DYNAMIC DEBT MATURITY

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Motivation

- Debt maturity and its associated rollover risk is at the center of post-crisis policy discussion
  - Krishnamurthy 2010: Financial firms shorten their debt maturity right before crisis
  - Xu 2014: Speculative firms lengthen debt maturity in good time
- We offer a model to analyze endogenous debt maturity (with predictions consistent with empirical pattern)
  - In a dynamic setting, in what scenarios might keep going short with earlier default be observed?
  - **no commitment by firm to its debt maturity structure**

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- Debt maturity and its associated rollover risk is at the center of post-crisis policy discussion
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- We offer a model to analyze endogenous debt maturity (with predictions consistent with empirical pattern)
  - In a dynamic setting, in what scenarios might keep going short with earlier default be observed?
  - **no commitment by firm to its debt maturity structure**
- **Baseline:** constant current cash-flows, waiting for upside event; equity tends to default inefficiently early
  - Trade-off never in favor of short-term debt; possibly multiple equilibria but never shortening to death
- **Sinking ship:** waiting for upside event but deteriorating cash-flows
  - Shortening ensues: debt holders' recovery value declines when defaulting later

An Illustrating Example of Trade-Off

- Using short-term debt today helps reduce rollover losses now, but needs to be refinanced sooner
- Two period $t = 0, 1, 2$, no discounting
  - At the end of each date, exogenous default probability $q = 0.1$
- At $t = 0$, the firm needs to repay 1 dollar of matured debt
  - either issuing one-period short-term debt price $D_S = 1 - q = 0.9$
  - or two-period long-term debt price $D_L = (1 - q)^2 = 0.81$
  - Equity holders cover the "rollover loss" $D_i - 1$
- Trade-off:
  - Issuing short-term: less rollover loss -0.1 at today $t = 0$, but tomorrow $t = 1$ facing rollover loss again
  - Issuing long-term: greater rollover loss -0.19 at today $t = 0$, but tomorrow $t = 1$ free of rollover concern
- Main model: link rollover losses to
  1. endogenous default decision q which in turn affects debt prices D_i
  2. which in turn will affect endogenous issuance decision (S vs L)

Preview & Roadmap

Key trade-off for equity:

- Issue more short-term debt today at a higher price compared to long-term debt... **BUT**
- As maturity structure shorter, face stronger rollover tomorrow and lower equity value

Question:

- Can this trade-off every lead to *shortening to death* by lacking of commitment?
- Welfare implications?

Cash-flows & Unlevered Asset Value

Assets under management:

- Instantaneous cash-flow of y_t
- Upside event with payoff $X > 1$ occurs with intensity ζ
- Free option to abandon project, e.g., when cash-flows y_t are too low

First-best asset value:

- Current management is best suited to run the firm
- Total first-best unlevered asset value given by

$$A(y) = \mathbb{E}\left[\int_0^{T_A} e^{-(r+\zeta)t}\left(y_t + \zeta X\right) dt\right]$$

where T_A is the first-best abandonment time

Recovery value:

- If management gets replaced, new management generates unlevered asset value (with new abandonment time T_B) of

$$B(y) = \mathbb{E}\left[\int_0^{T_B} e^{-(r+\zeta)t}\left(\alpha_y y_t + \zeta \alpha_X X\right) dt\right]$$

- Assume α_y, α_X such that $B(y) < A(y)$

Capital Structure

Two kinds of (random maturity) bonds:

- Long-term bonds with maturity intensity δ_L
- Short-term bonds with maturity intensity $\delta_S > \delta_L$
- Expected maturities $1/\delta_L$ and $1/\delta_S$ respectively
- Coupon rate $c = r$ so i) without default debt value is 1; ii) with default, debt value below 1 and hence rollover loss

Outstanding face-value normalized to 1, following Leland settings to rule out outright *dilution*

- **Assumption:** Firm commits to constant face-value of debt, but cannot commit to future debt maturity
- In practice we observe more debt covenants on (book) leverage rather than debt maturity
- Differ from Brunnermeier Oehmke 2013: they do not have commitment on outstanding face value

Debt Maturity Structure

Debt maturity structure

- ϕ_t: fraction of short-term bonds *outstanding*
- Every instant $m(\phi_t)\,dt$ dollars of bonds mature:

$$m(\phi_t) = \phi_t \delta_S + (1 - \phi_t)\,\delta_L$$

  - $m'(\phi_t) > 0$: the shorter the maturity structure, the faster the rollover

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Issuance policy. $f_t \in [0, 1]$: the fraction of short debt in *new issue*
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Maturity structure dynamics

$$\frac{d\phi_t}{dt} = \underbrace{-\phi_t \delta_S}_{\textbf{Short-term maturing}} + \underbrace{m(\phi_t)\,f_t}_{\textbf{Newly issued short-term}}$$

- if $f_t = 1$, keep issuing short-term debt, $\phi_t \uparrow$

Baseline: Equity and Endogenous Default

Benchmark: constant cash-flow $y_t = y$. Equity value

$$
\overbrace{rE(\phi; y)}^{\text{req. return}} = y - c + \overbrace{\zeta\left[(X-1) - E(\phi; y)\right]}^{\text{upside event}}
$$

$$
+ \max_{f \in [0,1]} \left\{ \underbrace{m(\phi)\left[fD_S(\phi; y) + (1-f)D_L(\phi; y) - 1\right]}_{\text{rollover losses}} + \underbrace{\left[-\phi\delta_S + m(\phi) f\right] E'(\phi; y)}_{\text{impact of shortening}} \right\}
$$

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Endogenous default boundary $\Phi(y)$

- Equity defaults optimally when net cash-flows are zero

$$y - c + \zeta E^{rf} + \max_{f\in[0,1]} m(\phi)\left[fD_S(\phi;y) + (1-f)D_L(\phi;y) - 1\right] = 0$$

- Irrespective of f at default, $D_S = D_L = B(y)$ imply

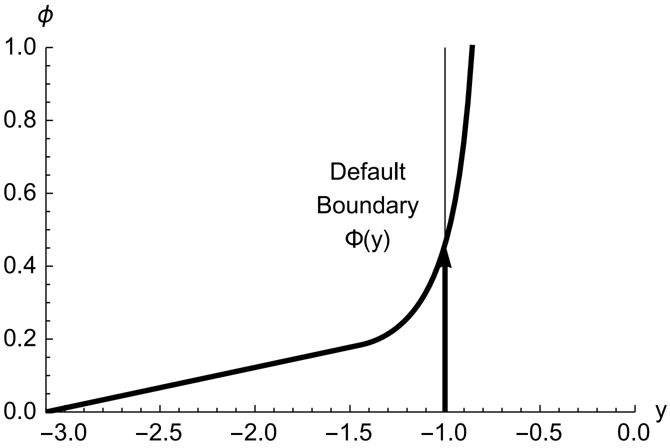
$$\Phi(y) = \frac{1}{\delta_S - \delta_L}\left[\frac{y - c + \zeta E^{rf}}{1 - B(y)} - \delta_L\right]$$

- $\Phi'(y) > 0$: **shorter maturity structure hastens default (default threshold cash-flow y is higher)**

Baseline: Shortening to Death?

Q: Is there ever "shortening-to-death" equilibrium default while equity keeps shortening maturity?

- Always shortening implies $f_t = 1$, so $\frac{d\phi_t}{dt} > 0$, while y is constant



Baseline: Bond Valuation Wedge Δ

Bond valuation:

- Value of debt $i \in \{S, L\}$ solves

$$\overbrace{rD_i(\phi; y)}^{\text{req. return}} = \overbrace{c}^{\text{coupon}} + \overbrace{\delta_i \left[1 - D_i(\phi; y)\right]}^{\text{maturity}} + \underbrace{\zeta \left[1 - D_i(\phi; y)\right]}_{\text{upside event}} + \underbrace{(1-\phi)\,\delta_L D_i'(\phi; y)}_{\text{maturity change}}$$

- At bankruptcy boundary, $D_i(\Phi(y); y) = B(y)$ for $i \in \{S, L\}$

Bond valuation wedge $\Delta \equiv D_S - D_L$

- At bankruptcy zero wedge $\Delta(\Phi(y); y) = B(y) - B(y) = 0$
- Away from bankruptcy, positive wedge $\Delta(\phi; y) > 0$ for $\phi < \Phi(y)$:
  - Short-term bonds more valuable as they mature with higher likelihood before default (i.e.,ϕ_t hits Φ)

Baseline: Optimal Issuance Strategies

- Rewrite **Equity** with bond valuation wedge Δ so that

$$\overbrace{rE(\phi; y)}^{\text{req. return}} = y - c + \overbrace{\zeta\left[E^{rf} - E(\phi; y)\right]}^{\text{upside event}}$$

$$+ \max_{f\in[0,1]}\left\{\underbrace{m(\phi)\left[D_L(\phi; y) + f\Delta(\phi; y) - 1\right]}_{\text{rollover losses}} + \underbrace{\left[-\phi\delta_S + m(\phi) f\right] E'(\phi; y)}_{\text{impact of shortening}}\right\}$$

- Optimal issuance policy f given by IC condition $\Delta + E'$:

$$f^* = \begin{cases} 1 & \text{if } \Delta(\phi; y) + E'(\phi; y) > 0 \\ 0 & \text{if } \Delta(\phi; y) + E'(\phi; y) < 0 \\ [0,1] & \text{if } \Delta(\phi; y) + E'(\phi; y) = 0 \end{cases} \tag{1}$$

- **Tradeoff:** Issuing more short-term bonds today (higher f)
  - **lowers today's rollover losses** as $\Delta(\phi; y) > 0$, but
  - shortens maturity structure (higher ϕ), which **hurts continuation value**, $E'(\phi; y) < 0$, as default occurs at some upper threshold Φ

Baseline: Never Shortening-to-Death!

Proposition: At default boundary, equity holders always want lengthening

Rough intuition:

- Suppose we are $2dt$ before default
- Fixing default policy⇒fixing bond prices. Savings of rollover losses today at time 0 exactly offsets greater rollover losses tomorrow at time dt
  - shortening just spreads rollover loss over two periods 0 and dt, but does not affect total rollover losses
  - the opening numerical example has this flavor
- Shortening worsens endogenous default, hurting bond price

Net effect on equity's IC equals the impact of shortening on bond price, which is negative in baseline case

Full Model: A Sinking Ship

- What if a firm faces declining profitability while maintaing the same upside potential X with intensity ζ?
- Consider a decreasing y with $\mu_y(y) < 0$ so that

$$dy_t = \mu_y(y)\,dt < 0$$

- Default occurs on all paths for sufficiently low $\mu_y(y)$
- 2 state variables (ϕ, y)
  - Introduce change-of-variables $(\phi, y) \iff (\tau, y_0)$ where τ is time-to-default and $y_{\tau=0}$ is CF at default

Full model: Default boundary

- Can we have shortening equilibrium (S equilibrium), i.e. $f_\tau = 1, \forall \tau$, where default is inefficiently early?
- Default at $\tau = 0$, defaulting cash-flow y_0 and corresponding maturity structure $\Phi(y_0)$
  - Equity default boundary still the same with $\Phi'(y_0) > 0$
- **Two-dimensional state space matters,** as equity's *IC* condition now involves partial derivative w.r.t. ϕ but not y

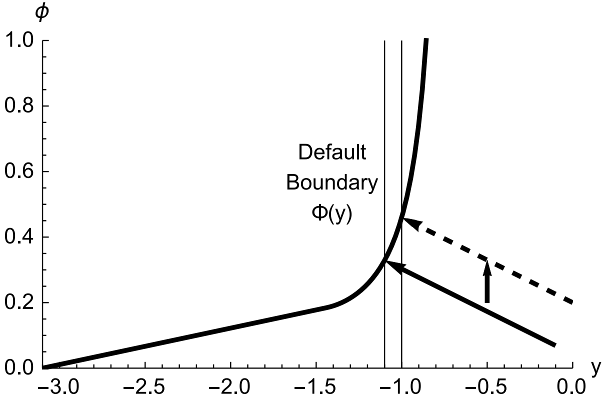
$$f_\tau = 1 \text{ requires } \Delta(\phi_\tau, y_\tau) + E_\phi(\phi_\tau, y_\tau) > 0$$

  - When purturbing ϕ, y_τ remains the same but the defaulting cashflow y_0 is affected!
  - This effect is absent in the baseline case with constant y

Full Model: Shortening Equilibrium

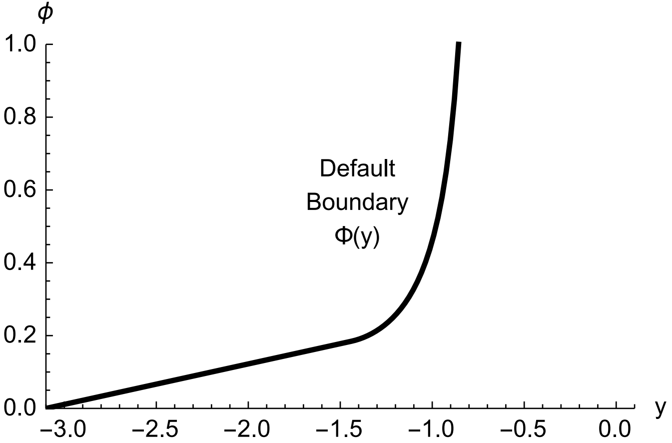
- Baseline: $f = 1$ required $\Delta + E' > 0 \iff D_S' (\Phi; y) > 0$ ↯
- Now, $f = 1$ requires $\Delta + E_\phi > 0$, which is equivalent to

$$\frac{\partial}{\partial \phi} D_S (\Phi(y_0), y_0) = \underbrace{\frac{\partial D_S(\tau, y_0)}{\partial \tau} \frac{\partial \tau}{\partial \phi}}_{(-),\text{ shortening hastens default}} + \underbrace{\frac{\partial D_S(\tau, y_0)}{\partial y_0} \frac{\partial y_0}{\partial \phi}}_{(+),\text{ higher recovery value}} > 0$$



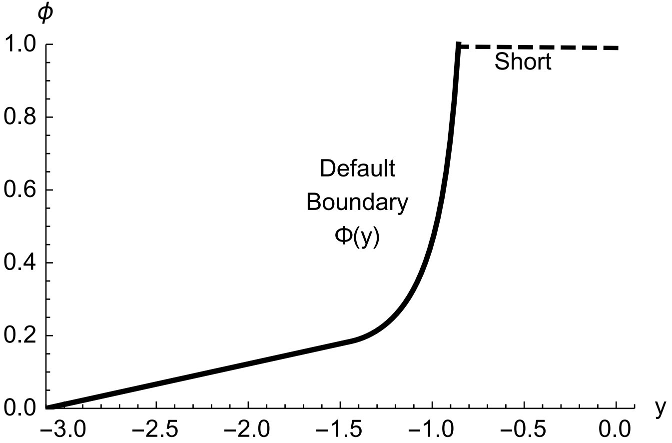
Full Model: Default Boundary

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths



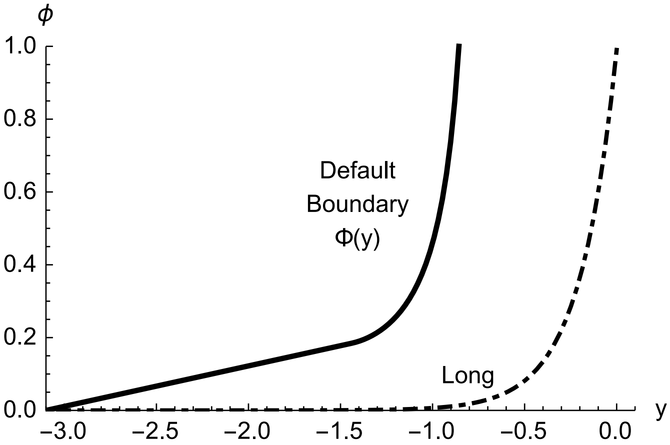
Full Model: A Shortening Equilibrium

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths



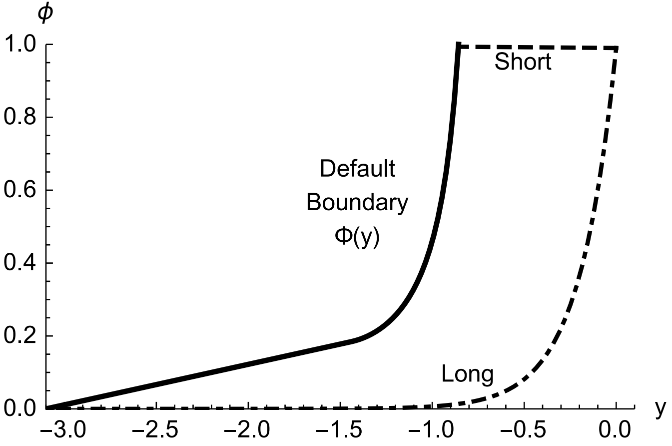
Full Model: A Lengthening Equilibrium

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths



Full Model: Pure Equilibria

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths



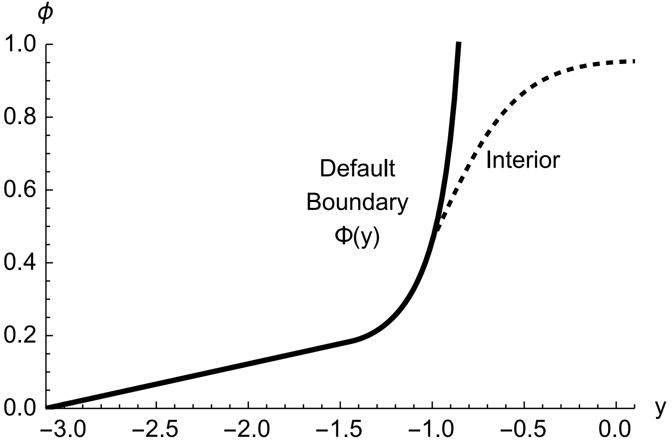
Full Model: Welfare

- Everyone might be worse off in shortening equilibrium T_b^S versus the lengthening equilibrium T_b^L

Initial $(.99, 0)$	Lengthening	Shortening
T_b	1.55	0.43
firm value $V(T_b)$	3.55	2.17
equity $E(\phi, y)$	2.55	1.17
short-term debt $D_S(\phi, y)$	0.99	0.99
long-term debt $D_L(\phi, y)$	0.89	0.82

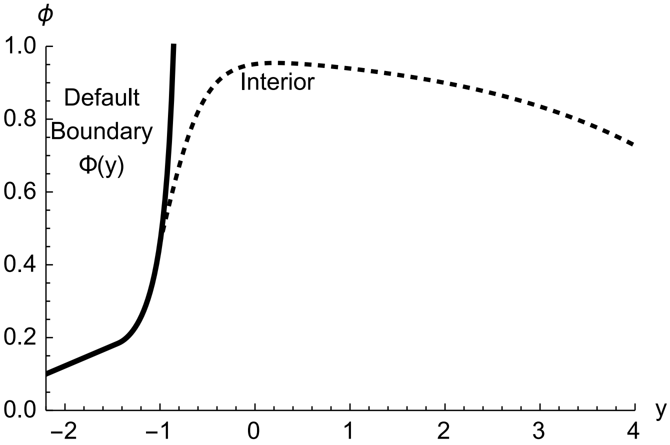
Full Model: An Interior Equilibrium

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths



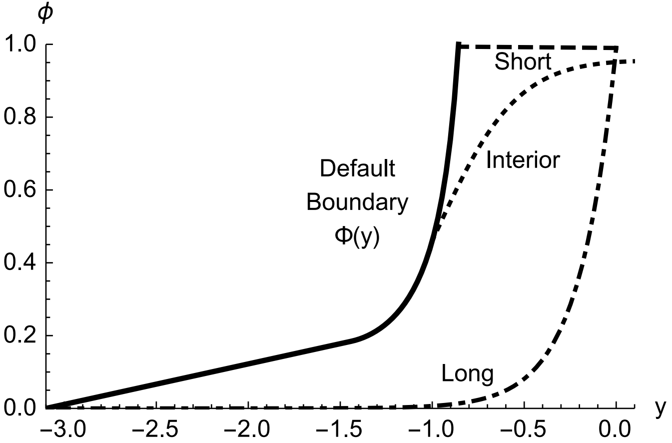
Full Model: An Interior Equilibrium (zoomed out)

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths



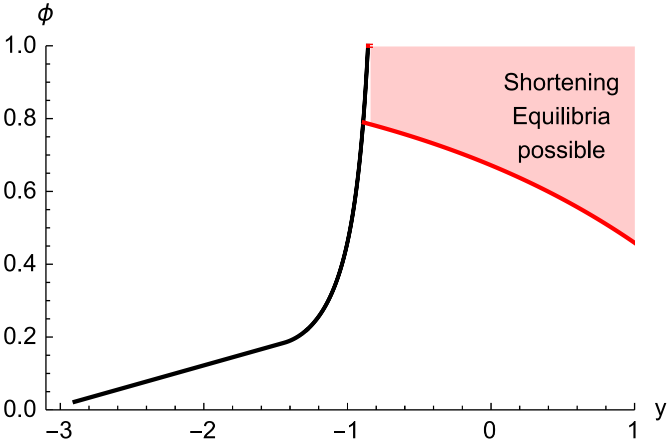
Full Model: All Types of Equilibria

- $\mu_y(y) = -\mu < 0$: negative drift leads to default on all paths



Full Model: Empirical predictions

1. More likely to observe shortening in response to deteriorating economic conditions
   - Consistent with empirical evidence of shortening before crisis while lengthening in good time
2. Conditional on deteriorating economic conditions, more likely in firms with *already* short maturity structures



Conclusion

- Can standard equity/debt agency conflicts lead to endogenous maturity shortening in a dynamic setting?
- Although issuing short-term debt reduces *today's rollover losses*, equity holders also care about their *continuation value* as rollover stronger in future. This is a strong force against shortening
  - **Baseline** (fixed recovery value)**:** Trade-off never in favor of short-term debt; possibly multiple equilibria but never shortening to death
  - **Sinking ship** (variable recovery value)**:** When equity holders locally prolong the length of the firm (recovery value changing), debt-holders may not appropriate enough of those benefits to acquiesce
- What we do not have: investors side mechanism.......
  - Diamond-Dybvig (1983), He-Milbradt (2014): short-term debt is better if bond investors have liquidity shocks