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1 Forecasting and GARCH(1,1) models

Consider the GARCH(1,1) k-step ahead forecast specification:

\[ h_{t+k|t} = \sigma^2 + (\alpha + \beta)^{k-1}(h_{t+1} - \sigma^2). \]

We know that the one-step ahead forecast is given by: \[ h_{t+1|t} = \omega + \alpha r_t^2 + \beta h_t. \]

In the session, we will be working with the volatility forecasts based on a GARCH(1,1) estimate of the daily return series for the S&P 500. Begin by loading the sp500daily.xls file into EViews. Estimate a GARCH(1,1) model using:

\[ \text{arch}(1,1) \text{ return} \]

Once the equation object appears, under the Proc tab, select Make GARCH variance series.... I’ve named this series garch11var in my workfile. Finally, since we will be doing out-of-sample forecasts, change the range of the workfile by double clicking of the Range at the top of the workfile. Change the End date to 1/15/2010. Also, open a new program file for the session.

1.1 Example 1

Suppose we want to construct the 10 period ahead forecast for the volatility under a GARCH(1,1) model for the stock returns data. In your program file, enter:

\[ \text{smpl @all} \]
\[ \text{series retsq=} \text{return}^2 \]
\[ c(4)=\text{@var(return)} \]
\[ \text{arch}(1,1) \text{ return} \]
\[ \text{smpl 1/16/2009 1/30/2009} \]
\[ \text{series forecast10}=0 \]
\[ \text{forecast10}(7330)=c(1)+c(2)*\text{retsq}(7329)+c(3)*\text{garch11var}(7329) \]
\[ \text{for } !i=1 \text{ to } 10 \]
\[ \text{forecast10}(7330+!i)=c(4)+(c(2)+c(3))^(!i)\text{*(forecast10}(7330)-c(4)) \]
\[ \text{next} \]
\[ \text{forecast10}=\text{@sqrt(forecast10)} \]
\[ \text{plot forecast10} \]

Notice that we begin by storing the unconditional variance of the original time series, \( \sigma^2 \), as the fourth entry in the time series of constant. The three GARCH(1,1) coefficients, \( \omega, \alpha, \beta \), have been stored as the first three entries of the constants time series. The last command, \text{plot forecast10}, automatically creates a graph object for the forecast.
1.2 Example 2

Suppose now we want to construct the one year ahead forecast for the volatility under a GARCH(1,1) model for the stock returns data. In your program file, enter:

smpl 1/16/2009 1/15/2010
series forecast252=0
forecast252(7330)=c(1)+c(2)*retsq(7329)+c(3)*garch11var(7329)
for !i=1 to 260
forecast252(7330+!i)=c(4)+(c(2)+c(3))^(!i)*(forecast252(7330)-c(4))
next
forecast252=@sqrt(forecast252)
plot forecast252

The resultant forecast is plotted in Fig. 2.

Question 1. Does our k-step ahead forecast converge? If so, under what condition(s)? Which value does it converge to?

Our k step ahead forecast converges to the unconditional variance $\sigma^2$ provided that

$$\alpha + \beta < 1.$$
Figure 2: 1 year volatility forecast