2013 Final Exam Answers

1. (a) Plugging the VAR into the identity, the terms multiplying \( dp_t \) must equate so

\[
b_r dp_t + \varepsilon_{t+1}^r = -\rho \left( \phi dp_t + \varepsilon_{t+1}^{dp} \right) + dp_t + (b_d dp_t + \varepsilon_{t+1}^d)
\]

that means \( b_r = 1 - \rho \phi + b_d \)

(b) The shock terms must also equate,

\[
\varepsilon_{t+1}^r = -\rho \varepsilon_{t+1}^{dp} + \varepsilon_{t+1}^d
\]

so multiplying be \( \varepsilon^{dp} \) and taking expectation,

\[
\text{cov}(\varepsilon_{t+1}^r, \varepsilon_{t+1}^{dp}) = -\rho \sigma^2 \left( \varepsilon_{t+1}^{dp} \right) + \text{cov}(\varepsilon_{t+1}^{dp}, \varepsilon_{t+1}^d) = -\rho \sigma^2 \left( \varepsilon_{t+1}^{dp} \right)
\]

(c)

\[
\begin{align*}
\sigma^2 (r_{t+1} + r_{t+2}) &= 2\sigma^2(r) + 2\text{cov}(r_{t+1}, r_{t+2}) \\
E (r_{t+1} + r_{t+2}) &= 2E(r)
\end{align*}
\]

\[
w = \frac{1}{\gamma \sigma^2 (r_{t+1} + r_{t+2})} = \frac{1}{\gamma \sigma^2(r) + \text{cov}(r_{t+1}, r_{t+2})} \geq \frac{1}{\gamma \sigma^2(r)} \quad \text{or} \quad \frac{1}{\gamma \sigma^2(r)} < 0
\]

(d) Using the VAR,

i. \( r_{t+1} = b_r dp_t + \varepsilon_{t+1}^r \)

\( r_{t+2} = b_r \phi dp_t + b_r \varepsilon_{t+1}^{dp} + \varepsilon_{t+2}^r \)

so

\[
\text{cov}(r_{t+1}, r_{t+2}) = \text{cov} \left( b_r dp_t + \varepsilon_{t+1}^r, b_r \left( \phi dp_t + \varepsilon_{t+1}^{dp} \right) + \varepsilon_{t+2}^r \right)
\]

ii. we turn off the \( \varepsilon_{t+1}^r \),

\[
\text{cov}(r_{t+1}, r_{t+2}) = \text{cov} \left( b_r dp_t, b_r \left( \phi dp_t + \varepsilon_{t+1}^{dp} \right) \right) + b_r^2 \phi \sigma^2(dp_t) > 0
\]

Intuition. \( dp \) is very slow moving. If \( \varepsilon_{t+1}^r = 0 \), returns have the same slow-moving and highly autocorrelated process as dividend yields themselves. This answer is good enough. You can also go one step further and write

\[
b_r^2 \phi \sigma^2(dp_t) > 0
\]

or even

\[
(1 - \rho \phi)^2 \frac{\phi}{1 - \phi^2} \sigma^2(\varepsilon^{dp}) > 0
\]

iii. If \( dp_t \) is a number, then

\[
\text{cov}(r_{t+1}, r_{t+2}) = \text{cov} \left( \varepsilon_{t+1}^r, b_r \varepsilon_{t+1}^{dp} + \varepsilon_{t+2}^r \right) = \text{cov} \left( \varepsilon_{t+1}^r, b_r \varepsilon_{t+1}^{dp} \right) = -b_r \rho \sigma^2(\varepsilon^{dp}) < 0
\]

Here is where stocks are like bonds. A positive shock to \( dp \) is a negative shock to returns which is a positive shock to expected returns.
(e) Now the whole thing.

\[ \text{cov}(r_{t+1}, r_{t+2}) = \text{cov} \left[ b_r dp_t + \varepsilon_{t+1}^r, b_r \left( \phi dp_t + \varepsilon_{t+1}^{dp} \right) + \varepsilon_{t+2}^r \right] \]

\[ = b_r^2 \phi \sigma^2(dp_t) + b_r \text{cov}(\varepsilon_{t+1}^{dp}, \varepsilon_{t+1}^r) \]

\[ = b_r^2 \phi \sigma^2(dp_t) - b_r \rho \sigma^2(\varepsilon^{dp}) \]

\[ = b_r \left( \frac{1 - \rho \phi}{1 - \phi^2} - \rho \right) \sigma^2(\varepsilon^{dp}) \]

with \( \rho = \phi \) we have \( \text{cov}(r_{t+1}, r_{t+2}) = 0! \) **Predictability does not affect the safety of stocks in the long run!** This is an initially counterintuitive result. We see here the two countervailing effects. You probably thought stocks safer in the long run because you thought of the bond-like effect, a negative shock to prices is a positive shock to returns. But this leaves out the smooth \( d \) effect: \( d \) is a very smooth variable, giving a very slow moving component to returns, which induces positive serial correlation. The two effects exactly offset!

2. The key here is that you must use the identity to find the return shock,

\[ \varepsilon_{t+1}^r = -\rho \varepsilon_{t+1}^{dp} + \varepsilon_{t+1}^d \]

in the first case \( \varepsilon^r = 1 \). In the second case \( \varepsilon^r = -\rho = -0.96 \). Then, just substituting the VAR you have

<table>
<thead>
<tr>
<th>( \varepsilon^d )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dp_t )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( r_t )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta d_t )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( dp_t )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_t )</td>
<td>-0.96</td>
<td>0.1</td>
<td>0.094</td>
<td>0.088</td>
<td>0.083</td>
</tr>
<tr>
<td>( \Delta d_t )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Dividend shock**

\[ \Delta d \]

\[ \phi \]

\[ r \]

35
3.

(a)

\[ f_t^{(2)} = E_t y_{t+1}^{(1)} = \delta + \rho (y_t^{(1)} - \delta) \]

\[ f_t^{(3)} = E_t y_{t+2}^{(1)} = \delta + \rho^2 (y_t^{(1)} - \delta) \]

\[ f_t^{(4)} = E_t y_{t+3}^{(1)} = \delta + \rho^3 (y_t^{(1)} - \delta) \]

\[ f_t^{(n)} = E_t y_{t+n-1}^{(1)} = \delta + \rho^{(n-1)} (y_t^{(1)} - \delta) \]

(b) \( \rho^2 = 0.25, \rho^3 = 0.125 \) so

<table>
<thead>
<tr>
<th>( y_t^{(1)} )</th>
<th>6%</th>
<th>5%</th>
<th>0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_t^{(2)} )</td>
<td>5.5</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>( f_t^{(3)} )</td>
<td>5.25</td>
<td>5</td>
<td>3.75</td>
</tr>
<tr>
<td>( f_t^{(4)} )</td>
<td>5.125</td>
<td>5</td>
<td>4.375</td>
</tr>
</tbody>
</table>
(c) If rates follow the assumed AR(1), then events like the graphed one should be very rare. We are always expecting yields to bounce back up, but it takes them forever. You don’t see that time series forecast error in this plot. Put another way, the $y_t^{(1)}$ process graphed is much more persistent than an AR(1) with $\rho = 0.5$. People are generating bond prices as if there is a quickly mean reverting AR(1), but the actual process doesn’t revert so fast, so you make money. You could assume $\rho = 1$ to generate the slow mean reversion, but then the forward rates would not be upward sloping. At $\rho = 1$, with the expectations hypothesis, all the forward rates collapse to the spot rate. So, to make a graph that looks like the forward rate data I have to assume people expect interest rates to revert back a lot faster than interest rates actually do revert back.

As another way to see the point, (far beyond what I expect on an exam) here is a plot of excess returns $rx_t^{(n)}$ through the episode, and the mean excess returns in the episode are as given in the table. The investor makes money through the episode. The market is “expecting” yields to rise, but it doesn’t happen fast enough.

<table>
<thead>
<tr>
<th>$E(rx^{(2)})$</th>
<th>$E(rx^{(3)})$</th>
<th>$E(rx^{(4)})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75%</td>
<td>1.12%</td>
<td>1.31%</td>
</tr>
</tbody>
</table>
4.

(a)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{t}^{(n)}$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$f_{t}^{(n)}$</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>$p_{t}^{(n)}$</td>
<td>0</td>
<td>-2</td>
<td>-6</td>
<td>-12</td>
<td>-20</td>
</tr>
<tr>
<td>$E_t(y_{t+1:n-1}^{(1)})$</td>
<td>0</td>
<td>0.4</td>
<td>2</td>
<td>4.8</td>
<td>7.2</td>
</tr>
</tbody>
</table>

(b) $y_{t}^{(1)}$ is always zero, so $f_{t}^{(n)} - y_{t}^{(1)} = f_{t}^{(n)}$. Numbers, 0.20×2 = 0.40; 0.50×4 = 2; 0.80×6 = 4.8; 0.90×8 = 7.2

5.

(a)

\[ R_{t}^{ei} = \alpha_{i} + b_{t}rmf_{t} + h_{t}hml_{t} + s_{t}smb_{t} + \varepsilon_{t}^{i} \quad T = 1..T \text{ for each } i \]

$R_{t}^{ei}$ = excess returns on 25 size and b/m sorted portfolios, $rmf_{t}$ = market excess return, $hml_{t}$ = long value short growth factor return, $smb_{t}$ = long small short big factor return.

(b)

i. The large $R^2$ is the most important statistic to say this is a good model of “returns” i.e. a factor model. The size and pattern of the $b, h, s$ along with their t statistics are good confirming evidence. The $\alpha$ don’t matter to this point

ii. The economically small alphas (mostly) are the most important statistics. The fact that $b, h, s$ vary in the same direction as $E(R^{ei})$ is good confirming evidence. The $R^2$ is irrelevant to this point.

(c) The large $R^2$ means $\Sigma$ is small, so $\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$ can be big even with small $\alpha$.

(d)

i. Dropping smb would lower $R^2$ and hence worsen the model of “returns.” It would also lower t statistics.

ii. Dropping smb would have precisely no effect on the FF alphas, and hence its model of expected returns.
(a) Fama and French’s returns are net of a matching portfolio, with the same book/market and size. Thus, this represents a multiple regression, the effect of NS independent of value.

(b) No, S shaped patterns are perfectly normal. There are three points to make here. First, when a variable is spread out, portfolios with even numbers of firms always have more extreme values in the tails. Thus, we expect S shapes in the anomaly variables – middle of the x axis buckets here.

Second, when we mix small and large firms, we are in effect drawing from two different distributions. The tails are going to be fatter, and represent the smallest firms. This fat tailed distribution makes the extremes even more extreme. The extremes are the smallest and most volatile firms.

However, this phenomenon should only lead to extremes in the tails of both anomaly variable and returns, if returns are really linear in the anomaly variable (left). How can the z variable be more spread out than the returns? Well, nobody said the function had to be linear. It’s fine if the function is S shaped. We’re only describing returns here. Theory says expected returns must be linear in betas, but not in anomaly variables.
7. (a) Carhart did sort on 5 year averages, and found weaker results – almost no expected return spread based on 5 year return averages
(b) If his one-year return continuation was really skill, then average returns should be higher in skilled portfolios for much longer times, as long as skill lasts. Let’s hope that’s more than a year.

8. (a) The key assumption under simulated is that no funds have any true alpha, positive or negative.
(b) 1.30 means that if all funds really have exactly zero alpha, then we expect to see that 10% of the funds in a sample will have an alpha t statistic greater than 1.30 just due to chance. In fact, 7% had a t stat greater than 1.30. Thus there are actually 3% too few funds with alpha greater than 1.30 than there should be.
(c) 5% of funds should have performance below -1.71. In fact, 5% of funds have performance below -2.84. This is a bit puzzling – why have negative alpha when you can just buy the index? But we have not chalked up all the costs here.
(d) Berk says we should measure skill by i) gross alpha (before fees) ii) alpha times assets under management – gross fees really, but with the assumption that alpha to investors is zero. iii) he wants tradeable benchmarks, available at the time. No cost-free hml factors in 1967.

9. For any payoff \( x = \{x_u, x_d\} \),
\[
p = E(mx) = \pi_u \frac{1}{c_u} x_u + \pi_d \frac{1}{c_d} x_d
\]
\[
p = E(mx) = \frac{1}{2} x_u + \frac{1}{2} \frac{1}{2} x_d = \frac{1}{4} x_u + \frac{1}{2} x_d
\]
(a) For the bond, \( x_u = x_d = 1 \)

\[
p = E(mx) = \frac{1}{4}1 + 1 = 1.25
\]

A bond price can exceed one meaning a negative real interest rate. \( 1/2 \) is such a terrible outcome that the consumer would really like to save to prevent it.

(b)

\[
p = \frac{1}{4}1 + (-1) = -0.75
\]

The price is negative. Well, losing a dollar in the state of the world that consumption goes down by half is a terrible idea, and you would pay not to take that bet.

(c)

\[
p = \frac{1}{4}(-1) + 1 = 0.75
\]

The situation is exactly reversed

(d) The mean \( E(x) = 0 \) is the same and the variance is the same. They differ by \textit{in which state of nature} you take losses. That matters \textit{This is important. Risk is not standard deviation, its covariance with consumption.}

(e)

\[
p = \frac{1}{4}1 + 0 = \frac{1}{4}
\]

\[
p = \frac{1}{4}0 + 1 = 1
\]

The claim that pays in the bad state is much more valuable. You’re hungrier in the bad state and willing to pay more

(f) Part b is +1 contingent claim to the good state and -1 contingent claim to the bad state. The value of this arbitrage portfolio is \( 1/4 - 1 = -3/4 \), the same value.

10.

(a) Palm had more turnover, a usual sign of ‘liquidity.” But it had a higher bid/ask spread. More quantity at higher price...we diagnosed this in class as a large demand for trading despite illiquidity, not a movement on the demand curve for trading induced by a big supply of liquidity. The facts:

\[
\begin{array}{|l|c|c|c|c|c|c|}
\hline
\text{Tech Stock Carve-Outs} & \multicolumn{2}{c|}{\text{Volume, Liquidity, and Institutional Ownership}} \\
\hline
\text{Turnover} & \text{Parent} & \text{Subsidiary} & \text{Bid/Ask Spread} & \text{Parent} & \text{Subsidiary} & \text{Institutional} \\
& (1) & (2) & (3) & (4) & (5) & (6) \\
\hline
\text{Creative/UBID} & 23.98 & 106.47 & .69 & .93 & 17.71 & 10.38 \\
\text{HNC/Reck} & 3.68 & 22.19 & .32 & .26 & 96.38 & 72.28 \\
\text{Dainobol/PPWWeb} & 2.42 & 25.53 & .62 & .31 & 71.88 & 69.95 \\
\text{Metanor/Xpeditor} & 2.13 & 11.79 & .42 & .49 & 53.96 & 35.96 \\
\text{3Com/Palm} & 4.54 & 19.18 & .09 & .14 & 52.22 & 46.01 \\
\text{Method/} & & & & & & \\
\text{Stratos} & 2.65 & 41.67 & .42 & .20 & 69.47 & 36.63 \\
\text{Average} & 6.56 & 37.99 & .43 & .47 & 60.12 & 45.29 \\
\text{Difference,} & & & & & & \\
\text{parent vs.} & & & & & & \\
\text{subsidiary} & & & & & & \\
\text{t-statistic} & 31.24 & .04 & -14.92 & 2.83 & .62 & -3.06 \\
\hline
\end{array}
\]

\text{Note:} Turnover is daily volume as a percentage of parent shares outstanding or subsidiary shares trading. Subsidiary shares trading are shares sold to the public in the IPO. Volume is average daily volume from the first 20 trading days after the IPO date (not including the first day of trading). The shares outstanding of the parent are taken from CSR files and the shares issued in the IPO are taken from company SEC filings. Bid/ask spread is the average percentage of prices from the first 20 trading days after the IPO date (not including the first day of trading). Institutional ownership from CSR filings are the SEC (via Securities Data Corp.), pertains to the first quarterly filing after the IPO. Institutional ownership refers to a percentage of parent shares outstanding or subsidiary shares trading.
(b) The $R^2$ is surprisingly low, and the tracking error huge – $\sigma(\varepsilon)$ is half the size of $\sigma(y)$! This means you can’t trade on palm news by buying 3 com. Palm is “special” for information trading, a key requirement for the monetary theory.

(a) Causality. Price pressure says that selling volume pushes prices down. Price discovery says there is a piece of news, which will depress prices eventually. The informed learn it first, and trade. Then the price goes down.

(b) The change in yield of each bond depends on the 2-5 year on the run order flow in a multiple regression, not its own order flow.

\[
\begin{bmatrix}
1 & -\rho \\
-\rho & 1
\end{bmatrix}^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix}
1 & \rho \\
\rho & 1
\end{bmatrix}
\]

\[
w = \frac{1}{\gamma} \times \frac{1}{0.1^{1.2}} \begin{bmatrix}
1 & -\rho \\
-\rho & 1
\end{bmatrix}^{-1} \begin{bmatrix}
1 \\
-0.5
\end{bmatrix}
\]

\[
= \frac{1}{\gamma} \times \frac{1}{1-\rho^2} \begin{bmatrix}
1 & \rho \\
\rho & 1
\end{bmatrix} \begin{bmatrix}
1 \\
-0.5
\end{bmatrix}
\]

\[
= \frac{1}{\gamma} \times \frac{1}{1-\rho^2} \begin{bmatrix}
1-0.5\rho \\
\rho - 0.5
\end{bmatrix}
\]

we just need the returns to be sufficiently negatively correlated, $\rho > 0.5$

(b) No. People who are not exposed should buy the value stocks.

(c) No. The hedging demand term is strong. Bond expected returns rise when bond prices fall.