A Macroeconomic Framework for Quantifying Systemic Risk

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Financial Crisis in the Model

Note: Capital constraint binds for $e < 0.435$
Outline of Presentation

1. Nonlinear macro model of a financial crisis
   - Recent work on financial intermediaries: He-Krishnamurthy, Brunnermeier-Sannikov, Rampini-Viswanathan, Adrian-Boyarchenko, Gertler-Kiyotaki
   - Our approach: occasionally binding constraint; global solution method (similar to Brunnermeier-Sannikov, Adrian-Boyarchenko)
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   ▶ Nonlinearity in model and data
   ▶ Match conditional moments of the data, conditioning on negative (i.e., recession) states
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2. Calibration and Data
   - Nonlinearity in model and data
   - Match conditional moments of the data, conditioning on negative (i.e., recession) states

3. Quantify systemic risk
   - Systemic risk: the state where financial intermediation is widely disrupted to affect real activities severely
     - In the model, states where capital constraint binds, crisis state
   - What is the ex-ante (e.g., initial conditions of 2007Q2) likelihood of crisis states? (... low)
   - What makes the probability higher?
   - Economics of stress tests (as opposed to accounting of stress tests)
Model

- Two classes of agents: households and bankers
  - Households:
    \[ \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \frac{1}{1 - \gamma} C_t^{1-\gamma} \, dt \right], \quad C_t = (c_t^y)^{1-\phi} (c_t^h)^{\phi} \]

- Two types of capital: productive capital \( K_t \) and housing capital \( H \).
  - Fixed supply of housing \( H \equiv 1 \)
  - Price of capital \( q_t \) and price of housing \( P_t \) determined in equilibrium
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- Production $Y = AK_t$, with $A$ being constant

- Fundamental shocks: stochastic capital quality shock $dZ_t$. TFP shocks
  \[
  \frac{dK_t}{K_t} = i_t \, dt - \delta dt + \sigma dZ_t
  \]
Model

- Two classes of agents: households and bankers
  - Households:
    
    $E \left[ \int_0^\infty e^{-\rho t} \frac{1}{1-\gamma} C_t^{1-\gamma} dt \right]$, \hspace{1cm} $C_t = (c_t^y)^{1-\phi} \left( c_t^h \right)^\phi$

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  \[
  \frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t
  \]

- Investment/Capital $i_t$, quadratic adjustment cost

  \[
  \Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t
  \]

  \[
  \max_{i_t} q_t i_t K_t - \Phi(i_t, K_t) \Rightarrow i_t = \delta + \frac{q_t - 1}{\kappa}
  \]
Aggregate Balance Sheet

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$
Equity $E_t$

Housing $P_t H$
Debt $W_t - E_t$

Household Sector

Financial Wealth
$W_t = q_t K_t + P_t H$
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$W_t = q_t K_t + P_t H$

$(1 - \lambda) W_t$

$\lambda W_t = "Liquid balances"$

benchmark capital structure
Equity Matters

Loans to Capital Producers $i_t$

Intermediary Sector

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Separation of ownership and control

Banker maximizes $E[ROE] - \frac{m}{2} \text{Var}[ROE]$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$

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$\lambda W_t = "\text{Liquid balances}\"$

benchmark capital structure
Equity Dynamics in GE

Loans to Capital Producers $i_t$

Intermediary Sector

- Capital $q_tK_t$
- Housing $P_tH$

-10%

Demands: $W_t = q_tK_t + P_tH$

Equity $E_t$

Debt $W_t - E_t$

Banker maximizes $E[ROE] - \frac{m}{2} \text{Var}[ROE]$

Household Sector

Financial Wealth

(1 - $\lambda$) $W_t$

$\lambda W_t = "Liquid balances"

Equity Constraint

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_tK_t$

Equity $E_t$

Constraint: $E_t \leq \mathcal{E}_t$

Debt $W_t - E_t$

Banker maximizes $E[ROE] - \frac{m}{2} \text{Var}[ROE]$

Household Sector

Financial Wealth

$W_t = q_tK_t + P_tH$

$(1 - \lambda)W_t$

$\lambda W_t = "\text{Liquid balances}"$

Aggregate intermediary equity constraint $\mathcal{E}_t$

$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m \times \text{ROE}, \text{ROE is endogenous}$
Equity constraint: $\epsilon_t$

- Bank can raise equity upto $\epsilon_t$ at zero cost
- Cost of raising equity more than $\epsilon_t$ is infinite.
- $\epsilon_t$ linked to intermediary performance (constant $m$)

$$\frac{d\epsilon_t}{\epsilon_t} = md\tilde{R}_t.$$
Equity constraint: $\epsilon_t$

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$$\frac{d\epsilon_t}{\epsilon_t} = md\tilde{R}_t.$$ 

- Poor returns reduce “reputation”: Berk-Green, 04; flow-performance relationship, Warther 95; Chevalier-Ellison, 97
- Or, $\epsilon_t$ as banker’s “net worth” fluctuating with past returns
  - Kiyotaki-Moore 97, He-Krishnamurthy 12, Brunnermeier-Sannikov 12
Equity constraint: \( \epsilon_t \)

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\[
\frac{d\epsilon_t}{\epsilon_t} = md\tilde{R}_t.
\]

- Poor returns reduce “reputation”: Berk-Green, 04; flow-performance relationship, Warther 95; Chevalier-Ellison, 97
- Or, \( \epsilon_t \) as banker’s “net worth” fluctuating with past returns
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- Aggregate dynamics of \( \mathcal{E}_t = \int \epsilon_t \)

\[
\frac{d\mathcal{E}_t}{\mathcal{E}_t} = md\tilde{R}_t - \eta dt + d\psi_t
\]

- Exogenous death rate \( \eta \). Endogenous entry \( d\psi_t > 0 \) of new bankers in extreme bad states
Equity Capital Constraint

- Representative household with $W_t$, split between bonds (at least) $\lambda W_t$ and equity (at most) $(1 - \lambda) W_t$

- Benchmark capital structure: $\lambda W_t$ of Debt, $(1 - \lambda) W_t$ of Equity
  - if there is no capital constraint ($E_t$ is infinite)...
Equity Capital Constraint

- Representative household with $W_t$, split between bonds (at least) $\lambda W_t$ and equity (at most) $(1 - \lambda) W_t$

- Benchmark capital structure: $\lambda W_t$ of Debt, $(1 - \lambda) W_t$ of Equity
  - if there is no capital constraint ($\mathcal{E}_t$ is infinite)...  

- Intermediary equity capital:  
  \[
  E_t = \min [\mathcal{E}_t, (1 - \lambda) W_t]
  \]

- Suppose a $-10\%$ shock to real estate and price of capital:
  - $W_t \downarrow 10\%$ (Household wealth = aggregate wealth)
  - Reputation: $\frac{d\mathcal{E}_t}{\mathcal{E}_t} = md\tilde{R}_t + \ldots$ Two forces make $\mathcal{E}_t \downarrow$ more than $10\%:$
    1. Return on equity $= d\tilde{R}_t < -10\%$: equity is levered claim on assets
    2. $m > 1$ in our calibration
## Calibration: Baseline Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Choice</th>
<th>Targets (Unconditional)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Intermediation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>Performance sensitivity</td>
<td>2</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Debt ratio</td>
<td>0.67</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Banker exit rate</td>
<td>13%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Entry trigger</td>
<td>6.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Entry cost</td>
<td>2.43</td>
</tr>
<tr>
<td><strong>Panel B: Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Capital quality shock</td>
<td>3%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>10%</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Adjustment cost</td>
<td>3</td>
</tr>
<tr>
<td>$A$</td>
<td>Productivity</td>
<td>0.133</td>
</tr>
<tr>
<td><strong>Panel C: Others</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Time discount rate</td>
<td>2%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1/EIS</td>
<td>0.15</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Housing share</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Results(1): State variable is $e_t = \mathcal{E}_t / K_t$

- Capital constraint binds for $e < 0.435$
• Capital constraint binds for \( e < 0.435 \)

• Without the possibility of the capital constraint, all of these lines would be flat. Model dynamics would be i.i.d., with vol=3%
State-dependent Impulse Response: -1% Shock ($= \sigma dZ_t$)

Investment

Sharpe ratio

Land price

He and Krishnamurthy (Chicago, Northwestern) Systemic Risk June 2014 15 / 30
Steady State Distribution
Nonlinearities in Model and Data

Model:

- Distress states = worst 33% of realizations of $e$ ($e < 1.27$)
- Compute *conditional* variances, covariances of intermediary equity growth with other key variables

Data:

- Distress states = worst 33% of realizations of (risk premium in) credit spread
  - We use Gilchrist-Zakrajsek (2011) Excess Bond Premium, which we convert to a Sharpe ratio
  - Excess Bond Premium: risk premium of corporate bonds, presumably reflects distress of financial sector
  - Similar results if using NBER recessions
- Compute *conditional* variances, covariances of intermediary equity growth with other key variables
EBS time series
Matching State-Dependent Covariances

<table>
<thead>
<tr>
<th></th>
<th>Distress</th>
<th>Non Distress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Baseline</td>
</tr>
<tr>
<td>vol ((Eq))</td>
<td>31.48%</td>
<td>34.45%</td>
</tr>
<tr>
<td>vol ((I))</td>
<td>8.05%</td>
<td>5.30%</td>
</tr>
<tr>
<td>vol ((C))</td>
<td>1.71%</td>
<td>3.54%</td>
</tr>
<tr>
<td>vol ((LP))</td>
<td>21.24%</td>
<td>21.04%</td>
</tr>
<tr>
<td>vol ((EB))</td>
<td>60.14%</td>
<td>74.20%</td>
</tr>
<tr>
<td>cov ((Eq, I))</td>
<td>1.31%</td>
<td>1.05%</td>
</tr>
<tr>
<td>cov ((Eq, C))</td>
<td>0.25%</td>
<td>-0.96%</td>
</tr>
<tr>
<td>cov ((Eq, LP))</td>
<td>4.06%</td>
<td>5.87%</td>
</tr>
<tr>
<td>cov ((Eq, EB))</td>
<td>-6.81%</td>
<td>-14.95%</td>
</tr>
</tbody>
</table>

- Note: without the capital constraint, all volatilities would be 3%, and have no state dependence.
- What we do badly on: Output vol is locally \(\sigma\) because \(Y_t = AK_t\). Financial friction only affects split between I and C.
Matching the 2007-2009 Crisis

He and Krishnamurthy (Chicago, Northwestern)
Based on EBS classification, economy crossed the 33% boundary \((e = 1.27)\) between 2007Q2 and 2007Q3. Assume \(e = 1.27\) in 2007Q2.

Then choose \((Z_{t+1} - Z_t)\) shocks to match realized intermediary equity series.

<table>
<thead>
<tr>
<th></th>
<th>07QIII</th>
<th>07QIV</th>
<th>08QI</th>
<th>08QII</th>
<th>08QIII</th>
<th>08QIV</th>
<th>09QI</th>
<th>09QII</th>
<th>09QIII</th>
<th>09QIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Total -15.5%. Capital constraint binds after 07Q4—systemic risk state
- In the model (data), land price falls by 50% (55%)
- In the model (data), investment falls by 23% (25%)
Based on EBS classification, we cross the 33% boundary ($e = 1.27$) between 2007Q2 and 2007Q3

What is the likelihood of the constraint binding (“systemic crisis”) assuming $e = 1.27$ currently:

- 0.32% in next 1 years
- 3.57% in next 2 years
- 17.30% in next 5 years

Small...
Stress testing: “Hidden” Leverage

- Financial sector aggregate leverage fixed at 3 in model
  - We measure across commercial banks, broker/dealers, hedge funds in 2007:
    - Assets = $15,703 billion; Liabilities = $10,545 billion
- Pushed to crisis boundary after a -7% shock. 3.57% prob. of crisis in next 2 years
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    ▶ Assets = $15,703 billion; Liabilities = $10,545 billion

Pushed to crisis boundary after a -7% shock. 3.57% prob. of crisis in next 2 years

Hidden leverage:
  ▶ ABCP (SIVs): $1,189 billion; Liabilities $1,189 billion
  ▶ Repo (MMFs and Sec Lenders): $1,020 billion; Liabilities $1,000 billion
    (assumed 2% haircut)

Hidden in sense that agents take as given price functions and returns at leverage=3
  ▶ 1 year: 6.73%
  ▶ 2 year: 23.45%
  ▶ 5 year: 57.95%
Stress testing

Key step: Need to map from stress scenario into underlying shock, $dZ_t$.

- Say stress scenario $\Rightarrow -30\%$ Return on equity
- Naive partial eqbm: leverage of 3, $\sigma(Z_{t+0.25} - Z_t) = -30/3 = -10\%$.
- Feed in $-10\%$ shock into the model over one quarter.
- Result: Beginning at $e = 1.27$ in 2007Q2, economy is immediately moved into crisis region, $e < 0.435$
- our model helps in figuring out the right shock $dZ_t$

In US stress tests, scenario was over 6 quarters. Feed in shocks quarter-by-quarter, over 6 quarters:

<table>
<thead>
<tr>
<th>Return on Equity</th>
<th>6 QTR Shocks</th>
<th>Prob(Crisis within next 2 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2%</td>
<td>-1.16%</td>
<td>5.25 %</td>
</tr>
<tr>
<td>-5</td>
<td>-2.53%</td>
<td>8.90</td>
</tr>
<tr>
<td>-10</td>
<td>-4.69%</td>
<td>22.88</td>
</tr>
<tr>
<td>-15</td>
<td>-6.71%</td>
<td>48.90</td>
</tr>
<tr>
<td>-30</td>
<td>-8.72%</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Map “stress test” into a shock to $e$. 

Probability of being distressed: hitting $e_{\text{distress}} = 1.27$

Probability of capital constraint being binding: hitting $e_{\text{crisis}} = 0.4354$
Other crises

Panel A: Savings and Loan Crisis

Panel B: 1998 Hedge Fund Crisis

Panel C: 2002 Corporate Bond Market Crisis

Equity-Data
Inv-Data
Spread-Data
Inv-Model
Sharpe-Model
Prob2-Model (Right axis)
We develop a fully stochastic model of a systemic crisis, with an equity capital constraint on the intermediary sector.

The model quantitatively matches the differential comovements in distress and non-distress periods.

It is able to replicate 2007/2008 period with only intermediary capital shocks.

It offers a way of mapping macro-stress tests into probability of systemic states.
Equity series

![Equity Chart](chart.png)
Panel A: Distress Periods

Panel B: Non Distress Periods