Agency Conflicts, Prudential Regulation, and Marking to Market*

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Abstract

We develop a theory to show how mark-to-market accounting and shareholder–debt holder agency conflicts interact to affect the prudential regulation of a financial institution. We demonstrate that, relative to a benchmark historical cost regime, mark-to-market accounting could alleviate the inefficiencies arising from asset substitution, but exacerbate those arising from the incentives to choose lower quality projects due to debt overhang. The inefficiencies due to debt overhang and asset substitution work in opposing directions. An increase in the propensity for asset substitution mitigates the debt overhang inefficiency, and this tradeoff is especially pronounced for highly levered financial institutions. The optimal choices of the accounting measurement regime and the prudential solvency constraint balance the conflicts between shareholders and debt holders.

From a policy standpoint, our results suggest that a uniform capital requirement across institutions could be sub-optimal. In fact, if the solvency constraint is too tight (i.e., the capital requirements is too strict), historical cost accounting dominates mark-to-market accounting. Our results therefore sound a note of caution given the recent proposals to require both mark-to-market accounting and stricter capital requirements in the Basel III accords.

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1 Introduction

An ongoing debate on the 2007-2008 financial crisis is about the role of fair-value-based prudential regulation of financial institutions (e.g., see Laux and Leuz (2009)). Proponents of fair value accounting argue that a balance sheet based on market prices leads to better insights into the current risk profiles of financial institutions. Regulators can intervene in a more timely and effective manner, and tools such as solvency constraints or capital requirements can be used to prevent the inefficient choices or continuation of bad projects. Opponents counter that market prices can only provide useful signals to outsiders if the assets and liabilities of institutions trade in frictionless competitive markets, which do not exist for several important items. Further, prudential regulation based on market values could increase the risks faced by institutions, and induce myopic corporate behavior by preventing the selection of efficient, long-term projects. To the best of our knowledge, however, a trade-off that is central to this debate—regulation based on fair value accounting could mitigate inefficient choices of bad projects, but simultaneously hamper the choices of good ones—has not been theoretically formalized.

We develop a theory of how accounting interacts with agency conflicts between the shareholders and debt holders of a financial institution to affect the optimal design of prudential capital regulation. We show that, relative to a benchmark historical cost regime in which all claims are measured at their origination values, prudential regulation based on market values could mitigate inefficiencies arising from asset substitution or risk-shifting (the choice of risky, negative NPV projects), but exacerbate inefficiencies due to debt overhang (the avoidance of risky, positive NPV projects). The conflicting effects of fair value accounting hold even if the institution’s claims are traded in frictionless, competitive markets. Put differently, even if prices fully reflect fundamentals, we show that fair value accounting may still be dominated by historical cost accounting. The inefficiencies due to debt overhang and asset substitution work in opposing directions in that an increase in the propensity for asset substitution alleviates the debt overhang problem. The optimal (total value-maximizing) choices of the accounting regime and prudential capital regulation balance the trade-off between debt overhang and asset substitution. Under fair value accounting, we show that the op-

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1 Throughout the paper, we use the terms mark-to-market accounting or fair value accounting synonymously. While mark-to-market accounting as embodied in our model is the use of observable market prices to measure the value of an asset, fair value accounting is a broader term in the sense that unobservable inputs might be used to measure the value if relevant observable inputs are unavailable.
optimal solvency constraint declines with the marginal cost of investment in higher project quality, and with the excess cost of equity relative to debt financing. From a policy standpoint, our results suggest that the excess cost of equity financing relative to debt financing—which generally varies across the business cycle—is a key determinant of the design of prudential regulation. Moreover, a uniform solvency constraint (or capital requirement) across institutions could be sub-optimal. Indeed, we show that if fair value accounting is used and the prudential solvency constraint is set too tight (alternately, the capital requirement is too strict), then historical cost accounting dominates fair value accounting. The latter result sounds a note of caution given the recent proposals to require both stricter capital requirements and fair value accounting in the Basel III accords.

Our theory focuses on financial institutions such as insurance firms and commercial banks that are subject to prudential regulation. Financial institutions differ from non-financial institutions in two important aspects. First, financial institutions are much more highly leveraged than non-financial firms. Second, in contrast to industrial firms, a relatively large proportion of the debt of a financial institution is held by uninformed and widely dispersed debt holders. As Dewatripont and Tirole (1994) argue, prudential capital regulation plays an important role of an ex ante commitment and coordination mechanism that enforces an ex post transfer of control from shareholders to creditors by imposing a solvency constraint that is ex ante optimal. By their “representation hypothesis,” the regulator serves as a representative of dispersed and uninformed debt holders by effecting such a transfer of control.

We capture the aforementioned distinguishing features of financial institutions in a two-period model in which a representative financial institution finances a long-term project (or a pool of projects) through a combination of debt and equity. We follow studies such as Heaton et al. (2010) by assuming that there are deadweight costs of equity financing that are represented by equity holders demanding an incremental (risk-adjusted) expected return or premium relative to debt holders. The excess cost of equity creates an incentive for the institution to choose a high leverage.²

The project’s cash flows are realized at the end of period 2. The cash flows depend stochastically

²Previous literature proposes a number of reasons for the high leverage levels of financial institutions (e.g., Allen and Gale (1999) and Santos (2001)). Because our results do not hinge on the particular mechanism that leads to a high leverage level, we assume for simplicity that the various mechanisms manifest in a higher effective cost of equity capital relative to debt capital.
on the project’s quality that is privately chosen by shareholders with a higher quality project entailing a higher expected investment by shareholders. At the end of period 1, there is a publicly observable signal about the cash flows of the project. The signal indicates a poor or good interim state. The probability of receiving the high signal increases with the project’s quality. Given the signal, if the institution meets the solvency constraint, its shareholders may act opportunistically by engaging in asset substitution in period 2. If the institution violates the solvency constraint, control transfers to the regulator who closely monitors its operations and ensures that the ex post efficient continuation strategy—no asset substitution—is chosen in period 2. Consequentially, the project’s terminal cash flows are affected by its quality choice in period 1 and potential asset substitution or control transfer in period 2. The institution’s capital structure reflects the trade-off between the excess cost of equity relative to debt financing and the agency costs of debt.

We analyze two accounting measurement regimes: a fair value (FV) regime in which the balance sheet of the institution—and, therefore, the solvency constraint—is marked to market every period, and a benchmark historical cost (HC) regime in which all claims are measured at their origination values. Given a solvency constraint, we first examine the institution’s optimal choices of capital structure, project quality, and asset substitution in each regime. We then derive the optimal prudential constraint, which is set by a regulator who maximizes the financial institution’s ex ante total value anticipating the institution’s capital structure, project quality, and asset substitution choices. Finally, we compare the two regimes.

Regardless of the measurement regime, there are two well known inefficiencies—the asset substitution problem and the debt overhang problem—that arise from agency conflicts between shareholders and debt holders. First, the higher the leverage, the greater are shareholders’ incentives to increase risk by engaging in asset substitution in the second period. Second, the higher the leverage, the lower are shareholders’ incentives to make a costly investment to increase project quality in the first period because a larger proportion of the increased total payoffs from a higher project quality accrues to debt holders. A novel outcome of our analysis, however, is that as leverage increases, there is a subtle trade-off between the asset substitution and debt overhang inefficiencies: an increase in the propensity for asset substitution in the second period alleviates the debt overhang

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3Our analysis does not change if we instead assume that the regulator liquidates the institution’s assets where the liquidation payoff equals the institution’s value under no asset substitution. Our results, therefore, hold even if the institution’s claims are traded in frictionless, competitive markets so that prices fully reflect fundamental values.
problem in the first period.

The solvency constraint plays an important role in mediating the inefficiencies arising from asset substitution and debt overhang, but its effectiveness depends on the prevailing accounting regime. Because the balance sheet is not re-measured in the $HC$ regime, the institution automatically meets the solvency constraint at date $1$ if it meets it at date $0$. Because there is no possibility of a transfer of control at date $1$, the solvency constraint has no bite so that the $HC$ regime is plagued with a high incidence of asset substitution. The high incidence of asset substitution in the second period, however, alleviates the debt overhang inefficiency in the first period. If asset substitution were hypothetically ruled out, the likelihood of choosing high project quality decreases. Further, the beneficial effect of asset substitution on project quality is especially pronounced at relatively high leverage levels that are typical of financial institutions. Stated differently, at high leverage levels, not only do the debt overhang and asset substitution become severe, but, more interestingly, these two inefficiencies move in opposing directions.

The intuition for the trade-off between the asset substitution and debt overhang inefficiencies is as follows. At low leverage levels, asset substitution occurs (if at all) only in the low state, i.e., when the interim signal is low. At low leverage levels, however, the debt overhang problem is also insignificant in that the institution chooses high project quality in the first period. Consequently, at low leverage levels, eliminating the possibility of asset substitution has little impact on the \textit{ex ante} project quality choice. At high leverage levels, however, asset substitution is pervasive in that it occurs in both the low and the high intermediate states. The reason is that, as leverage increases, the call option in the low state becomes more out of the money relative to the high state. Further, at higher leverage levels, the payoffs from asset substitution are much greater for the high state relative to the low state because the good outcome for the project is realized. Because the high state is more likely for the high quality project, this, in turn, increases the bank’s incentives to choose the higher project quality. Consequently, shutting down the possibility of asset substitution eliminates the \textit{incremental} rents from asset substitution in the good state relative to the bad state and, therefore, decreases shareholders’ \textit{ex ante} incentives to invest in the higher quality project, that is, the debt overhang problem worsens. Conversely, the high propensity for asset substitution in the $HC$ regime enhances \textit{ex ante} incentives to invest in higher project quality.

Consistent with the intuition expressed by proponents of fair value accounting that market prices
play a disciplining role, we show that the FV regime does indeed alleviate the asset substitution pervasive in the HC regime. Because claims are marked to market in the FV regime, the solvency constraint has bite at the intermediate date 1 so that transfer of control to the regulator occurs if it is violated. Further, such transfer of control occurs when the institution’s leverage is above a threshold. However, as discussed above, the incentives for asset substitution are particularly pronounced at high leverage levels, and this is precisely when shutting down the possibility of asset substitution through the transfer of control to the regulator has the biggest negative impact on the \textit{ex ante} project quality choice. In mitigating asset substitution, therefore, regulation based on fair value accounting exacerbates the debt overhang problem by inducing shareholders to choose a lower project quality.

From a normative perspective, the regulator faces a dilemma in choosing the optimal (total value-maximizing) solvency constraint in the FV regime. A lax solvency constraint aggravates asset substitution because the constraint has less bite. However, by increasing the likelihood of the transfer of control—and thereby curbing potential rents from \textit{ex post} asset substitution—a tight solvency constraint dampens incentives to invest in higher project quality. In choosing the solvency constraint, the regulator minimizes the expected inefficiencies arising from asset substitution and debt overhang.

We show that the optimal solvency constraint does not eliminate either inefficiency, that is, both debt overhang and asset substitution are possible at the optimum. Further, the optimal solvency constraint becomes more stringent when the marginal cost of investment in higher project quality or the excess cost of equity capital increase. As the excess cost of equity capital increases, the institution’s incentives to use debt financing increase so that both asset substitution and debt overhang inefficiencies are more likely. Nevertheless, it turns out that the asset substitution problem is relatively more pernicious than the debt overhang problem. Consequently, the solvency constraint becomes tighter to mitigate asset substitution at the expense of potentially curbing incentives to invest in project quality. If the marginal cost of investment in higher project quality increases, the debt overhang problem becomes less severe because shareholders have less incentives to raise project quality in the first place. The optimal solvency constraint, therefore, again becomes tighter to mitigate asset substitution.

Our results show that the optimal solvency constraint in the FV regime is \textit{institution-specific} in
that it depends on parameters that determine the payoff distribution of the institution’s projects. These parameters are likely to vary across institutions even if they belong to a particular category such as commercial banks or insurance firms. A uniform solvency constraint (or capital requirement) across institutions could, therefore, be suboptimal. In fact, if the solvency constraint is too tight (or the capital requirement is too strict), then $FV$ accounting is actually dominated by $HC$ accounting. The proposals to require both stricter capital requirements and fair value accounting in the Basel III accords should be implemented with caution. Our analysis, therefore, highlights the importance of tailoring the solvency constraint to the prevailing accounting regime.

The prediction that the optimal capital requirement becomes more stringent as the excess cost of equity capital increases also has relevant policy implications. The excess cost of equity capital relative to debt capital is likely to vary across the business cycle. During an upswing in the business cycle, credit becomes cheaper/easier to obtain so that it is plausible that the excess cost of equity capital increases with the reverse being true during a downswing in the business cycle. The result that the optimal capital requirement should increase with the excess cost of equity capital is, therefore, consistent with the proposal for higher capital requirements during booms compared with recessions that has been made by several academics and policy makers.

Our model and results are particularly pertinent to the prudential regulation of highly levered financial institutions that has become a hotly debated issue in the aftermath of the financial crisis. As discussed above, the key trade-off between asset substitution and debt overhang is particularly pronounced at high leverage levels when both problems are severe, and asset substitution is pervasive in that it occurs in both “good” and “bad” states. Indeed, one of the primary causes of the financial crisis was risky subprime mortgage lending by banks during a period when the economy was booming and credit was cheap. Subprime mortgage lending could be more generally viewed as asset substitution that occurred in “good” states. Our analysis highlights the fact that, at higher leverage levels that are more typical of financial institutions and where prudential regulation potentially plays a role, the option value of asset substitution is significantly higher in good states. Consequently, shutting down asset substitution through a prudential solvency constraint and the transfer of control to a regulator could have a much bigger negative impact on $ex$ $ante$ investment in project quality. Our study therefore sheds light on the trade-off between asset substitution and debt overhang problems, and the roles that prudential capital regulation and the accounting regime
play in balancing this trade-off.

2 Related Literature

We contribute to the growing stream of literature that theoretically analyzes the economic trade-offs of fair value versus historical cost accounting. O’Hara (1993) investigates the effect of market value accounting on project maturity and finds that mark-to-market results in a preference for short-term projects over long-term projects. Allen and Carletti (2008) (hereafter, AC) and Plantin, Sapra, and Shin (2008) (hereafter, PSS) are two recent studies that show how fair value accounting may have detrimental consequences for financial stability. In both studies, markets are illiquid and incomplete and therefore a reliance on price signals may lead to inefficiencies. We complement these studies in a number of ways. First, in contrast to the above studies, we analyze the effects of accounting measurement on the capital regulation of financial institutions. Because solvency constraints depend on how the values of assets and liabilities are measured, accounting measurement rules naturally have real effects. PSS, instead, assume that managers maximize expected accounting earnings so that accounting has real effects. Second, because the issues we examine are different, there are important distinctions in the tensions identified. In our setup, markets are frictionless and competitive so that price signals perfectly impound information about future cash flows. We focus on the effects of agency conflicts between a financial institution’s shareholders and its debt holders. We show that, even in the absence of liquidity risk so that prices fully reflect fundamentals, while fair value accounting curbs inefficient risk shifting, it could reduce incentives to invest in high quality projects.

Burkhart and Strausz (2009) (BS) and Heaton, Lucas, and McDonald (2010) (HLM) model the effects of fair value accounting on financial institutions and also assume frictionless and competitive markets so that prices fully reflect fundamentals. BS show that, unlike historical cost accounting, fair value accounting increases the liquidity of a financial institution’s assets, which, in turn, increases the institution’s asset substitution incentives. Our analysis identifies different frictions, and therefore generates very different conclusions. BS focus on the information asymmetry between the institution’s current shareholders and prospective shareholders, while we examine conflicts between debt holders and shareholders. In their environment, fair value accounting reduces information
asymmetry that induces asset substitution. In our environment, fair value accounting curbs asset substitution through the intervention of the regulator but unfortunately, the debt overhang problem is exacerbated. HLM build a general equilibrium model of an institution and study how accounting interacts with an institution’s capital requirements to affect the social costs of regulation. In their model, financial institutions invest in firms whose technologies are \textit{exogenous} and \textit{fixed}. In contrast, our analysis centers on how the optimal choices of the accounting regime and the solvency constraint anticipate the financial institution’s \textit{endogenous} project choices.

Our study is also related to the literature on the capital regulation of banks and, more generally, financial institutions (see Dewatripont and Tirole (1995) and Santos (2001) for surveys). We adopt the perspective in Dewatripont and Tirole (1995) who argue that the main concern of prudential regulation is the solvency of financial institutions that, in turn, is related to their capital structure. Capital structure is relevant because it implies an allocation of control rights (Aghion and Bolton (1992)) between shareholders and debt holders. Further, the importance of regulation stems from the fact that small, uninformed debt holders of institutions need a representative to protect their interests. In early studies, Merton (1978) and Bhattacharya (1982) show that capital requirements curb inefficient risk-shifting. However, studies such as Koehn and Santomero (1980), Kim and Santomero (1988), Gennette and Pyle (1991), and Rochet (1991) argue that capital requirements could alter the equilibrium scale of operations of an institution and, therefore, its optimal asset composition in ambiguous ways. Besanko and Kanatas (1996) show that conflicts of interest between a bank’s management and its shareholders could lower, and sometimes even reverse, the beneficial effects of capital regulation in curbing asset substitution. Kahn and Winton (2004) emphasize that risk-shifting incentives are particularly important for financial institutions. We contribute to this literature by showing how solvency constraints optimally balance the inefficiencies arising from asset substitution and debt overhang. More importantly, our study demonstrates how the trade–off between these inefficiencies is affected by the accounting measurement regime.
3 Model

3.1 Environment

A financial institution finances a long-term project through a combination of debt and equity. The term “project” could refer to a “pool” of projects. Because our theory is broadly applicable to institutions that are subject to prudential regulation such as insurance firms and commercial banks, we deliberately do not model a specific type of institution.\(^4\) Our focus is on agency conflicts between shareholders and debt holders so we assume that the financial institution’s insiders behave in the interests of shareholders.

The project’s payoff increases stochastically (in the sense of first-order stochastic dominance) in the project’s quality. The institution chooses the quality of the project through careful analysis and selection. The cost incurred by shareholders in choosing the quality of the project is a nonnegative random variable. The mean cost increases with project quality.

At some interim date before the project’s payoffs are realized, there is a publicly observable signal about the performance of the project. At this date, shareholders may act opportunistically by engaging in asset substitution or risk-shifting that results in the transfer of wealth from debt holders to shareholders, but lowers the value of the overall project. Asset substitution could be achieved by either engaging in off–balance sheet derivative transactions and/or altering the characteristics of the existing project.

The institution operates in a regulated environment. There is a prudential regulator who protects the interests of small and uninformed debt holders by ensuring that, at any point of time, the institution’s leverage ratio is not too high. The regulator imposes a prudential or solvency constraint to ensure that the value of the institution’s assets are sufficiently high relative to its liabilities (Dewatripont and Tirole (1994)). If the prudential constraint is violated at the interim date, control transfers to the regulator who closely monitors the institution and ensures that it chooses the efficient continuation strategy—no asset substitution—in the second period. Our analysis does not change in any way if we, instead, assume that the regulator sells or liquidates the institution’s assets where the total payoff is the market value of the assets assuming the efficient continuation

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\(^4\)If the institution is an insurance firm, its “creditors” include insurance policyholders. With the pooling of insurance risks, the insurance firm’s liabilities arising from insurance claims are similar to a debt obligation. If the institution is a bank, its creditors include depositors and other debt holders.
strategy—no asset substitution—is chosen in the second period. In other words, our results hold even if the institution’s claims are traded in frictionless, competitive markets, that is, there are no deadweight costs arising from the early sale or liquidation of the institution’s assets.

It is important to note that, in our environment, regulation plays the important role of serving as an *ex ante* commitment mechanism that enforces an *ex post* transfer of control that is optimal from an *ex ante* standpoint, that is, from the standpoint of maximizing the total enterprise value of the institution. The debt issued by non-financial firms typically has associated covenants that also play the role of effecting a transfer of control if they are violated. As discussed by Dewatripont and Tirole (1994), from a “high level” perspective, prudential regulation of financial firms and debt covenants for non-financial ones are “isomorphic” in that they are both mechanisms that achieve a transfer of control. As such, the main economic rationales for their presence are similar. As they emphasize, however, the debt issued by financial firms differs significantly from that issued by non-financial ones in that the former is held by widely dispersed, uninformed investors. Covenants are difficult (if not impossible) to enforce for such investors because it is costly for them to monitor the institution and coordinate to effect a transfer of control when covenants are violated. In this respect, regulation serves as a commitment and coordinating mechanism. Consequently, the mechanisms through which a transfer of control is achieved differ for financial and non-financial firms. Further, financial institutions have much higher leverage levels than non-financial firms (Gropp and Heider (2010)), and the trade-off we identify between asset substitution and debt overhang is particularly pronounced at high leverage levels. Consequently, the main implications of our study are much more pertinent to financial institutions.

We study two accounting measurement regimes: a historical cost (*HC*) regime in which the balance sheet of the institution is measured using the original prices of the claims; and a fair value (*FV*) regime in which the balance sheet of the institution—and therefore the prudential constraint—is marked to market every period using the current market prices of the claims. We view the *HC* regime as a benchmark against which we examine the effects of mark-to-market or fair value accounting. We carry out both positive and normative analyses. For each accounting regime, we first examine the effects of a given solvency constraint on the institution’s capital structure, its project quality, and its asset substitution strategy. We then derive the optimal (total enterprise value-maximizing) choice of the prudential constraint by the regulator. Finally, we analyze the
optimal choice of the accounting regime. We next describe the ingredients of the model in more detail.

### 3.2 Project and Capital Structure

There are two periods with three dates $0, 1, 2$. All agents are risk-neutral, but, as we discuss below, could have differing discount rates. At $t = 0$, the institution makes a fixed investment $A_0$ in a long-term project. The institution finances the investment through a combination of debt and equity. Our objective is to study shareholder-debt holder conflicts, especially when the institution’s leverage may be high. Similar to studies such as Giammarino et al. (1993), Heaton et al. (2010), and Mehran and Thakor (2010), there are deadweight costs of equity capital that we model by assuming that equity holders demand a higher (risk-adjusted) expected return on their investment than debt holders. For example, if the institution is an insurance firm, the lower cost of debt capital could arise from the fact that agents have a demand for insurance. The insurance firm’s core business is the provision of insurance so that it has a comparative advantage in supplying insurance that it does not possess in raising equity capital. If the institution is a bank, the lower cost of debt could arise from the fact that investors have a demand for information insensitive and liquid securities such as demand deposits that the bank has a comparative advantage in providing.

Previous literature suggests a number of reasons why financial institutions issue debt and, moreover, have relatively high leverage levels (e.g., see Dewatripont and Tirole (1994), Allen and Gale (1999), Santos (2001)). All these mechanisms have the effect of lowering the “effective” cost of debt relative to equity. Because the economic insights we focus on in this study do not hinge on the particular frictions that give rise to the excess cost of equity, we follow studies such as Giammarino et al. (1993), Heaton et al. (2010), and Mehran and Thakor (2010) by not modeling such frictions to simplify the exposition and analysis. Note that a fairly straightforward way to endogenize the excess cost of equity capital $\lambda$ in our framework would be to explicitly incorporate an agency conflict between the shareholders and the manager of the institution. Specifically, we could assume that the manager derives pecuniary private benefits that are proportional to the cash flows to equity. In such a scenario, debt is a “hard” claim that reduces the firm’s free cash flows that, in turn, provides ex ante incentives to issue debt (e.g., Jensen (1986)).

We normalize the cost of debt to 1 and the cost of equity to $1 + \lambda$ over the period between date
0 and date 2, where \( \lambda \) denotes the excess cost of equity. To simplify notation, we assume that the two periods are of equal length so that the cost of equity over the period between date 1 and date 2 is \( \sqrt{1+\lambda} \). Note that the parameter \( \lambda \) represents the additional deadweight loss of equity financing relative to debt financing; \( \lambda \) is not a risk premium.

Because financial institutions have substantially higher leverage levels relative to industrial firms, their “effective” \( \lambda \) is significantly higher in the context of our model. Gropp and Heider (2010) conduct an empirical analysis of the determinants of the capital structures of large U.S. and European banks. They document that the median book and market leverage ratios of banks in their sample are 92.6% and 87.3%, respectively, while the corresponding median ratios for non-financial firms are 24% and 23%, respectively. They argue that their findings are consistent with banks facing significantly higher excess costs of equity financing compared with non-financial firms, which could explain their substantially higher leverage levels.

For simplicity, we consider debt that pays off at date \( t = 2 \) with no intermediate interest payments. The amount of debt the institution chooses to issue is determined by its payoff/face value \( M \) at maturity. We later endogenize \( M \) when we analyze the institution’s capital structure. Let the market value of the debt at date \( t = 0 \) be \( D_0 \), which is endogenously determined. The institution therefore finances the remaining amount \( E_0 = A_0 - D_0 \) through equity. Capital markets are competitive.

If the institution is a bank, its depositors are protected by deposit insurance in practice. We do not incorporate the presence of deposit insurance in our analysis because, as mentioned earlier, we intend our theory to be applicable to a general financial intermediary whose liabilities need not be protected by deposit insurance. Further, even in the case of banks, a substantial portion of their debt is long-term and uninsured.

It turns out that, even if we restrict ourselves to the specific case in which the institution is a bank and all its debt comprises of insured demand deposits, our implications are unaffected as long as deposit insurance is fairly priced. The reason is that fairly priced deposit insurance—that is, the deposit insurance premium rationally incorporates the institution’s optimal choices of capital structure, project quality, and asset substitution—is merely a transfer of funds from shareholders to debt holders. Shareholders pay the deposit insurance premium to the deposit insurer who, in turn, compensates debt holders if the institution defaults. Consequently, although debt is risk-
free due to deposit insurance, the deposit insurance premium lowers the value of equity so that the value of the institution—the size of the total pie—is unchanged. Furthermore, the deposit insurance premium is a sunk cost that is incurred \textit{ex ante}. Consequently, the \textit{ex post} value of equity—that is, after deposit insurance and capital structure are in place—is identical to its value in the scenario in which there is no deposit insurance. The upshot of these implications is that none of the institution’s decisions—capital structure, project quality, and asset substitution—is affected by the presence of deposit insurance. Because the size of the total pie is unchanged by deposit insurance, the regulator’s objective function is also unaltered. The only result that changes is the magnitude of the optimal solvency constraint which increases with deposit insurance because the value of insured debt is higher than that of uninsured debt.\(^5\)

3.3 Project Quality

The terminal cash flows of the project are realized at date \(t = 2\). The terminal cash flows, which we describe shortly, are affected by both the quality of the project chosen in period 1 and potential asset substitution chosen in period 2. We denote the quality of the institution’s project by \(q \in \{0, q_H\}\) where \(0 < q_H \leq 1\). Without loss of generality, we normalize the low project quality to zero purely to simplify the notation. The project quality is only observable by the manager and shareholders. The institution can always invest in a default long-term project, i.e., in a project with a low quality level 0. By carefully analyzing and screening the type of project that it finances, the institution can raise the quality of its project from 0 to \(q_H\). The resources invested by the shareholders in choosing a project of quality \(q \in \{0, q_H\}\) is a nonnegative random variable \(\tilde{C}(q)\) with support \((0, \infty)\). The expected cost of choosing a project is increasing in its quality. Enhancing the project quality from 0 to \(q_H\) requires the institution’s shareholders to incur an additional expected cost of \(kq_H\). We alternately refer to the additional expected cost \(kq_H\) as the \textit{additional investment} to increase project quality. Consequently,

\[
E[\tilde{C}(q_H) - \tilde{C}(0)] = kq_H. \tag{1}
\]

\(^5\)An analysis of the model with deposit insurance is available upon request. Chan et al. (1992), Giammarino et al. (1993) and Freixas and Rochet (1995) examine the feasibility of fairly priced deposit insurance when there is \textit{adverse selection} regarding the bank’s projects.
3.4 Intermediate Signal and Prudential Constraint

At the interim date $t = 1$, there is a publicly observable signal of the final payoff of the project. The signal $y \in \{X_L, X_H\}$ where $X_H > X_L > 0$. If the quality of the project is $q \in \{0, q_H\}$, then

$$
\Pr(y = X_H) = q \text{ and } \Pr(y = X_L) = 1 - q.
$$

(2)

By (2), the high quality project first-order stochastically dominates the low quality project, that is, the probability of receiving a high intermediate signal is greater with the higher quality project.

At any date $t$, the institution faces a solvency constraint imposed by a regulator, which requires that the value of the institution’s assets be high enough relative to the value of its liabilities. In a fair value accounting regime, where all assets and liabilities are marked to market, the constraint takes the form

$$
\frac{D_t}{A_t} \leq c \text{ where } t \in \{0, 1\},
$$

(3)

where $D_t$ is the market value of the institution’s debt and $A_t = D_t + E_t$ is the market value of the institution’s total assets at date $t$. In (3), the interval $0 \leq c \leq 1$ implies that the institution’s leverage ratio must be below a threshold $c$.\(^6\)

If the prudential constraint is satisfied at date 1—that is, $\frac{D_1}{A_1} \leq c$—the institution’s shareholders maintain control for the second period. However, if it is not satisfied—that is, $\frac{D_1}{A_1} > c$—control transfers to the regulator who closely monitors the institution and ensures that it does not engage in asset substitution. We later describe a benchmark accounting regime that we refer to as the historical cost regime in which the institution’s assets and liabilities are not marked to market.

3.5 Terminal Payoffs

At the beginning of period 2, the shareholders decide whether or not to engage in asset substitution. In particular, given the signal $y = X_i$, where $i \in \{L, H\}$, the shareholders take a hidden action that is represented by the ordered pair $(r, z) \in \{(0, 0), (r_H, z_H)\}$ that alters the distribution of terminal payoffs of the institution. Given $y = X_i$, the terminal payoff of the institution, $\bar{X}$, takes

\(^6\)As we show later, the optimal threshold $c$ may depend on the accounting measurement regime.
two possible values, either \((1 + z)y\) or \((1 - z)y\), where

\[
\Pr(\tilde{X} = (1 + z)y) = \frac{1}{2} - r \\
\Pr(\tilde{X} = (1 - z)y) = \frac{1}{2} + r.
\]  

(4)

We assume that \(0 < r_H \leq \frac{1}{2}\) and \(0 < z_H \leq 1\). Given the asset substitution strategy \((r, z)\) and public signal \(y\), the expected value of the terminal cash flows of the institution is

\[
E(\tilde{X}|y) = (1 - 2rz)y.
\]

From the above, it is clear that the action \((0, 0)\) captures “no asset substitution” because the terminal payoff conditional on the intermediate signal is risk-free and equals the value of the signal. On the other hand, the action \((r_H, z_H)\) captures asset substitution because it injects uncertainty in the terminal payoffs, while simultaneously reducing the expected terminal cash flows of the institution from \(y\) to \((1 - 2r_Hz_H)y\). We choose the two strategies to be “no asset substitution” and “asset substitution” purely to simplify and sharpen the analysis.

To simplify the algebra, we assume a “recombining” binomial tree when asset substitution is chosen in the high and low states. More precisely, the best possible terminal payoff from asset substitution when the intermediate signal is low (i.e., when \(y = X_L\)) equals the worst possible terminal payoff from asset substitution when the intermediate signal is high (i.e., when \(y = X_H\)) so that

\[
(1 - z_H)X_H = (1 + z_H)X_L.
\]  

(5)

We also make the following standing assumption on project parameters:

\[
\frac{1}{1 + \lambda}(1 + z_H)X_L < A_0 < \frac{1}{1 + \lambda}X_H - kq_H.
\]  

(6)

The first inequality implies that, conditional on a low intermediate signal at date 1, even the best possible outcome under asset substitution is not sufficient to recover the initial investment \(A_0\). The second inequality ensures that, conditional on a high intermediate signal at date 1, engaging in no asset substitution has a positive net payoff in the sense that the corresponding terminal
payoff $X_H$ is greater than the sum of the initial investment $A_0$ and the expected incremental cost $kq_H$ of choosing high project quality. Assumption (6) ensures that the inefficiencies due to asset substitution are severe enough for prudential regulation to be relevant.

By (2) and (4), the distribution of terminal cash flows $\tilde{X}$ depends on both the unobservable project quality $q \in \{0, q_H\}$ chosen in period 1 and on the unobservable asset substitution strategy $(r, z) \in \{(0, 0), (r_H, z_H)\}$ chosen in period 2. We refer to period 1 as the investment stage and to period 2 as the asset substitution stage.

Figure 1 summarizes the sequence of events. Figure 2 illustrates how the distribution of terminal cash flows $\tilde{X}$ depends on the institution’s investment $q$ in period 1 and its asset substitution choice $r$ in period 2.

The payoffs of the shareholders and debt holders depend on whether the solvency constraint (3) is violated at the end of period 1 and, therefore, on whether the regulator takes control. If the regulator takes control at $t = 1$, it ensures that the institution chooses the ex post efficient strategy of no asset substitution, that is, it chooses $(r, z) = (0, 0)$ in the second period. The debt holders’ payoffs equal the lower of the face value $M$ of the debt or the terminal payoff $\tilde{X}(q; (r, z))$ of the institution, where we explicitly indicate the dependence of the terminal payoff on the project quality $q$ and the asset substitution strategy $(r, z)$. Shareholders receive the cash flows net of payments to debt holders minus the cost of investment in project quality. Table 1 summarizes the payoffs of the shareholders, the debt holders, and their combined payoffs:
Figure 2: Technology

\[ X_u \xrightarrow{\frac{1}{2} - r_u} (1 - z_u)X_u \]
\[ X_u \xleftarrow{\frac{1}{2} - r_u} (1 - z_u)X_u \]
\[ \frac{1}{2} + r_u \]

\[ X_s \xrightarrow{\frac{1}{2} + r_s} (1 - z_s)X_s \]
\[ X_s \xleftarrow{\frac{1}{2} + r_s} (1 - z_s)X_s \]

0 \quad quality investment stage \quad \text{asset substitution stage} \quad 1

---

Table 1: Payoffs of Debt Holders and Shareholders

<table>
<thead>
<tr>
<th></th>
<th>Institution Maintains Control</th>
<th>Regulator Takes Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date 0</td>
<td>Date 2</td>
<td></td>
</tr>
<tr>
<td>Debt holders’ Payoff</td>
<td>min{M, \tilde{X}(q; (r, z))}</td>
<td>min{M, \tilde{X}(q; (0, 0))}</td>
</tr>
<tr>
<td>Shareholders’ Payoff</td>
<td>{-\tilde{C}(q)} \max{\tilde{X}(q; (r, z)) - M, 0}</td>
<td>\max{\tilde{X}(q; (0, 0)) - M, 0}</td>
</tr>
<tr>
<td>Total Payoff</td>
<td>{-\tilde{C}(q)} \tilde{X}(q; (r, z))</td>
<td>\tilde{X}(q; (0, 0))</td>
</tr>
</tbody>
</table>

Note that the payoffs in the scenario where the regulator takes control reflect the fact that the regulator ensures that the institution chooses the \textit{ex post} efficient continuation strategy—no asset substitution—in the second period, that is, \((r, z) = (0, 0)\).

3.6 Measurement Regimes

We study two accounting measurement regimes. The first regime, which is a benchmark regime, is a \textit{historical cost} regime \((HC)\) in which the institution’s assets and liabilities are measured at their initial date 0 “origination” values. More precisely, in the context of our model, the prudential constraint is given by

\[
\frac{D_0}{A_0} \leq c^{HC}
\]  

(7)

at the initial date \(t = 0\) and the intermediate date \(t = 1\). In the above, \(D_0\) is the initial present value of the institution’s debt and \(A_0\) is the acquisition cost of its assets.
The second regime is the *fair value* regime (*FV*) in which the institution’s balance sheet is marked to market every period so that the solvency constraint is given by (3), that is,

\[
\frac{D_0}{A_0} \leq c^{FV} \text{ at } t = 0 \text{ and } \frac{D_1}{A_1} \leq c^{FV} \text{ at } t = 1. \tag{8}
\]

The superscripts on the solvency constraints in the two regimes reflect the fact that they could differ across the regimes.

In the first best scenario, all decisions are made to maximize the total value of the institution rather than just shareholder value, and the excess cost of equity \(\lambda\) is zero. In this scenario, it is easy to show that the institution always chooses the high quality project and does not engage in asset substitution. Because the incentives of shareholders are aligned with those of creditors, the institution’s capital structure and therefore its leverage play no role. Therefore, accounting measurement issues are moot.

In the second-best world, maximizing shareholder value is not necessarily equivalent to maximizing the total value of the institution (that is, the equity value plus the debt value). We analyze each accounting measurement regime using backward induction. We start at the beginning of period 2 when the public signal has been released. For a given solvency constraint, capital structure, and a given public signal, we first derive the transfer of control decision and asset substitution decision in period 2. Next, we derive the project quality decision in period 1, which anticipates the transfer of control and asset substitution decisions in period 2. Next, we determine the capital structure decision at date 0, which is determined by the choice of the face value of debt. Finally, given the institution’s optimal capital structure, investment, and asset substitution decisions, we derive the optimal solvency constraint that maximizes the *total enterprise value* of the institution.

4 Historical Cost Regime

Under the *HC* regime, because the institution’s assets and liabilities are not remeasured until terminal payoffs are realized, the solvency constraint \(\frac{D_0}{A_0} \leq c\) is used at both \(t = 0\) and \(t = 1\). Therefore, in this “benchmark” *HC* regime, if the solvency constraint is satisfied at date 0, it is automatically satisfied at date 1. Consequently, there is no transfer of control at date 1.
Note that we deliberately study an HC regime as a benchmark to highlight and sharpen the intuition behind our main result—namely the trade-off between asset substitution and debt overhang inefficiencies. In practice, the measurement regime is often a “hybrid” between HC and FV regimes in the sense that there is some reliance on price signals at date 1 either through impairment of the long lived asset and/or the realization of interim cash flows. This, in turn, would imply that the solvency constraint at date 1 would be different from the constraint at date 0 so that transfer of control would, in general, be feasible. Nevertheless, it would still be true that transfer of control is much less likely in such a hybrid regime relative to the FV regime because the solvency constraint is relatively insensitive to interim price signals. Our main implications, which hinge on the significantly higher likelihood of the transfer of control in the FV regime, are unlikely to be significantly altered. In this sense, the HC regime serves as a useful benchmark to analyze the effects of the potential transfer of control in the FV regime. We later show that such an HC regime may still dominate the FV regime under certain circumstances.

4.1 Asset Substitution

At date $t = 1$, given the public signal $y$ and the debt face value $M$, shareholders decide whether to engage in asset substitution by choosing the hidden action $(r, z) \in \{(0, 0); (r_H, z_H)\}$ to maximize

$$
E[\max\{X - M, 0\}|y] = (\frac{1}{2} - r) \max\{(1 + z)y - M, 0\} + (\frac{1}{2} + r) \max\{(1 - z)y - M, 0\}.
$$

**Proposition 1 (Asset Substitution in HC Regime)** Under the HC regime, shareholders choose asset substitution if and only if the maturity value $M$ of debt is sufficiently high, that is, $M > c_0 y$, where $c_0 \equiv 1 - \frac{1}{2} - r_H \frac{z_H}{2 + r_H}$. The intuition for Proposition 1 is well known. Because shareholders effectively hold a call option on the terminal payoff with strike price equal to the face value of debt, it is optimal for them to increase risk by choosing asset substitution when the intermediate signal is sufficiently low relative to the face value of debt.
We also note that, as \( \frac{1}{2} - r_H \) (the probability of a good outcome given asset substitution) and/or \( z_H \) (the spread of outcomes resulting from asset substitution) increases, asset substitution becomes more attractive to shareholders in period 2. Consequently, the threshold value \( c_0 \) of the face value of debt above which asset substitution takes place decreases, that is, asset substitution occurs for a larger range of debt face values.

It follows from the proposition that the propensity to choose asset substitution depends on the leverage level of the financial institution which is endogenous. Furthermore, for high leverage levels, asset substitution is likely in both the good state (\( y = X_H \)) and in the bad state (\( y = X_L \)); an observation that is important for our subsequent analysis.

### 4.2 Project Quality

At date 0, given the face value \( M \) of debt, shareholders choose the project quality \( q \in \{0, q_H\} \) anticipating the asset substitution decision in period 2 given by Proposition 1. In choosing the project quality, shareholders trade off their expected payoff incorporating the period 2 asset substitution decision against the expected investment in project quality. The following proposition describes the optimal choice of project quality by the shareholders.

**Proposition 2 (Project Quality in HC Regime)** Under the HC regime, shareholders choose low project quality if and only if the maturity value \( M \) of debt is sufficiently high: (i) for \( k \leq k^* \), \( q_L \) is chosen if and only if \( M > c_2 X_H \); (ii) for \( k > k^* \), \( q_L \) is chosen if and only if \( M > c_1 X_H \). In the above,

\[
c_1 = 1 - \frac{k(1 + \lambda)}{X_H} ; \quad c_2 = (1 + z_H) - \frac{k(1 + \lambda)}{(\frac{1}{2} - r_H)X_H} ; \quad k^* = \frac{1}{2} - \frac{r_H}{2 + r_H} z_H X_H / (1 + \lambda) .
\]

Proposition 2 states that shareholders choose the low quality project if the face value of debt is sufficiently high. This is essentially a consequence of the well known “debt overhang” problem. If the amount of debt in the institution’s capital structure is sufficiently high, shareholders’ incentives to make a costly investment in the higher quality project are curbed because a larger proportion of the increased total payoff from such an investment accrues to debt holders.

The novel and interesting implication of our analysis is that Propositions 1 and 2 together imply that the debt overhang problem in period 1 is actually alleviated by the possibility of asset
substitution in period 2. The following corollary makes this statement precise.

**Corollary 1 (Asset Substitution and Debt Overhang in the HC Regime)** If \( r_H \) decreases and/or \( z_H \) increases (i) the threshold level of the debt face value above which asset substitution occurs decreases for any value of the intermediate signal \( y \); (ii) for given \( k \), the threshold level of the debt face value above which the low project quality is chosen increases; and (iii) the threshold level \( k^* \) in Proposition 2 increases.

By the discussion following Proposition 1, a decrease in \( r_H \) and/or an increase in \( z_H \) increases the incentives for asset substitution in period 2, that is, the range of debt face values for which asset substitution occurs increases for any value of the intermediate signal. The corollary, however, shows that a decrease in \( r_H \) and/or an increase in \( z_H \) causes the range of debt face values for which the low quality project is chosen to shrink. In other words, an increase in the propensity for asset substitution in the second period increases the likelihood of choosing high project quality in the first period, that is, it alleviates the debt overhang problem by providing incentives to invest in higher project quality. Furthermore, as \( r_H \) decreases and/or \( z_H \) increases, the threshold \( k^* \) in Proposition 2 increases so that the region \( k \leq k^* \) expands while the region \( k > k^* \) shrinks. Therefore, not only does the range of debt face values for which the low quality project is chosen shrink in the presence of asset substitution, but as asset substitution becomes more attractive, the latter effect also persists for larger values of \( k \).
Figure 3 illustrates the corollary via a numerical example. In particular, Figure 3 demonstrates how an increase in the propensity of asset substitution via a decrease in the value of $r_H$ from $\frac{1}{10}$ to $\frac{1}{16}$ affects the incentives to invest in project quality in the HC regime. The top half of Figure 3 illustrates that, as $r_H$ declines from $\frac{1}{10}$ to $\frac{1}{16}$, the range of debt face values $M$ for which asset substitution occurs expands from the interval $M > 0.4y$ to the interval $M > 0.3y$. The bottom half of Figure 3 shows that, following a decline in $r_H$, the corresponding range of values of $M$ for which the low quality project is chosen shrinks from $M > 40$ to $M > 53$. Hence, an increase in the propensity for asset substitution enhances incentives to invest in higher project quality.

The intuition for these results is as follows. At low leverage levels, asset substitution is either non-existent or occurs only in the low state $X_L$. At low leverage levels, however, the debt overhang problem is also nonexistent in that the high project quality is chosen in the first period as shown by Proposition 2. Since asset substitution occurs (if at all) only in the low state where payoffs are low, a change in the incentives for asset substitution triggers little distortion from an ex ante perspective so that the project quality choice in the first period is unaffected. As leverage increases, however, the option to engage in asset substitution becomes more valuable in the high state relative to the low state because the call option in the low state becomes more out of the money relative to the high state. Further, at higher leverage levels, the payoffs from asset substitution are much greater for the high state relative to the low state because the good outcome for the project is realized. Given that the high state is more likely for the high quality project, an increase in the propensity for asset substitution in the second period increases the institution’s incentives to choose the higher project quality in the first period. Furthermore, as asset substitution becomes more profitable in period 2 (i.e., when $r_H$ decreases and/or $z_H$ increases), the call option in the high state becomes even more valuable so that the incentives to choose the higher quality project persist even for large values of $k$.

To summarize, at low leverage levels, the debt overhang and asset substitution problems are both minor so that the low ex post rents from asset substitution have little or no impact on the ex ante project quality choice. At high leverage levels, however, asset substitution also occurs in the good state. The potentially large ex post rents from asset substitution in the good state increase the institution’s incentives to choose the high quality project in the first period, thereby reducing the debt overhang problem.
4.3 Capital Structure

We now analyze the institution’s optimal choice of its capital structure. The bank’s original shareholders (that is, before capital structure is in place) optimally finance the project by issuing debt and equity rationally anticipating the *ex post* project quality and asset substitution choices. In particular, at $t = 0$, the bank’s original shareholders choose the institution’s capital structure to maximize their value subject to the date $t = 0$ solvency constraint (7). The value of original shareholders at date zero equals the market value of equity plus the market value of debt. Recall that the cost of debt is normalized to $1$ and that of equity to $1 + \lambda$. Consequently, the debt face value, which determines the institution’s capital structure, solves

$$
M^{HC} = \arg \max_M \frac{\text{market value of equity}}{1 + \lambda} \left( E(\max\{X - M, 0\}) + E(\min\{M, X\}) \right) - \text{expected investment in project quality} \left( \hat{E}[C(q^{HC})] \right) - \text{initial investment in project} \left( \hat{A}_0 \right)
$$

subject to the $t = 0$ solvency constraint

$$
\frac{D_0}{A_0} \leq c,
$$

where $D_0$ is the market value of debt at date 0. In (10), $q^{HC}$ is the optimal project quality choice and $M^{HC}$ is the optimal debt face value, where the superscripts denote that these are their values in the $HC$ regime.

By (10), $M^{HC}$ balances the trade–off between the excess cost of equity represented by $\lambda$ and the inefficiencies arising from debt overhang and asset substitution due to the presence of debt in the institution’s capital structure. The optimal face value of debt, $M^{HC}$, depends on the underlying parameters of our environment. In particular, as one would expect, $M^{HC}$ increases in $\lambda$, that is, the optimal amount of debt financing increases with the excess cost of equity. The formal characterization of the institution’s optimal capital structure choice is rather messy. Because it is not the central focus of our study, we have relegated the statement of the result on capital structure and its proof to Lemma 1 in the Appendix. The goal of analyzing the optimal capital structure decision is to emphasize that the central trade–off between asset substitution and debt overhang
inefficiencies that we identified earlier holds when capital structure is endogenized.

4.4 Prudential Constraint

From a normative perspective, the regulator chooses the optimal solvency constraint, $c^{HC}$, to maximize the total value of the institution rationally anticipating its subsequent capital structure, project quality, and asset substitution choices. Given that capital markets are competitive, the institution’s original shareholders extract all the surplus from its operations. Therefore, in choosing the optimal solvency constraint, $c^{HC}$, the regulator’s problem of maximizing the total value of the institution is equivalent to maximizing the value of original shareholders subject to the solvency constraint (7).

In the $HC$ regime, the solvency constraint does not have any bite at date $t = 1$ and thus transfer of control never occurs. The prudential constraint is only relevant at $t = 0$ because it constrains the shareholders’ optimal choice of $M$. As discussed in Section 4.3, however, the original shareholders optimally choose $M$ to maximize their value that, as we have argued above, is equivalent to maximizing the value of the institution. Constraining the choice of $M$ via the prudential constraint only reduces the institution’s date 0 value below the unconstrained maximum. Therefore, $c^{HC}$ should be set as high as possible, that is, it should be set to 1.

**Proposition 3 (Optimal Prudential Constraint in HC Regime)** *The optimal prudential constraint in the historical cost regime, $c^{HC}$, is 1.*

To summarize, in the $HC$ regime, because transfer of control does not occur at the end of period 1, there is a prevalence of asset substitution in period 2. Further, as leverage increases, not only does asset substitution become feasible in both the good and the bad state but the differential ex post rents from asset substitution for the high state relative to the low state also increase. These differential rents alleviate the debt overhang problem in period 1 by increasing shareholders’ incentives to invest in the high quality project. Therefore, the more severe the asset substitution problem is in the second period, the less severe the debt overhang problem is in the first period. Finally, in the $HC$ regime, because the prudential constraint only restricts the institution’s choice of capital structure and plays no role once the capital structure is in place, it should be set as high as possible.
To highlight the trade-off between asset substitution and debt overhang, we shut down the possibility of transfer of control by analyzing a benchmark HC regime. The absence of transfer of control implies that the benchmark HC regime is essentially equivalent to a regime with no prudential regulation, that is, we could also interpret the benchmark HC regime as a “no regulation” regime. However, as we mentioned earlier, if we consider a hybrid measurement regime that incorporates interim cash flows and/or impairment of the long-lived asset, then transfer of control becomes feasible at date 1. Nevertheless, it can be shown that, for a reasonable parameterization of the model, transfer of control would still be significantly less likely compared with the FV regime. Our main results hinge on the trade-off between the higher likelihood of transfer of control in the FV regime against the lower incidence of asset substitution.

5 Fair Value Regime

In the fair value regime (FV), the institution’s balance sheet is marked to market every period so that the prudential constraint is

\[
\frac{D_0}{A_0} \leq c \text{ at } t = 0 \text{ and } \frac{D_1}{A_1} \leq c \text{ at } t = 1,
\]

where \(D_t\) and \(A_t\), respectively, denote the market values of the institution’s debt and assets at \(t\). If \(\frac{D_1}{A_1} > c\), the regulator takes control and closely monitors the institution to ensure that there is no asset substitution in period 2.

5.1 Asset Substitution

The analysis of the fair value regime is significantly more complicated than the historical cost regime. By (12), the prudential constraint at date 1, which determines whether transfer of control occurs, depends on the market values of the institution’s debt and assets. These market values are, however, determined in equilibrium along with the institution’s asset substitution strategy.

More precisely, at \(t = 1\), the institution’s asset substitution decision \((r, z)\) is unobservable. Consequently, in order to value the institution’s debt, the capital market forms a conjecture \((\hat{r}, \hat{z})\)
about \((r, z)\). Given the date \(t = 1\) signal \(y\), the capital market values the institution’s debt at

\[
D_1(y, (\hat{r}, \hat{z})) = E[\min\{M, \tilde{X}\}|y, (\hat{r}, \hat{z})]
\]

\[
= (\frac{1}{2} - \hat{r}) \min\{M, (1 + \hat{z})y\} + (\frac{1}{2} + \hat{r}) \min\{M, (1 - \hat{z})y\}. 
\]

Similarly, at date \(t = 1\), the market value of equity is

\[
E_1(y, (\hat{r}, \hat{z})) = E[\max\{\tilde{X}(y, (\hat{r}, \hat{z})), -M, 0\}]
\]

\[
= \left(\frac{1}{2} - \hat{r}\right) \max\{(1 + \hat{z})y - M, 0\} + \left(\frac{1}{2} + \hat{r}\right) \max\{(1 - \hat{z})y - M, 0\}.
\]

These date \(t = 1\) market prices along with the prudential constraint determine whether transfer of control occurs. Given the continuation or control transfer outcome, the institution chooses \((r, z) \in \{(0, 0), (r_H, z_H)\}\) in period 2. If transfer of control occurs at date \(t = 1\), the regulator ensures that the \textit{ex post} efficient continuation strategy, i.e., \((r, z) = (0, 0)\) in period 2 is chosen so that the payoff at date 2 is \(y\). If transfer of control does not occur at date \(t = 1\), then shareholders could choose whether or not to engage in asset substitution. In a rational expectations equilibrium, the market’s conjecture regarding the chosen asset substitution strategy is correct. In other words, given \(D_1(y, (\hat{r}, \hat{z}))\) and \(A_1(y, (\hat{r}, \hat{z}))\) and the prudential constraint \(c\), the institution’s optimal asset substitution strategy \((r, z)\) is indeed \((\hat{r}, \hat{z})\).

The following proposition characterizes the optimal continuation/transfer of control and asset substitution decisions given the debt face value \(M\) and prudential constraint \(c\).

**Proposition 4 (Asset Substitution in FV Regime)** Under the fair value regime, shareholders choose asset substitution if and only if the prudential constraint is less than a threshold and the maturity value of debt lies in an intermediate interval. That is, asset substitution is chosen if and only if \(c_0 < T(c)\) and \(M \in [c_0 y, T(c) y]\), where

\[
 c_0 \equiv 1 - \frac{1 - r_H}{\frac{1}{2} + r_H}z_H; \; T(c) \equiv \frac{c}{\sqrt{1 + \lambda - c(\sqrt{1 + \lambda} - 1)}}.
\]

For \(M < c_0 y\), shareholders choose no asset substitution voluntarily. For \(M > T(c) y\), no asset substitution is chosen because the prudential constraint is violated and transfer of control occurs.
Note that unlike the HC regime, transfer of control occurs in the FV regime if $M$ is sufficiently large relative to $y$. In fact, the preceding proposition shows that, regardless of the value of $c$, as long as $M > T(c)y$, transfer of control occurs. This is a direct consequence of violating the date $t = 1$ solvency constraint. Transfer of control prevents the possibility of asset substitution in period 2.

Furthermore, in the FV regime, for values of $c$ below a threshold, i.e., $T(c) < c_0$ (note that $T(c)$ increases with $c$), either (i) transfer of control occurs or (ii) shareholders retain control and voluntarily do not choose asset substitution in period 2. Consequently, inefficiencies created by asset substitution are eliminated for low values of $c$. In fact, as $\frac{1}{2} - r_H$ and/or $z_H$ increases, $c_0$ shrinks. Therefore as shareholders find asset substitution more enticing in period 2, an even tighter solvency constraint is necessary to eliminate asset substitution.

For relatively high values of $c$, i.e., $T(c) > c_0$, asset substitution only occurs for intermediate values of $M$ relative to $y$. For large values of $M$ relative to $y$, the institution violates the prudential solvency constraint so that transfer of control occurs, and the regulator ensures that no asset substitution is chosen.

To summarize, compared to the HC regime, in which transfer of control is not feasible at $t = 1$, the prevalence of asset substitution is lower in the FV regime because transfer of control occurs for large values of $M$ relative to $y$. A low enough value of $c$ may completely rule out asset substitution. Conversely, a high value of $c$ exacerbates asset substitution. In fact, as $c$ increases so that the solvency constraint is very lax, the FV regime becomes effectively equivalent to the HC regime.

### 5.2 Project Quality

At date 0, given the face value $M$ of the debt, shareholders choose the project quality $q$ anticipating the transfer of control/continuation and asset substitution decisions described in Proposition 4. The following result characterizes shareholders’ optimal choice of project quality at date $t = 0$.

**Proposition 5 (Project Quality in FV Regime)** Under the fair value regime, shareholders choose the low project quality $q_L$ if and only if the maturity value $M$ of debt is sufficiently high.
Define

\[ c_1 \equiv 1 - \frac{k(1 + \lambda)}{X_H}; \quad c_2 \equiv (1 + z_H) - \frac{k(1 + \lambda)}{(1/2 - r_H)X_H}; \quad T(c) \equiv \frac{c}{\sqrt{1 + \lambda - c(\sqrt{1 + \lambda} - 1)}}, \]  \hspace{1cm} (15)

\[ k^* \equiv \frac{1}{\frac{3}{2} + r_H}z_HX_H/(1 + \lambda). \]

(i) For \( k \leq k^* \):

- If \( T(c) > c_2 \), \( q_L \) is chosen if and only if \( M > c_2X_H \).
- If \( T(c) \in [c_1, c_2] \), \( q_L \) is chosen if and only if \( M > T(c)X_H \).
- If \( T(c) < c_1 \), \( q_L \) is chosen if and only if \( M > c_1X_H \).

(ii) For \( k > k^* \): \( q_L \) is chosen if and only if \( M > c_1X_H \).

Note that, unlike the \( HC \) regime in which the solvency constraint plays no direct role in affecting the choice of project quality, Proposition 5 illustrates the crucial role that it plays in the \( FV \) regime in determining the project quality choice \( q \in \{0, q_H\} \) when the marginal cost of investment in project quality is below a threshold \( (k \leq k^*) \). Recall from Proposition 4 that, the smaller \( c \) is, the higher the likelihood of transfer of control. Proposition 5 implies that, the smaller \( c \) is, the higher the likelihood of choosing the low quality project \( q = 0 \); that is, the debt overhang problem is more severe. Taken together, these two propositions imply a positive relationship between transfer of control and the debt overhang problem.

For low values of \( c \) \( (T(c) < c_1) \), the prudential constraint is relatively tight so that the institution is very likely to exceed it. A high likelihood of transfer of control implies that the incidence of asset substitution is very low. Not surprisingly, the \( FV \) regime becomes equivalent to a world in which asset substitution is exogenously ruled out so that shareholders choose a lower project quality (that is, choose \( q = 0 \)) if and only if \( M > c_1X_H \).

For high values of \( c \) \( (T(c) > c_2) \), the prudential constraint is relatively loose so that transfer of control is highly unlikely and the \( FV \) regime becomes equivalent to the \( HC \) regime. In fact, for high values of \( c \), we recover the same result obtained in the \( HC \) regime: shareholders choose a lower project quality if and only if \( M > c_2X_H \).
For intermediate values of $c$ ($T(c) \in [c_1, c_2]$), the threshold $(T(c)X_H)$ of the face value of debt triggering a low project quality decreases. Thus, as the likelihood of transfer of control increases, the debt overhang problem worsens. To understand this result, note that transfer of control in the FV regime shuts down asset substitution and such transfer of control is more likely the higher the leverage of the bank. But as we discussed earlier, this is precisely when the option value of asset substitution is greater for the high state than for the low state! Consequently, shutting down asset substitution via a change in control in the FV regime has a significant negative impact on the project quality choice in the first period.

Furthermore, as shareholders find asset substitution more attractive in period 2, i.e., as $\frac{1}{2} - r_H$ and/or $z_H$ increases, both $c_2$ and $k^*$ increase. In other words, as the ex post rents from asset substitution increase, shareholders find asset substitution in the high state even more valuable so that the positive relationship between transfer of control and the debt overhang problem becomes more pervasive as it applies to a larger set of values of the ceiling $c$ and the marginal cost $k$. The following corollary makes this precise.

**Corollary 2 (Asset Substitution and Debt Overhang in the FV Regime)** As $r_H$ decreases and/or $z_H$ increases, (i) the range of debt face values for which asset substitution occurs increases for each value of the intermediate signal; (ii) for given $k$, the range of debt face values for which the low project quality is chosen shrinks; (iii) the threshold $k^*$ in Proposition 5 increases.
Figure 4 illustrates the corollary using the same numerical example introduced earlier in Figure 3. However, unlike the $HC$ regime, the value of the prudential constraint now matters in the $FV$ regime. We set the value of the prudential constraint, $c = 0.6$. For the chosen parameter values, when $r_H$ declines from $\frac{1}{10}$ to $\frac{1}{16}$, asset substitution incentives increase. For $r_H = \frac{1}{10}$, then asset substitution occurs for $M \in [0.40y, 0.58y]$. For $r_H = \frac{1}{16}$, asset substitution occurs over a larger intermediate range of $M \in [0.30y, 0.58y]$. For $M < 0.30y$, no asset substitution occurs because the debt face value is relatively low so that shareholders have no incentives to asset substitute regardless of the value of $y$. For $M > 0.58y$, the leverage is so high that the prudential solvency constraint is violated. Control transfers to the regulator and asset substitution is shut down. The bottom half of the figure shows that the decline in $r_H$ from $\frac{1}{10}$ to $\frac{1}{16}$ shrinks the range of debt face values for which the debt overhang problem occurs.

The discussion above along with the intuition for Proposition 4 suggests that, while transfer of control at date $t = 1$ mitigates inefficiencies created by asset substitution in period 2, it exacerbates inefficiencies arising from debt overhang due to which the likelihood of choosing low quality project in period 1 increases. As we discuss shortly, the optimal choice of the solvency constraint balances the trade-off between these two sources of inefficiencies while also incorporating the fact that equity capital is costlier than debt capital.

5.3 Optimal Capital Structure and Prudential Constraint

For a given solvency constraint $c$, the institution’s original shareholders choose its capital structure to maximize their value subject to the prudential constraint (11). Unlike the $HC$ regime, in which there is no transfer of control at $t = 1$, in the $FV$ regime, the shareholders’ payoffs are affected by the potential transfer of control at $t = 1$. In Lemma 2 in the Appendix, we analyze the optimization program that determines the institution’s optimal capital structure.

We now turn to the derivation of the optimal solvency constraint in the $FV$ regime. In other words, anticipating the institution’s optimal capital structure, project quality, and asset substitution decisions, how should a regulator set the optimal value of the solvency constraint that maximizes the total value of the institution? In choosing the optimal solvency constraint in the $FV$ regime, the regulator faces a dilemma. Choosing a high value of $c$ (a loose constraint) aggravates the asset substitution problem in period 2 while choosing a low value of $c$ (a tight constraint)
reduces incentives to invest in higher project quality in period 1. Further, choosing low values of $c$ imply that the institution must tilt its capital structure more towards costlier equity capital instead of debt capital.

To illustrate the trade-off in choosing the optimal prudential constraint in the fair value regime, Figure 5 uses the numerical example illustrated in Figure 4 except that we now fix $r_H$ at $\frac{1}{16}$ and change $c$ from 0.5 to 0.6. The top half of Figure 5 shows that as the prudential constraint $c$ increases from 0.5 to 0.6 (i.e., the constraint becomes looser), the range of debt face values for which asset substitution occurs expands. The bottom half of Figure 5 shows that such an increase in $c$ shrinks the range of values of $M$ for which the debt overhang problem occurs.

The next result characterizes the optimal solvency constraint in the $FV$ regime.

**Proposition 6 (Optimal Prudential Constraint in FV Regime)** Under the fair value regime, the optimal solvency constraint, $c^{FV}$ equals $\frac{1}{1+\frac{\lambda}{\sqrt{1+\lambda}}}$.

By setting $c^{FV} = \frac{1}{1+\frac{\lambda}{\sqrt{1+\lambda}}}$, it follows from Proposition 4 and from Proposition 5 that the regulator maximizes the expected value of the institution by reducing the incidence of asset substitution while tolerating the debt overhang problem arising from excessive transfer of control. With this constraint, however, neither inefficiency is completely eliminated, that is, debt overhang and asset substitution inefficiencies both occur at the optimum.

Proposition 6 also shows that the optimal solvency constraint becomes tighter as the excess cost of equity $\lambda$ or the marginal cost of investment in project quality $k$ increase. As $\lambda$ increases,
the institution’s incentives to use debt financing increase so that asset substitution and debt overhang inefficiencies both become more likely. Nevertheless, it turns out that the asset substitution problem is relatively more pernicious than the debt overhang problem. Consequently, the solvency constraint becomes tighter to mitigate asset substitution at the expense of potentially reducing incentives to invest in higher project quality. As \( k \) increases, the debt overhang problem becomes less severe because the NPV of the project decreases. The optimal solvency constraint, therefore, again becomes tighter to mitigate asset substitution.

To summarize, in the \( FV \) regime, because the balance sheet of the institution is marked to market every period, the solvency constraint reflects current market values. The institution therefore faces a threat of transfer of control at the end of period 1. If the institution violates the solvency constraint, transfer of control eliminates the possibility of asset substitution in period 2. However such transfer of control takes place precisely when the option value of asset substitution is potentially high. Therefore the regulator faces a dilemma in choosing the prudential constraint. The tighter (looser) the prudential constraint, the higher (lower) the likelihood of transfer of control. Therefore, to reduce the incidence of asset substitution, the solvency constraint must be tightened. Unfortunately, in doing so, the debt overhang problem is aggravated. The regulator trades off the asset substitution inefficiency against the debt overhang inefficiency.

Our results suggest that the key trade-off between asset substitution and debt overhang problems, and the role that prudential regulation in mediating the distortions arising from them, are particularly pronounced at high leverage levels where both problems are significant, and prudential regulation plays a role. As we mentioned in Section 3.2, financial institutions are characterized by much higher leverage levels (on average) than non-financial firms. Indeed, Gropp and Heider (2010) document that the average leverage ratio of banks is approximately 90%, while that of non-financial firms is only around 25%. In the context of our model, financial institutions have greater effective costs of equity capital \( \lambda \) that induces them to choose higher leverage levels, which is consistent with the empirical findings and discussion in Gropp and Heider (2010). Consequently, even though asset substitution and debt overhang are also relevant for non-financial firms, our results are especially pertinent to financial institutions. We further discuss the relevance of our results in the context of financial and non-financial firms in Section 7.
6 Accounting Measurement, Regulation, and Policy Implications

Our previous results show that, in both accounting measurement regimes, the institution’s total value is reduced by the inefficiencies arising from debt overhang and asset substitution. The solvency constraint mediates these two distortions. In the $HC$ regime, because the balance sheet is not remeasured in the interim date, the solvency constraint has no bite in the interim date. The institution faces no threat of transfer of control so that the incidence of asset substitution is high. In the $FV$ regime, because the balance sheet is marked to market, the solvency constraint serves as a credible threat of transfer of control, thereby alleviating the asset substitution inefficiency pervasive in the $HC$ regime.

Comparing Propositions 1 and 4, we easily see that the incidence of asset substitution is higher in the $HC$ regime than in the $FV$ regime. In fact, for low values of the solvency constraint $c$, $FV$ eliminates asset substitution. The following proposition compares the under-investment or debt overhang problem in the two regimes (recall the definitions of $k^*$, $T(c)$ and $c_2$ in (15).

Proposition 7 (Debt Overhang in the Two Regimes) (i) If the marginal cost of investment is low, i.e., $k \leq k^*$, and the solvency constraint is tight, i.e., $T(c) < c_2$, then (a) the debt overhang problem in the $FV$ regime is worse than that in the $HC$ regime and (b) as $1/2 - r_H$ and/or $z_H$ increases, the debt overhang problem deteriorates in the $FV$ regime relative to the $HC$ regime.

(ii) If $k \leq k^*$ and $T(c) > c_2$, or if $k > k^*$, the extent of debt overhang problem is the same in both regimes.

Note that as the asset substitution problem increases, i.e., $1/2 - r_H$ and/or $z_H$ increases, both $k^*$ and $c_2$ increase so that the debt overhang problem in the $FV$ regime relative to in the $HC$ regime occurs over a larger range of values of parameters. That is, the debt overhang problem in the $FV$ regime worsens.

It follows from the above results that the interplay between asset substitution and debt overhang inefficiencies could cause the $HC$ regime to dominate the $FV$ regime if the solvency constraints in the two regimes do not take their optimal values, $c^{HC}$ and $c^{FV}$, respectively. However, if the two constraints take their optimal values, it easily follows from our analysis that the $FV$ regime unambiguously dominates the $HC$ regime. This is because the regulator operating in the $FV$
regime can always replicate the HC regime by setting a loose enough solvency constraint so that it will not have bite at the interim date and there is no transfer of control as in the HC regime. Consequently, the FV regime can do at least as well as the HC regime.

**Proposition 8 (HC vs. FV)** Suppose that $c^\text{HC} = 1$ and $c^\text{FV} = \frac{1}{1 + \frac{k - X_H}{X_H^2(1 + X_H)}}$. The FV regime always dominates the HC regime.

Note that the optimal solvency constraint, $c^\text{FV}$, in the FV regime depends on the marginal cost of investment in project quality, $k$, and the value of the high signal $X_H$. These parameters are likely to vary across institutions even if they belong to the same category such as commercial banks or insurance firms. The optimal solvency constraint is, therefore, institution-specific. Further, the optimal solvency constraint also depends on the excess cost of equity $\lambda$ that could vary over time and, in particular, with the business cycle.

The above discussion implies that a uniform solvency constraint across institutions may not be optimal. Further, the above result crucially depends on the respective solvency constraints in the HC and FV regimes taking their optimal values. In fact, the following proposition shows that, if the solvency constraint in the FV regime is below a threshold, the HC regime dominates the FV regime.

**Proposition 9 (HC Versus FV)** Suppose that $c^\text{HC} = 1$. There exists $c_2 \in (0, c_1)$ such that for $c \in [0, c_2)$, the HC regime dominates the FV regime.

Proposition 9 shows that, if the solvency constraint in the fair value regime is below a threshold, the historical cost regime would be superior to the fair value regime. This result highlights the importance of tailoring the solvency constraint to the accounting measurement regime. In particular, this result suggests that the proposal to substantially raise capital requirements (tighten solvency constraints) in the Basel III accords and simultaneously require fair value accounting should be implemented with caution.

Our results also imply that the optimal solvency constraint becomes tighter (alternately, the optimal capital requirement increases) as the excess cost of equity $\lambda$ increases. The excess cost of equity could vary over time and, in particular, with the business cycle. In particular, during an upswing in the business cycle, credit is easier/cheaper to obtain so that the excess cost of
equity capital is likely to increase. The reverse is true during a downswing in the business cycle. Consequently, the prediction that the optimal capital requirement increases with the excess cost of equity suggests that capital requirements should be higher during upswings in the business cycle than downswings. Our theory, therefore, supports recent proposals by academics and policymakers for higher capital requirements during booms compared with recessions.

7 Conclusions

In the aftermath of the 2007-2008 financial crisis, the merits and demerits of prudential regulation based on fair value accounting are being actively debated by academics, practitioners, and regulators. Our study contributes to the debate by showing how prudential regulation and accounting measurement interact with the agency conflicts between a financial institution’s shareholders and debt holders. Relative to a benchmark historical cost accounting regime, fair value accounting could alleviate the inefficiencies arising from asset substitution, but exacerbate those due to debt overhang. The subtle, but important, opposing effects of fair value accounting on asset substitution and debt overhang inefficiencies are especially pronounced at high leverage levels that are typical of financial institutions.

The optimal choices of accounting regime and prudential solvency constraint balance the conflicts between shareholders and debt holders while also incorporating the fact that equity capital is costlier than debt capital. Under fair value accounting, the optimal solvency constraint declines with the marginal cost of investment in higher project quality and the excess cost of equity capital relative to debt capital. Our results suggest that a uniform solvency constraint across institutions could be sub-optimal. In fact, we show that, if the solvency constraint in the fair value regime is sub-optimally chosen to be too tight, historical cost accounting actually dominates fair value accounting.

To sharpen the analysis and to highlight the main results in the paper, we developed a two-period binomial model with binary actions. However, we believe that the central trade-off between debt overhang and asset substitution would generalize to a setting with multiple states and multiple actions even though the analysis would be much more complicated. Even in a general setting, the debt overhang and asset substitution problems are either both absent or insignificant at very low
leverage levels. At moderate leverage levels, asset substitution is present, but is not severe in that it occurs only in “bad” states. However, at these moderate leverage levels, the debt overhang problem is also not severe so that high quality projects are chosen anyway. Further, shutting down the possibility of asset substitution in bad states has only a minor impact from an \textit{ex ante} standpoint because payoffs in these states are low to begin with. Consequently, shutting down asset substitution would have only a minor impact on the project quality choice. But, at higher leverage levels, which are more typical of financial institutions and where prudential regulation is relevant, the asset substitution problem is more severe in that asset substitution also occurs in “good” states. Further, at these leverage levels, the expected payoff from asset substitution is much higher in the good states because the corresponding call option is deep out of the money in bad states. Consequently, shutting down asset substitution has a much bigger negative impact on the expected payoffs in the good states than in the bad states. Since payoffs are higher in the good states, this, in turn, has a significant negative impact on the \textit{ex ante} project quality choice.

As discussed above, the key trade-off between asset substitution and debt overhang is particularly pronounced at high leverage levels when both problems are severe, and asset substitution is pervasive in that it occurs in both “good” and “bad” states. The trade-off we identify is especially relevant in the context of the recent financial crisis. Indeed, one of the primary causes of the financial crisis was risky subprime mortgage lending by banks during a period when the economy was booming and credit was cheap. Subprime mortgage lending could be more generally viewed as asset substitution that occurred in “good” states. Our study sheds light on the interactions between pervasive asset substitution and debt overhang inefficiencies and the role that prudential capital regulation based on market values plays in balancing the trade-off between these two inefficiencies.

Appendix

\textbf{Proof of Proposition 1}

If the shareholders choose \((r_H, z_H)\), it follows from (9) that their value is

\[
\left(\frac{1}{2} - r_H\right) \max\{(1 + z_H)y - M, 0\} + \left(\frac{1}{2} + r_H\right) \max\{(1 - z_H)y - M, 0\}. \tag{16}
\]
However, if they choose \((0,0)\), their value is

\[
\max\{y - M, 0\}. \tag{17}
\]

Using expressions (16) and (17), the following table summarizes shareholders’ expected payoff from asset substitution (“AS”) and that from no asset substitution.

For example, for \(M \in [(1 - z_H)y, y]\), with probability \(\frac{1}{2} - r_H\), AS will produce \((1 + z_H)y\), which is larger than \(M\), so shareholders, as a residual claimant, will get \((1 + z_H)y - M\), and with probability \(\frac{1}{2} + r_H\), AS will produce \((1 - z_H)y\), which is smaller than \(M\), so shareholders will get nothing. Therefore, shareholders’ expected payoff from AS is \((\frac{1}{2} - r_H)((1 + z_H)y - M)\). In contrast, no AS will always produce \(y\), which is larger than \(M\), so shareholders will get \(y - M\). Comparing the expected payoff from AS with that from no AS, \((\frac{1}{2} - r_H)((1 + z_H)y - M)\) versus \(y - M\), yields the decision rule: AS if and only if \(M > c_0y\) for \(M \in [(1 - z_H)y, y]\).

<table>
<thead>
<tr>
<th>range of (M)</th>
<th>payoff from AS</th>
<th>payoff from no AS</th>
<th>decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M &lt; (1 - z_H)y)</td>
<td>((\frac{1}{2} - r_H)((1 + z_H)y - M)) (+ (\frac{1}{2} + r_H)((1 - z_H)y - M))</td>
<td>(y - M)</td>
<td>no AS</td>
</tr>
<tr>
<td>(M \in [(1 - z_H)y, y])</td>
<td>((\frac{1}{2} - r_H)((1 + z_H)y - M))</td>
<td>(y - M)</td>
<td>no AS if (M &lt; c_0y)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>AS if (M &gt; c_0y)</td>
</tr>
<tr>
<td>(M \in [y, (1 + z_H)y])</td>
<td>((\frac{1}{2} - r_H)((1 + z_H)y - M))</td>
<td>0</td>
<td>AS</td>
</tr>
<tr>
<td>(M &gt; (1 + z_H)y)</td>
<td>0</td>
<td>0</td>
<td>AS</td>
</tr>
</tbody>
</table>

The above table implies the following:

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of (M)</td>
</tr>
<tr>
<td>(M &lt; c_0y)</td>
</tr>
<tr>
<td>(M \in [c_0y, (1 + z_H)y])</td>
</tr>
<tr>
<td>(M &gt; (1 + z_H)y)</td>
</tr>
</tbody>
</table>

**Proof of Proposition 2**
The shareholders’ expected payoff from $q$ at date $t = 0$ is their expected payoff from asset substitution decision (given in Table 2 in the proof of Proposition 1) minus the cost of investment in $q$. The shareholders’ expected payoffs from choosing $q$ are summarized below for all the feasible values of $M$.

For example, for $M \in [c_0 X_H, (1 + z_H)X_H]$, we know from Table 2 that shareholders will engage in asset substitution both when $y = X_H$ and when $y = X_L$. If $y = X_H$ (which occurs with probability $q$), Table 2 tells us that the payoff is $(\frac{1}{2} - r_H)[(1 + z_H)X_H - M]$; if $y = X_L$ (which occurs with probability $1 - q$), Table 2 tells us that the payoff is $0$. Therefore, shareholders’ expected payoff from asset substitution is $q(\frac{1}{2} - r_H)[(1 + z_H)X_H - M]$. Discounting this payoff to its present value at Date 0 by the cost of equity of $1 + \lambda$ and subtracting the cost of quality investment of $kq$ yields the shareholders’ expected payoff from $q$ for this range of $M$: $-kq + \frac{1}{1+\lambda}q(\frac{1}{2} - r_H)[(1 + z_H)X_H - M]$.

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>asset substitution decision</th>
<th>shareholders’ expected payoff from $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; c_0 X_L$</td>
<td>$(0, 0)$ if $y = X_H$</td>
<td>$-kq + \frac{1}{1+\lambda}q[X_H - M]$</td>
</tr>
<tr>
<td></td>
<td>$(0, 0)$ if $y = X_L$</td>
<td>$+(1 - q)[X_L - M]$</td>
</tr>
<tr>
<td>$M \in [c_0 X_L, (1 + z_H)X_L]$</td>
<td>$(0, 0)$ if $y = X_H$</td>
<td>$-kq + \frac{1}{1+\lambda}q[X_H - M]$</td>
</tr>
<tr>
<td></td>
<td>$(r_H, z_H)$ if $y = X_L$</td>
<td>$+(1 - q)(\frac{1}{2} - r_H)[(1 + z_H)X_L - M]$</td>
</tr>
<tr>
<td>$M \in [(1 + z_H)X_L, c_0 X_H]$</td>
<td>$(0, 0)$ if $y = X_H$</td>
<td>$-kq + \frac{1}{1+\lambda}q[X_H - M]$</td>
</tr>
<tr>
<td></td>
<td>$(r_H, z_H)$ if $y = X_L$</td>
<td></td>
</tr>
<tr>
<td>$M \in [c_0 X_H, (1 + z_H)X_H]$</td>
<td>$(r_H, z_H)$ if $y = X_L$</td>
<td>$-kq + \frac{1}{1+\lambda}q(\frac{1}{2} - r_H)[(1 + z_H)X_H - M]$</td>
</tr>
</tbody>
</table>

Using the above table, we can investigate the shareholders’ quality decision by comparing the shareholders’ expected payoff from $q_H$ and that from $q_L$ and derive the following decision rules:

Case 1: $M < c_0 X_L$: Shareholders choose $q_H$ if and only if $k < \frac{1}{1+\lambda}(X_H - X_L)$, which is true by assumption in (6). Therefore, shareholders will choose $q_H$.

Case 2: $M \in [c_0 X_L, (1 + z_H)X_L]$: Shareholders choose $q_H$ if and only if $M < \frac{X_H - (\frac{1}{2} - r_H)(1 + z_H)X_H - k(1 + \lambda)}{\frac{1}{2} + r_H}$.

Case 3: $M \in [(1 + z_H)X_L, c_0 X_H]$: Shareholders choose $q_H$ if and only if $M < c_1 X_H$.

Case 4: $M \in [c_0 X_H, (1 + z_H)X_H]$: Shareholders choose $q_H$ if and only if $M < c_2 X_H$.

Using the above results, we derive the optimal choice of $q$ for different values of $k$.

(i) $k < k^*$: Shareholders choose $q_H$ if and only if $M < c_2 X_H$.

Proof: In Case 1, shareholders will always choose $q_H$. In Case 2, shareholders choose $q_H$ if
and only if $M < \frac{X_H - \frac{1}{2}(1+r_H)(1+z_H)X_L - k(1+\lambda)}{\frac{1}{2}+r_H}$. But even for the highest possible value of $M$ in Case 2, $(1 + z_H)X_L$, that inequality always holds as long as $k < X_H - (1 + z_H)X_L$. Because of the assumption of $(1 + z_H)X_L = (1 - z_H)X_H$, $k < X_H - (1 + z_H)X_L$ if and only if $k < z_H X_H$, which is satisfied because $k < k^*$. So $q_H$ will be chosen. In Case 3, shareholders choose $q_H$ if and only if $M < c_1 X_H$. But even for the highest possible value of $M$ in Case 3, $c_0 X_L$, that inequality always holds as long as $k < k^*$, which is true by the assumption for $k$ in this scenario (i). So $q_H$ will be chosen. In Case 4, shareholders choose $q_H$ if and only if $M < c_2 X_H$, which is exactly stated in the statements of this proposition.

(ii) $k \in [k^*, (X_H - (1 + z_H)X_L)/(1 + \lambda)]$: shareholders choose $q_H$ if and only if $M < c_1 X_H$. (The proof is analogous to that in case (i) and so is omitted.)

(iii) $k > (X_H - (1 + z_H)X_L)/(1 + \lambda)$: This case is infeasible by the assumption in (6).

We summarize the shareholders’ expected payoff from the optimal choice of $q$ in the following:

| Table 3 (the case of $k < k^*$) |
|--------------------------|----------|-------------------|
| range of $M$              | choice of $q$ | shareholders’ expected payoff from the optimal choice of $q$ |
| $M < c_0 X_L$             | $q_H$     | $-kq_H + \frac{1}{1+\lambda} \{q_H X_H + (1 - q_H)X_L - M\}$ |
| $M \in [c_0 X_L, (1 + z_H)X_L]$ | $q_H$     | $-kq_H + \frac{1}{1+\lambda} \{q_H X_H + (1 - q_H)(\frac{1}{2} + r_H)(1 + z_H)X_L$ |
| $M \in [(1 + z_H)X_L, c_0 X_H]$ | $q_H$     | $-kq_H + \frac{1}{1+\lambda} q_H [X_H - M]$ |
| $M \in [c_0 X_H, c_2 X_H]$ | $q_H$     | $-kq_H + \frac{1}{1+\lambda} q_H (\frac{1}{2} - r_H)(1 + z_H)X_H - M]$ |
| $M \in [c_2 X_H, (1 + z_H)X_H]$ | $q_L$     | 0 |

The payoff for $k > k^*$ is similar and so is omitted.

**Lemma 1** Under the historical cost regime, the shareholders’ optimal choice of the maturity value $M$ of debt is as follows:

<table>
<thead>
<tr>
<th>$\lambda &lt; \lambda_1$ :</th>
<th>$\lambda \in [\lambda_1 , \lambda_2]$ :</th>
<th>$\lambda &gt; \lambda_2$ :</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = c_0 X_L$</td>
<td>$M = c_0 X_H$</td>
<td>$M = c_2 X_H$</td>
</tr>
</tbody>
</table>
where

$$
c_0 \equiv 1 - \frac{1}{2} - r_H z_H; \ c_2 \equiv (1 + z_H) - \frac{k(1 + \lambda)}{(1 - r_H) X_H};
$$

$$
\lambda_1 \equiv \frac{2 r_H z_H}{(1 - 2 r_H z_H)} = -c_0 \frac{(1 - q_H) X_H / X_L}{1 - q_H}; \ \lambda_2 \equiv \frac{2 r_H z_H}{(k^* - k) / X_H}.
$$

**Proof of Lemma 1**

We use (10) to derive the shareholders’ expected payoff at the time when they make capital structure decision. This payoff is the shareholders’ expected payoff from quality decision (given in Table 3 in the proof of Proposition 2) minus $E_0$, the shareholders’ equity investment, which equals $A_0 - D_0$. Therefore, in the following, we first derive $D_0$, the equilibrium debt price at Date 0, and then substitute this price into the shareholders’ expected payoff, and finally derive the shareholders’ optimal choice of $M$.

(i) The case where $k < k^*$:

Under the historical cost regime, taking into consideration of the optimal choices of $(r, z)$ and $q$, we first derive $D_0$ using (10). Table 4 summarizes various values of $D_0$ for different ranges of $M$.

For example, for $M \in [(1 + z_H) X_L, c_0 X_H]$, we know from Table 3 that shareholders will choose $q_H$ and therefore $D_0 = q_H E[M, X(q_H, (0, 0)) + (1 - q_H) E[M, X(q_H, (r_H, z_H))]$. Because $X(q_H, (0, 0)) = X_H > M$, debt holders expect to receive $M$; because $X(q_H, (r_H, z_H)) = (1 - 2 r_H z_H) X_L < M$, debt holders expect to receive $(1 - 2 r_H z_H) X_L$. Therefore, $D_0 = q_H M + (1 - q_H)(1 - 2 r_H z_H) X_L$.

<table>
<thead>
<tr>
<th>Table 4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $M$</td>
<td>$D_0$</td>
</tr>
<tr>
<td>$M \leq c_0 X_L$</td>
<td>$M$</td>
</tr>
<tr>
<td>$M \in [c_0 X_L, (1 + z_H) X_L]$</td>
<td>$[q_H + (1 - q_H)(\frac{1}{2} - r_H)] M + (1 - q_H)(\frac{1}{2} + r_H)(1 - z_H) X_L$</td>
</tr>
<tr>
<td>$M \in [(1 + z_H) X_L, c_0 X_H]$</td>
<td>$q_H M + (1 - q_H)(1 - 2 r_H z_H) X_L$</td>
</tr>
<tr>
<td>$M \in [c_0 X_H, c_2 X_H]$</td>
<td>$q_H (\frac{1}{2} - r_H) M + q_H (\frac{1}{2} + r_H)(1 - z_H) X_H + (1 - q_H)(1 - 2 r_H z_H) X_L$</td>
</tr>
<tr>
<td>$M \in [c_2 X_H, (1 + z_H) X_H]$</td>
<td>$(1 - 2 r_H z_H) X_L$</td>
</tr>
</tbody>
</table>

Substituting $D_0$ in Table 4 into the shareholders’ expected payoff, which is the shareholders’ expected payoff from quality decision (given in Table 3 in the proof of Proposition 2) minus $E_0 = A_0 - D_0$, yields the following:
Comparing those three payoffs demonstrates that shareholders prefer $\pi^{HC}_I$ for low values of $\lambda$, $\pi^{HC}_{II}$ for high values of $\lambda$, and $\pi^{HC}_{III}$ for intermediate values of $\lambda$. Note that $\pi^{HC}_I$, $\pi^{HC}_II$, and $\pi^{HC}_{III}$ are all decreasing in $\lambda$, and that $\lambda = 0$, $\pi^{HC}_I > \pi^{HC}_{II} > \pi^{HC}_{III}$.

Because regional payoffs are increasing in $M$, the optimal value of $M$ for a given region is the upper bound of that region. Substituting the regional optimal $M$ into the regional payoff function yields the regional maximal expected payoffs for shareholders at Date 0. For example, for the region of $M \leq c_0X_L$, the shareholders’ expected payoff is increasing in $M$ and so the optimal value of $M$ for this particular region is $c_0X_L$. Inserting $M = c_0X_L$ into the payoff function for this region yields $-A_0 - kq_h + \frac{1}{1+\lambda}q_hX_h + (1 - q_h)(1 - r_h)X_L + \frac{\lambda}{1+\lambda}M$. Similar analyses apply to other regions. Note that the payoff in the third region is always larger than that in the second region, and therefore we combined those two regions. In addition, the last region is never optimal because the payoff of $-A_0 + (1 - 2r_Hz_H)X_L$ is negative by the assumption that a project with a low quality and asset substitution is a negative NPV project.

### Table 5

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>choice of $M$</th>
<th>Shareholders’ ex ante payoff</th>
</tr>
</thead>
</table>
| $M \leq c_0X_L$ | $c_0X_L$ | $\pi^{HC}_I \equiv -A_0 - kq_h + \frac{1}{1+\lambda}(q_hX_h + (1 - q_h)X_L)$  
+ $\frac{\lambda}{1+\lambda}(1 - \frac{1}{2+r_h}z_H)X_L$ |
| $M \in [c_0X_L, c_0X_H]$ | $c_0X_H$ | $\pi^{HC}_{II} \equiv -A_0 - kq_h + \frac{1}{1+\lambda}(q_hX_h + (1 - q_h)(1 - 2r_Hz_H)X_L)$  
+ $\frac{\lambda}{1+\lambda}(q_h(1 - \frac{1}{2+r_h}z_H)X_h + (1 - q_h)(1 - 2r_Hz_H)X_L)$ |
| $M \in [c_0X_H, c_2X_H]$ | $c_2X_H$ | $\pi^{HC}_{III} \equiv -A_0 - kq_h$  
+ $\frac{1}{1+\lambda}(q_h(1 - 2r_Hz_H)X_h + (1 - q_h)(1 - 2r_Hz_H)X_L)$  
+ $\frac{\lambda}{1+\lambda}(q_h[1 - 2r_Hz_H]X_h - k(1 + \lambda)) + (1 - q_h)(1 - 2r_Hz_H)X_L)$ |

Note that $\pi^{HC}_I$, $\pi^{HC}_{II}$, and $\pi^{HC}_{III}$ are all decreasing in $\lambda$, and at $\lambda = 0$, $\pi^{HC}_I > \pi^{HC}_{II} > \pi^{HC}_{III}$.
for high values of $\lambda$, and $\pi_{II}^{HC}$ for intermediate values of $\lambda$:

$$\pi_{I}^{HC} > \pi_{II}^{HC} \iff \lambda < \lambda_1;$$

$$\pi_{II}^{HC} > \pi_{III}^{HC} \iff \lambda > \lambda_2.$$

Therefore, the value of $\lambda$ dictates the choice of $M$. For example, when $\lambda \in [\lambda_1, \lambda_2]$, shareholders prefer $\pi_{II}^{HC}$. To induce it, they set $M$ to be $c_0X_H$.

(ii) The case where $k > k^*$: The analysis of this case is analogous to that of the preceding case and so is omitted. □

**Proof of Proposition 4**

The outcome of control transfer or continuation depends on the prudential constraint ($c$) and the debt/asset ratio at Date 1 ($\frac{D_1}{A_1}$). For the feasible range of values of $M$, using equations (??) through (??), Table 6 shows the corresponding market values $D_1$ and $A_1$:

<table>
<thead>
<tr>
<th>Table 6 Part A</th>
<th>$(\tilde{r}, \tilde{z}) = (r_H, z_H)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $M$</td>
<td>$(\tilde{r}, \tilde{z}) = (r_H, z_H)$</td>
</tr>
</tbody>
</table>
| $M < (1 - z_H)y$ | $D_1 = M$ and $E_1 = \frac{1 - 2r_H z_H y - M}{\sqrt{1 + \lambda}}$  
$\implies \frac{D_1}{A_1} = \frac{D_1}{D_1 + E_1} < c \iff M < \frac{c(1 - 2r_H z_H y)}{\sqrt{1 + \lambda - c(1 + \lambda - 1)}}$ |
| $M \in [(1 - z_H)y, (1 + z_H)y]$ | $D_1 = \frac{1}{2} - r_H)M + \frac{1}{2} + r_H)(1 - z_H)y$ and $E_1 = \frac{\frac{1}{2} - r_H)((1 + z_H)y - M)}{\sqrt{1 + \lambda}}$  
$\implies \frac{D_1}{A_1} = \frac{D_1}{D_1 + E_1} < c \iff M < \frac{c(\frac{1}{2} - r_H)((1 + z_H)y - (1 - c)\sqrt{1 + \lambda})}{\sqrt{1 + \lambda - c(1 + \lambda - 1)}}$ |
| $M > (1 + z_H)y$ | $D_1 = 1 - 2r_H z_H y$ and $E_1 = 0$  
$\implies \frac{D_1}{A_1} = \frac{D_1}{A_1 + E_1} = 1 \geq c$ |

<table>
<thead>
<tr>
<th>Table 6 Part B</th>
<th>$(\tilde{r}, \tilde{z}) = (0, 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of $M$</td>
<td>$(\tilde{r}, \tilde{z}) = (0, 0)$</td>
</tr>
</tbody>
</table>
| $M < y$        | $D_1 = M$ and $E_1 = \frac{y - M}{\sqrt{1 + \lambda}}$  
$\implies \frac{D_1}{A_1} = \frac{D_1}{D_1 + E_1} < c \iff M < T(c)y$ |
| $M > y$        | $D_1 = y$ and $E_1 = 0$  
$\implies \frac{D_1}{A_1} = \frac{D_1}{D_1 + E_1} = 1 \geq c$ |

Note that Table 6 implies the following three general facts:
(i) Given $(\tilde{r}, \tilde{\omega}) = (r_H, z_H)$, for $M > (1 + z_H)y$, transfer of control will occur for sure, for $M \in [(1 - z_H)y, (1 + z_H)y]$, transfer of control will occur if and only if

$$M > \frac{c(\frac{1}{2} - r_H)(1+z_H)-(1-c)\sqrt{1+\lambda - c(1+\lambda - 1)(1-z_H)}}{\sqrt{1+\lambda - c(1+\lambda - 1)(1-z_H)}}y,$$

and for $M < (1 - z_H)y$, transfer of control will occur if and only if $M > \frac{c(1-2r_H+2y)}{\sqrt{1+\lambda - c(1+\lambda - 1)}}y$.

(ii) Given $(\tilde{r}, \tilde{\omega}) = (0, 0)$, for $M > y$, transfer of control will occur for sure, and for $M < y$, transfer of control will occur if and only if $M > T(c)y$.

Claim: When $T(c)y \leq c_0 y$, shareholders will choose $(r, z) = (0, 0)$.

**Proof.** Suppose the conjecture is that $(\tilde{r}, \tilde{\omega}) = (0, 0)$.

For $M > y$, transfer of control will occur and so $(r, z) = (0, 0)$.

For $M < y$, transfer of control will occur and so $(r, z) = (0, 0)$ if $M > T(c)y$ and continuation will occur if $M < T(c)y$. In the latter case, because $T(c)y \leq c_0 y$, by Proposition 1, shareholders will choose $(r, z) = (0, 0)$.

Altogether, shareholders will choose $(r, z) = (0, 0)$ in all cases, thereby confirming the conjecture.

Claim: When $T(c)y > c_0 y$, shareholders will choose $(r, z) = (r_H, z_H)$ if $M \in [c_0 y, T(c)y]$ and choose $(r, z) = (0, 0)$ in all other cases.

**Proof.** (i) Suppose the conjecture is that $(\tilde{r}, \tilde{\omega}) = (0, 0)$ when $M > T(c)y$.

For $M > y$, transfer of control will occur and so $(r, z) = (0, 0)$. For $M \in [T(c)y, y]$, transfer of control will occur and so $(r, z) = (0, 0)$. Altogether, shareholders will choose $(r, z) = (0, 0)$ in both cases, thereby confirming the conjecture.

(ii) Suppose the conjecture is that $(\tilde{r}, \tilde{\omega}) = (0, 0)$ when $M < c_0 y$.

Continuation will occur because $M < c_0 y < T(c)y$. By Proposition 1, shareholders will choose $(r, z) = (0, 0)$, thereby confirming the conjecture.

(iii) Suppose the conjecture is that $(\tilde{r}, \tilde{\omega}) = (r_H, z_H)$ when $M \in [c_0 y, T(c)y]$.

Note that $[c_0 y, T(c)y] \in [(1 - z_H)y, \frac{c(\frac{1}{2} - r_H)(1+z_H)-(1-c)\sqrt{1+\lambda - c(1+\lambda - 1)(1-z_H)}}{\sqrt{1+\lambda - c(1+\lambda - 1)(1-z_H)}}y]$. Therefore, continuation will occur. Because $M > c_0 y$, by Proposition 1, shareholders will choose $(r, z) = (r_H, z_H)$, thereby confirming the conjecture.

The above two claims imply the following:
Table 7

When $T(c)y > c_0y$

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>decision</th>
<th>payoff from decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; c_0y$</td>
<td>no AS</td>
<td>$y - M$</td>
</tr>
<tr>
<td>$M \in [c_0y, T(c)y]$</td>
<td>AS</td>
<td>$(\frac{1}{2} - r_H)[(1 + z_H)y - M]$</td>
</tr>
<tr>
<td>$M \in [T(c)y, y]$</td>
<td>no AS</td>
<td>$y - M$</td>
</tr>
<tr>
<td>$M \in [y, (1 + z_H)y]$</td>
<td>no AS</td>
<td>0</td>
</tr>
</tbody>
</table>

When $T(c)y \leq c_0y$

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>decision</th>
<th>payoff from decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; y$</td>
<td>no AS</td>
<td>$y - M$</td>
</tr>
<tr>
<td>$M \in [y, (1 + z_H)y]$</td>
<td>no AS</td>
<td>0</td>
</tr>
</tbody>
</table>

Proof of Proposition 5

The shareholders’ expected payoff from choosing $q$ at date $t = 0$ is their expected payoff from asset substitution decision (given in Table 7 in the proof of Proposition 4) minus the cost of quality investment.

For low values of $T(c) \leq c_0$, the payoffs are summarized in Table 8 Part A, and for high values of $T(c) > c_0$, the payoffs are summarized in Table 8 Part B.

Table 8 Part A: $T(c) \leq c_0$

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>shareholders’ expected payoff from choosing $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; X_L$</td>
<td>no AS regardless of $y$</td>
</tr>
<tr>
<td>region 1</td>
<td>$-kq + \frac{1}{1+x}(q[X_H - M] + (1-q)[X_L - M])$</td>
</tr>
<tr>
<td>$M \in [X_L, X_H]$</td>
<td>no AS regardless of $y$</td>
</tr>
<tr>
<td>region 2</td>
<td>$-kq + \frac{1}{1+x}q[X_H - M]$</td>
</tr>
<tr>
<td>$M \in [X_H, (1 + z_H)X_H]$</td>
<td>no AS regardless of $y$</td>
</tr>
</tbody>
</table>
Table 8 Part B: $T(c) > c_0$

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>shareholders’ expected payoff from choosing $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &lt; c_0 X_L$ region 1</td>
<td>no AS regardless of $y$</td>
</tr>
<tr>
<td></td>
<td>$-kq + \frac{1}{1+\lambda}(q[X_H - M] + (1-q)[X_L - M])$</td>
</tr>
<tr>
<td>$M \in [c_0 X_L, T(c) X_L]$ region 2</td>
<td>no AS if $y = X_H$ and AS if $y = X_L$</td>
</tr>
<tr>
<td></td>
<td>$-kq + \frac{1}{1+\lambda}(q[X_H - M] + (1-q)(\frac{1}{2} - r_H)(1 + z_H)X_L - M)$</td>
</tr>
<tr>
<td>$M \in [T(c) X_L, X_L]$ region 3</td>
<td>no AS regardless of $y$</td>
</tr>
<tr>
<td></td>
<td>$-kq + \frac{1}{1+\lambda}(q[X_H - M] + (1-q)[X_L - M])$</td>
</tr>
<tr>
<td>$M \in [X_L, c_0 X_H]$ region 4</td>
<td>no AS regardless of $y$</td>
</tr>
<tr>
<td></td>
<td>$-kq + \frac{1}{1+\lambda}q[X_H - M]$</td>
</tr>
<tr>
<td>$M \in [c_0 X_H, T(c) X_H]$ region 5</td>
<td>AS if $y = X_H$ and no AS if $y = X_L$</td>
</tr>
<tr>
<td></td>
<td>$-kq + \frac{1}{1+\lambda}q[X_H - M]$</td>
</tr>
<tr>
<td>$M \in [T(c) X_H, X_H]$ region 6</td>
<td>no AS regardless of $y$</td>
</tr>
<tr>
<td></td>
<td>$-kq + \frac{1}{1+\lambda}q[X_H - M]$</td>
</tr>
<tr>
<td>$M \in [X_H, (1 + z_H) X_H]$ region 7</td>
<td>no AS regardless of $y$</td>
</tr>
<tr>
<td></td>
<td>$-kq$</td>
</tr>
</tbody>
</table>

Using Table 8, we derive the following optimal quality choices in the $FV$ regime.

We first analyze the case where $T(c) \leq c_0$. It is obvious that shareholders always choose $q_H$ in region 1 and $q_L$ in region 3. In region 2, the shareholders choose $q_H$ if and only if $M < c_1 X_H$.

For the case where $T(c) > c_0$, using Table 8, a similar analysis can be done. It is obvious that shareholders always choose $q_H$ in regions 1, 2, and 3 and $q_L$ in region 7. In regions 4 and 6, the shareholders choose $q_H$ if and only if $M < c_1 X_H$. In region 5, the shareholders choose $q_H$ if and only if $M < c_2 X_H$.

(i) $k < k^*$: In regions 1, 2, and 3, shareholders always choose $q_H$. Furthermore, $k < k^*$ implies that $c_2 X_H > c_1 X_H$, which implies that shareholders choose $q_H$ in region 4. The choices of $q$ in regions 5 and 6 depend on the values of $c$. When $T(c) < c_1$, shareholders always choose $q_H$ in region 5 but chooses $q_H$ in region 6 if and only if $M < c_1 X_H$. When $T(c) > c_2$, shareholders always choose $q_L$ in region 6 but chooses $q_H$ in region 5 if and only if $M < c_2 X_H$. When $T(c) \in [c_1, c_2]$, shareholders always choose $q_H$ in region 5 and $q_L$ in region 6; therefore, the cutoff value of $M$
dividing the $q_H$ and $q_L$ regions is the boundary of regions 5 and 6, that is, $T(c)X_H$. Finally, shareholders always choose $q_L$ in region 7.

We summarize the shareholders’ expected payoff from the optimal choice of $q$ in the following, where payoffs in Scenarios A, C, and D are expressed in terms of payoffs in Scenario B:

<table>
<thead>
<tr>
<th>Table 9 Scenario A: $T(c)$</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(c) \leq c_0$</td>
<td>$q_H$</td>
<td>$\pi_3$</td>
</tr>
<tr>
<td>$M &lt; X_L$</td>
<td>$q_H$</td>
<td>$\pi_6$</td>
</tr>
<tr>
<td>$M \in [X_L, c_1X_H]$</td>
<td>$q_H$</td>
<td>$\pi_7$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 9 Scenario B: $T(c)$</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(c) \in [c_0, c_1]$</td>
<td>$q_H$</td>
<td>$\pi_1 = \frac{1}{1+r} \left{ q_HX_H + (1 - q_H)X_L - M \right} - kq_H$</td>
</tr>
<tr>
<td>$M \in [c_0X_L, T(c)X_L]$</td>
<td>$q_H$</td>
<td>$\pi_2 = \frac{1}{1+r} \left{ q_HX_H + (1 - q_H)(\frac{1}{2} - r_H)(1 + z_H)X_L - [q_H + (1 - q_H)(\frac{1}{2} - r_H)]M \right} - kq_H$</td>
</tr>
<tr>
<td>$M \in [T(c)X_L, X_L]$</td>
<td>$q_H$</td>
<td>$\pi_3 = \frac{1}{1+r} \left{ q_HX_H + (1 - q_H)X_L - M \right} - kq_H$</td>
</tr>
<tr>
<td>$M \in [X_L, c_0X_H]$</td>
<td>$q_H$</td>
<td>$\pi_4 = \frac{1}{1+r} \left{ q_H[X_H - M] \right} - kq_H$</td>
</tr>
<tr>
<td>$M \in [c_0X_H, T(c)X_H]$</td>
<td>$q_H$</td>
<td>$\pi_5 = \frac{1}{1+r} \left{ q_H(\frac{1}{2} - r_H)((1 + z_H)X_H - M) \right} - kq_H$</td>
</tr>
<tr>
<td>$M \in [T(c)X_H, c_1X_H]$</td>
<td>$q_H$</td>
<td>$\pi_6 = \frac{1}{1+r} \left{ q_H[X_H - M] \right} - kq_H$</td>
</tr>
<tr>
<td>$M \in [c_1X_H, (1 + z_H)X_H]$</td>
<td>$q_L$</td>
<td>$\pi_7 = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 9 Scenario C: $T(c)$</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(c) \in [c_1, c_2]$</td>
<td>$q_H$</td>
<td>$\pi_1$</td>
</tr>
<tr>
<td>$M &lt; c_0X_L$</td>
<td>$q_H$</td>
<td>$\pi_2$</td>
</tr>
<tr>
<td>$M \in [c_0X_L, T(c)X_L]$</td>
<td>$q_H$</td>
<td>$\pi_3$</td>
</tr>
<tr>
<td>$M \in [T(c)X_L, X_L]$</td>
<td>$q_H$</td>
<td>$\pi_4$</td>
</tr>
<tr>
<td>$M \in [X_L, c_0X_H]$</td>
<td>$q_H$</td>
<td>$\pi_5$</td>
</tr>
<tr>
<td>$M \in [c_0X_H, T(c)X_H]$</td>
<td>$q_H$</td>
<td>$\pi_7$</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Table 9 Scenario D:</th>
<th>choice of $q$</th>
<th>shareholders’ expected payoff from the optimal choice of $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(c) &gt; c_2$</td>
<td>$q_H$</td>
<td>$\pi_1$</td>
</tr>
<tr>
<td>$M &lt; c_0 X_L$</td>
<td>$q_H$</td>
<td>$\pi_2$</td>
</tr>
<tr>
<td>$M \in [c_0 X_L, T(c) X_L]$</td>
<td>$q_H$</td>
<td>$\pi_3$</td>
</tr>
<tr>
<td>$M \in [T(c) X_L, X_L]$</td>
<td>$q_H$</td>
<td>$\pi_4$</td>
</tr>
<tr>
<td>$M \in [X_L, c_0 X_H]$</td>
<td>$q_H$</td>
<td>$\pi_5$</td>
</tr>
<tr>
<td>$M \in [c_0 X_H, c_2 X_H]$</td>
<td>$q_H$</td>
<td>$\pi_6$</td>
</tr>
<tr>
<td>$M \in [c_2 X_H, (1 + z_H) X_H]$</td>
<td>$q_L$</td>
<td>$\pi_7$</td>
</tr>
</tbody>
</table>

(ii) $k > k^*$: The analysis is analogous to that in (i) and thus is omitted.

Lemma 2 In the fair value regime, the shareholders’ optimal choice of the maturity value $M$ of debt is as follows:

<table>
<thead>
<tr>
<th>$T(c) &gt; c_2$ :</th>
<th>$\lambda &lt; \lambda_2 : \lambda &gt; \lambda_2 :$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M = c_0 X_H$ \quad $M = c_2 X_H$</td>
</tr>
<tr>
<td>$T(c) \in [c_1, c_2]$ :</td>
<td>$\lambda &lt; \lambda_3 : \lambda &gt; \lambda_3 :$</td>
</tr>
<tr>
<td></td>
<td>$M = c_0 X_H$ \quad $M = T(c) X_H$</td>
</tr>
<tr>
<td>$T(c) &lt; c_1$ :</td>
<td>$M = c_1 X_H$</td>
</tr>
</tbody>
</table>

where

$$c_0 \equiv 1 - \frac{1}{2} \left( 1 - \frac{r_H}{\lambda_2} \right) X_H; \quad c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H}; \quad c_2 \equiv (1 + z_H) - \frac{k(1+\lambda)}{\left( \frac{1}{2} - \frac{r_H}{\lambda_2} \right) X_H}; \quad T(c) \equiv \frac{c}{\sqrt{1 + \lambda - c(\sqrt{1 + \lambda} - 1)}};$$

$$\lambda_2 \equiv \frac{2r_H z_H}{(k^* - k)/X_H}; \quad \lambda_3 \equiv \frac{2r_H z_H}{(1 - r_H) T(c) + (\frac{1}{2} + r_H)(1 - z_H) - c_0}.$$

Proof of Lemma 2

We derive the shareholders’ expected payoff at the time when they make capital structure decision. This payoff is the shareholders’ expected payoff from quality decision (given in Table 9 in the proof of Proposition 5) minus $E_0$, the shareholders’ equity investment, which equals $A_0 - D_0$. Therefore, in the following, we first derive $D_0$, the equilibrium debt price at Date 0, and then substitute this price into the shareholders’ expected payoff, and finally derive the shareholders’ optimal choice of $M$.

Table 9 identifies four scenarios, A, B, C, and D. We first analyze Scenario B.
Scenario B: Taking into consideration of the optimal choices of \((r, z)\) and \(q\), we first derive \(D_0 = E[\min\{M, \bar{X}\}]\):

<table>
<thead>
<tr>
<th>range of (M)</th>
<th>(D_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M &lt; c_0X_L)</td>
<td>(M)</td>
</tr>
<tr>
<td>(M \in [c_0X_L, T(c)X_L])</td>
<td>(q_H + (1 - q_H)(\frac{1}{2} - r_H)]M + (1 - q_H)(\frac{1}{2} + r_H)(1 - z_H)X_L)</td>
</tr>
<tr>
<td>(M \in [T(c)X_L, X_L])</td>
<td>(M)</td>
</tr>
<tr>
<td>(M \in [X_L, c_0X_H])</td>
<td>(q_HM + (1 - q_H)X_L)</td>
</tr>
<tr>
<td>(M \in [c_0X_H, T(c)X_H])</td>
<td>(q_H(\frac{1}{2} - r_H)M + q_H(\frac{1}{2} + r_H)(1 - z_H)X_H + (1 - q_H)X_L)</td>
</tr>
<tr>
<td>(M \in [T(c)X_H, c_1X_H])</td>
<td>(q_HM + (1 - q_H)X_L)</td>
</tr>
<tr>
<td>(M \in [c_1X_H, (1 + z_H)X_H])</td>
<td>(X_L)</td>
</tr>
</tbody>
</table>

We now substitute the equilibrium values of \(D_0\) in Table 10 into the expected payoff at the time when shareholders make capital structure decision, which is the shareholders’ expected payoff from quality decision (given in Table 9) minus \(E_0 = A_0 - D_0\). We do it region by region. As it turns out, the regional payoff increases in \(M\), and so the regional optimal \(M\) is the upper bound of the region. Evaluating the regional payoff at the regional optimal \(M\) yields the regional maximal payoffs in the following:
Table 11: Scenario B: $T(c) \in [c_0, c_1]$

<table>
<thead>
<tr>
<th>range of $M$</th>
<th>choice of $M$</th>
<th>shareholders’ ex ante payoff</th>
</tr>
</thead>
</table>
| $M < c_0X_L$ | $c_0X_L$      | $\pi_{B1} \equiv -A_0 - kqH + \frac{1}{1+\lambda}[qH X_H + (1-qH)X_L]$
|              |               | $+ \frac{\lambda}{1+\lambda} \left(1 - \frac{\frac{3}{2} + r_H}{2 + r_H} z_H\right) X_L$ |
| $M \in [c_0X_L, T(c)X_L]$ | $T(c)X_L$ | $\pi_{B2} \equiv -A_0 - kqH$
|              |               | $+ \frac{1}{1+\lambda} \left(qH X_H + (1-qH)(1-2r_H z_H)X_L\right)$
|              |               | $+ \frac{\lambda}{1+\lambda} \left[(qH + (1-qH)(\frac{1}{2} - r_H)\right] \frac{c}{\sqrt{1+\lambda-c(\sqrt{1+\lambda}-1)}} X_L$
|              |               | $+(1-qH)(\frac{1}{2} + r_H)(1-z_H)X_L$ |
| $M \in [T(c)X_L, X_L]$ | $X_L$ | $\pi_{B3} \equiv -A_0 - kqH + \frac{1}{1+\lambda}[qH X_H + (1-qH)X_L]$
|              |               | $+ \frac{\lambda}{1+\lambda} X_L$ |
| $M \in [X_L, c_0X_H]$ | $c_0X_H$ | $\pi_{B4} \equiv -A_0 - kqH + \frac{1}{1+\lambda} \left[qH X_H + (1-qH)X_L\right]$ |
|              |               | $+ \frac{\lambda}{1+\lambda} \left(qH(1-\frac{\frac{3}{2} - r_H}{2 + r_H} z_H)X_H + (1-qH)X_L\right)$ |
| $M \in [c_0X_H, T(c)X_H]$ | $T(c)X_H$ | $\pi_{B5} \equiv -A_0 - kqH$
|              |               | $+ \frac{1}{1+\lambda} \left[qH(1-2r_H z_H)X_H + (1-qH)X_L\right]$ |
|              |               | $+ \frac{\lambda}{1+\lambda} \left[qH \left[\left(\frac{1}{2} - r_H\right)T(c) + (\frac{1}{2} + r_H)(1-z_H)\right]X_H$
|              |               | $+(1-qH)X_L\right]$ |
| $M \in [T(c)X_H, c_1X_H]$ | $c_1X_H$ | $\pi_{B6} \equiv -A_0 - kqH + qH X_H + (1-qH)X_L - \lambda kqH$ |
| $M \in [c_1X_H, (1+z_H)X_H]$ | $[c_1X_H, (1+z_H)X_H]$ | $\pi_{B7} \equiv -A_0 + X_L$ |

It is straightforward to show the following results:

$\pi_{B3} > \pi_{B1}; \pi_{B3} > \pi_{B2}; \pi_{B4} > \pi_{B3}; \pi_{B6} > \pi_{B4}; \pi_{B6} > \pi_{B5}; \pi_{B6} > \pi_{B7}$.

Therefore, $\pi_{B6}$ is the highest payoff in Scenario B. To induce $\pi_{B6}$, by Table 11, shareholders must choose $M = c_1X_H$.

Scenario A: By similar reasoning process demonstrated in Scenario B, we can derive $\pi_{A1}$ to $\pi_{A3}$ for Scenario A, using Table 9 Scenario A, where $\pi_{A1}$ to $\pi_{A3}$ are the regional maximal payoffs for the three regions in Scenario A. It is straightforward to show the following results:

$\pi_{A1} = \pi_{B3}; \pi_{A2} = \pi_{B6}; \pi_{A3} = \pi_{B7}$.
Therefore, \( \pi_{A2} = \pi_{B6} \) is the highest payoff in Scenario A. To induce \( \pi_{A2} \), shareholders must choose \( M = c_1X_H \).

Scenario C: By similar reasoning process demonstrated in Scenario B, we can derive \( \pi_{C1} \) to \( \pi_{C6} \) for Scenario C, using Table 9 Scenario C, where \( \pi_{C1} \) to \( \pi_{C6} \) are the regional maximal payoffs for the six regions in Scenario C. It is straightforward to show the following results:

\[
\pi_{C1} = \pi_{B1}; \pi_{C2} = \pi_{B2}; \pi_{C3} = \pi_{B3}; \pi_{C4} = \pi_{B4}; \pi_{C5} = \pi_{B5}; \pi_{C6} = \pi_{B7}.
\]

Therefore, \( \pi_{C4} \) and \( \pi_{C5} \) dominate the other payoffs. \( \pi_{C4} > \pi_{C5} \) if and only if \( \lambda < \lambda_3 \). To induce \( \pi_{C4} \), shareholders must choose \( M = c_0X_H \); to induce \( \pi_{C5} \), shareholders must choose \( M = T(c)X_H \).

Scenario D: By similar reasoning process demonstrated in Scenario B, we can derive \( \pi_{D1} \) to \( \pi_{D6} \) for Scenario D, using Table 9 Scenario D, where \( \pi_{D1} \) to \( \pi_{D6} \) are the regional maximal payoffs for the six regions in Scenario D. It is straightforward to show the following results:

\[
\begin{align*}
\pi_{D1} & = \pi_{B1}; \pi_{D2} = \pi_{B2}; \pi_{D3} = \pi_{B3}; \pi_{D4} = \pi_{B4}; \\
\pi_{D5} & = -A_0 - kq_H + \frac{1}{1+\lambda}\{q_H(1 - 2r_Hz_H)X_H + (1 - q_H)X_L\} \\
& \quad + \frac{\lambda}{1+\lambda}\{q_H[(1 - 2r_Hz_H)X_H - k(1 + \lambda)] + (1 - q_H)X_L\}; \\
\pi_{D6} & = \pi_{B7}.
\end{align*}
\]

Therefore, \( \pi_{D4} \) and \( \pi_{D5} \) dominate the other payoffs. \( \pi_{D4} > \pi_{D5} \) if and only if \( \lambda < \lambda_2 \). To induce \( \pi_{D4} \), shareholders must choose \( M = c_0X_H \); to induce \( \pi_{D5} \), shareholders must choose \( M = c_2X_H \).

**Proof of Proposition 6**

The proof of Lemma 2 shows that \( \pi_{B6} \) is the highest payoff in Scenario B and that \( \pi_{A2} = \pi_{B6} \) is the highest payoff in Scenario A.

It is easy to see that \( \pi_{A2} = \pi_{B6} \) exceeds any payoff in Scenario C. Specifically, region 6 in Scenario B does not exist in Scenario C.

It is easy to see that \( \pi_{A2} = \pi_{B6} \) exceeds any payoff in Scenario D. Specifically, not only region 6 in Scenario B does not exist in Scenario D, but also the payoff in region 5 in B is larger than the payoff in region 5 in D.
Therefore, the highest payoff among all the four scenarios is $\pi_{A2} = \pi_{B6}$, and so the regulator will choose $c$ to induce Scenarios A and B.

Recall that Scenario A will be viable when $T(c) \leq c_0$. Recall also that Scenario B will be viable when $T(c) \in [c_0, c_1]$. Therefore, to induce Scenario A and/or B, it suffice for the regulator to set $c \leq c_1$. However, any further reduction of $c$ below $c_1$ will constrain shareholders’ choice of $M$ at date 0 and therefore damage their ex ante welfare. This tension gives rise to the optimal constraint, $T(c) = c_1$ if $c = \frac{1}{1 + \frac{k \sigma_1 + X}{X_H \sigma(1 + \lambda)}}$.

Proof of Proposition 7

If $k > k^*$, from Propositions 2 and 5, the threshold value of $M$ that triggers investment in the low quality project in both regimes is $c_1 X_H$.

If $k \leq k^*$, from Proposition 2, the threshold value of $M$ that triggers investment on the low quality project in the $HC$ regime is $c_2 X_H$. From Proposition 5 it is $c_1 X_H$, $T(c) X_H$, or $c_2 X_H$ under the $FV$ regime. The region over which the low quality project is chosen is larger under $FV$ regime than that under $HC$ regime if and only if $c_2 c_1 < c_1 < c_2$, which is true by assumption. Furthermore, $T(c) X_H < c_2 X_H \iff T(c) < c_2$, which is true when $T(c) X_H$ is the threshold value.

When $\frac{1}{2} - r_H$ and/or $z_H$ increases, both $c_2$ and $k^*$ increases, thereby expanding the low quality $(q_L)$ region.

Proof of Proposition 9

We already know from the proof of Proposition 8 that at $T(c) = c_1$, the fair value regime dominates the historical cost regime. We show in the following that at $c = 0 \Rightarrow T(c) = 0$, the historical cost regime dominates the fair value regime. Taken together these two facts, by continuity, there must exist $c$ such that for $c \in [0, c]$ the historical cost regime dominates the fair value regime.

Under the fair value regime, when $c = 0$, it must be the case that the debt/asset ratio at date 0, $\frac{D_0}{A_0}$, exceeds or equals $c = 0$. For the business to continue beyond date 0, the shareholders must choose the minimal face value of debt in order to satisfy the solvency constraint. Therefore, $M(c = 0) = 0$. This face value of debt implies that the shareholders’ ex ante payoff, given in Table
9 Scenario A in the proof of Proposition 5, is

\[-A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1 - q_H)X_L - M] \]

\[= -A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1 - q_H)X_L],\]

which is less than its counterpart in the historical cost regime, given in Table 5 in the proof of Lemma 1,

\[\pi_I^{HC} = -A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1 - q_H)X_L] + \frac{\lambda}{1+\lambda}(1 - \frac{1}{2 + r_H}z_H)X_L.\]

Therefore, at \(c = 0\), the historical cost regime dominates the fair value regime.

References


