SOLUTIONS
Business 320  Midterm,  Spring 1999

You have 90 minutes.
You may use a calculator and one  8.5 by 11 “cheat sheet”.
Each part of each question is worth two points.

To answer the multiple choice questions just circle your choice.

NAME ____________________________________
Question 1

The Hockey News (I'm sure you all read it) selected the 50 greatest NHL players of all time. 7 of the players were goalies. The scatterplot below has goals vs assists for each of the 43 forwards and defensemen.

Below are dotplots for variables $x$ and $y$. 
(a) The variables $x$ and $y$ are the assists and goals variables displayed jointly in the scatterplot. Which is which?

$x$ is (answer “goals” or “assists”) ___goals______

(b) The average goals is

(i) 389  (ii) 613  (iii) 212  (iv) 743

(c) The standard deviation of the goals is

(i) 23.2  (ii) 577  (iii) 203  (iv) 400

(d) The average assists is

(i) 1024  (ii) 384  (iii) 613  (iv) 89

(e) The standard deviation of the assists is

(i) 350  (ii) -453  (iii) 1050  (iv) 27

(f) The correlation between goals and assists is

(i) -.75  (ii) .75  (iii) .1  (iv) .95

Let $T = $goals + assists (total points).

(g) What is the average value of $T$?

$389 + 613 = 1002$

(h) What is the covariance between goals and assists?

$.75*203*350= 53287.5$

(i) What is the variance of $T$?

$203^2 + 350^2 + 2*53287.5 = 270284$
Question 2

The table at right gives the joint probability distribution for the two random variables $X$ and $Y$.

<table>
<thead>
<tr>
<th></th>
<th>$X$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>.81</td>
<td>.09</td>
</tr>
<tr>
<td>1</td>
<td>.09</td>
<td>.01</td>
</tr>
</tbody>
</table>

We are making two parts. $X$ is a 1 if the first part is defective and 0 otherwise. $Y$ is a 1 if the second part is defective and 0 otherwise.

(a) What is the probability that both parts are defective? 
\[ .01 \]

(b) What is the marginal distribution of $X$? 
\[ p(X=0) = .9 \quad p(X=1) = .1 \]
\[ \text{or Bernoulli}(.1)\]

(c) What is the marginal distribution of $Y$? 
\[ p(Y=0) = .9 \quad p(Y=1) = .1 \]

(d) What is $E(X)$, the expected value of $X$? 
\[ .1 \]

(e) What is $\text{Var}(X)$, the variance of the random variable $X$? 
\[ .1 \cdot .9 = .09 \]

(f) What is the conditional distribution of $X$ given $Y=1$? 
\[ p(X=0|Y=1) = .09/(.09+.01) = .09/.1 = .9 \]
\[ p(X=1|Y=1) = 1-.9 = .1 \quad X|Y=1 \sim \text{Bernoulli}(.1)\]
(g) Are X and Y independent?
   yes

(h) What is the covariance between X and Y?
   0

(i) Are X and Y iid?
   yes

Let $Z = X + Y$

(j) What is the distribution of $Z$?
   $\text{Binomial}(2,.1)$

(k) What is $E(Z)$?
   $np=2*.1=.2$

(l) What is $\text{Var}(Z)$?
   $np(1-p)=.18$

Let $A=(X+Y)/2 = Z/2$

(m) What is $E(A)$?
   $E(Z)/2 = .1$

(n) What is $\text{Var}(A)$?
   $\text{Var}(Z)/4 = .045$
At right are four probability density functions labelled 1, 2, 3, and 4.

(a) What is the mean (expected value) for random variables having densities 1, 2, and 3?

(b) What is the mean for density 4?
2.5

(c) The standard deviations corresponding to densities 1, 2, and 3 are 1, 2 and .5. Which standard deviation goes with which density?

  density 1 has standard deviation .5
  density 2 has standard deviation 1

(d) What is the standard deviation of density 4?
1

Let X be a random variable have density 4.

(e) What is the probability X>.5?
1-.025=.975

(f) What is the probability 1.5< X < 3.5?
.68
Question 4

Suppose we are manufacturing microrods. The manufacturing process is such that the lengths of the rods look like iid draws from a normal distribution with mean 2 and variance .0001.

A microrod is considered defective if the length is greater than 2.01 or less than 1.99.

(a) What is the probability that a microrod is defective ?

\[ 1 - 0.68 = 0.32 \]

Suppose we make 1000 microrods. Let \( N \) denote the number of defective microrods.

(b) What is the distribution of \( N \) ?

\[ \text{Binomial}(n=1000, p=0.32) \]

(c) What is the expected value of \( N \) ?

\[ 320 \]

(d) What is the variance of \( N \) ?

\[ np(1-p) = 1000 \times 0.32 \times 0.68 = 217.6 \]
Suppose we obtain data on 20 newly manufactured microrods.

The histogram of the 20 numbers is at right.

A manager says “that histogram does not look normal to me, I think the process has changed and it is no longer producing parts whose lengths look normally distributed’.

The histogram does not look like the normal curve, but is the manager necessarily right?

No. With only 20 observations we cannot expect the histogram of the sample to look exactly normal.
(f) At right is the time series plot of the lengths of the 20 parts (microrods).
Could they be iid normal?

Again, from only 20 it could be tough to tell. Nothing obviously wrong.

Suppose the manager does not believe the information that the parts are iid $N(2, .0001)$ and wants to directly estimate the true probability that a microrod is defective using just the data on the twenty parts.

(g) How many of the twenty are defective?

5

(h) Using the data from part (g) give a 95% confidence interval for the true probability that a part is defective.

\[
\begin{align*}
0.25 + 2\sqrt{0.25 \times 0.75 / 20} &= 0.4436492 \\
0.25 - 2\sqrt{0.25 \times 0.75 / 20} &= 0.05635083
\end{align*}
\]

pretty big.
Question 5

Suppose return R1 and R2 over the next two period are iid with $E(R_i) = .1$ and $Var(R_i) = .01$.

Suppose you invest $100 at the beginning of period 1.

At the end of period one your wealth will be $100(1+R_1)$.
If you reinvest this wealth, the wealth at the end of two periods is $100(1+R_1)(1+R_2)$.

(a) What is the expected value of the wealth at the end of period one?

\[ 100 \times (1.1) = 110 \]

(b) What is the variance of the wealth at the end of period one?

\[ 100 \times 100 \times .01 = 100 \]

(c) Assuming you will reinvest no matter what happens in period 1, what is the expected wealth at the end of period 2.

\[ W = (1+R_1+R_2+R_1R_2) \times 100 \]

\[ E(W) = 100(1+.1+.1+.01) = 100 \times 1.21 = 121 \]

Note: since $R_1$ and $R_2$ are independent $E(R_1R_2) = E(R_1)E(R_2)$

(d) Given the return in period one is .2 and you reinvest, what is the expected wealth at the end of period 2.

\[ W = (1+.2)(1+R_2) \times 100 = 120 + 120R_2 \]

\[ E(W) = 120 + 12 = 132. \]