Rollover Risk and Credit Risk

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Motivation

- What determines a firm’s credit spread?
  - default premium;
  - liquidity premium, e.g., Longstaff, Mithal, and Neis (2005), and Chen, Lesmond, and Wei (2007).

- However, default premium and liquidity premium are typically treated as independent and measured separately.

- The recent credit crisis demonstrated an intricate interaction between them.
  - Deterioration of market liquidity can exacerbate default risk.
  - Lehman Brothers and Bear Stearns.

- This paper develops a model to study this interaction.
  - Firms’ rollover risk as the channel: deterioration of market liquidity causes equity holders to suffer losses in rolling over maturing bonds.
  - Conflict of interest between debt and equity holders causes equity holders to default earlier, e.g., Flannery (2005) and Duffie (2009).
Summary of the Paper

- We build on Leland (1994) and Leland and Toft (1996):
  - A firm has to constantly roll over its maturing debt by issuing new debt with the same maturity and face value at market price.
  - Equity holders of the firm bear the rollover gain/loss and endogenously default when the equity value drops to zero.
- Debt rollover exposes the firm to liquidity risk in bond markets.
  - Deteriorating liquidity exacerbates default risk.
  - Tradeoff: rollover loss vs option value of keeping the firm alive.
- Flight to quality:
  - liquidity deterioration has greater effects on weaker firms.
- Short-term debt also exacerbates the liquidity effect by forcing equity holders to quickly absorb the rollover loss.
- Implications:
  - 1) Liquidity can predict defaults; 2) caution in decomposing credit spreads; 3) maturity risk; 4) highly variable fundamental and liquidity risk.
Brief Literature Review

- Growing literature on rollover risk:
  - Diminishing debt capacity: Acharya, Gale, and Yorulmzer (2009)
  - Dynamic debt runs: He and Xiong (2009)

- Structural credit risk models focus on fundamental default risk:
  - Exogenous default threshold: Merton (1973), Longstaff and Schwartz (1995);

- Empirical evidence on important liquidity effects in credit spreads:
  - Interpreted as a liquidity-premium effect.
  - Our model: an increase in liquidity premium also leads to higher default premium.
Model (1)

- We build on Leland and Toft (1996) with an additional feature:
  - Illiquid secondary bond markets.
- A firm repays maturing bonds by issuing new bonds at market prices.
  - The rollover gain/loss is absorbed by equity holders;
  - The firm defaults when equity value drops to zero.
- The unlevered firm value follows a log-normal process under the $Q$-measure:
  \[
  \frac{dV_t}{V_t} = (r - \delta) \, dt + \sigma \, dZ_t. 
  \]
  - Riskfree rate $r$, payout rate $\delta$.
- In bankruptcy creditors recover $\alpha$ fraction of the firm value.
Model (2): Debt Structure

- The firm commits to a stationary debt structure \((C, P, m)\):
  - aggregate face value \(P\) and annual coupon payment \(C\);
  - each bond has maturity \(m\);
  - debt expirations are uniformly spread across time, i.e., over \((t, t + dt)\), \(\frac{1}{m} dt\) fraction of the bonds matures.

- The firm issues new bonds with the same face value, coupon rate and maturity to replace maturing bonds.

- Over \((t, t + dt)\), the net cash flow to equity holders is

  \[
  NC_t = \delta V_t - (1 - \pi) C + \frac{1}{m} [\bar{d}(V_t, m) - P].
  \]

- \(\bar{d}(V_t, m)\): market value of per unit newly issued bond;
- When the bond price drops, equity holders face rollover losses.
- Will show the loss is greater for short-term debt.
The firm defaults when $V_t$ drops to an endogenous threshold $V_B$.
- At $V_B$, equity value $E(V_B) = 0$, i.e., the firm cannot raise any equity financing;
- Optimality of $V_B$: smooth pasting $E' (V_B) = 0$.

Intrinsic conflict of interest between debt and equity holders:
- When the bond price falls (for either fundamental or liquidity reasons), equity holders bear the rollover loss while the maturing debt holders get paid in full.
- Equity holders face a tradeoff: rollover loss vs option value of keeping the firm alive.
Model (4): The Secondary Bond Markets

- The secondary markets of corporate bonds are highly illiquid.
  - Large bid-ask spreads and price impact.
  - Edwards, Harris, and Piwowar (2007): bid/ask spread on corporate bonds ranges from 4 to 75 bps.
  - Bao, Pan, and Wang (2009): trading cost (bid/ask spread & price impact) ranges from 74 to 221 bps; and the cost is higher for long-term bonds.
- When a bond holder sells a bond, he only recovers a fraction \((1 - k)\) of the value.
  - \(k\) represents the liquidity discount (trading cost, info problem, ...)
- Each bond investor is subject to Poisson liquidity shocks with intensity \(\zeta\), a la Amihud and Mendelson (1986).
  - Upon the arrival of a liquidity shock, he has to sell his bond holdings.
- We assume no cost for trading equity and issuing new bonds.
Solving the Equilibrium

- For a given $V_B$, PDE for the debt value $d (V_t, \tau; V_B)$:

$$\left( r + \xi k \right) d (V_t, \tau) = c - \frac{\partial d (V_t, \tau)}{\partial \tau} + (r - \delta) V_t \frac{\partial d (V_t, \tau)}{\partial V} + \frac{1}{2} \sigma^2 V_t^2 \frac{\partial^2 d (V_t, \tau)}{\partial V^2}.$$

  - At the bankruptcy, $d (V_B, \tau; V_B) = \frac{\alpha V_B}{m}$, for all $\tau \in [0, m]$.
  - At maturity, $d (V_t, 0; V_B) = p$, for all $V_t > V_B$.

- ODE for equity value $E (V)$:

$$rE = (r - \delta) V_tE_V + \frac{1}{2} \sigma^2 V_t^2 E_{VV} + \delta V_t - (1 - \pi) C + d (V_t, m) - p.$$

  with boundary condition $E (V_B) = 0$:

  - Closed-form solution for $E (V)$ using Laplace transformation.

- Smooth pasting $E' (V_B) = 0$: closed-form solution for $V_B$. 

Key Channels of Liquidity Effects

- Consider an unanticipated liquidity shock which increases $\zeta$ or $k$.
  - e.g., increased redemption risk, margin risk, or market illiquidity.
### Baseline Model Parameters for Illustration

- **Risk-free rate**: $r = 7.5\%$.
- **Tax rate**: $\pi = 35\%$.
- **Asset volatility** $\sigma = 15\%$; payout rate $\delta = 7\%$.
- **Trading cost** $k = 1.5\%$; Intensity of liquidity shocks $\xi = 1$.
  - Consistent with Bao, Pan, and Wang (2009) who focus on a relatively liquid sample.
- **Liquidation recovery rate**: $\alpha = 0.5$.
- **Debt maturities** $m = 1$; total principal $P = 24.09$; total coupons $C = 2.17$.
  - Optimal debt structure when initial $V_0 = 100$.
- **Current asset value**: $V_t = 44$. 
Market Liquidity and Endogenous Default

- Two channels of liquidity effects: liquidity premium and endogenous default risk.

Panel A: Rollover Loss
Panel B: Default Boundary
Panel C: Credit Spread
Panel D: Composition of Credit Spread
Is the Liquidity-driven Default Efficient?

- Liquidity deterioration increases the firms’ financing cost. Thus, an earlier default might be desirable to the joint interest of debt and equity holders.

- Suppose that the firm never defaults.
  - The present value of future tax shield is $\pi C $, while the present value of future bond transaction costs is $\xi k \frac{C}{r + \xi k}$.
  - Default hurts the joint interest if
    \[ \pi > \frac{\xi k}{r + \xi k}, \]
    which always holds in our illustration.

- Thus, the increased default risk caused by liquidity deterioration originates from conflict of interest between debt and equity holders.
  - i.e., debt overhang problem of Myers (1977).
Flight to Quality

- Flight to quality: after major liquidity disruptions prices of low quality bonds drop much more than high quality bonds.
Amplification by Short-term Debt

- Shorter maturity forces equity holders to quickly realize rollover loss.
  - Rollover loss per unit of time: \[ \bar{d} (V_t, m) - P \] / m.
  - More severe conflict b/w debt- and equity-holders.

- Short-term maturity makes an individual bond safer, but a firm with more short-term debt is riskier.
Optimal Debt Structure

- Bond market illiquidity reduces the firm’s initial leverage choice.
Implications: Predicting Defaults

- Our model predicts market liquidity as a new factor for predicting bond defaults, in addition to
  - Distance to default: leverage, asset volatility
  - Firms’ liquidity holdings: cash, credit lines

- The existing structural credit risk models have mixed successes:
  - Leland (2004): Leland model does a good job in capturing average default probabilities of bonds with different ratings.
  - Bharath and Shumway (2008): distance-to-default variable constructed from Merton model is not a sufficient statistic for default probability.
  - Davydenko (2007): distance to default cannot capture the cross section of bond spreads;

- Collin-Dufresne, Goldstein, and Martin (2001): standard variables cannot explain the changes of credit spreads.

- Das, Duffie, Kapadia, and Saita (2007): distance-to-default variables cannot fully capture default correlation observed in the data.
Both academics and policy makers have recognized the important effect of market liquidity on credit spreads, but tend to treat it as independent from default risk.

Several studies, e.g., Longstaff, Mithal, and Neis (2005), Beber, Brandt, and Kavajecz (2008), and Schwarz (2009), decompose credit spreads to assess contributions of liquidity premium and default risk:

\[
CreditSpread_{i,t} = \alpha + \beta \cdot CDS\_Spread_{i,t} + \delta \cdot LIQ_{i,t} + \epsilon_{i,t}
\]

Default risk explains a majority part of the cross-sectional variation, although the liquidity effect is also significant.

But these two effects are correlated through endogenous default.

How to classify the correlated part?

In the empirical analysis, the more precise measure of default risk (via traded prices) could have favored the default risk effect.
Implications: Measuring Liquidity Effects

- Several recent studies examine the impact of TAF on LIBOR-OIS spread.
- They tend to control for default risk using certain credit spread, such as CDS spread or LIBOR-REPO spread.
  - Example: Taylor and Williams (2009)

\[
(LIBOR - OIS)_t = a \cdot (LIBOR - REPO)_t + b \cdot TAF_t + \epsilon_t
\]

- The control variables can also absorb liquidity effects and thus leading to an under-estimation.
Implications: Maturity Risk

- Our model implies that firms’ debt maturity structure is an important determinant of credit risk.
- Evidence on non-financial firms with more maturing long-term debt during the recent credit crisis period had to cut down more investment and had greater credit spread increases.
- Evidence on credit ratings had ignored maturity risk.
  - Gopalan, Song, and Yerramilli (2009).
Implications: Managing Credit Risk

- Variability of fundamental beta and liquidity beta (like Gamma for options) is important for effectiveness of static hedges of bond price risk over a given period.
  - Transaction cost prevents institutions from continuously updating their hedges.
- Define fundamental and liquidity betas of \( d (V_t, \zeta; V_B (\zeta)) \):

  \[
  \beta_V \equiv \frac{\partial d (V_t, \zeta; V_B (\zeta))}{\partial V},
  \]

  and

  \[
  \beta_\zeta \equiv \frac{d d (V_t, \zeta; V_B (\zeta))}{d \zeta} = \frac{\partial d (V_t, \zeta; V_B (\zeta))}{\partial \zeta} + \frac{\partial d (V_t, \zeta; V_B (\zeta))}{\partial V_B} \cdot \frac{d V_B (\zeta)}{d \zeta}.
  \]
Fundamental and Liquidity Betas

- We compare betas implied by our model with those by a structural model with an exogenous default threshold (e.g., Merton (1973) and Longstaff and Schwartz (1995)).
  - Assume that the default threshold is fixed at the baseline level.
Variability of fundamental beta and liquidity beta is even greater when the firm is financed by short-term debt.
Extension

- A more elaborate secondary market:
  - Multiple types of bond investors with different frequencies of liquidity shocks;
  - Multiple classes of long-term and short-term bonds with short-term debt being more liquid.

- Endogenous market segmentation in spirit of Amihud and Mendelson (1986):
  - Investors with higher liquidity needs self-select to short-term bonds;
  - Liquidity effect spill over across different segments through investors’ required bond returns.

- Endogenous debt maturity structure:
  - The firm trades off short-term debt’s lower liquidity premium and the resulting higher default risk.
Conclusion

- A model of liquidity effects on credit spreads.
  - Two channels: liquidity premium and endogenous default.
  - The latter channel operates through firms' rollover risk.

- Several results:
  - Liquidity shocks increase credit spreads not only through higher liquidity premia, but also higher default probabilities.
  - Flight to quality: Bonds with weaker fundamentals are more exposed to liquidity shocks.
  - Shorter debt maturity exacerbates rollover risk and thus effects of liquidity deterioration on endogenous default.

- Implications:
  - 1) Liquidity can predict defaults; 2) caution against treat liquidity and default premia as independent; 3) maturity risk; 4) highly variable fundamental and liquidity risk.