Do not turn over this page until you are told to do so. You will have 1 hour and 20 minutes to complete this exam. The exam has a total of 100 points and is divided into three parts. The true and false questions are worth 10 points, the multiple choice are worth 30 points and the long answer questions are worth 60 points. You can use one side of an 8.5x11 sheet of notes during the exam. Please write clearly and provide answers in the space provided. If you need additional space use the back of the exam pages and clearly organize your work. A table for the cumulative distribution function (cdf) for a standard Normal is attached on the last page.

Students in my class are required to adhere to the standards of conduct in the GSB Honor Code and the GSB Standards of Scholarship. The GSB Honor Code also require students to sign the following GSB Honor pledge,

"I pledge my honor that I have not violated the Honor Code during this examination. I also understand that discussing the content of this exam with anyone prior to completion by all students would be a violation of the honor code."

Please sign here to acknowledge _______________________________
True or False
Clearly indicate the best answer by circling T or F indicating that the statement is true or false respectively. If neither T nor F is clearly indicated the problem will be marked as incorrect. Each problem is worth 1 point.

1. T  F For any discrete valued random variable, if you are given the cumulative distribution function (CDF) you can always recover the probability function.

2. T  F The correlation will always lie in the interval [-1,1]

3. T  F For any data set at least 95% of the data lies within 2 standard deviations of the mean.

4. T  F For any random variable \( F(1) \geq F(0) \)

5. T  F The sample standard deviation must always be less than the sample variance.

6. T  F If the sample covariance is negative then the sample correlation must also be negative.

7. T  F If \( X \) and \( Y \) are discrete random variables then \( \sum_{all \ x} P(X=x|Y=y) = 1 \)

8. T  F The Illinois state lottery introduces a new game. You win if you correctly pick all four numbers. The numbers range from 1 through 20 and numbers may be repeated. The probability that an individual correctly guesses all four numbers on a single ticket is \( \frac{1}{124,750} \)

9. T  F If you guessed the answer to every question in this true and false section (including this one) then the number of correct answers would be distributed as a Binomial random variable with \( n=10 \) and \( p=.5 \).

10. T  F If \( X_t \) follows a random walk then \( X_t \) is iid.
I. Multiple choice: Clearly circle the answer that is best. Each problem is worth 3 points for a total of 30. No partial credit will be given in this section. If no answer is clearly circled the problem will be marked as incorrect.

1. You have a sample with two observations, and calculate that $\bar{x} = 1$ and $s_x^2 = 2$. What are the values of the two original observations?
   a. -1 and 1
   b. 0 and 2
   c. -3 and 5
   d. -1 and 3
   e. None of the above.

2. Let $X \sim N(105, 5^2)$. What is the probability that $X > 97.5$?
   a. .0668
   b. .9332
   c. .6179
   d. .3821
   e. None of the above.

3. If $X$ and $Y$ are independent random variables then
   a. Knowledge of the outcome of $X$ would not change your beliefs about the outcome of $Y$.
   b. $P(x|y) = P(x)$
   c. $P(x,y) = P(x)P(y)$
   d. b) and c)
   e. a., b. and c.

4. A man says that he can correctly pick the winner of horse races 90% of the time. In order to test this claim, he picks winners (in advance) of 10 races. What is the probability that he picks the winner of all 10 races? (answers rounded to the nearest tenth)
   a. .50
   b. .24
   c. .90
   d. .35
   e. none of the above.

5. Let $Y = 2 + 3Z$ where $Z$ is a standard Normal random variable. Then $Pr(Y < 2)$ is:
   a. .95
   b. 0
   c. .5
   d. Not enough information is given.
   e. None of the above.
For the multiple choice questions 6-9 use the following probability table for the discrete random variable X.

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>.1</td>
<td>.2</td>
<td>.15</td>
<td>.2</td>
<td>.2</td>
<td>.1</td>
<td>.15</td>
<td>.05</td>
</tr>
</tbody>
</table>

Compute the following probabilities

6. What is the probability that X is greater than 8 ie Pr(X>8)?
   a. .5
   b. .05
   c. .25
   d. 0
   e. None of the above.

7. What is the probability that X is negative ie Pr(X<0)?
   a. .15
   b. .3
   c. .85
   d. .5
   e. None of the above

8. What is the probability that X is positive and even?
   a. .5
   b. .15
   c. .25
   d. .1
   e. None of the above.

9. What is the probability that X takes a value between 1 and 8 inclusive (ie \( \Pr(1 \leq X \leq 8) \))?
   a. .5
   b. .3
   c. .55
   d. .25
   e. None of the above.

10. (THIS PROBLEM IS RELATIVELY DIFFICULT SO I SUGGEST YOU WORK ON IT AFTER FINISHING THE REST OF THE EXAM)
    A chest has 2 drawers. It is known that one drawer has 3 gold coins and no silver coins. The other drawer is known to contain 1 gold coin and 2 silver. You don’t know which drawer is which. You randomly select a drawer and without looking pull out a single coin. It is gold. What is the probability that the remaining two coins in the drawer you selected are gold?
    a. .50
    b. .67
    c. .80
    d. .75
    e. None of the above.
Long answer questions. Try to do work in the space provided under each question and be show all work in order to facilitate partial credit. Also be sure to place the final answer in the space provided as indicated by the underline when present.

1. (18 points) A Gallup poll taken April 24 asked individuals if they approve, disapprove, or had no opinion about the federal government's action to seize Elian Gonzonlaz from his Miami relatives. Consider the following joint probability table constructed from the answers to this question and the sex of the respondent. (http://www.gallup.com/poll/index.asp)

<table>
<thead>
<tr>
<th></th>
<th>disapprove</th>
<th>No opinion</th>
<th>Approve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>.155</td>
<td>.015</td>
<td>.33</td>
</tr>
<tr>
<td>Female</td>
<td>.185</td>
<td>.075</td>
<td>.24</td>
</tr>
</tbody>
</table>

a. What is the probability that an individual is a male and approves of the action?______

b. What is the probability that an individual approves of the action?________

c. What is the probability of approval given that the individual is Male?________

d. What is the probability of approval given that the individual is Female?_________

e. Is the sex of the respondent independent of their views on the action? Be sure to state why or why not.

f. Interpret the results of part e in plain English. A sentence or two should be sufficient.
2. (4 points) Match the following scatter plots with the sample correlations below.

Sample correlations:

.711 ______

.882 ______

-.230 ______

.150 ______
2. (16 points) Consider a uniformly distributed random variable $X$ over the interval 0 to 1.

![Graph of f(x) and c](image)

a. What must be the value of $c$? \\

b. What is the probability that $X<3$? \\
c. What is the probability that $X=.25$? \\
d. What is the probability that $X<.25$? \\
e. What is the probability that $.5<X<.75$? \\
f. Graph the cumulative distribution function (cdf) $F(x)$ for the random variable $X$. Put $F(x)$ on the vertical axis and $x$ on the horizontal axis.

g. Let $Y=1+2X$ where $X$ is the random variable defined above. Graph the density function of $Y$ putting $f(y)$ on the vertical axis and $y$ on the horizontal axis.
3. (22 points) The following is a daily plot of the NASDAQ index at the end of 1999. Recall that an index is simply a portfolio of stocks.

![NASDAQ Index Plot](image1)

a. Do you think the NASDAQ index is iid? Clearly state why, or why not. You should directly refer to features of the plot in your answer.

b. The following is a plot of daily NASDAQ returns over the same time period. Do returns look iid? Why?

![NASDAQ Returns Plot](image2)
Here are the descriptive statistics for the NASDAQ returns:

**Descriptive Statistics**

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>TrMean</th>
<th>StDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASD</td>
<td>200</td>
<td>0.00118</td>
<td>0.00264</td>
<td>0.00144</td>
<td>0.01784</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00126</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASD</td>
<td>Minimum: -0.08565</td>
<td>Maximum: 0.06022</td>
<td>Q1: -0.00759</td>
<td>Q3: 0.01094</td>
<td></td>
</tr>
</tbody>
</table>

Let's suppose that the daily NASDAQ returns are iid Normal with mean ($\mu$) .00118 and standard deviation ($\sigma$) .01784. That is $X_i \sim N(.00118,.01784^2)$ and are iid.

c. What is the probability that the return tomorrow will be negative (ie what is the pr($X<0$))?

d. What is the probability that it is less than -5% (ie what is Pr($X<-.05$))?

e. Without doing any calculations, what can you say about the probability of a -14% return like we observed two weeks ago?

f. What is the distribution of the number of times that the market will go up (positive return) in the next n days? Be as specific as you can.

g. Find an interval for which we have a 95% chance that the return tomorrow will be in.