Clarida Gali Gertler

• Basic idea: if \( r \) increases more than 1-1 with \( \pi \), price level might be stable under an interest rate target.

• Explanation: 70s \( r \) increased less than 1-1; 80’s to now it increases more than 1-1 — victory over inflation.

• Note about systematic part of policy not shocks.

• The Fed responds to output as well as inflation and does try to stabilize output.

\[
r = \beta \pi + \gamma y_{gap}
\]

Thus, regressions of this in pre- and post 80

• Bottom line: TABLE II

• p. 150 This paper: a bit more sophisticated. Fed reacts to future output and inflation

\[
r^*_t = r^* + \beta (E \{\pi_{t,k}|\Omega_t\} - \pi^*) + \gamma E \{x_{t,q}|\Omega_t\} \tag{10}
\]

\( x_{t,q} \) = percent deviation between GDP and corresponding target.

• Important implication — why does the Fed react to so many variables (including interest rates)? — Maybe because those variables forecast the thing the Fed really cares about.

• 153 Fed smooths. (Note Woodford has a paper where smoothing is optimal.) Here ad hoc.

\[
r_t = \rho(L)r_{t-1} + (1 - \rho)r^*_t \tag{3}
\]

\[
\rho = \rho(1) \tag{11}
\]

• (11) in (10) gives\(^1\)

\[
r_t = (1 - \rho) [r^* - \beta \pi^* + \beta E_t (\pi_{t,k}) + \gamma E_t (x_{t,q})] + \rho(L)r_{t-1}
\]

• What to do about \( E_t \pi_{t+k} \)? Forecast them using some variables \( z_t \). First stage,

\[
\pi_{t,k} = a + bz_t + \varepsilon_{t+k}
\]

then use \( a + bz_t \) for \( \beta E_t (\pi_{t,k}) \).

---

\( r_t = \rho(L)r_{t-1} + (1 - \rho)[r^* + \beta (E \{\pi_{t,k}|\Omega_t\} - \pi^*) + \gamma E \{x_{t,q}|\Omega_t\}]
\]

\( r_t = (1 - \rho)[r^* - \pi^* + (1 - \beta) \pi^* + \beta E \{\pi_{t,k}|\Omega_t\} + \gamma E \{x_{t,q}|\Omega_t\}] + \rho(L)r_{t-1}
\]

\( r_t = (1 - \rho)[r^* - \pi^* + (1 - \beta) \pi^* + \beta \pi_{t,k} + \gamma x_{t,q}] + \rho(L)r_{t-1} + (1 - \rho)[\beta (\pi_{t,k} - E \{\pi_{t,k}|\Omega_t\}) + \gamma (x_{t,q} - E \{x_{t,q}|\Omega_t\})]
\]

\( r_t = (1 - \rho)[r^* - \pi^* + (1 - \beta) \pi^* + \beta \pi_{t,k} + \gamma x_{t,q}] + \rho(L)r_{t-1} + \varepsilon_t
\]
This is IV! Equivalent: Estimate

\[ r_t = (1 - \rho) \left[ r^* - \beta \pi^* + \beta E_t \pi_{t,k} + \gamma E_t x_{t,q} \right] + \rho(L) r_{t-1} + \epsilon_{t,k} \]

by IV.

- **IF** there are no policy shocks, \( \epsilon_{t,k} \) is a forecast error, orthogonal to time t information. Note: They are assuming no policy shocks, and especially not serially correlated shocks. This turns out to be crucial.

- If your \( z \) is a subset of the Fed’s \( z \), no problem – it just introduces a larger forecast error uncorrelated with the right hand variables\(^2\)

- (Digression on OLS/GMM)

\[ E \left[ (y_t - bx_t)x_t \right] = 0 \]

Implies \( b = E (xx')^{-1} E(xy) \). Thus, you use this moment condition to do OLS by GMM and get distribution theory that corrects for everything. Similarly the moment

\[ E \left[ (y_t - bx_t)z_t \right] = 0 \]

implies IV so you can map IV to GMM and get a good distribution theory

- “overidentification tests” check if you get the same number using different zs. (Power? Weak instruments will not reject.)

- 155 Data:

  1. Avg federal funds – not target; GDP deflator. Also CPI; CBO output gap; Instruments lags of ff, \( \pi \), \( y \), commodity prices, M2, spread. (JC why not level of 30 year yield to forecast inflation?)

  2. 156. Horizon one quarter

  3. 157. 4 lags of instruments!

  4. Should show first stage estimate, and forecasts. Are they reasonable? Is everything from a few data points? Episodes? Is there marginal information in the 4th lag of M2?

- TII estimates. **Big point of the paper** is \( \beta \) changing from \(< 1\) to \(> 1\).

\[
\begin{align*}
r_t &= (1 - \rho) \left[ r^* - \pi^* + (1 - \beta) \pi^* + \beta E (\pi_{t,k}|z_t) + \gamma E (x_{t,q}|z_t) \right] + \rho(L) r_{t-1} \\
&\quad + (1 - \rho) \left[ \beta E (\pi_{t,k}|z_t) - E \{ \pi_{t,k}|\Omega_t \} \right] + \gamma (E (x_{t,q}|z_t) - E \{ x_{t,q}|\Omega_t \})
\end{align*}
\]

if \( z \subset \Omega \), the right hand variable is uncorrelated with the error.
· Note: including output is crucial to this result.

Where is the evidence for $\beta = 2$ post but not pre? If you run $r = \beta \pi$ you get $\beta \approx 1$ because of Fisher (real rates vary much less than inflation). $\beta = 2$ as found here is impossible – we don’t have 12% interest rates at 6% inflation. Thus, the output gap and correlation of output and inflation are vital to get $\beta$ far from one. At 6% inflation you have 6% rates, but if output is low you can say “The fed would have raised rates to 12% if this recession weren’t going on.”

· Why should $\gamma$ rise? Thought pre was more concerned with stabilization? Different $\gamma$ make a difference on $\beta$!

· FI Don’t be too impressed. Lagged $r$ is a predictor, AR(1)s look pretty good.

· TIII TIV robust. This is important. Always put the regression equation in the table caption

· TV subsamples. Conclusion 164 of pure inflation targeting. But it’s weird that you add two subsamples $\gamma = 1$ and get 0.

· T VI 164 “backward looking” works too – the simple regression. But now what are instruments? This should now be a pure regression.(And we now have policy shocks!)

· 165.top. “volatile?” what models???

· Is the Fed really so smart? Would the Fed really raise interest rates to 20% if we replayed the 73 oil shock? Or would all this “inflation targeting” business go right
out the window? Was the Fed tough or lucky in the post period. Note its’ more about the level. Would the Fed really leave interest rates that high for a decade?

![Figure 5. Target Based On Estimated Post-October '79 Rule vs. Actual Funds Rate](image)

- JC final comments

1. I distrust these estimates (see problem set). Small changes in specification change $\phi_\pi$ a lot. Serially correlated policy shocks make IV invalid.

2. I doubt the Fed will be so tough when it sees a big gap again. Is the Fed really tougher than in the 70s or just a lot luckier?

3. I’m still not sure that you can see $\phi_\pi$ in sample. Does artificial data from CGG model produce CGG regressions? This depends on the rule, etc. but the usual theorem is “you can’t see off-equilibrium threats from an equilibrium”. The only way out: tie off-equilibrium behavior to some part of the equilibrium. For example, King $i = i^* + \phi(\pi - \pi^*)$ there is no hope of measuring $\phi_\pi$ since $\pi = \pi^*$ But maybe $i = \alpha + \phi\pi$?
153. Figure 1. Real interest rates are high now.

CGG conclusion that Fed may have thought $\bar{y}$ was higher than it was is inconsistent. We’re modeling the Fed’s behavior, so we should use their $\bar{y}$

156 Estimate just like CGG

Fig 2 and 3. Note how deep the output gap seemed in 1974.

Table 1 161. Big point. Use the right gap, you get the same $\beta$ in both periods.

163 Basic story: there was a productivity slowdown in 1973, and nobody caught on until much later. they were using 60s trends to calibrate 70s “potential”

164. what they thought at the time vs. current “detrended” (using ex-post information) output.
Woodford Ch. 1 The return of monetary policy rules.

Short version

Big points: 1) Taylor rules, of course! 2) Larger “doctrines” of monetary policy to be discussed. Read this all as claims of stuff to be modeled later. 3) Optimal interest rate rules.

1. Lots of talk about “communication” and “expectations” and one big reason for rules:

   p.2 bottom to 3 top, “rule based monetary policy ..is possible only in the case that the central banks can develop a conscious and articulate (!) account of what they are doing.” ..“sound monetary policy...dependent on the policy’s being understood and relied upon by the private sector in arranging its affairs”

   p.3. “...dependent up on the policy’s being understood and relied upon by the private sector...”

   p. 3, middle. Inflation targeting. “Commitment”. “Explain the central bank’s policy”... “communication”...“publication of the models used in producing forecasts...”

   p.14. Why a rule? a) “The effectiveness of monetary policy depends as much on the public’s expectations about future policy as upon the bank’s actual actions” b) (minor, to Woodford) – time-consistency

15 “Successful monetary policy is not so much a matter of effective control of overnight interest rates as it is one of shaping market expectations of the way of the way in which interest rates inflation and income are likely to evolve”

15 “Not only do expectations about policy matter, but...very little else matters”


16. Transparency is desirable (in contrast to long tradition) (Another “doctrine” to be defended by a model.)

17 most important thing is to “give clear signals about what the public should expect it to do in the future” (Contingent, I think, not the path of ff)

JC comment: Nowhere else in economics do we think people need to understand it for it to work. Usually, prices communicate all information a la Hayek.

What is this in the model? I think,

1) Taylor explosions, – believe they’d blow up off equilibrium – and

2) \[ y_t = E_t[y_{t+1} - s_t], \pi_t = \beta E_t \pi_{t+1} + \phi (y_t - y^*) - E \] on the right hand side affects stuff today, so if you think day by day, affecting \( E \) can have effects today. . .
2. Inflation targets.

p. 13. Stable p implies optimal y in many models, again justifying inflation targets. Don’t need also fight output.

(Will Woodford believe this when he’s Fed chairman and ignore unemployment?)

p. 14. Right inflation to target is the one with the stickiest prices.

Very interesting idea – that is of course the hardest to measure, and contrary to most measurement, which places emphasis on things that are quick to move. (Do quick-moving things forecast the sticky ones?)

3. More reasons for and character of rules:

a) solves time-consistency problem

19. Reoptimizing at each point in time can lead to a worse outcome than a rule. (“Should the Fed raise rates?”)

19 Rule as precommitment. Because economy not like “a system that evolves mechanically in response to the current action of the controller”

19 Because expectations matter.

20: Example: Inflation bias. Always tempted to use Phillips curve ex post.

20 a suboptimal response to shocks as well. Example. A short term disturbance. It would be good to have a smaller but longer lasting response. A reoptimizer can’t do it because after the shock ends, the reoptimizer optimizer will end too.

b) Good rules will not be purely forward looking

21. Thus, optimal policy must be “history dependent” not just “forward looking” = discretionary.

22 A justification for the strong $\rho r_{t-1}$ in Taylor rule estimates! That even Taylor seem not to think is good.

c) Rules can just feed back on $y, \pi$

22. How do we avoid a) having to write out a full state-contingent rule b) having to stick to a rule even though ex post it looks bad? (Unlikely to do that!)

23 Claim: a Taylor rule avoids both. a) It maps from output, inflation not underlying shocks. (JC is that really less “state-contingent” or just a very simple model?) b) the rule is time-consistent; you’d still choose that rule later even if not the time $t$ decision.
p.23. The bank “should consider what an optimal rule...would be rather than asking what an optimal action is on the individual occasion” Good luck!

4. Defense of the idea that a government can set the interest rate, even in a cashless economy.

3.2. Monetary policy in a cashless economy

For Woodford, does controlling the interest rate as in NZ require a money demand?

33 basically, like pegging a price “we borrow and lend as much as you want at r, come and get it”

But what about huge transactions will be needed, no “one doubts that such efforts will be effective. What’s different about controlling .. nominal interest rates”

33 “The difference is that there is no inherent equilibrium level of interest rates”

34 “No inherent value of a dollar.”

34 But wait, that’s nominal rates. We’re not talking about changing week to week inflation – how can the Fed change real rates with no money demand?

34 A: sticky prices. (JC: what do real rates mean if prices are sticky – at some sense something is non-allocative)

The rest is just dollar as definition of a security, as in “money as stock.”

5. Summary of Taylor etc.

38 Wicksell rule for price level stability

\[ i_t = \bar{i} + \phi p_t \]

to stabilize price level. 39, bottom, Woodford thinks base drift is optimal.

39 Taylor rule

\[ i_t = 0.04 + 1.5(\pi_t - 0.02) + 0.5(y_t - y_t^P) \]

40. Charge that Taylor coefficient below 1 “this failure may well have been responsible for the greater US macroeconomic instability during the 1960s and 1970s.”

41 Table 1 taylor rule estimates. Mostly, like CGG < 1 to > 1.

42 Note Orphanides dissents. (And JC thinks you can’t measure \( \phi > 1 \) in equilibrium)

43: ‘Questions” – is the taylor rule optimal? Is responding to only these variables enough? Is responding to \( y, \pi \) and not to underlying shocks enough? Again, dynamics? Presages Woodford program: find the model in which what is, is optimal.
6 Response to critics of rules.

45 Sargent Wallace. interest rate rules lead to indeterminacy.

45 Answer: Mccallum S W have exogenous interest rate rules not rules that respond to endogenous variables ($\pi$). Past criticisms are to fixed interest rate rules.
Woodford Ch 2 Price level determination under interest rate rules

64-74. Very careful but endlessly long setting up of the standard frictionless economy. Endowment economy with $C = Y$. There can be money, money can pay interest

67 Especially careful in writing no-Ponzi conditions

69. Household budget constraint

**Cashless economy**

all the buildup gets you to

$$1 + i_t = \frac{1}{\beta E_t \left[ \frac{uc(Y_{t+1})}{uc(Y_t)} \frac{P_t}{P_{t+1}} \right]}$$ \hspace{1cm} (1.21)

75 “Wicksellian” controls interest rates to keep the price level bounded

$$i_t = \phi(P_t/P^*_t; \upsilon_t)$$

$P^*$ = target path,

76 – As much as we say about fiscal: set D, tax rule follows

77 –looking only for local determinacy. Restricts to bounded solutions

78 Steady state

$$\frac{P^*_{t+1}}{P^*_t} = \bar{\Pi} = 1$$

$$1 + \phi(1, 0) = \bar{\pi}/\beta$$

(the latter from 1.21)

Woodford: Linear stuff comes from loglinearization near steady state. Loglinearize rule (80) and (1.21) (81)

$$\hat{i}_t = \phi_p \hat{P}_t + \upsilon_t$$

$$\hat{i}_t = \hat{r}_t + E_t \hat{\pi}_{t+1}$$ \hspace{1cm} (1.32)

$$\hat{r}_t = \sigma^{-1} E_t (\hat{Y}_{t+1} - \hat{Y}_t)(1.33)$$

($\sigma = -\frac{uc}{ucc}$. King called this the Fisher equation / IS curve. The latter can include shocks $g_{t+1}, g_t$. Here $Y$ are endowments, so (1.33) will set the real interest rate no matter what you do. Back to King but with a price level goal.)

81 As before

$$\hat{r}_t + E_t \hat{\pi}_{t+1} = \phi_p \hat{P}_t + \upsilon_t$$

$$\hat{r}_t + E_t \hat{P}_{t+1} = (\phi_p + 1) \hat{P}_t + \upsilon_t$$
With $\phi_P > 0$, as usual we solve forward

$$\hat{P}_t = \sum_{j=0}^{\infty} \left( \frac{1}{1 + \phi_P} \right)^{j+1} E_t(\hat{r}_{t+j} - v_{t+j}) (1.37)$$

82. the usual "$\phi_P > 0$ gives local determinate price level"

(Mysterious prop 2.4. If you add money with zero interest, then there must be deflation. But if there is deflation either $M \to 0$ or the value of $M$ explodes, violating the transversality condition. Thus we can’t have $M$ bounded away from zero forever, and $i_m = 0$. point?)

p. 84. Wicksellian flavor. “Increases in the .. price level ... result from exogenous increase in the.. natural rate of interest that are not sufficiently offset by an adjustment of the ..target.” Wicksell thought this held during the gold standard too – an excellent empirical question! (Foretaste of time varying intercept in taylor rules, e.g. King)

**Alternative interest rate rules**

**2.1 Exogenous interest rate targets**

86, prop. 2.5 87. How about the rule

$$1 + \hat{i}_t = 1 + \hat{r}_t$$

to give no inflation? This satisfies

$$\hat{i}_t = \hat{r}_t + E_t\pi_{t+1} \quad (1.32)$$

with $\pi_t = 0$. A la King, yes, but not enough for determinacy as any

$$\pi_t = \varepsilon_t; \quad E_{t-1} (\varepsilon_t) = 0$$

will also work.

p. 88 *Explicitly Ricardian Regime*. Moving to a Non-Ricardian regime would give a determinate price level! (Aching for a definition. What does “locally Ricardian” mean?)

**2.2 Taylor rules**

90 Log linear approximation

$$\hat{i}_t = \bar{i}_t + \phi_\pi \pi_t$$

91

$$\hat{i}_t = \hat{r}_t + E_t\pi_{t+1} \quad (1.32)$$

$$\phi_\pi \pi_t = E_t\pi_{t+1} + (\hat{r}_t - \bar{i}_t)$$

$$\pi_t = \sum_{j=0}^{\infty} \phi_\pi^{-(j+1)} E_t [\hat{r}_{t+j} - \bar{i}_{t+j}]$$
Prop 2.6: $\phi_\pi > 1$ gives “locally determinate” $\pi$

*Big news: Fed needs to move intercept $\bar{i}_t$ to match $\hat{r}_t$ at least in “present value” sense.*

92 read “Wicksellian flavor”

93 Acknowledgement that the regressions can’t tell! But ‘downward bias” not “mongrel coefficient” – Woodford doesn’t see the depth of the problem.

(An advantage of $\phi_\pi$ not much above 1?)

### 2.3 Inertial responses

Big picture: fancier taylor rules.

a) $$\begin{align*}
i_t &= \bar{i}_t + \phi_\pi \bar{\pi}_t \\
\bar{\pi}_t &= (1 - \delta)\pi_t + \delta \bar{\pi}_{t-1} = (1 - \delta) \sum \delta^j \pi_{t-j}
\end{align*}$$

Proposition 2.7. $\phi_\pi > 1$ is again what we need. (2.10) “Discounting” by $\delta + (1 - \delta)\phi_\pi = \phi_\pi - \delta(\phi_\pi - 1)$

b) $$\begin{align*}
i_t &= \bar{i}_t + \rho(i_{t-1} - \bar{i}_{t-1}) + \phi_\pi \pi_t
\end{align*}$$

Even with $\rho > 1$. My eyes are glazing

Comment. Inflation rules let $P$ wander arbitrarily far from $\bar{P}$. Wicksellian rule insisted on local equilibrium for $P$. But if you admit inflation rules, you admit nothing in economics restricts you to local movements in $P$.

### 3. Price-level determination with monetary frictions

Big Q: does adding money change this? A: No. You’ve all done $u(c,m)$ models. Let’s move on.

### 4. Self-fulfilling inflations and deflations

*Global* at last.

123 Promises a simple example and policy regimes that solve the problem.

$$i = \phi(\Pi_t) \quad (4.1)$$

A target $\Pi^*$, satisfies at steady state.

$$\phi(\Pi^*) = \beta^{-1} \Pi^* - 1$$

124. Explicitly Ricardian fiscal policy.
Let’s do the cashless version! Equilibrium must satisfy

\[ 1 + i_t = \frac{\beta u_c(c_{t+1})P_t}{u_c(c_t)P_{t+1}} = \frac{\beta P_t}{P_{t+1}} = \frac{\beta}{\Pi_{t+1}} \]

\[ \Pi_{t+1} = \beta(1 + \phi(\Pi_t))(4.6) \]

This is just the nonlinear version of

\[ \phi_\pi \pi_t = E_t \pi_{t+1} + (\hat{r}_t - \bar{r}_t) \]

Result: Infinite number of global solutions even if \( \phi_\pi > 1 \) Aha, he admits it!

SGU’s point: with \( i > 0 \), there must also be a lower bound and a “stable”
equilibrium. “The Taylor principle cannot be globally valid”

infinite equilibrium, and sunspots too (as you can add \( \varepsilon_t \) to any of them)

\( \Pi^* \) is “unique bounded local equilibrium” But that looks less appealing!

These results may make it seem that a Taylor rule is not a very reliable way
of ensuring a determinate... price level after all” You bet! “Several responses may be made...”

1. “The equilibrium in which inflation is stabilized at the target level is nonetheless
locally unique, which may be enough to allow expectations to coordinate upon that
equilibrium rather than one of the others.” That’s it? You gotta be kidding.

2. “\( \Pi^* \) is unstable implying that an economy should be expected almost inevitably
to experience a self-fulfilling inflation or deflation.” right.

“such reasoning implies a serious misunderstanding of the causal logic.” Whoops!
“The (difference) equation indicates how the...inflation rate in period \( t \) is determined by
expectations regarding inflation in the following period. ..thus the economy can only move
to one of these ..paths if expectations about the future change significantly something
that one may suppose would not easily occur” Right, because we know the Fed isn’t so
pig-headed as actually to follow the Taylor rule and set interest rates at 200% if inflation
is 100%. But then the equilibrium doesn’t hold in the first place. Like Francisco said.....

Note: “Old Keynesians” had a phillips curve like

\[ \pi_t = (\pi_{t-1}) + s y_t \]

Thus the price level (inflation) was determined by past prices (inflation), plus “aggregate
demand”. New Keynesians just change the data on the right hand side

\[ \pi_t = E_t \pi_{t+1} + s y_t \]

So now they think of prices (inflation) as set by expected future prices (inflation) modified
by “aggregate demand”. That’s where all this “mold expectations” stuff comes from –
you can change expectations, you can’t change past inflation.
129 Argument for learning dynamics and stability of rational expectations equilibria. Not worked out, let’s wait for section 2.3 of Chapter 4.

129-130 It’s just like the multiple equilibrium problem with hyperinflation dynamics. Right. And we didn’t believe that one either. Wasn’t it Woodford who told us to accept explosions and use the fiscal theory to pick the right equilibrium?

131. An argument that money will fix it.. let’s go on to the simple cashless case.

132. A non-Ricardian fiscal policy to the rescue! Model
\[
\frac{D_{t+1}}{D_t} = \bar{\mu} \geq 1
\]
\[i = \phi(\Pi) \quad (4.1)\]

\(\phi(\Pi)\) hits zero above the Friedman rule – the left crossing in the figure is at \(i = 0\).

Then we hit \(i = 0\) in a finite time, so \(\Pi = \beta\) deflation after that. But since the government keeps a fixed amount of nominal debt outstanding, the real value \(D/P\) of the nominal debt \(D\) explodes, which can’t happen.

Same point in equations: Simplifying the equation on 132 (\(\lambda(i_t) = \omega_c(c_t)\))
\[
\beta^T \omega_c(T) D_T / P_T = \beta^T \bar{\mu}^{T-t} \omega_c(T) D_t / P_t \geq \beta^T \omega_c(T) D_t / P_t > 0
\]
(First equality, because \(P_T = \beta^{T-t} P_t\) and \(D_T = \bar{\mu}^{T-t} D_t\). Second because \(\bar{\mu} \geq 1\).) \(\phi(\Pi)\) goes to infinity at some finite \(\Pi\). You gotta be kidding, right? The Fed will set an infinite interest rate?

138. Or, promise convertibility at some high rate of inflation. But you need the gold to do it. Back to the fiscal theory

JC conclusion: All the equilibria are valid. Taylor rules do not avoid interest rate indeterminacy. Once you look global, you need the FTPL to come to the rescue!

A deeper look at fiscal saving of SGU deflation.

Questions: couldn’t you have taxes rise with \(D/P\)? A: no, because taxes would buy back the D.

Does this mean fiscal theory is in charge from the get-go? Where did we lose (p.73) the endogeneity of taxes? Woodford’s answer (email to JC)

The SG&U 'balanced budget’ rule is not globally Ricardian (under the def. in my JMCB paper) because intertemporal government solvency would
not be satisfied for all possible paths of the price level; in particular, it is not satisfied in the case of paths that are sufficiently deflationary. It is nonetheless locally Ricardian, because i.g.s. is satisfied for all price-level paths that satisfy certain bounds (for example, if the inflation rate is forever within a certain interval, that contains zero as an interior point). One can tell this simply by asking whether condition (1.24) of chapter 2 is satisfied for various price-level paths.

From Ch. 4.

4. Fiscal requirements for price stability.

Big picture: fiscal theory plus interest rate targets. Ricardian vs non Ricardian.

So far: Government specifies $D$ ($M$ if there is any), $T$ evolves following one period budget constraint. This doesn’t tell us anything about ricardian or no – for that we have to know if the transversality condition is holding. Sometimes it does (SGU, explosive inflation, $D = constant$), sometimes it does not (SGU, implosive deflation, $D = constant$)

311 One period debt, cashless economy $D_t = \text{nominal value of government liabilities at end of } t.$

$$D_t = (1 + i_{t-1})D_{t-1} + (P_tG_t - T_t) \quad (4.1)$$

312 "$i_t$ affects the evolution of the public debt, even in the absence of seigniorage... through... debt service on existing government debt... this is the most important fiscal consequence of monetary policy

312 $G_t$ exogenous, alternative rules for $T_t$

312

$$b_t = (1 + i_t)D_t/P_t$$

= real face ("maturity") value of the debt.

Linearize (4.1), around zero inflation, constant $Y$, etc.

$$\frac{D_t}{P_t} = (1 + i_{t-1})\frac{D_{t-1}}{P_t} + \frac{P_{t-1}}{P_t} + (G_t - T_t) \quad (4.1)$$

$$\beta^{-1} \left[ \hat{b}_{t-1} - \hat{b}_t + \hat{G}_t - \hat{\pi}_t \right] + \hat{b}_t \quad (4.2)$$

312. Definition:
A Fiscal rule is locally Ricardian if when substituted into the flow budget constraint (4.1) or (4.2) it implies that [the real value of the debt] \{b_t\} remains forever within a bounded neighborhood of \(\bar{B}\) for all paths of the endogenous variables \{\pi_t, Y_t, i_t\} that remain forever within .. small neighborhoods of the steady-state values \(\bar{Y}, \bar{i}\) and all small enough values of the exogenous disturbances.

Read top of p. 313.

313. Locally ricardian policies throw out one equilibrium condition, (1.3)

\[
\lim_{T \to \infty} T E_t \left[ u_c(Y_T - G_T; \xi_T) D_T/P_T \right] = 0
\]

313 An example of a tax policy that is/is not locally Ricardian. (4.4) If the government raises taxes to pay off outstanding debt, then it’s locally Ricardian

\[
\hat{\tau}_t = \tau_b \hat{b}_{t-1}
\]

(More Woodfordian needless generality)

\[
\begin{align*}
\hat{b}_t &= \beta^{-1} \left[ \hat{b}_{t-1} - \hat{b}_{\pi_t} + \hat{G}_t - \tau_b \hat{b}_{t-1} \right] + \hat{b}_t \\
\hat{b}_t &= \beta^{-1} \left[ (1 - \tau_b) \hat{b}_{t-1} - \hat{b}_{\pi_t} + \hat{G}_t \right] + \hat{b}_t
\end{align*}
\]

always converges if

\[
\left\| \beta^{-1}(1 - \tau_b) \right\| < 1 \\
1 - \tau_b < \beta \\
\tau_b > 1 - \beta = 1 - \frac{1}{1+r} = \frac{r}{1+r}
\]

You just need to pay the interest on the debt and an extra penny, then debt will decline.

314. Proposition 4.11. Locally Ricardian \(\rightarrow\) \(\phi_\pi > 1\) for locally determinate. Locally non-Ricardian \(\rightarrow\) \(\phi_\pi < 1\) for locally determinate.

Intuition. As we did with fiscal theory, one of “Fed” or “Treasury” must be “active” and the other “passive”. If both are active, “overdetermined” and no equilibrium

Proof (rough) Add debt equation. We need two eigenvalues >1 since we have two equations with \(E_t x_{t+1}\) rather than \(x_{t+1}\) (IS, Phillips). Either \(\tau_b\) or \(\phi_\pi\) must be “explosive”.

Comment. notice the symmetry; there’s an off-equilibrium threat to let something explode. Woodford might say it’s more plausible to threaten a nominal explosion than a real explosion.

315-316. Is non-ricardian possible? Been there, done that.
317. Possibility to be locally Ricardian, globally not – extreme events test ricardian limits.

316. “bond price support regime” Why did the fed-treasury accord work so long? (A fixed peg at 1%) A: It’s ok if paired with a ricardian regime. But Korean war meant higher future deficits, caused inflation in the Ricardian regime.

318. Woodford’s choice to go Ricardian.

Back to SGU? Does it make sense to say $\Pi^*$ is “locally ricardian” i.e. ftpl is not behind $\Pi^*$? The threat to go non-ricardian with constant $D$ rules out the hyperdeflation, (and hyperinflation). Does it make sense to say $\Pi^*$ is locally ricardian?

Models with money in the utility function.

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