Optimal Investment with Stock Repurchase and Financing as Signals

George M. Constantinides
University of Chicago and NBER

Bruce D. Grundy
Stanford University

When management has private information it has an incentive to finance investment by issuing a security that is overpriced in the market. The market's valuation of the issued security may lead management either to forego profitable investments or to invest suboptimally. With investment fixed, there exist fully revealing signaling equilibria in which the covenants of the issued claim serve as signals. A straight bond issue cannot provide the signals but a convertible bond issue can. With investment endogenous, fully revealing equilibria exist in which the par value of a straight bond issue and the announced level of investment jointly serve as signals and investment is optimal. The article also investigates the role of a stock repurchase in these equilibria.

Two fundamental problems in corporate finance—investment and financing—become particularly interesting and challenging when management has information that the market does not have. Rational investors recognize that management would like to finance investment by issuing some security that is overpriced

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in the market. The empirical evidence is consistent with this notion. The stock price reacts negatively at the announcement of a security offering, and the reaction is stronger the more junior is the security being offered. [See Smith (1986, table 2).]

Myers and Majluf (1984) show that the informational asymmetry between management and the market may lead management to forego a positive net present-value project, if the firm is restricted to finance investment with equity. The first goal of this article is to investigate modes of financing that allow management to signal its information to the market and permit management to undertake a project if its net present value is positive. More generally, when the level of investment is a choice variable, we investigate modes of financing that permit management to invest at the level which is optimal in the absence of informational asymmetry. The investigation sheds light on why firms issue straight debt or convertible debt. Whereas taxes, agency/bankruptcy costs, and differential flotation costs may be part of the answer, this article highlights the signaling role of the chosen mode of financing.¹

The second goal is to investigate the signaling role of stock repurchases. Specifically we investigate how a stock repurchase, coupled with the issue of a senior security, permits management to signal its information to the market and accept a positive net present-value project or, more generally, invest at the level which is optimal in the absence of informational asymmetry. The empirical evidence summarized by Smith (1986, tables 2 and 3) is that the stock price reacts favorably to such announcements and is consistent with the notion that these announcements are motivated by the management's desire to signal good news.

The model is presented in Section 1. A known fraction of the stock of an all-equity firm is owned by management. Management has information about the firm's investment prospects that the outside stockholders and the market do not have. The firm has insufficient financial slack to invest optimally. The management's objective is to maximize the value of its shares by choosing the level of investment by the firm and the mode of its outside financing. Management may not sell its stock this period and may not purchase any of the securities to be issued by the firm. The uncertainty is resolved in the last period, at which time the stochastic outcome of the production process is revealed and the security holders are paid off.

In equilibrium, the announced level of investment and its mode of financing act as signals, which in turn determine the price which the market

assigns to the issued securities. In this model, unlike that of Leland and Pyle (1977), management is risk-neutral, so that the risk which management undertakes does not serve as the cost that makes the signal credible. In a fully revealing equilibrium, the issued securities are fairly priced so that management does not incur the cost of offering the securities at a discount. Furthermore flotation costs are assumed away. The only possible source of deadweight loss, which would give rise to dissipative signaling, is suboptimal investment. Motivated by the work of Bhattarcharya (1980) we seek modes of financing that result in nondissipative signaling equilibria, hence optimal investment.

In earlier related work, Heinkel (1982) demonstrated the existence of nondissipative equilibria in which the firm finances optimal investment by issuing equity and debt. These equilibria hinge on particular production functions that imply that the price of debt of given face value may be decreasing in the value of the firm. Brennan and Kraus (1984, 1987) address a similar problem in which the amount of investment is fixed. They conclude that a nondissipative signaling equilibrium requires the firm’s net payoff function to all claimants other than stockholders to be decreasing in the firm value, for at least some range of firm values.

General properties of fully revealing equilibria are investigated in Section 2. Proposition 1 states that under reasonable assumptions investment in a fully revealing equilibrium is optimal, that is, maximizes the firm’s profit.

The role of an open market stock repurchase which excludes the management’s shares is investigated in Proposition 2. It states that a signaling equilibrium with optimal investment requires that the firm issue a security the proceeds of which partly finance investment and partly finance a stock repurchase. Note that the stock repurchase differs from a cash dividend in that a cash dividend is paid to all stockholders, including management. Our model provides an economic justification for stock repurchases that is not based on a repurchase premium over the equilibrium stock price.

Proposition 3 states that, if investment is independent of the firm type, then in a fully revealing equilibrium the issued claim cannot be a straight bond. It must be a claim which is concave in the firm value for low values of the firm value and convex for high values of the firm value. A claim which conforms to these specifications is a convertible bond.

Propositions 2 and 3 are illustrated in Figure 1. The impatient reader is advised to go straight to the discussion of Figure 1 which follows the proof of Proposition 3.

Section 3 presents an example in which the equilibrium investment is a function of the firm type. Investment is a signal that, along with the announced par value of a straight bond, leads to a fully revealing equilibrium. Existence of the equilibrium is proved in Proposition 4. The balance of that section discusses the model’s empirically testable implications and their relation to previous empirical work.

Two alternative solutions to the problem posed in this article are pre-
sented in Section 4. The first is a rights issue of common stock in which management commits to fully participate. The second solution is a managerial compensation scheme, which, however, may require large negative compensation (punishment) for the manager for some realizations of the cash flow.

In Section 5 we discuss the limitations of the model and suggest directions for further research.

1. The Model

The model is informally introduced in Section 1.1. The formal definition of the model as a signaling game and the formal definition of a sequential Nash equilibrium are given in Section 1.2.

1.1 Informal description of the model

There are three periods. In the first period a firm has some assets in place. The only claim to the firm is one share of common stock. Management holds \( N \) shares, \( 0 < N < 1 \), and outside investors hold \( 1 - N \) shares. Management announces the firm's planned new investment, \( I \), and how it will be financed. Specifically, management announces that the firm will issue a new claim that matures in period 3. The claim may be common stock, straight bond, callable convertible bond, a combination of these claims, or a more general claim. The announced covenants of the claim (e.g., the number of shares issued, the par value of the bond, the call price, the conversion price) are summarized by a vector \( s \). The set of pairs \( (s, I) \) that management is permitted to announce is denoted by \( A \).

In the second period the new claim is sold by auction to competitive bidders. The outside stockholders may participate in the auction but management may not. It is known to all parties that if the proceeds from the new claim exceed \( I \), the firm will invest \( I \) in production and use the excess proceeds to repurchase stock from the outside stockholders at the prevailing stock price. The outside stockholders are assumed to have only publicly available information and to be competitive. If the proceeds from the new claim exceed the sum of \( I \) and the market value of the outsiders' entire holding of stock, the firm will invest the excess proceeds in a riskless security, payable to the firm in the third period. Management is not permitted to tender its stock in the repurchase.\(^2\)

We have to consider the possibility that the proceeds from the new claim fall short of the announced investment. In that case it is stipulated that

\(^2\) Outside stockholders are indifferent in the margin between tendering their shares or not because we focus on fully revealing equilibria in which the management's announcement of the mode of financing reveals to the outside stockholders the true value of the shares. Brennan and Thakor (1989) examine open market and tender offer repurchases in a model in which the stockholders are not fully informed and therefore have an incentive to incur the cost of collecting information about the true value of the shares. The firm may eliminate these costs by distributing cash through a pro-rata stock repurchase or cash dividend thereby imposing the cost of personal taxes on the shareholders. Depending on the relative size of information collection and personal tax costs alternative modes of cash disbursement become optimal.
the firm returns to the new claimants the capital raised from them, invests zero capital in production, and the game ends. This possibility does not arise in any of the equilibria that we consider, but needs to be stipulated so that the firm's response is defined in out-of-equilibrium conjectures. In the remainder of the article we suppress this possibility when it is non-essential.

In period 3 the firm's value $x$ becomes publicly known. The senior claimants to the firm are paid off first, and the remaining capital is divided equally among the common stockholders.

Viewed from the first period, the firm value in the third period is a random variable with cumulative probability distribution $F(\cdot; I, \theta)$. It depends parametrically on investment $I$ and the scalar interchange $\Theta$, which has domain $\theta$. The parameter $\theta$ represents the information that management has in the first period but that the market does not have.

We normalize prices so that the interest rate between periods 1 and 3 and between periods 2 and 3 is zero. We assume that the management and the market are risk-neutral, so that the price of a claim to some future cash flow equals the expectation of the future cash flow. Then the firm value in the first and second periods is

$$V(I, \theta) = \int x \, dF(x; I, \theta)$$  \hspace{1cm} (1)

$V(I, \theta)$, or simply $V$, is referred to as the "true" firm value, being the assessment of the firm value conditional on the true value of $\theta$.

The value of the new claim in the third period is a publicly known function of $x$ and of the covenants $s$ and is denoted by $b(x; s)$. Limited liability by the new claimants and by the stockholders restricts the choice of payoff functions by

$$0 \leq b(x; s) \leq x$$  \hspace{1cm} (2)

The value of the new claim in the first and second periods is

$$H(s, I, \theta) = \int b(x; s) \, dF(x; I, \theta)$$  \hspace{1cm} (3)

$H(s, I, \theta)$, or simply $H$, is referred to as the "true" value of the new claim, being the assessment of the value conditional on the true value of $\theta$.

In the first period and before management announces its action, the market has a prior distribution of the attribute $\theta$. In a partially revealing equilibirum the market treats the announcement, $(s, I) \in A$, as a signal and updates its distribution of $\theta$. In this article we focus on the subset of signaling equilibria that are fully revealing in the sense that the market's posterior distribution of $\theta$ is a mass point at $\hat{\theta}(s, I) \in \Theta$ with probability 1.

The market's assessment of the cumulative probability distribution of $x$ is $F(x; I, \hat{\theta}(s, I))$, based on the market's assessment, $\hat{\theta}(s, I)$, of $\theta$. Also the market's assessment of the firm value is $V(I, \hat{\theta}(s, I)) = \int x \, d\hat{F}(x; I, \hat{\theta}(s, I))$, sometimes denoted by $\hat{V}$. Finally, the market's assessment of the value of the issued claim is $H(s, I, \hat{\theta}(s, I))$, sometimes denoted by $\hat{H}$.
After management announces \((s, I) \in A\), but before these actions are implemented, the market perceives the value of the stock to be \(\hat{V} - I\), and the price of the one outstanding share of stock to be also \(\hat{V} - I\). To see this, note that the firm eventually repurchases (or issues) stock worth \(\hat{H} - I\) and that the remaining outstanding stock is perceived by the market to be worth \(\hat{V} - \hat{H}\). Therefore, before the repurchase, the stock is perceived to be worth \((\hat{V} - \hat{H}) + (\hat{H} - I) = \hat{V} - I\).3

If the market’s assessment of the value of the issued claim exceeds (or falls short of) the announced investment, that is, \(\hat{H} - I \geq 0\) (or, \(\hat{H} - I < 0\)), the firm repurchases from the outside stockholders (or sells to the outside stockholders or to a third party, but not to management) \((\hat{H} - I)/(\hat{V} - I)\) shares, where \(\hat{V} - I\) is the perceived price per share. The number of shares in the recapitalized firm is \(1 - (\hat{H} - I)/(\hat{V} - I)\). The true value of the recapitalized firm is \(\nu\), and the true value of the stock of the recapitalized firm is \(V - H\). The true value per share, denoted by \(u(s, I, \theta)\), is \((V - H)/(1 - (\hat{H} - I)/(\hat{V} - I))\), which, upon simplification, becomes

\[
u(s, I, \theta) = \frac{[V(I, \theta) - H(s, I, \theta)][V(I, \hat{\theta}(s, I)) - I]}{V(I, \hat{\theta}(s, I)) - H(s, I, \hat{\theta}(s, I))} \tag{4}
\]

We assume that management is risk-neutral. The case in which management is risk-averse, as in Leland and Pyle (1977), is of interest in its own right and may admit different equilibria than the ones discussed in this article. We also assume that management cannot buy or sell stock in periods 1 and 2. After the stock repurchase, the fraction of shares owned by management is \(N/[1 - (\hat{H} - I)/(\hat{V} - I)]\). Therefore management receives payoff \([x - b(x; s)]N/[1 - (\hat{H} - I)/(\hat{V} - I)]\) in period 3. Management maximizes the present value of this payoff conditional on \(\theta\), that is, management maximizes \(N(V - H)/(1 - (\hat{H} - I)/(\hat{V} - I)) = Nu(s, I, \theta)\). Since \(N\) is a constant, the management’s objective becomes the maximization of the true value per share, \(u\).

If management is permitted to sell a fraction of its \(N\) shares in the second period, then management has an additional incentive to overstate the firm value. If management is committed to purchase \(C\) dollars of stock in the second period, then management has a disincentive to overstate the firm value, and we may obtain an equilibrium in which it is unnecessary for the firm to repurchase outsiders’ stock. The case \(C > 0\) is dealt with in the discussion which follows Proposition 2.

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3 We consider separately the case where \(\hat{H} - I\) exceeds the market value of the outsiders’ stock by \(K\). Then the firm repurchases stock worth \(\hat{H} - I - K\) and invests \(K\) in a riskless security. The remaining stock is perceived by the market to be worth \(V - H + K\). Therefore, before the repurchase, the stock is perceived to be worth \((\hat{V} - \hat{H} + K) + (\hat{H} - I - K) = \hat{V} - I\). The firm repurchases \((\hat{H} - I - K)/(\hat{V} - I)\) shares and the number of shares in the recapitalized firm is \(1 - (\hat{H} - I - K)/(\hat{V} - I)\). The true value of the recapitalized firm is \(V + K\), and the true value of the stock of the recapitalized firm is \(V - H + K\). The true share price is \((V - H + K)/(1 - (\hat{H} - I - K)/(\hat{V} - I))\), which simplifies to \((V - H + K)/(\hat{V} - I)/(\hat{V} - H + K)\). This expression becomes identical to Equation (4) under the reinterpretation that the true value of the new claim is \(H - K\) and its perceived value is \(\hat{H} - K\). The analysis proceeds as in the main body of the article.
1.2 Formal definition of the model

We consider a signaling game with two players as in Cho and Kreps (1987). The first player is management. The second player is the collection of outside stockholders and the potential buyers of the new claim issued by the firm. There is no loss of generality in grouping as one player the outside stockholders and the potential buyers of the new claim. We refer to the second player as the "market."

Management learns its type $\theta$, drawn from a set $\Theta$, according to some probability distribution over $\Theta$ that is common knowledge. Management sends a message $(s, I)$ chosen out of a set $A$. The set $A$ may depend upon the management's type, and we write $A(\theta)$ for the set of messages available to type $\theta$. We also write $\Theta(s, I)$ for the set of types that have available the message $(s, I)$.

Having heard the message $(s, I)$, the market form their posterior beliefs on $\theta$, represented by the cumulative probability distribution $G(\cdot; s, I)$ over $\Theta(s, I)$. Whereas we are interested only in "fully revealing" equilibria in which the posterior distribution of $\theta$ is a mass point, we have to allow for more general posterior distributions in the statement of the game.

The game ends and a payoff $Nu(s, I, \theta)$ is made to management. $N$ is the fraction of equity originally held by management and is a constant. Therefore the management's maximization objective $Nu$ may be replaced by $u$ without loss of generality. The payoff $u$ depends on the variables $s, I, \theta$, on the posterior cumulative probability distribution $G(\theta; s, I)$, and on the functions $F$ and $h$, which are exogenously given and are common knowledge. $F(x; l', \theta)$ is the cumulative probability distribution of $x$, conditional on actual investment $l'$ (which may differ from the announced investment $l$), and type $\theta$. The function $u$ is

$$u(s, I, \theta) = \begin{cases} 
\frac{(V - H)(\hat{V} - I)}{\hat{V} - H} & \text{if } \hat{H} \geq I \\
0 & \text{if } \hat{H} < I 
\end{cases}$$  \hspace{1cm} (5)

where

$$V = \int x \, dF(x; I, \theta)$$  \hspace{1cm} (6)

$$\hat{V} = \int \int x \, dF(x; I, \theta) \, dG(\theta; s, I)$$  \hspace{1cm} (7)

$$H = \int h(x; s) \, dF(x; I, \theta)$$  \hspace{1cm} (8)

and

$$\hat{H} = \int \int h(x; s) \, dF(x; I, \theta) \, dG(\theta; s, I)$$  \hspace{1cm} (9)
We write the management's strategy as \( \rho(s, I; \theta) \), where, for each \( \theta \), \( \rho(\cdot, \cdot; \theta) \) is a cumulative probability distribution over \( A(\theta) \). The market's strategy is the null set. The market's role is in forming the posterior beliefs on \( \theta \) as \( G(\cdot, s, I) \), which determine the management's payoff through the functions \( V \) and \( H \).

A sequential Nash equilibrium is described by Conditions 1–3.

**Condition 1.** Each type \( \theta \) management evaluates the payoff from sending message \( (s, I) \) as \( u(s, I, \theta) \), and \( \rho(s, I; \theta) \) puts weight on \( (s, I) \) only if it is among the maximizing pairs \( (s, I) \).

**Condition 2.** For any message \( (s, I) \) that is sent with positive probability by some \( \theta \), the market uses Bayes' rule to compute \( G(\theta|s, I) \), the posterior assessment that \( (s, I) \) comes from each type \( \theta \in \Theta(s, I) \).

**Condition 3.** For every message \( (s, I) \) that is sent with zero probability by management, there must be some probability distribution over types \( \Theta(s, I) \), that is, there must exist some type \( \theta \) management that can send the message \( (s, I) \).

In this article we focus on the subset of fully revealing sequential equilibria. In a fully revealing sequential equilibrium there exists a function \( \hat{\theta}(s, I) \in \Theta(s, I) \), such that, having heard the message \( (s, I) \), the market's posterior beliefs on \( \theta \) are \( \hat{\theta}(s, I) \) with probability 1. The management's payoff, defined in Equations (5) to (9), collapses to \( u(s, I, \theta) \), defined in Equation (4).

Define \( (s^*(\theta), I^*(\theta)) \) as any pair in \( A(\theta) \) that maximizes \( u(s, I, \theta) \). Then Condition 2, the informational consistency condition, becomes

\[
\hat{\theta}(s^*(\theta), I^*(\theta)) = \theta
\]

We substitute Equation (10) in Equation (4) and obtain

\[
u(s^*(\theta), I^*(\theta), \theta) = V(I^*(\theta), \theta) - I^*(\theta)
\]

The latter result leads to Lemma 1, proved in the Appendix, which is useful in discussing the properties of fully revealing equilibria.

**Lemma 1.** For any \( \theta_i \in \Theta \), the global maximum of the problem

\[
\max_{\theta \in \Theta(s^*(\theta_i), I^*(\theta_i))} [u(s^*(\theta_i), I^*(\theta_i), \theta) - V(I^*(\theta_i), \theta) + I^*(\theta)]
\]

is zero and is attained at \( \theta = \theta_i \).

If investment is fixed and stock repurchases are banned, the lemma reduces to theorem 1 in Brennan and Kraus (1987).
2. Properties of Fully Revealing Equilibria

The first proposition motivates our focus on fully revealing equilibria.

**Proposition 1.** If there exists a fully revealing equilibrium, then the equilibrium investment is the investment from the set \( A(\theta) \) that maximizes \( V(I, \theta) - I \).

**Proof.** See the Appendix. ■

In our model there are two signals, investment with potential deadweight costs and financing with zero deadweight costs but potential costs to management. Proposition 1 states that, if a signaling equilibrium exists, then investment is optimal. This is consistent with earlier multidimensional signaling models by Kohlleppel (1983), Quinzii and Rochet (1985), Wilson (1985), Engers (1987), and Ambarish, John and Williams (1987), who find that the equilibrium is Pareto-optimal. Furthermore, in our model the equilibrium is nondissipative in the sense that the equilibrium investment is the investment from the set \( A(\theta) \) that maximizes \( V(I, \theta) - I \).

In issuing a claim to finance investment, management has an incentive to overstate the value of the issued claim and thereby benefit the existing equityholders. In a fully revealing equilibrium, management must have a countervailing incentive to understate the value of the issued claim so that the management’s overall incentive is compatible with the market’s, and management reveals the value of the issued claim.

The assumption that the firm must use any proceeds above the declared investment to repurchase outsiders’ equity provides such an incentive. If management overstates the value of the issued claim, it may also overstate the stock price, and dilute its own claim by following its declared policy to repurchase some of the outsiders’ stock. Given the announced investment, the more management inflates the perceived value of the claim, the more capital it raises and the more shares it must repurchase from the outsiders at the inflated price.

The next proposition highlights the role of a stock repurchase in a fully revealing equilibrium.

**Proposition 2.** Assume that:

1. The set \( A(\theta) \) is such that \( h(x, s) \) is nondecreasing in \( x \) for all \( s \).
2. The cumulative probability distribution \( F(x; I, \theta) \) is such that \( \theta \) orders \( x \) by first-degree stochastic dominance, that is, \( F_\theta(x; I, \theta) \leq 0, \forall x, I \).
3. A fully revealing equilibrium exists.

Then the firm engages in a stock repurchase whenever the stock price is positive.

**Proof.** See the Appendix. ■

Proposition 2 states that, under our model assumptions, a stock repurchase is necessary in a fully revealing equilibrium. The role of the stock
repurchase is to make the management's incentive compatible with the market's. Another way to achieve the same goal is to change the model and assume that management receives a fixed dollar compensation \( C \) in period 2 in the form of new equity issued by the firm. In the second period the perceived stock price is \( \hat{V} - I - C \) and management receives \( C/(\hat{V} - I - C) \) shares. In the second period, after the issue of new shares and after the stock repurchase, there are \( 1 - (\hat{H} - I - C)/(\hat{V} - I - C) \) shares outstanding. The true value of the outstanding stock is \( V - H \), hence the true share price is \( (V - H)/(1 - (\hat{H} - I - C)/(\hat{V} - I - C)) \). Management maximizes the true value of its \( N + C/(\hat{V} - I - C) \) shares. Upon simplification, the management's objective is to maximize

\[
u(s, I, \theta) = \frac{(V - H)(\hat{V} - I + C/N - C)}{\hat{V} - \hat{H}}
\]

(13)

This objective reduces to Equation (4) when \( C = 0 \). Retracing the proof of Proposition 2, Equation A9 is replaced by

\[
H - I + C/N - C = \frac{(V - I + C/N - C)H_0}{V_0} > 0
\]

(14)

and \( H - I \) need not be positive if \( C \) is sufficiently large. That is, a stock repurchase may be unnecessary at the equilibrium.

The next proposition characterizes the class of claims that the firm issues in equilibrium, if investment is fixed and, therefore, cannot serve as a signal.

**Proposition 3.** Assume that Conditions 1–3 of Proposition 2 hold and that investment is fixed. Then the issued claim is locally convex in \( V \) and concave for at least some value of \( V \) below the equilibrium value of \( V \).

**Proof.** See the Appendix. ■

In related work Heinkel (1982) and Brennan and Kraus (1984, 1987) ban stock repurchases and obtain fully revealing equilibria. In Heinkel's model the firm finances optimal investment by issuing equity and debt. The critical assumption is that the firm value is positively related to credit risk, implying that the price of debt of given face value is not everywhere nondecreasing in the firm value.

In Brennan and Kraus' (1987) model the required investment is exogenously fixed and publicly known. In their theorem 3 they assume that \( \theta \) orders \( x \) by first-degree stochastic dominance and prove that, for at least some values of \( x \), \( b(x; s) \) must be decreasing in \( x \). In their theorem 4 they assume that \( \theta \) orders \( x \) by a mean-preserving spread and prove that \( b(x; s) \) must be neither everywhere convex nor everywhere concave in \( x \). These results compare with our Proposition 3. Brennan and Kraus illustrate the existence of fully revealing equilibria in examples in which the firm issues a claim and may simultaneously repurchase debt which is outstanding at
time 1. By contrast, in our model the firm is all-equity financed at time 1 and therefore does not have the option to repurchase debt.

Proposition 3 is illustrated in Figure 1. We assume that investment is either 0 or 1 where I is exogenously fixed and is independent of θ. We parameterize by V the firm type and write $H(V)$ for the value of the claim issued by a firm of type $V$ and for given $s$. Suppose that management signals $\hat{V}$ through the covenants of the issued claim. The perceived value of the claim is $H(\hat{V})$, and the firm repurchases stock of perceived value $H(\hat{V}) - I$. The stock price is $\hat{V} - I$. Thus the firm repurchases $[H(\hat{V}) - I]/(\hat{V} - I)$ shares and this number is given by the slope of the heavy line which passes through the points $(I, I)$ and $(H(\hat{V}), \hat{V})$.

Suppose that the true value of the firm is $V_3, V_3 > \hat{V}$. We proceed to show that management is worse off in signaling $\hat{V}$ and understating the firm value, if and only if the point $(H(V_3), V_3)$ lies above the heavy line. In falsely signaling, management understates the stock price by $V_3 - \hat{V}$. In repurchasing $[H(\hat{V}) - I]/(\hat{V} - I)$ shares, management and the remaining outside stockholders (that have not tendered their shares) benefit by an amount shown in the figure as “gain in stock repurchase.” Also in falsely signaling, the firm issues a claim worth $H(V_3)$ but raises capital $H(\hat{V})$. The difference, $H(V_3) - H(\hat{V})$, is a loss to management and to the remaining stockholders, as shown in the figure. If $(H(V_3), V_3)$ lies above the heavy line, the loss in the issue of the claim and the gain in the stock repurchase result in a net loss and management is worse off in understating the firm value.

We repeat the analysis in the case that the firm value is $V_2, V_2 < \hat{V}$. We conclude that management is worse off in overstating the firm value, provided that the point $(H(V_2), V_2)$ lies above the heavy line.
We combine the two results to conclude that incentive compatibility requires that (1) \( H(V) \) be tangent to the heavy line at \( V = \hat{V} \), (2) \( H(V) \) be locally convex at \( V = \hat{V} \), and (3) the point \( (H(V), V) \) lie above the heavy line in the feasible range of \( V \). Formally condition (1) is the first-order condition of optimality of the management's maximization problem, (2) is the second-order condition, and (3) is the condition that management's local maximum is also global.

The example illustrates the role of a stock repurchase. If we ban stock repurchases and allow only equilibria in which the heavy line is horizontal, incentive compatibility requires that \( H(V) \) be locally decreasing in \( V \) for \( V < \hat{V} \).

The example also illustrates a case where the issue of equity or straight debt cannot lead to a fully revealing equilibrium but the issue of a convertible bond can. Note that \( H(0) = 0 \), the function \( H(V) \) intersects the heavy line at some value \( V_i, V_i < \hat{V} \), and \( H(V) \) cannot be everywhere convex.

Finally the example illustrates that local optimality is insufficient to imply global optimality. If \( V_i > I \), the management of a firm with value \( V, I < V < V_i \), has an incentive to falsely signal \( \hat{V}_i \). Thus we must restrict in an appropriate way the set \( A(s, I) \).

3. A Fully Revealing Equilibrium with Debt Financing

In the example of the last section we showed that the issue of equity or straight debt cannot lead to a fully revealing equilibrium, if investment is fixed; but the issue of a convertible bond attained a fully revealing equilibrium. In issuing a (noncallable) convertible bond management announces the face value of the bond and the conversion price, so that the dimensionality of the signal is two.

In this section we make the level of investment an endogenous decision that depends on the firm type. The announced level of investment becomes a signal that along with the announced face value of the issued straight bond leads to a fully revealing equilibrium. The dimensionality of the signal is again two. Essentially the investment signal replaces the signal that was earlier provided by the conversion price. Whereas the firm may still finance the investment with a convertible bond, a straight bond suffices because the signal provided by the conversion price becomes redundant.

In equilibrium, investment is optimal thereby illustrating the claim in Proposition 1.

We assume that the cash flow in period 3 is given by

\[
\hat{x} = (\theta I^n / n) \hat{e}
\]

where \( \hat{e} \) is uniformly distributed in \([0, 2]\), \( n \) is a known parameter, \( 0 < n < 1 \), and \( 0 \leq \theta \). The value of the firm this period is
\[ V(I, \theta) = \frac{1}{2} \int_0^2 (\theta I^n/n) e^{-\epsilon} d\epsilon = \frac{\theta I^n}{n} \]  \( \text{(16)} \)

Given \( I \) and \( \theta \), \( V(I, \theta) \) is determined by Equation (16) and \( \tilde{x} \) is uniformly distributed in \([0, 2V]\).

The firm issues a straight bond with zero coupon, maturing in period 3, and par value \( K, 0 < K < 2V \). The assumption \( K < 2V \) excludes the trivial case in which the stock is worthless in the first period. The vector of covenants, \( s \), consists of just one element, \( K \). The bond value is

\[ H(K, I, \theta) = \frac{1}{2V} \int_0^K x \, dx + \frac{1}{2V} \int_K^{2V} K \, dx = K - \frac{K^2}{4V} \]  \( \text{(17)} \)

The following proposition states conditions under which a fully revealing equilibrium exists and characterizes the equilibrium.

**Proposition 4.** Assume that

1. The set of messages is
   \[ A = \{K, I: 0 \leq K, 0 \leq I\} \]  \( \text{(18)} \)

where \( K \) is the par value of a zero coupon, straight bond.

2. The third-period cash flow satisfies Equation (15).

3. \[ N \leq \frac{1 - n}{(1 - n/2)^2} \]  \( \text{(19)} \)

Define the sets \( A_1, A_2 \) by

\[ A_1 = \{K, I: 0 < K, I = (2 - n)K/2\} \]  \( \text{(20)} \)

and

\[ A_2 = \{K, I: K = I = 0; \text{ or } 0 \leq K, 0 \leq I, I \neq (2 - n)K/2\} \]  \( \text{(21)} \)

Then a fully revealing equilibrium exists in which the management’s strategy is

\[ K^*(\theta) = \left(\frac{2}{2 - n}\right) \theta^{1/(1-n)} \quad 0 < \theta \]  \( \text{(22)} \)

\[ I^*(\theta) = \theta^{1/(1-n)} \quad 0 < \theta \]  \( \text{(23)} \)

\[ \{K^*(\theta), I^*(\theta)\} \in A_2 \quad \theta = 0 \]  \( \text{(24)} \)

where there is a nonzero probability of any message in \( A_2 \). The market’s inference is

\[ \hat{\theta}(K, I) = \begin{cases} 
\left(\frac{2 - n}{2}\right)^{1-n} & \text{if } (K, I) \in A_1 \\
0 & \text{if } (K, I) \in A_2 
\end{cases} \]  \( \text{(25)} \)
Proof. Substitute the market's inference \( \hat{\theta}(K, I) \) from Equation (25) in the management's objective function (4) using Equations (16) and (17). The reader may verify that, for \( \theta > 0 \), \( K^*(\theta) \) and \( I^*(\theta) \), given by Equations (22) and (23), are the unique solution to the management's unconstrained maximization of \( u(K, I, \theta) \). Also any \( (K, I) \in A_2 \) maximizes \( u(K, I, \theta) \), the maximum being zero. Condition (19) ensures that there are sufficient shares of stock held by the outsiders to limit the repurchase to outsiders. To see this, note that Equations (16), (22), and (23) imply that \( I = nV \) and \( K = 2nV/(2 - n) \). Then the number of shares repurchased from the outsiders is \( n^2/(2 - n)^2 \) and this is less than \( 1 - N \) by condition (19).

Since the union of the sets \( A_1 \) and \( A_2 \), the set \( A \), there do not exist out-of-equilibrium conjectures. Taking as given the management's strategy, as given by Equations (22) to (24), the market's inference, as given by Equation (25), implies \( \hat{\theta}(K^*(\theta), I^*(\theta)) = \theta \) and therefore it is the outcome of Bayes' rule.

The fraction of stock repurchased from outsiders is \( (H - I)/(V - I) \) and the fraction of stock held by insiders after the repurchase is

\[
\alpha(N, n) = \frac{N}{1 - (H - I)/(V - I)} = N \left[ \frac{(1 - n/2)^2}{1 - n} \right] \tag{26}
\]

Unlike the Leland and Pyle (1977) model, the fraction of stock held by insiders after the repurchase is independent of the management's private information \( \theta \) and does not serve as a signal; it is, however, increasing in the fraction of stock originally held by insiders \( N \) and increasing in the publicly known parameter \( n \).

In equilibrium, the stock price is

\[
V(I^*(\theta), \theta) = \left( \frac{1}{n} \right)^{1/(1-n)} \tag{27}
\]

If we restrict \( \theta \) to be less than or equal to 1, the equilibrium stock price is decreasing in \( n \). Hence the model predicts a negative cross-sectional association between the signaled \( \theta \) and the fraction of stock held by insiders after the repurchase.

The model predicts that the higher the value of the attribute \( \theta \), the larger the announced investment, the debt issue, and the stock repurchase. The latter follows from the fact that the stock repurchase is

\[
H(K^*(\theta), I^*(\theta)) = I^*(\theta) = \left[ \frac{n(1 - n)}{(2 - n)^2} \right] \theta^{1/(1-n)} \tag{28}
\]

The existing empirical evidence does not address directly the predictions of the model on the concurrent announcement of investment, debt issue, and stock repurchase.
An indirect prediction of the model is that the higher the announced investment, the higher the inferred attribute \( \theta \), which is consistent with the event study of McConnell and Muscarella (1985). A second indirect prediction is that the higher the announced stock repurchase, the higher the inferred \( \theta \), consistent with the event studies of Dann (1981) and Vermaelen (1984) on open market repurchases, and Masulis (1980), Dann (1981), Vermaelen (1984), and Rosenfeld (1982) on intrafirm tender offers. A third indirect prediction is that the announcement of a debt issue is good news, whereas the evidence in Dann and Mikkelsen (1984), Eckbo (1986), and Mikkelsen and Partch (1986) is that this announcement is neutral news. Finally, the model predicts indirectly that the issue of debt and the repurchase of equity is good news, consistent with the evidence of Masulis (1980, 1983).

For the production function assumed in this section, there exist other signaling equilibria in which the firm issues a convertible, rather than a straight bond. In these alternative equilibria the firm repurchases more shares from the outsiders than it does in the equilibrium of Proposition 4, and is more likely to violate the constraint that it cannot buy the insiders’ stock. Thus the conversion feature serves no useful purpose.

4. Alternative Solutions

4.1 Rights issue of common stock

A potential solution of the problem posed in this article is to have the firm issue common stock through a rights issue. Suppose that the management of a type \( \theta \) firm offers to the holder of each share of stock the option to purchase \( m \) newly issued shares of stock from the firm at the price \( K \) per share. The management makes the binding commitment that it will fully participate in the rights issue (i.e., that it will acquire \( mN \) shares by exercising its options) and that it will invest in production all the proceeds from the rights issue. If fraction \( \delta \), \( 0 \leq \delta \leq 1 \), of all the stockholders, including insiders and outsiders, exercise their options, the firm raises capital \( \delta mK \) and invests \( I = \delta mK \).

Suppose that the cash flow distribution is as in the example of Section 3, so that \( V(I, \theta) = \theta^p/n, \ 0 < n < 1, \ 0 \leq \theta \). The true value of the firm is \( V(\delta mK, \theta) \) and the true share price is \( V(\delta mK, \theta)/(1 + \delta m) \). The investment level that maximizes the firm’s profit, \( V(I, \theta) - I \), is \( I = \theta^{1/(1-n)} \). Suppose that the announced message \((m, K)\) satisfies

\[
mK = \theta^{1/(1-n)}
\]  

(29)

If the market inverts Equation (29) and infers \( \theta \) correctly, the stock price exceeds or equals \( P_{\min} \), where

\[
P_{\min} = \min_{\delta, N, \delta^2 \leq 1} \frac{\theta(\delta mK)^n}{(1 + \delta m)n}
\]  

(30)
The reader may verify that there exists a fully revealing equilibrium in
which the firm's strategy is a message \((m, K)\) satisfying Equation (29) and
\(P_{\min} > K\). This equilibrium hinges on the assumption that management has
capital \(N[1 - \alpha]\) and can commit to fully participate in the rights issue.

4.2 Managerial compensation
Since the cash flow is observable in the third period, another resolution
of the problem posed in this article, in the spirit of Ross (1977), is to offer
management, before management knows the firm type, compensation in
the third period equal to \(K - NP_3 + \alpha(V_3 - I)\), where \(K, \alpha\) are constants,
\(\alpha > 0\), and \(P_3, V_3\) are the stock price and firm value in period 3, respectively.
The sum of the managerial compensation and the proceeds from the man-
agement's share ownership is \(K + \alpha(V_3 - I)\) and the management's objec-
tive in the first period is to maximize the expectation of the firm's profit.
Since some realizations of \(K - NP_3 + \alpha(V_3 - I)\) may be negative, they
may have to be interpreted as nonpecuniary punishment, if management
has limited financial liability. If \(N\) is large, the size of the compensation/
punishment may be large for particular realizations of the cash flow.

5. Concluding Remarks
One generalization of the model is to allow the firm in the first period to
have not only equity but also debt subject to default risk. We consider
separately the cases in which the firm finances investment with a claim
senior or junior to the outstanding debt.

Suppose that the firm may only issue a claim subordinate to the out-
standing debt. We denote by \(\hat{y}\) the cash flow of the firm in the third period
and by \(g(\hat{y})\) the cash flow to the outstanding debt. The cash flow to the
equityholders and the holders of the subordinate claim is \(\hat{x} = \hat{y} - g(\hat{y})\).
If \(\theta\) orders \(\hat{y}\) by first-degree stochastic dominance, it also orders \(\hat{x}\) by first-
degree stochastic dominance, since \(\hat{x}\) is increasing in \(\hat{y}\). Proposition 2 still
says that in a fully revealing equilibrium the firm repurchases some of the
outsiders' stock. However, now we have introduced an agency problem
and investment may be suboptimal.

A variant of the above problem arises when we allow the firm to issue a
claim senior to the outstanding debt, as suggested by Stulz and Johnson
(1985). We can no longer claim that repurchase is necessary, and there
may exist a fully revealing equilibrium that does not require stock repur-
chase and that leads to optimal investment.

A stock repurchase plays an important role in the equilibria discussed
in this article. The next step is to ban stock repurchases and investigate
the equilibria, which, in general, will not be fully revealing and will not
lead to optimal investment.

In another extension we may allow the management to raise capital first
and then choose the level of investment. This introduces an agency prob-
lem in that the management may attempt to expropriate the senior claim-
ant's wealth. As the model stands it suppresses the agency problem in
order to isolate the signaling role of the chosen mode of financing. How-
ever, the agency problem is of interest in itself, and in particular the mode
of financing which mitigates this problem [see Green (1984)].

A problem related to the one addressed in this article assumes that there
is no asymmetry of information in the first period. The asymmetry of infor-
mation arises in the second and last period at which time the outsiders
(i.e., the holders of the claims) have less information than the management.
If the outsiders can observe the management's information only through
a costly verification technology (which may be exogenous or endogenous),
it is of interest to search for the form of a contract that raises the required
amount of capital and minimizes the expected verification cost. This prob-
lem has been addressed by Townsend (1979), Diamond (1984), Gale and
Hellwig (1985), Chang (1986a, 1986b), and most recently by Williams
(1987). Specifically, we would like to find conditions that imply that the
optimal contract consists of equity, debt subject to default risk, converti-
bles, and warrants. In a generalization of the above we may allow for
asymmetry of information between the management and outsiders both at
the beginning and the end of the period and investigate again the form of
the optimal contract.

Appendix

Proof of Lemma 1

Suppose that the lemma is false. Then there exists a \( \theta_2 \in \Theta(s^*(\theta_1), I^*(\theta_1)) \)
such that

\[
    u(s^*(\theta_2), I^*(\theta_2), \theta_2) - V(I^*(\theta_2), \theta_2) + I^*(\theta_2) > u(s^*(\theta_1), I^*(\theta_1), \theta_1) - V(I^*(\theta_1), \theta_1) + I^*(\theta_1) > 0 \quad (A1)
\]

where the latter inequality follows from Equation (11). By the definition
of \((s^*(\theta_2), I^*(\theta_2))\) we have

\[
    u(s^*(\theta_1), I^*(\theta_1), \theta_2) \leq u(s^*(\theta_2), I^*(\theta_2), \theta_2) \\
    \leq V(I^*(\theta_2), \theta_2) - I^*(\theta_2) \quad (A2)
\]

where the latter inequality follows from Equation (11). Equation (A2)
contradicts Equation (A1), completing the proof. \( \blacksquare \)

Proof of Proposition 1

We prove by contradiction that there does not exist a message \((s_1, I_1) \in
A(\theta)\) such that

\[
    \hat{\theta}(s_1, I_1) = \theta \quad (A3)
\]

and

\[
    V(I^*(\theta), \theta) - I^*(\theta) < V(I_1, \theta) - I_1 \quad (A4)
\]

Suppose that the proposition is false. Then
\[ u(s, I, \theta) \leq u(s^*(\theta), I^*(\theta), \theta) \]
\[ \text{[by the optimality of the signal } s^*(\theta), I^*(\theta)] \]
\[ \leq V(I^*(\theta), \theta) - I^*(\theta) \quad \text{[by (11)]} \]
\[ < V(I_1, \theta) - I_1 \quad \text{[by (A4)]} \]
\[ \text{(A5)} \]

But Equations (4) and (A3) imply
\[ u(s_1, I_1, \theta) = V(I_1, \theta) - I_1 \]
\[ \text{(A6)} \]
contradicting Equation (A5) and completing the proof. ■

**Proof of Proposition 2**

The first-order necessary condition implied by Lemma 1 is
\[ (V_\theta - H_\theta) \left( \frac{V - I}{V - H} \right) - V_\theta - (V_I - 1)I_\theta^* = 0 \]
\[ \text{(A7)} \]

Proposition 1 states that the equilibrium investment is the investment from the set \( A(\theta) \) that maximizes \( V(I, \theta) - I \). If the unconstrained value of \( I \), which maximizes \( V(I, \theta) - I \), is in the set \( A(\theta) \), then \( V_I(I^*(\theta), \theta) - 1 = 0 \). If the unconstrained value of \( I \) is not in the set \( A(\theta) \), then \( I_\theta(\theta) = 0 \). Combining these two results we obtain
\[ \{ V_I(I^*(\theta), \theta) - 1 \} I_\theta(\theta) = 0 \]
\[ \text{(A8)} \]

We use Equation (A8) to simplify Equation (A7), rearrange, and obtain
\[ H - I = \frac{(V - I)H_\theta}{V_\theta} > 0 \]
\[ \text{(A9)} \]

where the latter inequality follows from the fact that assumptions 1 and 2 of the proposition imply \( H_\theta/V_\theta > 0 \), and the assumption of positive stock price implies \( V - I > 0 \). Therefore \( H - I > 0 \), completing the proof. ■

**Proof of Proposition 3**

Since investment is fixed, the second-order necessary condition implied by Lemma 1 is
\[ (V_{\theta\theta} - H_{\theta\theta}) \left( \frac{V - I}{V - H} \right) - V_{\theta\theta} \leq 0 \]
\[ \text{(A10)} \]

Combining Equations (A10) and (A9) we obtain
\[ H_{\theta\theta} - \frac{V_{\theta\theta}H_\theta}{V_\theta} \geq 0 \]
\[ \text{(A11)} \]

The claim is locally convex in \( V \) because
\[
\frac{d^2 H}{dV^2} \bigg|_{s, t} = \frac{H_{t0} - H_0 V_{t0}}{V_0^2} - \frac{H_0 V_{t0}}{V_0^2} \\
= \frac{(H_{t0} - V_{t0} H_0 / V_0)}{V_0^2} \\
\geq 0 \quad \text{[by (A11)]}
\]  

We prove by contradiction that the claim cannot be convex in \( V \) in the entire range of \( V \) from 0 to the equilibrium \( V \). Otherwise,

\[
\frac{H_0}{V_0} \geq \frac{H}{V} \quad \text{(by convexity)}
\]

\[> \frac{H - I}{V - I} \quad \text{(since } H < V \text{)} \tag{A13}
\]
contradicting Equation (A9) and completing the proof. ■

References


