COMMENT ON CHEN, KIM AND KON

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Received September 1975, revised version received October 1975

In a recent paper Chen, Kim and Kon (1975) – hereafter CKK – explicitly introduced stochastic cash demand into the portfolio decision and derived a capital asset pricing model with cash demand and liquidation costs. In this note I argue that CKK are inconsistent in their application of liquidation costs and that the preference function postulated by CKK conforms neither to a single-period nor to a multi-period framework of maximizing the expected utility of consumption. I therefore question the validity of the CKK approach in incorporating stochastic cash demands and liquidation costs in a capital asset pricing model. I also discuss a consistent approach to this problem.

CKK consider a single-period model. At the beginning of the period the \( i \)th investor is endowed with wealth \( W_i \) which he invests in \( n \) risky assets and in a risk-free liquid asset. No transaction costs are incurred in investing wealth in risky assets. At the end of the period the stochastic returns on the risky assets and the stochastic demand for cash are realized. If the demand for cash exceeds the investor’s holding of the risk-free asset, then a portion of the investor’s risky assets is liquidated in order to meet the excess demand for cash, thereby incurring a liquidation cost. The ending portfolio value for the \( i \)th investor, \( \tilde{W}_i \), is the initial holding of the risky and risk-free assets, plus returns, less the liquidation cost in meeting the excess demand for cash out of the risky assets. The objective is to maximize a risk-averse preference function which depends on the expectation and variance of \( \tilde{W}_i \), subject to the initial budget constraint.

I analyze the CKK model first in a single-period and then in a multi-period framework of maximizing the expected utility of consumption:

(i) Assume that the investor consumes his entire wealth at the end of the period and after the demand for cash has been satisfied. Then the wealth which remains invested in risky assets after the demand for cash has been satisfied is liquidated for consumption and additional liquidation costs are incurred. The CKK preference function depends on moments of \( \tilde{W}_i \) only and thus ignores this additional liquidation cost.

(ii) Assume next that the wealth at the end of the period is partly consumed and partly invested in a portfolio to be used for consumption in subsequent time periods. Then the utility of wealth at the end of the period is the induced utility
of consumption at the end of the period plus consumption in all future periods. In the presence of liquidation costs the state of the portfolio is not fully described by the total wealth; one needs to specify the portfolio composition, since the portfolio composition may be changed only by incurring liquidation or transaction costs. Thus the induced utility at the end of the period is a function both of total wealth and of portfolio composition. By ignoring the dependence of the utility on portfolio composition, CKK are inconsistent in their application of liquidation costs.

The same issue may be viewed from a different angle. Consider the conditions under which the investor's observable behavior in the market in any period is indistinguishable from that of a risk-averse expected utility of wealth maximizer who has a one-period horizon. Fama (1970) proved that such an approach is justified if the markets for consumption goods and portfolio assets are perfect. In the presence of the imperfection of liquidation costs, CKK provide no justification for limiting their discussion to one-period expected utility of wealth maximization.

I consider next the conjecture that transaction costs are distinct from, and negligible compared to liquidation costs. The argument might run as follows: Transaction costs (price spread and brokerage commission) are negligible or may be approximately incorporated in the price of the asset. Liquidation costs are the costs associated with selling an asset within a very short time period without obtaining the best price in the market and these costs are large compared to transaction costs. If such an approach is taken by CKK, the authors must explicitly differentiate between instantaneous transactions and transactions with a time lag. The liquidation cost would then be an expediting cost. Clearly CKK do not draw this distinction, nor is it possible to do so in a single-period formulation.

Finally I offer some suggestions for correcting the CKK model. A single-period model must account for the transaction costs in forming the initial portfolio and liquidating the entire portfolio at the end of the period. A more satisfactory formulation is in terms of a multi-period model with consumption in every period or consumption of the entire wealth at the end of the horizon, with transaction costs incurred in some periods as the result of portfolio revision. In either case the resulting model is more complex than the one considered by CKK and it is doubtful whether the individual investors' demands for assets upon aggregation yield a meaningful model of pricing capital assets. The derivation of a capital asset pricing model in the presence of transaction costs remains a difficult and, as yet, unsolved problem in finance.

References