Directions

DO NOT START UNTIL WE TELL YOU TO DO SO. Read these directions in the meantime.

Please do not tear your exam apart. Answer the questions in the space provided. There are some extra pages at the end if you run out of space (but if you do, it means you’re writing too much.)

You can rip off the formula sheet and blank pages at the back for throw-away scrap paper if you wish.

Show your work. An answer that comes out of the blue or is the right answer but coming from the wrong equation will be graded as wrong. Also, by showing your work you may get partial credit.

Keep your answers short. We are only looking for the right answer; we will grade off for a memory dump of unrelated stuff as it reveals you don’t know what’s relevant to the question. Put your answers in a box or underline to make sure we find them. Make sure you answer each direct question. The questions are not clever or subtle. In each case, we just want to know the one obvious point.

For fact questions, quote the author and paper, or state that the fact comes from a problem set if such a source is relevant.

This is an closed-book, closed-note exam. You may use a calculator, but you do not need one; all the answers come out to simple numbers. You may not use a laptop computer, PDA, ipad, cell phone, etc.

Each question has a suggested time, which is also the number of points it will count in grading. Small times (5 min) require shorter answers. The total time is 2:15 = 135 so you have plenty of time.

Friday section: Do not discuss the contents of this exam with anyone until Sat 12 PM. There are two sections of this class, and any information passed to the other section is not only a serious honor code violation, it lowers your grade directly.

Booth honor code required statement: I pledge my honor that I have not violated the honor code during this examination.

Signature:
1. (5) You see the stock market fall by 10\%, $\Delta p_t = -0.10$. Does this fact imply that expected returns rise, fall, or stay the same relative to what you expected before the shock – i.e are prices are expected to “mean revert,” continue with “momentum” or stay the same? The answer is “it depends,” so explain what else you need to know, and say how much expected returns change in a few cases. Use the VAR we developed in class, see the formula sheet for a reminder.
2. (20) Suppose the regressions in logs had come out instead (ignoring constants) to

\[ r_{t+1} = 0.2 \times dp_t + \epsilon_{t+1}^r \]
\[ dp_{t+1} = 0.64 \times dp_t + \epsilon_{t+1}^{dp} \]

a) What value do you expect for \( \beta \) in \( \Delta d_{t+1} = b_d \times dp_t + \epsilon_{t+1}^d \)?

Hint: Use the return identity \( r_{t+1} \approx -\rho dp_t + \Delta d_{t+1} + dp_t \) (formula sheet) to connect coefficients. Give approximate answers, i.e. \( 0.96 \times 0.94 \approx 0.90 \) is fine. Is your number the “right” sign – high prices mean higher future dividend growth?

b) What value do you expect for long-run return and dividend growth forecast coefficients,

\[
\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = a + b_r^l \times dp_t + \epsilon^r_{t+1}^l
\]
\[
\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} = a + b_d^l \times dp_t + \epsilon^d_{t+1}^l
\]

c) Looking at the present value identity (formula sheet) We decided that all variation in price-dividend ratios corresponded to variation in expected returns and none to expected dividend growth. How is that conclusion altered by the fact that returns are even more predictable in this case? (Hint: think about running both sides of the present value identity on \( dp_t \), and multiplying by \( var(dp_t) \))
More room for #2
3. (5) Fama and French ("Multifactor explanations", "Dissecting anomalies") show that portfolios of smaller stocks (low market equity) earn higher average expected returns. This fact seems to offer an amazing profit opportunity: We’ll form a holding company ("Booth Hathaway"). We’ll buy lots of small stocks and earn their high expected returns. We’ll issue stock as a single company. Our total market equity will be so large that we’ll have to pay only a small expected return to our investors. We can pay ourselves huge salaries off the difference.

How would Fama and French respond? This seems like an awfully “inefficient” conclusion!
4) a(5) Which gets better returns going forward, stocks that had great past sales growth, or stocks of companies whose sales are going down? Are the high expected return stocks riskier, in the sense that they are more affected by market downturns? Cite evidence from a paper you read.

b(5) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? I.e. if value stocks go up, do momentum stocks tend to go up, down, or remain the same? Cite evidence from a paper you read.
5. (5) a) Some behavioral researchers claim that managers exploit “bubbles,” issuing stock when their stock is “overpriced,” and repurchasing when it’s “underpriced.” As a result, they say that high stock issues should forecast low returns. Leaving aside the explanations, is the fact right, or does the sign go the other way (high stock issues forecast high returns)?

b) Whatever the sign, do net stock issues add additional information about returns along with all the other forecasters?

In both cases, be specific, alluding to regression or portfolio evidence.
6. (10) The graph represents consumption over time, in percent (100 x log). Use the consumption-based model to find and plot the interest rate over time, also in percent, assuming people know ahead of time where consumption is going. Use discount rate $\delta = 2\%$, and risk aversion $\gamma = 2$, and approximate as necessary to get round (integer) answers. Hint: Start by making a table of interest rates for consumption growth -1,0,1, and 2%. Make sure you put the interest rate at the right moment in time. $t$ vs. $t + 1$ is vital here! What do you learn about how interest rates should move over the business cycle?
7a) (5) “The CAPM doesn’t work. You get much higher returns on small stocks than on big stocks.” Is this correct?

7b) (5) A friend brings in the following table of results

<table>
<thead>
<tr>
<th>(\hat{\gamma})</th>
<th>(\hat{\lambda})</th>
<th>(\sigma(\hat{\gamma}))</th>
<th>(\sigma(\hat{\lambda}))</th>
<th>(\sqrt{\frac{1}{N} \sum \alpha_i^2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.38</td>
<td>-0.57</td>
<td>0.40</td>
<td>0.19</td>
<td>0.15</td>
</tr>
</tbody>
</table>

CAPM, 1947-2010, FF 25 size and B/M portfolios. Estimate of \(E(R^{ci}) = \gamma + \lambda \beta_i + \alpha_i, i = 1, 2, ..25\) by cross-sectional regression.

You ask for a graph and he produces the following graph of \(E(R^{ci})\) (vertical axis) vs. predicted mean return, \(\gamma + \lambda \beta_i\) (horizontal axis). Ok, he says, it’s not perfect, but it’s not a total disaster either.

Did something go wrong here? Can you suggest a better procedure?
More space for 7
8. a) (5) A mutual fund manager complains, "Carhart’s results are bogus. He sorted mutual funds by their one-year past returns. Everyone knows that’s mostly luck. He should have looked at funds based on 5 year performance averages, like Morningstar does. Then he would have seen some alphas!" How would Carhart respond?

8. b) (5) Berk argues that there can be alpha even if mutual fund returns to investors do not persist over time, and that flows following past returns are not irrational. Which of the following considerations is key to this argument, and explain why. (Focus on the right one, briefly comment on the wrong ones).

   a) Managers can only achieve alpha up to a certain scale
   b) Managers raise their fees (as a percent of assets under management) if they do well
   c) Momentum in underlying stocks explains the appearance of persistent returns
9) (10) Below, find an excerpt from Fama and French’s Table Mutual Funds 3

a) What is the key assumption under “simulated?”

b) What does 1.68 mean in “simulated”? What does 2.04 mean? What does the relative position of 1.68 in “Actual” vs. “Simulated” mean?

c) How does this table address the claim, “the only reason you see some funds with really good performance is that they got lucky?”

d) What do -1.71 and -2.19 mean? Is this normal, or a puzzle?

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Simulated</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.71</td>
<td>-2.19</td>
</tr>
<tr>
<td>50</td>
<td>-0.01</td>
<td>-0.06</td>
</tr>
<tr>
<td>90</td>
<td>1.30</td>
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<td>91</td>
<td>1.38</td>
<td>1.68</td>
</tr>
<tr>
<td>95</td>
<td>1.68</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Table 3 - Percentiles of t(α) estimates for actual and simulated fund returns...[3-factor adjusted] gross fund returns....
10. (15)
a) On the day that Palm went public, what happened to 3Com’s price?
b) Did short sales constraints mean that nobody in fact was able to short Palm stock?
c) Was Palm more or less liquid than 3Com?
d) If you want to buy Palm because you think it will go up next week, why not buy 3Com instead? After all, 3Com owns 95% of Palm so it will go up too. (Be specific about facts.)
e) What implication did Cochrane draw from this graph?

Dollar volume on NYSE, NASDAQ and NASDAQ with SIC code 737. Series are normalized to 100 on Jan 1 1998.
11. (10)

a) A broker-dealer lost money and is running short of cash. Why does it not just issue more equity?

b) Derivatives are exempt from bankruptcy – they get paid first. Why does this make sense? Since the firm typically is running a matched book, with no overall derivative exposure, why does it case a problem in bankruptcy?

c) Why does it hurt the bank if you pull securities of your brokerage account? After all, they’re just executing trades for a fee; the securities you own are yours. Missing a few weeks of fee income isn’t going to make them bankrupt.

d) Why, according to Gorton and Metrick, did a run at Lehman spark a crisis, but a run at MF Global did not?
12. (15) The price of one, two and three year bonds is \( p_0^{(1)} = -0.05, p_0^{(2)} = -0.15, p_0^{(3)} = -0.30 \)

a) Find today’s yields and forward rates

b) Plot the expected evolution of these bonds’ prices over time, according to the expectations hypothesis.

c) Plot the expected evolution of these bonds’ prices for the first year, according to the Fama Bliss regressions, specializing all the coefficients to 1 and 0 as appropriate.
12. (5) Cochrane and Piazzesi run regressions

\[ r_{t+1}^{(n)} = a_n + \beta_{n,1}y_t^{(1)} + \beta_{n,2}f_t^{(2)} + \beta_{n,3}f_t^{(3)} + \beta_{n,4}f_t^{(4)} + \beta_{n,5}f_t^{(5)} + \varepsilon_{t+1}^{n} \]

They find betas in a tent shape across the right hand variables. What pattern do they find in these betas across maturity n? Write an equation that captures this pattern.
13. (15) Suppose the one-year rate is an MA(1),

\[ y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}. \]

\( E_t(\varepsilon_{t+1}) = 0; E(\varepsilon_t) = 0 \) as usual. Form a term-structure model, by supposing that the expectations hypothesis holds. (You’re looking for yields and forward rates of all maturities as a function of two “factors” \( \varepsilon_t \) and \( \varepsilon_{t-1} \).)

a) Find forward rates (at time \( t \), for maturity 2,3,4,...n)

b) Find yields (at time \( t \), for maturity 2,3,4,...n).

c) plot the yield and forward curves on a day in which \( \varepsilon_t = 1; \varepsilon_{t-1} = 1. \)

(Hint: You may think you got it wrong because the answer is too simple. Don’t worry, it really is simple. This problem does NOT involve a lot of algebra.)
14(5) You form an optimal portfolio of the 25 Fama French size and b/m sorted returns. You use the mean-variance formula

\[ \text{“optimal”: } w = \frac{1}{\gamma} \Sigma^{-1} E(R) \]

Here are the results, in percent. Did something go wrong, and if so what? Explain, using an equation or a graph.

<table>
<thead>
<tr>
<th></th>
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<th>2</th>
<th>3</th>
<th>4</th>
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<tr>
<td>smb</td>
<td></td>
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</tbody>
</table>
15. (5) You have risk aversion $\gamma = 1$, and returns are independent over time. Your best guess is that the mean annual premium $\mu$ is 4% with volatility $\sigma = 20\%$,

a) What should your allocation to stocks be?

b) In fact you don’t really know what the mean return is. Reflecting on it, your uncertainty about the mean return $\sigma(\mu)$ is 10 percentage points, and both the actual return and your uncertainty about it are normally distributed. (Numbers are easy to calculate, not realistic.) How does this consideration change your optimal allocation to stocks?
More space for any question 1
More space for any question 2
More space for any question 3
More space for any question 4
Formulas

Prediction and present value

If \( x_t = \phi x_{t-1} + \varepsilon_t \) then \( E_t(x_{t+j}) = \phi^j x_t \)

\[ r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t; \quad dp_t \equiv d_t - p_t \]

\[ p_t - d_t \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}; \quad \rho = \frac{1}{1+D/P} \approx 0.96 \]

VAR

\[ r_{t+1} = b_r \times dp_t + \varepsilon_{t+1}^r; \quad b_r \approx 0.1 \]
\[ \Delta d_{t+1} = b_d \times dp_t + \varepsilon_{t+1}^d; \quad b_d \approx 0 \]
\[ dp_{t+1} = b_{dp} \times dp_t + \varepsilon_{t+1}^{dp}; \quad b_{dp} \approx 0.94 \]

\[ \sum_{j=0}^{\infty} z^j = \frac{1}{1-z} \text{ if } \|z\| < 1 \]

Discount factors, consumption and models

\[ p_t = E_t(m_{t+1} x_{t+1}) = E_t \left( \beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right) \]

\[ m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \approx 1 - \delta - \gamma \Delta c_{t+1} \]

\[ 0 = E_t(m_{t+1} R_{t+1}^e); \quad 1 = E_t(m_{t+1} R_{t+1}) \]

\[ R_t^f = 1/E(m_{t+1}) = 1/E_t \left( \beta \frac{u'(c_{t+1})}{u'(c_t)} \right) \approx 1 + \delta + \gamma E_t(\Delta c_{t+1}) \]

\[ E(R_{t+1}^e) = -R_t^f \text{cov}(m_{t+1}, R_{t+1}^e) \approx \gamma \text{cov}(\Delta c_{t+1}, R_{t+1}^e) \]

Term structure

\[ p_t^{(n)} = \log \text{ price at } t \text{ of bond that comes due at } t + n, \text{ e.g. } -0.20 \]
\[ y_t^{(n)} \equiv -\frac{1}{n}p_t^{(n)}; \quad f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)}; \quad r_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)} \]
\[ y_t^{(n)} = \frac{1}{n} \left( y_t^{(1)} + f_t^{(2)} + f_t^{(3)} + \ldots + f_t^{(n)} \right) \]

Expectations:

\[ y_0^{(n)} = \frac{1}{n}E \left( y_t^{(1)} + y_t^{(1)} + y_t^{(1)} + \ldots + y_t^{(1)} \right) + \text{(risk premium)} \]
\[ f_t^{(n)} = E_t(y_t^{(1)} + \text{(risk premium)} \]
\[ E_t \left[ r_t^{(n)} \right] = y_t^{(1)} + \text{(risk premium)} \]
\[ r_t^{US} = r_t^{Euro} + s_t - E_t s_{t+1} + \text{(risk premium)} \]

25
Fama-Bliss regression

\[ r_{x_{t+1}}^{(n)} = r_{t+1}^{(n)} - y_t^{(1)} = a + b \left( f_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+1} \]

\[ y_{t+n-1}^{(1)} - y_t^{(1)} = a + b \left( f_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+1} \]

Cochrane-Piazzesi regression

\[ r_{\pi_{t+1}} = \frac{1}{4} \sum_{n=2}^{5} r_{x_{t+1}}^{(n)} = \gamma' f_t + \varepsilon_{t+1} \]

\[ r_{x_{t+1}}^{(n)} = b_n \left( \gamma f_t \right) + \varepsilon_{t+1}^{(n)} \]

**Portfolios**

Quadratic utility, independent returns, or mean-variance objective,

\[ w_0 = \frac{1}{\gamma} \Sigma^{-1} E(R^c); \Sigma = \text{cov}(R^c) \]

With a factor model

\[ R_{t+1}^p = R_f^l + w_m R_{t+1}^{em} + \omega_{\alpha} (\alpha + \varepsilon). \]

\[ w_m = \frac{1}{\gamma} E(\Sigma^{em}) = \frac{1}{\gamma} \Sigma^{-1} \alpha; \Sigma \equiv E(\varepsilon_{t+1} \varepsilon_{t+1}') \]

Multifactor, \( y = \) state variable, and relative to the market if everyone is like this

\[ w = \frac{1}{\gamma} \Sigma^{-1} E(R^c) + \beta_{R,y} \frac{\eta}{\gamma} \]

\[ R_i^l = R_f^l + \frac{\gamma_m}{\gamma^l} R^{em} + \frac{1}{\gamma^l} (\eta^l - \eta^m) R^{ez}; R^{e z} \equiv \beta_{y,R} R^c \]

\[ E(R^c) = \text{cov}(R^c, R^{em}) \gamma^m - \text{cov}(R^c, y') \eta^m \]

Bayesian portfolios

\[ \hat{\alpha} = \left( \Sigma^{-1}_{\alpha} + \Sigma^{-1}_{p} \right)^{-1} \left( \Sigma^{-1}_{\alpha} \alpha + \Sigma^{-1}_{p} 0 \right) \]

\[ f(R) = \int f(R|\mu) f(\mu) d\mu. \]

\[ R^* N(\mu, \sigma^2), \mu^* N(\mu, \sigma_{\mu}^2) \text{ then } f(R^*) N(\mu^*, \sigma^2 + \sigma_{\mu}^2) \]
2012 Final Exam Answers

1. (5) You see the stock market fall by 10%, $\Delta p_t = -0.10$. Does this fact imply that expected returns rise, fall, or stay the same relative to what you expected before the shock – i.e. are prices are expected to “mean revert,” continue with “momentum” or stay the same? The answer is “it depends,” so explain what else you need to know, and say how much expected returns change in a few cases. Use the VAR we developed in class, see the formula sheet for a reminder.

_**ANSWER:**_ $r_{t+1} = b_r \times (d_t - p_t) + \varepsilon_{t+1}$. It depends on what happened to $d_t$. If $p$ changes with no change in $d$, it means expected returns rise by about $0.1 \times 10\% = 1\%$. If $d$ changed 10% as well, then there is no change in expected returns.

2. (20) Suppose the regressions in logs had come out instead (ignoring constants) to

\[
\begin{align*}
r_{t+1} &= 0.2 \times dp_t + \varepsilon_{t+1}^r \\
dp_{t+1} &= 0.64 \times dp_t + \varepsilon_{t+1}^{dp}
\end{align*}
\]

a) What value do you expect for $b_d$ in

\[\Delta d_{t+1} = b_d \times dp_t + \varepsilon_{t+1}^{dp}?
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Hint: Use the return identity $r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t$ (formula sheet) to connect coefficients. Give approximate answers, i.e. $0.96 \times 0.94 \approx 0.90$ is fine. Is your number the “right” sign – high prices mean higher future dividend growth?

b) What value do you expect for long-run return and dividend growth forecast coefficients,

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\begin{align*}
\sum_{j=1}^{\infty} \rho^j r_{t+j} &= a + b_r^l \times dp_t + \varepsilon_{t+1}^{r_l} \\
\sum_{j=1}^{\infty} \rho^j \Delta d_{t+j} &= a + b_d^l \times dp_t + \varepsilon_{t+1}^{d_l}
\end{align*}
\]

c) Looking at the present value identity (formula sheet) We decided that all variation in price-dividend ratios corresponded to variation in expected returns and none to expected dividend growth. How is that conclusion altered by the fact that returns are even more predictable in this case? (Hint: think about running both sides of the present value identity on $dp_t$, and multiplying by $var(dp_t)$)

_**ANSWER:**_

a) Regressing both sides of the return identity on $dp_t$, $b_r = 1 + b_d - \rho b_{dp}$ Hence $b_d = b_r + \rho b_{dp} - 1$. In the old regression $b_d = 0.1 + 0.96 \times 0.94 - 1 = 0$. In the new regression $b_d = 0.2 + 0.96 \times 0.64 - 1 = -0.2$. Negative is the “right” sign.

b) $b_d^l = b_r (1 + \rho b_{dp} + \rho^2 b_{dp}^2 + \ldots) = b_r / (1 - \rho b_{dp}) = 0.2 / (1 - 0.96 \times 0.64) = 0.2 / 0.4 = 1 / 2$. Similarly, $b_r^l = -1 / 2$.

c) Running both sides of the present value identity on $dp$, $1 = b_d^l - b_d^l; 1 = 1 / 2 - (-1 / 2)$. We interpreted the two terms as fractions of var $dp$ explained, so with these numbers the variance of prices comes half from expected returns and half from expected dividend growth. (If you state the formula

$$var(p_t - d_t) \approx cov \left( p_t - d_t; \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} \right) - cov \left( p_t - d_t; \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} \right)$$
that’s even better, but just stating the answer in terms of regression coefficients is enough.

3. (5) Fama and French ("Multifactor explanations", "Dissecting anomalies") show that portfolios of smaller stocks (low market equity) earn higher average expected returns. This fact seems to offer an amazing profit opportunity: We’ll form a holding company ("Booth Hathaway"). We’ll buy lots of small stocks and earn their high expected returns. We’ll fund the purchase by issuing stock as a single company, and our total market equity will be so large that we’ll have to pay only a small expected return to our investors. We can pay ourselves huge salaries off the difference.

How would Fama and French respond? This seems like an awfully “inefficient” conclusion!

ANSWER: The new company would inherit the beta of small stocks, and, since expected returns are really a function of beta, not of market cap, our company would have to pay the expected return of small stocks. (For this answer it really doesn’t matter whether market beta is enough, or whether small firm beta gets a special premium. The point is that expected return is really a function of beta, not of size, and size is only coincidentally correlated with beta in the other firms.)

4a(5) Which gets better returns going forward, stocks that had great past sales growth, or stocks of companies whose sales are going down? Are the high expected return stocks riskier, in the sense that they are more affected by market downturns? Cite evidence from a paper you read.

ANSWER: the low sales growth stocks have higher expected returns. This does not correspond to higher market betas. It does correspond to larger hml betas. Fama and French "Multifactor anomalies"

4b(5) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? I.e. if value stocks go up, do momentum stocks tend to go up, down, or remain the same?

ANSWER

High momentum stocks have high h values. In “Multifactor anomalies” So momentum is negatively correlated with value.

5. (10) 5. (5) a) Some behavioral researchers claim that managers exploit “bubbles,” issuing stock when their stock is “overpriced,” and repurchasing when it’s “underpriced.” As a result, they say that high stock issues should forecast low returns. Leaving aside the explanations, is the fact right, or does the sign go the other way (high stock issues forecast high returns)?

b) Whatever the sign, do net stock issues add additional information about returns along with all the other forecasters?

In both cases, be specific, alluding to regression or portfolio evidence.

ANSWER FF dissecting anomalies. Yes, net issues do correspond to low returns and vice versa. Portfolios sorted by low stock issuance or repurchase have high subsequent returns and vice versa. Regressions $R_{t+1} = a + bNS_t + \varepsilon_{t+1}$ work. The portfolios are net of matched size and BM stocks; the regressions include size, bm and lots of other variables, so NS is an independent forecaster.

6. (10)The graph represents consumption over time, in percent (100 x log). Use the consumption-based model to find and plot the interest rate over time, also in percent, assuming people know ahead of time where consumption is going. Use discount rate $\delta = 2\%$, and risk aversion $\gamma = 2$, and approximate as necessary to get round (integer) answers. Hint: Start by making a table of interest rates for consumption growth -1,0,1, and 2%. Make sure you put the interest rate at the right moment in time. $t$ vs. $t + 1$ is vital here! What do you learn about how interest rates should move over the business cycle?
ANSWER This is from a problem set.

\[ r_t^f = \delta + \gamma E_t \Delta c_{t+1} = 2 + 2 \times E_t \Delta c_{t+1} \]

\[
\begin{align*}
E_t \Delta c_{t+1} & \quad -1 \quad 0 \quad 1 \quad 2 \\
2 + 2 \times E\Delta c & \quad 0 \quad 2 \quad 4 \quad 6
\end{align*}
\]

I graphed \( \Delta c_{t+1} \) in red and \( r_t^f \) in black. This is the interest rate quoted at time \( t \) for loans from \( t \) to \( t+1 \), and is conventionally dated as of time \( t \). I graphed it that way. That’s why the interest rate moves one period before the peaks of the consumption series. There’s a bit of a lesson here. See the “recession” in the second part of the plot. Interest rates move pretty much contemporaneously with the growth rate of consumption, with only the one-period advance notice. Interest rates move ahead of recessions as defined by the level of consumption. Much popular discussion confuses the level and growth views of where we are in economic cycles.
The hard part is the t vs t+1. The interest rate at t reflects consumption growth over the next year.

7a) (5) “The CAPM doesn’t work. You get much higher returns on small stocks than on big stocks.” Is this correct?

ANSWER: a) Two mistakes: i) higher average returns by themselves don’t mean anything, the question is whether they are matched by higher betas. ii) Actually, small stock average returns are matched by higher CAPM betas, as we saw in class

7b) (5) A friend brings in the following table of results

\[
\hat{\gamma} \quad \hat{\lambda} \quad \sigma(\hat{\gamma}) \quad \sigma(\hat{\lambda}) \quad \sqrt{\frac{1}{25} \sum \alpha_i^2}
\]

\[
1.38 \quad -0.57 \quad 0.40 \quad 0.19 \quad 0.15
\]

CAPM, 1947-2010, FF 25 size and B/M portfolios. Estimate of \( E(R^c) = \gamma + \lambda \beta_i + \alpha_i, i = 1, 2, \ldots, 25 \) by cross-sectional regression.

You ask for a graph and he produces the following graph of \( E(R^c) \) (vertical axis) vs. predicted mean return, \( \gamma + \lambda \beta_i \) (horizontal axis). Ok, he says, it’s not perfect, but it’s not a total disaster either.

Did something go wrong here? Can you suggest a better procedure?

ANSWER: This is from a problem set. This is the cross sectional regression with a free constant. Note the constant is huge and the market premium is negative. The actual performance of this model is awful. A graph like the following is an ideal answer, average returns vs betas,
A time series regression or including the factor portfolios (including rf) as test assets are ways to fix this.

8. a) (5) A mutual fund manager complains, "Carhart’s results are bogus. He sorted mutual funds by their one-year past returns. Everyone knows that’s mostly luck. He should have looked at funds based on 5 year performance averages, like Morningstar does. Then he would have seen some alphas!" How would Carhart respond?

ANSWER: Carhart also sorted funds on 5 year formation, and found even less result there than with sorts based on one-year performance.

8. b) (5) Berk argues that there can be alpha even if mutual fund returns to investors do not persist over time, and that flows following past returns are not irrational. Which of the following considerations is key to this argument, and explain why. (Focus on the right one, briefly comment on the wrong ones).

a) Managers can only achieve alpha up to a certain scale
b) Managers raise their fees (as a percent of assets under management) if they do well
c) Momentum in underlying stocks explains the appearance of persistent returns

ANSWER:

a is the right answer. As funds rush in, returns to investors decline. It’s important that b does not happen, otherwise we wouldn’t need new funds to give more money to the managers. c is irrelevant, that was Carhart’s point not Berk and Green’s.

9) (10) Below, find an excerpt from Fama and French’s Table Mutual Funds 3

a) What is the key assumption under “simulated?”

b) What does 1.68 mean in “simulated”? What does 2.04 mean? What does the relative position of 1.68 in “Actual” vs. “Simulated” mean?

c) How does this table address the claim, “the only reason you see some funds with really good performance is that they got lucky?”

d) What do -1.71 and -2.19 mean? Is this normal, or a puzzle?
<table>
<thead>
<tr>
<th>Percentile</th>
<th>Simulated</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.71</td>
<td>-2.19</td>
</tr>
<tr>
<td>50</td>
<td>-0.01</td>
<td>-0.06</td>
</tr>
<tr>
<td>90</td>
<td>1.30</td>
<td>1.59</td>
</tr>
<tr>
<td>91</td>
<td>1.38</td>
<td>1.68</td>
</tr>
<tr>
<td>95</td>
<td>1.68</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Table 3 - Percentiles of t(\(\alpha\)) estimates for actual and simulated fund returns...[3-factor adjusted] gross fund returns....

ANSWER

a) The key assumption under simulated is that no funds have any alpha, positive or negative.

b) 1.68 means that if all funds really have exactly zero alpha, then we expect to see that 5% of the funds in a sample will have an alpha t statistic greater than 1.68 just due to chance. In fact, 5% of funds had a t stat greater than 2.04, and 9% had a t stat greater than 1.68. Thus there are 4% too many funds with alpha greater than 1.68 than there should be.

c) Actually, there are more funds with very large alpha than there should be just due to luck. Not many, but a small number. (4%, above)

d) 5% of funds should have performance below -1.71. In fact, 5% of funds have performance below -2.19. This is a bit puzzling – why have negative alpha when you can just buy the index? But maybe they’re just on the way out.

10. (15)

a) On the day that Palm went public, what happened to 3Com’s price?

b) Did short sales constraints mean that nobody in fact was able to short Palm stock?

c) Was Palm more or less liquid than 3Com?

d) If you want to buy Palm because you think it will go up next week, why not buy 3Com instead? After all, 3Com owns 95% of Palm so it will go up too. (Be specific about facts.)

e) What implication did Cochrane draw from this graph?

Dollar volume on NYSE, NASDAQ and NASDAQ with SIC code 737. Series are normalized to 100 on Jan 1 1998.
ANSWER

a) Fell from 95 to 81.

b) No, at the peak palm was 147% shorted.

c) Fun question. Bid ask was larger, but turnover vastly more. We discussed and decided there was more demand to trade, despite higher costs, so less liquid. Mentioning the fact of high turnover and high bid ask spread is the key answer.

e) The volume here is visually nearly identical to a price graph. “Overpricing” comes with massive trading volume.

11. (10) a) A broker-dealer lost money and is running short of cash. Why does it not just issue more equity?

b) Derivatives are exempt from bankruptcy – they get paid first. Why does this make sense? Since the firm typically is running a matched book, with no overall derivative exposure, why does it case a problem in bankruptcy?

c) Why does it hurt the bank if you pull securities of your brokerage account? After all, they’re just executing trades for a fee; the securities you own are yours. Missing a few weeks of fee income isn’t going to make them bankrupt.

d) Why, according to Gorton and Metrick, did a run at Lehman spark a crisis, but a run at MF Global did not?

ANSWER

a) Debt overhang. Having lost money, the debt is trading below par. New equity first bails out that debt before making profits.

b) It makes sense to keep them from running. However, they get the right to replace their contracts, so the bank pays the bid ask spread on the whole book.

c) They are rehypothecated, and used by the firm as collateral for its own trading.

d) Gorton and Metric’s big point is that the problem is “systemic runs” when the “system” is insolvent. This happens when losses at one institution spark an e coli outbreak, people become worried about other institutions or assets. MF Global was transparently a bet on Greece, and since nobody learned anything about Greece or other investment banks from its failure, it didn’t cause any systemic problems. An outbreak you avoid the whole salad bar.

12. (15) The price of one, two and three year bonds is \( p_0(1) = -0.05, p_0(2) = -0.15, p_0(3) = -0.30 \)

a) Find today’s yields and forward rates.

b) Plot the expected evolution of these bonds’ prices over time, according to the expectations hypothesis.

c) Plot the expected evolution of these bonds’ prices for the first year, according to the Fama Bliss regressions, specializing all the coefficients to 1 and 0 as appropriate.
Answer: This includes a 4 year bond that I deleted from the question.

\[
\begin{align*}
y^{(1)} &= 0.05 \\
y^{(2)} &= 0.075 \\
y^{(3)} &= 0.10 \\
f^{(2)} &= 0.10 \\
f^{(3)} &= 0.15
\end{align*}
\]

I drew expectations just by having each line rise at the same rate as the one year rate that year. (green)

for FB, \( r_{t+1}^{(n)} = 0 + 1(f_t^{(n)} - y_t^{(1)}) \) as plotted
12. (5) Cochrane and Piazzesi run regressions

\[ r_{t+1}^{(n)} = a_n + \beta_{n,1}y_t^{(1)} + \beta_{n,2}f_t^{(2)} + \beta_{n,3}f_t^{(3)} + \beta_{n,4}f_t^{(4)} + \beta_{n,5}f_t^{(5)} + \varepsilon_{t+1} \]

They find betas in a tent shape across the right hand variables. What pattern do they find in these betas across maturity \( n \)? Write an equation that captures this pattern.

**ANSWER:** They found that the betas have the same shape for each maturity, just scaled up. So, they obey

\[ r_{t+1}^{(n)} = a_n + b_n \left( \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \gamma_3 f_t^{(3)} + \gamma_4 f_t^{(4)} + \gamma_5 f_t^{(5)} \right) + \varepsilon_{t+1} \]

13. (15) Suppose the one-year rate is an MA(1),

\[ y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1} \]

\( E_t(\varepsilon_{t+1}) = 0; E(\varepsilon_t) = 0 \) as usual. Form a term-structure model, by supposing that the expectations hypothesis holds. (You’re looking for yields and forward rates of all maturities as a function of two “factors” \( \varepsilon_t \) and \( \varepsilon_{t-1} \).)

a) Find forward rates (at time \( t \), for maturity 2,3,4,...\( n \))

b) Find yields (at time \( t \), for maturity 2,3,4,...\( n \)).

c) plot the yield and forward curves on a day in which \( \varepsilon_t = 1; \varepsilon_{t-1} = 1 \).

(Hint: You may think you got it wrong because the answer is too simple. Don’t worry, it really is simple. This problem does NOT involve a lot of algebra.)

**ANSWER**
\[ y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1} \]
\[ f_t^{(2)} = E_t y_{t+1} = \varepsilon_t \]
\[ f_t^{(3)} = E_t y_{t+2} = 0 \]
\[ f_t^{(n)} = E_t y_{t+n-1} = 0 \]

\[ y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1} = 2 \]
\[ y_t^{(2)} = \frac{1}{2} \left( y_t^{(1)} + f_t^{(2)} \right) = \frac{1}{2} \varepsilon_t + \varepsilon_{t-1} = 1.5 \]
\[ y_t^{(3)} = \frac{1}{3} \left( y_t^{(1)} + f_t^{(2)} + f_t^{(3)} \right) = \frac{1}{3} \varepsilon_t + \frac{2}{3} \varepsilon_{t-1} = 1 \]
\[ y_t^{(n)} = \frac{1}{n} \left( y_t^{(1)} + f_t^{(2)} + f_t^{(3)} + \ldots + f_t^{(n)} \right) = \frac{1}{n} \varepsilon_t + \frac{2}{n} \varepsilon_{t-1} = \frac{3}{n} \]

(Optional: Factors) The factors are \( \varepsilon_t \) and \( \varepsilon_{t-1} \). You can find them just by
\[ \varepsilon_t = f_t^{(2)} \]
\[ \varepsilon_{t-1} = y_t^{(1)} - f_t^{(2)} \]

You already have the loadings.

\[
\begin{bmatrix}
  y_t^{(1)} \\
  y_t^{(2)} \\
  y_t^{(3)} \\
  y_t^{(n)}
\end{bmatrix}
= \begin{bmatrix}
  1 \\
  \frac{1}{2} \\
  \frac{1}{3} \\
  \frac{1}{n}
\end{bmatrix}
\begin{bmatrix}
  \varepsilon_t \\
  \varepsilon_{t-1}
\end{bmatrix}
\]

14 (5) You form an optimal portfolio of the 25 Fama French size and b/m sorted returns. You use the mean-variance formula

\[ \text{“optimal”: } w = \frac{1}{\gamma} \Sigma^{-1} E(R) \]

Here are the results, in percent. Did something go wrong, and if so what? Explain, using an equation or a graph.

<table>
<thead>
<tr>
<th></th>
<th>low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>-149</td>
<td>51</td>
<td>69</td>
<td>96</td>
<td>52</td>
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<tr>
<td>2</td>
<td>-19</td>
<td>-57</td>
<td>190</td>
<td>-13</td>
<td>-60</td>
</tr>
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<td>-31</td>
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<td>41</td>
</tr>
<tr>
<td>4</td>
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<td>2</td>
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<td>77</td>
<td>-69</td>
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</tr>
<tr>
<td>smb</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ANSWER: This happens typically. What went “wrong” was taking mean returns at face value.
15. (5) You have risk aversion $\gamma = 1$, and returns are independent over time. Your best guess is that the mean annual premium $\mu$ is 4% with volatility $\sigma = 20\%$,

a) What should your allocation to stocks be?

b) In fact you don’t really know what the mean return is. Reflecting on it, your uncertainty about the mean return $\sigma(\mu)$ is 10 percentage points, and both the actual return and your uncertainty about it are normally distributed. (Numbers are easy to calculate, not realistic.) How does this consideration change your optimal allocation to stocks?

**ANSWER**

\[
\frac{0.04}{0.2^2} = \frac{0.04}{0.04} = 1
\]

\[
\frac{0.04}{0.2^2 + 0.1^2} = \frac{0.04}{0.04 + 0.01} = \frac{0.04}{0.05} = 0.8
\]

Verbal answers to the effect that “parameter uncertainty is extra variance and scale back the allocation to stocks” are worth some partial credit.
Name (Print clearly):

Section:

Mailfolder location:

Directions

DO NOT START UNTIL WE TELL YOU TO DO SO. Read these directions in the meantime.

Please do not tear your exam apart. Answer the questions in the space provided. There are some extra pages at the end if you run out of space (but if you do, it means you’re writing too much.)

You can rip off the formula sheet and blank pages at the back for throw-away scrap paper if you wish.

Show your work. An answer that comes out of the blue or is the right answer but coming from the wrong equation will be graded as wrong. Also, by showing your work you may get partial credit.

Keep your answers short. We are only looking for the right answer; we will grade off for a memory dump of unrelated stuff as it reveals you don’t know what’s relevant to the question. Put your answers in a box or underline to make sure we find them. Make sure you answer each direct question. The questions are not clever or subtle. In each case, we just want to know the one obvious point.

For fact questions, quote the author and paper, or state that the fact comes from a problem set if such a source is relevant.

This is an closed-book, closed-note exam. You may use a calculator, but you do not need one; all the answers come out to simple numbers. You may not use a laptop computer, PDA, ipad, cell phone, etc.

Each question has a suggested time, which is also the number of points it will count in grading. Small times (5 min) require shorter answers. The suggested times add up to 2:45; you have the full 3 hours to complete the exam.

AM section: Do not discuss the contents of this exam with anyone until 9 PM. There are two sections of this class, and any information passed to the other section is not only a serious honor code violation, it lowers your grade directly.

Booth honor code required statement: I pledge my honor that I have not violated the honor code during this examination.

Signature:
1. (10) a) You show a buddy the following table from class. (Numbers slightly simplified)

\[ R_{t+1} = a + b \times D/P_t + \varepsilon_{t+1} \]

<table>
<thead>
<tr>
<th>b</th>
<th>t</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>2.47</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Your buddy says “Come on now, that’s not important. Ok, you got a t statistic above two, but the \( R^2 \) is only 9%. That’s not important in any economic or practical sense. It’s tiny.” What facts might you cite to defend the importance of this regression? (Hint: What’s the definition of \( R^2 \)? Is there a better number? Use the numbers in the table where possible.)

b) Many commenters say that events such as the internet boom, with huge price multiples (P/E, P/D, M/B), are “bubbles” in stock prices; prices were high just because people expected stock prices to keep going up; they irrationally expected good returns forever, not because high prices rationally signaled higher profits. What facts from this course support this interpretation? What facts argue against it?
We studied a simple vector autoregression (VAR) which came out roughly as follows:

\[ r_{t+1} = 0.1 \times dp_t + \varepsilon^r_{t+1} \]
\[ \Delta d_{t+1} = 0 \times dp_t + \varepsilon^d_{t+1} \]
\[ dp_{t+1} = 0.94 \times dp_t + \varepsilon^{dp}_{t+1} \]

We identified combinations of the shocks $\varepsilon^r_{t+1}, \varepsilon^d_{t+1}, \varepsilon^{dp}_{t+1}$ that corresponded to “expected return” shocks and “expected cashflow” shocks. Sketch the response of $r_t, d_t, dp_t$, $t = 0, 1, 2, \ldots$ to each kind of shock (a) expected return, and (b) expected cashflow), occurring at $t = 1$. (Hint: the identity $\varepsilon^r_{t+1} = -\rho \varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1}$, resulting from $r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}$ will prove useful. Be sure to distinguish period 0, before the shock period 1, when the shock hits, and period 2,3,4,... which respond to the shock.)
3. (15) a) True/False and why. The “value” puzzle is not there in US data before 1963, because in that period “value” stocks did not outperform “growth” stocks.

b) Is the basic point of the Fama-French model that stocks with higher book/market ratios and smaller size (market capitalization) have higher average returns?

c) Which gets better returns going forward, stocks that had great past sales growth, or stocks that had poor past sales growth? Does this pattern correspond sensibly to betas?

d) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? I.e. if value stocks go up, do momentum stocks tend to go up, down, or remain the same?
4. (10) a) Companies should issue stock and invest when the cost of capital is low, meaning expected returns are low. Thus, portfolios of companies that are repurchasing stock should have higher returns going forward than portfolios of companies that are issuing stock. If you form a portfolio of companies that are issuing vs. repurchasing stock, do you see a spread of average returns in the right direction?

b) Wait a minute – those issuing companies have high stock prices and the repurchases low stock prices. Surely the big issuers are growth stocks and the repurchasers are value stocks. Is any pattern you found in a subsumed by value and growth effects?
5. (15) You’re valuing two private-equity investments, which will pay off $x_{t+1}$ when they go public next year. (Ok, lame attempt to make this look “real-world.”) Here is how they behave in a scenario analysis. $c_{t+1}$ is consumption next year. Consumption this year is $c_t = $1,000. (The numbers are unrealistic to make the problem easy to solve. You should not use approximations in this problem.)

a) Use the consumption-based model to find the value of each investment today. Use $\beta = 1, \gamma = 1$.

b) Compare the values i) to each other and ii) to the expected value of the payoffs. Explain why each one is higher, lower, or the same as the other, and higher, lower, or the same as the expected value of the payoff.

<table>
<thead>
<tr>
<th>Probability:</th>
<th>1/3</th>
<th>1/3</th>
<th>1/3</th>
<th>Expected value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{t+1}$</td>
<td>$1,500$</td>
<td>$1,000$</td>
<td>$500$</td>
<td>$1000$</td>
<td>—</td>
</tr>
<tr>
<td>$x^A_{t+1}$</td>
<td>$90$</td>
<td>$60$</td>
<td>$30$</td>
<td>$60$</td>
<td>$60$</td>
</tr>
<tr>
<td>$x^B_{t+1}$</td>
<td>$30$</td>
<td>$60$</td>
<td>$90$</td>
<td>$60$</td>
<td>$60$</td>
</tr>
</tbody>
</table>
6. (5) Your assignment is to evaluate the CAPM using the FF 25 portfolios in a subsample. One group member uses a pure time-series regression. She reports that the CAPM is lousy; the market premium is positive, but the alphas are huge; some alphas are even bigger than the average excess returns. The other group member uses a cross-sectional regression. He reports that no, the CAPM is doing fine. The alphas are small and a plot of actual $E(R^m)$ vs. predicted $\hat{\gamma} + \beta_i \hat{\lambda}$ returns looks great. How can both of these results happen? Illustrate your answer on a graph of average returns vs. betas.
7. (10) a) You investigate a hedge fund with high CAPM alpha, but find that you can explain its entire performance with loadings on hml, smb and umd (momentum) factors, which you measure using tradeable ETFs. The fund objects: “You’re not allowed to use momentum as a factor, it’s not even remotely a state variable of concern to investors.” Is this a valid objection?

b) Fund managers claim that fees and turnover do not reduce returns to investors, because higher fees and turnover generate higher alpha. A cynical Chicago economist predicts that higher fees and turnover have no relationship to returns to investors, because the fund will keep all the alpha. What are the facts (roughly)? How much does a 1% change in fees change returns to investors? How much does turnover – selling one stock and buying another – change returns to investors? What regression did Carhart run to measure these quantities?
8. (5) Berk argues that there can be alpha even if mutual fund returns to investors do not persist over time, and that flows following past returns are not irrational. Which of the following considerations is key to this argument, and explain why. (Focus on the right one, briefly comment on the wrong ones).

a) Managers can only achieve alpha up to a certain scale
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9. (15)
   a) On the day that Palm went public, what happened to 3Com’s price?
   b) Did short sales constraints mean that nobody in fact was able to short Palm stock?
   c) Was Palm more or less liquid than 3Com?
   d) If you want to buy Palm because you think it will go up next week, why not buy 3Com instead? After all, 3Com owns 95% of Palm so it will go up too. (Be specific about facts.)
   e) What implication did Cochrane draw from this graph?

Dollar volume on NYSE, NASDAQ and NASDAQ with SIC code 737. Series are normalized to 100 on Jan 1 1998.
a) Suppose your brokerage is part of an investment bank that might default on its debt. Why might you pull out your securities? After all, they're yours, not the bank's.

b) Why does it hurt the bank if you pull out of your brokerage account? After all, they're just executing trades for a fee; the securities you own are yours. Missing a few weeks of fee income isn't going to make them bankrupt.

c) Gorton and Metrick call short-term debt “information insensitive.” What does this mean, and what can change that fact (hint: Gorton and Metrick made an analogy to a food-poisoning outbreak.)
11. (15) The current log forward curve is (in years)

<table>
<thead>
<tr>
<th>$y_t^{(1)}$</th>
<th>$f_t^{(2)}$</th>
<th>$f_t^{(3)}$</th>
<th>$f_t^{(4)}$</th>
<th>$f_t^{(5)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>2%</td>
<td>3%</td>
<td>4%</td>
<td>5%</td>
</tr>
</tbody>
</table>

a) According to the expectations hypothesis, what is the expected interest rate four years from now $E_t \left( y_{t+4}^{(1)} \right)$?

b) Do Fama and Bliss’ regressions support or contradict this conclusion?

c) According to the expectations hypothesis, what is the expected value of next year’s three-year forward rate $E_t(f_{t+1}^{(3)})$?

d) According to Fama and Bliss’ one-year regressions, and approximating the numbers by 1 and 0, what is the expected return on three year bonds $E_t f_{t+1}^{(3)}$?

e) According to Fama and Bliss’ one-year return regressions, and approximating the numbers by 1 and 0, what is the expected value of next year’s three-year forward rate $E_t f_{t+1}^{(3)}$? Compare to c.
12. (10) What does this picture represent? Be explicit, with equations. What is the point of the picture?

![Graph showing unrestricted and restricted functions with different markers for values 2, 3, 4, 5.](image-url)
13. (15) Suppose the one-year rate is an MA(1),

\[ y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}. \]

\( E_t(\varepsilon_{t+1}) = 0; E(\varepsilon_t) = 0 \) as usual. Form a term-structure model, by supposing that the expectations hypothesis holds.

a) Find forward rates (at time \( t \), for maturity \( 2, 3, 4, \ldots, n \))

b) Find yields (at time \( t \), for maturity \( 2, 3, 4, \ldots, n \)).

c) Express the factors in terms of yields or forward rates. Then express the yield-curve model as a function of factors, i.e. loadings on the factors. (There are many ways to do this. Choose an easy one!)

(Hint: You may think you got it wrong because the answer is too simple. Don’t worry, it really is simple. This problem does NOT involve a lot of algebra.)
14. (5) You are a very risk averse investor with a 10 year horizon, and you have decided that a 10 year zero coupon TIP is the right security. A new financial crisis comes along. Long-term rates rise, so your bond plummets in value. Interest-rate volatility spikes. Your long-term bond investment has underperformed the 7-year bond benchmark by 5%.

Your investment adviser wants you to bail out to less risky short term bonds, or more profitable investments. He reminds you of $w = \frac{1}{\gamma}E_t(R^e)/\sigma_t^2(R^e_{t+1})$. The bond-return forecasting model says $E_t(R^e)$ is dreadful for next year, and with $\sigma_t(R^e_{t+1})$ higher surely you should rebalance away from this bond. Is he right? If not, why not and what should you do? (This is a short answer question, not a lot of algebra. Explain with reference to portfolio formulas, and intuition.)
15. (5) You have risk aversion $\gamma = 1$, and returns are independent over time. Your best guess is that the mean annual premium $\mu$ is 4% with volatility $\sigma = 20\%$.

a) What should your allocation to stocks be?

b) In fact you don’t really know what the mean return is. Reflecting on it, your uncertainty about the mean return $\sigma(\mu)$ is 10 percentage points, and both the actual return and your uncertainty about it are normally distributed. (Numbers are easy to calculate, not realistic.) How does this consideration change your optimal allocation to stocks?
16. (5) You create a “small business” factor, a portfolio of assets that has 90% correlation with shocks to the profits of small and privately-held businesses. Alas, after running regressions of your portfolio return on rmrf, smb, hml, you find the alpha of your factor return is zero.

\[ R_{factor} = 0 + \beta \times rmrf_t + h \times hml_t + s \times smb_t + \epsilon_{factor}; \quad R^2 = 0.4 \]

This means we do not need your new factor to price assets, and the expected returns of your factor can be achieved by the combination of FF 3 factors. Might a fund based on your new factor be interesting to investors nonetheless? If so, how? (Hint: The \( R^2 \) matters here)
More space for any question 1
More space for any question 2
More space for any question 3
More space for any question 4
Formulas

Prediction and present value

If  \( x_t = \phi x_{t-1} + \varepsilon_t \)  then  \( E_t(x_{t+j}) = \phi^j x_t \)

\[
r_{t+1} \approx -\rho d p_{t+1} + \Delta d_{t+1} + d p_t = \rho(p_{t+1} - d_{t+1}) + \Delta d_{t+1} - (p_t - d_t)
\]

\[
p_t - d_t \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}; \quad \rho = \frac{1}{1 + D/P} \approx 0.96
\]

\[
\text{var} (p_t - d_t) \approx \text{cov} \left( p_t - d_t, \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} \right) = \text{cov} \left( p_t - d_t, \sum_{j=1}^{k} \rho^{j-1} r_{t+j} \right) + \text{cov} (p_t - d_t, \rho^k (p_{t+k} - d_{t+k}))
\]

VAR

\[
\begin{align*}
r_{t+1} &= 0.1 \times d p_t + \varepsilon^r_{t+1} \\
\Delta d_{t+1} &= 0.0 \times d p_t + \varepsilon^d_{t+1} \\
d p_{t+1} &= 0.94 \times d p_t + \varepsilon^{d p}_{t+1} \\
\sum_{j=0}^{\infty} z^j &= \frac{1}{1 - z} \text{ if } \|z\| < 1
\end{align*}
\]

Discount factors, consumption and models

\[
p_t = E_t(m_{t+1} x_{t+1}) = E_t \left( \beta^\frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right)
\]

\[
m_{t+1} = \beta \frac{c_{t+1}}{c_t} \approx 1 - \delta - \gamma \Delta c_{t+1}
\]

0 = \( E_t(m_{t+1} R_{t+1}^e) \); 1 = \( E_t(m_{t+1} R_{t+1}^e) \)

\[
R^f = 1 \left/ E_t(m_{t+1}) \right. = 1 \left/ E_t \left( \beta^\frac{u'(c_{t+1})}{u'(c_t)} \right) \right. \approx 1 + \delta + \gamma E_t (\Delta c_{t+1})
\]

\[
E_t(R_{t+1}^e) = -R^f_t \text{ cov} (m_{t+1}, R_{t+1}^e) \approx \gamma \text{cov} (\Delta c_{t+1}, R_{t+1}^e)
\]

Term structure

\[
p_t^{(n)} = \text{log price at } t \text{ of bond that comes due at } t + n, \text{ e.g. } -0.20
\]

\[
y_t^{(n)} = -\frac{1}{n} p_t^{(n)} ; \quad f_t^{(n)} = p_t^{(n-1)} - p_t^{(n)} ; \quad r_{t+1}^{(n)} = p_t^{(n-1)} - p_t^{(n)}
\]

\[
y_t^{(n)} = \frac{1}{n} \left( y_t^{(1)} + f_t^{(2)} + f_t^{(3)} + \ldots + f_t^{(n)} \right)
\]

Expectations:

\[
y_0^{(n)} = \frac{1}{n} E \left( y_t^{(1)} + y_{t+1}^{(1)} + y_{t+2}^{(1)} + \ldots y_{t+n-1}^{(1)} \right) + \text{(risk premium)}
\]

\[
f_t^{(n)} = E_t(y_{t+n-1}^{(1)}) + \text{(risk premium)}
\]

\[
E_t \left[ r_{t+1}^{(n)} \right] = y_t^{(1)} + \text{(risk premium)}
\]

\[
r_t^{(n)} = r_t^{Euro} + s_t - E_t s_{t+1} + \text{(risk premium)}
\]
Fama-Bliss regression

\[ r_{t+1}^{(n)} = r_{t+1}^{(n)} - y_t^{(1)} = a + b \left( f_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+1} \]

\[ y_{t+n-1}^{(1)} - y_t^{(1)} = a + b \left( f_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+1} \]

Cochrane-Piazzesi regression

\[ \overline{r}_{t+1}^{(n)} = \frac{1}{n} \sum_{n=2}^{5} r_{t+1}^{(n)} = \gamma' f_t + \varepsilon_{t+1} \]

\[ r_{t+1}^{(n)} = b_n (\gamma' f_t) + \varepsilon_{t+1}^{(n)} \]

Eigenvalues

\[ \Sigma = QAQ'; \quad QQ' = I \]

\[ y_t = Qx_t; \quad x_t = Q'y_t; \]

\[ \text{cov}(y', y) = \Sigma, \quad \text{cov}(x, x') = \Lambda \]

Lognormal trick

\[ E(e^x) = e^{E(x) + \frac{1}{2} \sigma^2(x)} \text{ if } x \text{ is normal} \]

Portfolios

Quadratic utility, independent returns, or mean-variance objective,

\[ w_0 = \frac{1}{\gamma} \Sigma^{-1} E(R^c); \quad \Sigma = \text{cov}(R^c) \]

With a factor model

\[ R^p_{t+1} = R^f + w_m R^{em}_{t+1} + w'_\alpha (\alpha + \varepsilon). \]

\[ w_m = \frac{1}{\gamma} E(R^{em}); \quad w'_\alpha = \frac{1}{\gamma} \Sigma^{-1} \alpha; \quad \Sigma = E(\varepsilon_{t+1} \varepsilon_{t+1}') \]

Multifactor, \( y = \) state variable, and relative to the market if everyone is like this

\[ w = \frac{1}{\gamma} \Sigma^{-1} E(R^c) + \beta_{R,y} \eta - \frac{\gamma}{\gamma} \]

\[ R^i = R^f + \frac{\gamma}{\gamma'} R^{em} + \frac{1}{\gamma} (\eta' - \eta'' \gamma) R^{ez}; \quad R^{ez} \equiv \beta_{y,R} R^c \]

\[ E(R^c) = \text{cov}(R^c, R^{em}) \gamma^m - \text{cov}(R^c, y') \eta^m \]

Bayesian portfolios

\[ \hat{\alpha} = \left( \Sigma^{-1}_\alpha + \Sigma^{-1}_p \right)^{-1} \left( \Sigma^{-1}_\alpha \alpha + \Sigma^{-1}_p 0 \right) \]

\[ f(R) = \int f(R|\mu) f(\mu) d\mu. \]

\[ R^\sim N(\mu, \sigma^2), \mu^\sim N(\mu, \sigma^2_\mu) \text{ then } f(R) \sim N(\mu, \sigma^2 + \sigma^2_\mu) \]

60
2011 Final Exam Answers

1. (10)

a) You show a buddy the following table from class. (Numbers slightly simplified)

<table>
<thead>
<tr>
<th>$b$</th>
<th>$t$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>2.47</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Your buddy says “Come on now, that’s not important. Ok, you got a t statistic above two, but the $R^2$ is only 9%. That’s not important in any economic or practical sense. It’s tiny.” What facts might you cite to defend the importance of this regression? (Hint: What’s the definition of $R^2$? Is there a better number? Use the numbers in the table where possible.)

b. Many commenters say that events such as the internet boom, with huge price multiples (P/E, P/D, M/B), are “bubbles” in stock prices; prices were high just because people expected stock prices to keep going up so they irrationally expected good returns forever, not because high prices rationally signaled higher profits. What facts from this course support this interpretation? What facts argue against it?

ANSWER:

a) There are several observations we made to show the economic importance of forecastability. The one I was steering you towards was a, but b and c can also be mentioned.

i) The variation in expected returns, while small compared to the variation in total returns, is large compared to expected returns. From the table we can compute $R^2 = \frac{\text{var}(\Omega_t(R_{t+1}))}{\text{var}(R_{t+1})} = 0.09$ so $\sigma(b \times dp) = \sqrt{0.09\sigma(R)} = 0.3 \times 0.20 = 6\%$

ii) Both coefficient and $R^2$ are larger at longer horizons

iii) The volatility test says this predictability is just enough to account for the large variance of dividend yields.

b)  

i) It is true that price dividend ratios do not, in fact, forecast lower future dividends.

ii) a) Price dividend ratios do, however forecast lower returns. So the data are also consistent with lower discount rates. This is the main point – as stated the bubble said expected returns are constant.

Also possible to mention:

b) Price dividend ratios do not forecast forever higher price dividend ratios

c) we studied volatility tests

$$\text{var} \left( p_t - d_t \right) \approx \text{cov} \left( p_t - d_t, \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} \right) - \text{cov} \left( p_t - d_t, \sum_{j=1}^{k} \rho^{j-1} r_{t+j} \right) + \text{cov} \left( p_t - d_t, \rho^k (p_{t+k} - d_{t+k}) \right)$$

The return forecast coefficient on the right is just about the same as the variance on the left, so return predictability is enough to account for volatility.
The remaining argument is over whether the expected return variation is “rational.” An answer that mentions Cochrane Stocks as money and large amounts of trading and difficulty shorting houses gets an extra gold star.

2 (15)

We studied a simple vector autoregression (VAR) which came out roughly as follows

\[
\begin{align*}
    r_{t+1} &= 0.1 \times dp_t + \varepsilon^r_{t+1} \\
    \Delta d_{t+1} &= 0 \times dp_t + \varepsilon^d_{t+1} \\
    dp_{t+1} &= 0.94 \times dp_t + \varepsilon^{dp}_{t+1}
\end{align*}
\]

We identified combinations of the shocks \(\varepsilon^r_{t+1}, \varepsilon^d_{t+1}, \varepsilon^{dp}_{t+1}\) that corresponded to “expected return” shocks and “expected cashflow” shocks. Sketch the response of \(r_t, d_t, dp_t, t = 0, 1, 2, \ldots\) to each kind of shock (a) expected return, and b) expected cashflow), occurring at \(t = 1\). (Hint: the identity \(\varepsilon^r_{t+1} = -\rho \varepsilon^r_{t+1} + \varepsilon^d_{t+1}\), resulting from \(r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}\) will prove useful. Be sure to distinguish period 0, before the shock period 1, when the shock hits, and period 2, 3, 4, ... which respond to the shock.)

**ANSWER**

The “expected cashflow” shock moves \(\Delta d\) by 1 without changing \(dp\). Hence it moves \(r\) by 1 as well, by the \(\varepsilon\) identity. In subsequent periods, nothing changes. Thus, you get the plot on the left.

The “expected return” shock moves \(dp\) by 1, without changing \(\Delta d\). Hence, \(r\) moves by -0.96. \(dp\) then follows AR(1) reversion at rate 0.94. This means \(r\) also follows 0.1 \times 0.94. The right hand sketch shows a negative shock to \(dp\), flipped upside down is fine too.

3. (15) a) True/False and why. The “value” puzzle is not there in US data before 1963, because in that period “value” stocks did not outperform “growth” stocks.

b) Is the basic point of the Fama-French that stocks with higher book/market ratios and smaller size (market capitalization) have higher average returns?
c) Which gets better returns going forward, stocks that had great past growth in sales, or stocks that had poor past growth in sales? Is this a new factor or is the pattern of returns consistent with some pattern of factor exposures?

d) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? I.e. if value stocks go up, do momentum stocks tend to go up, down, or remain the same?

ANSWER

a) False. Yes, the value effect is not present in the earlier period, but expected returns are roughly the same. It’s the betas that changed and every puzzle is a joint puzzle of expected returns and betas.

b) NO. It is a fact, but the Fama French model explains this fact with size and b/m factors.

c) Fama and French multifactor. Good growth = bad returns. This is explained by h loadings.

d) High momentum stocks have high h values. So momentum is negatively correlated with value.

4. (10) a) Companies should issue stock and invest when the cost of capital is low, meaning expected returns are low. Thus, portfolios of companies that are repurchasing stock should have higher returns going forward than portfolios of companies that are issuing stock. If you form a portfolio of companies that are issuing vs. repurchasing stock, do you see a spread of average returns in the right direction?

ANSWER FF dissecting anomalies. Yes, net issues do correspond to low returns and vice versa.

b) Wait a minute – those issuing companies have high stock prices and the repurchasers low stock prices. Surely the big issuers are growth stocks and the repurchasers are value stocks, so any pattern you found in a is subsumed by value and growth effects?

ANSWER FF used characteristic adjusted portfolios, so this is above any BM effect.

5. (15) You’re valuing two private-equity investments, which will pay off x_{t+1} when they go public next year. (Ok, lame attempt to make this look “real-world.”) Here is how they behave in a scenario analysis. c_{t+1} is consumption next year. Consumption this year is c_t = $1,000. (The numbers are unrealistic to make the problem easy to solve. You should not use approximations in this problem.)

a) Use the consumption-based model to find the value of each investment today. Use β = 1, γ = 1.

b) Compare the values i) to each other and ii) to the expected value of the payoffs. Explain why each one is higher, lower, or the same as the other, and higher, lower, or the same as the expected value of the payoff.

<table>
<thead>
<tr>
<th>Probability:</th>
<th>1/3</th>
<th>1/3</th>
<th>1/3</th>
<th>Expected value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>c_{t+1}</td>
<td>$1,500</td>
<td>$1,000</td>
<td>$500</td>
<td>$1000</td>
<td>—</td>
</tr>
<tr>
<td>x^{A}_{t+1}</td>
<td>$90</td>
<td>$60</td>
<td>$30</td>
<td>$60</td>
<td>$60</td>
</tr>
<tr>
<td>x^{B}_{t+1}</td>
<td>$30</td>
<td>$60</td>
<td>$90</td>
<td>$60</td>
<td>$60</td>
</tr>
</tbody>
</table>

ANSWER

<table>
<thead>
<tr>
<th>Probability:</th>
<th>1/3</th>
<th>1/3</th>
<th>1/3</th>
<th>Expected value</th>
<th>Value</th>
<th>E(x)/R^f</th>
</tr>
</thead>
<tbody>
<tr>
<td>c_{t+1}</td>
<td>$1,500</td>
<td>$1,000</td>
<td>$500</td>
<td>$1,000</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>x^{A}_{t+1}</td>
<td>$90</td>
<td>$60</td>
<td>$30</td>
<td>$60</td>
<td>$60</td>
<td>$73.67</td>
</tr>
<tr>
<td>x^{B}_{t+1}</td>
<td>$30</td>
<td>$60</td>
<td>$90</td>
<td>$60</td>
<td>$86.67</td>
<td>$73.67</td>
</tr>
</tbody>
</table>

a)
\[
\frac{1}{3} \times \left( \frac{1500}{1000} \right)^{-1} \times 90 + \frac{1}{3} \times \left( \frac{1000}{1000} \right)^{-1} \times 60 + \frac{1}{3} \times \left( \frac{500}{1000} \right)^{-1} \times 30
\]
\[
\frac{1}{3} \times \left( \frac{1000}{1500} \right) \times 90 + \frac{1}{3} \times \left( \frac{1000}{1000} \right) \times 60 + \frac{1}{3} \times \left( \frac{1000}{500} \right) \times 30
\]
\[
\frac{2}{3} \times 30 + 1 \times 20 + (2) \times 10
\]
\[
20 + 20 + 20 = 60
\]
\[
\frac{1}{3} \times \left( \frac{1500}{1000} \right)^{-1} \times 30 + \frac{1}{3} \times \left( \frac{1000}{1000} \right)^{-1} \times 60 + \frac{1}{3} \times \left( \frac{500}{1000} \right)^{-1} \times 90
\]
\[
\frac{1}{3} \times \left( \frac{1000}{1500} \right) \times 30 + \frac{1}{3} \times \left( \frac{1000}{1000} \right) \times 60 + \frac{1}{3} \times \left( \frac{1000}{500} \right) \times 90
\]
\[
\frac{2}{3} \times 10 + 1 \times 20 + (2) \times 30
\]
\[
6.2/3 + 20 + 60 = 86.2/3
\]

b) They covary differently with consumption growth. Project A pays off when consumption is already good, and
does badly in bad times. B pays off when consumption is bad, so provides “consumption insurance.”

c) That again explains why B is more than its expected value, but what about A? It’s paying its expected value
even though it’s risky. The key here is that the risk free rate is negative. \( E(m) \) is not the same thing as expected
consumption growth.

\[
E(m) = \frac{1}{3} \times \left( \frac{1500}{1000} \right)^{-1} + \frac{1}{3} \times \left( \frac{1000}{1000} \right)^{-1} + \frac{1}{3} \times \left( \frac{500}{1000} \right)^{-1}
\]

\[
E(m) = \frac{1}{3} \times \left( \frac{1000}{1500} \right) + \frac{1}{3} \times \left( \frac{1000}{1000} \right) + \frac{1}{3} \times \left( \frac{1000}{500} \right)
\]

\[
E(m) = \frac{1}{3} \times \left( \frac{2}{3} \right) + \frac{1}{3} + \frac{1}{3} \times (2) = \frac{1}{3} \times \left( \frac{2}{3} + 1 + 2 \right) = \frac{11}{9}
\]

\[
R^f = \frac{9}{11}
\]

Thus, \( E(x) / R^f = 11/9 \) \( E(x) = 11/9 \times 60 = 11/3 \times 20 = 220/3 = 73.67 \)

6. (5) Your assignment is to evaluate the CAPM using the FF 25 portfolios in a subsample. One group member
uses a pure time-series regression. She reports that the CAPM is lousy; the market premium is positive, but the
alphas are huge; some alphas are even bigger than the average excess returns. The other group member uses a
cross-sectional regression. He reports that no, the CAPM is doing fine. The alphas are small and a plot of actual\( E(R^i) \) vs. predicted \( \hat{\gamma} + \beta_i \lambda \) returns looks great. How can both of these results happen? Illustrate your answer on
a graph of average returns vs. betas.

ANSWER

The problem is the free intercept. Here’s what happened. Either include the interest rate as a test asset, use
GLS, or don’t allow a free intercept.
7. (10) a) You investigate a hedge fund with high CAPM alpha, but find that you can explain its entire performance with loadings on hml, smb and umd (momentum) factors, which you measure using tradeable ETFs. The fund objects: “You’re not allowed to use momentum as a factor, it’s not even remotely a state variable of concern to investors.” Is this a valid objection?

b) Fund managers claim that fees and turnover do not reduce returns to investors, because higher fees and turnover generate higher alpha. A cynical Chicago economist predicts that higher fees and turnover have no relationship to returns to investors, because the fund will keep all the alpha. What are the facts (roughly)? How much does a 1% change in fees change returns to investors? How much does turnover – selling one stock and buying another – change returns to investors? What regression did Carhart run to measure these quantities?

ANSWER: performance evaluation factors are only about “can I program a computer to do that without paying your fees,” they don’t have to be “state variables of concern to investors.”

ANSWER: If fees and turnover produced more alpha. We should expect in fact that after fee returns are not affected by fees.

ANSWER: roughly 1%. roughly 1% roundtrip transactions costs. A cross sectional regression

\[
\alpha_i = a + b(\text{fees}_i) + c(\text{turnover}_i) + \varepsilon_i = 1, 2...N
\]

8. (5) Berk argues that there can be alpha even if mutual fund returns to investors do not persist over time, and that flows following past returns are not irrational. Which of the following considerations is key to this argument, and explain why. (Focus on the right one, briefly comment on the wrong ones).

a) Managers can only achieve alpha up to a certain scale

b) Managers raise their fees (as a percent of assets under management) if they do well

c) Momentum in underlying stocks explains the appearance of persistent returns

ANSWER:

a. As funds rush in, returns to investors decline. It’s important that b does not happen, otherwise we wouldn’t need new funds to give more money to the managers. c is irrelevant, that was Carhart’s point not Berk and Green’s.
9. (15)

a) On the day that Palm went public, what happened to 3Com’s price?

b) Did short sales constraints mean that nobody in fact was able to short Palm stock?

c) Was Palm more or less liquid than 3Com?

d) If you want to buy Palm because you think it will go up next week, why not buy 3Com instead? After all, 3Com owns 95% of Palm so it will go up too. (Be specific about facts.)

e) What implication did Cochrane draw from this graph?

![Dollar volume graph](image)

Dollar volume on NYSE, NASDAQ and NASDAQ with SIC code 737. Series are normalized to 100 on Jan 1 1998.

**ANSWER**

a) Fell, from 95 to 81.

b) No, at the peak palm was 147% shorted

c) fun question. Bid ask was larger, but turnover vastly more. We discussed and decided there was more demand to trade, despite higher costs, so less liquid. Mentioning the fact of high turnover and high bid ask spread is the key answer.

e) The volume here is visually nearly identical to a price graph. “overpricing” comes with massive trading volume.

10. (10)

a) Suppose your brokerage is part of an investment bank that might default on its debt. Why might you pull out your securities? After all, they’re yours, not the bank’s.

b) Why does it hurt the bank if you pull out of your brokerage account? After all, they’re just executing trades for a fee; the securities you own are yours. Missing a few weeks of fee income isn’t going to make them bankrupt.

c) Gorton and Metrick call short-term debt “information insensitive.” What does this mean, and what can change that fact (hint: Gorton and Metrick made an analogy to a food-poisoning outbreak.)

**ANSWER**

a) The bank has used your securities for its own loans, “rehypothecated” so it’s going to be hard to get them back in bankruptcy.
b) See above. If you pull out the bank loses financing for its own trading and must dump assets.

c) It means that most of the time you don’t do a lot of credit checking to lend overnight with full collateral. However, once one big bank is in trouble, it makes you wonder about all the others. Like e coli, once you hear there’s an outbreak you avoid the whole salad bar.

11. (15) The current log forward curve is (in years)

<table>
<thead>
<tr>
<th>$y_t$</th>
<th>$f_t^{(2)}$</th>
<th>$f_t^{(3)}$</th>
<th>$f_t^{(4)}$</th>
<th>$f_t^{(5)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>2%</td>
<td>3%</td>
<td>4%</td>
<td>5%</td>
</tr>
</tbody>
</table>

a) According to the expectations hypothesis, what is the expected interest rate four years from now $E_t\left(y_{t+4}\right)$?

b) Do Fama and Bliss’ regressions support or contradict this conclusion?

c) According to the expectations hypothesis, what is the expected value of next year’s three-year forward rate $E_t(f_{t+1}^{(3)})$?

d) According to Fama and Bliss’ one-year regressions, and approximating the numbers by 1 and 0, what is the expected return on three year bonds $E_t(f_{t+1}^{(3)})$?

e) According to Fama and Bliss’ one-year return regressions, and approximating the numbers by 1 and 0, what is the expected value of next year’s three-year forward rate $E_t f_{t+1}^{(3)}$? Compare to c.

**ANSWER**

a) $E_t y_{t+4}^{(1)} = f_t^{(5)} = 5\%$.

b) Yes, the regression was roughly $y_{t+4}^{(1)} - y_t^{(1)} = a + 1.0 \times (f_t^{(5)} - y_t^{(1)}) + \epsilon_{t+1}$

c) $E_t f_{t+1}^{(3)} = f_t^{(4)} = 4\%$

d) $E_t r x_{t+1}^{(3)} = f_t^{(3)} - y_t^{(1)} = 3\% - 1\% = 2\%$; $E_t r x_{t+1}^{(3)} = E_t r x_{t+1}^{(3)} + y_t^{(1)} = 3\%$

e) $E_t (f_{t+1}^{(3)}) = f_t^{(3)} = 3\%$. We did this on a problem set. Fama-Bliss is “prices don’t change.” To reach this conclusion you have to unwind return regressions to find the forward rate implications, $E_t (f_{t+1}^{(3)}) = E_t (p_{t+1}^{(2)} - p_t^{(3)}) = E_t (p_{t+1}^{(2)} - p_t^{(3)}) - (p_{t+1}^{(3)} - p_t^{(4)}) = E_t (f_{t+1}^{(3)} - f_t^{(4)}) - f_t^{(4)} = (\text{Fama-Bliss}) = (f_t^{(3)} - f_t^{(4)}) - f_t^{(4)}$

12. (10) What does this picture represent? Be explicit, with equations.
These are Cochrane-Piazzesi regressions. The top one is unrestricted
\[ r x_{t+1}^{(n)} = a^{(n)} + \beta_1^{(n)} y_t^{(1)} + \beta_2^{(n)} f_t^{(2)} + \ldots + \beta_5^{(n)} f_t^{(5)} + \varepsilon_{t+1}^{(n)} \]

this is a plot of \( \beta^{(n)} \), each line a different \( y_t \). The bottom one is restricted,
\[ r x_{t+1}^{(n)} = b_n \left[ \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \ldots + \gamma_5 f_t^{(5)} \right] + \varepsilon_{t+1}^{(n)} \]
The plot is \( b_n \gamma \). The point of the plot is that the two representations are very nearly identical.

13. (15) Suppose the one-year rate is an MA(1),
\[ y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}. \]

Form a term-structure model, by supposing that the expectations hypothesis holds.

a) Find forward rates (at time \( t \), for maturity 2,3,4,...,\( n \))

b) Find yields (at time \( t \), for maturity 2,3,4,...,\( n \)).

c) Express the factors in terms of yields or forward rates. Then express the yield-curve model as a function of factors, i.e. loadings on the factors. (There are many ways to do this. Choose an easy one!)

(Hint: You may think you got it wrong because the answer is too simple. Don’t worry, it really is simple. This problem does NOT involve a lot of algebra.)

ANSWER
\[ y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1} \]
\[ f_t^{(2)} = E_t y_{t+1}^{(1)} = \varepsilon_t \]
\[ f_t^{(3)} = E_t y_{t+2}^{(1)} = 0 \]
\[ f_t^{(n)} = E_t y_{t+n-1} = 0 \]

\[ y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1} \]
\[ y_t^{(2)} = \frac{1}{3} \left( y_t^{(1)} + f_t^{(2)} \right) = \frac{1}{3} \varepsilon_t + \varepsilon_{t-1} \]
\[ y_t^{(3)} = \frac{1}{3} \left( y_t^{(1)} + f_t^{(2)} + f_t^{(3)} \right) = \frac{1}{3} \varepsilon_t + \frac{2}{3} \varepsilon_{t-1} \]
\[ y_t^{(n)} = \frac{1}{n} \left( y_t^{(1)} + f_t^{(2)} + f_t^{(3)} + \ldots + f_t^{(n)} \right) = \frac{1}{n} \varepsilon_t + \frac{2}{n} \varepsilon_{t-1} \]

Factors The factors are \( \varepsilon_t \) and \( \varepsilon_{t-1} \). You can find them just by

\[ \varepsilon_t = f_t^{(2)} \]
\[ \varepsilon_{t-1} = y_t^{(1)} - f_t^{(2)}. \]

You already have the loadings.

\[
\begin{bmatrix}
  y_t^{(1)} \\
  y_t^{(2)} \\
  y_t^{(3)} \\
  \vdots \\
  y_t^{(n)}
\end{bmatrix} = \begin{bmatrix}
  1 \\
  -\frac{1}{3} \\
  -\frac{2}{3} \\
  \vdots \\
  -\frac{n-1}{2n-1}
\end{bmatrix} \varepsilon_t + \begin{bmatrix}
  1 \\
  \frac{1}{3} \\
  \frac{2}{3} \\
  \vdots \\
  \frac{n}{2n-1}
\end{bmatrix} \varepsilon_{t-1}
\]

14. (5) You are a very risk averse investor with a 10 year horizon, and you have decided that a 10 year zero coupon TIP is the right security. A new financial crisis comes along. Long-term rates rise, so your bond plummets in value. Interest-rate volatility spikes. Your long-term bond investment has underperformed the 7 year bond benchmark by 5%.

Your investment adviser wants you to bail out to less risky short term bonds, or more profitable investments. He reminds you of \( w = \frac{1}{2} \frac{E_t(R)^{c}}{\sigma_t^2(R_t^{c+1})} \). The bond-return forecasting model says \( E_t(R)^{c} \) is dreadful for next year, and with \( \sigma_t(R_t^{c+1}) \) higher surely you should rebalance away from this bond. Is he right? If not, why not and what should you do? (This is a short answer question, not a lot of algebra. Explain with reference to portfolio formulas, and intuition.)

ANSWER

No. He left out the “hedge demand” terms. One period mean-variance is wrong for long-term bond investors.
15. (5) You have risk aversion $\gamma = 1$, and returns are independent over time. Your best guess is that the mean annual premium $\mu$ is 4% with volatility $\sigma = 20\%$.

a) What should your allocation to stocks be?

b) In fact you don’t really know what the mean return is. Reflecting on it, your uncertainty about the mean return $\sigma(\mu)$ is 10 percentage points, and both the actual return and your uncertainty about it are normally distributed. (Numbers are easy to calculate, not realistic.) How does this consideration change your optimal allocation to stocks?

**ANSWER**

$$\frac{0.04}{0.2^2} = \frac{0.04}{0.04} = 1$$

$$\frac{0.04}{0.2^2 + 0.1^2} = \frac{0.04}{0.04 + 0.01} = \frac{0.04}{0.05} = 0.8$$

Verbal answers to the effect that “parameter uncertainty is extra variance and scale back the allocation to stocks” are worth some partial credit.

16. (5) You create a “small business” factor, a portfolio of assets that has 90% correlation with shocks to the profits of small and privately-held businesses. Alas, after running regressions of your portfolio return on rmrf, smb, hml, you find the alpha of your factor return is zero.

$$R_{\text{factor}} = \beta \times \text{rmrf}_t + h \times \text{hml}_t + s \times \text{smb}_t + \varepsilon_{\text{factor}}; \quad R^2 = 0.4$$

This means we do not need your new factor to price assets, and the expected returns of your factor can be achieved by the combination of FF 3 factors. Might a fund based on your new factor be interesting to investors nonetheless? If so, how? (Hint: The $R^2$ matters here)

**ANSWER**

Small business owners would love to short this portfolio!
MORE QUESTIONS

9) (10) Below, find an excerpt from Fama and French’s Table 3, which studies gross (before fees) mutual fund returns.

i) What does this table tell you about the median fund’s alpha?

ii) Are there more “good funds” with positive alpha than we would expect due to chance if all funds had zero alpha?

iii) Are there more “bad funds” with negative alphas?

iv) How does this calculation address the retort “sure, the average fund doesn’t have much alpha, but the good funds have alpha.”

Give some numbers in your answers: What does 1.68 mean? What does 2.04 mean? What do -1.71 and -2.19 mean? Showing you understand what “Sim” and “act” mean is important (and not just .

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Simulated</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.71</td>
<td>-2.19</td>
</tr>
<tr>
<td>50</td>
<td>-0.01</td>
<td>-0.06</td>
</tr>
<tr>
<td>90</td>
<td>1.30</td>
<td>1.59</td>
</tr>
<tr>
<td>91</td>
<td>1.38</td>
<td>1.68</td>
</tr>
<tr>
<td>95</td>
<td>1.68</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Table 3 - Percentiles of t(α) estimates for actual and simulated fund returns...[3-factor adjusted] gross fund returns....

ANSWER

i) The median fund has -0.06 alpha (before costs).

ii) There are slightly more good funds than you’d expect. 1.68 means that if all funds really have exactly zero alpha, then we expect to see that 5% of the funds in a sample will have an alpha t statistic greater than 1.68 just due to chance. In fact, 5% of funds had a t stat greater than 2.04, and 9% had a t stat greater than 1.68. Thus there are 4% too many funds with alpha greater than 1.68 than there should be.

iii) Similarly, -1.71 means that 5% of funds should have results this bad due to chance. The actual cutoff is -2.19.

iv) Thus, the before fee alpha distribution is very slightly wider than a view that all true alphas are zero and observed alphas are luck can account for. However, it’s “very slightly”. Yes there are some “good funds,” but they’re awfully hard to find.

5) (15) Suppose log consumption is as in the table, and suppose people know what’s going to happen ahead of time. (I’m trying to illustrate a "recovery," "boom," "recession" and "recovery," superimposed on growth.) Use the consumption-based model to find as many of the interest rates as possible. Use discount rate δ = 2%, and risk aversion γ = 2, approximate as necessary to get round answers.

<table>
<thead>
<tr>
<th>t</th>
<th>log(c_t)</th>
<th>r_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

ANSWER

\[ r_t = \delta + \gamma E_t \Delta c_{t+1} = 0.02 + 2 \times \Delta c_{t+1} \]
The trick is you have to compute $\Delta c_{t+1}$, not use the level of $c_t$.  

<table>
<thead>
<tr>
<th>t:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(c_t)$</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Delta c_{t+1}$</td>
<td>0.02</td>
<td>0.01</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>$r^f_t$</td>
<td>0.06</td>
<td>0.04</td>
<td>0.00</td>
<td>0.02</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$r^f_t$</td>
<td>6%</td>
<td>4%</td>
<td>0%</td>
<td>2%</td>
<td>4%</td>
<td></td>
</tr>
</tbody>
</table>

9) (10) You’re investigating a hedge fund that invests in illiquid securities. It offers a mean return of 10%, low market beta, and advertises a low annualized standard deviation of 10% and hence a very good annualized Sharpe ratio of 1.0. However, you discover that its returns follow an MA(1), with serial correlation $\text{cov}(r_t, r_{t+1})/\text{var}(r_t) = 0.5$; $\text{cov}(r_t, r_{t+2}) = 0$, $\text{cov}(r_t, r_{t+k}) = 0, k > 1$

a) Suggest two ways to get a more realistic beta.

b) Does the serial correlation suggest that the mean return is biased, and how?

c) Does the serial correlation suggest that the variance of returns is biased, and how?

d) If your answer to b and/or c is yes, find a better estimate of the fund’s i) mean, ii) variance and hence iii) Sharpe ratio? (Hint for b-d: Think about the mean and variance of long horizon returns ($r_{t+1} + r_{t+2} + \ldots + r_{t+k}$), ).

ANSWER

a) Add lagged market returns. Do betas at longer horizons

b) No

c) Yes

d) Mean long horizon returns still scale with $k$ as they should.

$$E(r_{t+1} + r_{t+2} + \ldots + r_{t+k}) = kE(r)$$

$$E(r_{t+1} + r_{t+2} + \ldots + r_{t+k})/k = E(r)$$

the variance of long horizon returns does not scale with $k$

$$\sigma^2(r_{t+1} + r_{t+2} + \ldots + r_{t+k})/k = \sigma^2(r) \left[1 + 2\frac{(k-1)}{k}\rho\right]$$

thus, the variance of long horizon returns is really ($\rho = 0.5$) twice as large as suggested by the variance of short horizon returns. Thus, i) the true mean is still 10% ii) the true variance is twice as large, $2 \times 0.10^2$. iii) the true annualized Sharpe ratio is $1/\sqrt{2} \approx 1/1.4 = 0.7$

5. (15) Suppose log consumption is as in the table, and suppose people know what’s going to happen ahead of time. (I’m trying to illustrate a "recovery," "boom," "recession" and "recovery," superimposed on growth.) Use the consumption-based model to find as many of the interest rates as possible. Use discount rate $\delta = 2\%$, $(\beta = e^{-0.02} \approx 0.98)$ and risk aversion $\gamma = 2$, approximate as necessary to get round answers, and give your answer in percent. You can add rows for intermediate results if you wish.
d) If you sort stocks into “winners” that went up from year -5 to one year ago, and losers that went down from year -5 to one year ago, which ones do better for the next year? Is this consistent with some pattern of factor exposures?

ANSWER: FF again. This is the long term reversal effects. It also corresponds to $h$ loadings – winners act like growth stocks.

5. (15) Should you keep the smb factor? You run regressions

\[ R^{\text{i}}_t = \alpha_i + b_i \text{rmrf}_t + h_i \text{hml}_t + s_i \text{smb}_t + \varepsilon_{it} \quad t = 1, 2, .. T \text{ for each i} \tag{1} \]

\[ R^{\text{e}}_t = \alpha_i + b_i \text{rmrf}_t + h_i \text{hml}_t + \varepsilon_{it} \quad t = 1, 2, .. T \text{ for each i} \tag{2} \]

\[ \text{smb}_t = \alpha_s + b_s \text{rmrf}_t + h_s \text{hml}_t + \varepsilon_{st} \quad t = 1, 2, .. T \tag{3} \]

(note $\alpha_i, b_i, h_i$ are not necessarily the same in (4) and (5).) You find that the $\alpha$ in (5) are about the same as in (4), and you find that $\alpha_s = 0$. On the other hand, $E(\text{smb})$ is quite high and statistically significant (well, suppose that is the case), the $t$ statistics on $s_i$ are very strong, the $R^2$ in (4) is much higher than in (5) and a joint test that all $s_i = 0$ decisively rejects. So, should you keep the smb factor or not?

ANSWER

What model you use depends on what the purpose is! Given these facts, you can drop smb for the purpose of understanding mean returns, i.e.

\[ E(R^{\text{e}}_t) = \alpha_i + b_i E(\text{rmrf}_t) + h_i E(\text{hml}_t) + s_i E(\text{smb}_t) \]

Even though $E(\text{smb}) \neq 0$, the results of (5) and (6) mean that the $b$ and $h$ will change, so the alphas do not change. For the purpose of understanding return variance however, the $s$ t stats and $R^2$ loudly warn you not to drop smb. Also, including smb will improve standard errors, including standard errors of $\alpha_i$, so it might be a good idea in any case.

8. (10) Mitchell and Pulvino ran regressions of merger arbitrage returns on the market. This is an excerpt from their Tables II and IV. These are monthly returns, so 0.0053 means 0.53% per month and 0.0101 means 1.01% per month.

a) Given these results, how should you benchmark a risk-arbitrage manager?

b) How much return does merger arbitrage earn after benchmarking?

c) Is it better to follow the VWRA strategy or the RAIM strategy?

<table>
<thead>
<tr>
<th></th>
<th>VWRA</th>
<th>RAIM</th>
<th>CRSP</th>
<th>Riskfree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average returns</td>
<td>16.05%</td>
<td>10.64%</td>
<td>12.25</td>
<td>6.22</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>9.29%</td>
<td>7.74%</td>
<td>15.08%</td>
<td>0.73%</td>
</tr>
<tr>
<td>Sharpe</td>
<td>1.06</td>
<td>0.57</td>
<td>0.40</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$\alpha_{\text{MktHigh}}$</th>
<th>$\beta_{\text{MktLow}}$</th>
<th>$\beta_{\text{MktHigh}}$</th>
<th>Adj.$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAIM returns</td>
<td>0.0053</td>
<td>0.4920</td>
<td>0.0167</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>(0.0011)***</td>
<td>(0.0673)***</td>
<td>(0.0292)***</td>
<td></td>
</tr>
<tr>
<td>VWRA returns</td>
<td>0.0101</td>
<td>0.4757</td>
<td>-0.0678</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td>(0.0013)***</td>
<td>(0.0840)***</td>
<td>(0.0364)***</td>
<td></td>
</tr>
</tbody>
</table>

VWRA = Value Weighted Risk Arbitrage portfolio; RAIM = Risk Arbitrage Index Manager Standard errors in parentheses. *** denotes significant at the 1% level.
ANSWER a) the point is that the payoffs behave like an index put, so you should benchmark to a strategy that writes puts. This also means to forgive them if they do badly in bad times! b) These are not returns, so the intercept is not alpha. The answer is “we don’t know from the provided information.” You need to do either a contingent-claim valuation or put actual option returns on the right hand side. c) The VWRA returns ignore transactions costs, so you’d follow them if you could but you can’t. Answer: that’s a silly question

11) (15) Here is a table of Fama-Bliss bond forecast regressions The coefficients in the first row add up to one. The coefficients in the other rows do not add up to one.

a) Why not? What does add up? Explain either in words or using a graph.

b) Be specific in the case of the second row and first column. If this coefficient is 1.12, exactly what coefficient is -0.12? (Hint: break up \( f_{t}^{(3)} - y_{t}^{(1)} = r_{x_{t+1}} \) + something else. Then if you run a regression of both sides on \( f_{t}^{(3)} - y_{t}^{(1)} \), you have \( 1 = b+ another coefficient.\)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( b )</th>
<th>( \sigma(b) )</th>
<th>( R^2 )</th>
<th>( b )</th>
<th>( \sigma(b) )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.83</td>
<td>0.27</td>
<td>0.12</td>
<td>0.17</td>
<td>0.27</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>1.12</td>
<td>0.36</td>
<td>0.13</td>
<td>0.47</td>
<td>0.31</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>1.34</td>
<td>0.45</td>
<td>0.14</td>
<td>0.75</td>
<td>0.32</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>1.02</td>
<td>0.52</td>
<td>0.06</td>
<td>0.87</td>
<td>0.16</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Forecasting one year returns on \( n \)-year bonds forecasting one year rates \( n \) years from now

ANSWER

a) The main point is to identify that the one-year change in longer-term yields is complementary on the left hand side, and the multi-year return on long term bonds held to one year before maturity is complementary on the right hand side. The graph is good.

b) To really get this right you have to identify the long/short positions. Left side:

\[
f_{t}^{(3)} - y_{t}^{(1)} = p_{t}^{(2)} - p_{t}^{(3)} - y_{t}^{(1)} = -(p_{t+1}^{(2)} - p_{t}^{(2)}) + (p_{t+1}^{(2)} - p_{t}^{(3)}) - y_{t}^{(1)} = 2(y_{t+1}^{(2)} - y_{t}^{(2)}) + r_{x_{t+1}}^{(2)}
\]
If you run a regression of left and right side on \( f^{(3)} - y^{(1)} \) you find that 1 (left) = (right) coefficient of \( 2 \left( y_{t+1}^{(2)} - y_t^{(2)} \right) \) on \( f^{(3)} - y^{(1)} \) plus the coefficient of \( r x_t^{(2)} \) on \( f^{(3)} - y^{(1)} \).

I did not ask for the right side, but here it is.

\[
f_t^{(3)} - y_t^{(1)} = p_t^{(2)} - p_t^{(3)} - y_t^{(1)} \\
= -p_{t+2}^{(1)} + p_t^{(2)} + p_t^{(1)} - p_t^{(3)} - y_t^{(1)} \\
= p_t^{(1)} - p_t^{(3)} + p_t^{(2)} + (y_t^{(1)} - y_t^{(1)}) \\
= \left( r_{t-t+2}^{(3)} - r_t^{(2-0)} \right) + (y_t^{(1)} - y_t^{(1)})
\]

Corresponding to the two-year change in \( y^{(1)} \) then is the excess return for buying a 3 year bond and holding for two years, over the return for buying a 2 year bond and holding for 2 years.

12) (15) You form a model of the term structure of interest rates by supposing the one-year rate is an AR(1),

\[
y_{t+1}^{(1)} - \delta = \rho(y_t^{(1)} - \delta) + \varepsilon_{t+1}
\]

and that the expectations hypothesis holds. If your model is right, what should the eigenvalue decomposition of forward rates look like? Specifically, if you took data as predicted from your model,

\[
f_t = \begin{bmatrix} y_t^{(1)} \\ f_t^{(2)} \\ f_t^{(3)} \end{bmatrix}
\]

and performed \( QAQ' = eig(cov(f_t, f_t')) \), what would the \( Q \) and \( \Lambda \) look like? Note: you do not have to give the exact values of \( Q \) and \( \Lambda \). It is enough to answer that columns of \( Q \) have a specific pattern, show where any zeros are, and say what if any parts of \( Q \) or \( \Lambda \) are arbitrary and don’t matter.

**ANSWER:** Forward rates should follow a one-factor model.

\[
\begin{align*}
f_t^{(2)} &= E_t(y_{t+1}^{(1)}) = \delta + \rho(y_t^{(1)} - \delta) \\
\end{align*}
\]

\[
\begin{align*}
f_t^{(3)} &= E_t(y_{t+2}^{(1)}) = \delta + \rho^2(y_t^{(1)} - \delta) \\
\end{align*}
\]

\[
\begin{align*}
f_t^{(N)} &= E_t(y_{t+N-1}^{(1)}) = \delta + \rho^{N-1}(y_t^{(1)} - \delta)
\end{align*}
\]

thus \( \Lambda \) will have only one non-zero element

\[
\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

and \( Q \) will look like

\[
Q = \begin{bmatrix} 1 & | & | \\ \rho & q_2 & q_3 \\ \rho^2 & | & | \end{bmatrix}
\]

where \( k \) is an arbitrary constant – the pattern is \( 1, \rho, \rho^2 \). Since they multiply zeros, the second and third columns of \( Q \) are irrelevant.

**Optional.** The exact values are harder, so I didn’t ask for them. The scale of \( Q \) is determined by \( Q'Q = 1 \) or \( k^2(1 + \rho^2 + \rho^4) = 1 \) so

\[
Q = \frac{1}{\sqrt{1 + \rho^2 + \rho^4}} \begin{bmatrix} 1 & | & | \\ \rho & q_2 & q_3 \\ \rho^2 & | & | \end{bmatrix}
\]
We can find $\lambda_1$ by the requirement that
\[
\sigma_{y^{(1)}}^2 = \frac{1}{1-\rho^2} \sigma_\varepsilon^2 = \eta_1^2 \lambda_1 = \frac{1}{1+\rho^2 + \rho^4} \lambda_1
\]
\[
\lambda_1 = (1 + \rho^2 + \rho^4) \sigma_{y^{(1)}}^2
\]
As the number of included maturities increases, these actually get simpler. In the limit of an infinite number of forward rates, we would have
\[
Q = \frac{1}{\sqrt{1 + \rho^2 + \rho^4 + \ldots}} \begin{bmatrix} 1 & | & \ldots \\ \rho & q_2 & q_3 \\ \rho^2 & | & \ldots \\ \ldots & | & \ldots \end{bmatrix}
\]
\[
= \sqrt{1-\rho^2} \begin{bmatrix} 1 & | & \ldots \\ \rho & q_2 & q_3 \\ \rho^2 & | & \ldots \\ \ldots & | & \ldots \end{bmatrix}
\]
\[
\lambda_1 = (1 + \rho^2 + \rho^4 + \ldots) \sigma_{y^{(1)}}^2 = \frac{1}{(1-\rho^2)} \sigma_{y^{(1)}}^2
\]

14) (5) Great news is published in the 2012 Journal of Finance. It turns out there was a bug in the programs, and the CAPM works beautifully. All the other “state variables” are unimportant for explaining mean returns. For example, the value premium $E(hml)$ is fully explained by its beta on the market $E(hml) = \beta_{hml,rmrf}E(rmrf)$. Furthermore, it turns out (one more bug!) that mean and variance of returns are constant after all. $E_t(R_{t+1}^\varepsilon)$ is the same all the time. Does this mean we should all go back to simple mean-variance portfolio theory and ignore all those troubling hedging and market timing terms? (You should point to some equations in your answer, but no big algebra is required.)

ANSWER:

\[
R^{i} = R^{f} + \frac{\gamma^m}{\gamma^f} R^{rm} + \frac{1}{\gamma^f} \left( \eta^{y'} - \eta^m \right) R^{ex} ; \quad R^{ex} \equiv \beta_{y,R^{r}} R^{ex}
\]
\[
E(R^{e}) = \text{cov}(R^{e}, R^{m}) \gamma^m - \text{cov}(R^{e}, y^f) \eta^m
\]
The capm working means $\eta^m = 0$. But you may still have $\eta^f \neq 0$. If you are in financial services, your portfolio should underweight financial services, even though there is no alpha.

10. (10) Below, find an excerpt from Fama and French’s Table 3.

i) What does this table tell you about the median fund’s alpha?

ii) Are there more “good funds” with positive alpha than we expect due to chance if all true alphas were zero?

iii) Are there more “bad funds” with negative alphas?

iv) How does this calculation address the retort “sure, the average fund doesn’t have much alpha, but the good funds have alpha.”

Give some numbers in your answers: What does 1.68 mean? What does 2.04 mean? What do -1.71 and -2.19 mean? Showing you understand what “Sim” and “act” mean is important.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Simulated</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.71</td>
<td>-2.19</td>
</tr>
<tr>
<td>50</td>
<td>-0.01</td>
<td>-0.06</td>
</tr>
<tr>
<td>90</td>
<td>1.30</td>
<td>1.59</td>
</tr>
<tr>
<td>91</td>
<td>1.38</td>
<td>1.68</td>
</tr>
<tr>
<td>95</td>
<td>1.68</td>
<td>2.04</td>
</tr>
</tbody>
</table>
Table 3 - Percentiles of $t(\alpha)$ estimates for actual and simulated fund returns...[3-factor adjusted] gross fund returns....

5. (15) Should you keep the smb factor? You run regressions

$$R^i_t = \alpha_i + b_i \text{rmrf}_t + h_i \text{hml}_t + s_i \text{smb}_t + \varepsilon_{it} \text{ } t = 1, 2, ..T \text{ for each } i \tag{4}$$
$$R^i_t = \alpha_i + b_i \text{rmrf}_t + h_i \text{hml}_t + \varepsilon_{it} \text{ } t = 1, 2, ..T \text{ for each } i \tag{5}$$
$$\text{smb}_t = \alpha_s + b_s \text{rmrf}_t + h_s \text{hml}_t + \varepsilon_{st} \text{ } t = 1, 2, ..T \tag{6}$$

(note $\alpha_i, b_i, h_i$ are not necessarily the same in (4) and (5).) You find that the $\alpha$ in (5) are about the same as in (4), and you find that $\alpha_s = 0$. On the other hand, $E(\text{smb})$ is quite high and statistically significant (well, suppose that is the case), the $t$ statistics on $s_i$ are very strong, the $R^2$ in (4) is much higher than in (5) and a joint test that all $s_i = 0$ decisively rejects. So, should you keep the smb factor or not?

d) If you sort stocks into “winners” that went up from year -5 to one year ago, and losers that went down from year -5 to one year ago, which ones do better for the next year? Is this consistent with some pattern of betas?

2 (15) a) Suppose that the stock market rises 10%, and dividends also rise 10%. Sketch how this event changes your forecast of future i) dividends ii) returns iii) prices. (All variables in logs)

b) Now suppose that the stock market rises 10% but dividends don’t change. Make the same sketch.

(Note: art is enough, but if you want to work it out with equations, the formula sheet has the ones you need.)
I hope you remember the results, but you can work it out from formulas as below.

$$dp_t = (0, -0.1, -0.1\phi, -0.1\phi^2...)$$

$$r_{t+1} = -\rho(dp_{t+1} + dp_t) = 0.1 \times \{\rho, -(1 - \rho), -(1 - \rho)\phi, -(1 - \rho)\phi^2\ldots\}$$

$$\sum r_t = 0.1 \times \{\rho, \rho - (1 - \rho), \rho - (1 - \rho)(1 + \phi), \rho - (1 - \rho)(1 + \phi + \phi^2)\ldots\}$$

$$\Delta p_{t+1} = -(d_{t+1} - p_{t+1}) + (d_t - p_t) + \Delta d_{t+1}$$

$$= -dp_{t+1} + dp_t$$

$$= \{0.1, -0.1(1 - \phi), -0.1\phi(1 - \phi)\}$$

$$p_t = \{0.1, 0.1\phi, 0.1\phi^2\}$$