Do not turn over this page until you are told to do so. You will have 1 hour and 20 minutes to complete the exam. There are a total of 100 points divided into three parts. The true and false questions are worth 10 points, the multiple choice are 3 points each for a total of 30 points and the long answer questions are worth 60 points. You can use one side of an 8.5x11 sheet of notes during the exam. No other notes are permitted. Please write clearly and provide answers in the space provided. If you need additional space use the back of the exam pages and clearly organize your work. Two probability tables are given on the last page of the exam that may be useful for completing the long answer questions.

Students in my class are required to adhere to the standards of conduct in the GSB Honor Code and the GSB Standards of Scholarship. The GSB Honor Code also requires students to sign the following GSB Honor pledge,

"I pledge my honor that I have not violated the Honor Code during this examination."

Please sign here to acknowledge _______________________________
I. True or False
Clearly indicate the best answer by circling T or F indicating that the statement is true or false respectively. If neither T nor F is clearly indicated the problem will be marked as incorrect. Each problem is worth 1 point.

1. T  F The covariance of two random variables X and Y is equal to the correlation between X and Y multiplied by the product of the standard deviations of X and Y.

2. T  F If X and Y are two independent, but not identically distributed random variables then Var(X+Y)=Var(X-Y).

3. T  F If the sample standard deviation associated with a data set is zero then the histogram must be perfectly symmetric.

4. T  F Let X~Bernoulli(.5) and Y~Bernoulli(.25) be independent. Then if Z=XY, Z is also Bernoulli with p=.125 (Z~Bernoulli(.125).

5. T  F After converting a data set from units of dollars to units of 1000’s of dollars the mean will decrease, but the variance will be unchanged.

6. T  F Suppose that there is a 15% chance that it snows tomorrow and a 50% chance that the S&P500 goes up tomorrow. Using common sense assumptions the probability that it snows and the S&P500 goes up tomorrow is 10%.

7. T  F The covariance is a good measure of linear dependence.

8. T  F Consider tossing a fair coin 100 times and getting heads on every single toss. The probability of a tail on the 101st toss must be higher than 50%.

9. T  F If the exam scores are normally distributed with a mean of 85 and a standard deviation of 5 then about 5% of the class must have scored above 95.

10. T  F If the probability that a student gets this question correct is .9 and the probability that the student gets this question correct and scores above a 85 overall on the exam is .8 then the probability a student scores above 85 on the exam given that the student got this question correct is 88.8%.
II. Multiple choice: Clearly circle the answer that is best. Each problem is worth 3 points for a total of 30. No partial credit will be given in this section. If no answer is clearly circled the problem will be marked as incorrect.

Use this information for the next 3 problems. For a particular company daily internet sales of Palm Pilots are iid averaging 120 and a standard deviation of 12. The unit sells for $229.00 each. Assume that daily sales in the future will be like sales daily in the past. Also assume that the sales are independent from one day to the next.

1. The total expected revenue over the next 30 days is:
   a. $269.00
   b. $8070
   c. 4035
   d. Need more information.
   e. None of the above.

2. The standard deviation of the total sales over the next 30 days is:
   a. $65.73
   b. $360
   c. $75
   d. Need more information
   e. None of the above.

3. The distribution of total sales over the next 30 days is:
   a. Binomial with n=30 and p=.5.
   b. Approximately Normal.
   c. Will be exactly Normally distributed
   d. Not enough information is given.
   e. None of the above.

4. If I play a slot machine 50 times and the probability that I win on any given attempt is .01, what is the probability that I win on at least one attempt?
   a. .5
   b. .395
   c. .605
   d. .2
   e. None of the above.
5. If the annual return on an asset is given by a $N(.1, .1^2)$ then the probability that the return is negative is
a. .5  
b. .25  
c. .025  
d. .16  
e. None of the above.

Consider the following scatter plot for the next two problems.

6. Which of the following statements is most incorrect:
   a. The mean of $X$ is smaller than the mean of $Y$.  
b. The variance of $X$ is larger than the variance of $Y$.  
c. $X$ and $Y$ are uncorrelated.  
d. $X$ and $Y$ are independent.  
e. None of the above.

7. Which of the following statements is the most correct:
   a. $X$ is approximately Normal  
b. $Y$ is approximately Normal  
c. Neither is approximately Normal.  
d. Both are approximately Normal.  
e. Need more information.
9. Suppose that X is a Binomial(n,p). Then the variance of \( nX \) is:
   a. \( n^3 p(1 - p) \)
   b. \( np(1 - p) \)
   c. \( \frac{p(1 - p)}{n} \)
   d. \( \frac{p(1 - p)}{n^2} \)
   e. None of the above.

10. A recent survey found that 50% of American households are connected to the internet. If we randomly select 4 households what is the probability that all of them are connected?
   a. .5
   b. .25
   c. .10
   d. .0625
   e. None of the above.
III. Long answer questions. Try to do work in the space provided under each question and be sure to clearly indicate your final answer.

1. (20 points) Mercedes is trying to attract a younger buyer to its line of cars. In response to BMWs success in this market Mercedes introduced the CLK line (in particular they are trying to compete with the BMW 328i model). The average sales price of the BMW 328i in 1998 was $34,745 and the average sales price of the CLK was $39,850. Managers at Mercedes felt that at $39,850 the CLK was a competitor to the BMW 328i, however, they were concerned about the large variation in sales price across different Mercedes dealers. A survey of Mercedes dealers found that the standard deviation of sales prices for the CLK was $2005 (with a mean of $39,850) and the histogram looked approximately Normal. You might need to consult a table on the last page to complete parts of this question.

   a. The Managers at Mercedes felt that a CLK sales price greater than $42,000 was outside of the BMW 328i market. What percent of the dealers were pricing outside of this market?

   b. Approximately what percentage of the CLKs are selling for less than the average price of a 328i?

   c. How competitive does this data suggest the CLK is with the 328i?

   d. The Mercedes managers considered measures to restrict the variation of price from one dealership to the next. What would the standard deviation need to be reduced to in order to insure that 97.5% of the dealer’s sales prices were less than $42,000?
2. (20 points) A CEO is faced with a tough human resource decision. Because the company is currently operating in a budgetary crisis the CEO will either lay off 1000, 5000, or no employees. The CEO cannot foretell exactly what the business climate will be over the next few months. If business improves, there will be a cost associated with laying off workers in terms of company morale and lost productivity. Obviously, if the business environment does not improve there will be a cost associated with not laying off workers today. Consider the following table where the payoff is given in $10,000’s:

<table>
<thead>
<tr>
<th>State of Business Climate</th>
<th>Improved (Pr=.10)</th>
<th>Same (pr=.40)</th>
<th>Worsened (Pr=.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Layoffs</td>
<td>100</td>
<td>-300</td>
<td>-1700</td>
</tr>
<tr>
<td>Lay Off 1000</td>
<td>-100</td>
<td>100</td>
<td>-700</td>
</tr>
<tr>
<td>Lay Off 5000</td>
<td>-200</td>
<td>300</td>
<td>600</td>
</tr>
</tbody>
</table>

a. Based on the payoff table above, make a suggestion about what the company should do. You should frame your discussion around expected payoffs, but other information may be relevant.

b. What is the most the CEO should be willing to pay for information about the occurrence of the various states of the business climate? Again, use numbers here to back up your claim.
3. (20 points) Airlines often intentionally overbook flights knowing that not everyone who purchased a ticket for the flight will actually show up. Suppose that there is a 98% chance that a passenger with a ticket will actually show up for the flight. Suppose that a particular flight has 85 seats and the airline sold 88 tickets for the flight (they overbooked by 5 seats). You might need to consult a table on the last page of the exam to complete parts of this problem.

a. What is the expected number of passengers that will show up?

b. Suppose that all tickets are purchased for $375. If more passengers show up than there are seats available for the flight the airline will have to pay $1000 to each passenger without a seat. Use the table below to determine how many seats the airline should sell in order to maximize expected net revenue (revenue less payout for overbooking). Assume that the airline keeps the money from tickets purchased even if the passenger doesn’t get on the flight. What is the expected net revenue for the optimal number of seats sold?
### Probability Density Function

**Binomial with n = 86 and p = 0.980000**

<table>
<thead>
<tr>
<th>x</th>
<th>P( X = x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>86.00</td>
<td>0.1760</td>
</tr>
</tbody>
</table>

**Binomial with n = 87 and p = 0.980000**

<table>
<thead>
<tr>
<th>x</th>
<th>P( X = x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>86.00</td>
<td>0.3062</td>
</tr>
<tr>
<td>87.00</td>
<td>0.1725</td>
</tr>
</tbody>
</table>

**Binomial with n = 88 and p = 0.980000**

<table>
<thead>
<tr>
<th>x</th>
<th>P( X = x)</th>
</tr>
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<tbody>
<tr>
<td>86.00</td>
<td>0.2694</td>
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<tr>
<td>87.00</td>
<td>0.3035</td>
</tr>
<tr>
<td>88.00</td>
<td>0.1690</td>
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</tbody>
</table>

### Cumulative Distribution Function

**Normal with mean = 39853.0 and standard deviation = 2005.00**

<table>
<thead>
<tr>
<th>x</th>
<th>P( X &lt;= x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33,000</td>
<td>0.0003</td>
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<tr>
<td>34,000</td>
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<td>35,000</td>
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</tr>
<tr>
<td>39,000</td>
<td>0.3353</td>
</tr>
<tr>
<td>40,000</td>
<td>0.5292</td>
</tr>
<tr>
<td>41,000</td>
<td>0.7164</td>
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<tr>
<td>42,000</td>
<td>0.8579</td>
</tr>
<tr>
<td>43,000</td>
<td>0.9417</td>
</tr>
<tr>
<td>44,000</td>
<td>0.9807</td>
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<tr>
<td>45,000</td>
<td>0.9949</td>
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