Name_______________________________

Financial Econometrics
Jeffrey R. Russell
Midterm Winter 2011

You have 2 hours to complete the exam. Use can use a calculator. Try to fit all your work in the space provided. If you find you need more space continue on the back of the page. The last page contains a set of formulas that might be useful on the exam. No other notes or texts are permitted.

Students in my class are required to adhere to the standards of conduct in the GSB Honor Code and the GSB Standards of Scholarship. The GSB Honor Code also require students to sign the following GSB Honor pledge,

"I pledge my honor that I have not violated the Honor Code during this examination."

Please sign here to acknowledge _______________________________
1. (10 points) Identify which series is an MA(1) model and which is an AR(1) model.

Series 1

Series 2

For series 1 and 2 below, identify which ACF and PACF corresponds to an MA model and which is an AR model.

Series 1

Series 2
2. (18 points) For the AR(1) model $y_{t+1} = 1 + .95 y_t + \varepsilon_{t+1}$ where $\varepsilon_{t} \sim iid \ N \left(0, 2\right)$ find the following.

a. What is the unconditional mean of $y_t$?

b. What is the unconditional variance of $y_t$?

c. What is the mean of $y_{t+1}$ given that $y_t = 9$?

d. What is the variance of $y_{t+1}$ given that $y_t = 9$?

e. Find the k-step ahead forecast of $y_{t+k}$ given $y_t = 9$.

f. Find the k-step ahead forecast error variance as a function of $k$. 
3. (12 points) Consider the MA(2) model
\[ y_t = 0.5 + 0.9 \varepsilon_{t-1} + 0.2 \varepsilon_{t-2} + \varepsilon_t \]
where \( \varepsilon_t \sim iid \ N(0, 2^2) \).

a. What is the unconditional mean of \( y_t \)?

b. What is the unconditional variance of \( y_t \)?

c. Find the 1, 2, 3, and 4 step ahead forecast of \( y_{t+k} \), given \( \varepsilon_t = 0.8 \) and \( \varepsilon_{t-1} = 0.7 \).

d. What would happen if you attempted to fit an AR(p) model to data generated by this MA(2) model above? Just give me a general idea.
4. (8 points) Consider the Augmented Dickey-Fuller test for a unit root associated with the Dollar/Euro foreign exchange rate.

Null Hypothesis: RATE has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic based on SIC, MAXLAG=12)

<table>
<thead>
<tr>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-0.033658</td>
</tr>
<tr>
<td>Test critical values:</td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-3.461967</td>
</tr>
<tr>
<td>5% level</td>
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<tr>
<td>10% level</td>
<td>-2.601546</td>
</tr>
</tbody>
</table>


Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RATE)
Method: Least Squares
Date: 02/11/06 Time: 11:21
Sample (adjusted): 4 100
Included observations: 67 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
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<td>D(RATE(-1))</td>
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<td>0.068483</td>
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<td>D(RATE(-2))</td>
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<tr>
<td>C</td>
<td>0.002723</td>
<td>0.021681</td>
<td>0.122436</td>
<td>0.9012</td>
</tr>
</tbody>
</table>

R-squared | 0.102556 | Mean dependent var | 0.001378 |
Adjusted R-squared | 0.071574 | S.D. dependent var | 0.062548 |
S.E. of regression | 0.009625 | Akaike info criterion | -7.79117 |
Sum squared resid | 0.022525 | Schwarz criterion | -7.722996 |
Log likelihood | 366.7257 | F-statistic | 3.461966 |
Durbin-Watson stat | 1.907050 | Prob(F-statistic) | 0.019333 |

What is the null hypothesis being tested here?

What do you conclude? Be specific.
5. (12 points) Consider a random walk model for the log of the S&P500 index. The monthly returns have a mean of .008 and a standard deviation of .0577. The index level (not log level) at the close on the last in sample day was 1331.29.

a. Find the k-step ahead forecast of the log S&P500 index level as a function of k and the initial log index.

b. Find the k-step ahead forecast error variance associated with part a. as a function of k.

c. Find the expected return over the next k days. You should be able to write the forecast at horizon k as a function of k.

d. Find a 95% CI for the return over the next k days. You should be able to write this interval as a function of k.
6. (15 points) Consider the following output from a threshold (asymmetric) GARCH(1,1) model estimated on daily returns data.

![Output table]

The last observed return in the sample is \( r_T = -0.035 \) and the volatility (standard deviation) on the last day in the sample \( \sqrt{h_T} \) is 0.024 where \( T \) denotes the last observation in the sample.

a. Find the one-step ahead out of sample forecast. You should get a number here.

b. Recall that for a standard normal, \( \Pr(Z < -2.33) = 0.01 \). Find the one-day-ahead 1% Value at Risk
c. For a t-distribution with 5.73 degrees of freedom we have $\Pr(t < 0.01) = -3.36$. The variance of a t-distribution is $\nu/(\nu - 2)$. If the GARCH model above used a t-distribution with 5.73 degrees of freedom, what would the one-day-ahead 1% Value at Risk be?

\[
\frac{r_i}{\sqrt{h_i}}
\]

and we find that the empirical fraction of $z_i$'s smaller than -2.8% is 1%. What is the bootstrapped one-day-ahead 1% Value at Risk?

e. If you used the Value at Risk from part b, would your risk be overstated or understated. Discuss this in a couple of sentence.
7. (13 points) Consider the returns of two assets that each follow a GARCH process: 
\[ r_{1,t} = \sqrt{h_{1,t}} z_{1,t} \] and 
\[ r_{2,t} = \sqrt{h_{2,t}} z_{2,t} \] for assets 1 and 2 respectively. \( z_{1,t} \) and \( z_{2,t} \) are iid mean zero and variance 1. Additionally, \( \text{cov}(z_{1,t}, z_{2,t}) = .5 \) but \( z_{1,t} \) and \( z_{2,t} \) are not correlated in different time periods i.e \( \text{cov}(z_{1,s}, z_{2,s}) = 0 \) for \( t \neq s \).

a. Find the conditional covariance between returns for asset 1 and asset 2, i.e. find 
\[ \text{cov}_t(r_{1,t}, r_{2,t}) = E\left(r_{1,t}, r_{2,t} \mid F_{t-1}\right). \]

b. What is the conditional correlation between returns on asset 1 and returns on asset 2 where 
\[ \text{cor}_t(r_{1,t}, r_{2,t} \mid F_{t-1}) = \frac{E\left(r_{1,t}, r_{2,t} \mid F_{t-1}\right)}{\sqrt{E\left(r_{1,t}^2 \mid F_{t-1}\right)E\left(r_{2,t}^2 \mid F_{t-1}\right)}}? \]

c. Suppose that you build a portfolio that puts .5 weight in asset 1 and .5 weight in asset 2. The returns on the portfolio in time \( t \) will be 
\[ p_t = .5r_{1,t} + .5r_{2,t}. \] Find the conditional variance of the portfolio \( E\left(p_t^2 \mid F_{t-1}\right). \)
8. (12 points) Consider the model for observed financial prices $p_t^o = p_t \xi_t$, where $p_t$ is the true price (the fair market value) and $\xi_t$ is a multiplier that moves the observed price up or down a little bit away from the fair market value due to market microstructure effects. The fair market value follows a random walk model: $\ln(p_t) = \ln(p_{t-1}) + \epsilon_t$. $\epsilon_t$ is iid $N(0, \sigma^2)$. $\eta_t = \ln(\xi_t)$ follows an AR(1) model $\eta_t = \beta \eta_{t-1} + \gamma_t$, where $\gamma_t \sim iid N(0, \sigma^2_\gamma)$ with $|\beta| < 1$. $\epsilon_t$ and $\gamma_t$ are independent.

a. What is the unconditional mean of the observed continuously compounded returns?

b. What is the unconditional variance of the observed continuously compounded returns?
c. What are the first 4 autocorrelations of the observed continuously compounded returns?

d. What do the autocorrelations tell you about which type of ARMA model would be appropriate?
|            | Stationary $|\beta_1| < 1$ | Non-stationary $|\beta_1| = 1$ |
|------------|-------------|------------------|
| **Forecasts** | Mean revert | Trend up or down depending on sign of $\beta_0$ |
|             | $Y_t^k = \beta_1^k Y_t + (1 - \beta_1^k) \mu$ | $Y_t^k = Y_t + k \beta_0$ |
| **Forecast errors** | Initially increase with the forecast horizon. | Increases with the forecast horizon. |
|             | $Var(e_t^k) = \frac{(1 - \beta_1^{2k})}{(1 - \beta_1^2)} \sigma^2$ | $Var(e_t^k) = k \sigma^2$ |