Dynamic Debt Runs

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Runs on Non-bank Financial Institutions

- Runs on the non-bank financial institutions was one of the main causes of the credit crisis of 2007-2008.
  - e.g., Bernanke (2008), Cox (2008), Geithner (2008), Brunnermeier (2009), Gorton (2008), Krishnamurthy (2009), and Shin (2009).
- The classic Diamond-Dybvig model on bank runs:
  - The simultaneous coordination problem among depositors leads to a self-fulfilling bank-run equilibrium.
- Global-games models of bank runs:
  - Rochet and Vives (2004) and Goldstein and Pauzner (2005) allow depositors holding noisy private signals about bank fundamental.
  - Signal noise leads to strategic uncertainty and prevents multiple equilibria, e.g., Carlsson and van Damme (1993) and Morris and Shin (2003).
- Many questions about debt runs involve time-varying fundamentals:
  - How does a firm's asset price volatility affect its debt run risk?
  - Do credit lines mitigate runs?
  - Do longer debt maturities mitigate runs?

Dynamic Debt Runs

- A model with time-varying fundamental w/o noisy private signals:
  - The firm fundamental is time-varying and all creditors share the same public information about fundamental.
  - The firm uses a staggered debt structure, i.e., creditors make rollover decisions at different times.
- A unique threshold equilibrium:
  - each creditor rolls over or not based on current fundamental;
  - a "rat race" among creditors in choosing rollover thresholds.
- Results similar to static global-games models:
  - Severe runs on firms with weaker fundamentals, greater illiquidity.
- New results:
  - Higher volatility increases strategic uncertainty and thus exacerbates runs.
  - When fundamental volatility is sufficiently high, stronger credit lines and longer debt maturities can exacerbate runs.

A Brief Literature Review

- Static global games models:
  - Carlsson and van Damme (1993), Morris and Shin (1998)
  - Rochet and Vives (2004), Goldstein and Pauzner (2005)
- Dynamic coordination games based on higher order beliefs:
  - Abreu and Brunnermeier (2003), Chamley (2003), Angeletos, Hellwig, and Pavan (2007)
- Dynamic coordination games based on observable fundamentals:
  - Frankel and Pauzner (2000)
- Growing literature on modeling rollover risk:
  - Acharya, Gale, and Yorulmzer (2009), Morris and Shin (2009), Brunnermeier and Oehmke (2009), He and Xiong (2009)

The Model Structure

- A firm holds a long-term risky asset by rolling over short-term debt.
  - Short-term debt acts as a disciplinary device on the firm, but also leads to a coordination problem among creditors.
- The environment of illiquid/imperfect capital markets:
  - The firm cannot find a single creditor (with deep pockets) to finance all the debt, and has to rely on a continuum of small creditors.
  - When some creditors choose to run, the firm needs to draw on unreliable credit lines.
  - The firm asset is illiquid, i.e., the firm can only recover a fraction of its fundamental value in a premature liquidation.
- Two key assumptions:
  - The asset fundamental is time-varying and publicly observable.
  - A staggered debt structure.

Long-Term Asset

- The firm holds a long-term asset:
  - The asset generates constant cash flow rdt over a period dt.
  - At a Poisson arrival time τ_ϕ, the asset matures with a final payoff equal to τ_ϕ value of a publicly observable process:

$$\frac{dy_t}{y_t} = \mu dt + \sigma dZ_t.$$

- Risk-neutral agents with discount rate ρ. Asset fundamental value:

$$F(y_t) = E_t\left[\int_t^{\tau_\phi} e^{-\rho(s-t)} r ds + e^{-\rho\left(\tau_\phi - t\right)} y_{\tau_\phi}\right] = \frac{r}{\rho+\phi} + \frac{\phi}{\rho+\phi-\mu} y_t$$

  - y_t is the firm fundamental.
  - Our model ignores complications from private information.

Staggered Debt Financing

- We assume a unit measure of short-term creditors (discount rate ρ).
  - Interest payment rdt.
  - $r > \rho$.
- A staggered debt structure:
  - Each contract matures with a probability of δdt, a la Calvo (1983).
  - In aggregate, δdt fraction of debt matures over $(t, t + dt)$.
  - This fraction is small and thus avoiding the Diamond-Dybvig type simultaneous coordination problem.
  - Rollover risk: during a contract period, other creditors might run.
- At τ_δ, an individual creditor decides to run or roll over.
  - Threshold strategy y_*: roll over if and only if $y \geq y_*$.

Debt Run and Liquidation

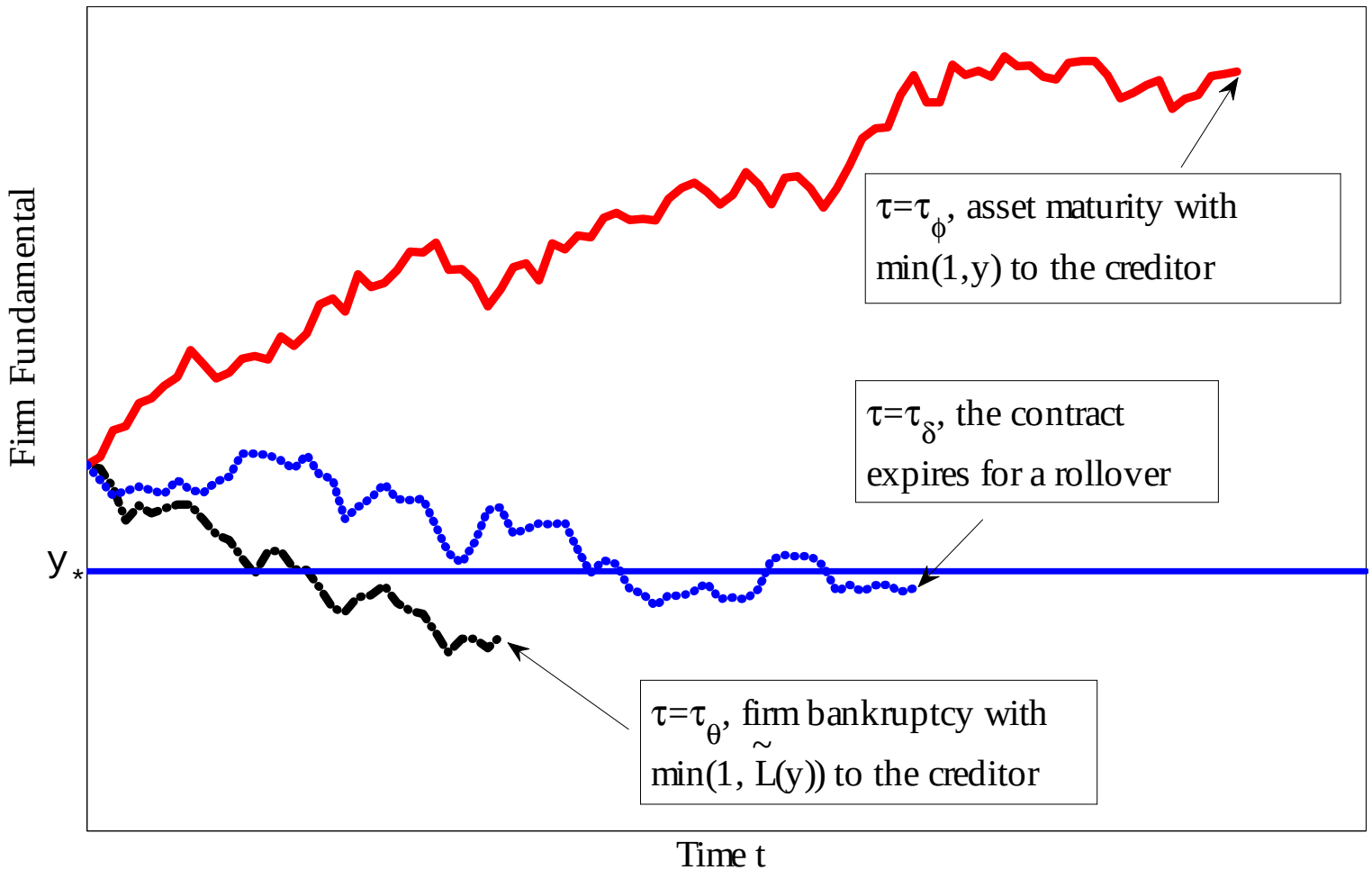
- Over $(t, t+dt)$, δdt fraction of contracts matures.
- If they choose to run, the firm needs to draw on its credit lines.
  - With prob $\theta\delta dt$, the credit lines fail, causing the firm to fail.
    - θ: unreliability of credit lines.
    - Can also be interpreted as imperfect government bailout.
  - With prob $1-\theta\delta dt$, the firm raises new fund and pays the creditors.
- Early liquidation recovers $\alpha \in (0,1)$ of the fundamental value:

$$\widetilde{L}(y_t) = \alpha F(y_t).$$

  - Liquidation decision is irreversible, no partial liquidation.
- The firm's liquidation value, $\widetilde{L}(y)$, is equally divided among all creditors, including the running ones.
- Because the probability of firm failing by one's own run is tiny, the expected payoff from running is 1.

Three Possible Paths for An Individual Creditor

- A creditor receives r until a random time $\tau = \min\left(\tau_\phi, \tau_\delta, \tau_\theta\right)$;
- Other creditors' rollover threshold y_*: rollover when $y > y_*$, run otherwise.



An Individual Creditor's Problem

- Given other creditors' threshold y_*, his value function is

$$
\begin{aligned}
V(y_t; y_*) \quad = \quad & E_t \left\{ \int_t^{\tau} e^{-\rho(s-t)} r ds \right. \\
& + \underbrace{e^{-\rho(\tau-t)} \min(1, y_\tau) \mathbf{1}_{\{\tau=\tau_\phi\}}}_{\text{Top path, the asset matures and pays off}} \\
& + \underbrace{e^{-\rho(\tau-t)} \max_{\text{rollover or run}} \{1, V(y_\tau; y_*)\} \mathbf{1}_{\{\tau=\tau_\delta\}}}_{\text{Middle path, make the rollover decision when contract expires}} \\
& \left. + \underbrace{e^{-\rho(\tau-t)} \min(1, L + l y_\tau) \mathbf{1}_{\{\tau=\tau_\theta\}}}_{\text{Bottom path, the firm fails due to other creditors' run}} \right\}.
\end{aligned}
$$

- Debt run externality: each creditor's run imposes an externality on the other creditors who are locked in.

The Unique Monotone Equilibrium

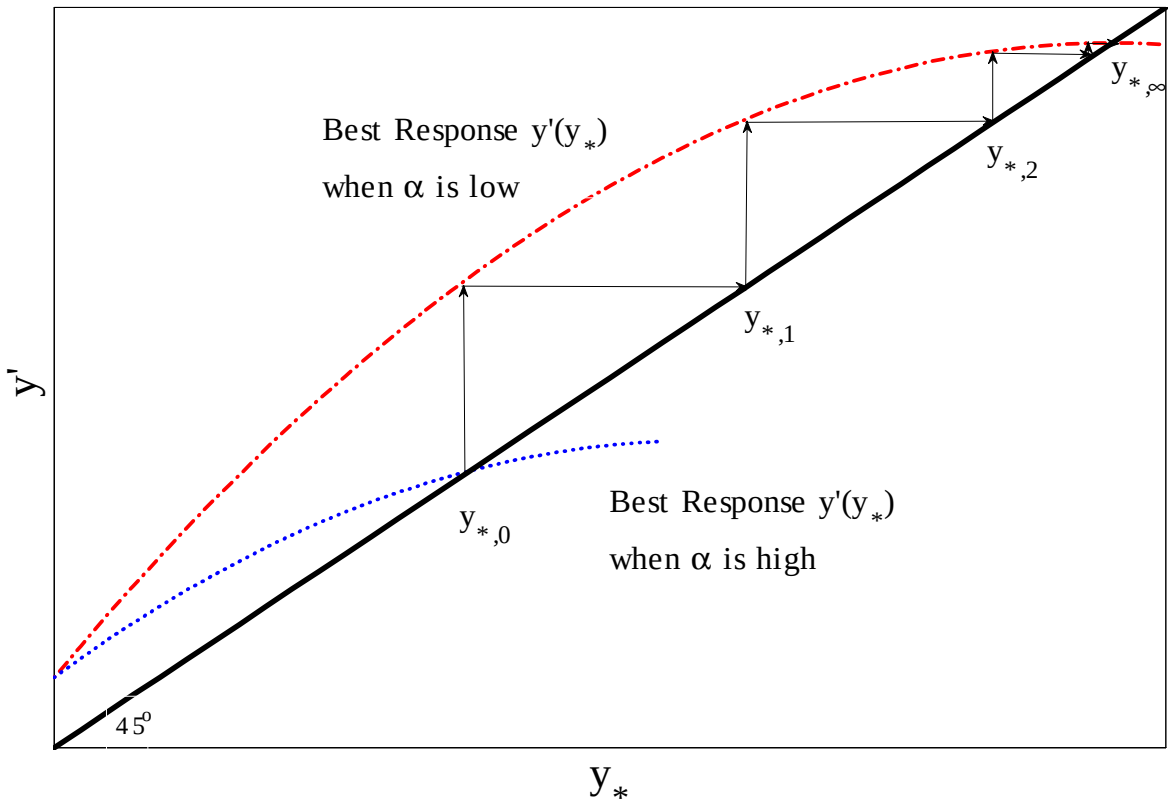
- There exists a **unique** equilibrium threshold y_* s.t. $V(y_*; y_*) = 1$.
  - Equilibrium uniquely defined in upper and lower dominance regions.
  - Knowing future maturing creditors will not run in dominance regions, backward induction uniquely determines equilibrium in the middle.
- Strategic uncertainty originates from time-varying fundamental.
  - e.g., Frankel and Pauzner (2000).
  - In contrast to Carlsson and van Damme (1993) and Morris and Shin (1998), strategic uncertainty arises from noise in private signals.
- Requires a well spread-out fundamental process.
  - Continuous time not essential.
  - Does not rely on specific information structure and immune from information revealed by market prices, e.g., Angeletos and Werning (2006) and Hellwig, Mukherji and Tsyvinski (2006).
- Difference from Frankel and Pauzner (2000).
  - Strategic complemetary exists in continuation values not in flow payoffs; deletion of dominated strategies not applicable.
  - We use a guess-and-varify approach.

Static-Rollover Benchmark

- Suppose that the firm's debt all matures at time 0, and each creditor simultaneously decides whether to rollover into a perpetual debt contract.
  - Multiple self-fulfilling equilibria emerge if the current fundamental is in an intermediate region.
  - Runs cannot occur when the firm's current liquidation value is higher than its liability, i.e., if $F(y_t) > 1/\alpha$.

Rat Race after a Drop in Liquidation Value

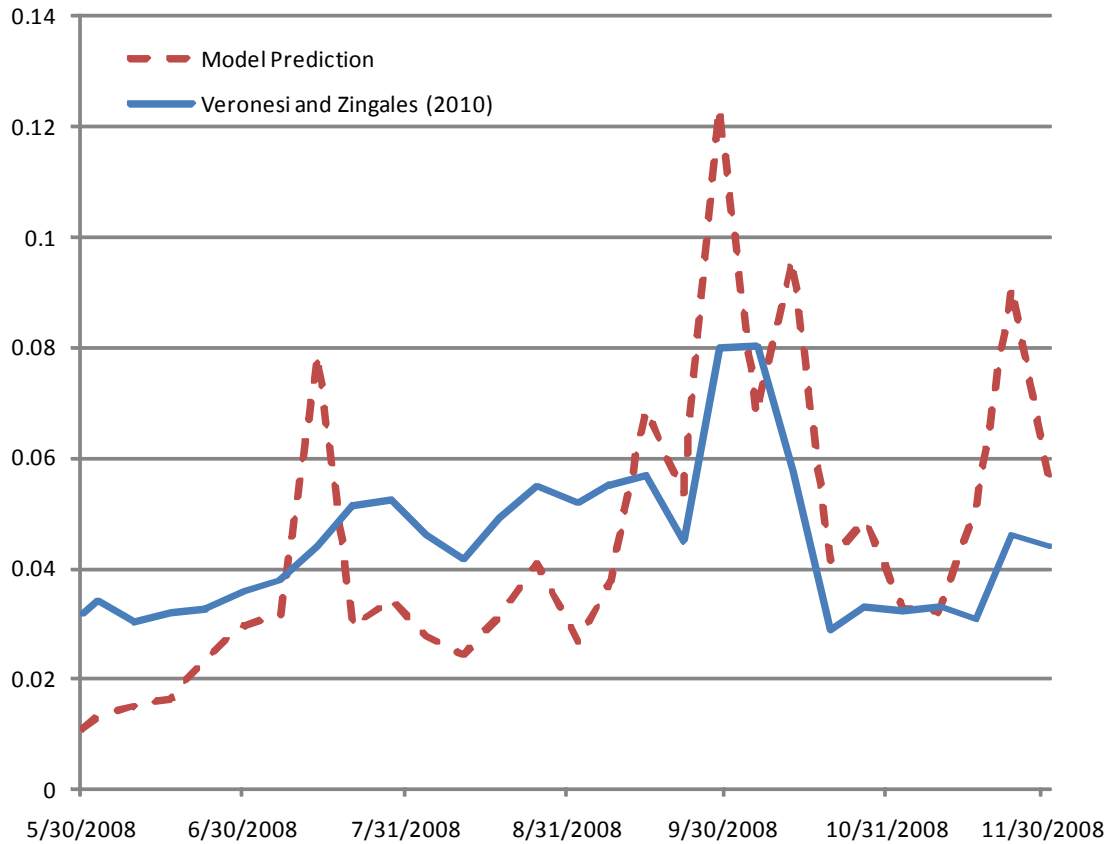
- Consider an unexpected drop in liquidation recovery rate α.
- A creditor's optimal response y' to other creditors' threshold y_*.



Calibrating Model Parameters

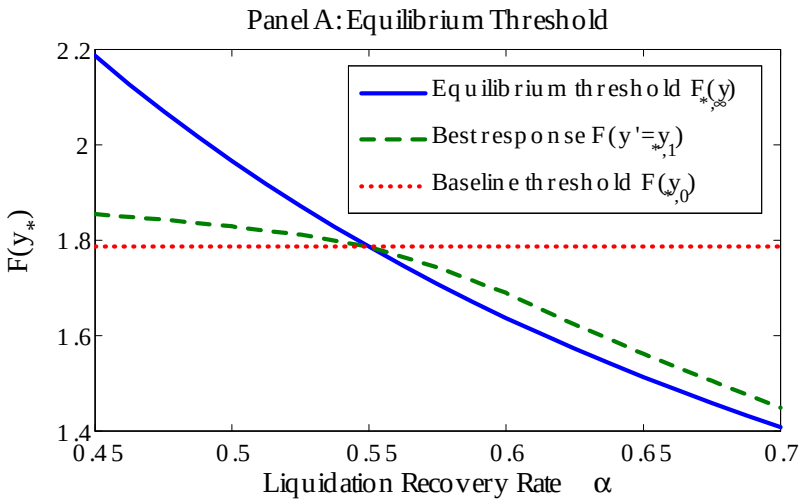
- We use a set of parameters for illustration.
  - Discount rate $\rho = 1.5\%$.
  - Asset cashflow $r = 7\%$; asset duration $1/\phi = 13$.
  - Asset's liquidation recovery rate $\alpha = 55\%$.
  - Asset's volatility $\sigma = 20\%$, growth rate $\mu = 1.5\%$, and current fundamental $y_0 = 1.4$.
  - Debt rollover frequency $\delta = 10$.
  - Unreliability of credit lines $\theta = 5$.

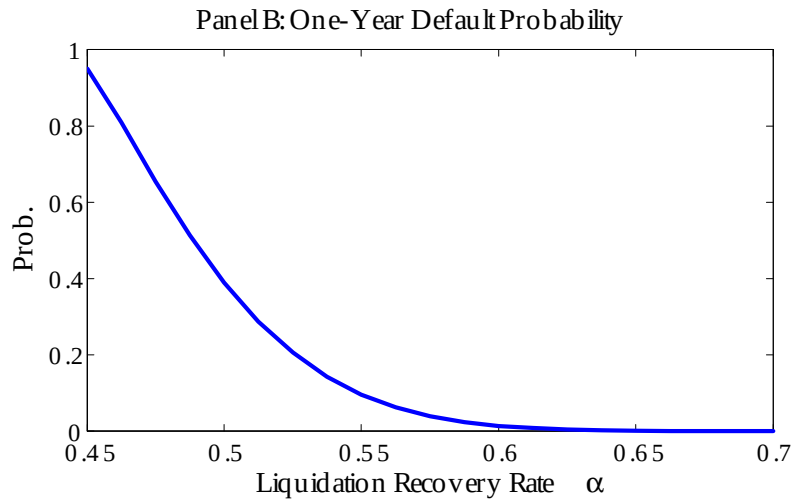
Predicting One Year Default Probability of Merrill Lynch



Effects of Liquidation Value

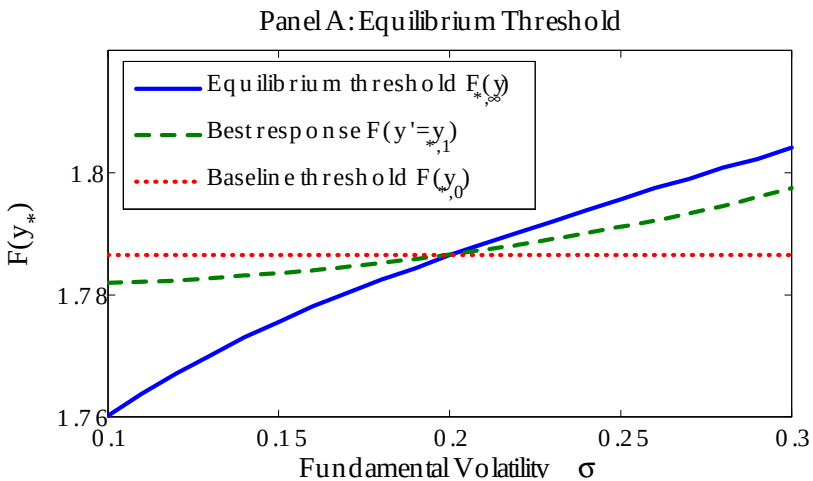
- Illiquidity exacerbates runs.
  - Similar to Rochet and Vives (2004).
- Threshold y_* sensitive to α.
  - Amplification effect by the rat race.

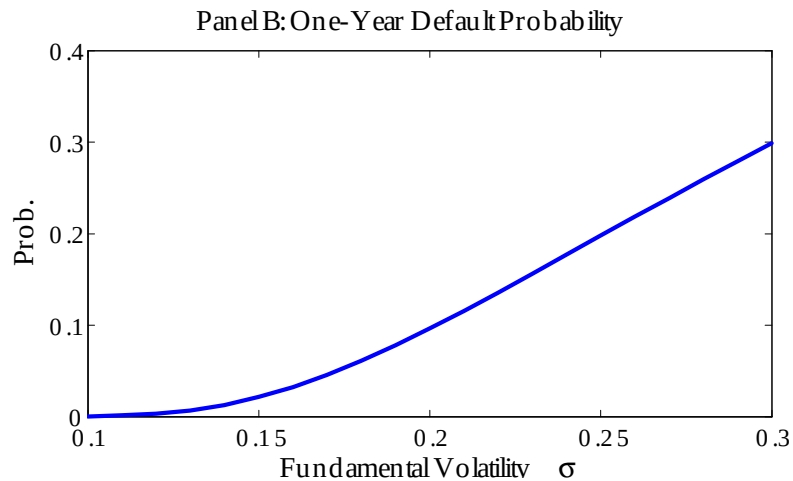




Effects of Fundamental Volatility

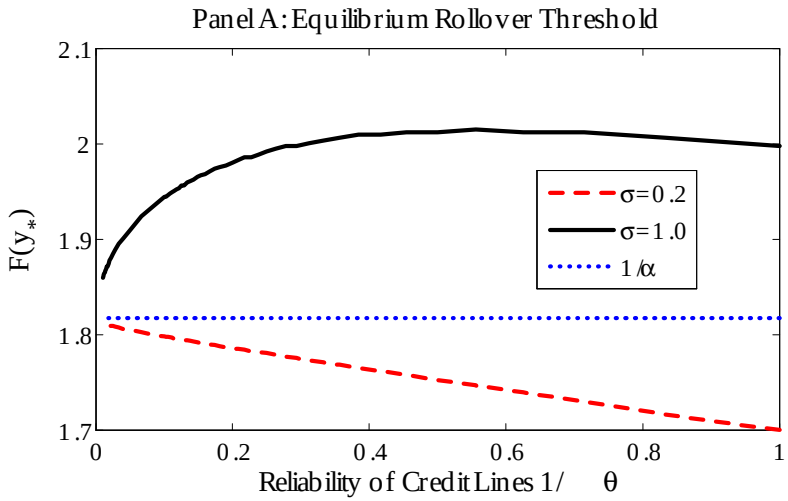
- Volatility affects each creditor in three channels:
  - Insolvency risk, causing y_* to increase with σ;
  - Rollover risk (strategic uncertainty), causing y_* to increase with σ;
  - Embedded option, causing y_* to decrease with σ.

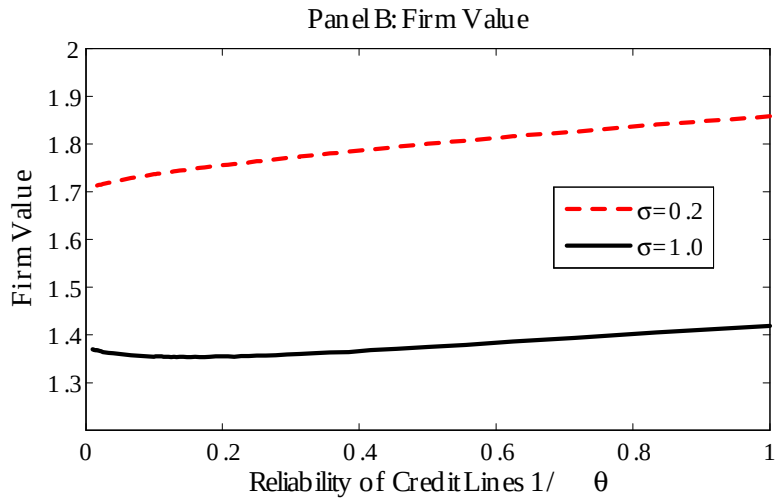




Effects of Credit Lines

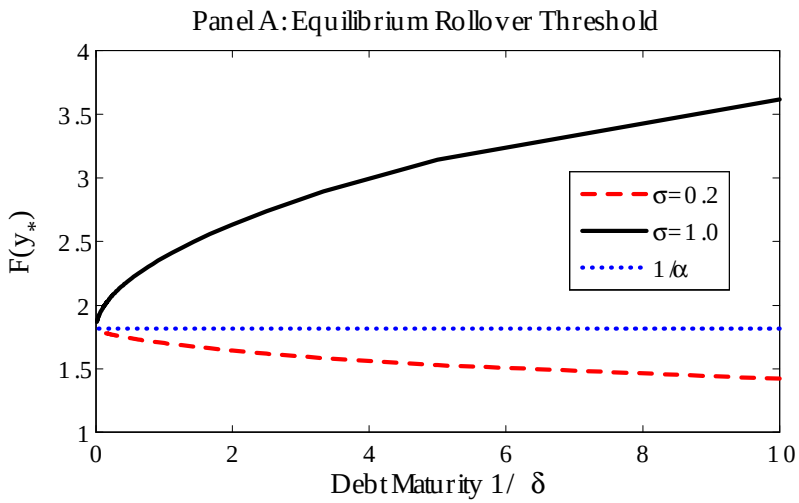
- Credit lines can temporarily sustain a firm under runs.
  - Common intuition: stronger credit lines should deter runs.
- When volatility is sufficiently large, credit lines exacerbate runs because fundamental can deteriorates during the period the firm lives on credit lines.
  - Uncertain government bailouts can be counter productive.

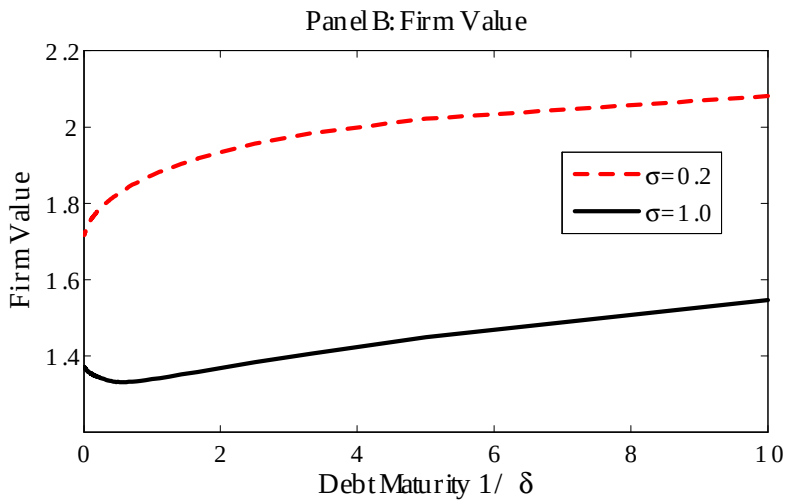




Effects of Debt Maturity

- Common intuition: longer debt maturities mitigate runs.
- Two offsetting effects of longer maturities:
  - 1) the firm faces less frequent rollover with other creditors and thus less likely to fail under runs.
  - 2) internally, longer lock-in effect for each creditor, which motivates runs, especially severe when voaltility is high.
- Longer maturities exacerbate runs when volatility is sufficiently high.
  - consistent with experience of runs on ABCP, e.g., Covitz, Liang, and Suarez (2009).





Further Discussion

- Synchronous vs Asynchronous Debt Structure
  - It is common for firms to spread out debt expirations.
  - The synchronous structure leads to more severe runs than the static-rollover benchmark when volatility is sufficiently high.
  - Which structure is optimal?
- Optimal Debt Maturity
  - Cheng and Milbradt (2010) extends our model to allow the firm switching b/w two projects: one with high growth and low volatility, the other with low growth and high volatility.
  - The optimal debt maturity trades off discipline on risk shifting and debt run risk.
- Spillover and Systemic Risk
  - When firms hold similar assets and face a downward sloping curve, runs on one firm can spill over to other firms.
  - Each firm's optimal debt structure and debt maturity depend on its own characteristics (fundamental volatility and asset illiqidity) and peer characteristics.

Conclusion

- We develop a dynamic model of debt runs.
  - Two basic ingredients: time-varying fundamental and staggered debt structure.
- The model highlights effects of volatility, debt maturity, and credit lines.
  - Fundamental volatility triggers strategic uncertainty (rollover risk.)
  - When volatility is sufficiently high, stronger credit lines and longer debt maturities exacerbate runs.
- Is readily extendable to address other dynamic corporate finance issues.
- Can be a building block in standard macro models to capture dynamic externalities.