What is Financial Econometrics?

- Financial econometrics is about the statistical tools that are needed to analyze and address the specific types of questions and modeling challenges that come up in analyzing financial data.
• Finance is about trading off risk and return.
  – Which risks are worth taking?
  – How much should I be compensated for taking a given risk?

• Financial Econometrics is mostly about quantifying rewards and risks associated with uncertain outcomes.
  – How do we predict rewards? What is the expected return on an investment?
  – How do we quantify and predict risk? What is the volatility of an asset, of a portfolio?
  – Answers to these questions may require forecasts of variances and covariances and expected rates of return.

General set up

• Let $p_{it}$ denote the value of asset $i$ at time $t$.

• Corresponding to the prices $p_{it}$ and dividend payments $d_{it}$ are the asset returns $r_{it}$, where

$$r_{it} = \frac{p_{it} - p_{it-1} + d_{it}}{p_{it-1}}$$
• Investment decisions in practice mean allocating your money across potentially many different investment alternatives to form portfolios.

• At the time of the investment, we don’t know what the return will be on each investment opportunity (they are random).

• However, we can still make smart investment decisions if we can quantify the uncertainty associated with all the investment opportunities. That is, we need a model for the returns in the future.

Problem: Everything is changing over time!
Time varying mean returns

Time varying volatility
Removing the outlier...

Absolute value of returns
Absolute value and sd

Time varying correlations across assets
• We can’t treat the means, variance, and covariances as constant.
• What is the “right” expected return for the tbills tomorrow?
• How risky is the S&P500 tomorrow?
• If I hold two assets, how diversified is my portfolio tomorrow? It depends on the correlation between the returns tomorrow.

• At the time we build the forecast, we observe in formation that may be useful in predicting future returns.
  – For example it could be past returns, Fed announcement data, or other macro information.
  – More generally, it is any available information that might be useful in predicting returns.
• We really want to know what is the right model for the outcomes of returns at time t+1 given information we observe at time t.
Another interesting question: sometimes prices (not returns) on two assets are related

- Sometimes economic theory tells us about long run relationships that must hold.
- The bid and ask price of a given asset cannot deviate substantially for a long period of time.
- The $/Euro exchange rate can’t be that different from the price of first purchasing yen, and then purchasing Euros. These ideas are common in statistical arbitrage strategies.

Long run relationships
We’ll look at some High frequency data too.

- Least squares models were adequate for most financial questions up until about 1980.
- This course is mostly about what happened since 1980.
We start with modeling a single asset:

- Often we assume that returns are conditionally normal. We begin with the case of a single asset. Since the Normal is determined by the mean and variance we write:

\[
f(r_{t+1} | F_t) = N(\mu_{t+1}, \sigma^2_{t+1})
\]

- Vertical bar denotes “given” or “conditional on”

\[F_t\] denotes relevant information observed at time \(t\)

- We first consider dynamic (time series) models for the conditional mean. **ARMA**

- **GARCH** is perhaps the most influential model in financial econometrics. This is a model for time varying volatility. (2003 Engle Nobel Prize in Economics)

- Models for the vector of returns are then considered. **VARs**
• We then consider cutting edge techniques for time varying variance covariance matrices. We look at **Factor models** and **Dynamic Conditional Correlations (DCC)** Engle (2003).

• Models for long run relationships are introduced (2003 Granger Nobel Prize in economics). **Cointegration**.

• We conclude with models and examples for **high frequency data**.

### 1. Introduction to Time Series and Autoregressive models.

1.1 Time-Series Data and Dependence  
1.2 Checking for Dependence  
1.3 The Autocorrelation Function  
1.4 The AR(1) Model  
1.5 More on the AR(1) Model  
1.6 The AR(p) Model  
1.7 The Partial Autocorrelation Function.
1.1 Time-Series Data and Dependence

**Time-series data** are simply a collection of observations gathered over time.

For now, consider only a single variable \( r \) (\( X \)’s will come later). To emphasize that the data are time-series, we index observations by \( t \) rather than \( i \).

The data (number of observations = \( T \)):

\[
\{ r'_1, r'_2, \ldots, r'_i, \ldots, r'_T \}
\]

The interval between observations can be any time interval (days, weeks, months, years, etc).

Examples of time-series data:

- monthly consumer price index
- daily Dow Jones index (at closing)
- weekly sales
- annual GNP
Example: Independent and Identically distributed data i.i.d. data

\[ r_t = \text{random number drawn from } N(0,1) \]

A time-series plot (\(r_t\) versus \(t\)):

There is no pattern in the data. There is no relationship between one outcome and the next.

Example: Tbill rates

\[ r_t = \text{annualized return on tbill} \]

A time-series plot (\(r_t\) versus \(t\)):

These data are not i.i.d.

The value of \(r_t\) is generally not far from the value of \(r_{t-1}\).
Suppose that we know the returns for all years from 1 through $T$ ($r_1, ..., r_T$).

What is your prediction for the next month’s return, $r_{T+1}$?

If the $r_T$’s were i.i.d. (independently and identically distributed), then the prediction would just be the average of all the $r_T$’s.

But, simply using $r_T$ as our prediction for $r_{T+1}$ will be much more accurate! Why? Because the data are not i.i.d.!

- If the data are not iid then there is a relationship between the past values and the future values. It makes sense to use information from the past to guide our predictions for the future.
- In this case, we can think of our relevant information as past values of the series (i.e. $F_t$ is just the past values of the series we would like to forecast).
1.2 Checking for Dependence

It’s not always easy to just look at a time-series plot and say whether or not the series is independent.

Saying that the $r_t$ series is independent means that knowing previous values doesn’t help you to predict the next value.

So, knowing $r_{t-1}$ doesn’t help to predict $r_t$.

More generally, knowing $r_{t-1}, r_{t-2}, \ldots, r_{t-L}$ doesn’t help to predict $r_t$.

To see if $r_{t-1}$ helps to predict $r_t$, just plot $r_t$ versus $r_{t-1}$ to see if there is a relationship.

To see if $r_{t-L}$ helps to predict $r_t$, just plot $r_t$ versus $r_{t-L}$ to see if there is a relationship.

$r_{t-L}$ is called “$r$ lagged L periods.”
each point corresponds to a pair of successive days

Corr(tbill, tbill(t-1)) = 0.991

still a strong relationship (but not as strong as between tbill and tbill(t-1))

Corr(tbill, tbill(t-2)) = 0.949
1.3 The Autocorrelation Function ("acf")

To summarize all of the plots of $r$ versus lagged $r$’s, we compute the correlations between $r_t$ and $r_{t-L}$ for $L = 1, 2, 3, \ldots$

These correlations between $r$ and lagged values of $r$ are called **autocorrelations**.

The **autocorrelation function** (or “acf”) is simply all of the autocorrelation values (for all $L$).

In Eviews:
There seems to be dependence of $r$ on past values. Also, the acf gets smaller as the lag gets larger --- $r_t$ is more strongly related to $r_{t-1}$ than $r_{t-2}$, more strongly related to $r_{t-2}$ than $r_{t-3}$, and so on.

An autocorrelation is significantly different from zero at 5% level if:

$$|\text{autocorrelation}| > \frac{2}{\sqrt{T}}$$

$T=696$ in tbill data---$>$ cutoff around 0.075. Since the autocorrelations are above 0.075, there is strong evidence of dependence.

1.4 The AR(1) Model

If there is dependence in returns, we would like a model that allows us to predict future outcomes from the past outcomes.

Since $r_t$ is related to the lagged values ($r_{t-1}$, $r_{t-2}$, $\ldots$), the obvious thing to try is a regression of $r_t$ on its lags.

A simple model, called AR(1) or the “autoregressive model of order 1”.
The model:

\[ r_t = \beta_0 + \beta_1 r_{t-1} + \varepsilon_t \]

where

\[ \varepsilon_t \sim N(0, \sigma^2) \text{ i.i.d.} \]

\[ \varepsilon_t \text{ independent of } r_{t-1}, r_{t-2}, r_{t-3}, \ldots \]

The model says that \( Y_t \) consists of two parts:

\[ r_t = \beta_0 + \beta_1 r_{t-1} + \varepsilon_t \]

the part that depends on the past

the part that is not predictable from the past

Notice that the AR(1) model says that \( r_t \) depends on the past only through \( r_{t-1} \)!

What this means is that knowing previous lag values (\( r_{t-2}, r_{t-3}, \ldots \)) doesn’t help to predict \( r_t \) if you already know the value of \( r_{t-1} \). We will denote the conditional mean by \( \mu_t \):

\[ \mu_t = E(r_t | r_{t-1}, r_{t-2}, \ldots) = \beta_0 + \beta_1 r_{t-1} \]

\[ f(r_t | r_{t-1}, r_{t-2}, \ldots) = f(r_t | r_{t-1}) = N \left( \frac{\beta_0 + \beta_1 r_{t-1}}{\mu_t}, \sigma^2 \right) \]
Suppose that the AR(1) model accurately describes the data and that the parameters are known:

\[ \beta_0 = 1, \beta_1 = 0.8, \sigma = 0.5 \]

If \( Y_T = 6 \), our prediction for \( r_{T+1} \) would be obtained by plugging the parameters into the model:

\[ \mu_{T+1} = 1 + (0.8)(6) = 5.8 \]

95% predictive interval for \( r_{T+1} \):

\[ (5.8 - 2\sigma, 5.8 + 2\sigma) = (4.8, 6.8) \]

How to estimate AR(1)?

To estimate the parameters, simply regress \( r_t \) on \( r_{t-1} \); that is, choose \( b_0 \) and \( b_1 \) to minimize the sum of squared errors:

\[ \sum_{t=2}^{T} \left( r_t - b_0 - b_1 r_{t-1} \right)^2 \]

start at \( t=2 \) since \( r_1 \) doesn't have a lagged value!
• Notice that we lose one observation when we estimate $b_0$ and $b_1$ by least squares.
• There is another estimation technique that almost does least squares but it also uses the first observation.
• As long as the sample size is not very small the two estimation techniques will yield nearly identical estimates.
• Here are some examples from Eviews.

• In estimate equation, use:
  \[ \text{tb6ms c tb6ms(-1)} \]
One-step ahead forecasts for the tbill data

Model checking: How do we know if the AR(1) model is a good model?

Given $b_0$ and $b_1$, we have

$$r_t = b_0 + b_1 r_{t-1} + e_t = \hat{r}_t + e_t$$

If the model is working well, we should not be able to predict our future mistakes from our past mistakes – the error should be uncorrelated with the past errors!
If the model correctly captures the dependence in the data, then the residuals should look i.i.d.

(There should be no dependence in the residuals! If there is dependence in the residuals, it means that we’ve failed to capture information from the past.)

As in regression, use the estimated residuals ($e_t$) as proxies for the true residuals ($\epsilon_t$).

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**Example SP500 daily returns**

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<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
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<tr>
<td>C</td>
<td>-3.38E-05</td>
<td>0.000230</td>
<td>-0.146687</td>
<td>0.8839</td>
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<tr>
<td>AR(1)</td>
<td>-0.08693</td>
<td>0.016094</td>
<td>-4.602213</td>
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</tbody>
</table>

R-squared                  | 0.007251    | Mean dependent var | -3.35E-05 |
Adjusted R-squared         | 0.007224    | S.D. dependent var  | 0.013837  |
S.E. of regression         | 0.013767    | Akaike info criterion | -5.729568 |
Sum squared resid          | 0.570118    | Schwarz criterion   | -5.725600 |
Log likelihood             | 8690.874    | Hannan-Quinn criter. | -5.728131 |
F-statistic                | 23.00125    | Durbin-Watson stat. | 2.006714  |
Prob(F-statistic)          | 0.000002    |                      |          |
Inverted AR Roots          | -0.09       |                      |          |
Plot of residuals

No obvious pattern

Here is the ACF of the residuals

Correlogram of Residuals

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<th>Sample: 1/02/2004 12/30/2005</th>
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<td>Q-statistic probabilities adjusted for 1 ARMA term(s)</td>
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<tr>
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<td>Partial Correlation</td>
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<tr>
<td>-----------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>1</td>
<td>-0.002</td>
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<tr>
<td>2</td>
<td>-0.042</td>
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<tr>
<td>3</td>
<td>-0.003</td>
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<tr>
<td>4</td>
<td>0.058</td>
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<tr>
<td>5</td>
<td>0.039</td>
</tr>
<tr>
<td>6</td>
<td>0.027</td>
</tr>
<tr>
<td>7</td>
<td>-0.013</td>
</tr>
<tr>
<td>8</td>
<td>-0.004</td>
</tr>
<tr>
<td>9</td>
<td>-0.012</td>
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<tr>
<td>10</td>
<td>-0.006</td>
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<tr>
<td>11</td>
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<td>12</td>
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<tr>
<td>13</td>
<td>0.019</td>
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<tr>
<td>14</td>
<td>-0.006</td>
</tr>
<tr>
<td>15</td>
<td>-0.049</td>
</tr>
</tbody>
</table>
The AR(1) model seems to fit the S&P500 data pretty well. Once yesterday’s return is used to explain today’s return, the part left over (the residual) has no significant dependence structure.

If the model were wrong, there would be evidence of dependence in the residuals.

To see this phenomenon, consider what happens when we try to fit an AR(1) model to the tbills data.
• For the tbill data, the errors in time t forecasts are correlated with the errors at time t-1. This means future mistakes are correlated with past mistakes so the model is not a good one.
• We need to try another model – more to come shortly.
1.5 More on the AR(1) Model

\[ r_t = \beta_0 + \beta_1 r_{t-1} + \epsilon_t \]

The AR(1) model can describe several different types of time-series data. The value of \( \beta_1 \) turns out to be very important in describing the nature of the data.

In this section, we will look at simulated data for several different \( \beta_1 \) values. Each data set has been created so that it follows the AR(1) model (for a given set of parameter values).

The simulated data sets:

**Series 1:** \( \beta_0 = 1, \beta_1 = 0.8, \sigma = 0.1 \)

**Series 2:** \( \beta_0 = 1, \beta_1 = -0.8, \sigma = 0.1 \)

**Series 3:** \( \beta_0 = 0.1, \beta_1 = 1, \sigma = 1 \)

**Series 4:** \( \beta_0 = 0, \beta_1 = 1.1, \sigma = 0.5 \)

Note: The value of \( \beta_1 \) is what’s important here. The other parameters (\( \beta_0 \) and \( \sigma \)) have been chosen so that the pictures will look nice.
Series 1:  $\beta_0 = 1, \beta_1 = 0.8, \sigma = 0.1$
The autoregression on Series 1:

![Autoregression Table]

The acf of the residuals:

![ACF Diagram]
Looking at the time-series plot of Series 1, it seems that the series sometimes wanders up and sometimes wanders down but that it seems to “revert” to some mean level.

We will be more explicit later in the notes, but for now, think of the mean as the value to which the series tends. Or if it’s at the mean value it will tend to stay there.

\[ \mu = \beta_0 + \beta_1 \mu \Rightarrow \mu = \frac{\beta_0}{1 - \beta_1} \]

For Series 1, the mean level is \( \frac{1}{1-0.8} = 5 \), which looks about right in the time-series plot.

Some algebra:

\[ \mu = \frac{\beta_0}{1 - \beta_1} \Rightarrow \beta_0 = (1 - \beta_1)\mu \]
\[ r_t = (1 - \beta_1)\mu + \beta_1 r_{t-1} + \epsilon_i \]
\[ r_t - \mu = \beta_1(r_{t-1} - \mu) + \epsilon_i \]

The result: \[ r_t - \mu = \beta_1(r_{t-1} - \mu) + \epsilon_i \]

For positive \( \beta_1 \), an above-average \( r \) tends to be followed by another above-average \( r \). This continues to happen until a large negative \( \epsilon \) comes along to knock \( r \) below the average. Then, a below-average \( r \) tends to be followed by another below-average \( r \) (until a large positive \( \epsilon \) comes along). This behavior produces the “wandering” evident in the time-series plot.
The fact that $\beta_1 < 1$ is what drives the **mean reversion** of Series 1.

From the formula

$$r_t - \mu = \beta_1 (r_{t-1} - \mu) + \varepsilon,$$

we see that, on average, $r_t$ tends to be closer to the mean value $\mu$ than $r_{t-1}$:

$$E(r_t - \mu | r_{t-1}) = \beta_1 (r_{t-1} - \mu) < r_{t-1} - \mu$$

Series 2: $\beta_0 = 1, \beta_1 = -0.8, \sigma = 0.1$
Because $\beta_1$ is negative, an above-average $r$ tends to be followed by a below-average $r$ (and vice versa). This explains the jaggedly times-series plot.

For Series 2, the mean level is

$$\mu = \frac{\beta_0}{1 - \beta_1} = \frac{1}{1 - (-0.8)} \approx 0.56$$

which looks about right in the plot.

Does the “flipping” in the autocorrelations make sense?
The autoregression on Series 2:

![Autoregression Table]

The acf of the residuals:

![ACF of Residuals]

65

66
Series 3: \( \beta_0 = 0.1, \beta_1 = 1, \sigma = 1 \)

This plot looks totally different. There is no mean reversion --- the series just seems to wander off.

Notice that the mean level formula doesn’t work here since

\[
\mu = \frac{\beta_0}{1 - \beta_1} = \frac{1}{1-1} = ???
\]

When \( \beta_1 =1 \), the AR(1) model is known as the **random-walk model**:  

\[
y_i = \beta_0 + y_{i-1} + \varepsilon_i
\]

or  

\[
y_i - y_{i-1} = \beta_0 + \varepsilon_i \sim N\left(\beta_0, \sigma^2\right)
\]
In the random-walk model, the “differenced series”. Financial theory predicts that asset prices (not returns) should follow a random walk so that \((p_t - p_{t-1})\) is i.i.d.

The parameter \(\beta_0\) is called the “drift parameter.”

If \(\beta_0\) is positive, there is “positive drift” and the series will wander upward.

If \(\beta_0\) is negative, there is “negative drift” and the series will wander downward.

If \(\beta_0\) is zero, there is no drift. The series will “meander” around its starting value with no particular trend, but it can take very long “excursions” away from its starting value.

Correlogram of SER03

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<td>0.474</td>
<td>-0.096</td>
<td>1547.5</td>
<td>0.000</td>
</tr>
</tbody>
</table>
The differenced series seems to be independent.

This differencing method is often a good way of checking if a series follows the random-walk model.

Series 4: $\beta_0 = 0, \beta_1 = 1.1, \sigma = 0.5$

This series “explodes.” The values seem to increase exponentially.
### Summary of AR(1) Model Behavior

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
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<tbody>
<tr>
<td>$</td>
<td>\beta_1</td>
</tr>
<tr>
<td>$\beta_1 = 1$</td>
<td>The series has no mean level and, thus, is called <strong>nonstationary</strong>. The drift parameter $\beta_0$ the direction in which the series wanders.</td>
</tr>
<tr>
<td>$</td>
<td>\beta_1</td>
</tr>
</tbody>
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### 1.6 The general AR(p) model

If the AR(1) model doesn’t capture the dependence in the data, you can try a multiple regression with higher lags of $r$ thrown in:

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \cdots + \beta_p r_{t-p} + \epsilon_t$$

(This model is an “autoregressive model of order $p$.”)
• We now have p lags of returns on the right hand side.
• We now have p+1 parameters to estimate: $\beta_0$, $\beta_1$, ..., $\beta_k$.
• We can again estimate this model by least squares.

1.7 The partial autocorrelation function (PACF)

• The first partial autocorrelation is simply the estimated regression coefficient $\beta_1$ from the regression

$$r_t = \beta_0 + \beta_1 r_{t-1}$$

• The second partial autocorrelation is the coefficient $\beta_2$ from the regression

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2}$$

• In general, the $j$th partial autocorrelation is the $j$th regression coefficient in a regression of $r_t$ on $r_{t-1}, r_{t-2}, ..., r_{t-j}$
Run this regression

\[ r_t = \beta_0 + \beta_1 r_{t-1} \rightarrow b_1 \text{ 1st PAC} \]

\[ r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} \rightarrow b_2 \text{ 2nd PAC} \]

\[ r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \beta_3 r_{t-3} \rightarrow b_3 \text{ 3rd PAC} \]

\[ r_t = \beta_0 + \sum_{i=1}^{j} \beta_i r_{t-i} \rightarrow b_j \text{ jth PAC} \]

\text{jth partial autocorrelation (the last coefficient of regression of } r_t \text{ on } r_{t-1}, r_{t-2}, \ldots, r_{t-j})\]

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How to use the PACF

- Suppose that the true time series process is an AR(1) model

\[ r_t = \beta_0 + \beta r_{t-1} + \epsilon_t \]  

\text{True model} \]

- What should the PACF look like?

- The first PACF, \( b_1 \), should be close to \( \beta \), the true slope coefficient. The second PACF, \( b_2 \), should be close to zero.

- Why? Because in the AR(1) model, \( r_t \) only depends on \( r_{t-1} \), not \( r_{t-2} \) and \( r_{t-3} \).
• Of course, even if the true model is an AR(1) the second PAC will not be exactly zero, but it should be close.

• We would reject the null that the jth PAC is zero if

$$|b_j| > \frac{2}{\sqrt{T}}$$

• It is therefore common to plot the first L PAC’s and compare their magnitude to $$\frac{2}{\sqrt{T}}$$.

• So for the AR(1) model, the first PAC should be significant and the remaining PAC’s should all be insignificant.

This is the PAC for series 1
This is the PAC for series 2