Contents

1 Historical Volatility 2
2 Exponential Smoothing 3
3 ARCH and GARCH models 5
In this review session, we will use the daily S&P 500 data.

1 Historical Volatility

We are interested in computing the annualized daily volatility using a moving average window of 30 days for the squared returns. In other words, we use the 30 most recent observations to get an estimate of volatility at a point in time. From the File menu, select New and then Program. Enter the following code in the program file, save it and click Run:

```plaintext
series retsq=sprtrn^2
smpl @first+30 @last
series vol30=0
for !i=0 to 2117
  smpl @first+!i @first+!i+30
  vol30(!i+31)=@sqrt(252*@mean(retsq))
next
smpl @all
```

**Question 1.** What is the above code achieving? Formally, how are we computing volatility at a given point in time?

We run a loop throughout all the historical squared return series. The average of the 30 squared returns is our estimation for the current variance. Then we annualize it by multiplying $\sqrt{252}$.

Consider now computing the annualized daily volatility using a moving average window of 250 days for the squared returns. In other words, we use the 250 most recent observations to get an estimate of volatility at a point in time.

```plaintext
smpl @first+250 @last
series vol250=0
for !i=0 to 1897
  smpl @first+!i @first+!i+250
  vol250(!i+251)=@sqrt(252*@mean(retsq))
next
smpl @all
```

**Question 2.** What is the above code achieving? Formally, how are we computing volatility at a given point in time?

We run a loop throughout all the historical squared return series. The average of the 250 squared returns is our estimation for the current variance. Then we annualize it by multiplying $\sqrt{252}$. 

2
Question 3. How does this compare to the previous plot?

This plot is smoother than the previous one. Also volatility dies more slowly since we are averaging over a large window.

2 Exponential Smoothing

We are interested in constructing the risk metrics exponential smoother for the daily S&P500 return series. In other words, we consider a specific updating rule to compute the annualized daily volatility series. Consider the case of $\lambda = 0.1$. Use the following code to compute the smoothed estimate:

\begin{verbatim}
series expsm1=0
expsm1(1)=@var(sprtrn)
smpl @first+1 @last
expsm1=0.1*expsm1(-1)+0.9*retsq(-1)
smpl @all
expsm1=@sqrt(252)*@sqrt(expsm1)
\end{verbatim}

Question 4. What is the above code achieving? How are we initializing the first variance (denoted $\sigma_1^2$)?
The updated variance is a weighted average of the previous daily squared returns and the previous daily variance. We use the unconditional variance as our initial variance.

**Question 5.** *What is the Risk Metrics updating rule formally given by?*

\[
\sigma_t^2 = 0.1\sigma_{t-1}^2 + 0.9r_{t-1}^2
\] (2.1)

**Question 6.** *How is our choice of \( \lambda \) seen in the plot?*

The larger the \( \lambda \) is, the less information we included from previous daily squared returns. In this case the plot becomes smoother and shocks in volatility die out more slowly.

Consider now the case of \( \lambda = 0.5 \). Use the following code to compute the smoothed estimate:

```
series expsm2=0
expsm2(1)=@var(sprtren)
smpl @first+1 @last
expsm2=0.5*expsm2(-1)+0.5*retsq(-1)
smpl @all
expsm2=@sqrt(252)*@sqrt(expsm2)
```
Question 7. What is the Risk Metrics updating rule formally given by in this case?

(2.2) \[ \sigma_t^2 = 0.5\sigma_{t-1}^2 + 0.5r_{t-1}^2 \]

Question 8. How does this plot compare with the previous plot?

This plot is smoother than the previous one.

3 ARCH and GARCH models

GARCH models have proven to be a remarkable success in modeling conditional variance dynamics. Specifically, GARCH models have been quite popular in modeling the volatility of stock returns. Let’s begin with motivating an ARCH(1) model via an applied example.

To estimate an ARCH(1) model for the S&P500 returns, use:

\texttt{arch(1,0) sprtn}\
Notice that, by default, EViews estimates a GARCH model, so, to estimate an ARCH model, we need to specify the GARCH order to be 0. Suppose that we want to plot the estimated time series of volatilities. Open the equation object corresponding to the ARCH(1) model and, under the View tab, select the GARCH graph... option. The estimated volatilities are plotted in Fig. 5.

**Question 9.** Is the lagged squared return significant?

Yes, the corresponding p-value for lagged squared return is 0, so we should reject the null that the coefficient equals to 0, which means it is significant.

**Question 10.** What is the unconditional variance given by?

\[
\sigma^2 = \frac{\alpha_0}{1 - \alpha_1} = \frac{0.0001}{1 - 0.23}
\]

**Question 11.** What is the conditional variance given by?

\[
\sigma^2_t \mid F_{t-1} = 0.0001 + 0.23 \sigma^2_{t-1}
\]
Question 12. How do we write out the estimated model?

\[ \sigma_t^2 = 0.0001 + 0.23 r_{t-1}^2 \]  \hspace{1cm} (3.3)

To evaluate the goodness of fit, consider plotting the original data series, together with the \pm 2 standard deviations. To do this, first save the conditional variances by selecting the Proc tab in the equation object and choosing Make Garch Variance Series. Then, to calculate the \pm 2 standard deviations, use:

\[
\begin{align*}
\text{series archbp}=2*\sqrt{\text{garch01}} \\
\text{series archbm}=-2*\sqrt{\text{garch01}}
\end{align*}
\]

The plot is presented in Fig. 6.

Consider now estimating a GARCH(1,1) model. Use:

\text{arch(1,1) sprtrn}

Suppose that we want to plot the estimated time series of volatilities. Open the equation object corresponding to the ARCH(1) model and, under the View tab, select the GARCH graph... option. The estimated volatilities are plotted in Fig. 5.
Figure 6: Data ± 2 standard deviations, ARCH(1) model

Question 13. Is the lagged squared return significant?

Yes, the corresponding p–value for lagged square return is 0, so we should reject the null that the coefficient equals to 0.

Question 14. What is the unconditional variance given by?

\[
\sigma^2 = \frac{\alpha_0}{1 - \alpha_1 - \beta_1} = \frac{9.99 \times 10^{-7}}{1 - 0.0649 - 0.9270}
\]

Question 15. What is the conditional variance given by?

\[
\sigma_t^2 | F_{t-1} = 9.99 \times 10^{-7} + 0.0649 r_{t-1}^2 + 0.9270 \sigma_{t-1}^2
\]

Question 16. How do we write out the estimated model?
Figure 7: Condition volatility, GARCH(1,1) model

\[ \sigma_t^2 = 9.99 \times 10^{-7} + 0.0649r_{t-1}^2 + 0.9270\sigma_{t-1}^2 \]  

(3.6)

To evaluate the goodness of fit, consider plotting the original data series, together with the ±2 standard deviations. To do this, first save the conditional variances by selecting the \textbf{Proc} tab in the equation object and choosing \textbf{Make Garch Variance Series}.

Then, to calculate the ±2 standard deviations, use:

\begin{verbatim}
series archbp=2*@sqrt(garch02)
series archbm=-2*@sqrt(garch02)
\end{verbatim}

The plot is presented in Fig. 8.

**Question 17.** Does the \textit{GARCH(1,1)} provide a better fit than the \textit{ARCH(1)} model?

The loglikelihood for ARCH(1) is 6623.6, which is lower than the loglikelihood for GARCH(1,1) 6857.2. So GARCH(1,1) is a better fit.
Figure 8: Data ± 2 standard deviations, GARCH(1,1) model