Financial Econometrics: Midterm Solutions

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Problem 1
(4 points)
a. 4 b. 2 c. 1 d. 3

Problem 2
(2 pts ea.)
a. 
\[ \frac{1.2}{1 - 0.8} = 0.6 \]
b. 
\[ \mathcal{N} \left( \frac{1.2}{1 - 0.8}, \frac{2^2}{1 - 0.8^2} \right) \]
c. 
\[ 1.2 + 0.8(5.1) \approx 5.28 \]
d. 
\[ \mathcal{N} (1.2 + 0.8(5.1), 2^2) \]
e. 
\[ (1 - 0.8^2) \frac{1.2}{1 - 0.8} + 0.8^2(5.1) \]
f. 
\[ \hat{r}_{t+2} + 2 \sqrt{\frac{(1 - 0.8^4)}{1 - 0.8^2} \cdot 2^2} \]
where the first term is your prediction from part e.
Problem 3
(5 pts ea.)

a.
1. Model selection involves trying multiple models
2. This makes it likely that the chosen model makes our selection criteria look good just for our sample of data (overfitting)
3. Sample splitting, reserve sample for testing that isn’t used in estimation. Overfit models will likely not perform well on the out of sample data

b.
1. Split the sample beforehand and use part of it for model selection.
2. Use the other set for model comparison.
   Your loss function should be based on profits somehow. For example:
   \[
   r_t = \begin{cases} 
   s_t - f_{t-1} & E[s_t|F_{t-1}] > f_{t-1} \\
   f_{t-1} - s_t & E[s_t|F_{t-1}] < f_{t-1} 
   \end{cases}
   \]
   and then compare mean profits or some risk-adjusted mean profit.

c.
Recall from homework, test
\[
H_0 : \bar{r} = 0, \ H_a : \bar{r} > 0
\]
where \(\bar{r}\) is the mean return on the strategy.

Problem 4
(3 pts. ea.)

a.
Null hypothesis that there is no correlation between the \(j^{th}\) lag and current inflation.

b.
Null is that conditional on first \(j-1\) lags, no additional info from lag \(j\).

c.
This is a joint test where the null is
\[
\rho_1 = \rho_2 = \cdots = \rho_{10} = 0
\]

d.
It’s actually really unclear from the ACF so we were pretty lenient. A decent answer would be ARMA(1,1) or even MA(1). The model is pretty clearly not AR(1), there’s some MA component. Exactly the degree is tough to tell, you’d need to try a few and look at residuals or other model selection criteria.
e.

\[ \mu_t = 0.002 + 0.134(0.00149) + 0.3536(-0.001721) \approx 0.00159 \]

f.

\[ N(\mu_t, 5.46 \times 10^{-6}) \]

or \(0.00235^2\) for the variance, where \(\mu_t\) is your prediction from part e.

g.

\[ \mu_t \pm 2\sqrt{5.46 \times 10^{-6}} \]

again, where \(\mu_t\) is your prediction from before.

**Problem 5**

(3 pts ea.)

**a.**
The null is that the series is a random walk.

**b.**
p-value greater than .05 so fail to reject random walk.

**c.**
Mean return is 0 so

\[ y_t^k = y_t + \beta_0 \ast k = y_t = 0.807 \]

d.

\(0.0011^2k\)

e.

\(0.807 \pm 2\sqrt{0.0011}\)

**Problem 6**

(3 pts ea. except for e,g,j which were 4 pts ea.)

**a.**
The ACF is showing autocorrelation in the magnitudes of the returns (absolute values). This is usually a good test that there’s some volatility clustering going on.
b. 
We look at ACFs of the standardized residuals as a goodness of fit test for the GARCH model. If the volatility model is correct, the residuals should not exhibit correlation when squared.

c. 
\[ h_t = 1.55 \times 10^{-6} + .1022(.023^2) + .8865(0.0033) \approx .000348 \]

d. 
\[ \pm \sqrt{h_t} \]
where \( h_t \) is your prediction from above.

e. 
We should be concerned about the normality assumption. In particular, fat tails and left skew.

f. 
\[ \sqrt{255h_t} \]
again, \( h_t \) from before. Some used long-term volatility and we allowed that as well.

g. 
\[ \frac{1.55 \times 10^{-6}}{1 - (.1022 + .8865)} + (.1022 + .8865)^{k-1}(h_t - \frac{1.55 \times 10^{-6}}{1 - (.1022 + .8865)}) \]
again, \( h_t \) from before.

h. 
for \( k = 1, \ldots, 5 \)

0.000348 
0.000345 
0.000343 
0.000341 
0.000339 

i. 
summing, you should get \( \approx .001716 \) Then the annualized volatility is given by \[ \sqrt{\frac{.001716 \cdot 255}{5}} \approx .2941 \]

j. 
If you take \[ \sqrt{.0017} \approx .0415 \], this is the standard deviation. 2 standard deviations is approximately \( \pm 8.3\% \) so the probability of the straddle paying off, i.e. exceeding this interval is about 5\%.