Business 320, Summer 1999, Midterm Solutions

name ____________________________________________

There are three questions.
Each part of each question is worth 2 points.

You may use a calculator and one “cheat sheet”.

#1, 6 parts (out of 12)________

#2, 10 parts (out of 20)_______

#3, 10 parts (out of 20)_______
Question 1

Zagats rates restaurants on food, decor, service, and price. Here is the average and standard deviation for each of these four variables for 114 restaurants.

Here are the correlations between all possible pairs.

Here are dotplots of data labeled Dave and Ellen.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>food</td>
<td>114</td>
<td>19.605</td>
<td>2.537</td>
</tr>
<tr>
<td>decor</td>
<td>114</td>
<td>16.579</td>
<td>4.464</td>
</tr>
<tr>
<td>service</td>
<td>114</td>
<td>17.772</td>
<td>2.847</td>
</tr>
<tr>
<td>price</td>
<td>114</td>
<td>33.32</td>
<td>11.12</td>
</tr>
</tbody>
</table>

Correlations

<table>
<thead>
<tr>
<th></th>
<th>food</th>
<th>decor</th>
<th>service</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>decor</td>
<td>0.213</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>service</td>
<td>0.707</td>
<td>0.590</td>
<td></td>
<td></td>
</tr>
<tr>
<td>price</td>
<td>0.599</td>
<td>0.669</td>
<td>0.753</td>
<td></td>
</tr>
</tbody>
</table>
(a) The variables Dave and Ellen are food and decor. Which is which?

Dave is _____food_______ (put either food or decor)

(b) What is the variance of the food?

\[ 2.54 \times 2.54 = 6.45160 \]

(c) One of the two scatterplots at right is food vs decor and the other is food vs service. Which is which?

Plot (A) (the top one is)

_____service_____

(put either decor or service)

(d) What is the covariance between decor and service?

\[ 4.464 \times 2.847 \times .59 = 7.49831 \]

Let \( I = \) decor + service

(e) What is the mean value of \( I \)?

\[ 16.58 + 17.77 = 34.35 \]

(f) What is the variance of \( I \)?

\[ 4.464 \times 4.464 + 2.847 \times 2.847 + 2 \times 7.5 = 43.0327 \]
A friend of mine works at a major Chicago hospital. His job is “patient satisfaction”. He is trying to convince doctors that they should be “nice” to patients (even harder than convincing professors to be nice to students).

Let C=1 if a patient judges a doctor to be courteous, and 0 otherwise. Let S=1 if a patient is highly satisfied with overall service and 0 otherwise.

My friend has come up with the probabilities in the table above.

(a) What is the probability that a patient will judge the doctor to be courteous and be highly satisfied?

\[
1 -.05 -.05 -.09 = .81
\]

(b) What is the probability that a patient is highly satisfied?

\[
.05 + .81 = .86
\]

(c) Given a patient judges the doctor to be courteous, what is the probability that he is highly satisfied?

\[
.81 / (.81 + .09) = 0.9
\]

(d) Given a patient does not judge the doctor to be courteous, what is the probability that he is highly satisfied?

\[
.05 / (.05 + .05) = .5
\]
(e) What is the distribution of S?

Bernoulli(.86)

(f) What is E(S) ?

.86

(g) What is Var(S) ?

.86*(1-.86)=0.1204

(h) Are C and S independent ?

no, the prob if being highly satisfied if the doctor is courteous.

(i) Is the covariance between C and S positive or negative?

positive, they go up and down together

(j) What is the covariance between C and S ?

\[ E(CS) = .81*1 + 0*.19, \text{cov} = E(CS)-E(C)E(S) = .81-.86*.9 = .036 \]
Question 3

Suppose the return on my portfolio is $R_m \sim N(0.1, 0.01)$. The return on your portfolio is $R_y \sim N(0.2, 0.04)$. Suppose the two returns are independent.

(a) What is the probability that my return is positive?

\[ P(R_m > 0) = 0.84 \]

(b) What is the probability that your return is positive?

\[ P(R_y > 0) = 0.84 \]

Suppose I am competing with you and thus interested in whether my return is bigger than yours.

Let $D = R_m - R_y$.

(c) What is $E(D)$?

\[ E(D) = 0.1 - 0.2 = -0.1 \]

(d) What is $Var(D)$?

\[ Var(D) = 0.01 + 0.04 = 0.05 \]

(e) Assuming that $D$ is normal what is $Pr(D > 0)$?

(i) 0.1 (ii) 0.33 (iii) 0.9 (iv) 0.001
Suppose $R_m$ and $R_y$ are iid over successive periods.

Let $N$ be the # of times my return is bigger than yours our of 10 periods.

(f) What is the distribution of $N$?

Binomial(10,.33)

(g) What is the expected value of $N$?

$3.3 = 10(.33)$

(h) What is the variance of $N$?

$10 \times (.33) \times (1-.33) = 2.211$

Suppose that I get a 10,000 bonus every time my return is bigger than yours over the next 10 periods.
Let $B$ denote by bonus.

(i) What is $E(B)$?

$B = 10000N$, $E(B) = 10000E(N) = 10000(3.3) = 33000$

(j) What is $Var(B)$?

$10000 \times 10000 \times (10 \times .33 \times (1-.33)) = 221100000$