Habits

\[ u(C) = (C - X)^{1-\gamma} \rightarrow -\frac{u''(C)}{Cu'(C)} = \gamma \left( \frac{C}{C - X} \right) = \frac{\gamma}{S} \]

As \( C \) (or \( S \)) declines, risk aversion rises.
Habits

Slow-moving habit. Roughly, \( X_t \approx \sum \phi^j C_{t-j} \); \( X_t \approx \phi X_{t-1} + C_t \)

\( \rightarrow \) Time-varying, recession-driven, risk premium drives return predictability from p/d; “excess” volatility, much else (correlation, CAPM vs CCAPM, volatility, etc.). “Bubble” story.
Habits

- $u'(C) = (C - X)^{-\gamma}$
- Precautionary savings offset intertemporal substitution.
- Expected returns and fear/hunger. Habits add $S =$ fear that stocks fall in recession.

$$1 = E_t(M_{t+1}R_{t+1}); \quad E(R_{t+1}) = -\text{cov}(R_{t+1}^e, M_{t+1})$$

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma}$$
Here, $X_t = k \sum_{j=0}^{\infty} \phi^j C_{t-j}$
Habits – successes and ... directions for improvement

- Yes: Equity premium, low \( \sigma(\Delta c) \), unpredictable \( \Delta c \), low and constant (or slow varying) risk free rate.
- No: ... and low risk aversion.
- Yes: return predictability, p/d volatility, \( \sigma(R) \) volatility, long run equity premium.

\[
M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma}
\]

- Needed: ....
The Standard VAR

\[ r_{t+1} \approx 0.1 \times dp_t + \varepsilon_{r_{t+1}} \]
\[ \Delta d_{t+1} \approx 0 \times dp_t + \varepsilon_{d_{t+1}} \]
\[ dp_{t+1} \approx 0.94 \times dp_t + \varepsilon_{dp_{t+1}} \]

\[ \text{cov}(\varepsilon \varepsilon') = \begin{pmatrix} r & \Delta d & dp \\ r & \sigma = 20\% & +\text{big} & -\text{big} \\ \Delta d & & \sigma = 14\% & 0 \text{ not } -1 \\ dp & & \sigma = 15\% & \end{pmatrix} \]

- Needed: Two shocks! Data \( \varepsilon^d, \varepsilon^{dp} \) uncorrelated. \( \Delta c \) is both a cashflow and a discount rate shock.
- \( \Delta d \) shock in model has less correlation. Match VAR? \( d, c \) need to be cointegrated.
(Identities)

- Note: $\Delta d, dp$ carry all information

\[
\begin{align*}
    r_{t+1} & \approx dp_t - \rho dp_{t+1} + \Delta d_{t+1} \\
    b_r & = 1 - b_{dp} + b_d \\
    \varepsilon_{t+1}^r & = -\varepsilon_{t+1}^{dp} + \varepsilon_{t+1}^d
\end{align*}
\]
Habits – successes and … directions for improvement

▶ Needed: More state variables (?)

1. Empirical

\[ R_{t+1}^i = a_i + b_i x_t + c_i y_t + \varepsilon_{t+1}^i; \ E_t(R_{t+1}^i) = a_i + b_i x_t + c_i y_t \]

How many state variables – independent linear combinations of \( x, y, z \) are there? Factor analysis of \( \text{cov}(E_t(R_{t+1}^i)) \)? Across stocks, bonds, fx, etc? (For example, one factor for all bonds.) For mean and variance (separate?)

2. Theoretical: If more than 1, need more state variables (S) in the model!

▶ Test; Other assets, 1 = \( E(mR^e_i) \)? Cross section (treating time aggregation right)?

▶ But, warning, all explicit models fail \( R^2 = 1 \) tests.

▶ Still low hanging fruit for all similar models.
Other directions

▶ A sampling

1. Recursive utility (Epstein-Zin)
2. Long run risks (e.g. Bansal Yaron)
3. Idiosyncratic risk (e.g. Constantinides and Duffie)
4. Rare Disasters (e.g. Reitz; Barro)
5. Nonseparable across goods (e.g. Piazzesi Schneider, housing)
6. Leverage; balance-sheet; “institutional” (e.g. Brunnermerier, ..)
7. Ambiguity aversion, min-max, (Hansen and Scheinkman)
8. Behavioral finance; probability mistakes. (e.g. Shiller, Thaler)
9. Many others

▶ Great unity of theoretical ideas.

\[
M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{Y_{t+1}}{Y_t} \right)^\theta
\]

\[
P_t U'(C) = \beta \sum_s \pi_s(Y?) U'(C_s) X_s
\]

Y varies with business cycle. “Fear of Y” drives asset prices. (Probability = marginal utility)

▶ Habits can still capture most of these ideas. Convenience?
Recursive utility / Long run risk

- Function

\[ U_t = \left( (1 - \beta) c_t^{1-\rho} + \beta \left[ E_t \left( U_{t+1}^{1-\gamma} \right) \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}. \]

\( \gamma = \text{risk aversion} \quad \rho = 1/eis. \) Power utility for \( \rho = \gamma. \)

- Fear = utility index

\[ M_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\rho} \left( \frac{U_{t+1}}{E_t \left( U_{t+1}^{1-\gamma} \right) \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma} \]

\[ = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\rho} \left( Y_{t+1} \right)^{\rho-\gamma}. \]
Recursive utility / Long run risk

- Fear: news of future long-horizon consumption. \((\rho \approx 1)\).

\[
\Delta E_{t+1} (\ln m_{t+1}) \approx -\gamma \Delta E_{t+1} (\Delta c_{t+1}) + (1 - \gamma) \left[ \sum_{j=1}^{\infty} \beta^j \Delta E_{t+1} (\Delta c_{t+1+j}) \right]
\]

- Features/thoughts
  1. iid \(\Delta c\), reduces to power utility. Needs predictable \(\Delta c\).
  2. Current conditions \(\Delta c_t\) are essentially irrelevant to fear. Only from coincidence / assumption that current \(\Delta c_t\) is correlated with long run \(E_t \Delta c_{t+j}\). (Not strong in data)
  3. Is there really a lot of news about long run future \(\Delta c\)? Is that really the fear in 2008? Or “Dark Matter?” (Chen, Dou, Kogan)
  4. Time-varying risk premium, return predictability volatility, etc. must come from exogenously changing \(\sigma_t (\Delta c_{t+1})\)
  5. Interesting phenomena all from hard-to-see features of exogenous consumption process. Habits: endogenous rise in RA.
  6. “Separates IES / RA.” “Solves risk free rate puzzle (high risk aversion, steady low \(R^f\)).” (Still needs high RA). But so do habits!
  7. “Preference for early resolution of uncertainty.” “Separate time vs. state separability” Feature or bug?
(Note: Bansal Yaron Kiku consumption process)

\[
\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1} \\
x_{t+1} = \rho x_t + \phi e \sigma_t e_{t+1} \\
\sigma^2_{t+1} = \bar{\sigma}^2 + \nu (\sigma^2_t - \bar{\sigma}^2) + \sigma_w w_{t+1} \\
\Delta d_{t+1} = \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \phi \sigma_t u_{d,t+1}
\]
Constantinides and Duffie – idiosyncratic risk

- Bottom line:

\[ M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( e^{\frac{\gamma(\gamma+1)}{2} y_{t+1}^2} \right) \]

\[ y_{t+1} = \text{cross-sectional variance of consumption growth.} \]

\[ \Delta c^i_{t+1} = \Delta c_{t+1} + \eta_{i,t+1} y_{t+1} - \frac{1}{2} y_{t+1}^2; \quad \sigma^2 (\eta_{i,t+1}) = 1 \]

- Needs \( y = \sigma(\text{cross-sectional variance}) \) large, varies with business cycles, conditional distribution varies over time. Exogenous, or needs new theory

- New work in data (Schmidt). Maybe individual rare “disasters” in recessions drives \( \sigma(\Delta c) \)?
As people / intermediaries lose money, closer to default, they get more risk averse
Debt can look just like habit

\[ U(C) = (C - X)^{1-\gamma} \]

risk aversion increases
Debt/intermediated objections

- Why do agents get *more* risk averse as they approach bankruptcy, not *less*?
- OK for obscure CDS. But why not buy S&P500 directly?
- Why get in so much debt in the first place? Why use agents?
- Where are unconstrained, debt-free rich people, Warren Buffet, endowments, sovereign wealth funds etc.? (Answer: selling in a panic just like everyone else.)
- Why the strong correlation to macroeconomics? (Will the true state variable please stand up?)
- Why are individual mean returns strongly associated with comovement (factors)?
- Data (2008): Widespread coordinated rise in all risk premiums, including easy-to-trade, held in your and my 401(k) and Vanguard’s website.
A common risk premium
Rare disasters

\[ E_t(R_{t+1}) - R_t^f = \text{cov}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}, R_{t+1} \right] \]

- A small chance of a very low \( C_{t+1}/C_t \) can drive the whole covariance, raise \( E_tR_{t+1} \) despite reasonable \( \gamma \), and despite samples with small \( \sigma(\Delta c_{t+1}) \).

- Objections:
  1. Shouldn’t we see them more often? (Data controversy)
  2. Beyond equity premium? To get return predictability, p/d volatility, varying volatility, we need time-varying probabilities of rare disasters. External measurement or dark matter?
  3. We seem to need different time-varying probabilities for different assets (Gabaix).
  4. Correlation with business cycles? Probability of rare disasters exogenously correlated with business cycles? Or causality from stocks to recessions?
Probability assessments

\[ P_t U'(C) = \beta \sum_s \pi_s U'(C_s) X_s \]

- \( \pi, U' \) always enter together. There is no way to tell them apart without a priori restriction – \( U'(C) \) or \( \pi(Y) \)
- Do surveys “what do you expect” reveal \( E = \sum \pi \) or \( E^* = \sum \pi U' \)?
- Some model restricting \( \pi \) to other data, \( \pi(Y) \), or dark matter?
- Why the business cycle correlation?
- Min - max; robust control

\[ P_t U'(C) = \beta \min_{\{\pi \in \Theta\}} \sum_s \pi_s(Y_s) U'(C_s) X_s \]

But what's \( \theta \)? Why time-varying and business cycle related?
Summary:

- Many ideas give about the same result. An extra, recession-related state variable,
  \[ M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} Y_{t+1} \]

- No model yet decisively improves on habit in describing time-varying, business-cycle related risk premia; return predictability; “excess” volatility; “bubbles” associated with business cycles, long-run equity premium.

- No other model does so without relying on exogenous variation in the consumption process, just-so correlations (\( \Delta c_t \) with long run news) “dark matter” (time varying rare probabilities, business cycle correlated “sentiment,” long run news), rather than endogenous variation in risk premiums.

- Habit, despite neglect, is at least still a convenient formalism for capturing the common ideas.
Risk averse recessions

- Time to unite with production, general equilibrium! Integrate finance and macro (alternative to frictions)
- Keynesian: Recessions are driven by static flows: 
  \[ C = a + mpcY; \ I = \bar{I} - br; \text{ etc.} \]
- New-Keynesian: Recessions are intertemporal substitution
  \[ c_t = E_t c_{t+1} - \sigma r_t = E_t c_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \]
- Habit vision: Recessions are driven by endogenous time-varying risk aversion, not intertemporal substitution.
  \[ r = \delta + \gamma \left( \frac{c}{c-x} \right) E \left( \frac{dc}{c} \right) - \frac{1}{2} \gamma (\gamma + 1) \left( \frac{c}{c-x} \right)^2 \sigma^2 \]

(Looks like “discount rate shock” of NK models.) Consumption declines. \( Edc / c \) rise. Risk aversion rises some more. .. Asset prices decline. Investment declines. \( C+I \). Output declines. Almost multiplier-accelerator.

- Does it work?
Simple GE model 1: PIH with habit

\[
\max \frac{(c_0 - x)^{1-\gamma}}{1-\gamma} + E \left[ \frac{(c_1 - x)^{1-\gamma}}{1-\gamma} \right]
\]

\[c_1 = (e_0 - c_0) + e_1\]

\[e_1 = \{ e_h, e_l \} \quad pr(e_l) = \pi_l.\]

\[(c_0 - x)^{-\gamma} = E(c_1 - x)^{-\gamma}\]

\[(c_0 - x)^{-\gamma} = \pi_l(c_l - x)^{-\gamma} + \pi_h(c_h - x)^{-\gamma}\]

- \(x = 1, \gamma = 2, e_h = 2, e_l = 0.9 (\prec x!), \pi = 0.01\) (endpoint)
- \(c_0\) falls drastically in bad times, to make sure \(c_l > x\)
- \(c_0\) acts like buffer stock, leverage, debt models: high \(mpc\) for low \(c\).
- \(u'(c_0) = \pi_h u'(c_h)\) for high \(e_0\), but \(u'(c_0) = \pi_l u'(c_l)\) for low \(e_0\). Like min-max, ambiguity aversion, rare disaster, salience models.
- Stock prices fall, expected returns rise. Investment to fall?
Rising mpc in bad times

Time zero endowment $e_0$ vs. Consumption $c$

$e_l = 0.9, e_h = 2.0, x = 1$

$c_0, x=0$ PIH

$c_l \quad c^h \quad c=y$
Minimax, rare disaster behavior

In the diagram, the marginal utility is plotted against income $y_0$. The curves represent different utility functions:

- $\pi_l u'(c^l)$
- $u'(c_0)$
- $\pi_h u'(c^h)$

The $x$-axis represents income $y_0$, and the $y$-axis represents the marginal utility of income. The curves illustrate how marginal utility changes with income for different scenarios, indicating the effect of rare disasters on consumer behavior.
Stock prices fall
Risk Premia Rise

Consumption claim expected return and riskfree rate

First period income $y$

Expected returns, percent

$E(R)$

$R^f$
Investment and Q

\[ 1 + \alpha \frac{i_t}{k_t} = \frac{\text{market}_t}{\text{book}_t} = Q_t \]
A risky investment opportunity

\[
\max \frac{(c_0 - x)^{1-\gamma}}{1-\gamma} + E \left[ \frac{(c_1 - x)^{1-\gamma}}{1-\gamma} \right]
\]

\[
c_1 = e_1 + \theta_1 i_0 + B_0
\]

\[
c_0 = e_0 - i_0 - B_0 / R_f
\]

\[
i_0 \geq 0
\]

\[
(c_0 - x)^{-\gamma} = E(c_1 - x)^{-\gamma}
\]

\[
(c_0 - x)^{-\gamma} = E \left[ (c_1 - x)^{-\gamma} \theta_1 \right] \text{ if } i_0 > 0.
\]

- \(x = 1, \gamma = 2, e_h = 2, e_l = 0.9 (< x!), \pi = 0.01,
- \(\theta_l = 0.9, \theta_h = 1.2 \leftarrow
- \text{Risky investment collapses}
On to recessions

- The main issue of all macro:
  1. “Demand” falls, but $Y = F(K, L)$. Why does output fall?
  2. If $u'$ rises, hungry, why not work more?
     $$\max (c - x)^{1-\gamma} + (h - n)^{1-\gamma} \quad \text{s.t. } c = wn$$
     $$(c - x) = w(h - n)$$
  3. Desire to save rises. Why does investment fall?

- Answers:
  1. Traditional: sticky prices, wages.
  2. Shift of investment from risky private opportunity to storage/government debt. (“$R^f$”) Only $i$ counts as $y$.
  3. $h$ habit?
  4. Private work contributes to risky project which is being scaled back.
     $$c_1 = e_1 + \theta_1 \min (i_0, n_0) + B_0$$
     $$c_0 = e_0 - i_0 - B_0$$
     $$i_0 \geq 0; h > n > 0$$
     $$\rightarrow i_0 = n_0 \text{ collapses}$$

- Summary: Private economy is a risky project. Everyone wants to put in less money and less labor effort.
- Real dynamic model...
Empirical: Asset prices are driven by a large, time-varying, business-cycle correlated risk premium.

Theory: Habit captures it, endogenously.

Lots of other models capture many of the same ideas. (Elegant? Exogenous? Dark Matter?)

Habits capture many of the same ideas of those models. (Convenient?)

Business cycle correlation; merge asset pricing and finance!

Recessions are phenomena of risk aversion. Precautionary saving; scale back risky production / investment projects; all try to hold government debt.

See you in 20 years?