Final exam review problems.

These are a collection of problems from old exams, which should guide your study and give you some sense of my style. No, I’m not going to provide written answers!

In general, you do not need to memorize formulas. You should know the 2 or 3 main points of each paper we read. “State the main point of x and how it was documented” is legitimate. You should also be able to recognize facts documented in papers we read (and when we talked about those in class). See the Fama-French questions for good examples.

1. Consider a VAR representation of returns and dividend growth with two right hand variables,

\[
\begin{align*}
    r_{t+1} &= a_r z_t + b_r dp_t + \varepsilon_{r,t+1} \\
    \Delta d_{t+1} &= a_d z_t + b_d dp_t + \varepsilon_{d,t+1} \\
    dp_t &= \phi_{dp,dp} dp_{t-1} + \phi_{dp,z} z_{t-1} + \varepsilon_{dp,t+1} \\
    z_t &= \phi_{z,dp} dp_{t-1} + \phi_{z,z} z_{t-1} + \varepsilon_{z,t+1}
\end{align*}
\] (5)-(6)

where as usual \( r \) represents log returns, \( \Delta d \) log dividend growth, \( dp_t = d_t - p_t \) log dividend yield, \( z_t \) is the extra forecasting variable, and all variables are de-meaned. (Assume that \( r_t \) and \( \Delta d_t \) as well as further lags of \( dp \) and \( z \) have zero coefficients.)

(a) If \( z_t \) is not present, what rough values do you expect to see for \( b_r, b_d \) and \( \phi_{dp,dp} \) in annual data?

(b) What restrictions on the coefficients \( a, b, \phi \) of this VAR flow from identities?

(c) Suppose the extra variable helps to forecast dividend growth, i.e. \( a_d \neq 0 \). We have some intuition that if a variable helps to forecast dividend growth, it should also help to forecast returns. Is this true in your system? If \( a_d \neq 0 \), does that imply that \( a_r \neq 0 \)?

(d) To investigate this issue a bit further, write a “structural” system rather than the “reduced form” VAR, in which \( z \) only exists to forecast dividend growth

\[
\begin{align*}
    x_{t+1} &= \phi_x x_t + \varepsilon_{x,t+1} \\
    z_{t+1} &= \phi_z z_t + \varepsilon_{z,t+1} \\
    E_t(r_{t+1}) &= x_t \\
    E_t(\Delta d_{t+1}) &= z_t
\end{align*}
\] (7)-(8)

People in the economy observe both \( x \) and \( z \); we observe \( z \), as well as prices, dividends, and returns. Find the coefficients in the VAR representation of the form (5)-(6) that results from this “structural” system.

(e) If the VAR resulting from this system displays \( a_d \neq 0 \) must it also display \( a_r \neq 0 \)?

(f) Your VAR representation from part c may have some zeros or other restrictions not present in (5)-(6). How would you modify the setup of (7)-(8) to remove those restrictions? (Just write down the system you think you need to use, don’t solve it.)

Hint: In case you forgot, the return linearization (ignoring constants as usual) and associated formulas are

\[
r_{t+1} \approx \rho (p_{t+1} - d_{t+1}) - (p_t - d_t) + \Delta d_{t+1}.
\]
\[ p_t - d_t \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}; \quad \rho = \frac{1}{1 + D/P} \approx 0.96 \]

\[ r_t - E_{t-1}r_t \approx (E_t - E_{t-1}) \left[ \Delta d_t + \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} \right] \]

2. (a) (5) What is a reasonable approximate value for the regression coefficient of market returns on the dividend price ratio in annual data? Answer both for log returns, log dp, and for percent returns on percent dp

(b) (5) We said that roughly speaking 100% of the variance of market dividend yields comes from returns and 0% from dividend growth, but only roughly 60% of the variance of market returns comes from expected returns, with 40% from dividend growth. How is this possible – do dividends matter, or don’t they? (Hint: What happens if \( \Delta d_t \) is iid?)

(c) (5) Cochrane claims that “long run” coefficients \( b_r/(1 - \rho \phi) \) are more powerful tests of return predictability. But you can’t beat maximum likelihood, and the ML estimate and test of \( b_r \) is just the OLS estimate. Which is right?

3. (5) We studied the following table, updating Fama and Bliss’s results

<table>
<thead>
<tr>
<th>n</th>
<th>a</th>
<th>b</th>
<th>( \sigma(a) )</th>
<th>( \sigma(b) )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.04</td>
<td>0.91</td>
<td>0.28</td>
<td>0.26</td>
<td>0.14</td>
</tr>
<tr>
<td>3</td>
<td>-0.15</td>
<td>1.20</td>
<td>0.50</td>
<td>0.35</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>-0.37</td>
<td>1.41</td>
<td>0.70</td>
<td>0.44</td>
<td>0.16</td>
</tr>
<tr>
<td>5</td>
<td>-0.09</td>
<td>1.10</td>
<td>0.95</td>
<td>0.52</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>forecasting one year returns</td>
<td>forecasting one year rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>on n-year bonds</td>
<td>n years from now</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I highlighted the number 0.85. What, exactly, is therefore equal to 0.15? (No need to prove, just state the answer. Be very careful where you put your ns and your ts. Feel free to draw a picture too. Clarify any notation you invent by defining it in terms of bond prices.)

4. (10)

(a) What do Cochrane and Piazzesi mean by a “one factor model of expected returns?” (A few equations are appropriate here.) How does this concept of “factor” relate to usual eigenvalue decompositions?

(b) Do Cochrane and Piazzesi find that, statistically, bond expected returns follow a one-factor model? Explain how one might test it and whether it does or does not pass the test. (If you can’t remember what we did, invent a new test. It is enough to say what regression you would run or moment condition you would look at – you do not have to derive the covariance matrix or test statistic.)
5. (5) Brandt, Cochrane and Santa Clara say that discount factors are highly correlated across countries. But people exhibit a lot of “home country bias,” and stock markets are not that well correlated across countries. How do we resolve this apparent contradiction? (Hint: it might be useful to think about two countries with uncorrelated stock markets and a constant exchange rate)

6. (5) If you run cross-sectional Fama-MacBeth regressions of average returns on full-sample betas and loglinear functions of the characteristics size and book/market ratio, which set of variables drives the other out?

7. (10) Start with the CAPM,

\[ R_{it}^e = \alpha_i + \beta_i R_{it}^{em} + \varepsilon_i \]

Now, let’s consider adding another factor \( F_t \), which is also an excess return (hml or smb for example)

\[ R_{it}^e = \alpha_i + \beta_i R_{it}^{em} + \gamma_i F_t + \varepsilon_i \]

(a) Suppose that the \( t \) statistic for \( \gamma_i \) is significant, for all \( i \), the \( R^2 \) of the regression improves, and \( E(F) > 0 \) and also is statistically significant. Does that mean we should adopt this multifactor model, i.e. that we should describe average returns by

\[ E(R_{it}^e) = \alpha_i + \beta_i E(R_{it}^{em}) + \gamma_i E(F_t) \]?

(b) Suppose the GRS test rejects the second model, but does not reject the first model. Does that mean that the pricing errors of the second model are larger?

8. (10)

(a) Which gets better returns going forward, stocks that had great past growth in sales, or stocks that had poor past growth in sales? Is this pattern consistent with some pattern of betas?

(b) If you sort stocks into “winners” that went up from year -5 to one year ago, and losers that went down from year -5 to one year ago, which ones do better for the next year? Is this consistent with some pattern of betas?

(c) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? If value stocks go up, do momentum stocks tend to go up, down, or remain the same?

9. (15)

(a) Here’s an idea: Companies should issue stock and invest when the cost of capital is low, meaning expected returns are low. Thus, portfolios of companies that are repurchasing stock should have a lot higher returns going forward than portfolios of companies that are issuing stock. Does this idea work?

(b) Wait a minute – those issuing companies have high stock prices and the repurchasers low stock prices. Surely the big issuers are growth stocks and the repurchasers are value stocks, so we are just finding that value stocks have high average returns?

(c) If the answer to b is no, does this fact mean that we need to form a new “issues factor,” a portfolio of all high issues firms minus low issues firms, and then run factor models that include this factor

\[ R_{it}^e = \alpha_i + b_i rmrf_t + h_i hml_t + a_i iiss_t + \varepsilon_{it} \]?

Explain exactly how an extra factor might not be necessary, even if the answer to b is no, and what regressions you would run to check.
10. (5) Your assignment is to evaluate the CAPM using the FF 25 portfolios on postwar data. One group member uses a pure time-series regression. She reports that the CAPM is lousy; the market premium is positive, but the alphas are huge; some alphas are even bigger than the average excess returns. The other group member uses a cross-sectional approach with a free intercept. He reports that no, the CAPM is doing fine. The alphas are reasonable, though in this sample it seems the market premium came out negative. Can both of these results happen, or did one of them make a mistake? If a mistake, who made the mistake? (Illustrate your answer with an appropriate graph. Label the axes.

11. (10)

(a) Lamont and Thaler think Palm investors are behaving irrationally. What should these investors have done with their money rather than buy Palm?

(b) Name at least two piece of evidence that Cochrane cites for the “convenience yield" theory as opposed to the “morons” theory or the theory that “short sales constraints means that pessimists can’t express their views” in explaining the high price of Palm over 3com.

12. (10) What does this picture represent? Be explicit, with equations.
Rank forward curves 1-4 by which provides the strongest signal of one year excess returns on 5 year bonds i) according to Fama and Bliss’ regressions ii) according to Cochrane and Piazzesi’s regressions. (There may be ties.) (A: FB look for slope, CP look for tent shapes.)

13. The current log yield on 1, 2 and 3 year bonds is 20%, 15%, 10% – an inverted yield curve
   
   (a) Find current log prices and forward rates.
   (b) Find the expected one year return on 2 and 3 year bonds, and the expected one and two year yields one year from now,
      1. According to the expectations hypothesis
      2. According to Fama and Bliss. Simplify their regression coefficients to 1 or 0, as appropriate. (Hint: you can figure out the expected two year yield one year from now from the expected return on the three year bond.)
   (c) Plot the expected bond prices through time in each case. (Your plot has time on the x axis and bond price on the y axis. You do not have to find the FB path for the 3 year bond past time 1).

14. Show that a discount factor linear in the market return

   \[ m_{t+1} = a - bR^m_{t+1} \]

   implies the CAPM

   \[ E(R^{e_i}) = \beta_i E(R^{em}) \]

   (A: Start with \[ 0 = E(mR^e) \], and use the definition of covariance. \[ 0 = E(m)E(R^e) + \text{cov}(m, R^e), E(R^e) = -\text{cov}(m, R^e)/E(m).... \]

15. Do you expect interest rates to be higher in good times or bad times? Back up your view with an equation and an explanation.
16. An investor lives for two periods, time 0 and time 1. He has a utility function over consumption $c_0$ in period zero and random consumption $c_1$ in period 1 given by

$$ -\frac{1}{2}E\left[(c^* - c_0)^2 + 0.95 \times (c^* - c_1)^2\right] $$

$c^*$ is a parameter (number), $c_0$ and $c_1$ are consumption in the first and second periods of life. We learn from a detailed statistical analysis that his consumption follows a random walk,

$$ c_1 = c_0 + \epsilon_1; $$

the random shock $\epsilon_1$ is normally distributed with mean 0 and variance $\sigma^2$. (A useful preliminary: As of time zero, i.e., knowing $c_0$, what is the mean and variance of $c_1$?) We observe consumption at period 0, $c_0$. It is less than $c^*$; $c_0 < c^*$. Your answers to the following questions can contain $c_0$. Find the price at time 0 (i.e. knowing $c_0$) of the following securities.

(a) A one period zero coupon bond. (You may assume zero inflation if this worries you.)
(b) A “Stock,” which pays a random dividend equal to $c_1$ and nothing thereafter.
(c)

1. Is the stock price greater or less than the price of a bond with $c_0$ face value?
2. How and why does the price depend $\sigma^2$?
3. How and why does the price depend on $c^*$? In particular, explain what happens as $c_0$ gets closer and closer to $c^*$?

17. Verdelhan, Lustig and Roussanov formed 6 carry trade portfolios. The mean annual returns on these 6 portfolios (percent per year) are

$$\begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 \\
-2.92 & 0.02 & 1.40 & 3.66 & 3.54 & 5.90
\end{bmatrix}$$

They performed an eigenvalue decomposition of the covariance matrix of these returns,

$$ Q\Lambda Q' = \text{cov}(\mathbf{R^e}, \mathbf{R'^e}) $$

Here are their results:

$$ Q = \begin{bmatrix}
0.43 & 0.41 & -0.18 & 0.31 & 0.72 & 0.03 \\
0.39 & 0.26 & -0.14 & -0.02 & -0.44 & 0.75 \\
0.39 & 0.26 & -0.46 & -0.38 & -0.31 & -0.57 \\
0.38 & 0.05 & 0.72 & -0.56 & 0.16 & -0.01 \\
0.42 & -0.11 & 0.38 & 0.66 & -0.37 & -0.31 \\
0.43 & -0.82 & -0.28 & -0.10 & 0.18 & 0.11
\end{bmatrix} $$

$$ 100 \times \frac{\text{diag}(\Lambda)}{\sum \lambda_i^2} = \begin{bmatrix}
70 & 12 & 6.2 & 4.5 & 3.8 & 3.2
\end{bmatrix} $$

What were their portfolios? What do these results mean?

18. (30) (This is from 35904, a bit harder than I am heading towards on this final, but we did talk about nonseparable utility, so it’s useful) Let’s think about how our asset pricing formulas would change if
we recognize that the consumption series we’re using is durable. Assume a single durable good, so the representative investor objective is

\[E \sum_{j=0}^{\infty} \beta^j u(k_{t+j}) \text{s.t.} \ k_t = (1 - \delta) k_{t-1} + c_t\]

c_t now represents durable good purchases. General hint: This problem does not require lots of algebra. I used no more than 3 lines for each part. I strongly advise you to work it out on the scratch paper at the end before answering it here!

(a) (5) State the investor’s first order conditions for buying an asset with price \(p_t\) and payoff \(x_{t+1}\). How is this equation different from the standard nondurable case \(p_0 u(c_t) = E_t \left[ \beta u(c_{t+1}) x_{t+1} \right]\)?

(b) (10) Now assume a constant riskfree rate \(R^f = 1/\beta\). Use the equation for pricing the risk free rate to collapse the new terms, so you have an asset pricing equation \(p = E(mx)\) expressed in terms of \(u(k_t)\) and \(u(k_{t+1})\).
(Hints: 1) Do the \(\delta = 1\) case first, then show this solution works for the \(\delta < 1\) case. You do not have to prove this is the only solution. 2) If you’re having trouble, start with quadratic utility, and then generalize to arbitrary \(u(k)\).

(c) (7.5) In the case of power utility, express the discount factor in terms of \(c_{t+1}/c_t\), purchases/stock \(c_t/k_t\) ratio. Suppose as in the Campbell/Cochrane model that the variance of purchases growth \(\sigma(c_{t+1}/c_t)\) is constant over time, When does this model generate high risk premia – in booms when purchases are high relative to the stock of durables or in recessions when purchases are low relative to the stock?

(d) (7.5) Express the model in continuous time,

\[\max E_0 \int_0^{\infty} e^{-\rho t} u(k_t) dt \]
\[dk_t = -\delta k_t dt + c_t dt\]

assume \(c\) follows a diffusion process and power utility \(u(k) = k^{-\gamma}\). Assume your results from part a, b go through so \(\Lambda_t = e^{-\rho t} u(k_t)\). By characterizing this discount factor, do risk premia increase or decrease in this model relative to the nondurable model? How might you modify this continuous-time setup to generate the opposite result (one sentence)?

19. Show that the asset pricing predictions of internal vs. external habit models are the same for power utility, an AR(1) habit and linear technology.

20. (A very simple version of Cochrane/Piazzesi) Let’s modify the basic Vasicek term structure model, and see if we can account for Fama-Bliss regressions. The basic model has a constant market price of risk. We need to have a time-varying price of risk. The obvious way to do that is just to make the price of risk depend on the single factor. So, let’s pursue the obvious extension, in which rather than just \(\lambda\) we have a time-varying \(\lambda_t = \lambda_0 + \lambda_1 x_t\),

\[x_{t+1} - \delta = \rho(x_t - \delta) + \varepsilon_{t+1}\]
\[\log m_{t+1} = -x_t - \frac{1}{2} (\lambda_0 + \lambda_1 x_t)^2 \sigma^2 - (\lambda_0 + \lambda_1 x_t) \varepsilon_{t+1}\]

(a) Find \(p_t^{(1)}, p_t^{(2)}\), hence \(y_t^{(1)}, f_t^{(2)}, r_{x_{t+1}}^{(2)}, E_t r_{x_{t+1}}^{(2)}\) in this model. Hint: they are still linear functions (stuff) + (stuff) \(x_t\)! You have to use \(E e^x = e^{Ex + \frac{1}{2} \sigma^2 x^2}\), exactly as we did in lecture.
(b) Find the predicted value of the Fama-Bliss coefficients, i.e. write \( E_t r x_{t+1}^{(2)} = (\cdot) + (\cdot) (f_t^{(2)} - y_t^{(1)}) \). (All you’re doing here is substituting out the previous results. You had \( E_t r x_{t+1}^{(2)} = a + bx_t \) and \( (f_t^{(2)} - y_t^{(1)}) = c + dx_t \), so if you just write

\[
E_t r x_{t+1}^{(2)} = a + b \frac{(f_t^{(2)} - y_t^{(1)}) - c}{d}
\]

\[
E_t r x_{t+1}^{(2)} = a - \frac{bc}{d} + \frac{b}{d} (f_t^{(2)} - y_t^{(1)})
\]

your’re done. )  Forget the mess in the constant, we’re only interested in the coefficient, \( b/d \). Can we find \( \lambda_0, \lambda_1 \) so that this model captures the Fama-Bliss slope coefficient of approximately 1? 

21. More Paper questions

(a) What is Goyal and Welch’s main complaint about return predictability regressions?

(b) Does “the dog that did not bark” show anything wrong with Goyal and Welch’s calculations, or does it admit them but counter in some other way?

(c) Cochrane and Piazzesi AER decisively reject their single-factor model. Yet they ignore this rejection and trumpet the single factor model as a great success. Why?

(d) Expected returns are always earned for covariance of returns with shocks. According to Cochrane and Piazzesi’s “Decomposing the yield curve” what is the important shock, covariance with which drives expected bond returns? Do Lustig, Verdelhan and Roussanov find that the same structure works across countries?

(e) Campbell and Cochrane claim that they produce imperfect correlation between consumption growth and stock returns. Yet their model has a single shock – doesn’t this mean every variable has to be perfectly correlated?