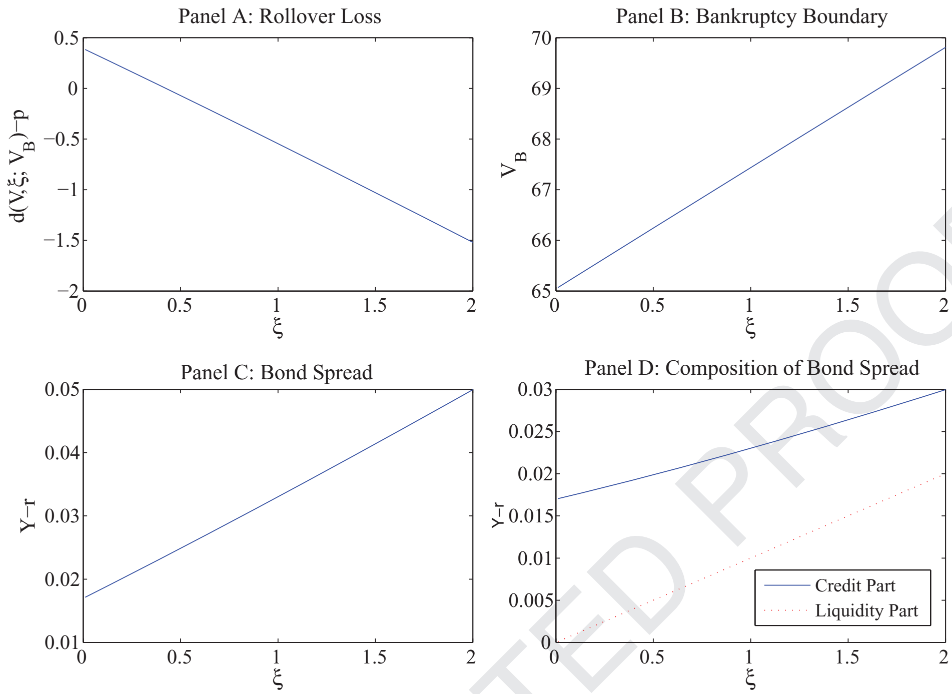

Figure 2. Effects of bond investors' liquidity demand intensity. This figure uses the baseline parameters listed in Table I. Panel A depicts equity holders' aggregate rollover loss per unit of time, $d(V_t, m; V_B) - p$, which has the same scale as the firm's fundamental; Panel B depicts their default boundary V_B; Panel C depicts the credit spread of the firm's newly issued bonds; and Panel D decomposes the credit spread into two components, the liquidity premium ξk and the remaining default premium. All the panels are with respect to ξ.

As deterioration in market liquidity increases the firm's debt financing cost, it is reasonable to posit that the resulting earlier default might be consistent with debt and equity holders' joint interest. To clarify this issue, suppose that the firm never defaults. Then the present value of the future tax shield is $\frac{\pi C}{r}$, while the present value of future bond transaction costs is $\frac{\xi k}{r}\frac{C}{r+\xi k}$, where $\frac{C}{r+\xi k}$ is the firm's bond value (i.e., coupon payments discounted by the transaction-cost-adjusted discount rate). The present value of the future tax shield is higher than that of future bond transaction costs if

$$\pi > \frac{\xi k}{r + \xi k}. \tag{13}$$

Under the condition in (13), default damages the joint interest of debt and equity holders because even in the absence of any bankruptcy costs, the tax shield benefit dominates the cost incurred by future bond trading.

The condition in (13) holds under the different sets of parameters that are used to generate Figure 2. Thus, the default boundary depicted in Panel B originates from the conflict of interest between debt and equity holders: when the bond price falls (even for liquidity reasons), equity holders have to bear all of the rollover losses to avoid default while maturing debt holders are paid in full. This unequal sharing of losses causes the equity value to drop to zero at V_B, at which point equity holders stop servicing the debt. If debt and equity holders were able to share the firm's losses, they would avoid the deadweight loss induced by firm default. See Section I.C for a discussion of various realistic considerations that can prevent the use of debt restructuring in this situation.

The asset pricing literature recognizes the importance of bond market liquidity on firms' credit spreads. However, most studies focus on the direct liquidity premium channel. For instance, Longstaff et al. (2005) find that while default risk can explain a large part of firms' credit spreads, there is still a significant nondefault component related to measures of bond-specific illiquidity; and Chen et al. (2007) show that bonds with lower market liquidity tend to earn higher credit spreads. In contrast, our model identifies a new channel—the rollover risk channel, through which the liquidity premium and default premium interact with each other. Our channel is also different from the bankruptcy renegotiation channel emphasized by Ericsson and Renault (2006), who show that market illiquidity can hurt bond holders' outside option in bankruptcy negotiation.

C. Amplification of Short-Term Debt

A standard intuition suggests that shorter debt maturity for an individual bond leads to lower credit risk. However, shortening the maturities of all bonds issued by a firm intensifies its rollover risk and makes it more vulnerable to deterioration in market liquidity. According to our model, a shorter debt maturity for the firm implies a higher rollover frequency. Directly from the rollover loss expression $d(V_t, m) - P/m$, if the market value of the firm's newly issued bonds $d(V_t, m)$ is below the principal of maturing bonds P/m, a higher rollover frequency forces equity holders to absorb a greater rollover loss per unit of time. This means a higher cost of keeping the firm alive, which in turn motivates equity holders to default at a higher fundamental threshold.

To illustrate this maturity effect, we compare two otherwise identical firms, one with debt maturity of 1 year and the other with debt maturity of 6 years. Note that the second firm has the same fundamental, coupon payment, and face value of debt as the first firm; in other words, we do not calibrate its credit spread to any benchmark level. As a result, this firm is different from the calibrated BB-rated firm with 6-year debt maturity in Section III.D.

Figure 3 demonstrates the different impacts of a change in ξ on these two firms with different maturities. Panel A shows that as bond investors' liquidity shock intensity ξ increases, both firms' rollover losses (per unit of time) increase. More importantly, the rollover loss of the firm with shorter debt maturity increases more than that of the firm with longer maturity. Panel B further confirms that while both firms' default boundaries increase with ξ, the boundary of the shorter maturity firm is uniformly higher. Panel C shows that as ξ increases from one to two, the credit spread of the shorter maturity firm

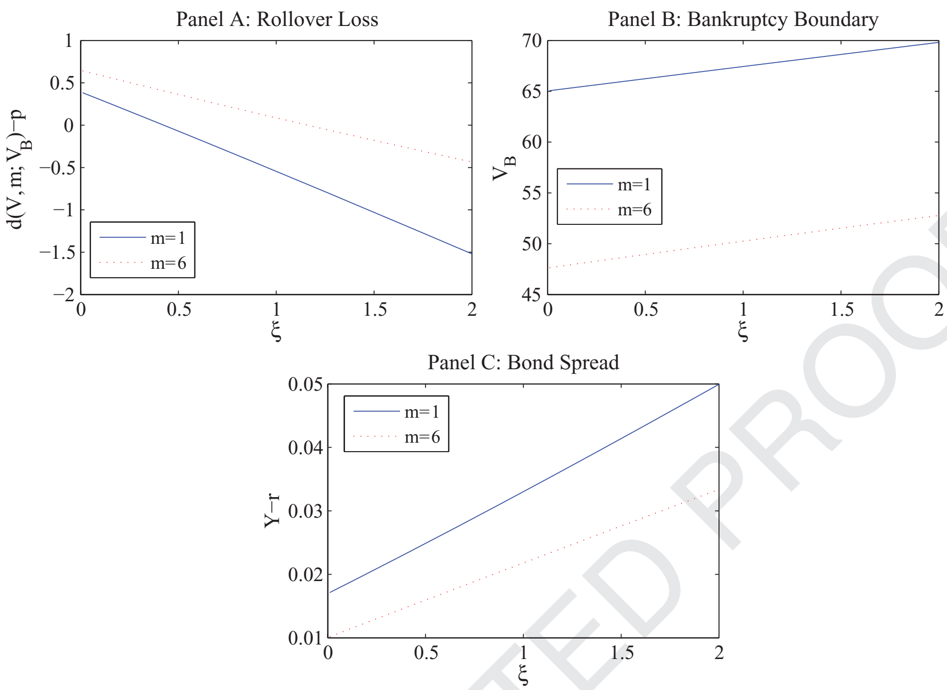

Figure 3. Effects of debt maturity m. This figure uses the baseline parameters listed in Table I, and compares two firms with different debt maturities $m = 1$ and 6. Panels A, B, and C depict equity holders' rollover loss $d(V_t, m; V_B) - p$, the endogenous default boundary V_B, and the credit spread of the firm's newly issued bonds, respectively. All panels are with respect to bond investors' liquidity shock intensity ξ.

increases by 170 basis points from 330 to 500, while that of the longer maturity firm increases only by 119 basis points from 215 to 334. As these firms share the same liquidity premium in their credit spreads, the difference in the changes in their credit spreads is due to the default component of credit spread.

We can formally prove the following proposition regarding the effect of debt maturity on the firm's rollover risk under the conditions that the principal payment due at debt maturity and bankruptcy costs are both sufficiently high.

PROPOSITION 3: *Suppose* $(r + \xi k)P - C \geq 0$ *and* $\frac{C}{r+\xi k}\frac{\delta}{\eta-1} > \alpha\frac{(1-\pi)C+((r+\xi k)P-C)}{\eta}$. *Then the firm's default boundary* V_B *decreases with its debt maturity m.*

From a contracting point of view, the effect of debt maturity on rollover gains/losses originates from short-term debt being a "harder" claim relative to long-term debt. Essentially, short-term bond holders do not share gains/losses with equity holders to the same extent as long-term debt holders do. As a result, short-term debt leads to greater rollover losses borne by equity holders in bad times. This is similar in spirit to the debt overhang problem described by Myers

(1977). See Diamond and He (2010) for a recent study that further analyzes the effects of short-term debt overhang on firms' investment decisions.[12]

In the aftermath of the recent financial crisis, many observers (e.g., Brunnermeier (2009) and Krishnamurthy (2010)) have pointed out the heavy use of short-term debt financing by many financial institutions leading up to the crisis. In the months preceding its bankruptcy, Lehman Brothers was rolling over 25% of its debt every day through overnight repos, a type of collateralized lending agreement with an extremely short maturity of 1 day. Consistent with the rollover difficulty faced by Lehman Brothers, Figure 3 and Proposition 3 demonstrate that short-term debt can significantly amplify a firm's rollover risk and make it vulnerable to shocks to bond market liquidity. Our model thus highlights firms' debt maturity structure as an important determinant of credit risk.

D. Calibration of Different Firms

Our model shows that liquidity premia and default premia are intertwined and work together in determining firms' credit spreads. In particular, an increase in liquidity premium can exacerbate default risk and make firms with weaker fundamentals more susceptible to default risk. To illustrate this effect, we compare responses of a set of firms with different credit ratings and debt maturities to the same liquidity shock represented by an increase in ξ. This exercise also allows us to show that deterioration in market liquidity can have a significant effect on the credit risk of a variety of firms through debt rollover.

We focus on firms with two particular credit ratings: investment-grade A and speculative-grade BB. For each credit rating, we consider firms with four different debt maturities: $m = 1, 3, 6,$ and 10. We let these firms share the same baseline values given in Table I for interest rate r, debt tax benefit rate π, bankruptcy recovery rate α, payout rate δ, current firm fundamental V_0, and investor liquidity shock intensity ξ. We let A-rated firms have fundamental volatility $\sigma = 21\%$ and bond trading cost $k = 0.5\%$, while BB-rated firms have $\sigma = 23\%$ and $k = 1.0\%$. For each A-rated firm, we calibrate its leverage (i.e., coupon payment C and face value of debt P) so that the firm issues new bonds at par and these bonds have a credit spread of 100 basis points at issuance. For each BB-rated firm, we calibrate its leverage so that its newly issued par bonds have a credit spread of 330 basis points. These parameter choices are discussed in Section III.A.

For each of the firms, Table II reports its bond spread when $\xi = 1$ (the baseline), 2, and 4, together with the total spread change from the baseline and the part caused by increased default risk. As ξ changes from one to two, the liquidity premium doubles from 100 basis points to 200 for the credit spread of

[12] This result is also similar to that in Manso, Strulovici, and Tchistyi (2010), who show that performance-sensitive debt, which corresponds to a rising refinancing rate for short-term debt when the firm's fundamental deteriorates, leads to earlier endogenous default. For other debt overhang effects in the Leland setting, see Lambrecht and Myers (2008) and He (2011).

Table II

Responses of Different Firms' Credit Spreads to a Liquidity Shock

The common parameters are $r = 8\%$, $\pi = 27\%$, $\alpha = 60\%$, $\delta = 2$, and $V_0 = 100$. For A-rated firms, $\sigma = 21\%$, $k = 50$ basis points. For BB-rated firms, $\sigma = 23\%$, $k = 100$ basis points. We calibrate a firm's leverage (C, P) so that its newly issued par bonds with the specified maturity have an initial credit spread of 100 basis points for A-rated firms and 330 basis points for BB-rated firms.

	$\xi = 1$	ξ rises to 2				ξ rises to 4			
				Default Part				Default Part	
Maturity (yrs)	Spread (bps)	Spread (bps)	ΔSpread (bps)	(bps)	(fraction)	Spread (bps)	ΔSpread (bps)	(bps)	(fraction)
Panel A: Firms with Speculative-Grade BB Rating									
$m = 1$	330	499.6	169.6	69.6	41.0%	853.0	523.0	223.0	42.6%
$m = 3$	330	474.6	144.6	44.6	30.8%	752.1	422.1	122.1	28.9%
$m = 6$	330	458.9	128.9	28.9	22.4%	699.8	369.8	69.8	18.9%
$m = 10$	330	450.3	120.3	20.3	16.9%	671.9	341.9	41.9	12.3%
Panel B: Firms with Investment-Grade A Rating									
$m = 1$	100	161.7	61.7	11.7	18.8%	290.7	190.7	40.7	21.3%
$m = 3$	100	157.2	57.2	7.2	12.6%	274.3	174.3	24.3	13.9%
$m = 6$	100	156.4	56.4	6.4	11.3%	266.9	166.9	16.9	10.1%
$m = 10$	100	153.7	53.7	3.7	6.9%	259.7	159.7	9.7	6.1%

a BB-rated firm and from 50 to 100 for that of an A-rated firm. Similarly, As ξ changes from one to four, the liquidity premium quadruples. According to Bao et al. (2011), the trading costs of corporate bonds more than quadrupled during the recent financial crisis. We thus interpret the change of ξ from one to two as a modest shock to market liquidity and from one to four as a severe crisis shock.

Table II shows that the credit spreads of BB-rated firms are more sensitive to the same shock to market liquidity than those of A-rated firms. Furthermore, for a given debt maturity, increased default risk contributes to a greater fraction of the credit spread increase for the BB-rated firm. This is because the weaker BB-rated firm is closer to its default boundary and thus more vulnerable to any increase in default boundary caused by the shock to market liquidity. This result sheds some light on the so-called flight-to-quality phenomenon. After major liquidity disruptions in financial markets, prices (credit spreads) of low quality bonds drop (rise) much more than those of high quality bonds.[13]

[13] Recent episodes include the stock market crash of 1987, the events surrounding the Russian default and the LTCM crisis in 1998, the events after the attacks of 9/11 in 2001, and the credit crisis of 2007 to 2008. See Bank for International Settlements report (1999) and Fender, Ho, and Hordahl (2009) for reports of flight to quality during the 1998 LTCM crisis and the period around

Table II also offers the calibrated magnitude of the effect of the market liquidity shock on different firms' credit risk. For firms with 1 year debt maturity (financial firms), the modest liquidity shock of ξ from one to two increases the default component of the credit spread of a BB-rated firm by 69.6 basis points (which contributes to 41% of the net credit spread increase) and that of an A-rated firm by 11.7 basis points (18.8% of the credit spread increase). While the effect is smaller for the A-rated firm, it is nevertheless significant. The shock can also have a significant effect on the credit risk of firms with 6 year debt maturity (nonfinancial firms). Specifically, the effect on the default component of the credit spread of a BB-rated firm is 28.9 basis points (22.4% of the credit spread increase), and the effect on an A-rated firm is 6.4 basis points (11.3% of the credit spread increase). For the more severe liquidity shock of ξ from one to four, increased credit risk contributes to similar fractions of these firms' credit spread increases.

IV. Optimal Leverage

Given the substantial impact of market liquidity on the firm's credit risk, it is important for the firm to incorporate this effect in its initial leverage choice at $t = 0$. We now discuss the firm's optimal leverage. Like Leland and Toft (1996), we take the unlevered asset value V_0 as given and compute the levered firm value by

$$v(C, P, V_0) = E(C, P, V_0; V_B(C, P)) + D(C, P, V_0; V_B(C, P)), \tag{14}$$

where the equity value $E(\cdot)$, debt value $D(\cdot)$, and default boundary $V_B(\cdot)$ are given in (A7), (8), and (12), respectively. For a given annual coupon payment C, we choose the aggregate face value of debt $P(C)$ such that the bond is issued at par at $t = 0$, that is, $P = D(C, P, V_0; V_B(C, P))$. We then search for the optimal C^* that maximizes (14) and calculate the optimal leverage ratio as

$$\frac{D(C^*, P(C^*), V_0; V_B(C^*, P(C^*)))}{E\left(C^*, P^*, V_0; V_B\left(C^*, P\left(C^*\right)\right)\right) + D\left(C^*, P\left(C^*\right), V_0; V_B\left(C^*, P\left(C^*\right)\right)\right)}.$$

In analyzing the firm's optimal leverage, we focus on the effects of three model parameters: bond trading cost k, debt maturity m, and the firm's asset volatility σ. Figure 4 depicts the firm's optimal leverage with respect to bond trading cost k (Panel A) and debt maturity m (Panel B) for two firms, one with asset volatility $\sigma = 15\%$ and the other with $\sigma = 23\%$. Both panels show that the optimal leverage of the firm with the lower asset volatility is uniformly higher than that of the other firm, because the former firm can afford to use a higher leverage due to its smaller credit risk.

the bankruptcy of Lehman Brothers in September 2008. Several recent studies (e.g., de Jong and Driessen (2006), Chen et al. (2007), and Acharya, Amihud, and Bharath (2009)) provide systematic evidence that the exposures (or betas) of speculative-grade corporate bonds to market liquidity shocks rise substantially during times of severe market illiquidity and volatility.

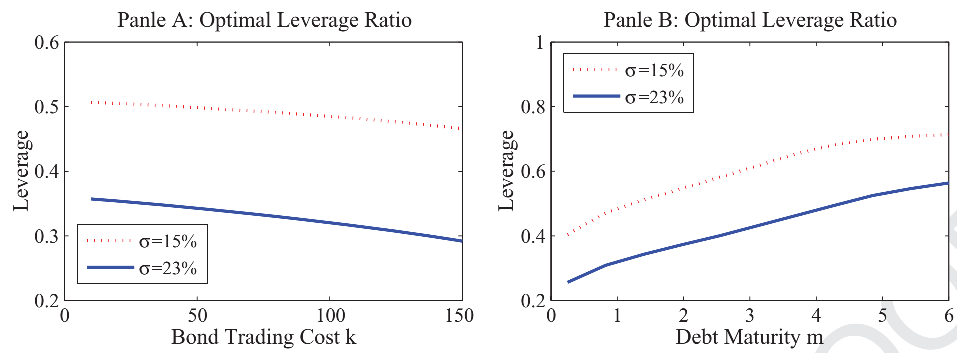

Figure 4. The firm's optimal leverage. This figure uses the baseline parameters listed in Table I. Panel A depicts the optimal initial leverage with respect to the bond trading cost k for two firms, one with asset volatility $\sigma = 15\%$ and the other with $\sigma = 23\%$. Panel B depicts the optimal leverage with respect to debt maturity m for these two firms.

Panel A shows that the optimal leverage of both firms decreases with bond trading cost. As k increases from 10 to 150 basis points, the optimal leverage of the firm with the higher asset volatility drops from 35.7% to 29.2%. This pattern is consistent with the key insight of our model that as the debt market becomes more illiquid, the firm's default risk rises, which in turn motivates the firm to use lower leverage.

Panel B shows that each firm's optimal leverage increases with its debt maturity. As m increases from 0.25 to six, the optimal leverage of the firm with 23% asset volatility increases from 25.6% to 56.4%. This pattern is again consistent with our earlier result that short-term debt amplifies firms' rollover risk. As a result, it is optimal to use a lower leverage for shorter debt maturity. This implication raises a question about firms' optimal debt maturity. In practice, bonds with shorter maturities tend to be more liquid (e.g., Bao et al. (2011)) and thus demand smaller liquidity premia. In the earlier version of this paper (NBER working paper #15653), we allow the firm to issue two types of bonds with different maturities and trading costs, and then analyze the tradeoff between the lower liquidity premium and higher rollover risk of short-term debt in determining the firm's optimal maturity structure. To save space, we do not present this analysis in the current version and instead refer interested readers to the earlier version.

It is well known that firm leverage predicted by the Leland model tends to be too high relative to the level observed in the data (e.g., Goldstein et al. (2001)). Given the presence of realistic rollover risk faced by firms, our analysis implies that illiquidity in the secondary bond market motivates firms to use lower leverage, and thus helps reconcile the observed leverage level with standard structural models.

While our model treats market liquidity as independent of a firm's fundamental, market liquidity tends to be cyclical with the aggregate economy. One can formally analyze this effect by extending our model to allow for time-varying liquidity regimes that are correlated with investors' pricing kernels. It is intuitive that a firm's optimal leverage and maturity choices should depend on the aggregate bond market liquidity regime, which in turn may have useful implications for leverage/credit cycles that we have observed in the past. Suppose that bond market liquidity follows a binary-state Markovian structure, and that firms may adjust their leverage and debt maturity at a certain adjustment cost. Then, in the high liquidity state, we expect firms to use relatively high leverage with shorter debt maturity because of the lower rollover risk they face. When the liquidity condition switches to the low regime, firms are likely to encounter mounting rollover losses, which, as we analyzed in our model, can lead them to default earlier rather than reduce their leverage at the expense of equity holders. Although a thorough examination of this credit cycle is challenging, the economic mechanism is important and worth pursuing in future research.

V. Model Implications

A. Predicting Default

Structural credit models (e.g., Merton (1974), Leland (1994), and Longstaff and Schwarz (1995)) are widely used to predict firms' default probabilities. The models share the common feature that a firm defaults when its fundamental drops below a default boundary. In the Merton model, the default occurs only at debt maturity if the firm's fundamental is below its debt level. In the Longstaff-Schwarz model, a firm defaults when its fundamental drops below an exogenously specified threshold for the first time. In the Leland model, the default boundary is endogenously determined by the equity value. These models together highlight distance to default, which is essentially a volatility-adjusted measure of firm leverage, as the key variable for predicting defaults.

Several empirical studies examine the empirical performance of the distance-to-default measure constructed from these models. Leland (2004) calibrates the Leland-Toft model and finds that it can match the average long-term default frequencies of both investment-grade and noninvestment-grade bonds. Bharath and Shumway (2008) find that while the Merton model implemented by the KMV corporation provides a useful predictor of future default, it does not produce a sufficient statistic for default probability. Davydenko (2007) compares firm characteristics at the time of bankruptcy and finds rich heterogeneity. Some firms default even when their fundamentals are still above the default boundary calibrated from the Leland-Toft model, while other firms manage not to default for years even though their fundamentals are below the boundary.

Our model provides a new perspective: secondary bond market liquidity can act as an additional factor in explaining the heterogeneity in firm default. In particular, our model modifies distance to default, defined in a standard structural credit framework, by incorporating the effect of market liquidity through firms' endogenous default boundary.

A crucial issue for predicting the default of bond portfolios is the default correlation between different firms. Duffie et al. (2009) find that commonly used variables, such as distance to default, trailing stock returns of firms and the market, and the risk-free interest rate, can only capture a small fraction of firms' default correlation. Instead, they introduce common latent factors to model correlated defaults.

Our model shows that correlated shocks to the liquidity of different firms' bonds, which have been largely ignored in this literature, can help explain correlated defaults. In our model, it is intuitive to interpret a shock to bond investors' liquidity shock intensity ξ as common to all firms, while a shock to the trading cost of a bond k as firm specific. Our model is thus suitable for employing the bond market liquidity factors identified in the empirical literature (e.g., Chen et al. (2007), and Bao et al. (2011)) to predict firm default.

On a related issue, Collin-Dufresne, Goldstein, and Martin (2001) find that proxies for changes in the probability of future default based on standard credit risk models and for changes in the recovery rate can only explain about 25% of the observed changes in credit spread. On the other hand, they find that the residuals from these regressions are highly cross-correlated, and that over 75% of the variation in the residuals is due to the first principal component. The source of this systematic component still remains unclear. Our model suggests that aggregate shocks to the liquidity of bond markets are a possible candidate.

B. Decomposing Credit Spreads

Academics and policy makers alike have recognized the important effect of the liquidity premium on credit spreads, but tend to treat it as independent from the default premium. This is probably due to the fact that the existing structural credit risk models ignore liquidity effects. Our model demonstrates that market liquidity can affect firms' default risk through the rollover risk channel. If market liquidity deteriorates, not only is the liquidity premium greater, but the default premium is also greater as increasing rollover losses cause equity holders to default earlier. This implies that the default premium and liquidity premium in firms' credit spreads are correlated. The existence of this correlation has important implications for empirical studies that aim to decompose credit spreads and test liquidity effects in credit spreads.

Several studies (e.g., Longstaff et al. (2005), Beber et al. (2009), and Schwarz (2009)) decompose firms' credit spreads to assess the quantitative contributions of the liquidity premium and default premium. These studies typically use the spread in a firm's credit default swap (CDS) to proxy for its default premium as CDS contracts tend to be liquid. A commonly used panel regression is

$$Credit\_Spread_{i,t} = \alpha + \beta \cdot CDS_{i,t} + \delta \cdot LIQ_{i,t} + \epsilon_{i,t}, \tag{15}$$

where $Credit\_Spread_{i,t}$ and $CDS_{i,t}$ are firm i's credit spread and CDS spread, and $LIQ_{i,t}$ is a measure of the firm's bond liquidity. Longstaff et al. (2005), and

Beber et al. (2009) find that a majority of the cross-sectional variation in credit spreads can be explained by the CDS spreads, although the coefficients on the liquidity measures (such as bid-ask spread and market depth) are also significant. Schwarz (2009) reports a greater contribution by the liquidity measures.

Our model cautions against overinterpreting quantitative results from such a decomposition. As the CDS spread also captures the premium related to endogenous default driven by market liquidity, the coefficient on the liquidity measure underestimates the total effect of liquidity on the credit spread. Formally, our model implies the following data-generating process for a firm's CDS:

$$CDS_{i,t} = f(V_{i,t}) + (\gamma_0 + \gamma_1 V_{i,t}) \cdot LIQ_{i,t} + v_{i,t}.$$

The firm's CDS is determined not only by the firm's fundamental $V_{i,t}$, but also by its $LIQ_{i,t}$. Here, $\gamma_0 > 0$ captures the higher default boundary when liquidity deteriorates, and $\gamma_1 < 0$ captures the potential flight-to-quality property illustrated in Section III.D. Suppose the firm's fundamental V_t is fixed and, without loss of generality, set at $V_t = 0$. Then the effect of liquidity on the firm's credit spread is $\delta + \beta\gamma_0$, where δ and β are given in equation (15). However, an econometrician who runs a regression in the form of equation (15) will only attribute δ as the effect of liquidity on the firm's credit spread.

This critique is especially relevant for tests of liquidity effects on credit spreads. Several recent studies (e.g., Taylor and Williams (2009), McAndrews, Sarkar, and Wang (2008), and Wu (2008)) test whether the term auction facility (TAF) created by the Federal Reserve during the recent credit crisis improved the funding liquidity of banks and financial institutions. These studies all interpret this potential effect as a liquidity effect, which should lead to a lower spread between the LIBOR rate and overnight index swap (OIS) rate. Because the LIBOR-OIS spread may include default risk, these studies all control for the default premium in the LIBOR-OIS spread by using certain measures of banks' credit risk, such as the CDS spread. Taylor and Williams (2009) use the following regression:

$$(LIBOR - OIS)_t = a \cdot CDS_t + b \cdot TAF_t + \epsilon_t,$$

where CDS_t is the median CDS spread for 15 of the 16 banks in the U.S. dollar LIBOR survey and TAF_t is a dummy used to represent activities of the TAF. They find that the regression coefficient b is insignificant and thus conclude that the TAF had an insignificant effect on the LIBOR-OIS spread.[14] Again, as our model suggests that the liquidity effect created by the TAF should also feed back into the default premium in the LIBOR-OIS spread. As a result, by controlling for the CDS spread, the coefficient on the TAF dummy underestimates the liquidity effect of TAF.

[14] McAndrews et al. (2008) and Wu (2008) use similar regression specifications but different dummy measures of the TAF and find more significant regression coefficients.

C. Maturity Risk

Several recent empirical studies find that firms with shorter debt maturity or with more short-term debt faced greater default risk during the recent credit crisis. This so-called maturity risk effect essentially reflects firms' rollover risk and has been largely ignored by both academics and industry practitioners. Almeida et al. (2009) use the fraction of long-term debt that is scheduled to mature in the near future as a measure of the rollover risk faced by firms. This measure avoids the potential endogeneity problems related to firms' initial debt maturity choice. They find that during the recent credit crisis, firms facing greater rollover risk tend to have a more pronounced investment decline than otherwise similar firms. Hu (2010) further shows that these firms also have higher credit spreads. Our model explains this phenomenon (Proposition 3) and thus highlights firms' debt maturity structure as a determinant of their credit risk.

In assigning credit ratings, rating agencies tend to ignore the effects of firms' debt maturity structures. Gopalan, Song, and Yerramilli (2009) find that firms with a higher proportion of short-term debt are more likely to experience multi-notch credit rating downgrades. Their evidence suggests that credit ratings underestimate maturity risk. Interestingly, rating agencies have recently started to incorporate this risk into credit ratings. For example, one of the major rating agencies, Standard & Poor's, has recently improved its approach to rating speculative-grade credits by adjusting for maturity risk:

> *"Although we believe that our enhanced analytics will not have a material effect on the majority of our current ratings, individual ratings may be revised. For example, a company with heavy debt maturities over the near term (especially considering the current market conditions) would face more credit risk, notwithstanding benign long-term prospects."* Standard & Poors Report "Leveraged finance: Standard & Poor's revises its approach to rating speculative-grade credits" (May 13, 2008, page 6).

D. Managing Credit and Liquidity Risk

Our model also has an important implication for managing the credit and liquidity risk of corporate bonds. We can measure the exposures of a bond to fundamental shocks and liquidity shocks by the derivatives of the bond price function with respect to V_t and ξ, which we call the fundamental beta and liquidity beta:

$$\beta_V \equiv \frac{\partial d(V_t, \xi; V_B(\xi))}{\partial V},$$

and

$$\beta_\xi \equiv \frac{\mathrm{d}d(V_t, \xi; V_B(\xi))}{\mathrm{d}\xi} = \frac{\partial d(V_t, \xi; V_B(\xi))}{\partial \xi} + \frac{\partial d(V_t, \xi; V_B(\xi))}{\partial V_B} \cdot \frac{dV_B(\xi)}{d\xi}.$$

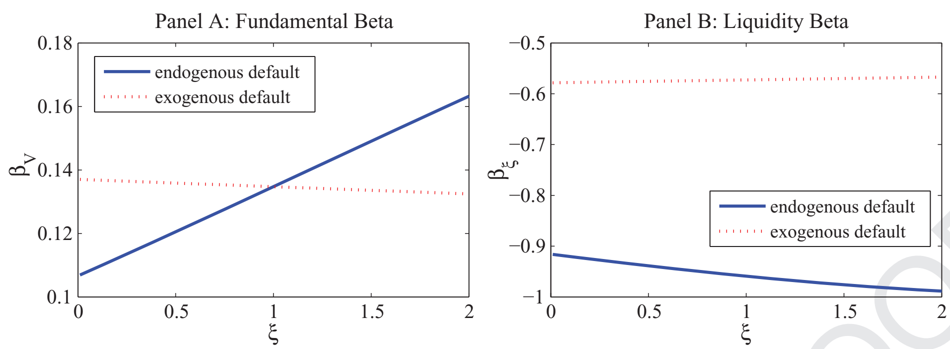

Figure 5. Variability of fundamental beta and liquidity beta. This figure uses the baseline parameters listed in Table I. Panel A depicts the fundamental beta of newly issued bonds with respect to bond investors' liquidity shock intensity ξ and Panel B depicts the liquidity beta of newly issued bonds with respect to ξ.

Note that the liquidity beta contains two components, which capture the effects of a liquidity shock through the liquidity-premium channel and the rollover risk channel.

As investors cannot constantly revise hedges of their portfolios, variability in the fundamental beta and liquidity beta directly affects the residual risk that remains in their portfolios even if they initially hedge away the fundamental beta and liquidity beta. To hedge a stock option, the celebrated Black-Scholes model requires a continuous revision of the delta hedging position in order to maintain a perfect hedge when its underlying stock price fluctuates. However, such a strategy requires infinite trading and is thus precluded by transaction costs (e.g., Leland (1985)). To reduce transaction costs, institutions often choose to follow discrete revisions of their hedging positions. The gamma of the option (i.e., variability of its delta) is thus important in determining the residual risk—the higher the gamma, the greater the residual risk in using the discrete delta-hedging strategy. The same argument implies that the variability of a bond's fundamental beta and liquidity beta determines the residual risk in applying discrete hedges of the bond's fundamental and liquidity risk.

To highlight the variability of the fundamental beta and liquidity beta implied by our model, we use a benchmark structural credit risk model, which is otherwise identical to our model except that the default boundary is exogenously specified (as in Longstaff and Schwarz (1995)). We fix the exogenous default boundary at the level derived from our model under the baseline parameters.

Figure 5 depicts the fundamental beta and liquidity beta with respect to bond investors' liquidity shock intensity ξ. The dotted lines in Panels A and B show that if the firm's default boundary is fixed at the baseline level, the bond's fundamental beta and liquidity beta do not vary much with ξ. However, when the default boundary is endogenously determined by equity holders, both betas (plotted in the solid lines) vary substantially with ξ. This figure demonstrates that through the rollover risk channel, fluctuations in debt market liquidity can cause large variability in bonds' fundamental beta and liquidity beta. As a result, investors should expect substantial residual risk even after an initially perfect hedge.

VI. Conclusion

This paper provides a model to analyze the effects of debt market liquidity on a firm's credit risk through its debt rollover. When a shock to market liquidity pushes down a firm's bond prices, it amplifies the conflict of interest between debt and equity holders because, to avoid bankruptcy, equity holders have to absorb the firm's losses from rolling over maturing bonds at the reduced market prices. As a result, equity holders choose to default at a higher fundamental threshold even if the firm can freely raise more equity. This implies that deterioration in debt market liquidity leads to not only a higher liquidity premium but also a higher default premium. This implication justifies market liquidity as a predictor of firm default, and cautions against treating the credit spread as the sum of independent liquidity and default premia. Our model also shows that firms with weaker fundamentals are more exposed to deterioration in market liquidity and thus helps explain the flight-to-quality phenomenon. The intricate interaction between a bond's liquidity risk and fundamental risk also makes its risk exposures highly variable and difficult to manage. Finally, our model highlights the role of short-term debt in amplifying a firm's rollover risk, and thus calls for more attention to be given to debt maturity structure when assessing credit risk.

Appendix: Technical Proofs

Proof of Proposition 1: We omit the time subscript in V_t in the following derivation. The equity value satisfies the following differential equation:

$$rE = (r-\delta)VE_V + \frac{1}{2}\sigma^2 V^2 E_{VV} + d(V,m) + \delta V - [(1-\pi)C + p].$$

Define

$$v \equiv \ln\left(\frac{V}{V_B}\right). \tag{A1}$$

Then we have

$$rE = \left(r - \delta - \frac{1}{2}\sigma^2\right)E_v + \frac{1}{2}\sigma^2 E_{vv} + d(v,m) + \delta V_B e^v - [(1-\pi)C + p], \tag{A2}$$

with the boundary conditions

$$E(0) = 0 \text{ and } E_v(0) = l,$$

where the free parameter l is determined by the boundary condition that as $v \to \infty$, the equity value is linear in V.

Define the Laplace transformation of $E(v)$ as

$$F(s) \equiv L[E(v)] = \int_0^\infty e^{-sv} E(v)\, dv.$$

Then, applying the Laplace transformation to both sides of (A2), we have:

$$rF(s) = \left(r - \delta - \frac{1}{2}\sigma^2\right) L[E_v] + \frac{1}{2}\sigma^2 L[E_{vv}] + L[d(v,m)] + \frac{\delta V_B}{s-1} - \frac{(1-\pi)C + p}{s}.$$

Note that

$$L[E_v] = sF(s) - E(0) = sF(s)$$

and

$$L[E_{vv}] = s^2 F(s) - sE(0) - E_v(0) = s^2 F(s) - l.$$

Thus, we have

$$\left[r - \left(r - \delta - \frac{1}{2}\sigma^2\right)s - \frac{1}{2}\sigma^2 s^2\right] F(s) = L[d(v,m)] - \frac{1}{2}\sigma^2 l + \frac{\delta V_B}{s-1} - \frac{(1-\pi)C + p}{s}.$$

Define $\eta > 0$ and $-\gamma < 0$ to be the two roots of the following equation with respect to s:

$$r - \left(r - \delta - \frac{1}{2}\sigma^2\right) s - \frac{1}{2}\sigma^2 s^2 = 0.$$

That is, $-\frac{1}{2}\sigma^2 (s - \eta)(s + \gamma) = 0$. Direct calculation gives

$$\eta = z - a > 1 \quad \text{and} \quad \gamma = a + z > 0,$$

where

$$a \equiv \frac{r - \delta - \sigma^2/2}{\sigma^2} \quad \text{and} \quad z \equiv \frac{(a^2\sigma^4 + 2r\sigma^2)^{1/2}}{\sigma^2}.$$

Then,

$$\begin{aligned} \frac{1}{2}\sigma^2 F(s) &= -\frac{1}{(s-\eta)(s+\gamma)} \left\{ L\left[d(v,m)\right] + \frac{\delta V_B}{s-1} - \frac{(1-\pi)C + p}{s} - \frac{1}{2}\sigma^2 l \right\} \\ &= -\frac{\dfrac{1}{s-\eta} - \dfrac{1}{s+\gamma}}{\eta + \gamma} \left\{ L\left[d(v,m)\right] + \frac{\delta V_B}{s-1} - \frac{(1-\pi)C + p}{s} - \frac{1}{2}\sigma^2 l \right\}. \end{aligned} \tag{A3}$$

Recall that $d(v,m)$ is given in (8). By plugging it into (A3), we have

$$\frac{1}{2}\sigma^2 F(s)$$
$$= -\frac{\frac{1}{s-\eta}-\frac{1}{s+\gamma}}{\eta+\gamma}\left\{\frac{\delta V_B}{s-1} - \frac{(1-\pi)C + (1-e^{-(r+\xi k)m})\left(p-\frac{c}{r+\xi k}\right)}{s} - \frac{1}{2}\sigma^2 l\right\}$$
$$-\frac{\frac{1}{s-\eta}-\frac{1}{s+\gamma}}{\eta+\gamma}\left\{-e^{-(r+\xi k)m}\left(p-\frac{c}{r+\xi k}\right)L[F(m)] + \left(\frac{\alpha V_B}{m}-\frac{c}{r+\xi k}\right)L[G(m)]\right\}. \tag{A4}$$

Call the first line in (A4) $\widehat{F}(s)$. It is easy to work out its Laplace inverse by using (A1) to derive the condition that $\frac{\delta V_B}{(\eta-1)(\gamma+1)}e^v = \frac{\sigma^2}{2}V$:

$$\widehat{E}(v) = \frac{\sigma^2}{2}V - \frac{\delta V_B}{\eta+\gamma}\left[\frac{1}{\eta-1}e^{\eta v} + \frac{1}{\gamma+1}e^{-\gamma v}\right]$$
$$+\frac{(1-\pi)C + (1-e^{-(r+\xi k)m})\left(p-\frac{c}{r+\xi k}\right)}{\eta+\gamma}\left[\frac{1}{\eta}\left(e^{\eta v}-1\right) - \frac{1}{\gamma}\left(1-e^{-\gamma v}\right)\right]$$
$$+\frac{1}{2}\sigma^2 l\frac{1}{\eta+\gamma}(e^{\eta v}-e^{-\gamma v}).$$

Call the second line in (A4) $\overline{F}(s)$. One can show that

$$(\eta+\gamma)\overline{F}(s) = e^{-(r+\xi k)m}\left(p-\frac{c}{r+\xi k}\right)\frac{1}{\eta}\left(\frac{1}{s-\eta}-\frac{1}{s}\right)$$
$$\times\left[N(-a\sigma\sqrt{m}) - e^{\frac{1}{2}((s+a)^2-a^2)\sigma^2 m}\right]$$
$$-e^{-(r+\xi k)m}\left(p-\frac{c}{r+\xi k}\right)\frac{1}{\gamma}\left(\frac{1}{s}-\frac{1}{s+\gamma}\right)$$
$$\times\left[N(-a\sigma\sqrt{m}) - e^{\frac{1}{2}((s+a)^2-a^2)\sigma^2 m}\right]$$
$$+e^{-(r+\xi k)m}\left(p-\frac{c}{r+\xi k}\right)\frac{1}{2a+\eta}\left(\frac{1}{s-\eta}-\frac{1}{s+2a}\right)$$
$$\times\left[N(a\sigma\sqrt{m}) - e^{\frac{1}{2}((s+a)^2-a^2)\sigma^2 m}\right]$$
$$-e^{-(r+\xi k)m}\left(p-\frac{c}{r+\xi k}\right)\frac{1}{\gamma-2a}\left(\frac{1}{s+2a}-\frac{1}{s+\gamma}\right)$$
$$\times\left[N(a\sigma\sqrt{m}) - e^{\frac{1}{2}((s+a)^2-a^2)\sigma^2 m}\right]$$
$$-\left(\frac{\alpha V_B}{m}-\frac{c}{r+\xi k}\right)\frac{1}{a-\widehat{z}+\eta}\left(\frac{1}{s-\eta}-\frac{1}{s+a-\widehat{z}}\right)$$

$$
\begin{aligned}
&\times [N(-\widehat{z}\sigma\sqrt{m}) - e^{\frac{1}{2}((s+a)^2-\widehat{z}^2)\sigma^2 m}] \\
&+ \left(\frac{\alpha V_B}{m} - \frac{c}{r+\xi k}\right)\frac{1}{\gamma - a + \widehat{z}}\left(\frac{1}{s+a-\widehat{z}} - \frac{1}{s+\gamma}\right) \\
&\times [N(-\widehat{z}\sigma\sqrt{m}) - e^{\frac{1}{2}((s+a)^2-\widehat{z}^2)\sigma^2 m}] \\
&- \left(\frac{\alpha V_B}{m} - \frac{c}{r+\xi k}\right)\frac{1}{a + \widehat{z} + \eta}\left(\frac{1}{s-\eta} - \frac{1}{s+a+\widehat{z}}\right) \\
&\times [N(\widehat{z}\sigma\sqrt{m}) - e^{\frac{1}{2}((s+a)^2-\widehat{z}^2)\sigma^2 m}] \\
&\times \left(\frac{\alpha V_B}{m} - \frac{c}{r+\xi k}\right)\frac{1}{\gamma - a - \widehat{z}}\left(\frac{1}{s+a+\widehat{z}} - \frac{1}{s+\gamma}\right) \\
&\times [N(\widehat{z}\sigma\sqrt{m}) - e^{\frac{1}{2}((s+a)^2-\widehat{z}^2)\sigma^2 m}].
\end{aligned}
$$

We need to calculate the Laplace inverse of $\overline{F}(s)$, which we call $\overline{E}(v)$. To this end, we define

$$
\begin{aligned}
&M(v; x, w, p, q) \\
&\quad\equiv L^{-1}\left\{\left(\frac{1}{s+p} - \frac{1}{s+q}\right)[N(y\sigma\sqrt{m}) - e^{\frac{1}{2}((s+x)^2-w^2)\sigma^2 m}]\right\} \\
&\quad= \{N(w\sigma\sqrt{m}) - e^{\frac{1}{2}[(p-x)^2-w^2]\sigma^2 m} N((p-x)\sigma\sqrt{m})\}e^{-pv} \\
&\qquad + e^{\frac{1}{2}[(p-x)^2-w^2]\sigma^2 m} e^{-pv} N\left(\frac{-v + (p-x)\sigma^2 m}{\sigma\sqrt{m}}\right) \\
&\qquad - \{N(w\sigma\sqrt{m}) - e^{\frac{1}{2}[(q-x)^2-w^2]\sigma^2 m} N((q-x)\sigma\sqrt{m})\}e^{-qv} \\
&\qquad - e^{\frac{1}{2}[(q-x)^2-w^2]\sigma^2 m} e^{-qv} N\left(\frac{-v + (q-x)\sigma^2 m}{\sigma\sqrt{m}}\right).
\end{aligned}
$$

We then have

$$
\begin{aligned}
M(v; x, w, x+w, q) &= -K(v; x, w, q), \\
M(v; x, w, p, x+w) &= K(v; x, w, p),
\end{aligned}
$$

where

$$
\begin{aligned}
K(v; x, w, p) \equiv\ & \{N(w\sigma\sqrt{m}) - e^{\frac{1}{2}[(p-x)^2-w^2]\sigma^2 m} N((p-x)\sigma\sqrt{m})\}e^{-pv} \\
& + e^{\frac{1}{2}[(p-x)^2-w^2]\sigma^2 m} e^{-pv} N\left(\frac{-v + (p-x)\sigma^2 m}{\sigma\sqrt{m}}\right) \\
& - e^{-(x+w)v} N\left(\frac{-v + w\sigma^2 m}{\sigma\sqrt{m}}\right).
\end{aligned}
\tag{A5}
$$

Note that $\frac{2}{\sigma^2}\frac{1}{\eta+\gamma} = \frac{1}{z\sigma^2}$. Then,

$$
\begin{aligned}
E(v) &= \frac{2}{\sigma^2}(\widehat{E}(v) + \overline{E}(v)) \\
&= V - \frac{\delta V_B}{z\sigma^2}\left[\frac{e^{\eta v}}{\eta - 1} + \frac{e^{-\gamma v}}{\gamma + 1}\right] + \frac{l}{2z}(e^{\eta v} - e^{-\gamma v}) \\
&\quad + \frac{(1-\pi)C + (1 - e^{-(r+\xi k)m})\left(p - \frac{c}{r+\xi k}\right)}{z\sigma^2}\left[\frac{1}{\eta}(e^{\eta v} - 1) - \frac{1}{\gamma}(1 - e^{-\gamma v})\right] \\
&\quad + \frac{e^{-(r+\xi k)m}\left(p - \frac{c}{r+\xi k}\right)}{z\sigma^2}\left[\begin{array}{l} \frac{1}{\eta}K(v; a, -a, -\eta) + \frac{1}{\gamma}K(v; a, -a, \gamma) \\ \quad + \frac{1}{\gamma}K(v; a, a, -\eta) + \frac{1}{\eta}K(v; a, a, \gamma) \end{array}\right] \\
&\quad + \frac{\left(\frac{\alpha V_B}{m} - \frac{c}{r+\xi k}\right)}{z\sigma^2}\left[\begin{array}{l} -\frac{1}{z-\widehat{z}}K(v; a, -\widehat{z}, -\eta) - \frac{1}{z+\widehat{z}}K(v; a, -\widehat{z}, \gamma) \\ \quad -\frac{1}{z+\widehat{z}}K(v; a, \widehat{z}, -\eta) - \frac{1}{z-\widehat{z}}K(v; a, \widehat{z}, \gamma) \end{array}\right].
\end{aligned}
$$

Now we impose the boundary condition at $v \to \infty$. The equity value has to grow linearly when $V \to \infty$. Since $e^{\eta v} = (\frac{V}{V_B})^{\eta}$ and $\eta > 1$, to avoid explosion we require the coefficient on $e^{\eta v}$ in $E(v)$ to collapse to zero. By collecting the coefficients of $e^{\eta v}$ and noting that $-\eta - a = -z$, $\gamma = 2a + \eta$, and $\frac{1}{2}[z^2 - a^2]\sigma^2 m = rm$, we have

$$
\begin{aligned}
0 &= -\frac{\delta V_B}{\eta - 1} + \left[(1-\pi)C + (1 - e^{-(r+\xi k)m})\left(p - \frac{c}{r+\xi k}\right)\right]\frac{1}{\eta} + \frac{\sigma^2}{2}l \\
&\quad + e^{-(r+\xi k)m}\left(p - \frac{c}{r+\xi k}\right)\left[\begin{array}{l} \frac{\{N(-a\sigma\sqrt{m}) - e^{rm}N(-z\sigma\sqrt{m})\}}{\eta} \\ + \frac{\{N(a\sigma\sqrt{m}) - e^{rm}N(-z\sigma\sqrt{m})\}}{\gamma} \end{array}\right] \\
&\quad + \left(\frac{\alpha V_B}{m} - \frac{c}{r+\xi k}\right)\left[\begin{array}{l} -\frac{\{N(-\widehat{z}\sigma\sqrt{m}) - e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}N(-z\sigma\sqrt{m})\}}{a - \widehat{z} + \eta} \\ -\frac{\{N(\widehat{z}\sigma\sqrt{m}) - e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}N(-z\sigma\sqrt{m})\}}{a + \widehat{z} + \eta} \end{array}\right]. \qquad \text{(A6)}
\end{aligned}
$$

This equation allows us to solve l.

We then get a closed-form expression for the equity value:

$$E(V_t) =$$

$$V_t - \frac{\delta V_B}{z\sigma^2}\frac{e^{-\gamma v_t}}{\gamma+1} - \frac{(1-\pi)C + (1-e^{-(r+\xi k)m})\left[p - \dfrac{c}{r+\xi k}\right]}{z\sigma^2}\left[\frac{1}{\eta} + \frac{(1-e^{-\gamma v_t})}{\gamma}\right]$$
$$+ \frac{1}{z\sigma^2}\left\{e^{-(r+\xi k)m}\left(p - \frac{c}{r+\xi k}\right)A(a) - \left(\frac{\alpha V_B}{m} - \frac{c}{r+\xi k}\right)A(\widehat{z})\right\}, \quad \text{(A7)}$$

where

$$A(y) \equiv \frac{1}{z-y}(K(v_t; a, y, \gamma) + k(v_t; a, -y, -\eta)) + \frac{1}{z+y}(K(v_t; a, -y, \gamma) + k(v_t; a, y, -\eta)),$$

with $K(v_t; a, y, \gamma)$ defined in equation (A5) and

$$k(v_t; a, y, -\eta) = e^{\frac{1}{2}[(-\eta-a)^2 - y^2]\sigma^2 m} e^{\eta v} N\left(\frac{-v + (-\eta - a)\sigma^2 m}{\sigma\sqrt{m}}\right)$$
$$- e^{-(a+y)v} N\left(\frac{-v + y\sigma^2 m}{\sigma\sqrt{m}}\right).$$

Basically, $k(v_t; a, y, -\eta)$ is $K(v_t; a, y, -\eta)$ but without the first term

$$[N(y\sigma\sqrt{m}) - e^{\frac{1}{2}[(-\eta-a)^2-y^2]\sigma^2 m} N((-\eta - a)\sigma\sqrt{m})]e^{\eta v},$$

because this part has to be zero as E cannot explode when $v \to \infty$.

The smooth-pasting condition implies that $E'(V_B) = 0$, or $E'_v(0) = l = 0$. We can then use condition (A6) to obtain V_B, which is given in (12). Q.E.D.

Proof of Proposition 2: We first fix the default boundary V_B. According to the Feynman-Kac formula, PDE (5) implies that at time 0, the price of a bond with time-to-maturity τ satisfies

$$d(V_0, \tau; V_B) = \mathbb{E}_0\left[\int_0^{\tau\wedge\tau_B} e^{-(r+\xi k)s} c\,ds + e^{-(r+\xi k)\cdot(\tau\wedge\tau_B)} d(\tau \wedge \tau_B)\right], \quad \text{(A8)}$$

where $\tau_B = \inf\{t : V_t = V_B\}$ is the first time that V_t hits V_B. V_t follows (1), and $d(\tau \wedge \tau_B)$ is defined by the boundary conditions in (6) and (7):

$$d(\tau \wedge \tau_B) = \begin{cases} \dfrac{1}{m}\alpha V_B & \text{if } \tau \wedge \tau_B = \tau_B \\ p & \text{if } \tau \wedge \tau_B = \tau. \end{cases}$$

As an increase in ξ leads to a higher discount rate for the bond's coupon payment and principal payment, a path-by-path argument implies that the bond price d decreases with ξ.

Similarly, the equity value can be written as

$$E(V_0, \tau; V_B) = \mathbb{E}_0\left\{\int_0^{\tau_B} e^{-rs}[\delta V_s - (1-\pi)C + d(V_s, m-s; \xi) - p]ds\right\},$$

where we write the dependence of d on ξ explicitly. Again, a path-by-path argument implies that when V_B is fixed, the equity value E decreases with ξ.

We now consider two different values of ξ: $\xi_1 < \xi_2$. Denote the corresponding default boundaries as $V_{B,1}$ and $V_{B,2}$. We need to show that $V_{B,1} < V_{B,2}$. Suppose that the opposite is true, that is, $V_{B,1} \geq V_{B,2}$. Since the equity value is zero on the default boundary, we have

$$E(V_{B,1}; V_{B,1}, \xi_1) = E(V_{B,2}; V_{B,2}, \xi_2) = 0,$$

where we expand the notation to let the equity value $E(V_t; V_B, \xi)$ explicitly depend on V_B (the default boundary) and ξ (the bond holders' liquidity shock intensity). Also, the optimality of the default boundary implies that

$$0 = E(V_{B,1}; V_{B,1}, \xi_1) > E(V_{B,1}; V_{B,2}, \xi_1).$$

Since E decreases with ξ, $E(V_{B,1}; V_{B,2}, \xi_1) > E(V_{B,1}; V_{B,2}, \xi_2)$. Because $V_{B,1} \geq V_{B,2}$ according to our counterfactual hypothesis, $E(V_{B,1}; V_{B,2}, \xi_2) < 0$. This contradicts limited liability, which says that

$$E(V_t; V_{B,2}, \xi_2) \geq 0 \text{ for all } V_t \geq V_{B,2}.$$

Therefore $V_{B,1} < V_{B,2}$. Q.E.D.

Proof of Proposition 3: We first consider the case in which $P = \frac{C}{r+\xi k}$. Under this assumption, the endogenous bankruptcy boundary V_B is given by

$$V_B(m) = \frac{\frac{(1-\pi)C}{\eta} + \left\{\frac{C}{r+\xi k}\frac{1}{m}[B(-\widehat{z}) + B(\widehat{z})]\right\}}{\frac{\delta}{\eta - 1} + \frac{\alpha}{m}[B(-\widehat{z}) + B(\widehat{z})]}, \tag{A9}$$

where

$$B(x) = \frac{1}{z+x}[N(x\sigma\sqrt{m}) - e^{\frac{1}{2}[z^2-x^2]\sigma^2 m}N(-z\sigma\sqrt{m})].$$

Define

$$\begin{aligned} Y(m) \equiv\ & \frac{1}{z-\widehat{z}}[N(-\widehat{z}\sigma\sqrt{m}) - e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}N(-z\sigma\sqrt{m})] \\ & + \frac{1}{z+\widehat{z}}[N(\widehat{z}\sigma\sqrt{m}) - e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}N(-z\sigma\sqrt{m})], \end{aligned} \tag{A10}$$

and $X(m) \equiv \frac{1}{m}Y(m)$.

It is clear that $Y(0) = 0$. Note that

$$\begin{aligned}
Y'(m) &= \frac{1}{z-\widehat{z}}\left[\begin{array}{l} -n(-\widehat{z}\sigma\sqrt{m})\widehat{z}\sigma\dfrac{1}{2\sqrt{m}} - e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}\dfrac{1}{2}[z^2-\widehat{z}^2]\sigma^2 N(-z\sigma\sqrt{m}) \\ +e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m} n(-z\sigma\sqrt{m})z\sigma\dfrac{1}{2\sqrt{m}} \end{array}\right] \\
&\quad + \frac{1}{z+\widehat{z}}\left[\begin{array}{l} n(\widehat{z}\sigma\sqrt{m})\widehat{z}\sigma\dfrac{1}{2\sqrt{m}} - e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}\dfrac{1}{2}[z^2-\widehat{z}^2]\sigma^2 N(-z\sigma\sqrt{m}) \\ +e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m} n(-z\sigma\sqrt{m})z\sigma\dfrac{1}{2\sqrt{m}} \end{array}\right] \\
&= \frac{\sigma}{\sqrt{2\pi m}}e^{-\frac{1}{2}\widehat{z}^2\sigma^2 m} - e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m} z\sigma^2 N(-z\sigma\sqrt{m}) \\
&= \frac{\sigma}{\sqrt{m}}e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}[n(z\sigma\sqrt{m}) - z\sigma\sqrt{m}N(-z\sigma\sqrt{m})], \qquad \text{(A11)}
\end{aligned}$$

where $n(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$. The following lemma shows that $Y'(m) > 0$. Q.E.D.

LEMMA 4: For all $m > 0$, $j(m) \equiv n(z\sigma\sqrt{m}) - z\sigma\sqrt{m}N(-z\sigma\sqrt{m}) > 0$, and $j'(m) < 0$.

Proof: Let $t = z\sigma\sqrt{m}$. When $t \to \infty$, $n(t) - tN(-t)$ converges to zero. When $t = 0$, it is $n(0)$, which is positive. Its derivative is

$$n'(t) - N(-t) + tn(-t) = -N(-t) < 0$$

as $n'(t) = -tn'(t)$. Because the derivative is always negative, $n(t) - tN(-t) > 0$ for $t \in (0, \infty)$. Q.E.D.

This lemma shows that $Y(m) > 0$. Therefore, $X(m) > 0$. We need to show that

$$V_B(m) = \frac{\dfrac{(1-\pi)C}{\eta} + \dfrac{C}{r+\xi k}X(m)}{\dfrac{\delta}{\eta-1} + \alpha X(m)}$$

is decreasing with m. Since $\frac{1}{r+\xi k}\frac{\delta}{\eta-1} > \frac{(1-\pi)\alpha}{\eta}$, it suffices to show that

$$X'(m) = \frac{Y'(m)m - Y(m)}{m^2} < 0.$$

We now show that $S(m) \equiv Y'(m)m - Y(m) < 0$. Note that

$$S'(m) = Y''(m)m + Y'(m) - Y'(m) = Y''(m)m,$$

where

$$Y''(m) = \frac{d}{dm}\left(\frac{\sigma}{\sqrt{m}}e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}\right)\left[\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2\sigma^2 m} - z\sigma\sqrt{m}N(-z\sigma\sqrt{m})\right] + \frac{\sigma}{\sqrt{m}}e^{\frac{1}{2}[z^2-\widehat{z}^2]\sigma^2 m}\frac{d}{dm}\left[\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2\sigma^2 m} - z\sigma\sqrt{m}N(-z\sigma\sqrt{m})\right].$$

The first term is negative because $z^2 - \widehat{z}^2 < 0$. The second term is also negative because the derivative is

$$\begin{aligned}
&-\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2\sigma^2 m}\frac{1}{2}z^2\sigma^2 + z\sigma\sqrt{m}n(-z\sigma\sqrt{m})\frac{z\sigma}{2\sqrt{m}} - \frac{z\sigma}{2\sqrt{m}}N(-z\sigma\sqrt{m}) \\
&= -\frac{z\sigma}{2\sqrt{m}}N(-z\sigma\sqrt{m}) < 0.
\end{aligned}$$

Thus, $Y''(m) < 0$ and $S'(m) = Y''(m)m < 0$. We therefore conclude that $S(m) < 0$ for all m, which in turn implies that $V_B'(m) < 0$ in the case of $P = \frac{C}{r+\xi k}$.

Now we consider the case in which $P > \frac{C}{r+\xi k}$. Let $u \equiv P - \frac{C}{r+\xi k} > 0$, $w(m) \equiv \frac{(1-e^{-(r+\xi k)m})}{m}$, and $W(m) = \frac{b(-a)+b(a)}{m}$. We know immediately that

$$w'(m) < 0, \text{ and } w(m) \le w(0) = r + \xi k. \tag{A12}$$

We then have

$$\begin{aligned}
V_B(m) &= \frac{\dfrac{(1-\pi)C + (1-e^{-(r+\xi k)m})\frac{1}{m}\left(P - \frac{C}{r+\xi k}\right)}{\eta} + \left\{\begin{array}{c}\frac{1}{m}\left(P - \frac{C}{r+\xi k}\right)[b(-a)+b(a)] \\ +\frac{C}{r+\xi k}\frac{1}{m}[B(-\widehat{z})+B(\widehat{z})]\end{array}\right\}}{\dfrac{\delta}{\eta-1} + \dfrac{\alpha}{m}[B(-\widehat{z})+B(\widehat{z})]} \\
&= \frac{\dfrac{(1-\pi)C + uw(m)}{\eta} + uW(m) + \dfrac{C}{r+\xi k}X(m)}{\dfrac{\delta}{\eta-1} + \alpha X(m)}.
\end{aligned}$$

By taking the derivative with respect to m, we have

$$\begin{aligned}
V_B'(m) \propto{} & \left(\frac{uw'(m)}{\eta} + uW'(m)\right)\left(\frac{\delta}{\eta-1} + \alpha X(m)\right) + \frac{C}{r+\xi k}X'(m)\frac{\delta}{\eta-1} \\
& - \left(\frac{(1-\pi)C + uw(m)}{\eta} + uW(m)\right)\alpha X'(m) \\
& < uW'(m)\frac{\delta}{\eta-1} + X'(m)\left(-\alpha\frac{(1-\pi)C + uw(m)}{\eta} + \frac{C}{r+\xi k}\frac{\delta}{\eta-1}\right) \\
& + u\alpha[W'(m)X(m) - W(m)X'(m)].
\end{aligned} \tag{A13}$$

We will show that

$$W'(m) < 0 \;\text{ and } W'(m)X(m) - W(m)X'(m) < 0. \tag{A14}$$

Given these two results, the first and third terms of (A13) are negative. The second term is negative given the sufficient condition that

$$\frac{C}{r+\xi k}\frac{\delta}{\eta-1} > \alpha\frac{(1-\pi)C + u(r+\xi k)}{\eta} > \alpha\frac{(1-\pi)C + uw(0)}{\eta},$$

using the properties given in (A12). Thus, $V_B^{'}(m) < 0$.

We now prove the first part of (A14). Note that

$$W(m) = \frac{b(-a)+b(a)}{m} = \frac{e^{-(r+\xi k)m}}{m}\left\{\begin{array}{l}\frac{1}{z-a}[N(-a\sigma\sqrt{m}) - e^{rm}N(-z\sigma\sqrt{m})]\\ \quad +\frac{1}{z+a}[N(a\sigma\sqrt{m}) - e^{rm}N(-z\sigma\sqrt{m})]\end{array}\right\}.$$

Let

$$Q(m) \equiv \frac{1}{z-a}[N(-a\sigma\sqrt{m}) - e^{rm}N(-z\sigma\sqrt{m})] + \frac{1}{z+a}[N(a\sigma\sqrt{m}) - e^{rm}N(-z\sigma\sqrt{m})].$$

Note the above equation's resemblance to the function $Y(m)$ defined in (A10) by recalling the definitions of z and a in (9) and thus that $rm = \frac{1}{2}(z^2 - a^2)\sigma^2 m$. Therefore, similar to the derivation for $Y(m)$, we have

$$Q'(m) = \frac{\sigma}{\sqrt{m}}e^{rm}[n(z\sigma\sqrt{m}) - z\sigma\sqrt{m}N(-z\sigma\sqrt{m})]. \tag{A15}$$

Define $F(m) \equiv \frac{\sigma}{\sqrt{m}}[n(z\sigma\sqrt{m}) - z\sigma\sqrt{m}N(-z\sigma\sqrt{m})]$. Then, Lemma 4 implies that $F(m) = \frac{\sigma j(m)}{\sqrt{m}}$. Note that

$$Y'(m) = e^{-\xi km}F(m). \tag{A16}$$

Q.E.D.

LEMMA 5: $F(m) > 0$ and $F'(m) < 0$.

Proof: Lemma 4 implies that the numerator of $F(m)$ is positive and decreasing. Since its denominator $\sqrt{m}$ is positive and increasing, the claim holds true. Q.E.D.

By taking the derivative with respect to m, the claim that $W(m) = \frac{e^{-(r+\xi k)m}Q(m)}{m}$ is decreasing is equivalent to

$$mQ'(m) < (1 + (r+\xi k)m)\,Q(m).$$

When $m = 0$, this holds in equality. Taking the derivative again on both sides and canceling the term $Q'(m)$, the claim becomes equivalent to

$$mQ''(m) < (r+\xi k)mQ'(m) + (r+\xi k)\,Q(m).$$

Note that

$$mQ'(m) = -\frac{\sigma}{2\sqrt{m^3}}e^{rm}j(m) + \frac{r\sigma}{\sqrt{m}}e^{rm}j(m) + \frac{\sigma}{\sqrt{m}}e^{rm}j'(m),$$

where the first and third terms are negative according to Lemma 4, and the second term is just $rQ(m)$. Thus, $mQ'(m) < rQ(m)$, which in turn leads to the claim.

We now prove the second part of (A14): $W'(m)X(m) - W(m)X'(m) < 0$, which is equivalent to

$$(e^{-(r+\xi k)m}Q(m))'Y(m) - e^{-(r+\xi k)m}Q(m)Y'(m) < 0.$$

Using (A15) and (A16), it suffices to show that

$$F(m)[e^{rm}Y(m) - e^{-\xi km}Q(m)] - (r+\xi k)Q(m)Y(m) < 0.$$

When $m = 0$, this holds in equality. Its derivative is

$$F'(m)[e^{rm}Y(m) - e^{-\xi km}Q(m)].$$

It is clear that $e^{rm}Y(m) - e^{-\xi km}Q(m) > 0$, because it is equal to zero when $m = 0$, and its derivative is always positive. Thus, according to Lemma 5 the claim holds true. Q.E.D.

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Queries

Q1 Author: Please provide the complete bibliographic details for Reference Duffie (2009).

Q2 Author: Please provide the complete bibliographic details for Reference Fender et al. (2009).

Q3 Author: Please provide the city location of publisher for Reference Flannery (2005).

Q4 Author: Please update Reference He and Xiong (2010).

Q5 Author: Please provide the page range for Reference Leland (2004).

Q6 Author: Please provide the complete bibliographic details for reference McAndrews et al. (2008).