DYNAMIC DEBT MATURITY

Zhiguo He (Chicago Booth and NBER)
Konstantin Milbradt (Northwestern Kellogg and NBER)

May 2015, OSU
Motivation

- Debt maturity and its associated rollover risk is at the center of post-crisis policy discussion
  - Krishnamurthy 2010: Financial firms shorten their debt maturity right before crisis
  - Xu 2014: Speculative firms lengthen debt maturity in good time
- We offer a model to analyze endogenous debt maturity (with predictions consistent with empirical pattern)
  - In a dynamic setting, in what scenarios might keep going short with earlier default be observed?
  - **no commitment by firm to its debt maturity structure**
Motivation

- Debt maturity and its associated rollover risk is at the center of post-crisis policy discussion
  - Krishnamurthy 2010: Financial firms shorten their debt maturity right before crisis
  - Xu 2014: Speculative firms lengthen debt maturity in good time
- We offer a model to analyze endogenous debt maturity (with predictions consistent with empirical pattern)
  - In a dynamic setting, in what scenarios might keep going short with earlier default be observed?
  - **no commitment by firm to its debt maturity structure**
- **Baseline:** constant current cash-flows, waiting for upside event; equity tends to default inefficiently early
  - Trade-off never in favor of short-term debt; possibly multiple equilibria but never shortening to death
- **Sinking ship:** waiting for upside event but deteriorating cash-flows
  - Shortening ensues: debt holders’ recovery value declines when defaulting later
An Illustrating Example of Trade-Off

- Using short-term debt today helps reduce rollover losses now, but needs to be refinanced sooner
- Two period \( t = 0, 1, 2 \), no discounting
  - At the end of each date, exogenous default probability \( q = 0.1 \)
- At \( t = 0 \), the firm needs to repay 1 dollar of matured debt
  - either issuing one-period short-term debt price \( D_S = 1 - q = 0.9 \)
  - or two-period long-term debt price \( D_L = (1 - q)^2 = 0.81 \)
  - Equity holders cover the “rollover loss” \( D_i - 1 \)
- Trade-off:
  - Issuing short-term: less rollover loss \(-0.1\) at today \( t = 0 \), but tomorrow \( t = 1 \) facing rollover loss again
  - Issuing long-term: greater rollover loss \(-0.19\) at today \( t = 0 \), but tomorrow \( t = 1 \) free of rollover concern
- Main model: link rollover losses to
  1. endogenous default decision \( q \) which in turn affects debt prices \( D_i \)
  2. which in turn will affect endogenous issuance decision (S vs L)
Preview & Roadmap

**Key trade-off** for equity:

- Issue more short-term debt today at a higher price compared to long-term debt... **BUT**

- As maturity structure shorter, face stronger rollover tomorrow and lower equity value

**Question:**

- Can this trade-off every lead to *shortening to death* by lacking of commitment?

- Welfare implications?
Cash-flows & Unlevered Asset Value

Assets under management:
- Instantaneous cash-flow of $y_t$
- Upside event with payoff $X > 1$ occurs with intensity $\zeta$
- Free option to abandon project, e.g., when cash-flows $y_t$ are too low

First-best asset value:
- Current management is best suited to run the firm
- Total first-best unlevered asset value given by

$$A (y) = \mathbb{E} \left[ \int_0^{T_A} e^{-(r+\zeta)t} (y_t + \zeta X) \, dt \right]$$

where $T_A$ is the first-best abandonment time

Recovery value:
- If management gets replaced, new management generates unlevered asset value (with new abandonment time $T_B$) of

$$B (y) = \mathbb{E} \left[ \int_0^{T_B} e^{-(r+\zeta)t} (\alpha_y y_t + \zeta \alpha_X X) \, dt \right]$$

- Assume $\alpha_y, \alpha_X$ such that $B (y) < A (y)$
Capital Structure

Two kinds of (random maturity) bonds:
- Long-term bonds with maturity intensity $\delta_L$
- Short-term bonds with maturity intensity $\delta_S > \delta_L$
- Expected maturities $1/\delta_L$ and $1/\delta_S$ respectively
- Coupon rate $c = r$ so i) without default debt value is 1; ii) with default, debt value below 1 and hence rollover loss

Outstanding face-value normalized to 1, following Leland settings to rule out outright dilution
- **Assumption:** Firm commits to constant face-value of debt, but cannot commit to future debt maturity
- In practice we observe more debt covenants on (book) leverage rather than debt maturity
- Differ from Brunnermeier Oehmke 2013: they do not have commitment on outstanding face value
Debt Maturity Structure

Debt maturity structure

- $\phi_t$: fraction of short-term bonds outstanding
- Every instant $m(\phi_t) \, dt$ dollars of bonds mature:

$$m(\phi_t) = \phi_t \delta_S + (1 - \phi_t) \delta_L$$

- $m'(\phi_t) > 0$: the shorter the maturity structure, the faster the rollover
Debt Maturity Structure

Debt maturity structure

- $\phi_t$: fraction of short-term bonds outstanding
- Every instant $m(\phi_t) \, dt$ dollars of bonds mature:

$$m(\phi_t) = \phi_t \delta_S + (1 - \phi_t) \delta_L$$

- $m'(\phi_t) > 0$: the shorter the maturity structure, the faster the rollover

Issuance policy. $f_t \in [0, 1]$: the fraction of short debt in new issue

Assumption. Equity holders cannot commit to issuance policy $\{f_t\}$

- Leland setting has pre-chosen constant $f$
Debt Maturity Structure

Debt maturity structure

- $\phi_t$: fraction of short-term bonds outstanding
- Every instant $m(\phi_t)\,dt$ dollars of bonds mature:
  
  $$m(\phi_t) = \phi_t \delta_S + (1 - \phi_t) \delta_L$$

- $m'(\phi_t) > 0$: the shorter the maturity structure, the faster the rollover

Issuance policy. $f_t \in [0, 1]$: the fraction of short debt in new issue

Assumption. Equity holders cannot commit to issuance policy $\{f_t\}$

- Leland setting has pre-chosen constant $f$

Maturity structure dynamics

$$\frac{d\phi_t}{dt} = -\phi_t \delta_S + \underbrace{m(\phi_t) f_t}_{\text{Newly issued short-term}}$$

- Short-term maturing

if $f_t = 1$, keep issuing short-term debt, $\phi_t \uparrow$
Baseline: Equity and Endogenous Default

Benchmark: constant cash-flow $y_t = y$. Equity value

$$rE(\phi; y) = y - c + \zeta [(X - 1) - E(\phi; y)]$$

$$+ \max_{f \in [0,1]} \left\{ m(\phi) [fD_S(\phi; y) + (1 - f) D_L(\phi; y) - 1] + [-\phi\delta_S + m(\phi) f] E'(\phi; y) \right\}$$

rollover losses

impact of shortening
Baseline: Equity and Endogenous Default

**Benchmark:** constant cash-flow \( y_t = y \). Equity value

\[
\hat{r}E(\phi; y) = y - c + \zeta [(X - 1) - E(\phi; y)] \\
+ \max_{f \in [0,1]} \left\{ m(\phi) \left[ fD_S(\phi; y) + (1 - f) D_L(\phi; y) - 1 \right] + \left[ -\phi \delta_S + m(\phi) f \right] E'(\phi; y) \right\}
\]

**Endogenous default boundary** \( \Phi(y) \)

- Equity defaults optimally when net cash-flows are zero
  \[
y - c + \zeta E^{rf} + \max_{f \in [0,1]} m(\phi) \left[ fD_S(\phi; y) + (1 - f) D_L(\phi; y) - 1 \right] = 0
  \]

- Irrespective of \( f \) at default, \( D_S = D_L = B(y) \) imply
  \[
  \Phi(y) = \frac{1}{\delta_S - \delta_L} \left[ \frac{y - c + \zeta E^{rf}}{1 - B(y)} - \delta_L \right]
  \]

- \( \Phi'(y) > 0 \): shorter maturity structure hastens default (default threshold cash-flow \( y \) is higher)
Q: Is there ever “shortening-to-death” equilibrium default while equity keeps shortening maturity?

- Always shortening implies $f_t = 1$, so $\frac{d\phi_t}{dt} > 0$, while $y$ is constant.
Baseline: Bond Valuation Wedge $\Delta$

**Bond valuation:**
- Value of debt $i \in \{S, L\}$ solves:
  \[
  rD_i (\phi; y) = \underbrace{c + \delta_i [1 - D_i (\phi; y)]}_{\text{coupon}} + \underbrace{\zeta [1 - D_i (\phi; y)] + (1 - \phi) \delta_L D'_i (\phi; y)}_{\text{maturity change}} + \underbrace{(1 - \phi) \delta_D D_i (\phi; y)}_{\text{maturity}}
  \]
- At bankruptcy boundary, $D_i (\Phi (y); y) = B (y)$ for $i \in \{S, L\}$

**Bond valuation wedge** $\Delta \equiv D_S - D_L$
- At bankruptcy zero wedge $\Delta (\Phi (y); y) = B (y) - B (y) = 0$
- Away from bankruptcy, positive wedge $\Delta (\phi; y) > 0$ for $\phi < \Phi (y)$:
  - Short-term bonds more valuable as they mature with higher likelihood before default (i.e., $\phi_t$ hits $\Phi$)
Baseline: Optimal Issuance Strategies

- Rewrite **Equity** with bond valuation wedge $\Delta$ so that

\[
\text{req. return} \quad rE(\phi; y) = y - c + \zeta \left[ E^{rf} - E(\phi; y) \right]
\]

\[
+ \max_{f \in [0,1]} \left\{ m(\phi) [ DL(\phi; y) + f \Delta(\phi; y) - 1] + [-\phi \delta s + m(\phi) f] E'(\phi; y) \right\}
\]

- Optimal issuance policy $f$ given by IC condition $\Delta + E'$:

\[
f^* = \begin{cases} 
1 & \text{if } \Delta(\phi; y) + E'(\phi; y) > 0 \\
0 & \text{if } \Delta(\phi; y) + E'(\phi; y) < 0 \\
[0, 1] & \text{if } \Delta(\phi; y) + E'(\phi; y) = 0
\end{cases}
\]

(1)

- **Tradeoff:** Issuing more short-term bonds today (higher $f$)
  - lowers today’s rollover losses as $\Delta(\phi; y) > 0$, but
  - shortens maturity structure (higher $\phi$), which hurts continuation value, $E'(\phi; y) < 0$, as default occurs at some upper threshold $\Phi$
**Proposition:** At default boundary, equity holders always want lengthening

**Rough intuition:**
- Suppose we are $2dt$ before default
- Fixing default policy $\Rightarrow$ fixing bond prices. Savings of rollover losses today at time 0 exactly offsets greater rollover losses tomorrow at time $dt$
  - shortening just spreads rollover loss over two periods 0 and $dt$, but does not affect total rollover losses
  - the opening numerical example has this flavor
- Shortening worsens endogenous default, hurting bond price

**Net effect on equity’s IC equals the impact of shortening on bond price,** which is negative in baseline case
What if a firm faces declining profitability while maintaining the same upside potential $X$ with intensity $\zeta$?

Consider a decreasing $y$ with $\mu_y (y) < 0$ so that

\[ dy_t = \mu_y (y) \, dt < 0 \]

Default occurs on all paths for sufficiently low $\mu_y (y)$

2 state variables $(\phi, y)$

Introduce change-of-variables $(\phi, y) \iff (\tau, y_0)$ where $\tau$ is time-to-default and $y_{\tau=0}$ is CF at default
Can we have shortening equilibrium (S equilibrium), i.e. $f_\tau = 1, \forall \tau$, where default is inefficiently early?

Default at $\tau = 0$, defaulting cash-flow $y_0$ and corresponding maturity structure $\Phi(y_0)$
  - Equity default boundary still the same with $\Phi'(y_0) > 0$

Two-dimensional state space matters, as equity’s IC condition now involves partial derivative w.r.t. $\phi$ but not $y$

$$f_\tau = 1 \text{ requires } \Delta(\phi_\tau, y_\tau) + E_\phi(\phi_\tau, y_\tau) > 0$$

- When perturbing $\phi$, $y_\tau$ remains the same but the defaulting cashflow $y_0$ is affected!
- This effect is absent in the baseline case with constant $y$
Full Model: Shortening Equilibrium

- Baseline: $f = 1$ required $\Delta + E' > 0 \iff D'_S(\Phi; y) > 0$
- Now, $f = 1$ requires $\Delta + E_{\phi} > 0$, which is equivalent to

$$\frac{\partial}{\partial \phi} D_S(\Phi(y_0), y_0) = \left( \frac{\partial D_S(\tau, y_0)}{\partial \tau} \frac{\partial \tau}{\partial \phi} + \frac{\partial D_S(\tau, y_0)}{\partial y_0} \frac{\partial y_0}{\partial \phi} \right) > 0$$

(-), shortening hastens default  (+), higher recovery value
Full Model: Default Boundary

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths
Full Model: A Shortening Equilibrium

\[ \mu_y(y) = -\mu < 0: \text{negative drift leads to default on all paths} \]
Full Model: A Lengthening Equilibrium

- $\mu y (y) = -\mu < 0$: negative drift leads to default on all paths
Full Model: Pure Equilibria

- $\mu y(y) = -\mu < 0$: negative drift leads to default on all paths
Everyone might be worse off in shortening equilibrium $T_b^S$ versus the lengthening equilibrium $T_b^L$.

<table>
<thead>
<tr>
<th></th>
<th>Initial (0.99, 0)</th>
<th>Lengthening</th>
<th>Shortening</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_b$</td>
<td></td>
<td>1.55</td>
<td>0.43</td>
</tr>
<tr>
<td>firm value $V(T_b)$</td>
<td></td>
<td>3.55</td>
<td>2.17</td>
</tr>
<tr>
<td>equity $E(\phi, y)$</td>
<td></td>
<td>2.55</td>
<td>1.17</td>
</tr>
<tr>
<td>short-term debt $D_S(\phi, y)$</td>
<td></td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>long-term debt $D_L(\phi, y)$</td>
<td></td>
<td>0.89</td>
<td>0.82</td>
</tr>
</tbody>
</table>
Full Model: An Interior Equilibrium

\[ \mu_y(y) = -\mu < 0: \text{negative drift leads to default on all paths} \]
$\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths
Full Model: All Types of Equilibria

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths
Full Model: Empirical predictions

1. More likely to observe shortening in response to deteriorating economic conditions
   ▶ Consistent with empirical evidence of shortening before crisis while lengthening in good time
2. Conditional on deteriorating economic conditions, more likely in firms with *already* short maturity structures
Conclusion

- Can standard equity/debt agency conflicts lead to endogenous maturity shortening in a dynamic setting?

- Although issuing short-term debt reduces *today’s rollover losses*, equity holders also care about their *continuation value* as rollover stronger in future. This is a strong force against shortening
  
  - **Baseline** (fixed recovery value): Trade-off never in favor of short-term debt; possibly multiple equilibria but never shortening to death
  - **Sinking ship** (variable recovery value): When equity holders locally prolong the length of the firm (recovery value changing), debt-holders may not appropriate enough of those benefits to acquiesce

- What we do not have: investors side mechanism........
  
  - Diamond-Dybvig (1983), He-Milbradt (2014): short-term debt is better if bond investors have liquidity shocks