1 Preliminaries

1.1 Class management

Review course outline, website. Bring namecards. The course is based on problem sets, exam, class participation. I will call on you, and may ask for 10 minutes w. chalk. Register somehow. Participate in class. Don’t get lost either on history and jargon or on equations. I will never be mean on a class question – but I will be mean if you ask it after class! Will we do makeup tues before Thanksgiving?

1.2 Big Picture

The focus of the course is the price level and hence inflation. Money, monetary policy also associated with output fluctuations, but they will be a lesser concern here. First things first – we need to understand the price level before understanding its non-neutralities. The fiscal theory high on my agenda. I think it may replace MV=PY and shake the foundations of monetary theory. We will mix theory, empirical work, historical experience. As reading Sargent and Velde shows, our forebears were much better at remembering long historical experience. We need to broaden our horizons past 20 years of US time series. Monetary economics is to be taken seriously, as an understanding of the world around us, not as a game for writing clever equations. You should take that attitude in your own work.

Money is hidden, which is why monetary economics is cool. Some people say “Economics is obvious,” and this is a good counterexample. Imagine an investigative reporter trying to understand inflation. He goes from the grocer to the wholesaler to the farmer to the seed supplier to the worker to the grocer...that (if!) money supply is the root cause is far from obvious! We’ll study lots of other far-from obvious things.

2 Overview: Theories of the price level

2.1 Commodity monies or standards.

The most obvious theory of the price level derives from gold coins or a commodity standard. Gold nuggets, gold coins then fully redeemable tokens. The price level is obvious.

It seems simple but...there is no real commodity use for money! Perhaps this is a commitment mechanism? Or perhaps we should understand it as an instance of the quantity theory (comes next), with nature giving us the limited supply? (Perhaps gold was worth a lot more when used as money than it would be in a fiat money economy.)
The same basic economics applies now in pegged exchange rate or currency boards. (Though where base country price level comes from still to be determined, and the option to devalue the peg or abrogate the board make it interesting.)

2.2 Monetarism: 20th century from Fisher to Friedman.

- MV = PY. Read this causally from M to P, Y. “Inflation is always and everywhere a monetary phenomenon.”
  - Read Friedman: How does M affect Y then P.
  - Fiat money. Thus can apply to 20th century institutions in a way gold/commodity theory cannot.
  - Ingredients of the “quantity theory”:
    a) Money demand as an inventory for making transactions. You hold money despite interest loss, as you hold a peanut butter inventory.
    b) Money supply is limited. (Though it’s free for the government to print!)
  - There is much more to “monetarism.” Among other propositions:
    a) The Fed should pay attention to and control money supply. Period. 4% rule. It may peek at inflation but really ignore output and interest rates.
    b) The Fed not be “activist” and try to offset shocks. (4% because Friedman didn’t trust the Fed, not because a stochastic optimal policy exercise would yield 4%. There is a whiff of commitment and rules vs. discretion here.)
    c) Most output and inflation variability is due to Fed mistakes in controlling the money supply. (False, as we’ll see.)
    d) “Operating procedures” The Fed should focus on money supply, not interest rates as instrument, signal, etc.
  - Problems:
    a) The theory relies on a rigorous distinction between “money” and “non-money” assets. It has always foundered the question, What is money anyway? Cash, reserves, checking accts. (M1). Money market – can write checks but few do (and pays interest)? Unused credit card balances? Sweep accounts? The Marie Antoinette theory applies – there are a lot of liquid assets that could be used as money though their velocities are typically low.. A martian would think that actual institutions look much more like unlimited private banknotes or electronic accounting system of exchange than like the textbook base + checking accounts limited by reserve requirements vs non-transactions “bonds.”
    b) The theory requires government control of the monetary assets. For example, if people can issue circulating IOUs (banknotes) the price level is indeterminate. How
should the Fed control “money” Should it eliminate financial innovation? This also requires the government to rigorously control “money substitutes” such as privately-circulating IOUs (checks, banknotes), and to pass laws forbidding the use of other objects (foreign currency). If these are free, demand for government money can go to zero and the price level to infinity.

c) Theory requires that the Fed does control the money supply. But in fact it pegs interest rates (for good reasons, “unstable money demand”) The theory no more applies to the current institutional setting than the gold standard theory applies to a fiat money setting.

2.3 “Fed view.” “Old-Keynesian”

This is the view implicit in every statement by the Fed, newspapers, Op-ed, and most applied macroeconomists writing for non-professional audiences.

a) Fed controls Federal funds rate. Not directly in US, but directly in some countries. This is the overnight rate for reserve requirements (in US) reserves (in other countries).

b) The causal change to inflation: ff → short term rates → long term rates → “demand” → “gaps” “unused capacity” → “price pressure” → inflation. (Since prices are stickly, control of nominal ff gives the fed control of real rates with a nominal lever)

c) Price level determination? $p_{t-1}$ and stickiness determines $p_t$

d) Problem: there is not a shred of evidence or theory for any step of the chain.

This basic Keynesian “model” is still with us. All the discussion of Bush tax cuts is over taxes (if we give them an extra $600, how much will they spend?) not tax rates. More “spending” is thought to be good. The idea seems to be that C/I affects Y. What is ”Aggregate demand”? Money vs “Credit”? How can consumption go up without investment going down? (The saving = investment condition seems to disappear.)

2.4 Fiscal theory

\[
\frac{\text{money base} + \text{nominal debt}}{\text{price level}} = Epv \text{ future primary surpluses.}
\]

This equation is in every model, including Friedman (46) - Sargent. It is often read as the “Government budget constraint” from left to right. The Fed sets P, base and debt are what they are, so this tells the treasury how much taxes to raise in order to pay off the debt. For example, if the Fed runs a deflation, the treasury has to raise taxes to pay off the increased value of nominal bonds.

In the fiscal theory, works from right to left, as expected present value determines the stock price (“money as stock.”) Both “readings” – “regimes” are possible.
- This can determine the price level with
  
  a) NO monetary frictions whatever

  b) Arbitrary private “moneys” (IOUs, banknotes, etc.) Unlike MV=PY only
government liabilities are on the LHS, not M1)

  c) Arbitrary financial and transactions-technology innovation. If we all want to
use debit cards, no problem.

  d) If the Fed pegs interest rates. (We’ll show that later.)

Thus it applies transparently to the current institutional environment.

It is also a natural “Frictionless benchmark” on which to build more complex theories.
Allows us to start with a Chicago "free market" approach to money, and add frictions
only if we need them.

A few questions and objections right off the top:

Q: What about helicopter drops?

A: Helicopter drops do raise the price level in both quantity and fiscal theories. It
would be a “wealth” or “pigou” effect. Note that helicopter drops are a fiscal operation.
Accounting would say this is “spending” financed by borrowing, and the Fed buys the
bonds issuing money.

Q: What about open market operations?

A: the Fed does not do helicopter drops, the Fed does open market operations. In an
open market operation, you get money but you give up the same amount of bonds.

BIG QUESTION #1: Is an open market operation the same as a helicopter drop
(money financed deficit)? Monetarists say YES. Really? This violates the MM theorem.
(Equity-debt swap, or short vs. long term debt swap). Put that way, it’s not so obvious.
Of course, it depends on the proposition that M is special.

FT: NO. Open market operations make NO difference to LHS! To first order, open
market operations have no effect on the price level. (First order: no frictions. A maturity
structure allows open market operations to affect the timing of inflation – “long term
debt and the fiscal theory”)

Q: What about the “Stability of Money Demand” For example, From Lucas “Money
demand in the united states: a quantitative review)
or Luas’ Nobel lecture

Figure 4: 1900-85

* - denotes observations from 1900 to 1957.
0 - denotes observations from 1958 to 1985.

Figure 1

A: Add to the theory

\[ MV = PY \]
Now we have two equilibrium conditions. Both hold. Causality can run $M \rightarrow MV = PY \rightarrow P \rightarrow FT \rightarrow surplus$. Or it can run $surplus \rightarrow FT \rightarrow P \rightarrow MV = PY \rightarrow M$. Money demand stability does not mean that money causes inflation. Rich guys smoke cigars; smoking cigars will not necessarily make you rich. There is a deeper theorem which we will study: no test based on times series of M, B, P, Y, etc. can tell you which regime works.

(Extra if there is time. The theories are really not all that distinct. There are fiscal underpinnings of even monetarism. Why does a peg or currency board work. If you have a peg or board (100% reserve peg), a strapped government will eventually raid the reserves and abandon the peg. OTOH, a government in good standing with no reserves can always borrow them and keep a peg. Thus a peg is ultimately a fiscal question, not a question of will. Even a gold standard is just a peg with gold, and thus requires fiscal backing.)

2.5 Interest rate rules.

I admit the fiscal theory is not that popular. Most monetary economic discussion today is conducted in the context of “New Keynesian” models and the study of interest rate rules. This is thus the “standard theory” (Woodford’s book), all academic policy-oriented advice. “Optimal policy” is cast in terms of Taylor-rule coefficients.

We have to face the fact: we live in an economy in which the Fed is pegging interest rates. We do not observe galloping inflation, so we need a theory of the price level that accommodates an interest rate rule.

The classic criticism: an interest rate peg does not determine the price level. (Friedman 68 stresses the experience of the Fed-Treasury accord.)

New theoretical “solution”: Rather than a fixed peg, what if the Fed raises interest rates more than 1-1 with inflation. If inflation starts up, the Fed will then end up raising real rates which should drive inflation back down again!

Since we have fiat money and interest rate targets, this is the only viable theory other than FT that can be applied to the current institutional arrangement. (This one is also quite immune to financial innovation, the other problem with the quantity theory.)

“Taylor rule” claims:

a) If the fed raises r more than 1-1 with inflation, the price level will be determinate (unlike classic interest rate pegs)

b) 70s (most recent and vital history for US inflation): The Fed did not raise interest rates enough (estimates less than 1-1). In the 80s the Fed “learned” estimates give more than 1-1, this stopped inflation.
Important note: read Friedman, and see how vital it is for him to have a stylized story for recent events (great depression and deflation, post WWII inflation with Fed-Treasury accord, and, amazingly enough, the 70s’ stagflation, which he called ahead of time.) A great part of Taylor rule success is this story for our most important recent episode.

Issues:

a) Empirical. Can we measure Taylor rule coefficients? (JC will argue no) Is there a 70-80 difference (Orphanides paper will argue no, if you account for output right.) Is this really true of US? Or has the Fed just not faced a serious challenge like 73? With inflation 10% unemployment 8% will they really go to 20% interest rates?

b) Theoretical. Is claim a) true? That will of course depend on the model of the economy you have in mind.

i) It’s easy in old-keynesian models — but those have no economics, so it’s hard to call this a “theoretical” understanding of the price level.

ii) Frictionless models rely on the threat of explosive inflation, since the “raise real rates, open gaps, reduce inflation through the Phillips curve” channel does not exist. If inflation rises to 10%, and the Fed must then raise interest rates to 20%, the only way to do this is to raise inflation to 20%-real rate. But then the Fed will have to raise inflation 40%,...Seriously, this is how the models work. The Taylor rule is $\pi_{t+1} = \phi \pi_t$. If $\phi > 1$, the only “bounded solution” is $\pi = 0$. Thus, the Taylor rule is said to determine the price level. The words around this are that the threat of explosive inflation will “align expectations” in the right way. We have to study this and see if we believe it.

iii) It’s a subtle issue in New-Keynesian models. In most such models, the timing conventions (do the output gap depend on $\pi_t$ vs. $\pi_{t-1}$ or $\pi_t$ vs. $E_t \pi_{t+1}$) mean they work like the frictionless model above.

New Keynesian models

All this leads us to studying New Keynesian models The central question is the Phillips curve, and central there is what is structural on the y axis. (Causality is also an issue – do gaps cause inflation or does inflation cause gaps)

a) Historical correlation (left axis variable inflation),

b) Unexpected inflation, adaptive expectations (Friedman)

c) Unexpected inflation, rational expectations (Lucas)

d) New Keynesian models. Rational expectations plus sticky prices. results in forward looking Phillips curve. Micro foundations are extremely spelled out, and satisfy the Lucas-Sargent challenge to old Keynesian macro in spades. From optimization, price = expected future price. Thus inflation less expected future inflation

e) Mankiw thinks we should go back to mechanical expectations and Friedman
to better fit data.

Fact: NK timing means like frictionless models, the theory relies on off-equilibrium threats.

As you see, I am still deeply sceptical that the New-Keynesian models plus Taylor rule can overturn the old proposition that interest rate pegging is not enough to determine the price level. If I’m right, the fiscal theory is the only theoretically coherent way of understanding the price level in a fiat money economy that is following an interest rate target. But this is just a conjecture, and is stuff I’m working on now.

2.6 Commitment

Central banks do a lot for “reputation,” “commitment,” “transparency” etc. Much of the “inflation targeting” and “rules-vs. discretion” is to get more “commitment” Why? How is this important?

a) Old (Barro-gordon) – Inflation is like capital levy (precisely, in fiscal theory!). We need commitment to keep them from inflating away bonds after they are sold.

b) More deeply now. Taylor rules rely on off-equilibrium threat explosions, so you have to make those never-observed off-equilibrium threats credible.
3 VAR preamble

The big point is the empirical question: What are the “effects of monetary policy?” Let’s stop theorizing and look at the data. It’s remarkably hard to do! We want to know the sign, size and duration of effects.

The point of CEE is to produce things like first graph of impulse responses. (Show in class) These summarize historical experience of postwar policymaking. “On average, 12 months after we tightened, what happened?”

3.1 Background

Ways to quantify experience following monetary policy moves.

1) Run regressions \( y_t = \sum a_j m_{t-j} + \varepsilon_t \) (From Friedman and Schwartz; St. Louis Fed Equation). Use \( \{a_j\} \) to measure the effects of monetary policy

2) Tobin, Solow, etc. critiques:
   a) Does casualty run from m to y or from y to m? Maybe people are demanding more m in anticipation of y?
   b) Maybe this just reflects policy reactions? (CEE make this point, p. 4, pp.2). If \( z_t \) (exchange rates, say) induces the Fed to change \( m_{t+1} \) and \( y_{t+2} \) you measure the effect of exchange rates as an “effect of monetary policy.” Example:

   \[
   \text{weather}_{t+1} = a + b \times \text{weather forecast}_t + \varepsilon_{t+1}
   \]

   but shooting the weather man will not produce a sunny day.

   c) In a controlled system, you might see nothing. Think of the steering wheel analogy: as you drive down the road, the car stays in the middle, the steering wheel moves all over to counter the wind. You see nothing in the regression, though the wheel really does control the car. (This is the ‘right hand variable correlated with error term since right hand variable is endogenous” problem. The former two are the “causal interpretation of regressions” problem.)

3) Sims/Granger: split \( m = \text{expected} + \text{unexpected} \), a “Rule” plus “shocks”: \( m_t = f(x_t) + \varepsilon_t \). \( \varepsilon_t \) is not a response to current or expected future \( y \). Then average \( y \) following \( \varepsilon \) can measure ”effect” of monetary policy. \( y = \sum a_j \varepsilon_{t+j} \) is not affected by the above problem. Thus, proceed in two steps 1) \( m = \sum b_j x_{t-j} + \varepsilon_t \), 2) \( y_t = \sum a_j \varepsilon_{t-j} \). This is equivalent to a VAR.

Does this really work? When? What are the hidden assumptions? All good questions, coming later.
3.2 VAR procedure

You have a vector of variables \( z_t \). Run

\[
z_t = \mu + A_1 z_{t-1} + A_2 z_{t-2} + \ldots + \varepsilon_t
\]
equation by equation OLS. (The \( \varepsilon \) are forecast errors, so orthogonal to right hand variables.) Then simulate the system’s response to shocks \( \varepsilon \).

For example,

\[
\begin{bmatrix} m_t \\ y_t \end{bmatrix} = \mu + \begin{bmatrix} a_{1,mm} & a_{1,my} \\ a_{1,ym} & a_{1,yy} \end{bmatrix} \begin{bmatrix} m_{t-1} \\ y_{t-1} \end{bmatrix} + \varepsilon_{mt} + \varepsilon_{yt}
\]

\[
m_t = \mu_m + a_{1,mm} m_{t-1} + a_{1,my} y_{t-1} + a_{2,mm} m_{t-2} + \ldots + \varepsilon_{mt}
\]

\[
y_t = \mu_y + a_{1,ym} m_{t-1} + a_{1,yy} y_{t-1} + a_{2,ym} m_{t-2} + \ldots + \varepsilon_{yt}
\]

The top equation is the "Fed rule." It decomposes \( m \) into an expected part, i.e. a part that is a response to the economy, and unexpected part; a shock, a part that is not a response to the economy. Then simulate forwards responses to shocks. Done.

**Orthogonalization**

There is a little issue. What if \( \varepsilon_m \) and \( \varepsilon_y \) are correlated? It doesn’t make sense to move one without the other. One solution is to incorporate current \( y \) in the \( m \) equation,

\[
\begin{bmatrix} m_t \\ y_t \end{bmatrix} = \mu + \begin{bmatrix} 0 & a_{0,my} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_t \\ y_t \end{bmatrix} + \begin{bmatrix} a_{1,mm} & a_{1,my} \\ a_{1,ym} & a_{1,yy} \end{bmatrix} \begin{bmatrix} m_{t-1} \\ y_{t-1} \end{bmatrix} + \varepsilon_{mt} + \varepsilon_{yt}
\]

\[
m_t = \mu_m + a_{0,my} y_t + a_{1,mm} m_{t-1} + a_{1,my} y_{t-1} + a_{2,mm} m_{t-2} + \ldots + \varepsilon_{mt}
\]

\[
y_t = \mu_y + a_{1,ym} m_{t-1} + a_{1,yy} y_{t-1} + a_{2,ym} m_{t-2} + \ldots + \varepsilon_{yt}
\]

Now \( \varepsilon_{mt} \) and \( \varepsilon_{yt} \) are uncorrelated, so it does make sense to move one without the other.

(Why are they uncorrelated? \( \varepsilon_{mt} \) is orthogonal to \( \{m_{t-1}, m_{t-2}, \ldots y_t, y_{t-1}, \ldots\} \). \( \varepsilon_{yt} = y_t - \mu_y - a_{1,ym} m_{t-1} - a_{1,yy} y_{t-1} \ldots \) is in \( \{m_{t-1}, m_{t-2}, \ldots y_t, y_{t-1}, \ldots\} \).)

Note this does affect the IR. Now a \( \varepsilon_y \) shock will affect \( m \) contemporaneously, but a \( m \) shock does not affect \( y \) contemporaneously. Before, since the shocks were correlated and you had to move both at once, both move in response to a shock.

Problem: there are two ways to do it. You’re making an assumption here. The assumption does not affect the model’s ability to describe the data, so data can’t help. The assumption is only about the interpretation of the resulting impulse response functions. The question is, does the contemporaneous \( m \) \( y \) correlation mean \( m \) affects \( y \) in the month – and conversely that the Fed does not look at \( y \) when setting \( m \) – or does it mean \( y \) affects \( m \) in the quarter –and conversely that the economy doesn’t react at all to policy
within the month. You may say neither assumption is plausible. That’s fine, and we’ll think about other ways to orthogonalize the VAR later.

This is the “identification problem” Note that it goes away if the shocks are not correlated.

A more general version: Write the regression errors (with no contemporaneous right hand variables) $\varepsilon$ and the underlying orthogonal shocks $v$ (including the “monetary policy shock” we’re after),

\[
\begin{align*}
  z_t &= \mu + A_1 z_{t-1} + .. + \varepsilon_t \\
  z_t &= \mu + A_1 z_{t-1} + .. + Cv_t
\end{align*}
\]

I.e. $\varepsilon_t = Cv_t$. The off-diagonal elements of $C$ will capture the contemporaneous correlation

\[
\begin{bmatrix}
  m_t \\
  y_t
\end{bmatrix} = \mu + \begin{bmatrix}
  a_{1,mm} & a_{1,my} \\
  a_{1,ym} & a_{1,yy}
\end{bmatrix} \begin{bmatrix}
  m_{t-1} \\
  y_{t-1}
\end{bmatrix} + .. + \begin{bmatrix}
  C_{mm} & C_{my} \\
  C_{ym} & C_{yy}
\end{bmatrix} \begin{bmatrix}
  v_{mt} \\
  v_{yt}
\end{bmatrix}
\]

and specify $E(vv') = I$.

Now, how do we Specify $C$? We need

\[
CIC' = E(\varepsilon\varepsilon') = \Omega
\]

That gives 3 conditions. We need one more condition – equivalent to which variable appears contemporaneously above. One way to do it is a zero restriction – that’s what we did above. (The Choleski decomposition – matlab command choleski – can find the C matrix for you. Thus a common way to program VARs is to run the regression using only lagged variables and then use the choleski decomposition to spread the shocks across variables.)

Other orthogonalization schemes are possible, and are far more attractive ways of “imposing theory” than the recursive scheme:

-Long run restrictions. Choose $C$ so that the limiting response of $y$ to an $m$ shock is zero. (Blanchard and Quah)

-Sign or shape restriction. Choose $C$ so that output always declines following a shock. “Use theory” (Uhlig)

Again, I think much too much is made of this. Many people dismiss VAR evidence as a result. For example,

“... with a specific parameterization of preferences the theory would place many restrictions on the behavior of endogenous variables. But these predictions do not take the form of locating blocks of zeros in a VAR description of these variables.”

(Quoted, and Bob and Nancy might say quoted a bit out of context and too harshly, in Fernandez-Villaverde, Rubio-Ramirez and Sargent “A,B,C, (and D)’s for Understanding VARs.”) As we will see, it is often the case that not much hinges on identification since the shocks are pretty orthogonal. (A LOT hinges on which variables you include). All VARs do is capture the autocovariance function; if you like second moments there is no reason not to like these!

**Impulse response, moving average representation and variance decomposition.**

The impulse-response function is the same as the moving average representation. You can view the simulation as just an easy way to calculate the moving average representation. (As a simple example, suppose you start with a scalar AR(1),

\[ z_t = \phi z_{t-1} + \varepsilon_t. \]

If you simulate the response to the shock you get \(1, \phi, \phi^2, \phi^3\ldots\) But these are exactly the coefficients of the moving average representation

\[ z_t = \sum_{j=0}^{\infty} \phi^j \varepsilon_{t-j}. \]

That’s a lot easier than factoring and inverting lag polynomials!

Thus, we can write the IR or moving average representation

\[ z_t = \mu + \sum_{j=0}^{\infty} B_j v_{t-j} \]

Think of one line of this representation

\[ y_t = \mu_y + b_{0ym} v_{mt} + b_{0yy} v_{yt} + b_{1ym} v_{mt-1} + b_{1yy} v_{yt-1} + \ldots \]

All the shocks are orthogonal and unit variance, so we can write

\[ \text{var}(y_t) = b_{0ym}^2 + b_{0yy}^2 + b_{1ym}^2 + b_{1yy}^2 + \ldots \]

Finally, we can group the terms and express the variance of \( y \) as the part “due to \( m \) shocks” and the part “due to \( y \) shocks.”

\[ \text{var}(y_t) = \{ b_{0ym}^2 + b_{1ym}^2 + \ldots \} + \{ b_{0yy}^2 + b_{1yy}^2 + \ldots \} = \text{variance due to } m \text{ shocks} + \text{variance due to } y \text{ shocks} \]

(It’s common to divide by \( \text{var}(y_t) \) to express the percent of variance due to the various shocks.) This is the variance decomposition. It is one way of quantitatively addressing Friedman’s question: how much volatility of output is due to monetary policy shocks? If we turned the shocks off and made monetary policy totally predictable, how much would output variance go down?
The variance here is an infinite sum which may explode (especially if \( y \) is a level). For this reason we often calculate variance decompositions at different horizons. How much of the \( k \)-step ahead forecast error variance of output is due to monetary policy shocks? To calculate this quantity start again with the moving average representation

\[
y_{t+k} = \mu_y + b_{0y} v_{mt+k} + b_{0yy} v_{yt+k} + b_{1y} v_{mt+k-1} + b_{1yy} v_{yt+k-1} + \ldots + b_{kym} v_{mt} + b_{kyy} v_{yt} + b_{kym} v_{mt-1} + b_{kyy} v_{yt-1} + \ldots
\]

Now, the shocks at \( t \) and before are known at time \( t \), so

\[
E_t y_{t+k} = \mu_y b_{kym} v_{mt} + b_{kyy} v_{yt} + b_{kym} v_{mt-1} + b_{kyy} v_{yt-1} + \ldots
\]

and

\[
y_{t+k} - E_t y_{t+k} = b_{0y} v_{mt+k} + b_{0yy} v_{yt+k} + b_{1y} v_{mt+k-1} + b_{1yy} v_{yt+k-1} + \ldots + b_{k-1m} v_{mt+1} + b_{k-1y} v_{yt+1}
\]

\[
\text{var}(y_{t+k} - E_t y_{t+k}) = [b_{0y}^2 + b_{1y}^2 + \ldots + b_{k-1m}^2] + [b_{0yy}^2 + b_{1yy}^2 + \ldots + b_{k-1y}^2]
\]

Again, I’ve grouped the coefficients into a part due to \( m \) and a part due to \( y \). You could graph this as a function of \( k \), and you can see how you can eyeball the results once you understand the impulse-response function.

The AR(1) representation

The AR(1) representation is really useful for programming. Write

\[
z_t = A_1 z_{t-1} + A_2 z_{t-2} + A_3 z_{t-3} + \varepsilon_t
\]

as

\[
\begin{bmatrix}
z_t \\
z_{t-1} \\
z_{t-2}
\end{bmatrix} =
\begin{bmatrix}
A_1 & A_2 & A_3 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
z_{t-1} \\
z_{t-2} \\
z_{t-3}
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_t \\
0 \\
0
\end{bmatrix}
\]

Now, the forward simulation just consists of

for i=2:horizon;
    y = A*y;
    response(i,:) = y(1:nvars)';
end;