Dynamic Debt Runs

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 Runs on Non-bank Financial Institutions

- Runs on the non-bank financial institutions was one of the main causes of the credit crisis of 2007-2008.
  - e.g., Bernanke (2008), Cox (2008), Geithner (2008), Brunnermeier (2009), Gorton (2008), Krishnamurthy (2009), and Shin (2009).

- The classic Diamond-Dybvig model on bank runs:
  - The simultaneous coordination problem among depositors leads to a self-fulfilling bank-run equilibrium.

- Global-games models of bank runs:
  - Signal noise leads to strategic uncertainty and prevents multiple equilibria, e.g., Carlsson and van Damme (1993) and Morris and Shin (2003).

- Many questions about debt runs involve time-varying fundamentals:
  - How does a firm’s asset price volatility affect its debt run risk?
  - Do credit lines mitigate runs?
  - Do longer debt maturities mitigate runs?
Dynamic Debt Runs

- A model with time-varying fundamental w/o noisy private signals:
  - The firm fundamental is time-varying and all creditors share the same public information about fundamental.
  - The firm uses a staggered debt structure, i.e., creditors make rollover decisions at different times.

- A unique threshold equilibrium:
  - each creditor rolls over or not based on current fundamental;
  - a “rat race” among creditors in choosing rollover thresholds.

- Results similar to static global-games models:
  - Severe runs on firms with weaker fundamentals, greater illiquidity.

- New results:
  - Higher volatility increases strategic uncertainty and thus exacerbates runs.
  - When fundamental volatility is sufficiently high, stronger credit lines and longer debt maturities can exacerbate runs.
A Brief Literature Review

- Static global games models:
  - Rochet and Vives (2004), Goldstein and Pauzner (2005)

- Dynamic coordination games based on higher order beliefs:

- Dynamic coordination games based on observable fundamentals:
  - Frankel and Pauzner (2000)

- Growing literature on modeling rollover risk:
  - Acharya, Gale, and Yorulmzer (2009), Morris and Shin (2009), Brunnermeier and Oehmke (2009), He and Xiong (2009)
The Model Structure

- A firm holds a long-term risky asset by rolling over short-term debt.
  - Short-term debt acts as a disciplinary device on the firm, but also leads to a coordination problem among creditors.
- The environment of illiquid/imperfect capital markets:
  - The firm cannot find a single creditor (with deep pockets) to finance all the debt, and has to rely on a continuum of small creditors.
  - When some creditors choose to run, the firm needs to draw on unreliable credit lines.
  - The firm asset is illiquid, i.e., the firm can only recover a fraction of its fundamental value in a premature liquidation.
- Two key assumptions:
  - The asset fundamental is time-varying and publicly observable.
  - A staggered debt structure.
The firm holds a long-term asset:

- The asset generates constant cash flow $r dt$ over a period $dt$.
- At a Poisson arrival time $\tau_\phi$, the asset matures with a final payoff equal to $\tau_\phi$ value of a publicly observable process:

$$\frac{dy_t}{y_t} = \mu dt + \sigma dZ_t.$$ 

- Risk-neutral agents with discount rate $\rho$. Asset fundamental value:

$$F(y_t) = E_t \left[ \int_t^{\tau_\phi} e^{-\rho(s-t)} rds + e^{-\rho(\tau_\phi-t)} y_{\tau_\phi} \right] = \frac{r}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t$$

- $y_t$ is the firm fundamental.
- Our model ignores complications from private information.
Staggered Debt Financing

- We assume a unit measure of short-term creditors (discount rate $\rho$).
  - Interest payment $rdt$.
  - $r > \rho$.
- A staggered debt structure:
  - Each contract matures with a probability of $\delta dt$, a la Calvo (1983).
  - In aggregate, $\delta dt$ fraction of debt matures over $(t, t + dt)$.
  - This fraction is small and thus avoiding the Diamond-Dybvig type simultaneous coordination problem.
  - Rollover risk: during a contract period, other creditors might run.
- At $\tau_\delta$, an individual creditor decides to run or roll over.
  - Threshold strategy $y_*$: roll over if and only if $y \geq y_*$.
Debt Run and Liquidation

- Over \((t, t + dt)\), \(\delta dt\) fraction of contracts matures.
- If they choose to run, the firm needs to draw on its credit lines.
  - With prob \(\theta \delta dt\), the credit lines fail, causing the firm to fail.
    - \(\theta\): unreliability of credit lines.
    - Can also be interpreted as imperfect government bailout.
  - With prob \(1 - \theta \delta dt\), the firm raises new fund and pays the creditors.
- Early liquidation recovers \(\alpha \in (0, 1)\) of the fundamental value:
  \[
  \tilde{L}(y_t) = \alpha F(y_t). 
  \]
  - Liquidation decision is irreversible, no partial liquidation.
- The firm’s liquidation value, \(\tilde{L}(y)\), is equally divided among all creditors, including the running ones.
- Because the probability of firm failing by one’s own run is tiny, the expected payoff from running is 1.
Three Possible Paths for An Individual Creditor

- A creditor receives $r$ until a random time $\tau = \min(\tau_\phi, \tau_\delta, \tau_\theta)$;
- Other creditors’ rollover threshold $y_*$: rollover when $y > y_*$, run otherwise.
An Individual Creditor’s Problem

- Given other creditors’ threshold $y_*$, his value function is

$$V(y_t; y_*) = E_t \left\{ \int_t^\tau e^{-\rho(s-t)} rds \right.$$  

$$+ e^{-\rho(\tau-t)} \min (1, y_\tau) 1\{\tau=\tau_\phi\}$$  

Top path, the asset matures and pays off

$$+ e^{-\rho(\tau-t)} \max \{1, V(y_\tau; y_*)\} 1\{\tau=\tau_\delta\}$$  

Middle path, make the rollover decision when contract expires

$$+ e^{-\rho(\tau-t)} \min (1, L + Ly_\tau) 1\{\tau=\tau_\theta\}$$  

Bottom path, the firm fails due to other creditors’ run

- Debt run externality: each creditor’s run imposes an externality on the other creditors who are locked in.
The Unique Monotone Equilibrium

- There exists a unique equilibrium threshold $y^*_s$ s.t. $V(y^*_s; y^*) = 1$.
  - Equilibrium uniquely defined in upper and lower dominance regions.
  - Knowing future maturing creditors will not run in dominance regions, backward induction uniquely determines equilibrium in the middle.

- Strategic uncertainty originates from time-varying fundamental.
  - e.g., Frankel and Pauzner (2000).
  - In contrast to Carlsson and van Damme (1993) and Morris and Shin (1998), strategic uncertainty arises from noise in private signals.

- Requires a well spread-out fundamental process.
  - Continuous time not essential.
  - Does not rely on specific information structure and immune from information revealed by market prices, e.g., Angeletos and Werning (2006) and Hellwig, Mukherji and Tsyvinski (2006).

- Difference from Frankel and Pauzner (2000).
  - Strategic complementarity exists in continuation values not in flow payoffs; deletion of dominated strategies not applicable.
  - We use a guess-and-varify approach.
**Static-Rollover Benchmark**

- Suppose that the firm’s debt all matures at time 0, and each creditor simultaneously decides whether to rollover into a perpetual debt contract.
  - Multiple self-fulfilling equilibria emerge if the current fundamental is in an intermediate region.
  - Runs cannot occur when the firm’s current liquidation value is higher than its liability, i.e., if $F(y_t) > 1/\alpha$. 
Consider an unexpected drop in liquidation recovery rate $\alpha$.

A creditor’s optimal response $y'$ to other creditors’ threshold $y_\ast$. 

Best Response $y'(y_\ast)$ when $\alpha$ is low

Best Response $y'(y_\ast)$ when $\alpha$ is high
Calibrating Model Parameters

- We use a set of parameters for illustration.
  - Discount rate $\rho = 1.5\%$.
  - Asset cashflow $r = 7\%$; asset duration $1/\phi = 13$.
  - Asset’s liquidation recovery rate $\alpha = 55\%$.
  - Asset’s volatility $\sigma = 20\%$, growth rate $\mu = 1.5\%$, and current fundamental $y_0 = 1.4$.
  - Debt rollover frequency $\delta = 10$.
  - Unreliability of credit lines $\theta = 5$. 
Predicting One Year Default Probability of Merrill Lynch

Model Prediction

Veronesi and Zingales (2010)
Effects of Liquidation Value

- Illiquidity exacerbates runs.
  - Similar to Rochet and Vives (2004).
- Threshold $y^*$ sensitive to $\alpha$.
  - Amplification effect by the rat race.

![Graphs showing equilibrium threshold and one-year default probability](image-url)
Effects of Fundamental Volatility

- Volatility affects each creditor in three channels:
  - Insolvency risk, causing $y_*$ to increase with $\sigma$;
  - Rollover risk (strategic uncertainty), causing $y_*$ to increase with $\sigma$;
  - Embedded option, causing $y_*$ to decrease with $\sigma$. 

Panel A: Equilibrium Threshold

Panel B: One-Year Default Probability
Effects of Credit Lines

- Credit lines can temporarily sustain a firm under runs.
  - Common intuition: stronger credit lines should deter runs.
- When volatility is sufficiently large, credit lines exacerbate runs because fundamental can deteriorates during the period the firm lives on credit lines.
  - Uncertain government bailouts can be counter productive.
Effects of Debt Maturity

- Common intuition: longer debt maturities mitigate runs.
- Two offsetting effects of longer maturities:
  - 1) the firm faces less frequent rollover with other creditors and thus less likely to fail under runs.
  - 2) internally, longer lock-in effect for each creditor, which motivates runs, especially severe when volatility is high.

- Longer maturities exacerbate runs when volatility is sufficiently high.
  - consistent with experience of runs on ABCP, e.g., Covitz, Liang, and Suarez (2009).
Further Discussion

- Synchronous vs Asynchronous Debt Structure
  - It is common for firms to spread out debt expirations.
  - The synchronous structure leads to more severe runs than the static-rollover benchmark when volatility is sufficiently high.
  - Which structure is optimal?

- Optimal Debt Maturity
  - Cheng and Milbradt (2010) extends our model to allow the firm switching b/w two projects: one with high growth and low volatility, the other with low growth and high volatility.
  - The optimal debt maturity trades off discipline on risk shifting and debt run risk.

- Spillover and Systemic Risk
  - When firms hold similar assets and face a downward sloping curve, runs on one firm can spill over to other firms.
  - Each firm’s optimal debt structure and debt maturity depend on its own characteristics (fundamental volatility and asset illiquidity) and peer characteristics.
We develop a dynamic model of debt runs.

- Two basic ingredients: time-varying fundamental and staggered debt structure.

The model highlights effects of volatility, debt maturity, and credit lines.

- Fundamental volatility triggers strategic uncertainty (rollover risk.)
- When volatility is sufficiently high, stronger credit lines and longer debt maturities exacerbate runs.

Is readily extendable to address other dynamic corporate finance issues.

Can be a building block in standard macro models to capture dynamic externalities.