Uncertainty and Incentives

Canice Prendergast, University of Chicago and National Bureau of Economic Research

Empirical work testing for a trade-off between risk and incentives has had, at best, mixed success. This article provides two simple reasons, associated with subjectivity of performance appraisals, why we might not expect to see any negative relationship. Both reasons relate to empirically observed problems associated with monitoring: (i) supervisors sometimes bias their evaluations based on their personal feelings toward their subordinates, and (ii) supervisors will sometimes offer evaluations that reduce their costs. These aspects of monitoring are ignored in the standard model and can reverse the usual negative trade-off between risk and incentives.

I. Introduction

The premise of agency theory is not simply that individuals respond to incentives but that contracts reflect the costs and benefits of inducing appropriate behavior. Foremost among issues addressed by the economics literature is the trade-off of risk and incentives where the cost to firms of basing pay on noisy performance measures is that it imposes risk on the compensation of employees, which will be reflected in higher levels of compensation. The risk imposed is increasing in the uncertainty of the environment, so incentive pay will be muted in uncertain environments as compared with situations where there is little uncertainty. This trade-off of risk and uncertainty has occupied center stage in the literature on

Thanks to Jim Baron, Wouter Dessein, Luis Garicano, Bob Gibbons, Tom Hubbard, Tano Santos, Michael Raith, and participants at the University of Chicago and the American Compensation Association conference, for helpful comments. This work was supported by the National Science Foundation. Any errors are my own.

© 2002 by The University of Chicago. All rights reserved.
0734-306X/2002/2002-0005$10.00
compensation since such early contributions as Holmstrom (1979). However, despite the prominent position of this trade-off in the agency literature, empirical research has not shown a convincing relationship between pay-for-performance and observed measures of uncertainty.

In a recent survey (Prendergast 1999), I examined the empirical evidence on this relationship at some length. Evidence on the trade-off between risk and incentives comes from four major occupations. First, the most commonly cited literature is on executives, where there is mixed evidence on the trade-off: some authors find evidence in favor of it, and others find none. There is also, at best, weak evidence that firms use relative performance evaluation for executives, an auxiliary implication of the theory. Second, among sharecropping farmers, the data suggest a positive relationship between observed measures of uncertainty and incentives. Similarly, there is consistent evidence of a positive relationship between risk and incentives in the franchising literature. Finally, the literature on the compensation of salesforce workers finds little evidence of any relationship between observed measures of uncertainty and incentives.

I find these observations somewhat disturbing: the trade-off of risk and incentives is the workhorse of agency theory, yet its empirical underpinnings are hardly overwhelming. Given the simple and persuasive argument for the negative trade-off, this article is largely concerned with understanding why we might expect to see little evidence in the data by showing effects which lead to more pay-for-performance as the environment becomes more uncertain. Specifically, I argue that the existing theories are missing some simple reasons why we would expect to see a positive relationship between uncertainty and incentives. For each of these, I do so in the context of risk neutral agents to ignore the standard trade-off.

I provide two reasons why incentive provision becomes more desirable in risky environments. Both reasons relate to the fact that workers are typically evaluated in a subjective fashion by supervisors. The economics literature has pointed to two problems that can arise with subjectivity, which occurs whenever supervisors have discretion over the rewards of

1 See, e.g., Antle and Smith (1986); Lambert and Larcker (1987); Gibbons and Murphy (1990); Janakiraman, Lambert, and Larcker (1992); Garen (1994); Bushman, Indeikian, and Smith (1996); Ittner, Larcker, and Rajan (1997); Aggarwal and Samwick (1999b); and Core and Guay (1999).
2 See Antle and Smith (1986); Barro and Barro (1990); Gibbons and Murphy (1990); Janakiraman, Lambert, and Larcker (1992); Aggarwal and Samwick (1999a).
4 See Lafontaine and Slade (2001) for appropriate references.
5 See Anderson and Schmittlein (1984); Coughlin and Narasimhan (1992); and John and Weitz (1989).
Uncertainty and Incentives

their subordinates, by paying based on factors other than the worker's performance. First, there is evidence of favoritism in ratings, where personal preferences toward workers affect ratings: workers who are liked do better for reasons that have little to do with performance. I show that the exercise of favoritism affects contracts such that uncertainty and incentives are likely to be positively correlated. Second, supervisors often wish to control their costs, which induces incentives to offer poor evaluations to workers to save on labor costs. Similarly, they have incentives often to carefully monitor performance only when performance is believed to be poor. I show that allowing for these realistic extensions to the standard agency model generates effects where uncertainty and incentives are positively rather than negatively related.

First, consider the effect of favoritism by supervisors on the provision of incentives. A common assumption in the economics literature on agency is that the only reason for doing performance evaluations is to reward the agent for high performance or to penalize him for perceived infractions. Yet the human resources management literature, typically carried out by organizational psychologists, has an enormous amount of information on the variety of reasons for carrying out performance appraisals. The desire to tie merit pay to the evaluations usually ranks quite low on the list of such reasons. As discussed in some of the principal textbooks on human resources management, such as Milkovich, Newman, and Milkvovich (1999), Mondy, Noe, and Premaux (1999), and Sherman, Bohlander, and Snell (1999), a more common reason for doing evaluations is to provide feedback to employees on their perceived strengths and weaknesses and to identify talent within the firm. For example, employees often are unaware of whether their actions are satisfactory; an important purpose of such evaluations is to tell them what areas they can improve in, what they need to do to be promoted, and so on.6

These other reasons for appraising the performance of workers would have little effect on the implications of agency theory without interactions among the purposes of these evaluations. However, I show that there are important interactions in the presence of supervisor favoritism because supervisor distortions in performance evaluations are more extreme where money is on the line than when evaluations have no effect on worker pay. Put simply, if a supervisor's evaluations affect the subordinate's pay, he is less likely to tell the truth.7 Thus, providing incentives causes less

6 See Cleveland, Murphy, and Williams (1989) on the importance of feedback and selection in performance evaluations.

7 For example, consider the extreme case where a bad evaluation results in a worker being fired. If the supervisor has a personal relationship with his subordinate, as seems plausible, it is unlikely that he will give a bad evaluation to the worker as compared with a case where there is little cost to the employee. Similarly, supervisors admit to lobbying more intensively for their own employees
truth telling by supervisors. In the presence of these effects, I show that
the optimal incentive contract is affected by these other reasons for per-
formance appraisal in such a way that there is a positive relationship
between observed measures of risk and incentives.

These facts, which were also considered in Prendergast and Topel
(1996), offer an alternative calculus when designing pay for performance,
namely, the trade-off between sorting and incentives. Unlike the standard
agency model, I assume that a firm uses supervisor evaluations both to
help assign workers to tasks based on their talents and to induce effort
by the workers. Those whose ratings are good will be assigned to different
positions than those with poor reported performance, where the quality
of the supervisor's recommendation improves the efficiency of the allo-
cation decision. Whereas in the usual agency setting, incentives are traded
off against risk imposed on workers, here the problem is that sorting gets
worse when workers are offered more incentives; thus, the trade-off be-
tween sorting and incentives. Consider the use of pay-for-performance
when a firm wishes (i) to provide incentives and (ii) to identify the talents
of workers. Supervisors can distort evaluations to reward favored em-
ployees and harm disfavored ones. Such distortions come at a personal
cost to the supervisor, as the supervisor may have to cover his tracks or
be penalized if he is caught. It follows that when incentive pay for the
worker is tied to supervisor evaluations, supervisors distort the evaluations
more, as their actions have more effect on the pay of their favored and
unfavored employees. So supervisors lie more when their reports carry
weight. But remember that the firm is using performance appraisals to
allocate workers to tasks based on their perceived talents. Then, as su-
pervisors lie more, more mistakes are made in the assignment of workers
to tasks, as sometimes the "teacher's pet" gets a job that he is not talented
enough to perform. In other words, better incentives are associated with
worse sorting. When choosing compensation plans, this issue naturally
generates a trade-off between incentives and sorting, where high incentive
pay causes agents to work hard, but at the cost of supervisors' reports
being less informative about talents.

The optimal degree of incentive pay depends on both the value of the
supervisor's information and the importance of the agent's efforts. When-
ever the marginal value of the supervisor's information for allocating
workers to jobs is high, a firm realizes that offering incentives is partic-
ularly costly because the quality of those reports falls. Consequently,
incentive provision for workers will be eschewed. As a result, few incen-
tives will be provided when sorting is greatly affected by the supervisor's
at promotion time than at other times, precisely because there is considerable
money on the line. See Landy and Farr (1980); Bjerke et al. (1987); and Larkey
and Caulkins (1992) on this relationship between pay and evaluation.
Uncertainty and Incentives report. By contrast, if the supervisor has little useful input for the allocation of workers to jobs, more incentives to workers will be offered as the marginal cost of further distortions in these reports is small.

The relationship between incentive pay and risk is, then, straightforward. Consider a “risky” environment, one where the supervisor’s estimate of the worker’s performance is particularly noisy. Even if the supervisor tells the truth in his evaluation, the information is of limited use for identifying the worker’s talent because it is so noisy. Consequently, the marginal cost to the firm of the supervisor distorting his evaluation is low in noisy settings, so the firm provides the worker with considerable pay for performance. By contrast, in less risky environments, supervisors have more valuable information, and incentive pay is eschewed because the value of truthful reporting of that information is high. Thus, when sorting issues arise, there is a natural positive relationship between uncertainty and incentives.

The second reason to expect a positive relationship between incentives and uncertainty concerns the use of formal investigations to monitor workers. One of the assumptions typically used in the agency literature is that the principal gets costless signals on the agent’s efforts and always monitors his performance. But this is not realistic; in many situations, monitoring is sporadic and costly. For example, with many contractual issues, disputes may result in costly court actions. Similarly, in many universities, an accusation of cheating by a student must be verified by a formal disciplinary committee. Finally, many firms offer employees the possibility of a due process hearing before they can be fired for disciplinary reasons. Investigations of this type are often used both to constrain the discretionary power of evaluators and to improve the quality of decision making. Section IV considers incentive provision when formal investigations are the only credible way to verify the performance of workers, because the verdicts of supervisors cannot be trusted. There are a myriad of reasons why such evaluations should not be trusted. The reason offered in Section IV as to why investigations are necessary is that supervisors have an incentive to offer bad evaluations on workers to reduce the amount they have to pay them.

In situations such as those described above, investigations into the agent’s performance are often infrequent because they are costly to administer and are endogenously chosen by the supervisor. My interest in Section IV is to address agency issues in such cases, rather than the more commonly studied situation where signals on performance come costlessly. I begin with a standard agency setting where a principal offers the worker a contract, which generates a subjective assessment by the supervisor. This subjective assessment cannot be credibly used to reward the agent, as the supervisor always cheats by claiming that performance was worse than the truth to save on wage costs. However, the supervisor
can launch a (possibly) costly investigation, which yields a verifiable measure of performance. However, subjectivity still plays a critical role as the supervisor chooses whether to launch an investigation on the basis of his subjective impression of the worker’s performance. Unlike the standard costly monitoring model of Becker (1968), it is efficient to allow the supervisor to make this choice; random monitoring independent of the supervisor’s information is simply not efficient, as it may be too costly to carry out an investigation. The supervisor uses his information to decide whether to investigate the performance of the worker, where the agent is rewarded and penalized on actual performance relative to its expected level. My interest here concerns the effectiveness of sporadic endogenous monitoring and how uncertainty affects the contract offered to the worker. I first show an independence result: if monitoring is independent of the impression that the supervisor has of the agent’s performance, risk has no effect on the optimal contract offered. However, it is not credible to monitor in this way, as the supervisor has an incentive to monitor only when the performance of the agent is poor. The reason that he will not monitor if his impression is that performance is high is that he believes that the outcome will increase the worker’s pay, which he would like to avoid. As a result, investigations occur only if performance is believed to be below some level. Thus, investigations (which are necessary for incentive provision in this setting) are used only when bad performance is suspected.

Without investigations, the agent never exerts effort, as investigations are the only credible way of revealing information on the performance of the worker. What matters, then, for incentive provision is not only the penalties induced by the investigation’s outcome but also the likelihood of bad performance being found, conditional on an investigation occurring. Not surprisingly, uncertainty in the environment affects the costs of investigation. I show that, when the initial impression that induces the investigation is riskier (so there is more noise in the initial information), the agent suffers less (in expectation) from an investigation. This arises simply because there are more cases where an investigation is erroneously launched. As a result, for a given contract, incentives are lower in riskier settings, even for risk neutral agents, as the supervisor is more likely to have made a mistake in launching the investigation in a more uncertain environment. In order to counteract this disincentive to exert effort in riskier settings, the firm then chooses higher pay-for-performance in cases where the environment is riskier. Once again, incentives and uncertainty are positively related.

I begin in Section II by outlining a simple model of subjective performance appraisal, which will be used throughout the article. I extend this model in Section III, where I consider a principal who has two uses for performance appraisal—to induce effort and to allocate workers to
tasks—but who relies on a supervisor who is prone to exercising favoritism in his evaluations. I show that the marginal value of truthful performance appraisals is decreasing in the riskiness of the environment, so that, again, the principal responds to an uncertain environment by increasing pay for performance. Section IV considers the case of sporadic endogenous monitoring and shows again the need for increasing incentives in uncertain environments. I conclude in Section V with a brief discussion.

II. A Model of Subjective Performance Appraisal

This section offers a simple model of subjective performance appraisal that will be used in the later sections to address the trade-off between uncertainty and incentives. As with the standard model of agency theory, a supervisor hires an agent to exert effort. The effort that ensues translates into a measure of observed output. However, the output measure itself cannot be contracted on. Instead, the supervisor can make a report on the performance of the worker. This report can be used to reward the agent, if desired. The difficulties that subjectivity gives rise to concern the relationship between the worker’s observed performance and the evaluation provided on the worker, where the noncontractability of output implies that the supervisor carries out actions that are in his interests, but not necessarily in the interests of maximizing surplus. In the later sections, these distortionary activities take the form of either favoritism or the desire to save on wage costs.

Assume that a risk neutral agent exerts effort $e$ at cost $C'(e) > 0$, $C''(e) > 0$, and $C'(0) = 0$. Expected output is given by $e$. However, the output observed by the supervisor is a noisy signal given by

$$y = e + \alpha + \epsilon.$$  \hspace{1cm} (1)

The variable $\alpha$ refers to the ability of the agent within the firm and $\epsilon$ is measurement error. I assume that $\epsilon \sim \mathcal{N}(0, \sigma^2)$. The worker’s skills are drawn from a (normalized) normal distribution $\alpha \sim \mathcal{N}(0, 1)$, and talent (how well the worker is matched to his current job) is unknown to all parties. I assume that $e$ and $\alpha$ are uncorrelated and that the outside option of the worker is normalized to zero.

The supervisor can make a report on the worker’s observed output, given by $y$. But the observation of output cannot be contracted on, so that $y$ need not equal $y$, as the supervisor can bias his report. The supervisor’s report is given by

$$y_s = y + b(y),$$  \hspace{1cm} (2)

where $b$ is the extent to which the supervisor distorts his report.

All that I have offered so far is a model where a supervisor can offer a report on the agent that can differ from the truth, but I have given no
reason why he would do so. In the following two sections, I offer some plausible reasons why a supervisor would want to distort his evaluations in ways that are potentially harmful from an efficiency perspective. I deal with each of these incentives (to exercise favoritism and to save on wage costs) in turn.

III. Favoritism and the Trade-Off of Incentives and Sorting

In this section, I consider one form of distortion that arises with subjective performance appraisal: favoritism. There is considerable evidence that workers who are liked by the boss are more apt to get a good rating, independent of performance. See Cardy and Dobbins (1986) and Varma, Denisi, and Peters (1996) for examples. This is important for the design of incentive contracts with two additional ingredients. First, the exercise of favoritism is more common when there is “money on the line”: when pay for subordinates is tied to performance, accuracy of evaluations seems to fall.8 Second, agency models assume that firms collect information on their workers only for the purpose of rewards or penalties. However, in reality, performance appraisals are used for many other reasons; according to the human resources management literature, identification of talent or identifying training needs typically dominate the use of performance appraisals for allocating merit pay. See Milkovich et al. (1999), Mondy et al. (1999), and Sherman et al. (1999) for details of this literature.9

This is where the cost of ignoring the effects of the other purposes of ratings becomes important for the design of agency contracts. Firms that offer high incentive pay risk distorting performance ratings, which is harmful on other dimensions, such as identifying talent or providing honest feedback. This concern has led to the suggestion of separating performance appraisals from merit pay.10 For example, Milkovich and Wigdor

---

8 For example, Landy and Farr (1980) and Bjerke et al. (1987) document supervisors distorting ratings more when high ratings will produce a greater return to workers. Bjerke et al. survey supervisors in the Navy, who admit to inflating evaluations at promotion time, in order to improve promotion prospects, as compared with times when there is less money at stake for their subordinates.

9 For example, in a survey carried out by Cleveland, Murphy, and Williams (1989), such items as “recognition of individual performance” and “determination of promotion” (standard agency-like concerns) are ranked as significantly less important factors than “identification of individual strengths and weaknesses” and “performance feedback,” which are more traditional concerns of human resources management.

10 In some ways, this insight is similar to that of the “influence costs” literature of Milgrom and Roberts (1988), which considers how rent seeking increases when supervisors hold considerable discretion over the allocation of resources. However, in this case there is an important interaction with the riskiness of the environment, which, once again, generates a positive correlation between risk and incentives.
(1983) and others argue that it is sensible for firms to separate appraisals from compensation, because once appraisals are used to reward people, problems with truthful revelation of information arise.

To formalize the multiple purposes of evaluation, I adapt Prendergast and Topel (1996) and consider the trade-off between incentives and selection as follows. Remember that the principal can make a report \( y_i = y + b(y) \). To generate a reason why a supervisor would distort ratings, I assume that he has personal preferences for his subordinates. Specifically, to reflect a taste for bias, the supervisor’s utility depends partly on his own compensation, \( w_s \), but also on the compensation of his subordinate, \( w_i \):

\[
\nu_i = w_i + \mu w - \frac{b^2}{2}. \tag{3}
\]

There are three unusual aspects of the supervisor’s utility function. First, the supervisor has likes and dislikes for employees, where \( \mu \) denotes the intensity of the supervisor’s preferences for the worker. Favoritism here takes the form of positive \((\mu \geq 0)\) or negative \((\mu < 0)\) altruism. We assume that \( \mu \) is unknown to all parties except the supervisor and is learned by the supervisor only after he joins the firm and encounters the worker.\(^1\) The ex ante distribution of \( \mu \sim \mathcal{N}(0, \sigma^2_\mu) \); \( \mu \) is uncorrelated with \( \epsilon \) and \( \sigma \). Second, the supervisor can distort his report from the truth by incurring a cost of \( b^2/2 \). This cost reflects both the costs of “covering up” information and the potential costs of being fired if found to have distorted the truth.\(^2\) Third, I assume that the supervisor is paid a salary independent of his report, where the salary is set before observing \( \mu \) to reach his reservation utility.\(^3\) Note that, with these preferences, the supervisor is not a residual claimant on profits \( y - w \), which is, instead, received by “the firm.” This implies that the supervisor does not have an incentive to offer bad evaluations on workers simply to save costs, which would be the case if he were a residual claimant.\(^4\) This issue is studied in Section IV.

The performance appraisal is used for two purposes. First, it is used

\(^1\) This means that the ex ante expected value of the returns to favoritism for the supervisor can be extracted from her salary.

\(^2\) For simplicity, I model these costs as a monetary fine that must be paid to the firm, and so are not part of surplus calculations.

\(^3\) See Prendergast and Topel (1996), where this assumption is relaxed.

\(^4\) The motivation for this assumption is that, in many firms, supervisors are not held responsible for the wage costs of their employees; instead, their only real source of discretionary power is to reward those that they like and penalize those that they dislike.
to reward the agent on perceived effort, as in a standard agency model. The worker can be rewarded by a linear contract

$$w = \beta_0 + \beta_1 y_i.$$  \hfill (4)

The issue here is how $\beta_1$ varies with the uncertainty of the environment. Second, the firms would like to select workers to their most appropriate tasks. On the basis of the supervisor's report, the agent can be assigned to a different task in an unmodeled future period if his skills match better to that task. This assignment is done by the firm, using the supervisor's assessment of the worker. With the technology given above, the marginal productivity of $\alpha$ is unity. The marginal product of $\alpha$ on the different task is given by negative one; on all other dimensions, the jobs are the same. Consequently, $\alpha$ refers to the worker's comparative advantage within the firm. Workers with $\alpha \geq 0$ should be retained in the current job, while those with $\alpha < 0$ should be reallocated to the other job. Let $\hat{\alpha}(y_i)$ refer to the firm's estimate of the worker's expected ability after receiving a report of $y_i$. Then, workers with expected ability $\hat{\alpha} < 0$ will be reassigned to another position at the end of the current period; otherwise, they are retained in the current position. We assume, only for simplicity, that the worker's reservation utility is independent of $\alpha$ so that skills are firm specific. Here, $\alpha$ is meant to reflect how well a worker matches to a particular task, for example, whether he is better at technical or personnel issues.

To summarize, the supervisor makes an evaluation, $y_i$, on the agent, but where the report may depend on whether he is liked by the supervisor. Contracts can be based on $y_i$, and the issue becomes the optimal incentives to provide to the worker, taking into account how incentive provision affects the exercise of favoritism and the allocation of the worker to jobs in the future.

Before considering the optimal contract, it is worthwhile to consider the incentives of the agent and supervisor. First, maximizing (3) implies that the supervisor chooses favoritism equal to $b(\mu, \beta_1) = \mu \beta_1$. Therefore, the supervisor distorts upward when the agent is liked ($\mu > 0$) and downward when the agent is disliked ($\mu < 0$). Note also that the extent to which the supervisor distorts the rating depends on incentives for the worker, $\beta_1$, and is unbiased, in that $E(b) = 0$ for all $e$. Second, as favoritism is unbiased, the agent chooses an effort level, $e^*(\beta_1)$, where $C'[e^*(\beta_1)] = \beta_1$ in the standard way. Given these outcomes, the firm forms its opinion on the agent's ability, $\hat{\alpha} = z(y_i - e^*)$, where $z = 1/(1 + \sigma^2 + \beta_1^2 \sigma_x^2)$. Note that this depends on the noisiness of the environment, the underlying distribution of bias, and the contract received by the worker.\footnote{Therefore, favoritism affects the equilibrium impression that the firm has on the agent, unlike the standard career concerns model of Holmstrom (1982).}
The problem for the firm is as follows. The supervisor knows two pieces of information, \( \mu \) and \( y \), which are not known by the firm. The supervisor's report is given by

\[
y_s = y + \mu \beta_1,
\]

which depends on both pieces of unobserved information; the supervisor makes a report which is not a one-to-one mapping of observed performance to reported performance but instead is muddied by the supervisor's preferences.\(^{16}\) But the principal would like to know \( \alpha \) as accurately as possible. Note that, from an ex ante perspective, the distribution of \( \hat{\alpha} \) given \( y \), is normal with mean \( \alpha \) but with variance \( \bar{\sigma}^2 \), where

\[
\bar{\sigma}^2 = \frac{\sigma^2 + \beta_1^2 \sigma_\mu^2}{1 + \sigma^2 + \beta_1^2 \sigma_\mu^2}.
\]

It is important that \( \bar{\sigma}^2 \), the uncertainty about the ability of the agent (after receiving the supervisor's report), is increasing in \( \beta_1 \). The cost of incentive pay in this model relates to this increased variance of the distribution of \( \hat{\alpha} \). This relationship generates the trade-off between sorting and incentives below.

Two features of this agency problem are unusual. First, supervisors have likes and dislikes for their employees, as manifested by the unknown parameter, \( \mu \). Second, there is a selection problem, where the firm would like to allocate workers to tasks based on their talents. If neither were present, the optimal contract to offer the risk-neutral agent would be the standard one of \( \beta_1 = 1 \). Another useful benchmark is to consider the case where there is no selection problem, but where the supervisor can exercise favoritism. It is straightforward to show that the expected ex ante utility obtained by the supervisor from the ability to show favoritism is given by \( \Lambda = \beta_1^2 \sigma_\mu^2 \). This surplus arises from the utility the supervisor receives from rewarding those she likes and penalizing those that she dislikes. As a result, the supervisor receives ex ante expected returns to being boss. Thus, there is an endogenous return to having the power to allocate rewards in this model.

If there is no sorting problem, the firm maximizes expected surplus \( E[y - C_e(\hat{\beta}_1)] + \Lambda] \). (Note that the returns to favoritism constitute sur-

---

\(^{16}\) Care needs to be taken in describing the activities of the supervisor as lying, as the Revelation Principle applies here. What I mean by lying is that the report from the supervisor is a less accurate representation of \( y \), and greater lying refers to reports where the variance of \( \hat{\alpha} \) around \( \alpha \) is greater.
plus and must be included in the maximization problem.) If the problem is well behaved,\(^{17}\) this yields optimal incentives, given by

\[
\tilde{\beta}_i = \frac{1}{1 - 2C''\alpha^2} > 1. \tag{6}
\]

Note that (i) the piece rate exceeds unity and (ii) the riskiness of the environment plays no role.\(^{18}\) Therefore, in the absence of the selection issue, there is no reason why incentives should vary with underlying uncertainty of the environment, \(\sigma^2\).

The importance of noise relates to the surplus created from allocating workers to tasks. Specifically, the firm would like to reduce measurement error, not in order to reduce the risk on workers (as is standard) but to improve the allocation of workers to jobs. Let \(S(\beta_i)\) be the surplus that is created from allocating workers to their correct jobs in the future period. This is measured relative to random assignment of workers. The optimal strategy is to allocate workers with perceived ability \(\hat{\alpha} > 0\) to the current job and to reassign all those with \(\hat{\alpha} \leq 0\) to the other job. As there are no other signals and the favoritism of the supervisor is unbiased, this implies that all those workers for whom \(\gamma_i - e^{\infty}(\beta_i) > 0\) should be retained in the current position and all others reassigned. Then, following Prescott and Visscher (1980) and Prendergast and Topel (1996), the surplus from reallocating workers to new positions is given by

\[
S(\beta_i) = 2 \int_{-\infty}^{\infty} \frac{t}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \left( \frac{1}{1 + \sigma^2 + \beta_i^2\alpha^2} \right), \tag{7}
\]

which is equivalent to

\[
S(\beta_i) = \frac{k}{1 + \sigma^2 + \beta_i^2\alpha^2}, \tag{8}
\]

where \(k = 0.7898\) is the mean deviation of the standard normal. Note that the surplus obtained from reallocating agents depends on each of the monitoring errors, with surplus decreasing in each variance. Remember that, in the absence of the sorting problem, the optimal piece rate is given

\(^{17}\) The problem yields an interior solution only if \(C'\sigma^2\) is small enough; otherwise, the convex (in \(\beta_i\)) returns to being boss dominate the costs of inefficient effort and the optimal solution is \(\beta_i = \infty\). To rule out this case, I assume \(1 - 2C''\sigma^2 > 0\).

\(^{18}\) This result that the piece rate exceeds unity arises from the fact that the ability to exercise favoritism is bundled with the incentive contract and the agent is risk neutral.
Uncertainty and Incentives

by $\tilde{\beta}_1 = 1/(1 - 2C''\sigma_n^2)$. Here the optimal piece rate $\beta_i^*$ is characterized by

$$\beta_i^* = \frac{1 + (\partial S/\partial \beta_1)}{1 - 2C''\sigma_n^2} = \frac{1 - [(2k\beta_i^*\sigma_n^2)/(1 + \sigma^2 + \beta_i^*2\sigma_n^2)]}{1 - 2C''\sigma_n^2} < \tilde{\beta}_1. \quad (9)$$

Note that $(\partial S/\partial \beta_1) < 0$, so that selection concerns reduce incentive pay from that given in (6). In other words, the desire to sort workers appropriately attenuates incentive provision. Thus, there is a trade-off of selection and incentives: any attempts to increase effort (by increasing $\beta_i$) come at the cost of more erroneous sorting of workers to jobs, as $(\partial S/\partial \beta_1) < 0$. The mechanism is that stated above: higher pay-for-performance results in more effort, but at the cost of supervisors distorting their evaluations, which harms the allocation of workers to jobs.

The subject of this article is how optimal incentives vary with the uncertainty of the environment. To address this, consider how riskiness in the environment ($\sigma^2$) affects the optimal design of incentives. Straightforward differentiation of (9) shows that

$$\frac{\partial^2 S}{\partial \beta_1 \partial \sigma^2} = \frac{4k\beta_i^*\sigma_n^2}{(1 + \sigma^2 + \beta_i^*2\sigma_n^2)^3} > 0, \quad (10)$$

so that the marginal cost of increasing incentives is decreasing in $\sigma^2$. As a result, if the problem is well behaved, $\beta_i^*$ is increasing in $\sigma^2$, illustrating a positive relationship between observed measures of uncertainty and incentives.

What is the source of this correlation? When choosing incentives, firms are weighing the benefits of inducing higher effort against the drawbacks of losing information about worker talent because of distorted evaluations. In risky environments, where $\sigma^2$ is large, the supervisor’s information is not particularly valuable for sorting purposes, so that the marginal cost of increased incentives is lower. As an extreme case, consider the outcome as $\sigma^2 \to \infty$: in that case $\partial S/\partial \beta_i^* \to 0$, and so $\beta_i \to 1/(1 - 2C''\sigma_n^2)$. So, in low-risk settings, the value of more truthful reporting by supervisors is high and incentives are eschewed; incentives and uncertainty are positively correlated.
IV. Subjectivity and Investigations

A. A Model of Investigations

There are many reasons not to trust subjective assessments of workers.19 In the previous section, I focused on the endogenous exercise of favoritism. Another problem with subjective assessments is that a supervisor who is residual claimant on income may have an incentive to renege on promised payments, even when appropriate efforts have been put forth. This issue, previously addressed in Kahn and Huberman (1988) and Prendergast (1993), often implies that, in the absence of credible reputation mechanisms, such as in Bull (1987) and Baker, Gibbons, and Murphy (1994), subjective assessments may not be used to reward the agents because agents realize that, even if they exert effort, they are likely to be “short-changed.” In this section, I consider such a case, where the supervisor is residual claimant on profits, which results in no role for effective pay-for-performance based on the subjective assessment of the supervisor because he always has an incentive to claim that performance is poor.

In the absence of subjective assessments being credible, firms often seek more reliable measures. In this section, I consider the effect of further investigations that are used to generate verifiable outcomes.20 These investigations are carried out by third parties, who do not face the same incentives to reduce wages. Becker (1968) has addressed the role of costly investigations and has shown that random monitoring can resolve the agency problem at little cost. But, in this model, investigations are (optimally) nonrandom; instead, the supervisor is delegated the choice of when to monitor, as he knows the profitability of the investigation. I show that the supervisor will investigate only when he thinks that performance is poor. In other words, investigations, which are necessary to

19 There is, by now, an enormous amount of evidence on various types of errors that supervisors make, such as errors of halo, leniency, centrality, and recency. These are defined as follows. Halo errors arise when a supervisor extends good performance on one measure of output to others, even when such a correlation is unwarranted. In effect, a “halo” is attached to an employee that covers all aspects of that employee’s contribution. Leniency errors concern an unwillingness to give a bad evaluation. Centrality errors arise when supervisors give all employees the same ratings, independent of their performance, while recency errors concern the case where supervisors care only about the most recent contributions.

20 The type of investigation that I have in mind here is a standard breach-of-contract situation, where one party is employed on a task for another and payment is based on the performance of a subjectively assessed task. If the buyer is allowed to reward based on his characterization of the task, he likely will claim that the job was not adequately done, and so will renege on the required payments. But some credible assessment is needed, or else the agent exerts no effort. In order to resolve these issues, courts can be used, which, while costly, may provide an objective (though possibly inaccurate) estimate of the outcome.
induce incentives, will occur only when the agent's performance is believed to be bad, based on the investigator's subjective impression. My interest is, then, in how the incentive to monitor using such signals is affected by the noisiness in the environment. I show below that, with such endogenous monitoring, there is a direct positive link between noise and optimal incentives.

The setting is as follows. An agent is employed to exert effort, as in the previous section. The supervisor obtains information about the performance of the agent; on the basis of this information, he decides whether to investigate his performance. This decision is delegated to him, as he is the only one who knows the costliness of an investigation. The investigation, if carried out, yields a verifiable measure of the worker's performance on which to base pay. If she does not investigate the worker's performance, the supervisor's assessment cannot credibly be used and the worker is offered a salary corresponding to the expected level of performance. However, if investigated, the agent is rewarded or penalized on the outcome of the investigation relative to expected performance.

The decision to launch an investigation is based on the supervisor's impression of the worker's performance, but this initial impression cannot be verified. Instead, an investigation is needed to enforce the incentive contract. The main result of this section is that (i) poor initial impressions cause investigations to be launched, but (ii) the likelihood of the agent being penalized from such an investigation is decreasing in the noisiness of the environment. As a result, in order to induce the agent to exert the optimal level of effort, the agent must be offered higher pay-for-performance in riskier environments: the theme of this article.

As in the previous section, the risk-neutral agent exerts effort $e$ at cost $C(e)$ above and produces expected output given by $e$. The supervisor then receives a subjective nonverifiable signal of the agent's performance, given by $y = e + \alpha + \epsilon$. In this section, there is no need for any divergence between the firm and the supervisor. Only the supervisor knows the outcome of the signal. In this section, the objective function of the supervisor is to maximize profits at all points in time, based on the information available at the point that the decision is made. As $y$ is nonverifiable, it cannot be used to induce incentives, as the supervisor will always offer the evaluation that offers the lowest pay to the worker to save on wage costs. Incentives, therefore, are provided through the use of investigations.

I assume that the principal can carry out an investigation of the agent's performance at cost $c$. This yields another observation,

$$y_I = e + \alpha,$$

which can be used to reward the agent. Thus, at some cost, a verifiable measure of output, which eliminates the measurement error, can be at-
tained.\footnote{It does not matter for the results that the supervisor can filter out all the measurement error. All that is necessary is that there be some imperfect correlation between the initial observation and the final outcome of the investigation, where the degree of correlation is decreasing in the uncertainty of the environment, $\sigma^2$. This is essentially the role of $\alpha$ in the model. It is also not necessary here that the investigation yield a better observation on performance than the subjective assessment. Similar results hold in the case where the investigation adds noise to $y$, on the assumption that the noise added by the investigation is higher in more uncertain settings.} In other words, the principal investigates and can eliminate the measurement noise. In the absence of an investigation, the agent is rewarded by a salary, $\beta_0$. Specifically, the contract has the following form:

\[
    w = \begin{cases} 
    \beta_0 & \text{if no investigation occurs,} \\
    \beta_0 + \beta_1 [y - Ey] & \text{if an investigation occurs,}
    \end{cases}
\]  

(12)

where the expected level of output $Ey$ is its ex ante level, $e^*$. Thus, investigations take the form of settling-up, at a rate $\beta_1$. Note that contracts can be written only on the outcome of an investigation. Although this contract may seem restrictive, it yields first best utility, as shown below.

This model is one where (possibly) costly monitoring is required to get a credible measure of performance. The literature on costly monitoring, beginning with Becker (1968), points to the importance of random monitoring to solve this problem, where the probability of monitoring is exogenous, that is, where investigations are independent of the supervisor's information. However, an important part of this section is the assumption that such monitoring is simply not efficient: instead, the supervisor must be delegated the power to decide when to carry out an investigation. I allow the supervisor to (optimally) control the decision on whether to launch an investigation by assuming that there is uncertainty about the cost of the investigation and the supervisor has private information on that cost. I take a particularly simple case here, where $c$, the cost of investigation is zero with probability $p$ and $\infty$ with residual probability $1-p$. This simple setting is one where the decision to investigate must be placed in the hands of the supervisor to avoid investigating in very costly states. The supervisor learns the cost $c$ at the same time as he observes $y$. The issue at hand is whether the supervisor uses that discretion in an efficient way.

The supervisor chooses a contract $(\beta_0, \beta_1)$ to maximize expected profits minus costs, $E(y - w - c)$. It does so in the realization that investigations will be launched after $y$ and $c$ are observed. For simplicity, I restrict attention to the case where $p$ is small (close to zero), so that investigations occur with low probability in equilibrium. This implies that the firm chooses $\beta_0$ to (at least) satisfy the agent's individual rationality constraint.
\( \beta_0 = C(e^*), \) where \( e^* \) is the agent’s equilibrium level of effort. The agent is rewarded for exerting effort \( e^* \), which, in this model, will be the first best level of effort, where \( C'(e^*) = 1 \). If no monitoring occurs, he is paid a salary based on the premise that this level of effort was realized. If on the other hand, he is monitored, then he will be rewarded based on how observed effort, \( y_s \), differs from the expected level, \( E_y = e^* \), where the penalty is given (per unit) by \( \beta_1 \). My interest is in how \( \beta_1 \) varies with \( \sigma^2 \): in other words, how does riskiness affect contracts?

**B. Exogenous Monitoring: An Independence Result**

Begin by considering the counterfactual where the supervisor always investigates if \( c = 0 \) and never monitors when \( c = \infty \). If investigations occur independent of \( y \), there would be no relationship between noise, as measured by \( \sigma^2 \), and the optimal contract. This can be seen as follows. If monitoring is independent of \( y \) and the principal expects effort \( e^* \), the agent has expected rewards conditional on monitoring, given by

\[
3 \cdot E(y, - e^*) = 3 \cdot (e - e^*)
\]

As a result, the firm can offer a contract where \( \beta_1 = 1/p \) and induce the first best level of incentives, independent of the level of noise. This is in effect the solution of probabilistic monitoring suggested by Becker (1968). This solution is, however, not credible, as it does not take account of the supervisor’s incentives to investigate.

**C. Endogenous Monitoring and the Relationship to Uncertainty**

Consider the incentives of the principal to monitor the performance of the worker. The supervisor does not always investigate conditional on observing \( y \). Instead, he will investigate if he believes that the net transfer from the investigation will favor him since this maximizes profits, given the information available to him at this point. This implies that he investigates only if \( E[\alpha|y] < 0 \), which occurs if and only if \( y - e^* < 0 \). If this is the case, consider the expected penalty for the agent conditional on an investigation. This is given by

\[
\beta_1 E(y, - e^*|y - e^* < 0) = \beta_1 E(\alpha|y - e^* < 0)
\]

\[
= \beta_1 E(\alpha|\alpha + \epsilon < 0) = \frac{k\beta_1}{(2 + \sigma^2)}, \quad (13)
\]

where

\[
k = 2 \int_0^\infty \frac{t}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = 0.7898
\]

as above. The term in (13) is nothing more than \( \beta_1 \) times the expected value of \( y_t - e^* \), conditional on \( y - e^* < 0 \). But from an ex ante perspective, investigations occur with probability \( p/2 \) (when both \( c = 0 \) and
\( y - e^* < 0 \), which are independent), so that the expected ex ante penalty is

\[
\frac{p \cdot k \beta_1}{2(2 + \sigma^2)}.
\]

(14)

In order to induce the optimal level of effort, \( 1 = C'(e^*) \), the firm then sets

\[
\beta_1^* = \frac{1}{(p/2)[(k/(2 + \sigma^2))] = \frac{4 + 2\sigma^2}{pk},}
\]

(15)

and \( \beta_0 \approx C(e^*) \). Note that

\[
\frac{d\beta_1^*}{d\sigma^2} = \frac{2}{pk} > 0.
\]

Thus, once again, incentives and noise are positively related, as can be immediately seen from (15).

Again, it is worth considering a more intuitive version of this result. In many situations, employees are rewarded and penalized only when an investigation occurs. For example, a bureaucrat who may be acting corruptly knows that a formal investigation is often required before he can be penalized. But investigations occur only when bad news for the agent is expected. The role of noisy environments then arises in the relationship between a suspicion of malfeasance and the truth. In noisy environments, this link is less clear than in simple settings, where initial impressions are rarely overturned. Consequently, in noisy environments, agents realize that they will sometimes "get away with" a bad evaluation, because the outcome of the investigation deviates more from the assessment. This reduces incentives: to overcome this, greater pay-for-performance is necessary in riskier environments.

It is also worth considering the kinds of jobs for which I think this is likely to be a problem. After observing performance, the supervisor’s incentive for launching an investigation is to save money; he intervenes if he thinks that the agent’s performance is poor so that he has to pay out less. But often supervisors are not residual claimant for the wages of their subordinates: a more plausible reason for intervention may be that the supervisor thinks that the wrong outcome arose from the agent’s

\[\text{This is where the importance of } \lambda \text{ close to zero arises. In equilibrium, the agent expects to be penalized } (p/2)\left[(k\beta_2)/(2 + \sigma^2)\right]. \text{ For } p \text{ close to zero, this is negligible, and so the fixed pay is approximately equal to the amount of pay that makes the individual rationality constraint bind.}\]

\[\text{It is easily shown that this result extends to an obvious nonlinear contract, where the agent is penalized a penalty } P \text{ if performance is below average } \alpha < 0 \text{ and is offered } \beta_0 \text{ otherwise.}\]
decision. As a result, an investigation is used to correct an incorrect allocation, such as a worker being wrongly fired. This is often the case with government bureaucrats, for example. Consider the employees of the Department of Child and Family Services in the U.S. welfare system, who place children in foster care. In a typical day, their performance in placing foster children with parents rarely has an effect on their compensation or employment status. Only in cases where mistakes are made (such as when foster children are abused or killed) does an investigation occur. Thus, a suspicion of poor performance focuses attention on the bureaucrats. Another example would be officials in the Food and Drug Administration, who rarely are praised for a job well done, but who garner considerable negative attention when a drug (such as Thalidomide) with dangerous side effects is allowed on the market. Again, external monitoring only occurs with the suspicion of poor performance. As a result, I think that this model applies best to cases where principals focus their attention only on cases where poor performance is particularly noteworthy, positions that Baron and Kreps (1999) call guardian jobs.

V. Conclusion

There is not much evidence to support the negative trade-off of risk and incentives. The objective of this article has been to provide a pair of theoretical reasons why we might not expect to find such a relationship; instead, I propose reasons for a positive relationship. First, there is considerable evidence that supervisors do not report evaluations truthfully; instead, they reward those that they like and penalize those that they dislike. Misreporting evaluations is a particular problem when this information is used to allocate workers to positions based on their ability. These problems become worse in settings where the supervisor’s evaluations count for a lot for compensating workers, so that there is a cost of increasing incentives in the form of less truthful reporting of information, thus generating a trade-off between sorting and incentives. This trade-off is affected by the uncertainty of the environment because in uncertain settings the supervisor’s evaluations are of little use anyway, as her honest reports are tainted by much measurement error. As a result, in risky settings, firms offer high pay-for-performance because supervisors’ reports count for little in the allocation process anyway and distorting them further has little cost. Second, in many occupations, the subjective impressions of superiors are not sufficient to reward and punish employees. Instead, more formal oversight is required. Section IV was concerned with the effectiveness of such investigation mechanisms. When choosing effort decisions, workers are then concerned with the accuracy of these investigations; the more accurate are the decisions to launch investigations, the greater will be the worker’s incentives. I showed that
the effectiveness of these investigations is lower in riskier settings, because more mistakes are made in launching investigations in more uncertain settings. As a result, workers are less worried about the effectiveness of this method of providing incentives in risky settings, and so firms must increase pay-for-performance in riskier settings.

There are other reasons why incentives may be more intense in uncertain settings, and these are described in more detail in Prendergast (2000). First, inferences are weaker in uncertain environments, as, for example, failure can be put down to bad luck rather than low ability. Yet inferences play an important role in generating incentives, as the literature on career concerns, such as Holmstrom (1982), has emphasized. In effect, reputational concerns can substitute for explicit incentive provision. But if reputational concerns are lower in more uncertain settings, incentives are likely to be affected in a variety of ways. For instance, agents may be more willing to take on risky endeavors, as they see the cost of failure on their reputations as lower than in a more stable environment. Alternatively, they may not be willing to work as hard in uncertain settings, because they know that their reputation is little affected by the outcome of their efforts in uncertain settings; more of the performance will be put down to good luck. As a result, firms may compensate by offering more explicit incentives in riskier settings. A second alternative reason for a positive relationship between risk and incentives concerns how tasks are delegated in firms. Firms often choose between assigning tasks to workers or delegating decision-making power over how they spend their time. In unstable environments, firms find that the optimal actions of their employees change rapidly. As a result, it is costly to simply assign actions to their workers, because those actions rapidly go out of date. Instead, they are more likely to delegate decision-making power in uncertain environments. Yet to constrain incentive problems when power is delegated, firms often use output-based contracts for the simple reason that there are no other good measures of performance that can be used to reward agents. Finally, one suspects that another reason why incentives may be more common in uncertain environments concerns selection issues. It seems plausible that the marginal returns to talent are greatest in uncertain environments, for the simple reason that the terrain is less well charted. As a result, it may be particularly important to attract high-ability people in uncertain environments, which requires high pay-for-performance to induce the right applicant pool. Thus, once again, there is a positive correlation between measures of uncertainty and incentives.

References
Uncertainty and Incentives


Coughlin, Anne, and Narasimhan, Chakravarti. “An Empirical Analysis


